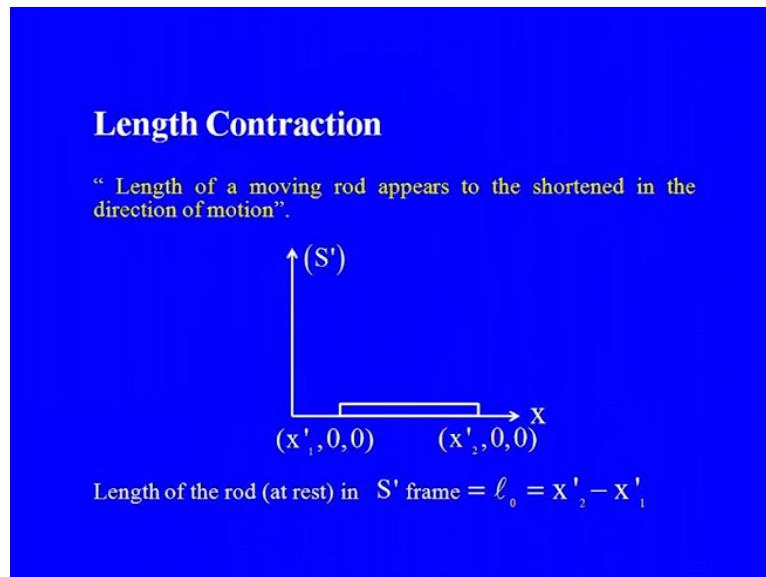


**Engineering Physics 1**  
**Dr. Rajdeep Chatterjee**  
**Department of Physics**  
**Indian Institute of Technology-Roorkee**

**Module-07**  
**Lecture-03**  
**Introduction of Special Relativity - II**

Hello, everybody, so today we come over to the third part of lecture on special relativity and having done the postulates of relativity and seen consequences like length contraction. Let us move over to another interesting consequence that is called a time dilation okay. So, just to set up the thing a little bit more let us just have the results which we had in our last lecture and it is mainly on line contraction.

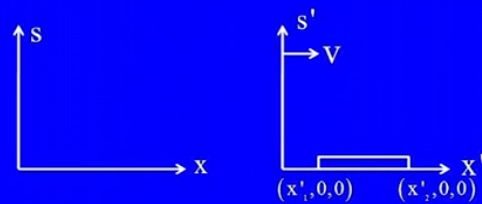
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So, what did you see there? So what we saw there was if you have borrowed you know if you have some object at rest in a certain frame. You would not measure its length okay find that the length of this rod let us say it is 10 okay. Now if you go and measure this in a different frame from which this rod appears to be moving okay with a uniform velocity. The length of the moving rod appears to be shortened in the direction of its motion okay.

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- Length of the rod,  $\ell$ , as seen from the S frame



$$\ell = \ell_0 \sqrt{1 - v^2/c^2}$$

Of course so, so that the result that we got so if you see the length of this object is rod from S frame okay, remember that the rod is at rest in the S prime frame which is moving with a certain uniform velocity v. Then what is the length that we are going to see in the S frame its 10 times the root over of 1 - v square by c square. So, obviously 1 is less than 10 okay. So, the length of a moving rod or a moving object appears to be shortened as compared to the measured length in its rest time okay.

So, well so having seen something to do with the space coordinates I mean that is the length contraction. What we do I mean what is it that we have with the time component in special relativity. Does something happen there okay remember in Galilean transformation of course time in both these frames in S and S prime frames are the same and they are moving with a certain uniform velocity with each other.

But is it so in special relativity if you if you remember your Lorentz transformations, so there is space component even in the time vision okay. And that in a sense leads to a another interesting concept in a consequence in special relativity it is called time dilation okay.

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## TIME DILATION

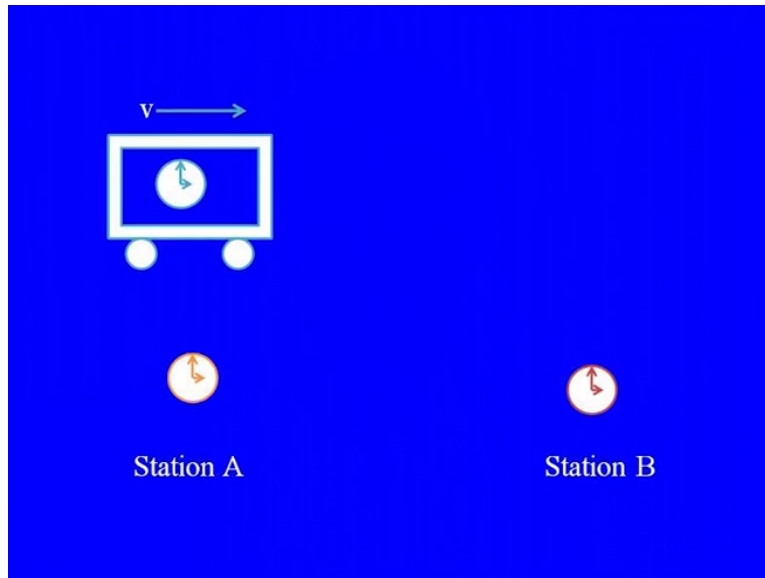
- Moving clock appears to run slow. Time “expands” for a moving body.
- Ex. Let a person move in a train which is moving uniformly with velocity ‘ $v$ ’. When the person in the train *matches* his own watch with the station clock he will observe that the times are not matching.

So, what is it way in what is the in long and short of it is just moving clocks appear to run slow okay. So, it is nothing to do with how good the machine inside the clock was it is the concept of physics here okay or in other words the time expands for a moving body okay right. So, let us take an example by short one what how do you how do you measure such a thing, how do you get the feel of such a thing of things like time dilation okay.

