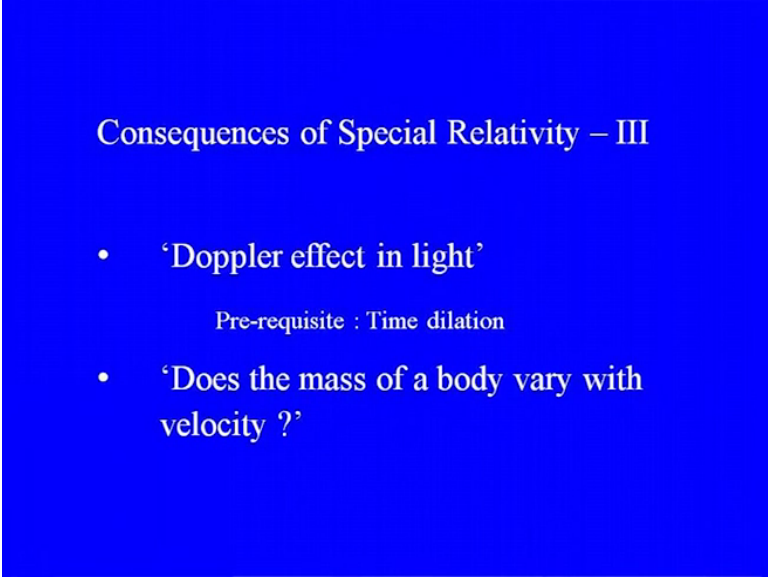


Engineering Physics 1
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Module-07
Lecture-04
Introduction of Special Relativity – III

Hello, everybody so but today we are going to talk of the consequences of special relativity. I am sure this portion is not new to you.

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Consequences of Special Relativity – III

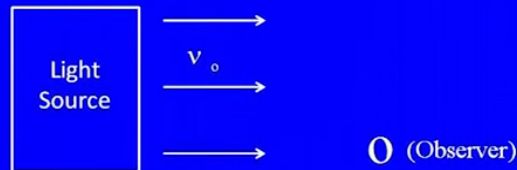
- ‘Doppler effect in light’
Pre-requisite : Time dilation
- ‘Does the mass of a body vary with velocity ?’

But we will talk of the Doppler effect in light the prerequisite for that is of course you know a bit off from time dilation things that we have covered earlier. Well I am sure you have heard of Doppler effect in sound ah let us see what it is there in light when we talk of electromagnetic waves example light okay. And after that we will check or rather we will see whether the mass of a body actually way better in values with velocity we may have heard of such things but let us see where it comes from okay.

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DOPPLER EFFECT IN LIGHT

- Consider a light source emitting light of frequency ν_0 .



So, let us first of all consider a you know we consider a light source and then it is emitting foot or by emitting light of a certain frequency let us just let us call that frequency ν_0 okay. And you have an observer here we will show it by oh here on to towards the right of your screen. I mentioned that of the Doppler Effect in sound, I am sure where instead of this light source you will be having someone some source which is emitting sound okay.

Which is making some noise and then emitting certain frequencies and then it is all fine when they are at both of them both the observer and the source is at rest at each other. But what happens if all of a sudden one of them starts moving okay. So, let us say the one of the source start moving and then the observer starts moving. Well if you have the observer starts moving towards these the source of the sound.

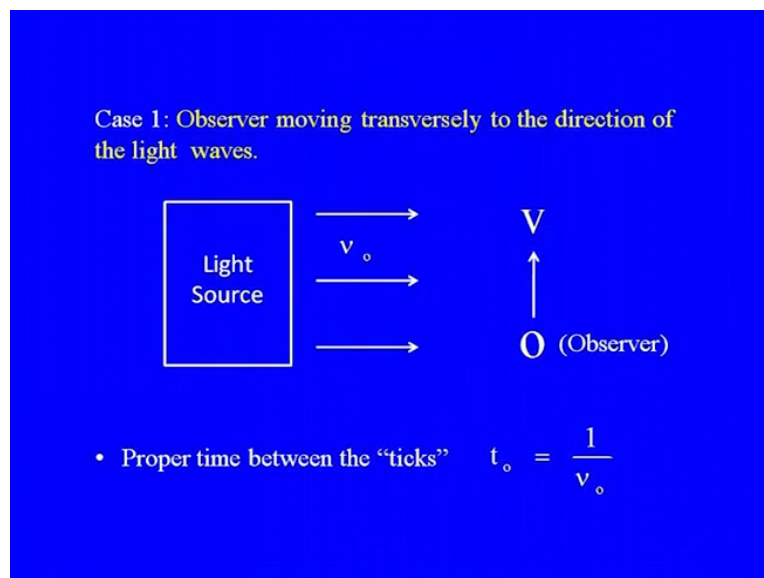
Do not you feel that you know the frequency of the sound that you are hearing has increased it is more like a car coming towards you, you know and it blows the car blows its horn then the more it comes towards you, you find the pitch of the sound increasing. Well so what is it with light do we observe certain the same things or is it a little bit different, let us check that out? Of course need to mention the very beginning so for propagation of light here you do not need a material medium.

Because it can propagate to vacuum but for sound of course you need a material medium okay. So, that is what we do, we consider a light source which is emitting light of a certain frequency that is ν_0 and then we have an observer okay. Now what we can also do you can also consider this a light source you know as a clock that is ticking like this tick, tick, tick and then added each tick each one of these ticks okay and it is emitting a light wave ok.

So, it is so what you are considering? You are considering a light source as a clock in a sense that is ticking ν_0 times per second and so on each tick its emitting a wave ok. So, it is now why do we put all these clocks and things here then because if you put this clocks you know time dilation, you really you can make the correlation and then we can use the all the principles of relativity that you have learnt earlier.

