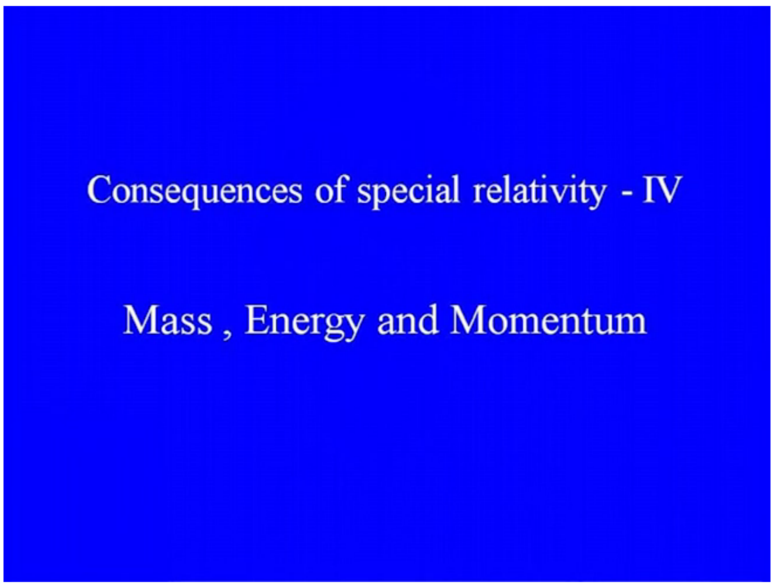
Engineering Physics 1

Dr. Rajdeep Chatterjee
Department of Physics
Indian Institute of Technology-Roorkee

Module-07
Lecture-05
Introduction of Special Relativity - IV

Okay, hello, everybody so, today we are going to talk of the some other new consequences value in the sense that the new to our course of course of special relativity.

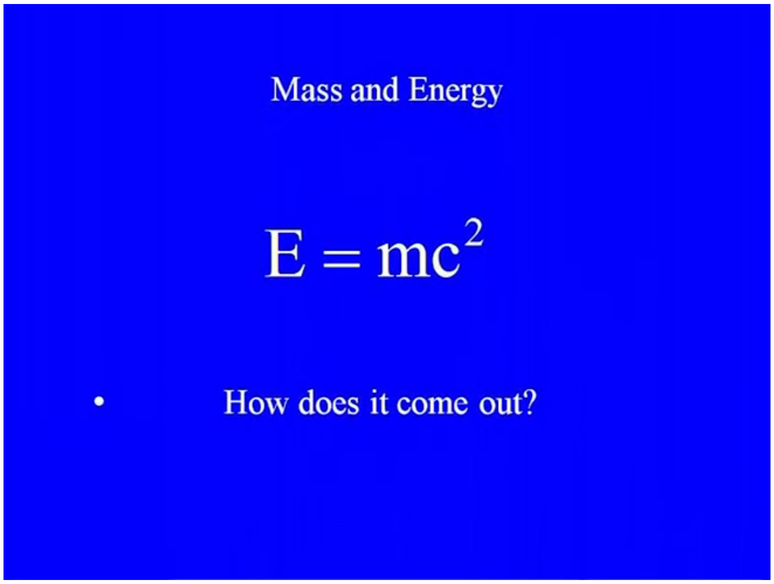
(Refer Slide Time: 00:46)



We are going to talk of mass energy and momentum in special relativity. I suppose remember how mass was varying with velocity right. So, in unlike in classical mechanics where you know if you whatever the mass of a body is, if you move, if the body moves its mass does not change. But here of course we have this concept of what a rest mass is and the fact that you are in a frame which is moving with some you know some uniform velocity with something else.

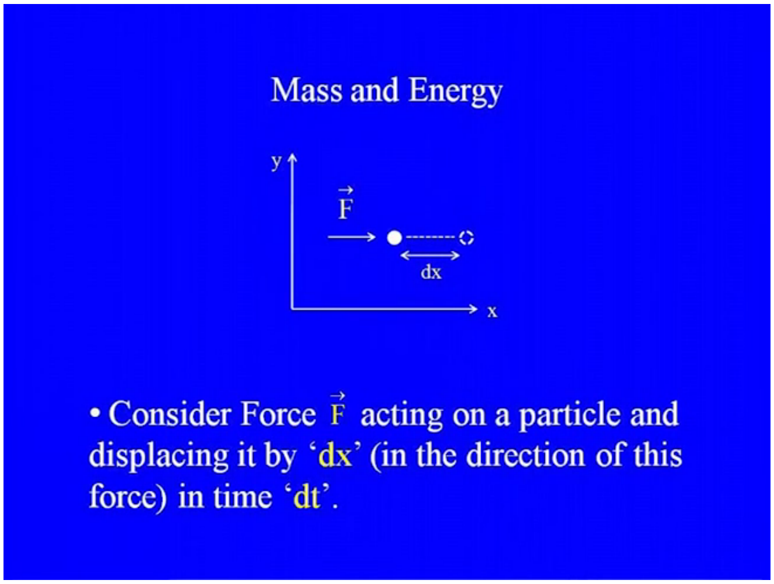
And in a sense the mass itself is moving at a certain velocity then you do find the mass of the body has changed. Well we did see some consequences of that. Today what we are going to do is relate this that relates the mass that is to the total energy and also with the momentum okay.

(Refer Slide Time: 01:56)



Now having said talked of mass and energy I mean have it is very certain that we are going to talk of E = mc squared. We are going to spend some time to see how this comes about. It is I suppose one of the most ah well known equations in physics. Anybody who has even opened a textbook in physics or about read about scientists and what physics is has heard about this equation. So, today what we are going to do is to see their simple way how to derive this from principles which are known to us okay.

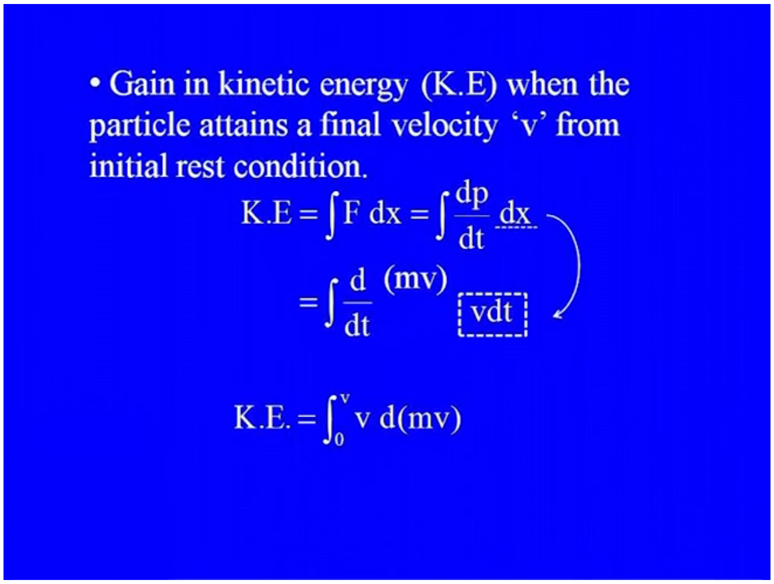
(Refer Slide Time: 02:37)



So, first you know let us consider a force which is being applied to body and when you know when this force acts. What happens is that it displaces the body by an amount by a small amount dx let us say in time dt okay. So, the small displacement dx and that in the direction of the force, why so you should take the dot product, you know if you just take you do not need to take the vector part in such F dx okay.

And then it also happens in some small time dt right. So, what happens? I mean what happens when one applies this force. Well body starts moving, I mean if the body was initially at rest and then after the application of the force which attains velocity v let us say small v then what would be the total gain in the kinetic energy of the body. Well by definition its t work done is not it I mean.

(Refer Slide Time: 03:45)



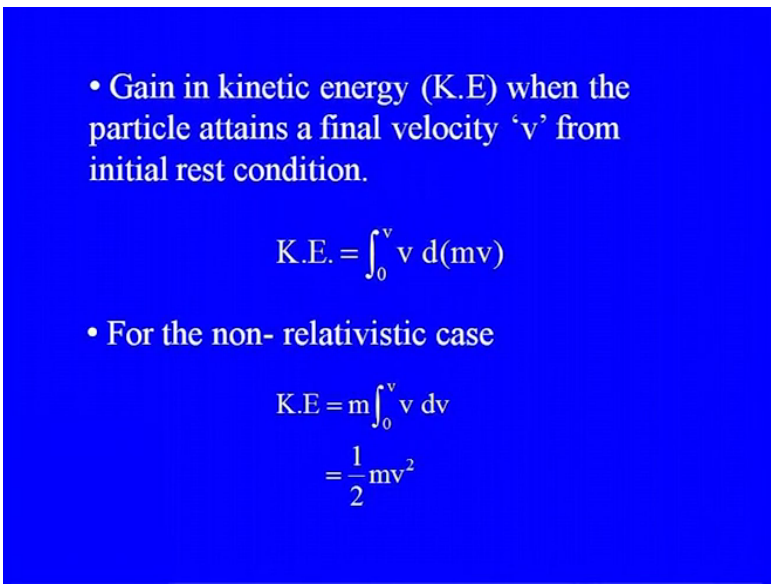
So, that is the energy that that kinetic energy that the body has gained once it reaches a final velocity v from its initial rest position. Again what is that? So that is integration of Fdx okay of course well you know in a proper sense one should of taking the vector F of dot dx but as we said the displacement is in a direction of the force, so, we talk about this here. An interesting point here is that we write this force now in terms of the rate of change of momentum.

You can just see the region next to F dx and you see that it is under the integral sign you have dp dt so that p, p being the momentum gained okay and dx is a displacement anyway. So, how do you write this momentum you know that momentum is mass times the velocity. So, we write d of dt of mv okay and then for dx you just write v dt now, why is that? So you know v be the velocity of the body that is equal to dx by dt. And then you just replay just dx as v dt right.

So, how does I mean what does the equation for the expression for the kinetic energy boils down to. Well it is as we said I mean be you are finding the gain in kinetic energy when the particle is you know from has moved from its rest position to its and has attained the final velocity v. It is the integral within the limits 0 to v, v being the final velocity gained. And in the integrand is v of d of mv.