So, if you consider a person who is inside a train and the train is moving with a certain velocity  $v$  okay with respect to the station, of course, station or stations in this case okay then for the person who is sitting in this train he matches his own watch I mean this he has synchronized his watch with one of the station clocks earlier okay. Let us say then in the next station when the train passes, he looks at the station clock and then he will see that the times in his watch and the station clock will not match okay.

So, that is again what is meant by you know an example of a time dilation but let us have a more, more pictorial example of this of this concept okay, so, which will clarify it a little bit more.

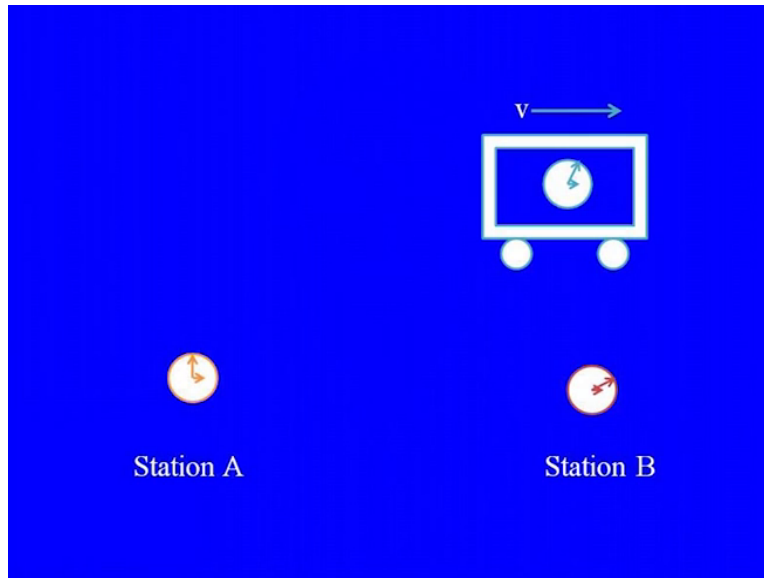
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See that you have two stations and then you have a synchronized clocks in these two stations okay you synchronize in a sense that so they give the same they same that the time so synchronized okay. So, so well I should say that if you have two stations you need two clocks there okay. So, we have our two clocks one on the on the left of your screen and one of the right of your screen as written as station A and station B.

And then their clock stay there keeping fine then what happens suppose let us say a person moves with a certain velocity you know the uniform velocity  $v$  okay. You know in a train coach and then he has a watch on his wrist let us say so he is talking of a wristwatch and then he checks his time in distress what only adjusted according to the to the time in station A, so that he synchronizes his clock with station A okay.

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Now when he passes station B okay what he will observe is that the time or the whatever time it is in his wristwatch and whatever time it is in station B and in station clock they will not match okay. So, this is a more toriel way of saying the thing okay. Well why is it so? Now for that we have to be a little bit more quantitative.

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- Consider two events ( $E_1$  and  $E_2$ ) seen from two different frames  $S$  and  $S'$  (which moves with some uniform velocity ' $v$ ' with respect to  $S$ )

<u>S-frame</u>	<u>S'-frame</u>
$E_1: (x_1, y_1, z_1; t_1)$	$(x'_1, y'_1, z'_1; t'_1)$
$E_2: (x_2, y_2, z_2; t_2)$	$(x'_2, y'_2, z'_2; t'_2)$

And so let us consider two events explain what these events are let us just define what is the space time coordinates of these events first? Okay. So, so we consider two events we call it event 1 and event 2 seen from two different frames of reference  $S$  and  $S$  prime okay. And as always we take the  $S$  prime frame that is moving with a certain uniform velocity with respect to  $S$  okay.

If the space time coordinates of these two events in either of these two events in S frame okay that is  $x_1, y_1, z_1$  and  $t_1$  so that is measured at  $t_1$ . And then the second event occurs at position  $x_2, y_2, z_2$  and at time  $t_2$  ok and correspondingly these two events occurs in the in the primed frame with  $x_1$  prime,  $y_1$  prime and  $z_1$  prime at this position and time  $t_1$  prime and then at the second event occurs at the positions  $x_2$  prime,  $y_2$  prime,  $z_2$  prime at time  $t_2$  prime.

Then what we have or if you want to be a little bit more specific, now if you wish to define these events in terms of our train example.

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- Let us be more specific and take our train example.

Let the frame in which the person with his clock is at rest be  $S'$  which is moving with uniform velocity relative to stations A and B fixed in frame 'S'

- When the person passes station 'A' call that event as  $E_1$  and similarly when the person passes station 'B' call that event as  $E_2$

We take the frame in which the so first we define what our S frame and this S prime frames are okay. So, the frame in which the person with his clock is at rest is the S prime frame okay. Now this frame so it is moving with a certain uniform velocity relative to stations A and B okay. And these are fixed in frame S okay. So, now you define what our frames are. So, the stations are so you somebody's watching from the station, so that is S frame okay.

And then you and then the person in the train sitting in the train looking at his wristwatch okay so that is the S prime frame. So, so when the person passes station A let us call that as event 1 okay so it is got a certain position at certain time okay. And similarly when the person passes station B let us call that as event 2. Seen from two different frames of course the positions are different and the times are also different okay.

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- In  $S'$ -frame (in which the person with his clock is at rest) the events  $E_1$  and  $E_2$  has occurred at the same space coordinate, but at different times.
- Thus  $x'_1 = x'_2$ ,  $y'_2 = y'_1$ , and  $z'_2 = z'_1$ , but the “times” are different.

So, what we have remember in the  $S$  prime frame in which the person with his clock is at rest the events  $E_1$  and  $E_2$  has occurred at the same space coordinate when the different times, why is that? You see he is at rest within the within the train carriage, let us say and then he just looks at his watch at the same position. So, his position coordinates are the same okay but then he from his point of view he has looked at it at different times and that is why he gets different times okay.

