**Engineering Physics 1**
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**Module-08**
**Lecture-01**
**Introduction**

Hello, everybody so, today we will be starting our first lecture on geometrical optics okay. So, what is the first thing that comes to your mind when we talk of?

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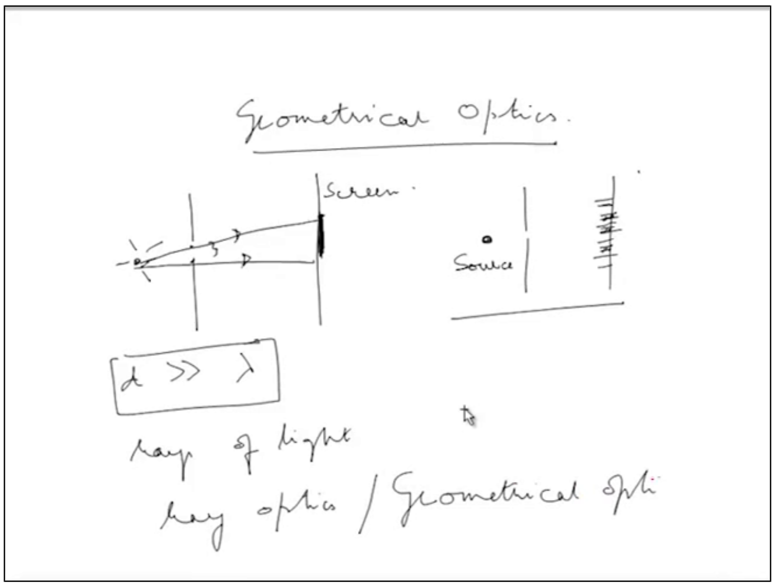


Well geometrical optics I suppose the first thing that comes to our mind is you know the passage of light through lenses. How curve lenses that convex lens or a concave lens or maybe reflection from curved mirrors. So, but one has to realize that this is rather pretty old subject in physics. We have been we have been doing things for 100's of years. In fact it is the use of these lenses and mirrors which had a huge role in the development of astronomy.

For example Galileo was the one who used just two lenses. He just used two lenses and then put it in a cylindrical tube and he watched the moons of Jupiter and he made the certain astronomical observations. So, in a sense well this subject is it is it is it is one of the primary things which was which was responsible for advancements in astronomy and not only in astronomy not only in physics for that matter.

I mean if you think of the simple magnifying glass okay to look at small things and I bet the modern biology of owes its origin to the observation of small things right. So, the study of geometrical optics had rather far reaching consequences not only in the development of optics but also in other branches of physics and also in other branches of science right, so the, now that we know that it is an important thing to study although it is an old subject.

What were rather what is the of applicability okay so rather at what wavelength range should we apply geometrical optics. Now why do I say that because of the back of your mind you also have things like diffraction, interference which have been explained so well with electromagnetic waves okay? So, what is the area of applicability here? For that let us draw a a simple diagram and define our area of applicability.

Let us say we have a small slit or a small hole okay and we are going to put some source in front of that okay some source of light in front of that right. Now we put a screen a little distance away from this hole or from the slit. Now what we expect to observe well I mean we have a reasonably big hole here. So, we can draw these two lines shall come to this point that we are drawing lines here.

And then we observe the image of this of this slit of this hole on this on the screen so on this screen. Now which is kept at a certain distance from this slit okay? Now if you observe you have sharp boundaries for this image okay and you just observe only one image of this hole. Then I am sure you are going to say that we are in this regime of geometrical optics. Now why I mean look at the other look at the other extreme I mean if you if your hole is small enough you will have.

And let us say let me draw another picture if your hole is small enough you and you can have and then you put a source of light here. I call this some source. You are going to have you are going to have alternate regions of light and dark regions okay. Now that is diffraction, so you have alternate regions of light and dark regions here. So, but the field of geometrical optics would not concern such a thing but not with not concerned with not be too concerned with this diffraction okay.

But would be talking of things in which when you have a slit okay the image of the slit is very well defined on the on the on the screen. Now when does that happen, that happens if the size of this slit that you have taken okay. Let us say the size of the slit is d now that is much, much greater than the wavelength of light that you have used okay. So, this is a one important one important criterion for the application of geometrical optics okay.

And well if it is actually if it is not, if the slit dimension is some way it is very near or comparable to the dimensions of the wavelength okay. You will as I said you will have to deal with things like diffraction okay. So, this is one thing about the sizes. The second thing is a bit involved but then it is not a conceptual it is a conceptual thing. It is that the light or the energy of the of the photon in this case okay should be lesser than the energy sensitivity of the instruments that you are using.

