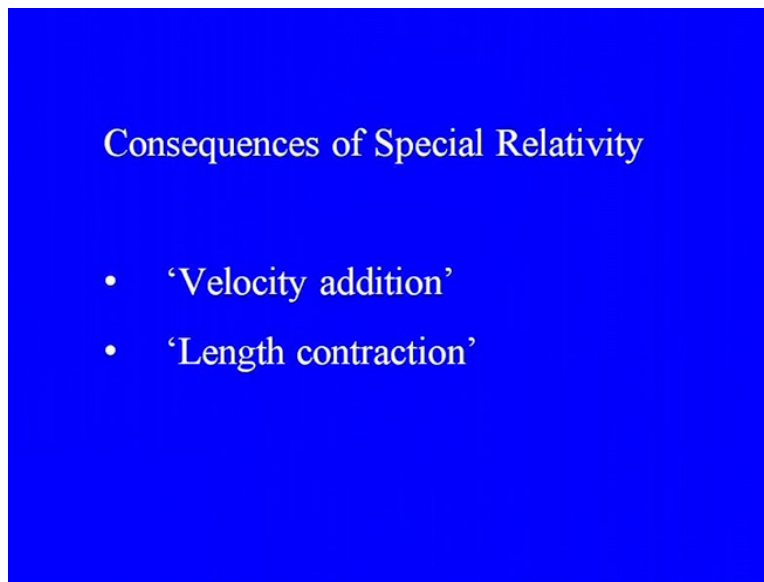


**Engineering Physics 1
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**Module-07
Lecture-02
Consequence of Special Relativity - 1**

Okay, so we will be talking of the consequences of special relativity today having been introduced to this subject in a previous lecture okay.

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So, we will be talking mainly on length contraction ah but of course before that we will spend some time in learning how ah velocities are added in special relativity okay as opposed to human you know this the Galilean velocity addition formula that we have so well used to, right. So, you know if you want to come to the subject we just summarized a little bit of what we did in last time.

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- In the previous talk we had seen Newtonian mechanics to be invariant under Galilean transformation and Electrodynamics under Lorentz transformation.
- Reconciliation of the transformation of mechanics and electrodynamics \Rightarrow special theory of relativity.

So, we were seeing we were talking of Newtonian mechanics being invariant under Galilean transformations. Electrodynamics I mean the laws of electrodynamics specially Maxwell's equations. So, we saw that they were invariant under Lorentz transformations so it was a peculiar situation I mean a branch of physics another branch of physics not being been variant on the same transformation okay.

And it is actually this reconciliation of transformations of transformation laws of mechanics and electrodynamics from a theoretical sense which led Einstein to the special theory of relativity. Of course having mentioned this I mean I should also mention the Michelson Morley experiment because had that not being there it would have been very difficult to see the they really need for for such a for such a theory okay, right.

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Postulates of special relativity

Postulate 1: The principle of relativity

The laws of physics are the same in all inertial systems. No preferred inertial system exists.

So, we recapitulate the postulates of relativity once again. The principle of relativity states of course that it is a very, very basic principle that the laws of physics are to be the same in all the systems. So, there is no preferred inertial system okay. So, what does it mean? It means that if you are in a system and you do an experiment there and then there is another system which is moving with uniform velocity with your system, you will not be able to differentiate that?

So, basically systems which move with uniform velocity relative to each other are absolutely similar in the sense that the laws of physics are going to be the same in all these systems okay.

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Postulates of special relativity

Postulate 2: The principle of the constancy of the speed of light.

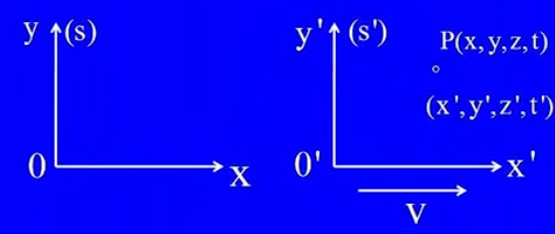
The speed of light in free space has the same value c in all inertial systems.

The second postulate and it is very important theory it is the principle of the constancy of the speed of light okay. So, well I mean in simpler words it says that the speed of light in free space it has to be the say it has to have the same value I mean we can see here and if you know that it is 3 into 10 to power 8 meter per second I mean which is almost 3 into 10 power 8 meter per second. We know that it is right 2.99 something in all inertial systems.