And just to keep the mathematics a little bit simpler he also take, you know so moving in the common  $x$  and  $x$  prime direction. So, that the  $y$  prime's are also  $y$ 's and  $z$ 's are also the same here but in terms of coordinates in the  $S$  prime frame we have  $x_1 \text{ prime} = x_2 \text{ prime}$  and then  $y_2 \text{ prime} = y_1 \text{ prime}$  and  $z_2 \text{ prime} = z_2 \text{ prime}$ ,  $z \text{ prime} = z_2 \text{ prime}$  that is it but the times are different okay. So, what are these times quote unquote the times in  $S$  frame that is seen from the station okay?

So, from the unprimed frame if we call this time  $t_1$  ok, so we just have the Lorentz contraction equations or Lorentz transformation equations to be more precise here;

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In fact by Lorentz transformation

$$t_1 = \frac{t_1' + vx_1' / c^2}{\sqrt{1 - v^2 / c^2}} \quad t_2 = \frac{t_2' + vx_2' / c^2}{\sqrt{1 - v^2 / c^2}}$$

Time interval between the two events according to the observer in S is,

$$\Delta t = t_2 - t_1 = \frac{t_2' - t_1' + \frac{v}{c^2}(x_2' - x_1')}{\sqrt{1 - v^2 / c^2}}$$

So, so  $t_1$  that is  $t_1'$  +  $vx_1'$  by  $c^2$  divided by root over of  $1 - v^2$  by  $c^2$  and  $t_2$  is  $t_2'$  +  $vx_2'$  by  $c^2$  into root over of  $1 - v^2$  by  $c^2$  ok. So, the time interval between these two events according to the observer in S ok, so, according to the measurement I made in S frame so that is  $\Delta t$ . Let us say so what is  $\Delta t$  now.

So,  $\Delta t$  is just  $t_2 - t_1$  okay and that is  $t_2' - t_1' + v$  by  $v^2$  into  $x_2' - x_1'$  prime divided by the usual the denominator root over  $1 - v^2$  by  $c^2$  ok. Now what is the time interval according; so that is the time interval of these two events according to the observer in S frame ok. But in the S prime frame remember the positions of these two events were the same ok. So, we have ah  $x_2'$  and  $x_1'$  are the same. So  $x_2' - x_1'$  see you okay, so that will give us what?

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Time interval between the two events according to the observer in S is,

$$\Delta t = t_2 - t_1 = \frac{t_2' - t_1' + \frac{v}{c^2}(x_2' - x_1')}{\sqrt{1 - v^2/c^2}}$$

But  $x_2' - x_1' = 0$

Thus  $\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \quad (t_2' - t_1' = \Delta t')$

$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2}$$

That will give us that this delta t is delta t prime, so we have defined delta t prime as the time difference as measured by the observer in the S prime frame. Remember the S prime frame is the one in which the person sitting in this train carried is looking at his watch okay. So, what we get? So what we get is delta t prime is delta t into root over of 1 - v square by c square okay. So, just if you get a feeling that you use immediately see that delta t prime is actually less than this delta t here okay.

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$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2}$$

• Time interval measured by the moving clock relative to S will be smaller than the time interval measured by the clock stationary in S.

• “Moving clocks run slower”

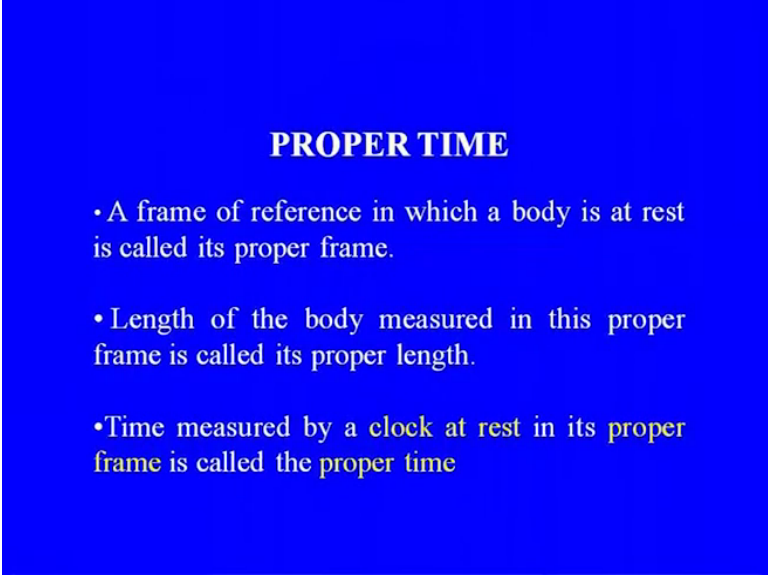
• However for  $v \ll c$ ,  $\Delta t' \approx \Delta t$

Now what does it mean physically it means that the time interval measured by the moving clock relative to S will be smaller than the time interval measured by the clock stationary S okay. So, in other words what we have is it is confirmation of the statement with which we started with that

moving clocks run slower. However I should remind you that if the velocity is much, much less than the speed of light then you will immediately see that these two time intervals are the same okay.

So, they are approximately the same, so, next time you come from let us say Delhi to Roorkee and then you look at the station clocks supposedly I mean you are moving with a uniform velocity and all these things with respect to the station in all cases. You should not you well you should not and you would not see this distinct this time dilation okay. So, so having seen this thing let us have a look at what proper time is okay.

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**PROPER TIME**

- A frame of reference in which a body is at rest is called its proper frame.
- Length of the body measured in this proper frame is called its proper length.
- Time measured by a clock at rest in its proper frame is called the proper time

So, what is the proper frame was a proper frame of reference, so a reference frame in which body is at rest it is called its proper frame we know that. And we have also measured we know what a proper length is. So, the length of the body measured in this proper frame is called its proper length. So, the time measured by a clock at rest in the proper frame is the proper time okay. So, just like the proper length is the proper time invariant.