And now let us see what happens if you have the source here you have the light source which is emitting a certain frequency and the observer starts moving. Now we can have three situations here either the observer is moving transversely or perpendicular to the direction of these light waves or it is moving towards a light source or it is moving away. So, let us let us consider each of them one by one ok.

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So, case number one so in this what we have is that the observer that is we denote by O here and it is moving uniformly with a certain velocity v okay, transverse to the direction of the light

waves ok. Now if you are you know in the frame of the light source itself where the things were at rest okay. And what is the proper time in the sense between the ticks you know you consider the light source to be a clock which is ticking. And each of these ticks emitting a it is a emitting a light wave okay. So, what is the proper time here it is call it t_0 then how is it related with the frequency well you know frequency is in Hertz and time it is in second.

So, it is $1/\nu_0$ okay, so that is the proper time interval between the tick in the light source frame okay. Now what about; how would they observer then find this time to be okay.

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- In frame of reference of the observer, the time elapsed between two ticks is

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

- Frequency as observed by the observer

$$\nu_t = \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0} = \nu_0 \sqrt{1 - v^2/c^2}$$

- $\nu_t < \nu_0$

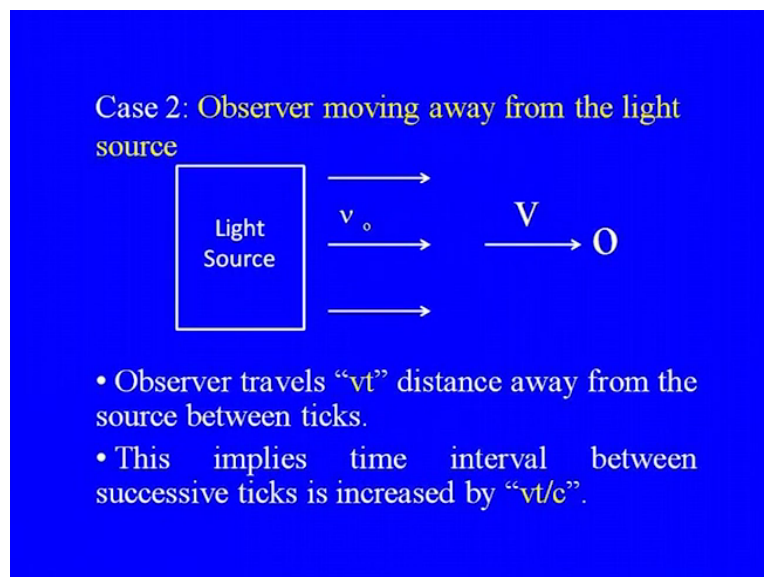
Now in the frame of the observer and the time elapsed between two ticks would be t . Now t will not be same as t_0 which was the proper time remember where t will be t_0 divided by root over of $1 - v^2/c^2$ okay. It is this very simple time dilation formula that you used it okay. Now what would then be what would then be the frequency of the light as observed by the observer. Well you take just the inverse of the time here.

Remember this time is the one measured by the observer in his or her frame ok. So, let us call this frequency as ν_t it is going to be $1/t$ and then you know what $1/t$ is you are going to you just see in the previous paragraph. So, you will see that its root over $1 - v^2/c^2$ whole divided by t_0 . Now t_0 is also related with the frequency at which the light is being emitted ok. So, you see that, so ν_t that is that is the frequency as measured by the observer.

That will be ν_0 times root over of $1 - v^2/c^2$. Now since v is always less than c here okay. It is going to be ν_t or the since the velocity of the of the observer is always going to be less than and the velocity of light okay. So, what you find as ν_t that is the frequency as measured by the observer will always be less than the proper frequency okay, proper time interval and then you know the proper frequency if I can put it that way.

So, ν_t here is less than ν_0 , so that is the frequency that is that will be measured by an observer who is moving transversely to the to the source of light okay. So, that leaves us with two other cases one in which the observer is moving towards the towards the light source and in the other when the observer is moving away from the light source ok. So, let us consider the other one first.

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Either namely the one in which the observer is moving away from the light source okay. So, here to you know what is the proper time between ticks effect it is going to be $1/\nu_0$ where t_0 which is like $1/\nu_0$ that is when ν_0 is the frequency at which the light is being emitted at this source okay. And remember here the observer is moving with a velocity v away from the light source. So, in the frame of the observer if t is the time that you know small t is the time that the observer will measure between each of the ticks so as to say.

Then for in time t remember the observer is also moving with a certain uniform velocity v . So, the observer has already traveled vt that is that is the unit of distance here, away from the source between the ticks ok. So, this implies that the time interval between two successive ticks as measured by the observer ok. Will be what? Will be vt by how much will it increase? It will increase by vt by c where t is the time as measured by, you know by the observer here, right.

And vt by c and c is the velocity of light and that has the same you know that so the value of c is going to be the same in both frames okay.

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- This implies time interval between successive ticks is increased by " vt/c ".
- Total time between the arrival of successive waves is

$$\begin{aligned}
 T &= t + \frac{vt}{c} \\
 &= t(1 + v/c) \\
 &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} (1 + v/c) \\
 &= t_0 \sqrt{\frac{1 + v/c}{1 - v/c}}
 \end{aligned}$$

Now what will be the total time between the arrival of successive waves or the successive light waves in the frame of the observer? It will be first the time between two ticks as measured by, you know by the observer plus the amount of time you know the observer has moved away in a sense by which time I saw was already moved by a certain distance between these two ticks ok. So, the time lapse between the arrival of successive waves will be capital T which is small $t + vt$ by c ok.

Now you simplify this as $t +$ and then take a bracket or $1 + v$ by c ok. Now how is t related with the proper time small t that is small t is related with the proper time t_0 by small t being $= t_0$ by root over of $1 - v$ square by c square and then you have the bracket $1 + v$ by c , so that simply

simplifies to the one given at the bottom of your screen that is t_0 into root over of so $1 + v$ by c divided by $1 - v$ by c .