So, it should not interact with the with the instruments okay. So, basically you are not in the quantum region quantum regime here. So, we are still in this classical regime and be using you know what we call as these lines or these rays of light let me draw it once again. So, these are the rays of light by which you are going to get some sharp images on the screen okay. Now what are these rays of light these rays of lights are nothing but infinite infinitesimally thin.

So, these will be using the rays of light okay, so and they are the infinitesimally thin beams of light okay. Now for that reason alone for that reason geometrical optics is also called ray optics okay. So, that is what we are going to study. We are going to study ray optics or geometrical optics here okay. Now let us do one more thing, now that we know that we have we are going to deal with rays ok. So, the next question to ask is what part do these ray's decay ok.

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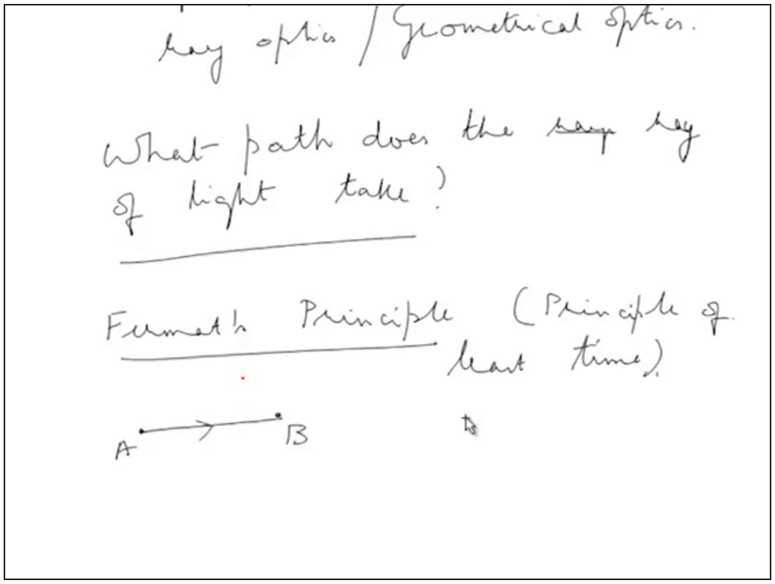


So, the question that we ask ourselves is what path does the does the let us say the ray of light take okay. Now this was something which bothered many people and quite early and apparently people had lots of ideas well they say that, it will follow the least distance between two points. Well if it is the same medium that is fine I mean the least distance is fine. But that may not be the case in which you have you have in media of different refractive indices in between okay; I will come to that again.

Now apparently there is a principle which tells you what part the ray of light will take. Well it is called Fermat’s principle or in its original form it is also called the principle of least time okay. So, we will be talking of the Fermat's principle or the principle of least time. Well it simply states that well the ray of light will follow that path from one point to another. In which you know I have point A and point B.

Well I you can take whatever refractive index in between well it is for it will follow from point A to point B the path in which the time taken will be the least will be the minimum one ok. So, that was Fermat’s principle in its original form. So, that is what Fermat said that light is going to travel from A and B ok and the path it is going to take and it is going to take is that for which the time is going to be minimum.

So, instead of this part or the other path the one from A to B will be the one, the one the light will take and the ray of light will take will be the one in which the time taken is a least ok. Now apparently the whole of geometrical optics can be can be derived in principle from this one simple concept ok, so that is how important it is ok, are there the well in this is the statement in its original form it is the principle of least time.

But I will come a little later on some modifications but let me just you know tell you two or three consequences of Fermat's principle ok.

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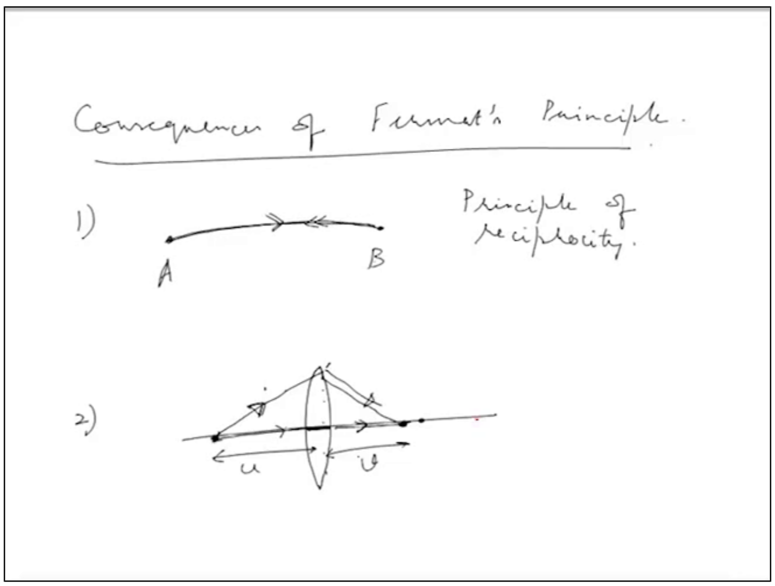


So, it is just do one of two consequences of Fermat's principle. Well the first is that if from point A to point B sets a light is traveling and which has taken the path of least time, so point A to point B. Suppose this is the path and this is the path of least time from point A to point B. Well if I start from point B to Point A the light will actually take the same path back okay.