Well I think it is but let us just have a look at it once again okay. So, what is so the time interval so what is the proper time interval? To be more precise;

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- Time interval measured between two events by a *single clock at rest* at the same place is the **proper time interval** between the two events.
- If this time interval is measured from another frame, having a relative velocity with respect to the proper frame; *two clocks will be needed* at different places. This time interval is **non-proper**

So, it is the time interval measured between two events via single o'clock at rest at the same place is the proper time interval between two events. We just had a example of a person sitting in a train looking at his watch at two different stations. So, that is an example of you know position is not changing according to him in he is sitting in the same place in a carriage, so his position is not changing and then he looks at his own watch.

So, that is a single clock and he is sitting at rest in the train. So, according to him he is measuring the proper time okay by a single clock. However I mean at the same time interval you know am I going to measure it from another frame having a relative velocity with respect to the proper frame. So, you need two clocks at different places so that is exactly what we had we had two stations and two clocks and then they had to be synchronized a beginning.

Well in the sense this time sometimes is also this kind of time is also time interval rather is also called non proper okay.

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- Let the velocity of the second frame be 'v' relative to the proper frame with proper time interval  $\Delta\tau$  between two events.
- The non-proper time interval  $\Delta t$  measured from the second frame (by two different clocks) is given by

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-v^2/c^2}}$$

$$\Delta\tau = \Delta t \sqrt{1-v^2/c^2}$$

So, if the velocity of the second frame we another frame be v relative to the proper frame with proper time interval delta Tau between these events then the non proper time interval delta t measured from the second frame of course by subtracting the time measured from two different clocks will be delta Tau divided by root over of 1 -v square by c square okay.

And so what is this delta Tau that is the proper time. So, it is delta t into root over of 1 - v square by c square okay.

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- So, in another frame; moving with velocity v' with respect to the proper frame if the time interval between the same two events is  $\Delta t'$ , then

$$\Delta t' = \frac{\Delta\tau}{\sqrt{1-v'^2/c^2}}$$

Now if in another frame in which it is moving with the velocity v prime its respect to the proper frame. If the time interval measured between the same two events is delta t prime then obviously

delta t prime is again delta Tau divided by root over now v prime squared by v squared. So, you immediately see that if you just write out the delta Tau here.

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- So, in another frame; moving with velocity  $v'$  with respect to the proper frame if the time interval between the same two events is  $\Delta t'$ , then

$$\Delta t' = \frac{\Delta \tau}{\sqrt{1 - v'^2/c^2}}$$

The delta Tau is nothing but delta t into root over of  $1 - v^2/c^2$  that is the time corresponding to this; in a measured in the frame, the time interval measured in the frame it is moving with a certain velocity  $v$  then  $t$  prime this time interval measured in the frame which it is moving with a certain speed  $v$  prime. So, that is equal to delta Tau okay and so delta Tau is invariant quantity okay.

So, the proper time just like the proper length is an invariant quantity okay. So, let us look at an example okay let us look at some examples here on these consequences of time dilation, proper time and see what else we can do from here okay. Let us take a simple but interesting sample. So, you know a Muon is that is a subatomic particle okay.

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Example:

- A muon is observed to move 800m on an average during its lifetime in the laboratory. The mean lifetime of a muon is also known to be  $2 \times 10^{-6}$  seconds.
- Does that mean that the speed of the muon is

$$v = \frac{800\text{m}}{2 \times 10^{-6}\text{s}} = 4 \times 10^8 \text{ms}^{-1} > c!!$$

And a Muon is observed to move approximately something like 800 meters on an average during its lifetime in a laboratory okay. So, if one looks up the mean lifetime of a Muon from one of these particle physics books are these data tables. You see that it is something like 2 into 10 power - 6 seconds okay. So, if we just go on write the velocity as the distance covered and then its mean lifetime it is okay 800 meters by 2 into 10 power -6 seconds we end up with 4 into 10 to the power 8 meters per second okay.

This is much bigger than the speed of light okay. Now this is this is Harrison in special relativity and what do we do, I mean obviously we have not considered something. So, what is that?

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- One needs to consider that the mean lifetime of the muon that we quoted is actually its “Proper lifetime”.
- So the “improper lifetime” of a muon as measured in a frame moving with velocity ‘v’ with respect to the proper frame is

$$\Delta t = \frac{2 \times 10^{-6} \text{s}}{\sqrt{1 - v^2/c^2}}$$

Well one needs to consider and that what is what given as mean lifetime of this Muon is actually its proper lifetime okay. So, when you are measuring the length in the laboratory okay. So, you need to divide this by this quote unquote improper lifetime. So, the measurement of the time and the measurement of the distance this Muon has travelled should be done in the same frame that is it okay.

So, so this quote unquote improper lifetime of this Muon has measured in a frame moving with this velocity  $v$  with respect to this proper frame. So, what is that? So, if you remember this few slides back we talked of the proper time interval that is divided by root over  $1 - v^2/c^2$ . So, the proper time here is  $2 \times 10^{-6}$  seconds. So, we divide that by root over  $1 - v^2/c^2$  okay.

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• So, what is the speed of the muon?

$$v = \frac{800\text{m}}{\Delta t}$$

$$= \frac{800\text{m}}{\frac{2 \times 10^{-6}\text{s}}{\sqrt{1 - v^2/c^2}}}$$

$$v = \frac{4}{5}c$$

And that is what we get the speed of the Muon is now 800 meters divided by delta  $t$  that is this improper lifetime. And if you put in this expression which we obtained earlier you land up with the speed of this Muon to be a 4/5th of at the speed of light. So, this is still a less than the speed of light okay. And it does not you know not more than speed of light as we had done as we had seen it would be if you do it you know in a wrong sense of course okay.