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$$T = t_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

Thus observed frequency

$$\nu_- = \frac{1}{T} = \frac{1}{t_0} \sqrt{\frac{1 - v/c}{1 + v/c}} = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

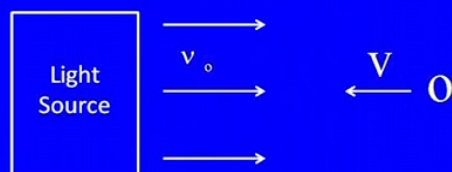
$$\nu_- < \nu_0$$

Now how would you find the you know the frequency here is that all you need to take is the inverse of the time, time interval in this case you see here I put it as V by ν_- - so that is 1 by capital T , now that you just you just invert the expression that you see in the top of screen and then you relate t_0 with the frequency that is the frequency with which the light is being emitted by the light source.

And then you immediately find what is a ν_- okay and then you can relate ν_- to ν_0 and then you see that ν_- is actually less than ν_0 here ok. So, in every case you see that the change in frequency so far we have seen that it is depending on the velocity of the observer ok, the velocity with which the observer is moving away from the light source here ok, so, what about the last case?

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Case 3: Observer moving toward the light source.



- Observer travels 'vt' distance towards the light source between successive ticks.
- Total time between the arrival of successive waves is

So, in this case let us say the observer is moving towards the light source ok. So, so by the same logic which we had applied earlier the observer travels a distance which is v times small t towards the light source between successive ticks of the imagine the clock that we have considered here ok. So, the total time interval between the arrival of successive waves here light waves here would be that is the capital T .

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$$T = t - \frac{vt}{c}$$

$$= t_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

Thus observed frequency

$$\nu_+ = \frac{1}{T} = \frac{1}{t_0} \sqrt{\frac{1 + v/c}{1 - v/c}} = \nu_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$\nu_+ > \nu_0$$

Would be small t and then remember it is $-vt$ by c ok, y - because the observer is moving towards the; it is moving towards the light source and then we have kept the velocity of light is the same in both frames I mean way whether you are in the moving frame uniformly moving frame or in a

rest frame ok. So, that is why it is the same c . So, it is it is natural, now so you do the simplification you know you convert the small t into t_0 , t_0 being the proper time ok.

And here what we will find is that when you take the observed frequency when you when you find the observed frequency it is going to be put ν plus here that is $\nu + 1$ by capital T and we are going to see that $\nu +$ here is more than ν_0 okay. It is going to be more than the frequency of and the emitted light okay. So, we have studied the three cases in which the first case the observer was moving perpendicular to the light source okay.

Then the one was moving away from the light source and then towards the light source. However I should also mention here that in all cases you have seen that enough in all these cases the shift in the frequency as observed by the observer okay depends on the velocity of the observer here okay. So, you do see it in all these in all these formulae here. Another important point that I should mention here is that in each of the formula that you have seen so far I have always considered to light source to be at rest.

Why? Because had I not for example the light source was moving and then the observer was at rest okay that would have been a similar situation. Because it is the relative velocity which is important here unlike the case of Doppler Effect in sound okay. So, it is not only in the Doppler Effect in sound it is not the relative velocity. You have to take into consideration the velocity of the speed of the source and speed of the observer here.

But here what we have it is the relative velocity of between the observer and the source which is going to be important here. So, it is if the source is moving or the observer is moving it does not matter as long because the velocity that we are considering here is actually the velocity v that you see in all the formula will be will always be the relative velocity okay. So, having done this let us spend a couple of minutes on where possible applications of this derivation would be okay.

Well as I told you see in the shift in frequency is related with the relative velocity with which the object is moving okay.

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Applications :

Measuring speeds of distant stellar objects

- Photo of light spectra emitted from the star taken
- Compare with known spectra of elements present in star
- Shift in spectral frequency – related with speed of the star

Temperature of hot plasma in nuclear fusion experiments

So, that immediately told people that maybe it could be used to measure the speeds of distance Taylor objects okay like stars. So, what do what do people do? What do astronomers do in that case? What they do is that they take a photograph of you know the spectra of elements which is in the star okay. So, from the light which is emitted from the star the light spectra is analyzed. So, and you know where the frequencies are people find out okay.

And then they compare it with the known specter of elements present in the star ok. So, these known spectra of elements you can find the spectera of elements by some experiment in the laboratory here on earth. And then they compare that spectra with the spectra that was obtained from this star. Now if the star is moving away or you know there is a relative velocity it is there is a relative motion between earth and the star there.

And then the star we are going to have a shift in spectral frequency okay. Now from the shift in spectral frequency you can then find the speed of the star okay. So, it is it is got lots of applications in astronomy of course. Then many other you know some other discoveries were also based on this okay. Another application actually a very interesting application of this would be to measure the temperature of hot plasma in nuclear fusion experiments or ok the temperature of very hot gases okay.

Now what they do is that well the principle is almost similar you know to what had, what I have outlined earlier it is that they take the spectra of you know when things are moving at such high speeds so they are emitting radiation and this radiation is analyzed and then if it is a known gas. So, the from the known spectra you can find out what is the shift in the spectral frequency okay.

And then you and then you can relate that with the speed of the molecule or the gas so which is there inside your experiment. And then if you relate, if you if you remember your kinetic theory of gases. So, from this velocity or you know the mean velocity of the root mean square velocity you can always relate that to the temperature okay that that can always be done from the kinetic theory okay. So, that will give an estimate of the temperature involved here okay.

So, we have studied the Doppler Effect in light as against it you know that now as against Doppler Effect in sound ok. And then we have also talked a bit about few applications. Fine, so now we are going to shift gear a little bit and do something a little bit different and go to another consequence of special relativity.