Now check that here we have not talked of whether m is constant or not okay. So, this form of the kinetic energy is very extended. We can use it for example let us let us why do not you use it to find the non relativistic equation non relativistic expression for the kinetic energy that will clarify the situation a bit. So, for the non relativistic case what do we do?

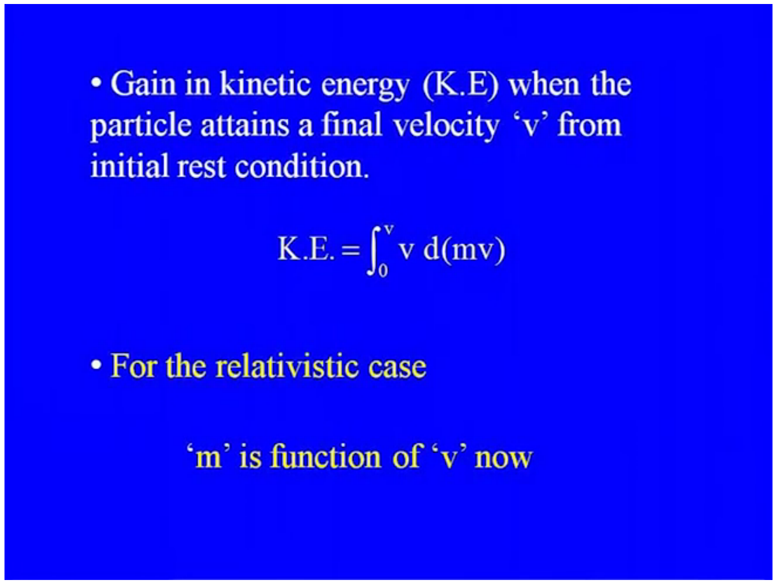
(Refer Slide Time: 06:05)



Here we know that you know mass of a body is constant I mean it is a fixed thing. So, we can take it out of the differential d of mv, take it out of the integral sign. So, that the integration of or the kinetic energy is simply v dv and then from 0 to the final velocity attained. It is a very simple integration to do, the integration the integral rather becomes v square by 2 v dv 0 to v that becomes v square by 2.

So the total kinetic energy is nothing but half mv square. Now this is a known expression to us even though we know this expression half mv square it is it is it is used almost everywhere in classical physics the classical mechanics by the by. So, of course what have we done here we have taken to mass to be a constant thing.

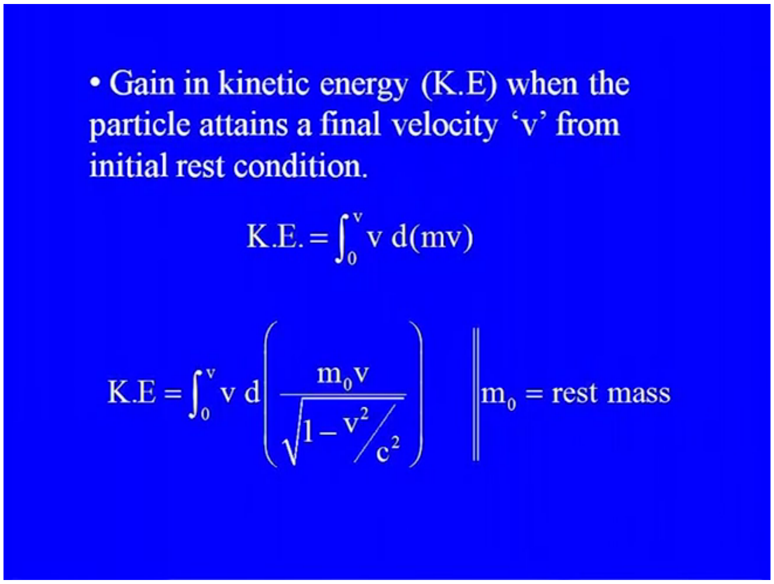
(Refer Slide Time: 07:02)



So, as expected now what happens to the relativistic case the relativistic case member that the mass is indeed a function of velocity okay. So, if the mass of course moves if the body moves with a certain velocity with a large velocity we do. You see actually you know large is compared to what and if it is of course all velocities. We are will be considering is less than the speed of light. Well then if it is approaching the speed of light we do see that the mass has been increased by a lot okay.

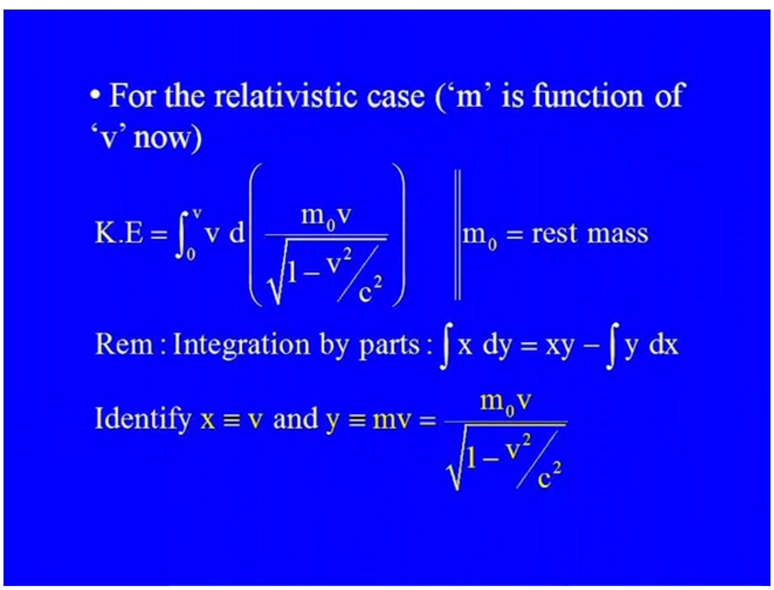
Of course I mean when we are at very low speeds it does not matter okay. So, we are still in the non relativistic limit. But then we have appreciable speeds then we do start feeling that the mass of the body has increased okay. So, what happens if we consider that so the mass of a body being a function of a velocity now, okay. Then we can no longer take the lake take the mass for the m out of the differential and then we have to use the entire thing okay. Now what would be the expression okay.

(Refer Slide Time: 08:12)



You are going to put in the exact value of the momentum or the exact value of the bass here that is m0 divided by root over of 1 - v square by c square of course the v is there okay. So, the kinetic in it and what is m0 here? M zero is the rest mass okay that is the mass of the body that is measured when the body is at rest okay. So, what is the expression for the kinetic energy then?

(Refer Slide Time: 08:40)

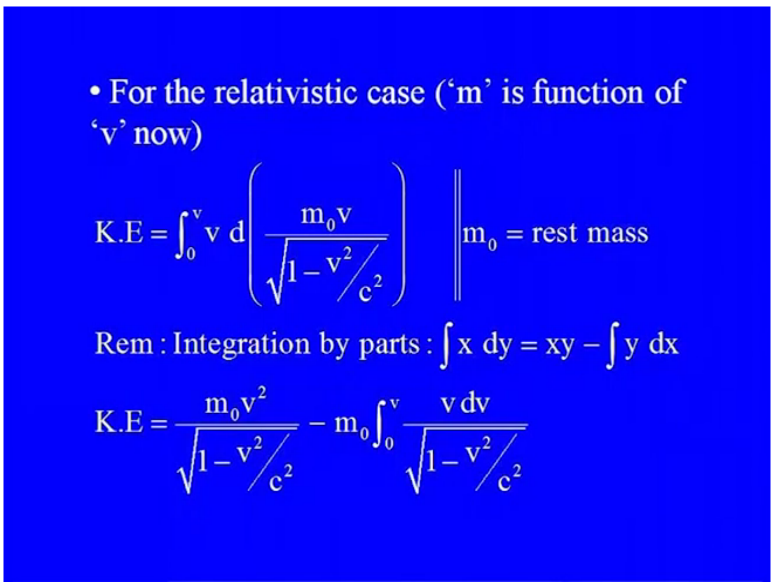


Well it is v times d of m0 v divided by root over of 1 - v square by c square okay. Now this might look like a bit complicated but we do have thing called integration by parts in mathematics. And so I think it is so proper case to utilize such a trick okay. So, what is this integration by parts? So it is that if you have an integration of x dy and then you can write this as you know and then y is some complicated function that you have.

So, it is nothing but xy - of integral of y dx okay. Now what we do is that we identify x as the velocity here. And then for y we take you know it is m times that is the momentum. So, m times v that is nothing but m0 v divided by root over of 1 - v square by c square okay. So, let us just do this integration and then get the result. If you are interested in the algebra of it I am going to spend one more slide on it.

So, just to show you some of these steps okay so a one or two slides that is so the integration that we are going to do is one written at the bottom of your screen okay. So, that is the expression for the relativistic case when mass is a function of velocity.

(Refer Slide Time: 10:16)

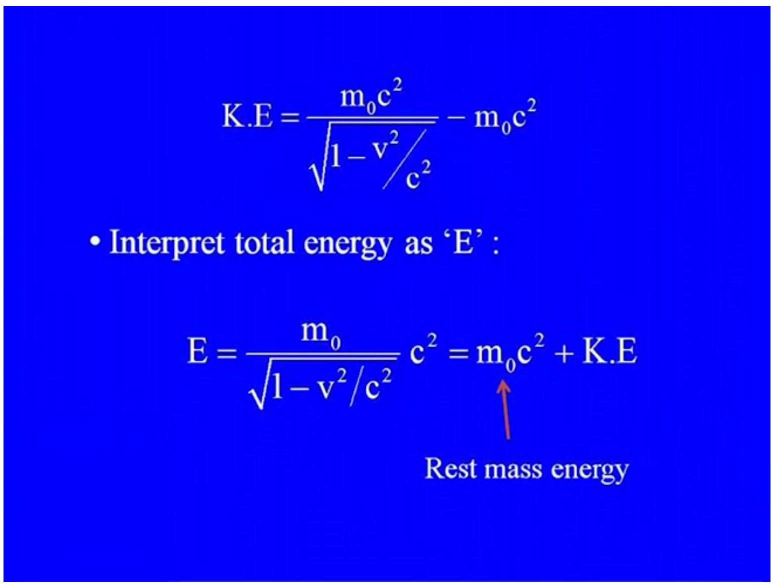


So, the kinetic energy happens to be m0 v square divided by 1 - v square by c square root of that - of m0 and then you take the integral of v dv divided by root over 1 - v square by c square. So, if you do this integral and then put in the proper limits okay ah you know just look at the second line so when you do the integral and put in the proper limits so you get the one that is given in the brackets there.