So, let me now move over to another interesting concept it is the relativity of simultaneity ok. So, and then I shall later on talk of things called earlier you know the concepts of earlier a little later

okay. So, the concepts like if you if a certain event is you know following a sequence I mean some event has occurred earlier and then later and then does it appeared to be in the same sequence in some other frame or not okay.

But before that we do a little you know more basic thing it is called this relativity of simultaneity. So, what is it that is what that we are going to say here?

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### **Relativity of Simultaneity**

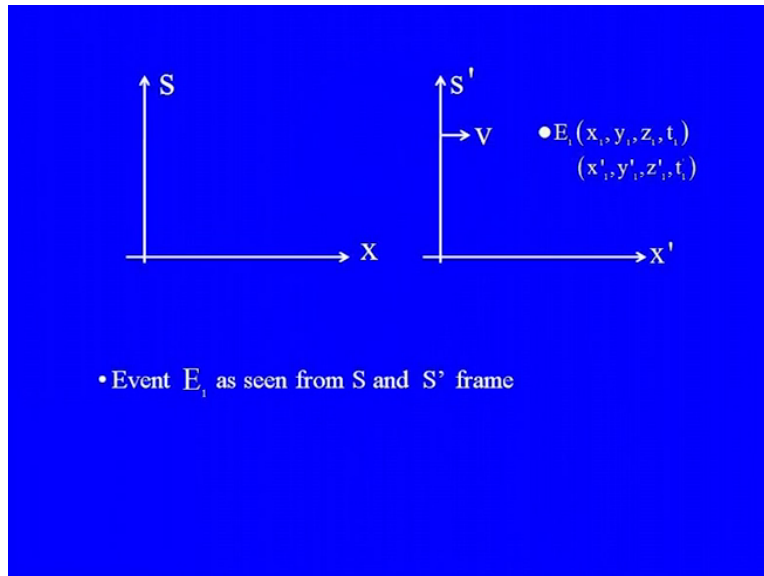
- Two events are said to be simultaneous if they occur at the same instant of time.
- If two events appear to be simultaneous in one inertial frame will it be simultaneous in another frame of reference.

What we are seeing is that two events are said to be simultaneous if they occur at the same instant of time of course I mean that is the basic definition of what you would see for a simultaneous event okay. But the question is if these events are to be simultaneous in one inertial frame. So, is it going to be simultaneous in another frame of reference which is you know I mean talk of inertial frames so we the frames here I am moving with uniform velocity with respect to each other okay.

So, if it is simultaneous in one frame is it going to be simultaneous in another frame okay? So, for that I think it will be good if you see there are more pictorial fashion, yes we talk of event one okay.

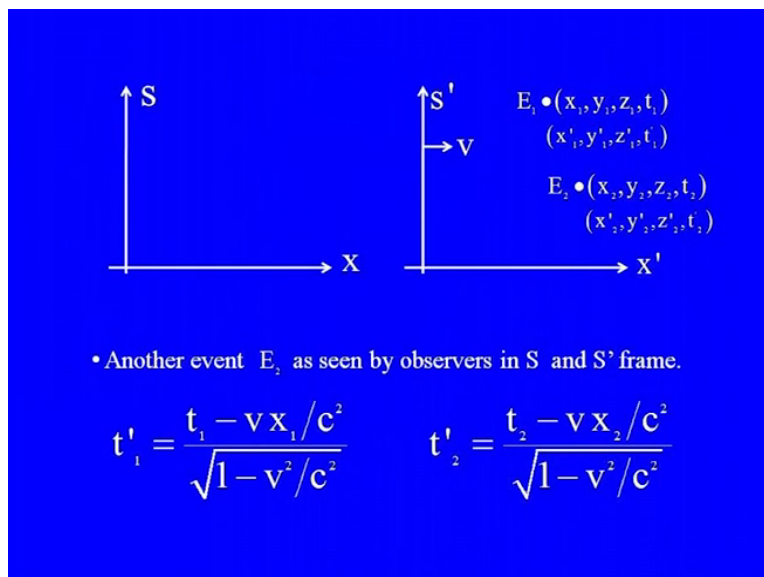
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So, we say event one, so now it is occurred at a certain position, so the  $x_1, y_1, z_1$  say at certain time in S frame let us say okay. And then the same event is seen to be at positions at position  $x_1$  prime,  $y_1$  prime,  $z_1$  prime and then the time measured is  $t_1$  prime ok. So, that is the event seen from the frames S and S prime. So, remember S prime is also moving with a certain uniform velocity with respect to the S frame ok.

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Now if you talk of another event okay seen by the observer in S and S prime frame, so it is occurred at a different position and time bolting it again. So, ok so what is it what is the relation between this between these time components. So, we know it we know it is  $t_1$  prime that is

=  $t_1 - vx_1$  by  $c^2$  divided by  $\sqrt{1 - v^2/c^2}$  ok. So, and same thing for the second measured time in the frame S prime.

So, that  $t_2$  prime is just  $t_2 - vx_2$  by  $c^2$  divided by  $\sqrt{1 - v^2/c^2}$ . Remember this unprimed things are for the S frame and then this primed things are for the S prime frame ok. So, what is the difference between these two times ok? As seen well the difference between these two primes is obvious.

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The diagram illustrates two reference frames, S and S', moving relative to each other with velocity  $V$ . Frame S has axes  $x$  and  $y$ , and frame S' has axes  $x'$  and  $y'$ . Two events are shown:  $E_1$  at  $(x_1, y_1, z_1, t_1)$  and  $E_2$  at  $(x_2, y_2, z_2, t_2)$  in frame S, and their coordinates in frame S' are  $(x'_1, y'_1, z'_1, t'_1)$  and  $(x'_2, y'_2, z'_2, t'_2)$  respectively.