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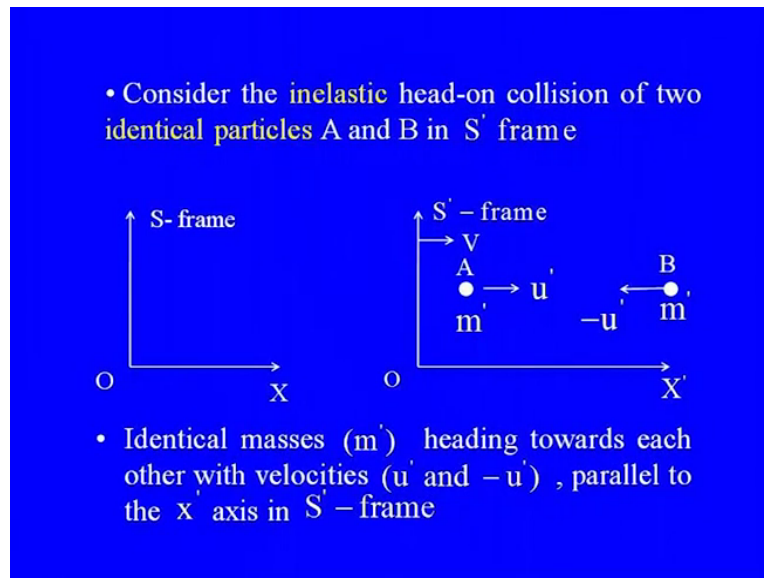
Does the mass of a body vary with velocity ?

And study if the mass of a body actually varies with the velocity with which it is moving when I know that this is one of the things that many of many of us are rather aware of when we study physics or any of the elementary courses of science fatigue mass increasing with velocity and E

= mc^2 ok. But all of it in due time ok. Now to do this, do this problem of where the mass of a body varies with velocity.

Let us start with up with an example ok and then we will come to a conclusion ok. So, concentrate on the on the on the thing written down as S prime frame ok.

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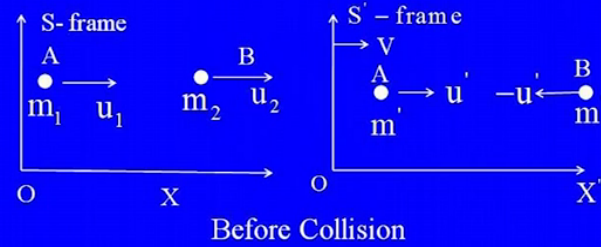


So, what we have here is that we are considering the inelastic head on collision of two identical particles A and B in S prime frame let us see. So, what is this S prime frame well it is a frame which is let us say it is moving with a certain velocity, certain uniform velocity v with respect to another frame S let us say okay. Now in it we have the inelastic collision of two bodies here named as A and B so they are of identical masses, m' both are of m' and they are moving with the same speed you know opposite to each other.

And then they are heading towards and head on collisions mean speed is u' here. The wall put prime's here because you know to keep in to keep you know in to make since we are talking of s prime frame. So, we are dealing with prime coordinates here let us put it that way okay. And then all these velocities u' you know they are moving parallel to the x axis okay. Fine on the way are identical masses also.

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- How is the event observed in S- frame?



From Einstein's velocity addition formula,

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad \text{and} \quad u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}}$$

Now how is the same event observed in S frame okay. Well you have different below, you have a you know in the S frame you going to see the same thing as some you know the velocities may not be the same the velocity is although they are moving the speeds are all the same but if you take the modulus of the speeds in S prime frame they are the same the directions may be different but the speeds are the same.

But then the velocities of the speeds are not going to be the same in S frame a simple reason it is that you need to invoke Einstein's velocity addition formula to find what is the velocity in S frame okay. And you go to see immediately that it is not going to be the same it is these written in you know in, for your convenience at towards the end of the slide. And for example if you just consider you one of the mass of the velocity of mass okay one mass A rather okay.

You are going to see in terms of the S prime quantities and then the velocity v its $u' + v$ divided by $1 + \frac{u'v}{c^2}$. However if it was $-u'$ since was moving in the other direction okay u_2 that is the other you know the other quantity that is $-u' + v$ divided by $1 - \frac{u'v}{c^2}$. So, I am going to be the same okay. So, you might be a little bit confused why you have written a different masses for identical masses ok.

So, in the S prime frame you had identical masses both of m_p okay. But why is it that they are different in the S frame. Well at this stage I do not know why they are different rather I just take

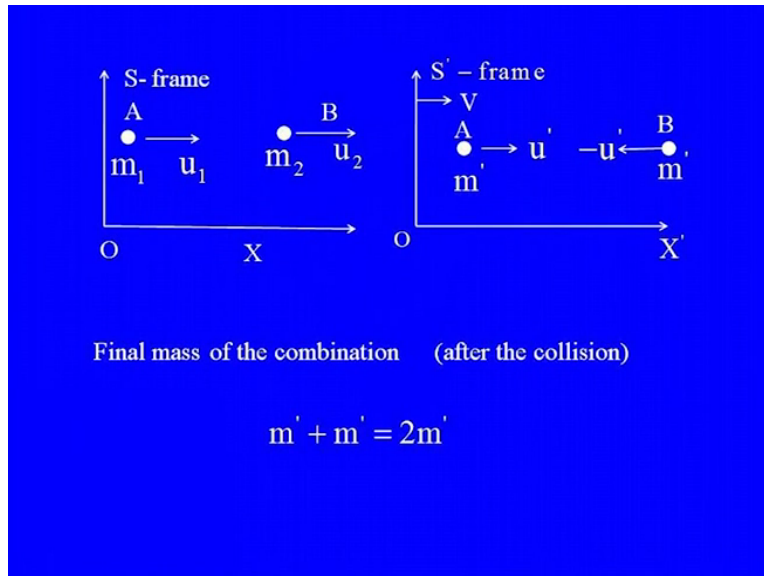
them to be different so just to keep myself in conjunction with the idea that I have different velocities okay I do not know what is going to the masses, so if it is if my y relief from my relativity from whatever I do it turns out from all this conservation laws a marine work on if it turns out that these masses are the same they will turn out to be same m_1 will be $= m_2$ but right now I am going to take m_1 and m_2 different ok.