Now this in the first instance may sound very strange because if you if you think of everyday experience. Suppose you shine a light say you light a torch and then in the direction of this light let us say you are moving in a vehicle with there an imaginary vehicle of course in a very fast way. So, let us say you are moving with 2 into 10 to power 8 meter per second. So, what happens, so what speed will you see the light will see this light from your normal experience of Galilean relativity okay.

So, Galilean velocity addition formula and all these things, so the light is moving forward 3 in 10 power 8 you are moving in the same direction in 2 into 10 power 8 so you are supposed to see the light in 3 - 2 that is 1 into 10 power 8 meter per second. Well relativity tells you that no you ever see it 3 into 10 power 8 meter per second and so. Where we will see how this is possible because if you believe in this Galilean transformations if you will be if you do the thing in gradient transformation of course this does not come about.

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(S) Frame in terms of (S') frame

$$\begin{aligned} x &= \gamma(x' + \beta ct') \\ y &= y', z = z' \\ ct &= \gamma(ct' + \beta x') \end{aligned} \quad \left| \quad \begin{aligned} \beta &= v/c \\ \gamma &= \frac{1}{\sqrt{1 - \beta^2}} \end{aligned} \right.$$

So, there is another transformation where such thing is possible and that is we thought what that was that was actually Lorentz transformations. And if you I am just summarizing the transformations once again for you. If you have let us say a frame S prime which is moving with a certain velocity v that is certain uniform velocity I should have with respect to another frame S , okay. So, they have let us say the common x , x prime x axis here.

And then a point a general point is xyz is there and then you put in the fourth component that time here and then at the same point from the S prime frame that is x prime, y prime, z prime and t prime and then those relations are given by the transformation equations which you see here. So, it is x how does the quantities in the primed frame relate the quantities in the unprimed frame okay.

So, that is so that is given by the Lorentz transformations. So, here you see $x = \gamma x' + \beta ct'$. Now I have defined what this β and the γ s are so β is also it is v by c . So, it is dimensionless and γ is $1/\sqrt{1 - \beta^2}$ okay. If you write it in this form you will see that when you write the time component it has almost similar looking expression.

So, of course instead of time I multiplied that by c that is the velocity of light. So, that it has the dimension of length okay, so I got ct , so you see you see compare this equation for x and compare this equation for ct okay. You see that it contains in the equation x you have both x' and t' and then in the equation for ct you have both t' and x' of course in this opposite fashion.

So, it looks very symmetric, so it is quite useful to write it in this way. But for the sake of simplicity and because we are doing this for the first time I will use all these expansions whenever we come across it okay. So, what are the limits I mean does this in some limit go back to the Galilean transformation? Well actually it does, where does it do? So, at small speeds that is if one is moving with speed much, much lesser than the speed of light here.

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<p>Limits for speeds $\ll c$</p> $x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$ $y' = y, z' = z$ $t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$ <p style="text-align: center;">Lorentz transformation</p>	<p style="text-align: center;">$v \ll c$ $v^2/c^2 \rightarrow 0$</p> $x' = x - vt$ $y' = y, z' = z$ $t' = t$ <p style="text-align: center;">Galilean transformation</p>
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So, you see v is much, much lesser than c so of course v^2/c^2 tends to 0 here. Now you can immediately figure out that x' becomes you know this the quantity below the denominator for x' , $x' = x - vt$ divided by root over of $1 - v^2/c^2$. So, this turns out to be just the Galilean equivalent that is $x' = x - vt$ and since you are moving along the x, x' direction.