So, if it was the least time on one way that will be on the other way also it will be the least time compared to other parts okay. Now this is called the principle of reciprocity okay. So, it retraces its path back, so this is the principle of reciprocity okay. Second thing is about well, I mean what comes to my mind is I mean I started this talking on you with doc right by talking of some lenses okay. So, let us call lens, let us draw a simple convex lens okay.

And on the axis, so light let us say I have some point on the axis on the left hand side here okay. So, this is my convex lens and then let us say the light from this point actually comes and then he comes here and then well I have well it comes on the other side okay. So, this is the object distance and this is the, let us say the image distance which I know denote v okay. Now there are well there are infinite parts of light.

Let us just stick two or three of them you know there is one path which goes entirely through air okay so the one up at this point and then comes and comes back here. There is one which goes partly through air which is actually along the axis okay and then there is one part which goes through the lens and then comes through and then comes and travels in air to reach the image point okay.

Now if you look at these two points the length of these two points are different the path lengths are different okay. The actual length from on the axis that is the least compared to compare to the length when it goes to the tip of the lengths and then comes to the image point okay. Now however by the principal release time you can you can argue that the time taken by these two light rays okay.

But this one the one in which I have drawn the one goes to the tip and then comes to the image point on the one in which it passes through the, you know which is along the axis of the length okay. The both of them take the same time okay otherwise you are not going to have an image on the other side okay. So, of course I can also derive the simple laws of reflection and refraction using Fermat's principle okay.

But before that I wish to remind you once again about well maybe I have not have not said this before is about the statement of Fermat's principle. Well now we do not see that the time taken is minimum what we see is that the light will follow that path.

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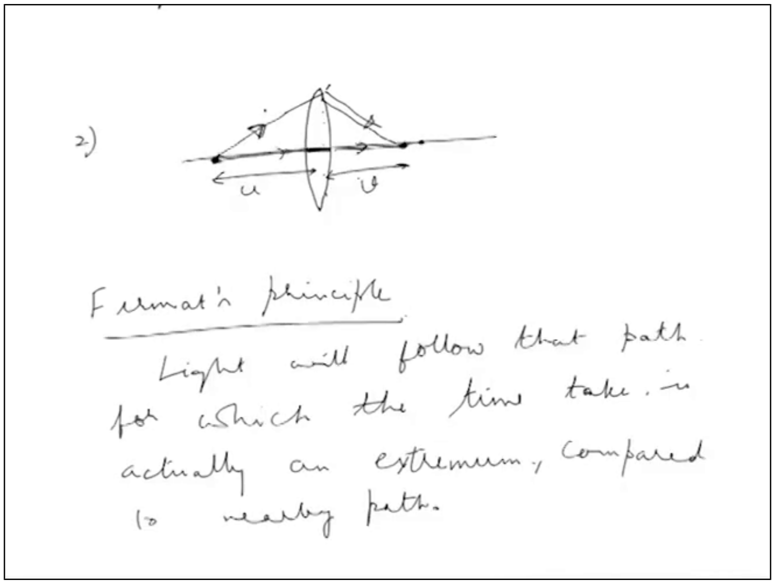


So, from A and B so formats principle the way we say it now formats principle is that the light will follow that path for which the time taken is actually an extremum compared to the nearby paths okay. Extremum compared to what, compared to two nearby paths when I say an extremum I do not actually specify whether the time is actually minimum or maximum it is an extremum okay.

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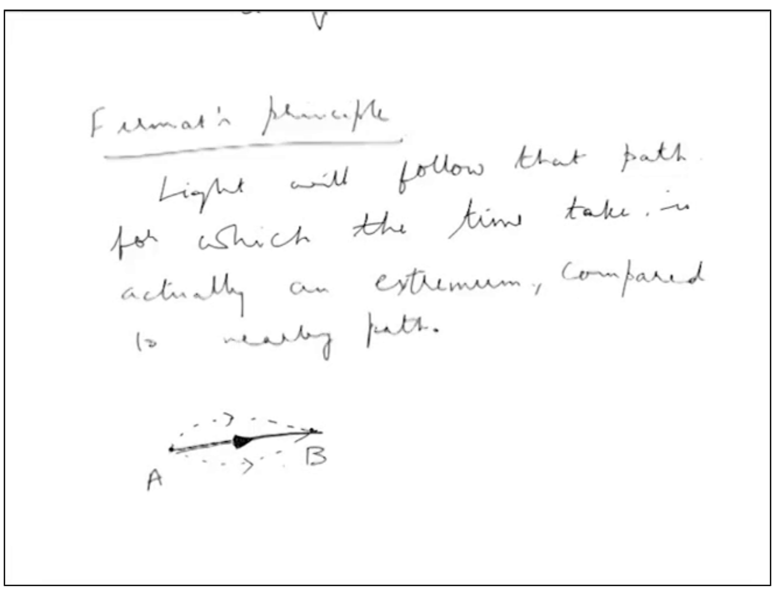