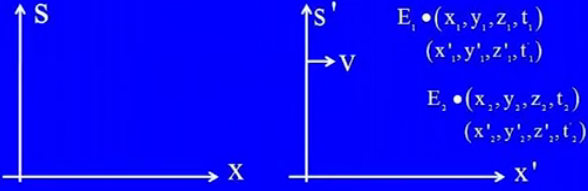
$$t'_2 - t'_1 = \frac{(t_2 - t_1) - v(x_2 - x_1)/c^2}{\sqrt{1 - v^2/c^2}}$$

- Assume that the events are simultaneous in S-frame.

Then  $t'_1 = t'_2$

So, it is  $t_2$  prime -  $t_1$  prime, so we find that the same as  $t_2 - t_1$ , so the things on the right hand side are the coordinates time that we have in the S frame okay. So, that is  $t_2 - t_1 - v$  of  $x_2 - x_1$  by  $c^2$  and divided by  $\sqrt{1 - v^2/c^2}$  okay. So, if these events are simultaneous in S frame ok then of course it is occurred you know at the same time. So, we take  $t_1 = t_2$  okay. So, that would mean that we have from this equation of difference of times in the prime frame. So, we have on the right hand side we have taken out  $t_1 - t_2$  okay.

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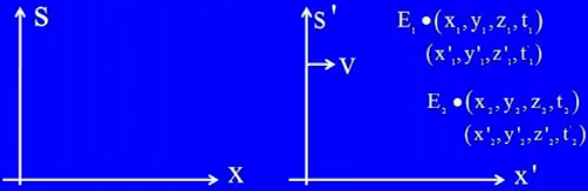


- But  $x_1 \neq x_2$  (otherwise the events will not be distinct)
- $t'_2 - t'_1 = \frac{-v(x_2 - x_1) / c^2}{\sqrt{1 - v^2 / c^2}} \neq 0$
- $t'_2 \neq t'_1$  (although  $t_1 = t_2$ )

However so the position so that is  $x_1 - x_2$ , so that well it is not 0, why because if it is 0 so you would not be able to distinguish between these events okay. So, that is not 0 okay, so that would mean that the difference of measured times of these two events in the prime frame is not 0 because none of the quantities here are 0. You can see here okay. So, what you get? So, what we get is the measured in the S prime frame need not be the same.

Even if the times are the same in you know the measured times are the same in the S frame okay. So, in other words to be more precise what we have is that the events in E1 and E2 okay.

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- Events  $E_1$  and  $E_2$  need not be simultaneous in  $S'$  frame even though they may be simultaneous in S frame.

Although they may be simultaneous in one frame of reference they need not be simultaneous in another inertial frame ok. So, we come then to another interesting concept it is the thing called the sequence of events okay. The concept of earlier and later, as we said I mean what we saw is that if something is happening you know earlier in a certain frame and then another event is happening a later in a certain frame.

Is this order of events preserved okay, so, why do we say that? Because we have just seen that you know this simultaneous itself is a relative concept here. So, it depends on I mean it need not be a cement something which is simultaneous in one frame of reference need not be simultaneous in a another frame of reference. So, will these walkers here too I mean in the sequence of events.

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**SEQUENCE OF EVENTS**

Concepts of “Earlier” and “Later”

- Consider two events ( $E_1$  and  $E_2$ ) seen from two different frames  $S$  and  $S'$  (which move with some uniform velocity with respect to  $S$ )

<u>S – frame</u>	<u>S' – frame</u>
$E_1 : (x_1, y_1, z_1, t_1)$	$(x'_1, y'_1, z'_1, t'_1)$
$E_2 : (x_2, y_2, z_2, t_2)$	$(x'_2, y'_2, z'_2, t'_2)$

Q: If  $E_1$  and  $E_2$  occur in a certain sequence in one frame can it occur in the reverse sequence in another frame.

So, we consider two events you  $E_1$  and  $E_2$  again and we see these events from two different frames  $S$  and  $S$  prime okay. So, this  $S$  prime frame it is moving with a certain uniform velocity with respect to  $S$  okay. So, we written down what the space time coordinates of these two events are in  $S$  frame and the  $S$  prime frame okay. So, the question that we now ask ourselves is that if these events occur in a certain sequence in one frame.

So, can it occur in the reverse sequence in another frame? So, for that we again have to look at the difference of times as measured in the  $S$  and this  $S$  prime frame okay.

(Refer Slide Time: 32:39)

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - v(x_2 - x_1)/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Say  $E_2$  has “occurred” after  $E_1$  in frame S.
- Then  $t_2 > t_1$

So, what is the difference of time measured in the S prime frame, so that is  $t_2 - t_1$  that is the time measured in the difference of time measured in the S frame minus of the it is the velocity  $v$  into  $x_2 - x_1$  that is the those are the positions as measured in the s frame by  $c$  squared. Then the usual factor to power of  $1 - v$  square in the denominator okay. Now if let us say event 2 has the second event has occurred later than event 1 in the frame S. So, obviously  $t_2 - t_1$  is positive okay.

(Refer Slide Time: 33:21)

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - v(x_2 - x_1)/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Now, for the *reverse* to happen in S' frame
- ( $E_1$  occurring after  $E_2$ ) we need  $t'_2 < t'_1$
- This implies

$$(t_2 - t_1) - v(x_2 - x_1)/c^2 < 0$$

Now that means that for the reverse to happen in S frame okay, what we have? We have you know  $t_2 - t_1$  was positive in S frame but  $t_2$  prime -  $t_1$  prime, so that is not positive okay. So, in that case what it means is that we have nothing in the numerator. So, that is  $t_2 - t_1 - v$  into  $x_2 - x_1$

divided by c square that has to be negative okay that is obvious and denominator here is positive ok.