That is m0 c square divided by into root over of 1 -v square by c square and then the limits from 0 to v. And then a whole expression simplifies whole expression for the kinetic energy simplifies into two parts. See that it simplifies into an expression for the mass of a body which is moving with a certain velocity v times c square. Why the mass of the body being m0 divided by root over of 1 - v square by c square?

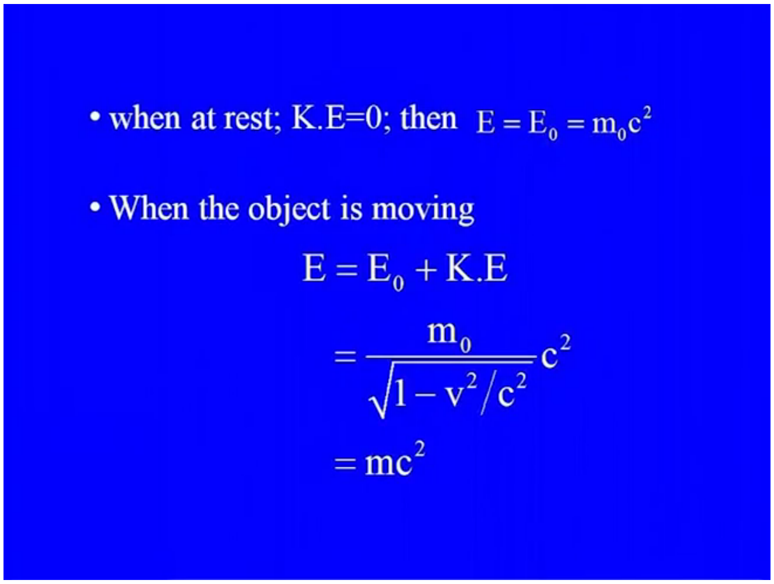
That is the total mass of a body when it is moving a certain velocity v, let us multiply it by c square - then m0 down m0 is what m0 is? Just a rest mass okay of the body times c, c being the velocity of it okay. So, that is what we have that is the total kinetic energy of the body okay, now kinetic energy of the body being that.

(Refer Slide Time: 11:52)



We can interpret the total energy of the body has something like m0 divided by root over 1 - v square by c square okay times the c square okay. So, that being, that will be then equal to m0 c square now we call it as the rest mass energy. If we consider mc square as the total energy plus the kinetic energy okay that sounds a bit confusing, might a little bit. Let us just have a look at the next slide and that will clarify our concept.

(Refer Slide Time: 12:29)



What is it so at rest when the body is at rest we know the kinetic energy is 0 okay? But what happens to the total energy. Remember if you go back to our old slide if you go back to rule set, we see that when the body is at rest the total kinetic energy is 0. But the total energy is still m0 c square when the body is at rest. So, that justifies the world and that justifies the term rest mass energy. That is the intrinsic energy associated with the body even when it is at rest.

So, when the body starts moving the total energy would be this intrinsic rest mass energy plus whatever kinetic energy the body has gained, so that will be then the total energy okay. So, that is exactly what we meant by saying that we interpret the total energy as mc square and m being m0 divided by 1 - v square by c square okay. So, having known this concept of what a rest mass energy is let us just write that as E0.

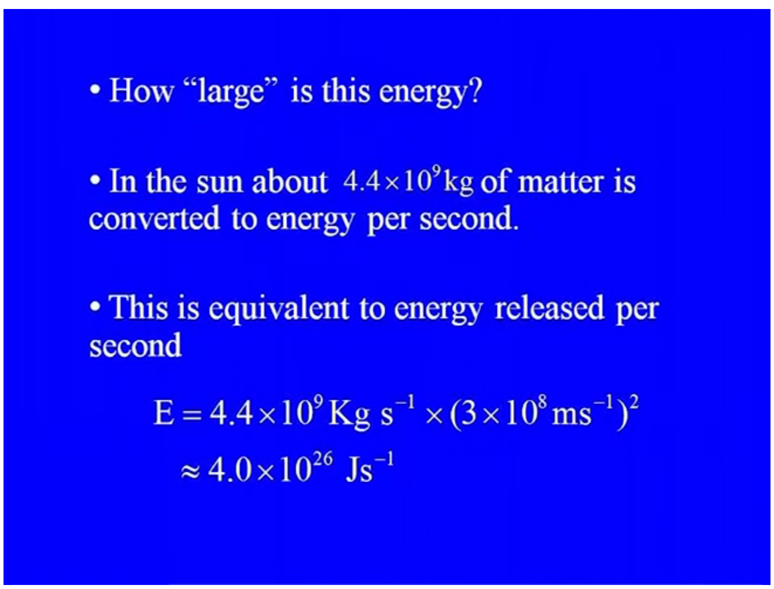
Just to signify that it is you know the rest frame of the body or the energy of the body where the body is at rest. So, what as you said, what is the total and the body when the object is moving? It will be E and that will be equal to E0 times plus the kinetic energy that is and what is that? That is nothing but m0 root over 1 - v square by c square times c square okay. And what is that at simply mc square where m is the relativistic mass okay.

So, you see how this equal to mc square has come about okay. Now of course this is a profound result. I mean it has not only altered physics, it has altered the world actually okay. So, it is the first time you are seeing that you know the intrinsic mass is being related with the energy of the body. And it has immense consequences in fields of in other branches of physics okay, for example if we consider the energy production in the Sun, exact their energies been produced by fusing hydrogen into helium okay.

So, when you so if you add up the masses of hydrogen and you add up the masses of helium okay. You would find that in the final product the total mass is not the same I mean whatever you started the total amount of hydrogen you know the total mass of the hydrogen you started okay and the total mass of helium that is produced you will see it is less. So, where has this mass gone into? This mass has been converted into energy okay.

So, hydrogen is being fused into helium and then some amount of energy is released okay. And that is the source of that; well, that is more or less the you know the source of energy production in the Sun.

(Refer Slide Time: 15:51)



And just to give you an example you know just a feel of the numbers or the amount of energy that is being converted from matter per second okay. About 4.4 into 10 to the power 9 kilograms of matter is being converted to energy per second in the Sun okay. So, how are you going to find the total energy released per second that is the total power actually. So, you are going to multiply this number 4.4 into 10 to the power 9 okay kg per second into 3 into 10 to the power 8 meter per second square okay.

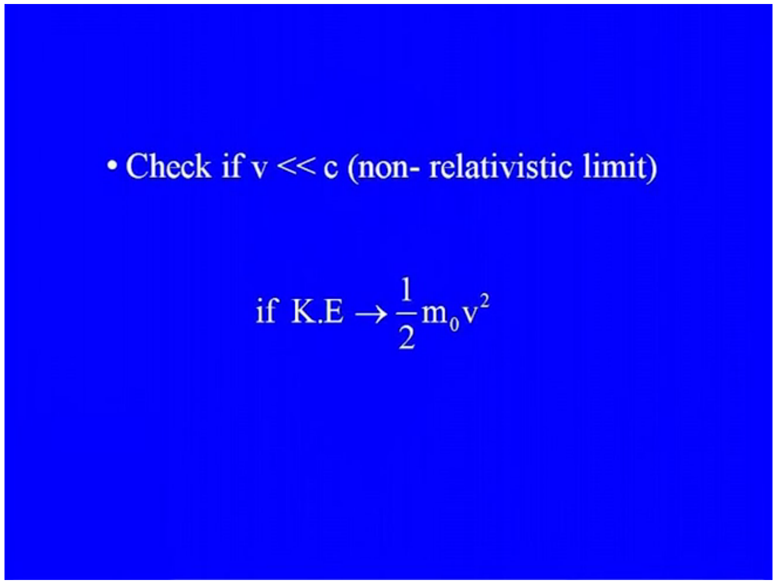
Meter per second that is I am sorry meter per second and take the square of that then what do you find you would find that the total energy is nothing but 4 into 10 to power 26 joule per second. Now if you just consider the thousand watt bulb okay, nobody uses thousand world bulbs you know their homes anymore. You would rather use more efficient sources okay. But let us you know how bright a thousand watt bulb can be?

Okay that is 10 to the power 3 okay send over 3 watts, if you divide 4 into 10 to power 26 by 20 power 3 it is not it is nothing but 4 into 10 to the power 23. So, per second there are the sun is shining like 4 into 10 to the power 23 thousand watt bulbs. Just have a feel and I just have a feeling of how large this power is well actually this is also an important lesson for us to utilize the solar energy in our daily lives.

I mean it will you know if you work more on research and solar energy and try to harness this energy which is just coming to us free from the Sun. The lots of this energy problem will be solved ok. So, I think you know appreciate how this E = mc square comes about and what the consequences of it are okay. Of course I did not so far talk about a nuclear fusion of you know in a very direct way I did not talk of nuclear fusion.

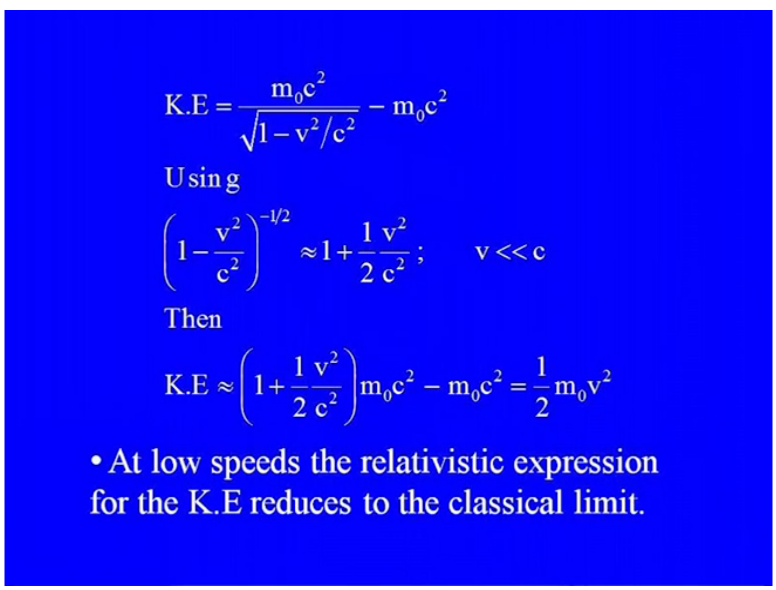
But then I give you an example of fusion of hydrogen to helium in stars okay. But some other applications in nuclear physics let us see if you can do such things a little bit later okay. So, having talked of energy and mass, let us see what the expression for the kinetic energy, I mean whatever things that we have discussed so far whether it actually boils down to two our known relativist and non relativistic formulae.