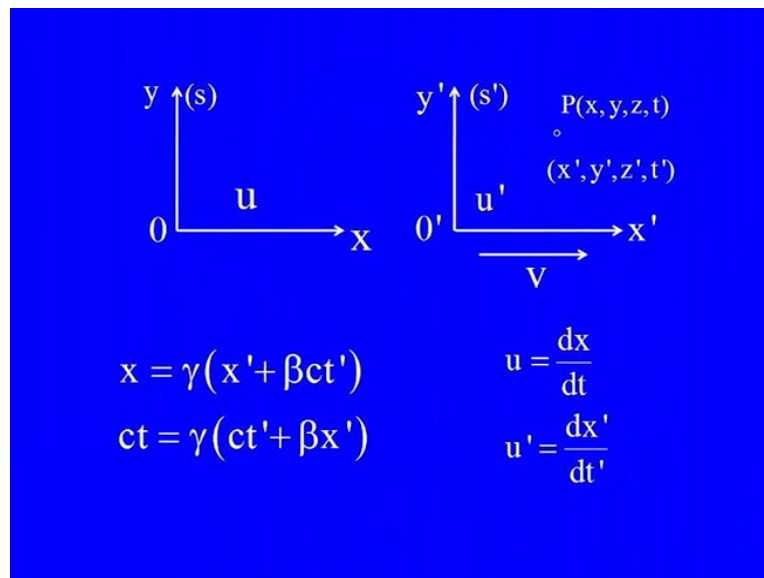
So, the x, y' prime for the z' prime are the same okay. Now what happens for the t' prime's okay? Now it is interesting when Lorentz transformation the time's in both these frames are not the same. They are also dependent on the coordinates and all the space coordinates of the system it is evident from this equation itself okay, now at small speeds at small speeds smaller much smaller than the speed of light that is.

You will see that these times t' and t they become equal they become approximately equal actually if you do this you take v by c to be much, much less than, so it is almost tending to 0, it is much, much less than 1 instead to 0 that is okay. So, that is the thing that we get Lorentz transformations. They go to Galilean transformations add small speeds, so that is the thing.

So, at smaller velocities Newtonian mechanics is still the thing that one can follow and that is that is it, that we that is what we do in our daily lives I mean we in our daily lives rarely do we

go at speeds such high speeds and speed of light okay. So, the Newtonian mechanics is valid is correct at small speeds okay.

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But having said that let us now have a look at how velocities are going to be added in these two frames okay. So, what are the relative velocities or what is the Galilean velocity addition formula here okay. I am sorry let us see the Lorentz velocity at the Einstein velocity addition formula according to Lorentz transformations because the Galilean velocity transformations are addition laws we have already done okay.

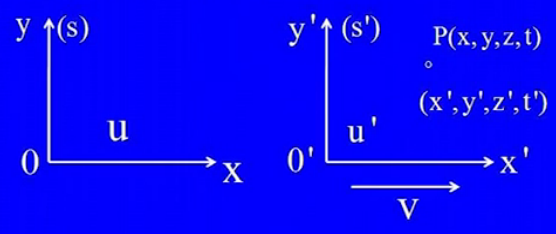
But how do you do it I mean the process is similar if you remember I have drawn this diagram for these frames once again see x_0 by the way the x axis is always there it is protruding out of the screen I am just not drawn it so that the picture looks a less clumsy that is all okay. So, so how do you find the velocities in each of these frames, you are going to do you are going to take this differential this dx by dt .

Now remember the coordinates are going to take for each frame and then the time which is suitable for that frame itself okay. Now so in the S frame it is dx by dt and in the prime frame it is obviously dx' by dt' , remember in when we were doing this Galilean velocity addition formula this t and t' were the same but here they are not okay. I mean just for the for the sake of simplicity and just for your remembrance I have listed it on the left.

So, you see that they are not the same so if you take dx prime by dt it is not the same as dx prime by dt prime okay but you need to take this properly. Now if you do this if you do dx by dt and du prime dx prime and u prime = dx prime by dt prime we are going to land up with the velocity addition formula.

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Einstein's velocity addition formula



$$u = \frac{u' + v}{1 + u'v/c^2} \quad u' = \frac{u - v}{1 - uv/c^2}$$

And so how does it look like so, so what is the velocity in S frame having known the velocity of a particle or some velocity in the prime frame. So, that is the one given on the left hand side of your screen you will see that if the S prime frame is moving with a certain uniform velocity v and then in this frame a velocity is measured as u prime. This velocity in the unprimed frame is going to be measured as u prime + v divided by $1 + u$ prime v by c square okay.

So, this is the Einsteinian velocity addition formula okay, just to remind you had this been the normal or the more familiar a Galilean velocity addition. You would not have the denominator here you just have u would be just u prime + v and so on okay. But then in some limit as we said I mean these transformations go at small speeds the Lorentz transformations become the Galilean transformation.