**(Refer Slide Time: 34:13)**

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - v(x_2 - x_1)/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- This implies  $(t_2 - t_1) - v(x_2 - x_1)/c^2 < 0$

Or  $\frac{x_2 - x_1}{t_2 - t_1} > \frac{c^2}{v}$

So, now if that is so we would require  $x_2 - x_1$  divided by  $t_2 - t_1$  to be greater than  $c^2$  by  $v$  okay. Now if these events are causally connected that is in a sense that event E2 is happening as a consequence of event 1 happening okay. It is more like you know you in a cricket field and then thinks of your favorite cricketer and then what he does is? He autographs a ball and it throws it at you okay and so you catch the ball.

So, he of the cricket of your favorite cricketer autographing a ball and then that is called an event one and then you catching you know the throw that he throws that event 2. So, this has to be done at a certain you know so certain that the message has to be passed at a certain speed here okay. So, you catching an autograph ball is in a sense dependent on your cricketer signing the ball okay.

So, unless the cricketer signs the ball and throws it to you you cannot catch the ball okay. So, in a sense some messages been passed in this case the example of two events which are causally connected okay.

**(Refer Slide Time: 35:52)**

- Now, if the two events are causally connected (e.g. event  $E_2$  “happening” as a consequence of event  $E_1$ )
- Then some message, originating from the 1<sup>st</sup> event must reach the place of the 2<sup>nd</sup> event, travelling with some speed ‘u’(say)
- Then again, 
$$\frac{x_2 - x_1}{t_2 - t_1} = u > \frac{c^2}{v}$$

So, in a sense what we have is that some message which is originating from the first event reaches the second place the position of the second event traveling with some speed let us say is some speed u okay. Then the minimum speed that this you should have is actually  $x_2 - x_1$  divided by  $t_2 - t_1$  difference of these positions and the difference of this measure time has a minimum thing that value that you should have okay, u that is the alphabet u, I mean.

Now what does it mean now this u in this case would have  $c^2$  would be more than  $c^2$  square by v okay. Why because  $x_2 - x_1$  divided by  $t_2 - t_1$  that is more than  $c^2$  square by v. So, that is the condition one has to satisfy remember. If the sequence of event is reversed in another frame and the events are causally connected okay. Then we will end up with a problem.

**(Refer Slide Time: 37:01)**

- $\frac{x_2 - x_1}{t_2 - t_1} = u > \frac{c^2}{v}$

- Now as  $v < c$  ( $v$  is the speed  $S'$  frame moves uniformly with respect to frame  $S$ ); This implies  $u > c$  !!

i.e. the message between the events  $E_1$  and  $E_2$  is 'travelling' with speed greater than the speed of light.

The problem is that  $v$  here is distance  $c$  okay or at most it is  $c$ , so at most its  $c$  but so you cannot go more than the speed of light well then what about  $u$ ? The alphabet with  $u$  that is the speed  $u$  see that it is more than  $c$  ok. So, this again it is not allowed, so because why I said again because  $v$  is the speed  $S$  prime frame is moving uniformly with respect to a frame  $S$  ok. So, in a sense it would mean that the message between the events  $E_1$  and  $E_2$  is travelling with a speed greater than the speed of light.

So, this is not allowed ok, so this condition is not fulfilled this condition is the wrong condition okay.

**(Refer Slide Time: 37:57)**

- Since this is not possible the condition  $\frac{x_2 - x_1}{t_2 - t_1} > \frac{c^2}{v}$  cannot be fulfilled if the

events ( $E_1$  and  $E_2$ ) are causally connected

- Thus, the concept of earlier and later for causally connected events will be preserved in all inertial frames.
- However, the opposite can happen if the events are not causally connected.



Now since it is not possible to fulfill this condition if the events E1 and E2 are causally connected then it means that the concept of earlier and later between events will be preserved in all inertial frames okay. So, that is the point here, so if the events are causally connected then it is going to be a pisser. But if the events are not causally connected ok, so event the second event let us see is happening independent of whether event occurs not ok.

In that case the opposite can be true okay, so that is the case when they are not causally connected ok. So, what we saw is that all these things called the simultaneity of the relativity of simultaneity tells us that if two events are simultaneous in certain inertial frame. So, it did not be simultaneous in another inertial frame ok. But the concepts of earlier and later so that is preserved if the events that we are talking of they are causally connected okay.

So, as usual let us have a few examples to clarify our concepts okay. So, let us take two events here E1 and E2 and in S frame and we give the position coordinates of these events here in the S frame and also write down what is the times here.

**(Refer Slide Time: 39:58)**

Ex: In S-frame two events( $E_1$  and  $E_2$ ) are such that

$$E_1 : x_1 = x_0, y_1 = 0, z_1 = 0, t_1 = x_0/c$$

$$E_2 : x_2 = 2x_0, y_2 = 0, z_2 = 0, t_2 = x_0/2c$$

The event however occurs at the same time ( $t'_1 = t'_2$ ) in another frame S' moving with velocity 'v' along common  $x - x'$  axis relative to s-frame. What is 'v' and  $t_1$  ?

So, here let us say E1 is happening at position x1 is x0 and then the, you know the y and z's are 0 here, so ok. And the x2 is happening at a position 2 x0 and then the time t1 that is x0 by c, so c being the speed of light and t2 is x0 by 2c okay. Now if this event is simultaneous, so see that in S frame this event is not simultaneous it is happening at different times okay. But if this event is

simultaneous in another frame, so which is moving with a certain velocity  $v$ . Let us say along the common  $x, x'$  axis ok relative to the  $S$  frame.

What is this velocity? And what is the time that; what is the simultaneous time where that is being measured in  $S'$  prime frame? Okay, so how do you go about doing these things? So, let us look at the time intervals had seen from the  $S$  frame and the  $S'$  prime frame. So, since the events are simultaneous in the  $S'$  prime frame.