(Refer Slide Time: 19:06)



If we take velocities less much, much less than the velocity of light, so, let us check if the mass times the energy, you know mass times the speed of light squared from where from you know we have derived the kinetic energy. Let us see whether we can get the proper limit here okay. So, we know the expression for the kinetic energy that is the total energy minus the rest mass energy of a body okay.

(Refer Slide Time: 19:43)

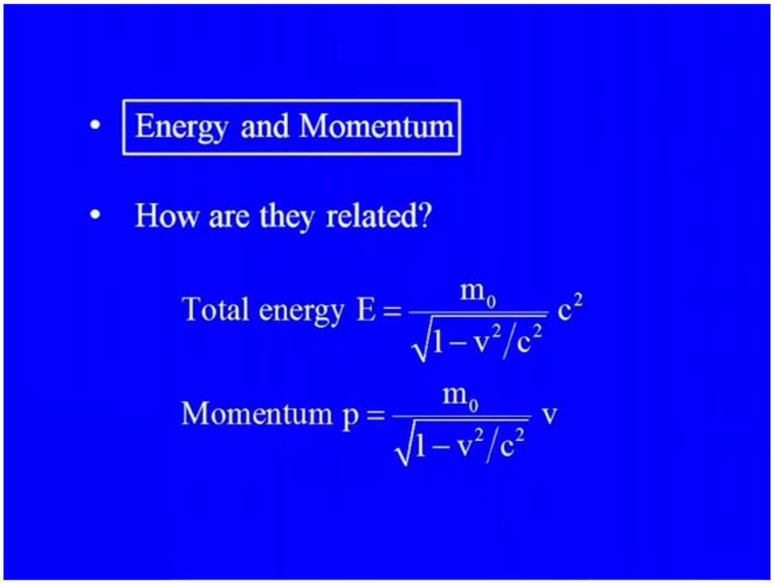


Now since we know that the velocity that we are considering here is much, much less than the velocity of light. So, we can do an approximation here we do a binomial expansion of 1 - v square by c square to the power - half here okay. That is the denominator of m0 c square divided by root over 1 - v square by c square what is that? Well the first term is 1 + you know the term of importance half times v square by c square.

So, you plug this in the expression for the kinetic energy okay it becomes 1 + half v square by c square names m0c square of course and then you subtract m0c square from here okay. What do you get? It is very simple you get half m0 v squared okay. So, indeed since the mass of the body does not change in non relativistic classical physics. It is half mv squared and then m doing the rest mass rest mass of the body okay.

So, we do see that it reduces to the classical limit at low speeds okay. So, having talked of mass energy and then seen and then having seen that it actually does give us the proper limit of the kinetic energy at low speeds. Let us now move a bit and bring in the momentum the okay. Remember we started this module by talking of mass energy and momentum.

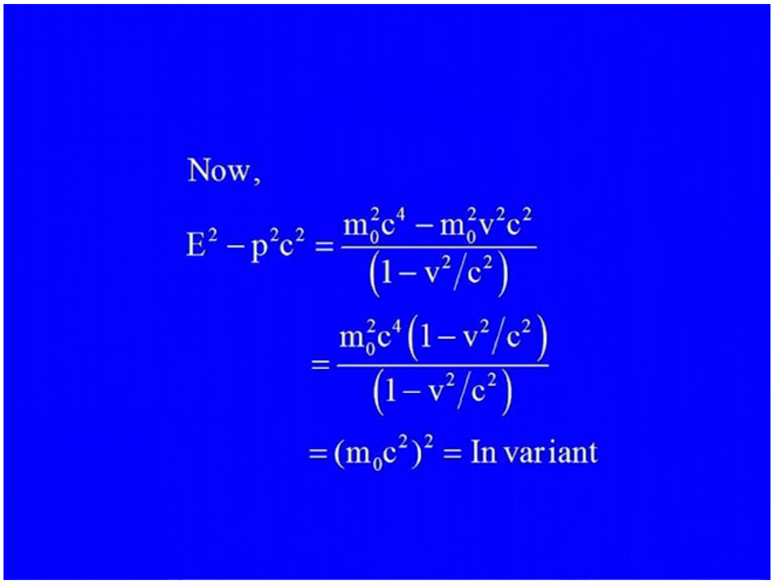
(Refer Slide Time: 21:34)



Let us talk of energy and momentum okay, so how are they related? Well you know from your previous slides here that the total energy is m0 divided by 1 - v square by c squared times e square that is mc squared. What is the momentum? Well that is m times vm being your familiar m0 divided by 1 - v square by c square okay. Well but looking at these two expressions mathematically you see that you know the mass portioned means this means the same.

But then you know the only thing different is the c square for the energy part and a velocity v for the momentum part. So, what will happen if we you know try to put them together on the same footing okay?

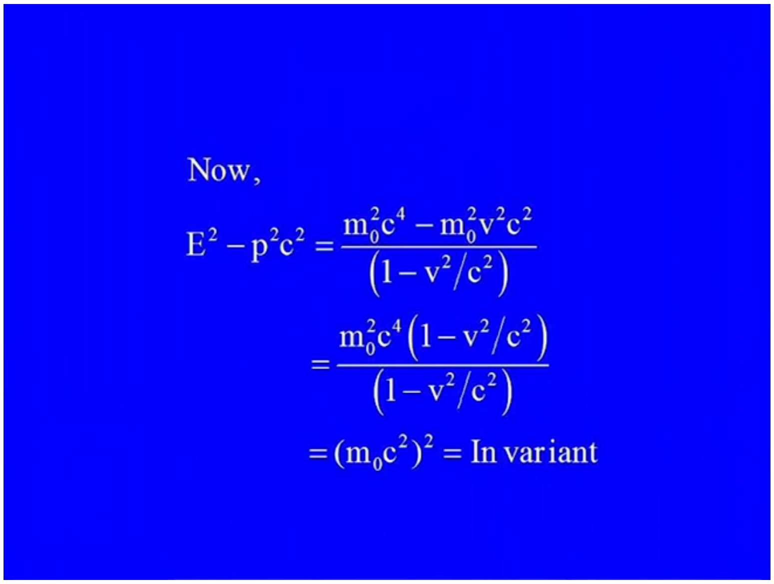
(Refer Slide Time: 22:30)



What happens for example if we just do e squared - v square c square. So, what we do is that we square the energy and then from that we subtract p squared times c square okay. Now you will check that of course that will the dimension of d energy square okay. But what is it? Walter does it give us something very interesting actually it does I mean if you do the algebra it is rather one or two line as algebra.

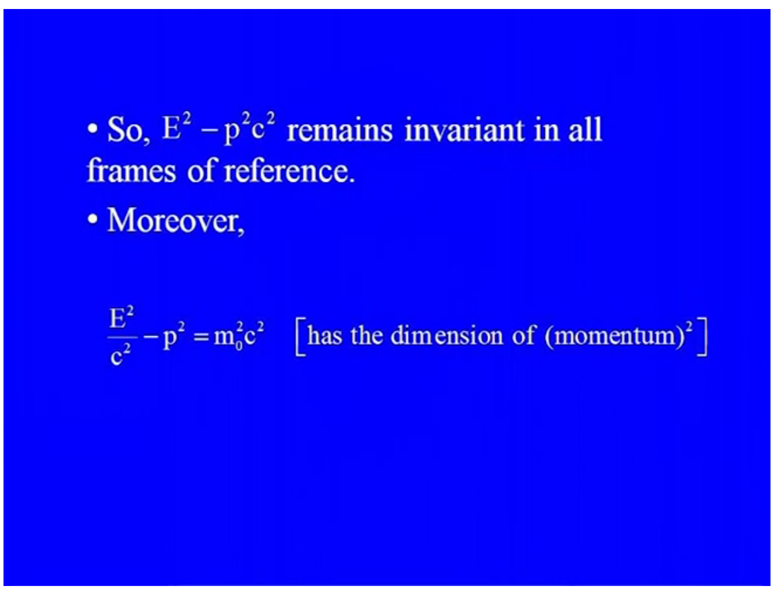
You will see that it is nothing but when you do E square that is the total energy and this from that you subtract P Square that is the momentum squared times the velocity of light square. We are going to get something very interesting.

(Refer Slide Time: 23:19)



You are going to get m0 c squared whole of that square of that whole square of that that is. So, that is nothing but the rest mass squared okay of the body okay. So, if the body has a total energy E and it is also having total momentum p its hat it is moving with a certain momentum p then E square - p square c square is nothing but the rest mass square of the body. It is an interesting concept actually. It is a because what it tells you?

(Refer Slide Time: 23:58)

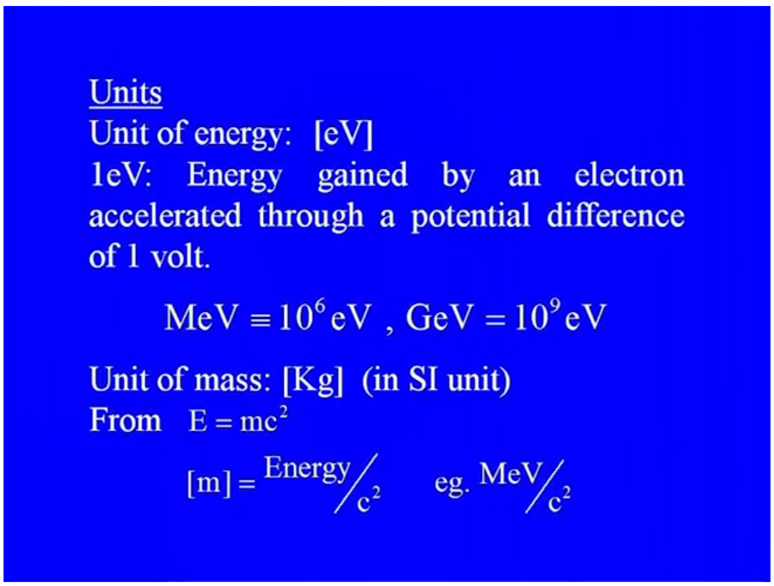


Is that this group of variables E square - p square c square remains invariant in all reference frames of course these reference frames are moving with constant velocity with respect to each other okay. So, that is what we have we have E square - p square c square to remain invariant okay. Now well that was the dimension of energy squared.