So, pictorially and diagrammatically how do I write that I mean I have 0 point A and point B okay let us say point A and point B now there are actually infinite number of paths which connect point A and point B ok. Now what it tells what for much principle tells is that light will follow that path for which the time taken is actually an extremum compared to these parts compared to for example it will be it will follow the path in which I have shown by a solid line ok.

As opposed to and that will be an extremum path compared to let us say the ones which I show by the dashed lines ok. So, that is that is how we know formats principle now. Now this is actually a far reaching consequence of the principle of least action or again it is the Hamilton's principle mechanics ok. Now let us as I said we can derive the laws of reflection and refraction from Fermat's principle let us see how we can do that ok.

And so that will give us some more confidence about the applicability of this of this theorem of this principle rather ok. So, we want to check what happens during reflection.

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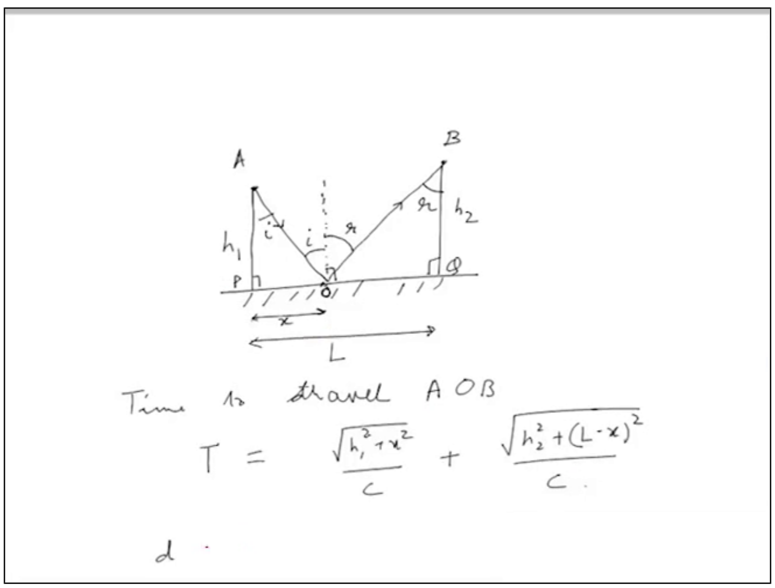


Draw a figure first so here we have a mirror a plane mirror let us say and say we have a point a and A point B. So, let us say we have a point A and point B, now the thing that we want to do here is that we want a ray of light to come and hit the mirror reflection and go to B. Now which direction should it go? Well common sense and what we have learnt in school is that well it should go in a such a way that the angle of reflection should be equal to the angle of incidence, now there should be equal to the angle of reflection ok.

Now why is that because you see from going from path, going from point A to B via the mirror I can actually have an infinite number of paths ok. Now why should light choose the one in which the angle of incidence is equal to the angle of reflection. Let us find out using formats principle ok. So, let us draw let us draw a normal at the point of incidence ok and consider that this point A is at a height h1 from the mirror.

Point B is at an height h2 and the and the distance on the mirror you know the distance of the mirror is L let us say and let us also think that the distance at which the light strikes the mirror that that is x okay. So, let me see you and have I drawn all the all the distances, yes I have taken the distance in the mirror the distance where it hits the light hits the mirror is incident on the mirror okay.

So, I have not drawn the angle of incidence let me call this i ok and the other one here is the angle of reflection let me call this r, so and this is the normal ok. So, the one this dotted line is normal I bet you are going to say that what will be this angle this angle is also r and this angle is i ok. These are all right angle triangles ok. So, let me okay, so, let me; I have in term labeled these points height of this is P.

Let me call the point at which the light is incident has O and let me also call this height so B this is Q okay. So, PQ is L be 0 is x some distance okay and the height from the mirror is h1 here and the height from the mirror I to the mirror of B is h2 okay. Now the thing and now the question that I am trying to address here is I mean light comes and hits the mirror at a distance x here. Well it could have it could have hit a little bit when if it hits at x.