So, to here it must be, so here these velocity addition formula must go to the Einsteinian formula must go to the Galilean formula small speed, so we will check that we will check that okay. So,

similarly of course can do the reverse transformations and find u' to $u - v$ divided by $1 - uv/c^2$. So, that you find, so if you know the velocity in the S , so how is it so what is the velocity u' you are going to measure in a frame S' that is which is moving away with a uniform velocity v ok.

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$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{Limits for speed} \ll c$$

$$u \ll c, v \ll c$$

$$u' \approx u - v \quad \text{Galilean limit}$$

So, as we are talking of the limits, so at small limits when u is small v small, you immediately see what you immediately see that this denominator, so uv by c^2 this becomes negligibly small so called limit speed = 0 you might you might say. So, that $u' = u - v$ or if you feel added your arms you are fastidious about this partition. So, $u = u' + v$ that is all so that gives us the Galilean limit at small speeds right.

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$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{Limits for speed} \ll c$$

$$u \ll c, v \ll c$$

$$u' \approx u - v \quad \text{Galilean limit}$$

How about some examples let us see what happens if in with this Einsteinian velocity addition formula where you have a frame which is moving let us say your S prime frame is moving away with the velocity $0.9c$ that is c 's being the speed of light okay. And in this S prime frame you measure ah some speed of some object you know things can have such large speeds that is relativistic speed some subatomic particles do have such relativistic speeds to be like $0.9c$ right.

So, if you directly go and apply the Galilean formula you know it is fine $.9 + 0.9$ you want to get more than c of course you know that that is forbidden relativity. So, how is it that comes what is it that comes here. So, for u prime you are going to substitute $.9c$ and for v $0.9c$ and then in a denominator you see $1 + 0.81 c^2$ square by c^2 square and then value you get something like $.99c$ okay.

So, it is still, still less than c right, so we have checked that even if individual velocities we have in this particular case is almost touching c the added velocity is the from the Einsteinian formula it still gives you less than c , I mean has it should okay.

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What happens if one of the speeds is c itself ? i.e., if $u' = c$ (S' frame), what speed is seen in (S frame)

$$u = \frac{u' + v}{1 + u'v/c^2}$$

$$u = \frac{c + v}{1 + cv/c^2}$$

$$= c$$

Speed of light is the same in all inertial frames

But the interesting case is you might say is what happens if one of these speeds is c itself okay that is if u' is c okay. In S' frame which is moving with the velocity some v with respect to the S frame. So, what is the speed that is seen in u , so in other words, so this is the case you have let us say your light focus, so you have a photon or speed light which is moving with speed or the cost of the speed of light c in one frame.

What are you going to observe in another frame okay with respect to which your frame is moving with a certain velocity v okay? So, this is a direct consequence you want to test that is a direct consequence of the second postulate of relativity. So, u' you are going to substitute by c , v is of course the speed with which the S' frame is moving with respect to the S frame and then the denominator you have $1 + cv/c^2$ and what does it give you.

It gives you just c okay, so that is the speed of light is the same in all inertial frames I mean this we had started with postulates no wonder that is so, but then at least you are verifying here that the Einsteinian velocity addition formula that we just use right now is consistent with this postulates okay, so, just one final example perhaps to find the relative velocities of particles.

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Ex. If two particles 'a' and 'b' are moving in opposite directions with speed $c/2$, what are the relative velocities of each with respect to the other?

Diagram illustrating two particles, 'a' and 'b', moving in opposite directions. Particle 'a' is moving to the right with velocity $c/2$, and particle 'b' is moving to the left with velocity $-c/2$.

So, let us say you have two particles a and b which are moving in opposite directions okay one moving towards left with point with $-c$ by 2 right well this plus and minus is just tells you the direction the direction is different that is on the vector direction is different. So, they are moving with speeds half the speed of light okay in opposite directions okay. The question is what is the relative velocity of 1 with respect to the other?

What is the relative velocity of a with respect to b and what is the relative velocity of b with respect to a yeah it is very common so I mean we all have travel in train, so you see when you travel in a train in a certain direction. So, you if you can you can always feel that a train which is moving in the other direction to you, is somehow moving what is the relative velocity of that train it moves it moves quite fast is not it okay. And in this so we are going to wind up you were to find out a similar situation here, okay.