Actually it need not be when we are also having momentum okay. So, we could also in a sense have this as the dimension of momentum squared that is very simple all you do is that you divide and the entire stuff by c squared and then you have E squared by c squared - p squared and then the invariant quantity will come out to be m0 squared c squared. Remember m0 is the rest mass of the body and c of course is the velocity of light okay.

And this has the dimension of m0 c that is has the dimension of momentum and then m0 square c square has the dimension of momentum square okay. So, I am going to spend some time on this relation okay and can then find out certain interesting things but before that let us continue a bit on these units okay. Remember in relativity and in like other to mean so physics we are quite interested in units in which you measure everything in terms of something you know in terms of the units of energy okay.

(Refer Slide Time: 25:48)

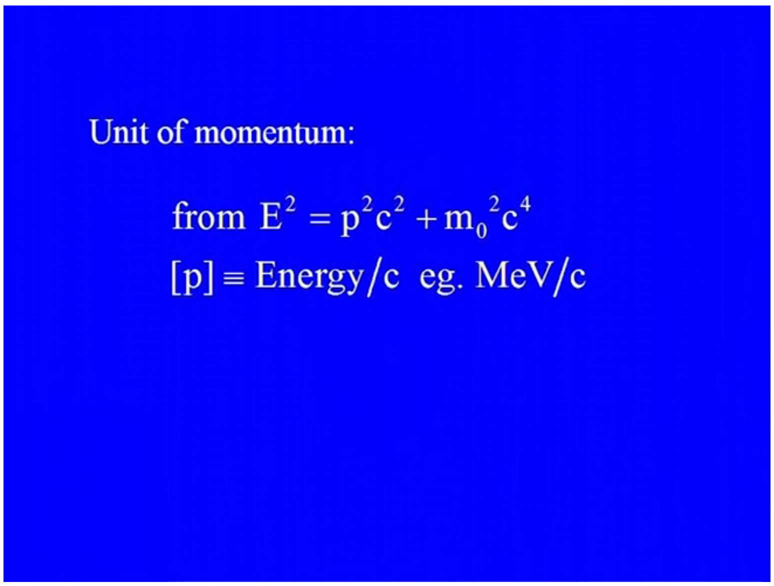


In atomic physics for example the unit of energy is generally electron volt okay physics MeV okay million electron. So, what is that what is MeV? So, MeV is nothing but the energy gained by an electron accelerated to a potential difference of 1 volt okay. So, if you take a million times of this that is 10 to the power 6 eV that gives you MeV. And then when you take 10 to the power 9 that is 1 billion eV you get 1 GeV that is Giga electron volts okay that being the unit of energy.

What about the mass well normally you would measure everything in grams or kilograms, its SI unit okay. So, however if you if you if you consider the expression E = mc squared you can very well measure mass in terms of energy by c square. Then you can measure mass everything in terms of in example like MeV by c square or eV by c square and GeV by c square okay.So, that actually helps a lot.

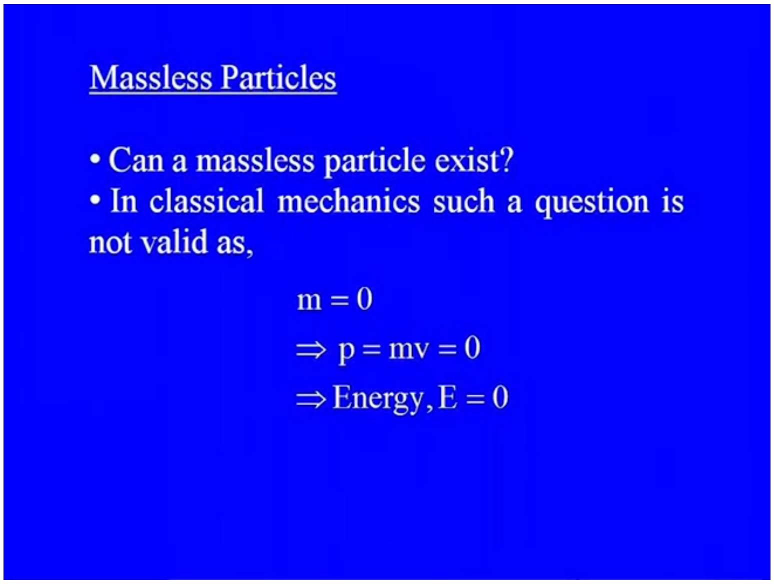
So, what about the; you know what about the expect, what about the unit for and the momentum? Okay, mass times the velocity. If you measure velocity terms of you know how much of it is a percentage of velocity of light. So, if it is MeV by c square then you multiply by c, it should be MeV by c or something like energy by c. Well that is that is very that is very normal.

(Refer Slide Time: 27:29)



Because if you just consider E square = p square c square + m0 square c to the power 4 then the momentum does turn out to be energy by c okay. And then you can you can measure it in terms of MeV by c or let us say the MeV by c, if it is an ultra relativistic momentum that you are considering okay.

(Refer Slide Time: 28:01)

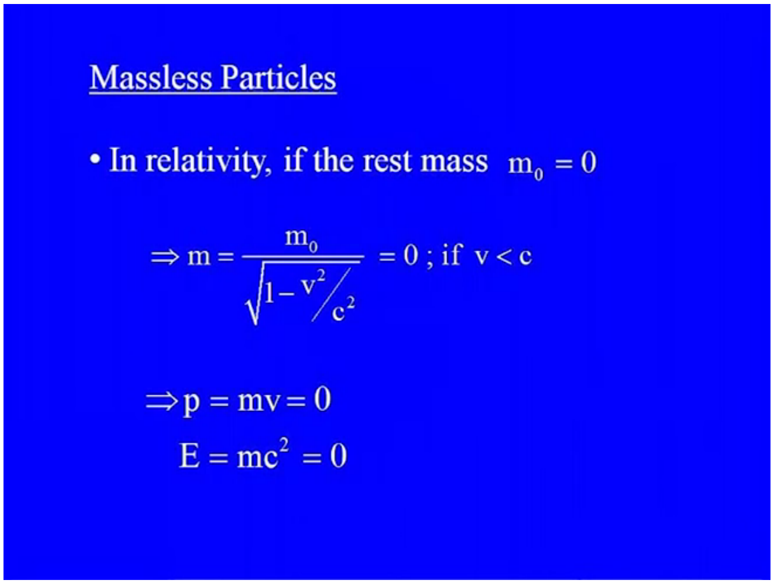


Now let us consider something else so what one other application okay, it is whether one can have mass less particles okay. And I see this as an application of relativity we are going to realize that in classical mechanics we kind of never ask such a question okay. Why? Because well once you have a mass less particle or you do not have you have matter which is mass less, well in classical mechanics such a thing does not exist at all.

Because you see if you have of something mass less I mean what is mass less in classical mechanics? Nothing I mean you consider unless you know you have an object which is a material matter and then you say it is mass less of course then object is not there okay. So, but in any case if you want to take this argument a little bit further. If m is 0, the momentum is 0, the energy is 0. So, such a question does not happen that does not arise at all.

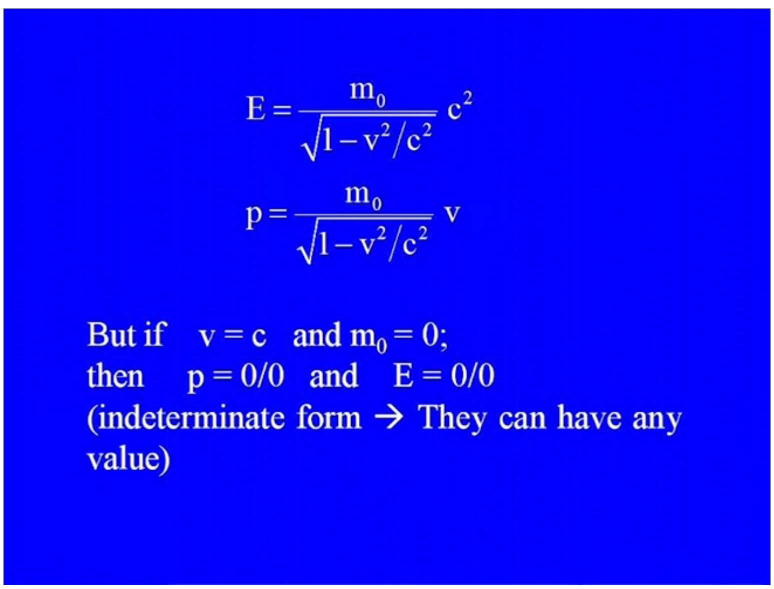
So, mass less particle cannot exist in classical physics or in on in a non relativistic situation however in relativistic situation do we have a mass less particle to exist does it you know. Where it is it is more like in classical physics it forbids the existence of mass less particle but that is relativity forbids such an existence okay. Let us let us just check it out.

(Refer Slide Time: 29:38)



Well if indeed the rest mass is 0 you would find the total mass to be 0 if velocity is less than c why because m0 is 0 in the denominator is 1 - v square by c square is finite and then that is so the total mass m0. And then immediately you are going to figure out that the momentum is 0 and the energy of course is 0.