What happens to the angles this is one thing or two to put it in another way down I mean why should it hit only at exit can hit at the other places and come to B okay. Now which is the one according to Fermat's principle should be the path of the light here ok. So, let us let us do this what is the time it takes for light to go from, so time to travel let us say to travel the distance AOB this distance AOB let me say let me call that time T.

And the velocity of light in this medium okay, so let me say this medium itself is air. So, velocity of light let us say you see how the speed of light is c ok. So, what is the distance AO well the distance AO is nothing but h1 square + x squared PI by see that is the time it takes to travel the distance AO. And the distance OB, so that will be square root of h2 square + L - x squared divided by c okay.

Now what do I do now, now I say that the time will be compared to different paths compared to its neighboring odd or  nearby paths so this is going to be a extremum okay. So, that is what I have,

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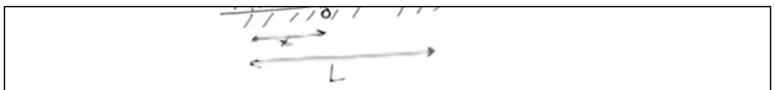


$$T = \frac{\sqrt{h_1^2+x^2}}{c} + \frac{\sqrt{h_2^2+(L-x)^2}}{c}$$

$$\frac{dT}{dx} = \frac{1}{c}\frac{2x}{2\sqrt{h_1^2+x^2}} - \frac{1}{c}\frac{2(L-x)}{2\sqrt{h_2^2+(L-x)^2}} = 0$$

$$\frac{x}{\sqrt{h_1+x^2}} = \frac{L-x}{\sqrt{h_2^2+(L-x)^2}}$$

So, I have let me let me do this dT dx, so this is equal to watch dt dx I am going to take and 1 by c here that is the speed of light as a constant thing. I am going to differentiate the one in the in the square root, so I will have a 2x that the top and then if i differentiate this entire thing. So, I am going to have 2 times root over h1 square + x1 x squared in the denominator what about the one on the for the for the path OB.

So, this is 2L – x, so, I am going to differentiate this point x h2 squared + L - x whole square and the square root of that so that is equal to twice of h2 squared + L - x whole square okay. So, I have not taken the minus sign out so because L - x ok fine, I put a minus sign here and then there is an 1 by C ok, so when I say it is an extremum I am going to put this equal to 0, so which immediately tells me that x by root over h1 + x square = L1 - x h2 squared + L - x root of that.

So, what does this tell us, so it tells us that if I want the time to be an extremum okay compared to all different paths light can travel now if this is the path light should travel ok I better have x well I should have put an h1 square + x square root over of that that should be equal to L - x h2 squared + L - x whole square square root of that. What does this mean let us look at this figure let us look at this triangle APO ok. Now what is x divided h1 square by x1 square by x square ok. So, let us look at triangle APO ok.

Let us look at the figure triangle APO. Now in triangle APO what is the sine of the angle i, well it just happens to be x by the distance h1 square by x square ok. Now compare that in triangle BOQ what is the sine of the angle r ok. Now these are what so sine of the angle r is L - x divided by h2 square + L - x whole square and take the square root of that in the denominator. So, which tells you immediately here that light will travel that path in which I am going to have;

Just the, you know in this same medium the sine of this angle i is = the sine of this angle r or in other words i need this angle i to be equal to the angle r which is actually the familiar laws of the reflection that you have ok. So, well to think of this as a mechanic's problem is also quite interesting. Actually if you give if you have set a problem like this and I want to run from point A to let us say point B by touching some wall ok which is PQ.

Now which path should I take so that the time taken will be the least here ok? well actually will be taking that path for which you know the point at which you hit the wall and then you draw in well to that then you find an angle okay that should be equal and you still call that an angle of incidence okay should be equal to the angle that you make with the normal when you run away from the wall to reach point B okay.

This again will actually follow from the principle of least action in mechanics but right now we are doing geometrical optics we are dealing with light rays of light. So, and we have just now verified again the laws of reflection using Fermat’s principle okay. Now with this a basic verification how about doing it for the refraction okay, so, you can always say that okay fine I have done this for a medium in which the refractive index is the same right.

So, you are the reflection is in this in this region in which you know in the region AOB that is in a region where the refractive index is the same ok. So, let us find out what it is going to be when you have two different medium.