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$\overleftarrow{\hspace{1cm}}$ $\overrightarrow{\hspace{1cm}}$
 $b, (-c/2)$ $a, (c/2)$

- Take particle 'b' to be at rest in frame S' which is moving relative to S with a uniform velocity
 $v = -c/2$
- So, here the velocity of 'a' relative to S is $u=c/2$ and therefore relative to S' (hence 'b') is

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{c/2 - (-c/2)}{1 + \frac{c^2/4}{c^2}} = \frac{4c}{5}$$

Velocity of 'b' relative to 'a' = $-4c/5$

So, here how do you do that so we take particle b to be at rest in frame S prime, so which is so which is moving with S with the other frame S with a uniform velocity c by 2. Here so again the velocity of a relative to s is whatever what is measured in S is c by 2 okay. So, what is the relative velocity so what you are going to find is the velocity of a relative to S is c by 2 and therefore relative to S prime and you see S prime is the frame in which b is taken to be at rest.

So, that is the frame you if you find this velocity of a with respect to S prime you want to find the relative velocity with b that is it. So, you use the velocity this addition formula here so that you are using the primed coordinates here. So, u prime is $u - v / 1 - uv / c^2$ and then you just put in the numbers c by 2 the perfect the proper numbers here and what do you get you get 4c by 5 still less than c.

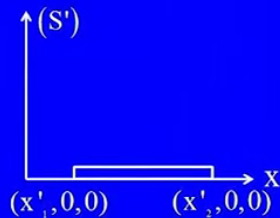
I mean if it had been the Galilean way it would be c but here it is 4 by c and what is the other way now. The velocity of b relative to a it should be exactly the opposite right. So, it is -4c by 5 you could have also used the velocity addition formula to get this speed, I mean that you can do it as an exercise of course ok, right. So, having now seen how velocities are done let us go to some of the other corollaries of special relativity.

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Corollaries of special relativity

Length Contraction

“Length of a moving rod appears to be shortened in the direction of motion”.



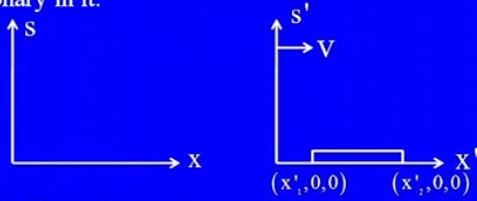
Length of the rod (at rest) in S' frame $= \ell_0 = x'_2 - x'_1$

And especially an interesting thing called length contraction okay so what is it so it just tells you that the length of a moving body appears to be shortened in the direction of its motion okay. So, for moving bodies appear to be shorter okay. We want to find out how? Ok. So, for this let us consider a rod okay, the end points of this rod are in S' prime let us say so this rod is in S' prime and it is at rest on this x axis ok.

So, just consider an unidirectional thing here, one dimension thing here and you have the x the you have the ends of this rod as x_1 prime and the other end as x_2 prime okay. So, the length of the rod which is at rest in this frame is recall that at t_0 , this is you just measure this what how do you do, so it is not you just measure you can take a measuring tape put it on one end of this sort and just put the other one at the other end okay. So, you how do you what do you find that is all, ℓ_0 is x_2 prime, $-x_1$ prime okay.

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- Here we need not bother about the time of measurement in (S') frame because the rod is stationary in it.



- So what is the length of the rod to an observer in the S-frame?
- Remember, the rod is moving parallel to $X - X'$ axes.
- To measure length of the rod in S- frame its end coordinates must be measured at the same instant of time.

However one thing is to be noted here is that we are not bothered about the time at which these two measurements are done because this thing is at rest, so it does not we are not bothered about the time okay. So, you put that is what you do you put you will lie surely put one end of your measuring tape on at x_1 and then go and put the other end at x_2 and then see that the length that is all, okay.

Now what happens I mean if you want to measure this in some other frame S let us say? So, you want to measure the length of this rod S. Now remember relative to S, S prime is moving with a certain velocity and hence the rod which is at rest in S prime is also moving with a certain velocity with respect to S prime okay. So, that is it so in this frame S right remember that the rod is now moving parallel to the x, x' axis with a certain uniform velocity v ok.