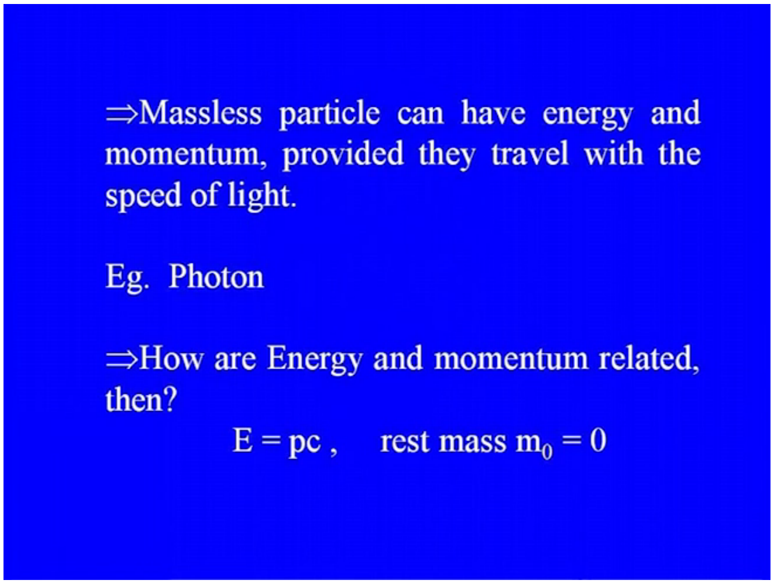
(Refer Slide Time: 30:01)



Fine, so if the rest mass of a body is indeed 0 and it is moving with a velocity less than the velocity of light. Then vastly spot it should not exist okay, but what happens if we still have the rest mass to be 0. But the particle is moving with a velocity of right itself okay. It is strained is it, so, then what you see is that the expression for the momentum and the energy becomes 0 by 0 it is an indeterminate form okay.

It is an indeterminate, you cannot determine, it is 0 by 0 form. So, it is basically you cannot say that it is 0 neither can you say it is actually infinite. It is actually indeterminate can have any value okay.

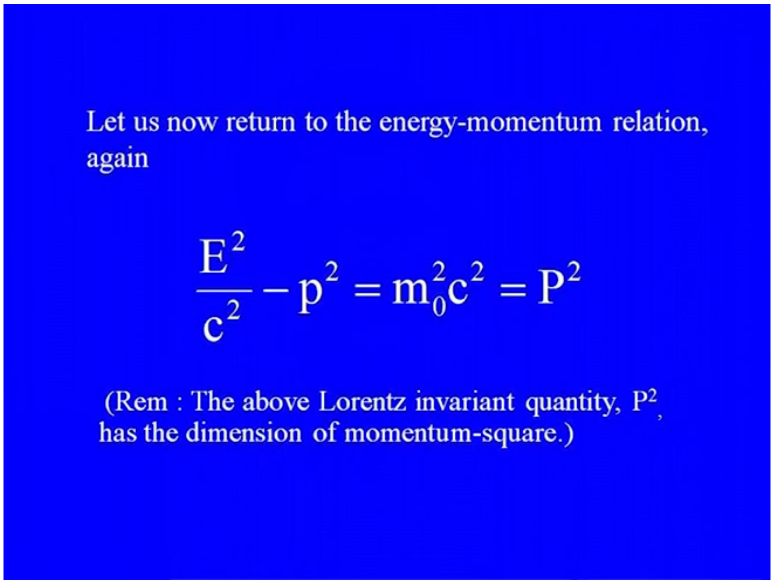
(Refer Slide Time: 30:47)



So, in a sense relativity does not forbid the existence of mass less particle if then in this case and only if it moves with the velocity of light ok. So, that is a very important conclusion that you can draw it is that mass less particle will be existing only and they do not have energy momentum provided they travel at the speed of light ok. Well actually can you can you can you name such a thing exactly you can it is called the photon ok.

Now it is the quanta of light ok. Then how are the energy and momentum related in this case? Well since the rest mass is 0 then energy is equal to p times velocity of light. Here c times p because otherwise a square is equal to p square c square + m naught square c power 4. So, I m naught square being 0, so you have E square = p square c square and here. So, E = p into c that is what okay. So, that is an important conclusion that we have a special relativity okay.

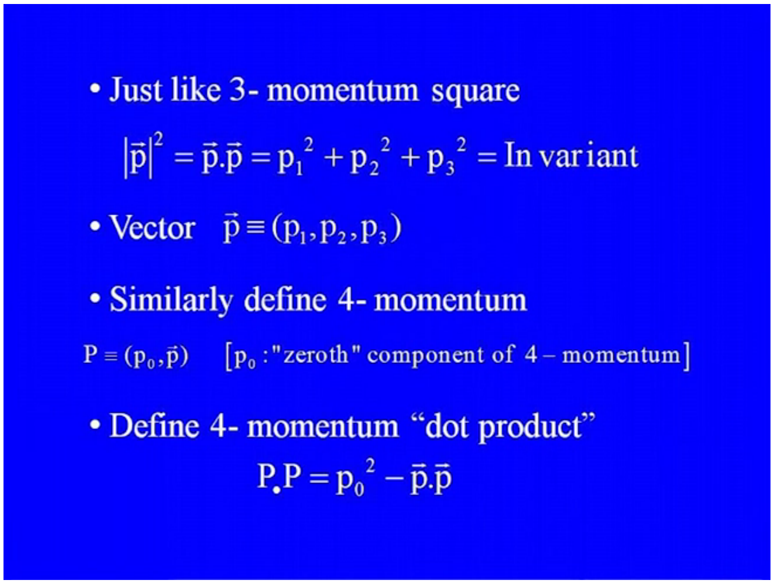
(Refer Slide Time: 32:06)



So, now let us change tack a little bit and return back to our energy momentum relation and see something quite interesting okay. So, what we have here? We have basically E square by c square - the momentum square remember this what we write at the small p that is the 3 momentum okay that is equal to m0 squared c squared and then I write that as capital P square just to you know give you an idea that the invariant quantity that you have has a dimension of momentum.

What kind of momentum is that it is definitely not the 3 vector momentum that we do in our daily lives okay. So, that is what I said so the capital P here has a dimension of momentum and then a P square of course has the dimension of momentum square here it is different from small p.

(Refer Slide Time: 33:07)



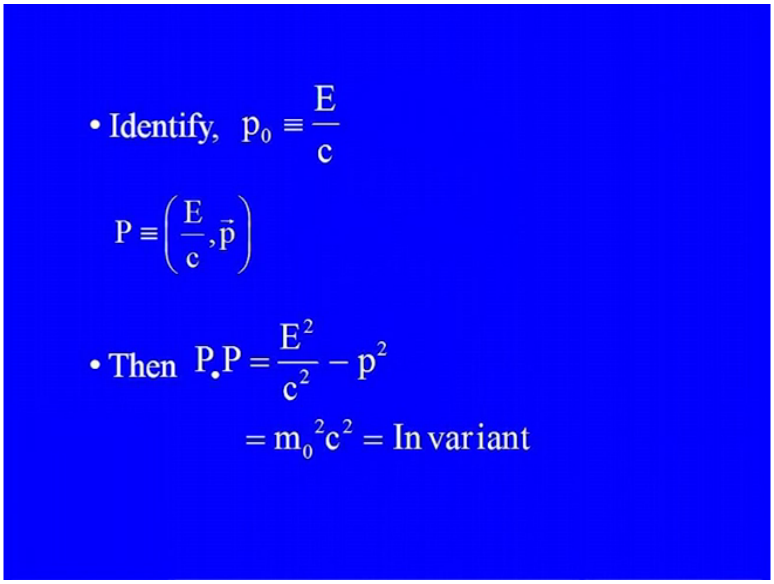
Now what is small p? Now small p let us say it is the momentum well it is the 3 momentum of your particle okay. Now if you take the 3 momentum squared so norm of the momentum I have put a vector sign on top of small p, what does it mean? It means that you take a dot product p dot p, small p dot small p. Then what does it turn out to be it is nothing but p1 square + p2 square + p3 square. If I write for example px square + py square + pz square in Cartesian coordinates and that is invariant, you know that.

Because that is equal to the length of a vector that is invariant okay even if you rotate the vector delay the length remains the same. So, the vector the 3 vector is the one in which we put the vector sign and then we have 3 components as there are 3 independent components okay. Now check that the momentum or the, you know we can define a 4 momentum which capital P whose which has of course 4 components.

And to keep in you know in consonance with the 3 momentum that we had earlier p1, p2, p3 which is denoted by vector p, we have the first part as p0 we just call this at the 0th component of the 4 momentum okay. And then if we define something like the for momentum dot product I put the dot product in inverted commas and then you put a bigger dot okay P big dot P okay and all capital P's.

Then we define that as p0 squared - p dot p the vector p then we can get something quite interesting actually. We can relate what we have found here with the energy momentum in special relativity.

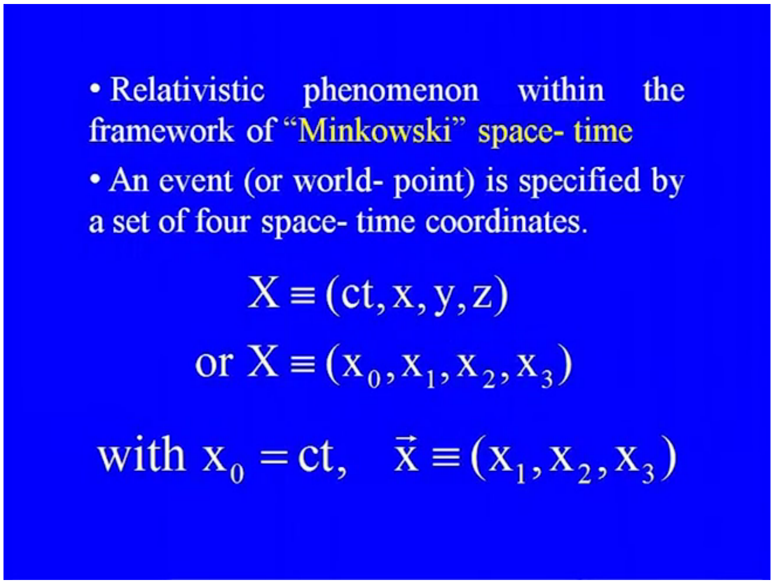
(Refer Slide Time: 35:12)



So, if you identify p0 as E by c okay and so the total P becomes the capital so the capital P becomes E by c and then the other vector is the 3 vector. Then you know the 4 vector dot product becomes E square by c square - small p squared and we know that that is invariant okay. Now just like the dot product of a vector was indeed invariant. Here to the dot product or the big dot product of the 4 vector is also invariant okay.