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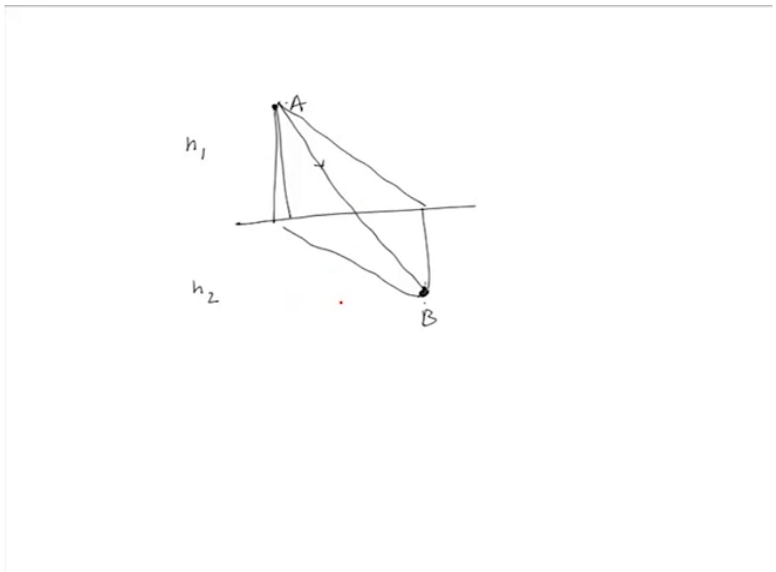


Let us draw figure once again so okay so we have two media okay which is infinite on both sides let us say. So, in one side the refractive index is n1 and on the other side the refractive index is n2 okay. Now you have you have you at point A let us say and you wish to go to point some other point ok. Now what is the point what is the direction that you need to follow okay, in case you want to go to point A to point B, is it is it this straight line.

Well I should draw the point P here is it the straight line well that is the distance that is the least distance. But then remember that the speed of light is different in this to media okay. So, the time taken is necessarily not the same if you go which is definitely not the same when you if you if you follow a straight line. Let us say you travel this much in one media and then the other here all let us say you would travel this much in medium one.

And then the one here well it is more like you all let us say n 1, so this is region 1 let us say this is land and then n2 this is some water okay you want to go from point A to point B okay. What do you do, do you jump straight you go straight into do go straight to the you go into the show where what where water is and then swim all the way to B okay. What do we go do you want to minimize your swimming because you are not swimming that that is a bit tougher than just running on land okay because of resistance?

So, do you maximize your path on land okay let us say you come here and then you swim a little bit in sea or in water will not let us not bring in sea or anything else in water. So, which part should you take rather problem is the same.

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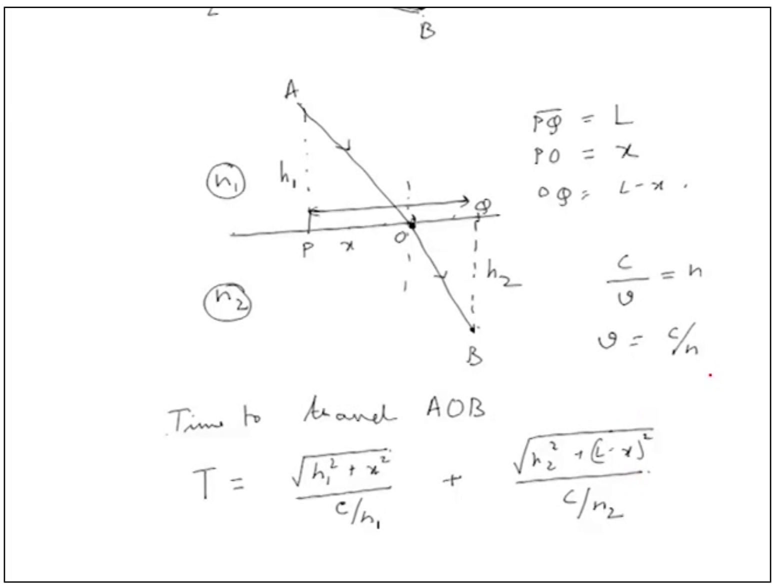


So, let us redraw this figure once again and label it. So, let us say you need this to be the path and this is how you wish to go, when you are in point A and this is the point you are incident on the interface of both the media. So, you want to go to point B here, let us say again I call this distance let me call this distance where the distance h1 this point as P, this point of contact to within these two media where light comes and hits as O.

And then B and this to be h2 and when I call this PQ, so this point is Q okay. Now I again take this distance, so this distance is fixed this distance PQ, this distance PQ this is let us say let us call that as L distance L. Let me call PO, so that is x okay so that is variable why do I want where do I want to hit the surface here do I want to hit it here, here, here or here. So, that the time it takes compared to all these paths to go from point A to point B is an extremum let us do that.

So, definitely what is point OQ, so this is L - x okay. So, what is the time it takes to travel this distance AO ok, so the time again to travel AOB this distance AOB. So, this is let us say capital T, so what is this distance by the way so this distance is AO in medium 1 okay where I have the refractive index n1. Now how is the velocity or the speed of light related with the refractive index? Well if the speed of light in vacuum air in vacuum let us say is C and then I divide that in a medium where the speed of light is V.