Now it is very important that to measure the length of the door in the S frame its end coordinates must be measured at the same instant of time okay.

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- For this two different observers with synchronized clocks are needed.
- Let at time $t=t_0$ (in S- frame) the two ends are at $(x_1, 0, 0)$ and $(x_2, 0, 0)$

Thus, measured length in S- frame

$$\ell = x_2 - x_1$$

- So what does Lorentz transformations give us

$$x'_1 = \frac{x_1 - vt_0}{\sqrt{1 - v^2/c^2}}$$

and $x'_2 = \frac{x_2 - vt_0}{\sqrt{1 - v^2/c^2}}$

This is very important, now ok, now for this, what you need of course you need synchronize clocks and then they have and you have two different observers who are going to measure the endpoints of the rod here okay. So, let us say at this particular time $t = t_0$ in S prime in S prime in S frame that is not in the S prime, frame. The two ends are measured as x_1 and x_2 okay, so what is the length of this rod in S frame.

You want to just subtract these two coordinates, so, $l = x_2 - x_1$ and say okay. So, this is my length in the S frame okay, relative to which new rod is moving with a certain velocity. But remember there is a relation between the quantities in the S frame and the prime frame in the in the frame in the x, x prime coordinates and the unprimed coordinates x which are unprimed. So, they are given by the Lorentz transformations okay.

So, we use the Lorentz transformations, so what is it again just to remind you, so what is x, x prime. So, that is $x - vt$, now here at a certain so on the right hand side of this equation so all these quantities are in the quantities measured in the S frame okay. So, on the left hand side the quantities which are measured in the primed frame okay. So, from this so you can find the relations of x_1 prime and x_2 prime in terms of the corresponding quantities in the S frame okay.

But remember along with these positions the time also enters into this picture. But then the point is that the time you measure this at the same time you have to measure the ends of the rod in S frame at the same time, so it is going to be the same here.

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- $$x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$\ell_0 = \frac{\ell}{\sqrt{1 - v^2/c^2}}$$

$$\ell = \ell_0 \sqrt{1 - v^2/c^2}$$
- Clearly $\ell < \ell_0$
- Length of the rod appears to be shortened in a frame relative to which the rod is moving.

So, if you subtract these two equations what you get, so you get $x'_2 - x'_1 = x_2 - x_1$ okay. What does this give you what is $x'_2 - x'_1$ that is, ℓ_0 ok. So, that is the length of this rod in the prime frame and then what is $x_2 - x_1$ that is ℓ that is the length of the rod in the S frame. So, and then notice that the denominator is still there okay, so that is root over of $1 - v^2/c^2$. So, you get this expression.

So, what is the length of this rod in the S frame remember it is moving in the S frame it is as if the rod is moving with a certain velocity v okay. You see that is this ℓ is nothing but ℓ_0 to root over of $1 - v^2/c^2$ clearly you see that this ℓ the length as measured in the frame in which the rod is moving is less than ℓ_0 , okay. Well to be very exact it should be less than equal to ℓ_0 , you can even have $v = 0$ in some sense okay.

So, if $v = 0$, $\ell = \ell_0$ okay, so this shows that the length of the rod it appears to be shortened in a frame relative to which the rod is moving okay. So, this is length contraction okay. But having said that let us just be familiarized with another concept of the thing called proper length. Well

you have just seen that if you know if this rod is moving with a certain velocity, if it moves at different velocities okay. It is going to be the measured length is going to be different.

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PROPER LENGTH

- If a rod has a length ℓ_1 in a frame relative to which it is moving with a velocity v_1 along its length.
- And similarly in another frame relative to which it is moving with velocity v_2 , the length is ℓ_2 , then...

So, if a rod has a length l in a frame relative to which it is moving uniformly with a velocity v_1 along its length this is important it is moving along its length. Because there could be a case where it is not moving along its length it is making some angle with the x axis. We will try to see if you can do that too. And similarly let us say in another frame this rod is moving with a certain velocity v_2 and then the measured length is l_2 .