Let us just see what happens okay. So, that is what we have we have the S prime frame and then in the S frame we are going to see what it is going to be of the same even the same collision how is going to how one is going to see in S frame. And remember S prime frame is moving with a certain uniform velocity v with respect to the S frame ok. So, on top of that of course on the back of our minds we have the conservation of linear momentum in our in the collisional process.

Well this is rather something very sacrosanct I mean we are not going to violate all these conservation principles ok whether it is non relativity or relativity of classical mechanics I mean the conservation laws you are supposed to you are supposed to con you supposed to maintain them okay. And then on top of that you also have the total mass to be conserved that the total variable mass you know the total mass.

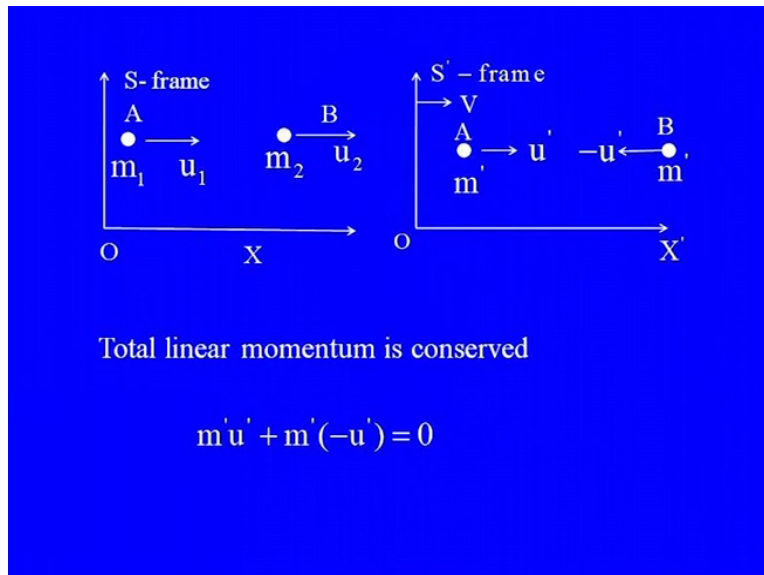
For example if it is considered S prime frame you have m prime A and then the other mass B also of m prime.

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So, after collision the mass is going to be m prime + m prime that is 2m prime that is what I mean the total mass remains conserved in the collision process in each of these frames okay. So, so what does it mean here well from the linear momentum conservation what we have in this conditional process, if you look at the S prime frame.

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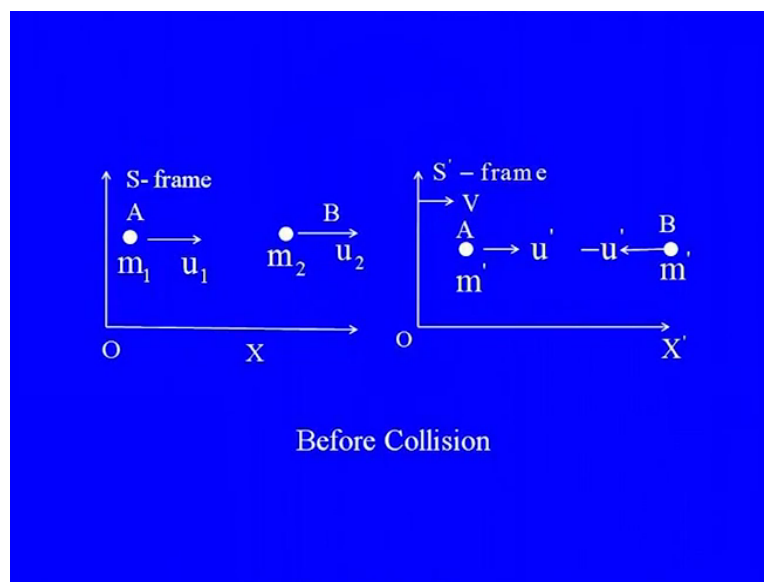


It is very simple it is going to be 0 is not a terminates they are moving that you have an identical masses the speeds are identical and opposite to each other moving parallel to the x and x prime axis. So, u prime m prime u prime + m prime - of u prime that is going to be 0, so, momentum so even if the, if so that tells you that you are going to have an inelastic collision in the end. So,

you know the mass is going to whatever it is the velocity or it has to be at rest in the S prime frame after the collision something of that later on.

But what about this to the final mass of the combination there is final mass of the combination m prime + m prime it is going to be $2m$ prime there is nothing rocket science in that but the only thing is that it is going to be maintained after collision that is what I want to say ok. So, that is what we have the thing ok. So, just one look one more slide to recapitulate on this collisional process before the collision okay.