Then the refractive index is n right. So, what do I get so the velocity of light in a certain medium which has a refractive index n which has a refractive index n is nothing but C by n where C is the velocity of light and all the speed of light in vacuum and then n is the 1 in n is the refractive index here ok. So, the time it takes to go to do the distance AO what is that? Well let us find the distance or the distance AO, So this is x ok.

So, this is nothing but again h1 square by x square now we need to divide this by the velocity in medium 1 or the speed in medium 1 which is C by n1 because you are a medium one. Now we need to add again so this is the time it takes for light to cover traverse AO ok. Now what about this distance OB well that is h2 squared + L - x whole square divided by C by n to remember the speed of light in the in medium to why call that velocity v2.

And that is again dependent on the refractive index the speed and the C is the speed of light in free space or the speed of light in vacuum is not right.

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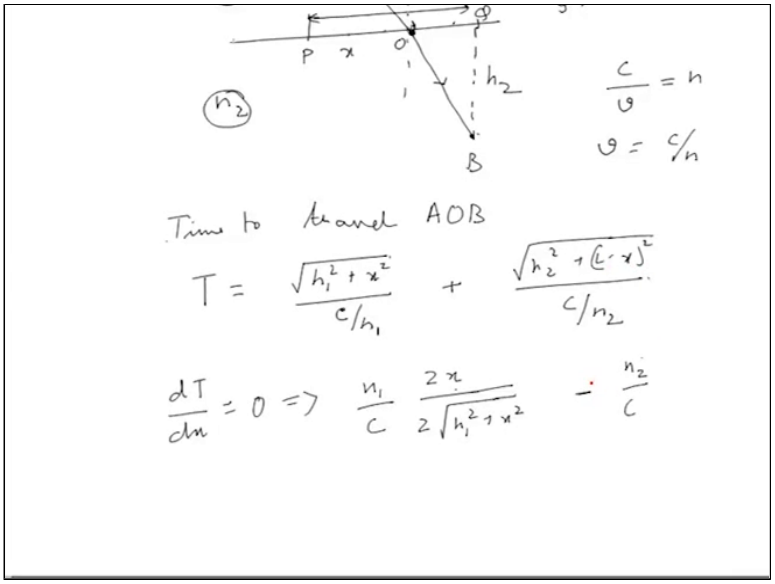


Now we need to take the next team of this so which means we simply take dT dx = 0, so, which tells us that well C and n and all these things are constants here. So, we need to differentiate this entire expression with respect to with respect to x, so what is that, so that is n1 by C, I am going to take it to x upstairs and so and then a differentiate that that will be, so that is h1 square + x square. So, differentiate 2x squares I get a 2x here and then again h1 square by x square, so, this half by this entire thing to the bottom 1 –half, so that is ok.

So, square root comes down and then I have again in 2 by C the other one, I get ok so L - L - x so I will have a minus sign here and so I get a 2 of L - x ok and again h2 squared + L - x whole squad ok. So, so what is the point? Well what is the time light will take once it goes from point A to B and via this from one medium to another. So, we will be calculating their time and the time it takes to travel from A to AOB, we have calculated that.

Now we are extra miser we are doing dT dx = 0 we are trying to find out what is the extremum of that, we found a condition ok. Now what is that condition?

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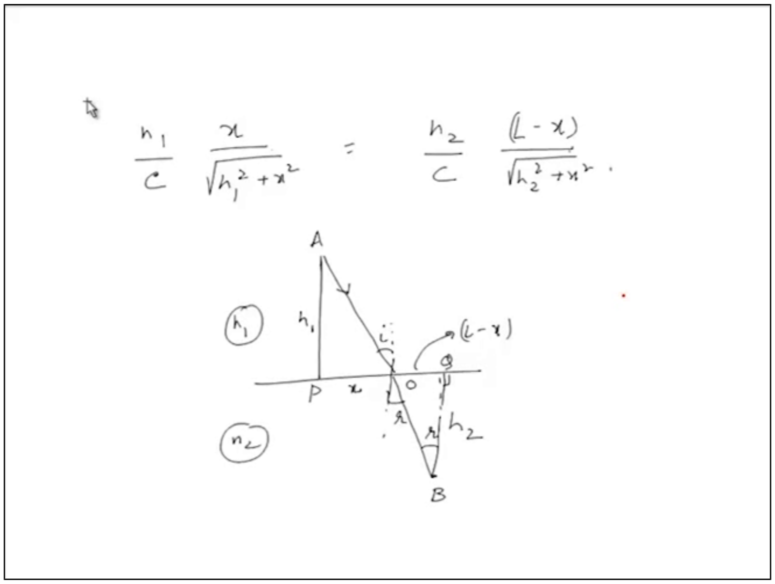


Once again you just found the condition that n1 over C remember n1 is the refractive index in medium 1. So, time x by root over h1 squared + x squared and that should be equal to n2 over C root over in to over C and then L - x root over h2 squared + x square ok. Now what was the thing that we were doing, we were looking at refraction case from medium1 to medium2 ok.