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$$\frac{\ell_1}{\sqrt{1-v_1^2/c^2}} = \frac{\ell_2}{\sqrt{1-v_2^2/c^2}} = \ell_0 = \frac{\ell_0}{\sqrt{1-0^2/c^2}}$$

- The quantity $\frac{\ell}{\sqrt{1-v^2/c^2}}$ is invariant in all inertial frames.
- $\ell_0 = \frac{\ell}{\sqrt{1-v^2/c^2}}$ is called the “proper length”
- Proper length of a rod is its length measured in a frame in which it is at rest. This is also the largest of all possible length measurements of the rod.

Then you must have figured out by now that l_1 divided by root over of $1 - v^2/c^2$ = l_2 divided by root over of $1 - v^2/c^2$ = l_0 okay and what is l_0 you well you can also write l_0 in the form of the other things that you have written on the left $l_0 = l_0 \sqrt{1 - v^2/c^2}$. So, it is as if moving in a frame which is it is not which is that in which the rod is at rest okay.

But what do you see? You see that the quantity l by root over of $1 - v^2/c^2$ is invariant it is not changing, you change it, so if you change v the length changes but then the whole quantity of the length divided by root over of $1 - v^2/c^2$ that is becoming invariant in all inertial frames okay. And this is called the proper length okay. So, the proper length of a rod is of an object is a length measured in a frame in which it is at rest.

And obviously figured out that this is also the largest of all possible length measurements of this rod okay, having done this theory I think it might be useful if we have a few examples.

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Ex. Suppose an observer 'A' finds a rod to be of length 1m moving with a velocity $0.8c$. If the same rod appears to have a velocity $0.6c$ relative to an observer 'B' in another frame, what is (i) the proper length of the rod (ii) and the length as measured by B.

$$(i) \text{ Proper length, } l_0 = \frac{l}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.64c^2}{c^2}}} = 1.67\text{m}$$

(ii) Length as measured by B,

$$l_s = l_0 \sqrt{1 - \frac{0.36c^2}{c^2}} = 1.33\text{m}$$

So, let us again have an observer A and let this person find the length of rod to be one meter and then and also to be moving with velocity let us say $0.8c$ okay. Now the same rod if it appears to have a velocity 0.6 the speed of light relative to another observer in another frame okay. So, see that we have not specified the length here of this rod.

So, the first question that we ask ourselves is what is the proper length of the rod okay and secondly what will be the length measured by the second observer okay, delayed relative to which it is moving with velocity $.6c$, $0.6c$ naught point cm. So, what is the proper length we just saw in our previous slide that it is the length divided by the root over of $1 - \frac{\text{velocity}^2}{\text{speed of light}^2}$.

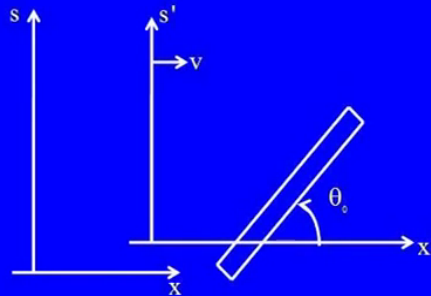
Now you do that you do the length of the rod was 1 meter so you have put one meter there and then for the velocity was $.8$, so which was like 0.6 for c square takes square of that that is and then you see that this proper length is l_0 and l_0 is 1.67 meters ok. So, this is the land that would be measured in a frame in the rest frame of the rod ok. So, what would be the length then measured by the other observer relative to which it is moving with a certain velocity okay.

That is simple the recall that as $l_{\text{subscript } b}$, l_b that is l_0 , l_0 in the sense that that is the proper length and l_0 into over $1 - v^2 \text{ by } c^2$ and then the v here is $.6$ okay, $.6c$ that is so you find here it is 1.33 meters that is all. So, so when it was moving with $0.8c$ the length was point 1.8 , the length was 1 meters. When it was when the when it is moving with a lesser speed of $0.6c$ the length measured appears to be 1.33 meters okay. And then so you see the higher the speed the shorter is the length that you want to measure okay, right.

So, we come over to another interesting case, now in the previous cases we had this rod to be parallel to the x axis okay. But now we are having distort let us see it is it is moving making a certain angle with the x axis okay.