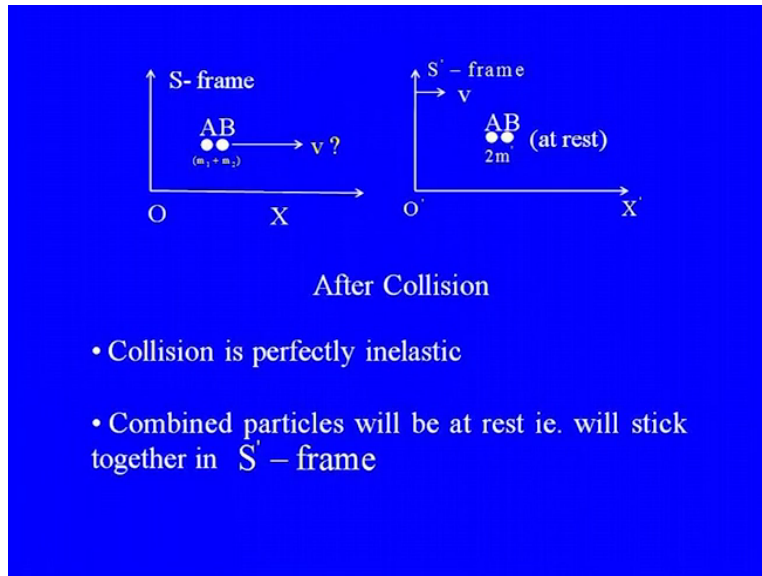
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So, we have the S prime frame where we have identical masses A and B okay they have similar masses they have the same mass m prime and they are moving opposite to each other parallel to the x prime axis. And then one is observing the same thing from the S frame and then the S prime frame is moving in a certain velocity v with respect to the S frame. And from the S frame we are going to see the same collisional process.

But then the velocities are not going to be different and then we have taken different masses here we do not know what we do not know whether the masses will be same or different but let us see if from all these conservation laws, what turns out to be? What whether we have indeed different masses whether m_1 a separate is different from m prime ok.

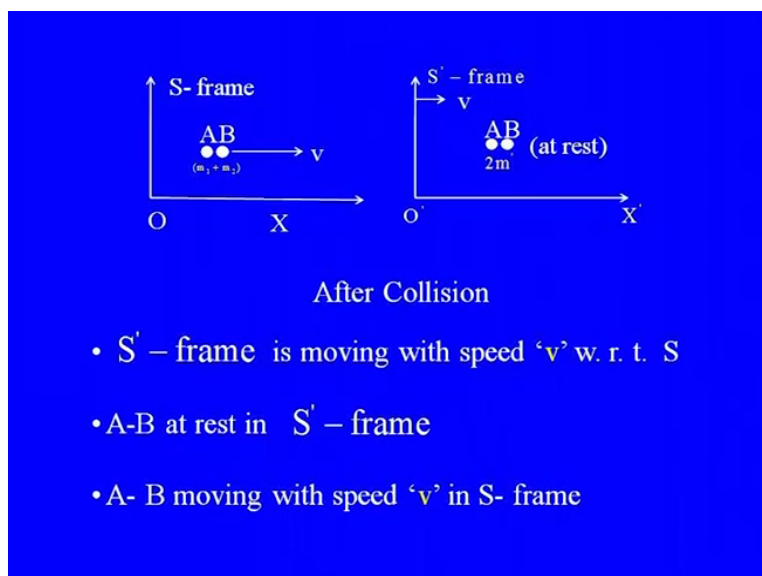
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So, what happens after collision remember the collision is perfectly inelastic. So, what it means is a combined particle the combined particles will be at rest and I will stick together in the S' prime frame. So, that is what they are they will stick together in S' prime frame the total mass is $2m$ prime and they are at rest. Now what will be the velocity with which these things will be seen to move from S frame ok? So, is it v we are going to find that out ok?

But remember here what will be the total mass of this AB system here at in S frame it is going to be $m_1 + m_2$ ok, his total mass is conserved in this coalitional process right. So, whatever the velocity as we said so remember after collision will still after collision.

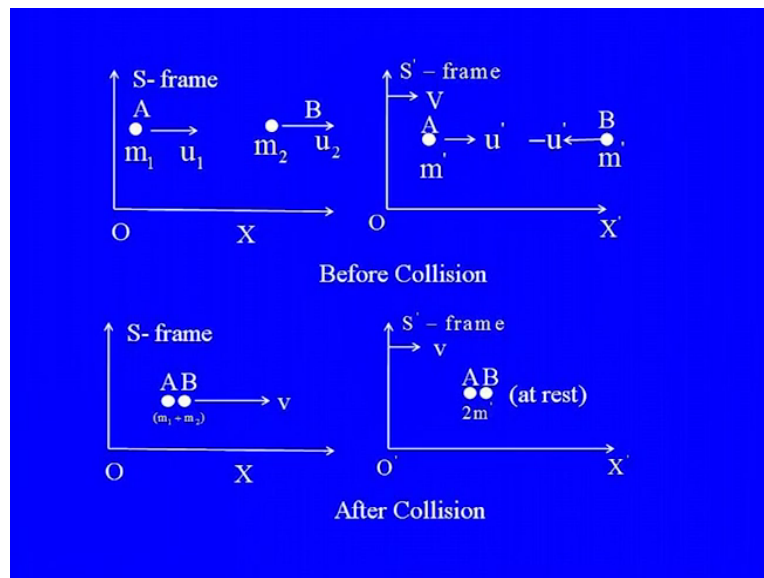
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We have we are still considering s prime frame to be uniformly moving with a certain speed v with respect to the air spray. Now in S prime frame A and B is at rest right is at rest. So, what does it tell you and then the s prime frame is moving with velocity v with respect to S. So, what is the velocity with which A and B will be seen to be moving from S frame. Well it is B that is what we have so in S frame we have these two particles they are sticking together okay.

Since it is sticking together prime frame that is sticking together is going to have mass $m_1 + m_2$ and it is going to move with a certain velocity v which is same as the relative velocity between S and the S prime okay, so, just to recapitulate the entire collision process.

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What you see in the top of the slide is the situation before the collision okay. Both in S prime frame and the S frame just have a nice look on the situation what is happening in the S prime frame and the situation that we have in the S frame okay. That is the situation before collision and then after collision it is perfectly inelastic collision. So, it is sticking together in s prime frame and total mass is $2m$ prime and it is at rest.