So, there is something which is happening here, I mean, so I think if you if you have not got it in this 2 slides do not but you are going to talk a little bit more about the 4 vectors and space time a little bit later okay. So, I just want to tell you that there is something interesting that you can relate this energy momentum with things called the 4 vectors okay. Let us see what that is? Okay.

(Refer Slide Time: 36:18)

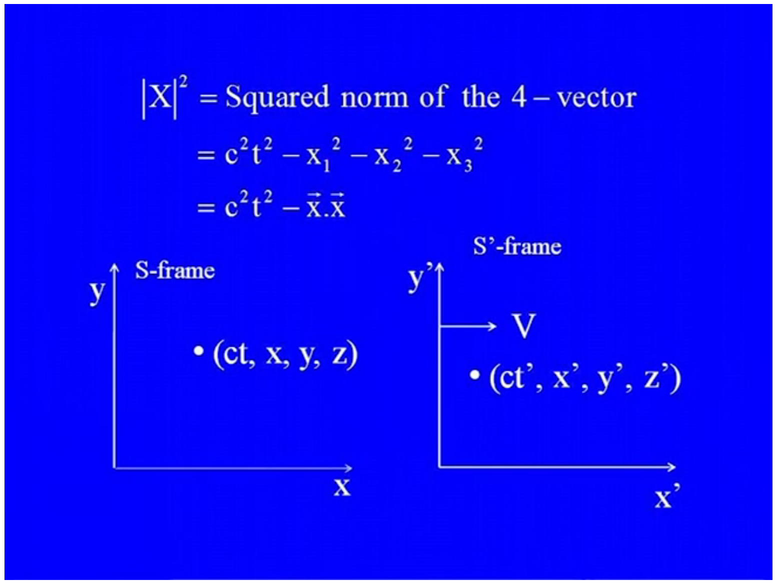


Well to change stack yet again and do a little bit of formal stuff in terms of Minkowski you are doing relativity. One does relativity in terms of Minkowski space time, remember in special relativity. So, it is not only the space part but we also have the time part which is which is equally important here. So, because here time is no longer an independent quantity it also depends on the space quantities that is what we saw in our Lorentz transformations okay.

So, if we define a quantity okay and event let us say or the world point by 4 space time coordinates okay. Since the velocity of light is c which is constant in all frames, so what we have here is that we take capital X as you know ct and then x, y, z. So, why have you taken ct? So that we preserve the same dimension as these space coordinates okay or you could even have make it look like, you know you, just say you have 4 coordinates x0, x1, x2, x3 and then x0 is the 0th, in a sense 0th component of the thing called X capital X here.

And then small x1, x2, x3 these are the space coordinates okay. So, that is what we said X0 is = ct and then the vector x is x1, x2, x3 or x, y, z. If you wish in Cartesian coordinates okay.

(Refer Slide Time: 38:17)

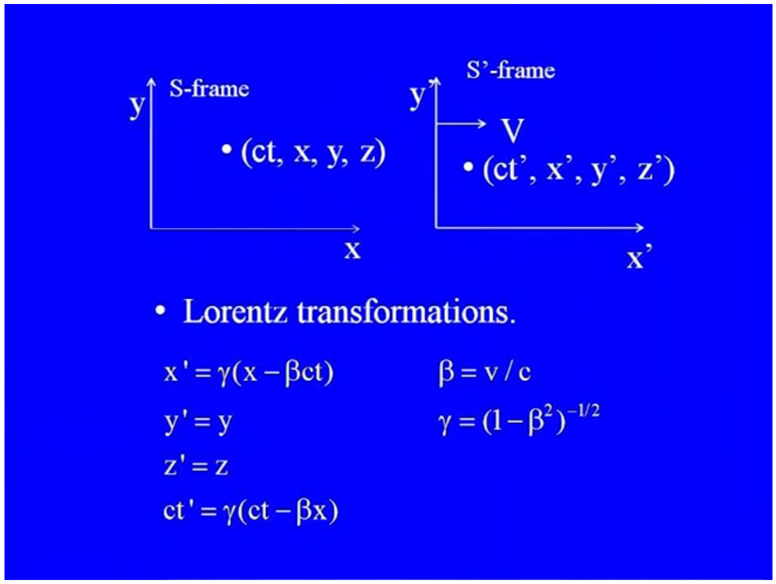


Now what is the squared norm of this quantity as we had defined it is c square t square - x1 squared - x2 squared - x3 squared okay. So, it is nothing but c squared t squared - vector small x dot vector small x okay. So, what do we have? So, what we have is that in S frame if we have a point which is denoted by you know one of this world space time events denoted by ct x, y, z okay. So, certain event is occurring at a certain time in a certain position in space.

Now the same event will be viewed by someone in the S prime frame which is moving with velocity v with respect to the S frame as ct prime and then at that time t prime that is and then the at position x prime, y prime, z prime okay remember at t = and t = t prime = 0, these two frames where you know coinciding and then it started with the movement velocity uniform velocity v that is okay, so how are these things related so these quantities are actually related by Lorentz transformations okay.

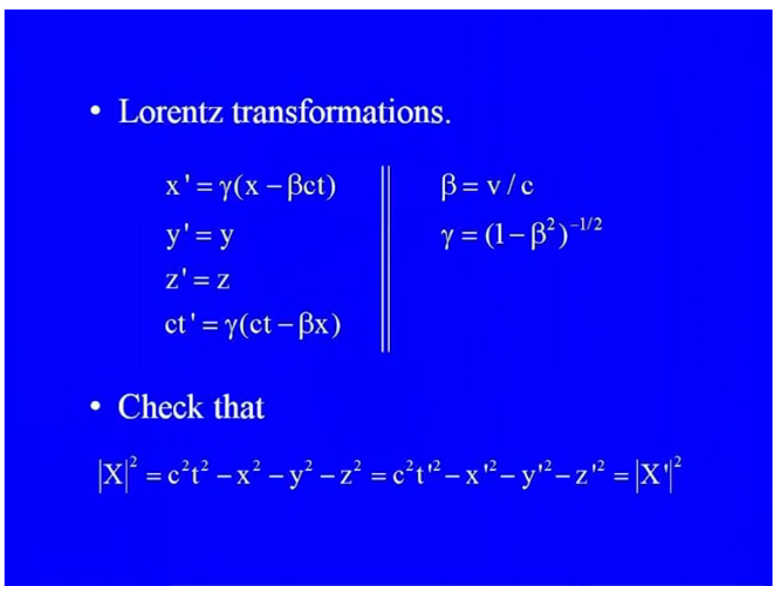
So, what is x prime? Well x prime is nothing but I mean if you use the notations gamma and beta remember we had used you know we had we are using them so that these expressions become less cumbersome okay. So, beta beings v by c okay and then gamma becomes 1 - beta square what root over R to the power – half. Basically 1 by root over 1 - v square by c square okay.

(Refer Slide Time: 40:20)



Since we are moving uniformly along S prime frame is moving uniformly along x, x prime axis and so what is x prime? So, x prime is nothing but in terms of the x quantities its gamma times X - beta ct okay and y prime of course is equal to y and then z prime is z and then ct prime is gamma ct - beta x ok, fine. So, what happens to the quantity capital x prime? Okay.

(Refer Slide Time: 41:07)

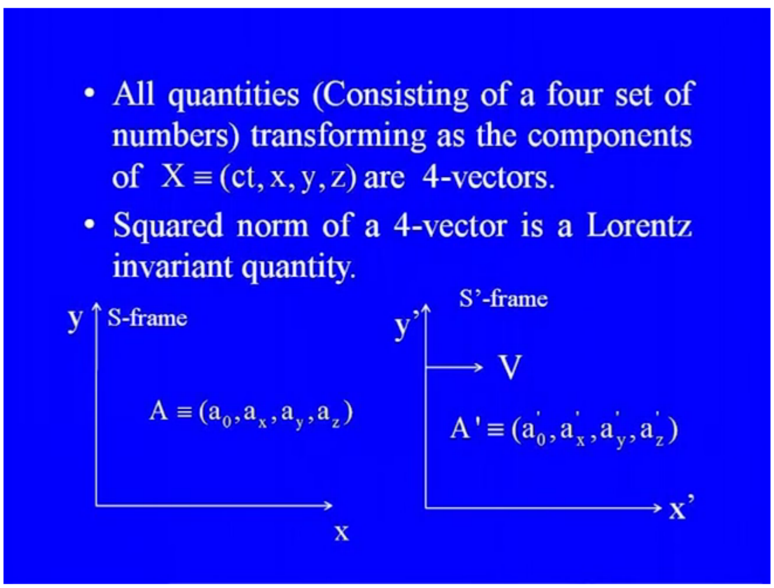


So, if you do c square t squared - x square minus y square - z square you should check that this becomes equal to c Square t prime squared - x prime square - y prime square - z prime square. So, the norm of the quantity x ok capital X that is of the 4 vector remains invariant whether you are doing it in the S frame or the S prime frame okay. So that is what we have and in fact that gives us the idea that if we have if we can if you can figure out our set of 4 quantities or 4 numbers that transform as the components of the 4 vector X okay capital X that is.

They will be 4 vectors okay so and then not only that I mean just just about any 4 quantities will not be any 4 numbers you just come them together they will not be 4 vectors well their component should transform like the components of the space time coordinate capital X okay which is basically they should transform by the same Lorentz transformation. Not only that the norm of the 4 vector that one has constructed should be invariant under any frame okay.

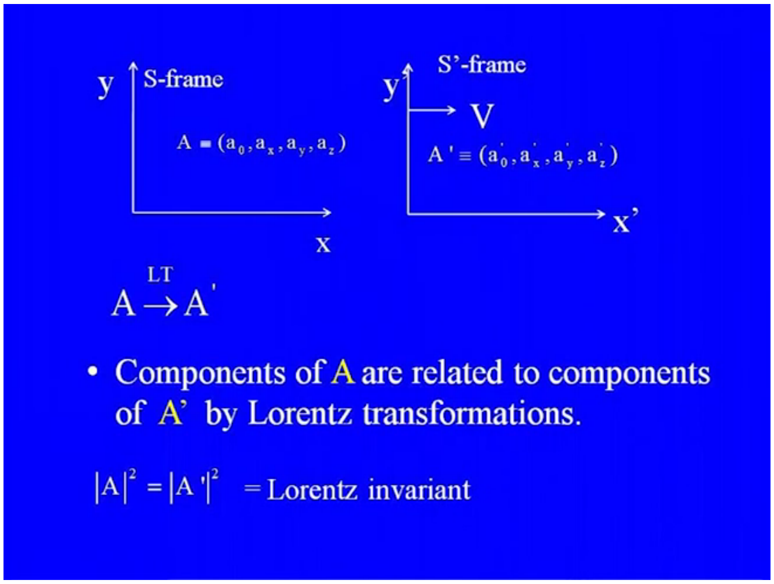
To give you an example just think of any other 4 vector save a general 4 vector some vector A, remembers since I am using the word 4 vector and not a 3 vector that is what I have not put a vector sign on top of a capital A that is.