Now this angle was i, let us say that is the angle of incidence and then this was height h1 so this was x, so this was A, it said B and then the angle of refraction was r ok. So, this distance was h2 and so this was B and this was Q and OQ this was L - x definitely this angle OBQ that is also r. Now you have got our relation regarding some the refractive index times something times x by something some distance ok.

So, let us find what the angle is by the way. So, this if this angle is the angle of incidence the angle PAO ok that angle is also i right.

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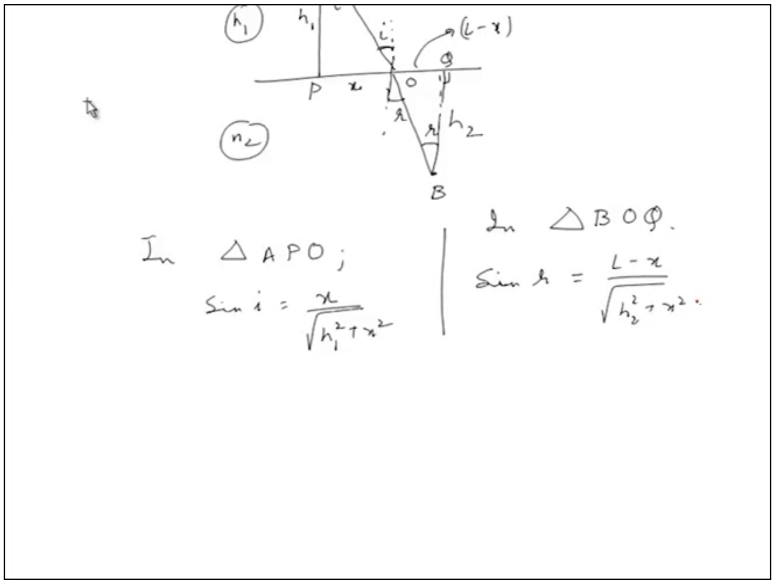


So, in triangle APO, what do I have sign i to be I have sign i to be x by the distance AO which is nothing but root over h1 squared + x squared and in triangle BOQ what is the sine of angle r. Well sine of angle r, nothing but L - x divided by root over of h2 squared + x square, therefore when we come back to our old equation which we have got by equating the time to be by making the time to be an extremum okay. Is if we substitute it back into our old equation whenever this equation here this one.

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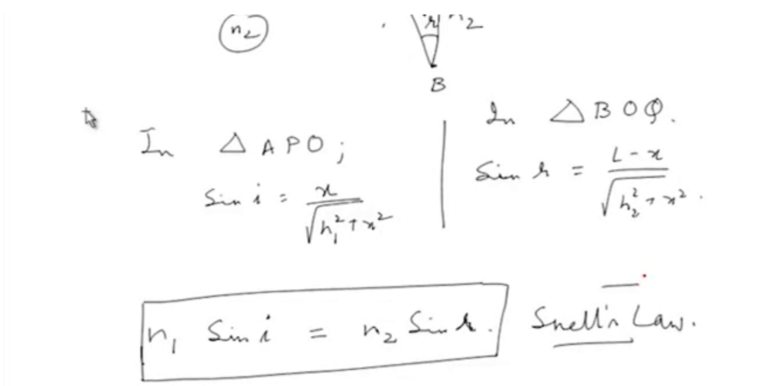


What we get is n1 sine of i = this value n2 sine of r okay, where i is the angle of incidence that is the angle at which the light makes or the ray of light mix with normal when it strikes the boundary of this to medium. And then r is the angle that is the angle in which light emerges in medium2 okay. Now this you know everybody knows as this as Snell's law in optics.

Now see that we have simply obtained the Snell's law in optics using Fermat's principle okay. And remember this mechanics problem is telling you so if you wish to go from point A to point B, let us say and then the region one was land and region two was water okay. So, what is the what is the root you need to take when you better take the route A or B okay that is also the thing you can work out using the principle of least action of the in mechanics right.

So to summarize what we have done today, so we have talked of the beginning of geometrical optics, we have talked of the fundamental guiding principle in geometrical optics which is Fermat's principle which actually tells you how the path of a ray of light okay. So, we have done that for reflection from a plane mirror. And we have also seen the path of light the ray of light will take when it goes from one medium to another okay.

And in doing so what we have also found is that we are right because we have reproduced the familiar laws of reflection and the familiar laws of refraction here find that Snell's law, so thank you very much.