And then from S frame we are going to have the total mass to be $m_1 + m_2$ that total mass is conserved here and is moving with velocity v okay. So, next what we are going to do is that we are going to invoke the conservation of momentum in the S frame. Remember if you invoke the

conservation of momentum in the S prime frame it is going to be 0 is not it. I mean the total momentum total linear momentum in the S prime frame is 0 okay.

And what is it if you invoke that same conservation of momentum in the S frame what is it going to be okay.

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Conservation of linear momentum in S- frame

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

Rearrange terms

$$\frac{m_1}{m_2} = \frac{v - u_2}{u_1 - v}$$

Substitute u_1 and u_2 from velocity addition formula

$$u_1 = \frac{u' + v}{1 + \frac{u' v}{c^2}} \quad \text{and} \quad u_2 = \frac{-u' + v}{1 - \frac{u' v}{c^2}}$$

It is going to be $m_1 u_1 + m_2 u_2$ that is the total momentum in one in the S frame before the collision and then after the collision it is going to be $m_1 + m_2$ whole times v that is the velocity with which this combination if $A + B$ or the same $m_1 + m_2$ is moving ok. Rearrange the term do as little simplifications immediately get what $m_1 + m_2$, is $m_1 + m_2$ is nothing but $v - u_2$ divided by $1 - v$ well that is very simple you just take all the m_1 's, you know all the terms involving m_1 's on one side all the terms involving m_2 's and then do simplifications okay.

And then what you do is that you substitute for u_1 and u_2 the expressions from Einstein's velocity addition formula. So, you know that very well, so u_1 is $u' + v$ divided by $1 + \frac{u' v}{c^2}$ and what is u_2 , u_2 is $-u' + v$ divided by $1 - \frac{u' v}{c^2}$. So, you want to substitute these two velocities u_1 and u_2 okay in the expression given in the middle of your screen, and then given by this m_1 by m_2 .

Well I am doing a little bit of the algebra a bit rather little bit of the algebra which only but if you miss this slide other next not going to miss the most of the physics okay. I am just doing it so that you will have the expression you can you can derive all these things by yourselves okay. So, what I have done is that now what will be $m_1 + m_2$ in terms of all the quantities in the prime frame okay.

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$$\frac{m_1}{m_2} = \frac{1 + \frac{u'v}{c^2}}{1 - \frac{u'v}{c^2}}$$

Again

$$\frac{1 - \frac{u_1^2}{c^2}}{1 - \frac{u_2^2}{c^2}} = \frac{\left(1 - \frac{u'v}{c^2}\right)^2}{\left(1 + \frac{u'v}{c^2}\right)^2} = \left(\frac{m_2}{m_1}\right)^2$$

So, it is $1 + u'v/c^2$ divided by $1 - u'v/c^2$ again what we do is that ok so we have derived up to the expression the top of the slide. Again what we see is that a little more simplification actually you can derive this if you invoke again the Einstein's velocity addition formula $1 - u_1^2/c^2$ and what is $1 - u_2^2/c^2$. So, you put in what is u_1 and u_2 in terms of u' and v 's and all these things.

You see rather very interesting expressions, you can you can you can actually check that out okay. It is $1 - u'v/c^2$ whole square divided by $1 + u'v/c^2$ whole square. Now from the top of the slide you can see that this is nothing but m_2 by m_1 whole squared ok. Now check what have check the one on the left hand side okay. So, it is $1 - u_1^2/c^2$ divided by $1 - u_2^2/c^2$ and then on the extreme right hand side you have this m_2 by m_1 and whole of square.

Is very interesting these are the two expression of the left hand side and the right hand side disregards the one in the middle okay. What do you see you see the masses you see the velocities okay. What do you again see?

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Therefore

$$m_1 \left(1 - \frac{u_1^2}{c^2} \right)^{1/2} = m_2 \left(1 - \frac{u_2^2}{c^2} \right)^{1/2} = \text{Invariant}$$

- m_1 and m_2 are masses of identical particles when their velocities are u_1 and u_2 , respectively

What you see is m_1 into $1 - u_1^2$ by c^2 and you take a square root of that is equal to m_2 into $1 - u_2^2$ by c^2 take square root of that and that is invariant and that is not changing okay. So, what you see is that what you have here, so m_1 and m_2 are the masses of identical particles okay. When the velocities are u_1 and u_2 respectively I mean when I say m_1 and m_2 are masses of identical particles.

And you know does not carry much of a meaning and then you could say that identical particles how come they have different masses okay. But you see that they are moving at different speeds okay. So, this combination some rather interesting combinations of m_1 and u_1 of these speeds with which they are moving and then use that combination given by m_1 times $1 - u_1^2$ by c^2 that or that square root of that.

That is the thing that is going to be invariant. Change the velocity the expression for the mass changes okay that is becoming invariant okay. So, what is invariant is this combination but in whatever you know whatever mass and then its corresponding velocity that is possible here.

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- So in a frame if 'v' is the velocity of the particle, and the mass is 'm', then the quantity

$$m \left(1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} = \text{Invariant}$$

So, what we have; so in a frame if v is the velocity of the particles v is the velocity of the particle and m is its mass then the quantity that will have m into 1 - v square by c square root over of that, that is going to be invariant okay. Now this is very interesting, why? Now watch this well rather what should be the invariant quantity? Let us check that out? Now if you take the velocity with which the mass is moving to be 0.

So, it is basically the mass is at rest, so you consider you considered a frame in which the particle is at rest and then you measure the mass there.

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- So in a frame if 'v' is the velocity of the particle, and the mass is 'm', then the quantity

$$m \left(1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} = \text{Invariant}$$

Now if the mass is m_0 there okay, so you plug in those quantities into the formula so m into $1 - v$ square by c square, so the mass is m_0 and the velocity is 0, see the one in the bracket $1 - 0$ square by c square root of that, so that is going to be 1, no problem and then the right hand side you have m_0 . So, that is going to be invariant quantity, m_0 is a mass of the same particle in a frame in which it is at rest.