(Refer Slide Time: 42:56)



So, in the S frame that is it has components a0, ax, ay, az and in the S prime frame it has components a0 prime my ax prime, ay prime and az prime and that is the vector A prime ok that is 4 vector A prime. Now if A is a 4 vector in S frame ok and S prime is the 4 vector seen from rather A prime is the same 4 vector seen from the S prime frame. Then there are certain conditions that should be obeyed.

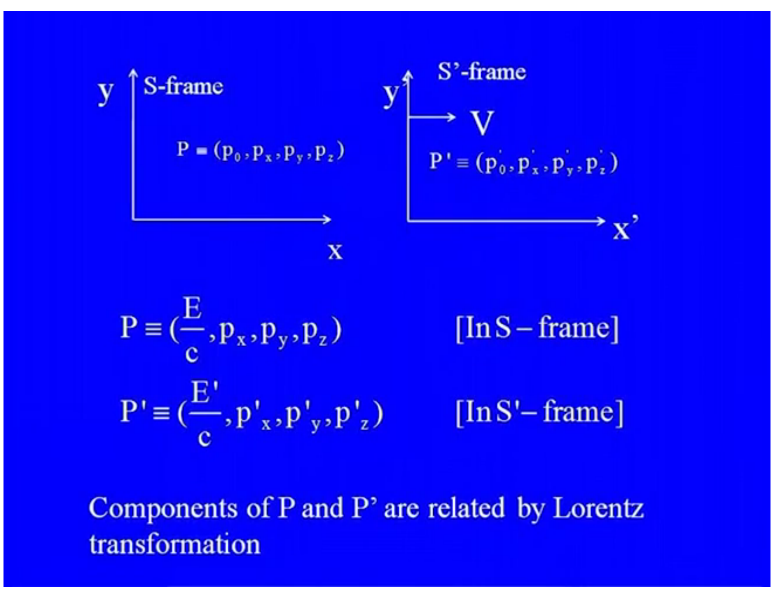
(Refer Slide Time: 43:44)



Well first of all the components of A and the components of A prime should be related by the Lorentz transformation okay and not only that the squared norm of A and A prime they should be equal and it is actually Lorentz invariant ok, fine. So, that is exactly what we had for our energy and momentum if we define the 4 momentum as E by c and that is the 0th part and then the other 3 parts are the if you are the familiar 3 momentum vectors ok.

Let us say the 3 linear moment of px, py, pz, so, if capital P that is the 4 momentum vector in S frame.

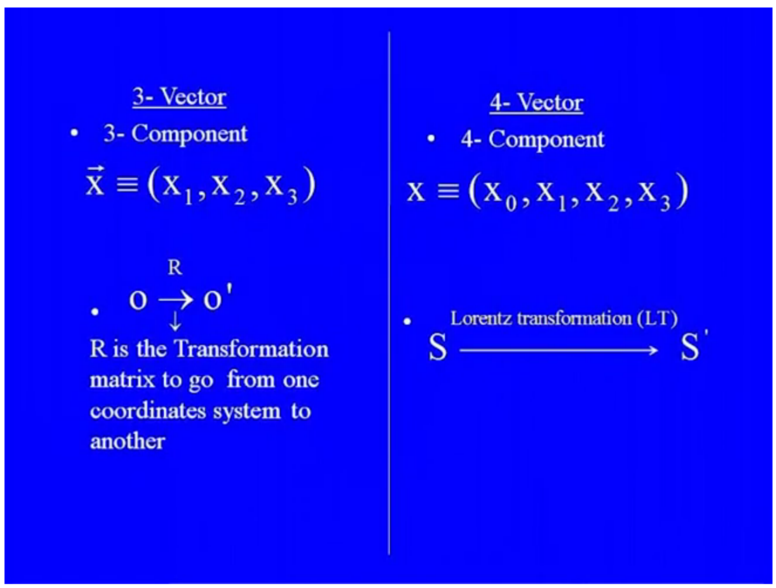
(Refer Slide Time: 44:37)



Which has components E by c and px, py, pz and then the same thing seen from the prime frame is capital P prime and that is E prime by c, px prime py prime and pz prime the small px prime, py prime and pz prime that is the 3 momentum vector in the prime frame. Then the components of P and P prime are related by none other than the Lorentz transformations. Well this has immense consequences actually.

Because in a sense this will help us in relating the energy and momentum in one frame with the energy and momentum in another frame and not only that you can also relate energy momentum conservation directly in the S frame and the S prime frame okay. So, more of that a little later but then to summarize what we have talked of 3 vector and the 4vector let us just spend two more slides on it.

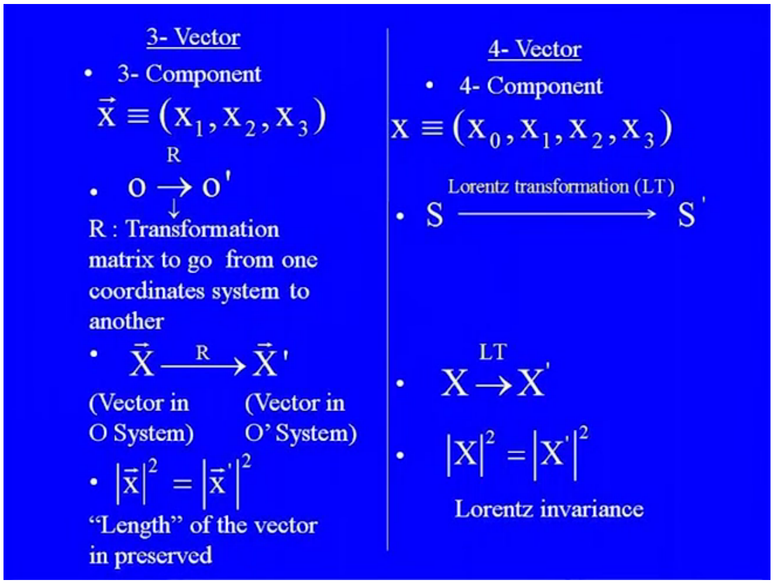
(Refer Slide Time: 45:53)



For a three vector and so let us say the velocity vector radial vector things like that okay a 3 vector of course has 3 components, so we put a vector sign on top of that and 4 vector of course will have four components okay. Now why is it that we call this three component thing as a vector? By the way well if it is because if you changing your coordinate system okay.

And then you are following you know the change of coordinate system is due to the change of coordinate system is due to this transformation when you need this transformation matrix R to go from one coordinate system to another. And similarly in this case of a vector it is the Lorentz transformation which is transforming from S to S prime okay. Now a vector I mean we define a vector to be that to be that physical entity is that look at the 3 vector part okay.

(Refer Slide Time: 47:07)



So, if you look at the 3 vector part and then you see that X vector X is transform to vector X prime okay. Now it is the same transformation matrix which will do the transformation. So, the transformation matrix which is transforming a coordinate system O to O Prime okay, the same transformation matrix will be responsible. It is the same transformation matrix which will transform X to X Prime okay.

So, that is a vector okay not only that the length of this vector in the whole system and the length of the vector in the O prime system will be preserved. Similarly what you see is that it is a Lorentz transformation which is transforming the quantities in the S frame to the S prime frame okay. So, it is the same Lorentz transformation which is going to transform the components of the 4 vector in S frame to the components of the 4 vector in the S prime frame.

Just like the transformation matrix R transform the components of the vector X in O system to the trans components of the vector X prime in the O prime system okay. And just like the length of the 3 vector was constant. Here what you see that the squared norm is Lorentz invariant okay. So, that gives you an idea why we had used this word vector for this 4 component entity in special relativity okay.

Now of course there are lots of applications to using 4 vectors we have just been introduced to 2 of the 4 vectors one of course is the space time vector and another is the 4 momentum 4 vector okay. Now these are going to have applications in relativistic kinematics and things like relativistic collisions. So, let us talk a bit about such a thing about our coordinate systems in you know in a case where such collisions can happen.

Namely the laboratory and the center of mass system, so, what is it? So, basically we are considering the collision of two particles A and B. let us say okay they are of certain they are of rest masses mA and mB okay. And let us say that well you can you can look at this either in the laboratory system and the center of mass system well. In case you want to know what these systems are the lab system is one.

In which one of the particles is at rest and the other comes and hits it with a certain momentum you know certain momentum. Let us say 3 momentum okay P or a vector P. In this case let particle B be a crest and then particle A is moving with a certain momentum in the lab frame P AL note is the vector sign and then the subscript AL, L stands for the laboratory quantity. So, whenever you see this L it means that it is the lab quantity.

So, in a sense what is the momentum for vectors here well capital PAL? That is; we will read 4 quantities. The first quantity relates to the energy and it is basically the dimension of momentum it is but involves the energy that is EA L by c. And then the 3 vector with which the momentum with which and the particle A is moving. What about PB L that is the 4 momentum vector for particle B in the laboratory frame.

Well that is EB L by c and then remember it is at rest so its total moment that is a linear momentum is 0 okay. Now what is EA L? EA L you know that it is m A into c square okay and then what about EB L is nothing but mB into c square okay. Now remember here EA L and now when I write ma here its mA divided by 1 - v square by c square actually that is assumed within when I write EAL here.

But when I write EBL here, remember that it is just the rest mass squared okay. So, we conclude our discussions here today once we have seen the differences between the 3 vector and the 4 vector. We now we realize why we call it as a 4 vector itself keeping it you know some sort of consonants with this component of a 3 vector ok. We are going to see in our next topic the applications of this of the 4 vector.

In special relativity especially in relativistic collisions and we will also do some more problems in relativistic kinematics in our next discussion thank you very much.