**(Refer Slide Time: 41:28)**

Events are simultaneously in  $S'$  - frame

$$t_2' - t_1' = \frac{(t_2 - t_1) - v(x_2 - x_1)/c^2}{\sqrt{1 - v^2/c^2}}$$

$$0 = \frac{\left(\frac{x_0}{2c} - \frac{x_0}{c}\right) - vx_0/c^2}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{-x_0/c - vx_0/c^2}{\sqrt{1 - v^2/c^2}}$$

So,  $t_2 - t_1$ , so that is 0, so if you look at the second step of the second equation, so, on this numerator you put in the times that we had for the event in for the events other that we had in the  $S$  frame. And then also the difference of their positions and divided by the root over of  $v$  square minus  $1 - v$  square by  $c$  square,  $v$  being the velocity with which  $s'$  prime frame is moving with respect to the  $S$  frame. So, we immediately get what we get the left hand side to be 0 of course.

**(Refer Slide Time: 42:15)**

$$\Rightarrow v = -\frac{c}{2}$$

$$\text{Time } t_1' = \frac{t_1 - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\text{Substituting } t_1 = x_0/c, x = x_0, v = -c/2$$

$$t_1' = \sqrt{3} x_0/c$$

And we can immediately find what  $v$  is and you see that  $v$  is actually  $-c$  by  $2$  okay. So, how do you find the time in the prime frame remember in the prime frame the events are simultaneous. So, so  $t_1$  prime that is nothing but  $t_1 - vx$  by  $c$  squared ok divided by root over  $1 - v$  square by  $c$  square and then what you are going to do is they are going to substitute  $t_1$  by the measured time, you know for a certain position.

So, if  $x$  is  $x_0$  we look this problem that this was measured at time  $t_1 = x_0$  by  $c$  ok and we know the velocity now with which this prime frame is moving. So, that is  $-c$  by  $2$ , so immediately get what this time is in the prime frame of reference. So,  $t_1$  prime is root  $3$   $x_0$  by  $c$  ok. So, let us go to a another problem ok. Well let us look at this problem first okay. So, in a certain inertial frame let us say so we have two events which occur at the same place.

**(Refer Slide Time: 43:43)**

Ex: In a certain inertial frame **two events** occur at the **same place** and are separated by a **time interval of 4 seconds**. In another inertial frame what is the *spatial separation between these two events* if these are separated by a **time interval of 6 seconds**

And they are separated by a time interval of four seconds okay. Now in another inertial frame the question is what is the spatial separation what is the spatial separation in these coordinates here between these two coordinates if these are separated by a time interval of 6 seconds okay, so that is what we do we. First try to find what is the velocity with which these frames are moving with respect to each other okay.

So, call the prime frame that is the; you know so the frame in which the time separation between these two events are 6 seconds. So, that is the prime frame see it colleague like that. So, what is the corresponding you know, portion of this time difference in the S frame how do you do it?

(Refer Slide Time: 44:50)

Ex: In a certain inertial frame **two events** occur at the **same place** and are separated by a **time interval of 4 seconds**. In another inertial frame what is the *spatial separation* between these two events if these are separated by a time interval of 6 seconds

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - \frac{v(x_2 - x_1)}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$6 = \frac{4 - 0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v = \frac{\sqrt{5}}{3} c$$

So, it is  $t_1 - t_2$ ,  $t_2 - t_1$  rather  $-v x_2 - x_1$  divided by  $c$  square root over of  $1 - v$  square by  $c$  square okay. So, here we have; so you put instead of  $t_2 - t_1$  you put that to be 6 and  $t_2 - t_1$  you put that to be 4 and you find what  $b$  is and this  $v$  in this particular case would be root over 5 by 3 into the speed of light okay. Now having found that it is very easy to find what the spatial separation is. So, we just find what the spatial separation is.

**(Refer Slide Time: 45:42)**

• Spatial separation

$$x'_2 - x'_1 = \frac{(x_2 - x_1) - v(t_2 - t_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{0 - v(4)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x'_2 - x'_1 = -2\sqrt{5}c$$

$$\text{Thus } |x'_2 - x'_1| = 2\sqrt{5}c$$

So, that is  $x'_2 - x'_1$  that is  $x_2 - x_1 - v$  into  $t_2 - t_1$  to root over of  $1 - v$  square by  $c$  square, we know the you know the spatial separations in the  $S$  frame that is 0 it is happened at the same place. So, what is the spatial separation in the prime frame? So, that actually is you put in the proper times here, so I mean the proper times not in the sense of proper time proper length I mean you did put in the right times here.

So, that is  $t_2 - t_1$  and you get the difference to be  $-2$  into root over of  $5c$  okay. So, the proper length the proper separation is the modulus of this of course, so that is twice of  $5c$  okay. So, we have seen so far it is another consequence of time dilation. We have talked of you know we have talked how or why moving blocks of run slow. You know things are, we derived this consequence from the Lorentz transformation.

Then we talked of the relativity of simultaneity, so in which we showed that two events which are simultaneous in an inertial frame need not be a simultaneous in another frame okay. And then in the third part we saw that the sequence of events they are preserved in all inertial frames if the events are if this events are causally connected okay.

**(Refer Slide Time: 47:57)**

### Consequences of Special Relativity - III

- 'Time dilation'
- 'Doppler effect in light'

So, in the next lecture what you want to see is talk of Doppler effect in light and see what next we can do from there okay. Doppler effect I think we yeah bit familiar with Doppler effect in sound where you know if the you have a source moving you know use your standing some you standing at a distance and then from a distance a train is coming at you okay and the train is flowing it is on or a giving a whistle.

Then you see that as the train moves towards you the frequency that changes okay. So, that is the Doppler Effect so the source is moving here so or if the source is at rest and the observer is moving the observed frequency of sound is also different. So, we want to look at whether Doppler Effect is there in light or not well that is a thing for the next lecture, thank you very much.