So, that is the formula we are looking for see that if you know that the invariant quantity is actually m_0 here. You can now relate the mass of a body which is moving with a certain velocity v .

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Thus

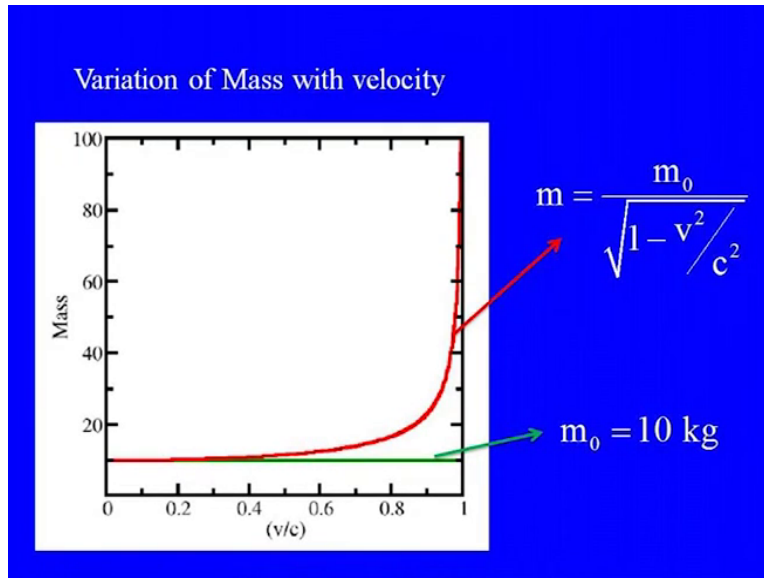
$$m \left(1 - \frac{v^2}{c^2} \right)^{1/2} = m_0 \left(1 - \frac{0^2}{c^2} \right)^{1/2} = m_0$$

or

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As $m = m_0$ which is the mass of the body in the inner frame in which is at rest and then divided by root over of $1 - v$ square by c square. So, so what we have is the mathematical formula and then this is very interesting of what the mass of the body will be, if it is moving with a certain velocity v ok. Now let us see how this is related. So, let us have a graphical illustration of this variation of mass and velocity.

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It is if you if you look at at the graph on your screens what I have plotted here on the y axis is the mass and on the x axis I have v by c ok. But is basically it is it is the velocity in the units of velocity of light. So, it is someone you know it is starting from 0 to the speed of light. So, when v by c is one it is the speed of light. So, okay and when v by c is 0 that you know that the body is at rest okay.

Now if we consider a mass of you know of which is of at rest which is m_0 to be off let us say 10 kg okay. Now that is the green line okay, so that is the green line and so whether if you increase the velocity or not so that is m_0 it is the rest mass so that is 10 kg. But as you increase the velocity okay see the red curve see that at small speeds when the velocity of when the velocity of the body is not too high or it is too high not too high compared to the speed of light.

You see that it is increasing and then all and then when it when it reaches towards the speed of light you see all of a sudden it increases very fast I mean is it is almost explodes okay. So, you do see that the mass of the body is increasing in a certain way when the velocity is increased okay. Now I also wish to remind you one more thing is that when you are at very small speeds when on basically when you are non relativistic speeds okay.

When v is much, much less than c you can immediately figure out look at the formula for example let us first look at the formula.

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Variation of Mass with velocity

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

m_0 : Rest mass of a body (also an invariant quantity like proper length or proper time)

So, $m = m_0$ divided by root over of $1 - v$ square by c square. So, when v is much, much less than c , v by c almost tends to 0, so, the denominator in this fraction is going to be 1 okay. So, m will roughly be equal to m_0 when the velocity is much, much less than the velocity of light and that is exactly what you see in the graph ok. So, when velocity is much, much less than the velocity of light.

You have you are in the non relativistic limit in the classical limit, the classical world that we all live in okay. You see that the red line actually converges with the green line, so it actually fits with our experience okay. So, now let us see what have another example of what the consequences of this of this formula would be. Let us just consider the, you know the rest mass of an electron okay.

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Example: The rest mass of an electron is 9.11×10^{-31} kg. Supposing that it is moving with a velocity $0.8c$, what would be its mass?

$$m_0 = 9.11 \times 10^{-31} \text{ kg.}$$

$$\text{Velocity } v = 0.8c$$

The rest mass of an electron if you look at the tables it is something like 9.11×10^{-31} kilograms. Now suppose this is moving with; it is a pretty high velocity it is eighty percent the speed of light ok. So, will its mass be equal to, you know how much will it what we must be will it be a very near to 9.11×10^{-31} kilograms. Let us see so that is the rest mass and the velocity v here is 80% of the speed of light.

So, what you see here will be m , so that is the measured mass will be equal to m_0 divided by root over of $1 - v^2/c^2$ ok. Now m_0 you know what that is 9.11×10^{-31} kilograms ok. And then $1 - 0.8^2$ and c^2 divided by c^2 ok. Now this turns out to be something like 1.5×10^{-31} kilograms. So, you do see an increase in mass ok when you when the velocity is increasing.

And that too when the velocity is much, much; well which it is actually 80% the speed of light is quite a huge amount of which is quite a large velocity ok. So, by hope today I have you know you be able to cover some part of the consequences of special activity. We talked of Doppler Effect in light and then the interesting phenomenon of the mass of a body varying with its velocity.

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Next topic :

mass and energy

In the next section what we will talk off is the relation between mass and energy okay. And I will heavily draw upon what we have done so far about the consequences of relativity specially about the variation of mass with its velocity. Well thank you very much.