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- what happens if the rod (proper length ℓ_0) makes an angle θ_0 with the x' -axis

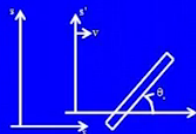


So, it is not moving along its length, so there is a component along its length along the x axis and then there is a component along the y axis also okay. So, the y axis and also I mean the z axis is also there it is protruding out of the screen again I have not drawn it, so that the figure does not look too clumsy okay. Now what we take this rod in the xy plane itself okay and then this rod it is in the s prime frame where is it at rest?

And then it is making a certain angle θ_0 and then this S prime frame is moving with a uniform speed v with respect to the S frame.

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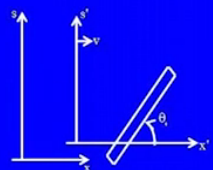
- S' is moving with a uniform velocity 'v' relative to S along common X-X' axis.
- Question 1: what is the length of the rod as measured in S-frame ?
- Question 2: what angle does the rod make with the x-axis in S-frame ?



So, the question that we asked ourselves is that so what is the length of the rod as measured in the S frame and what angle does this rod make with the x axis in the S frame okay. So, if it is with a certain angle θ_0 does it is it θ_0 again in the S frame where we were to find out okay. So, remember of course that this S is moving with a certain velocity v relative to the common x, x prime x axis here. I mean S prime is moving with a uniform velocity v okay.

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S' – frame
 $(x_1', y_1', 0); (x_2', y_2', 0)$
 $x_2' - x_1' = \ell_0 \cos \theta_0$
 $y_2' - y_1' = \ell_0 \sin \theta_0$



S – frame (End coordinates of the rod at time, $t = t_0$)
 $(x_1, y_1, 0); (x_2, y_2, 0)$
 $x_2 - x_1 = \ell \cos \theta$
 $y_2 - y_1 = \ell \sin \theta$

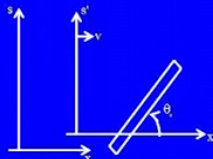
So, what we do in the S prime frame measure what is the length of the rod along the x axis its x_2 prime - x_1 prime and you are going to find this out to be $\ell_0 \cos$ of θ_0 okay. And then if ℓ_0 is the proper length of the rod that is and what will be the component perpendicular to this direction of motion so it is y_2 prime - y_1 prime that is simply $\ell_0 \sin$ of θ_0 okay. Now when measured in the S frame okay.

Now remember in the S frame this rod is moving with a certain velocity with a certain uniform velocity v with respect to the S frame. S the end coordinates of this rod at time you are measuring this end coordinates of this rod at a certain time $t = t_0$ here. So, you find them to be x_1 y_1 and x_2 y_2 , so what is this difference $x_2 - x_1$ here. Well it is $\ell \cos \theta$ okay, so ℓ is the length as measured in the what you know that with the length of the rod in the in the S frame and then the θ angle is the angle that will make with the x axis in the S frame okay.

Well so $x_2 - x_1$ that will give you what the length along the x axis that is $l \cos \theta$ and then $y_2 - y_1$ that will be $l \sin \theta$ okay. So, how are they related well we just figured out again that it is well by the Lorentz transformations of course.

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Now from Lorentz transformations



$$x_1' = \frac{x_1 - vt_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y_1' = y_1$$

$$y_2' = y_2$$

So, you use the transformations equations between the prime coordinates and the unprimed coordinates and you find them to be well it is given on your screen, so the things you it is done at as the prime coordinates on the left hand side and the unprimed coordinates on the right hand side okay. And then the unprimed coordinates in the S frame the measurement of the length is done at a certain instant okay.

So, you definitely require to observe us with synchronize clocks to do this and then synchronize and then measurement is done exactly at $t = t_0$ here and since it is the rod is moving parallel to the x, x prime axis here my coordinates are the same. So, $y_1' = y_1$ is actually y_1 and y_2' is also equal to y_2 okay.

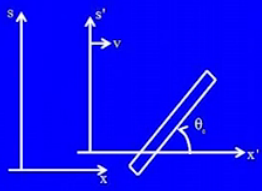
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$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$y_2' - y_1' = y_2 - y_1$$

Thus,

$$\ell_o \cos \theta_o = \frac{\ell \cos \theta}{\sqrt{1 - v^2/c^2}}$$

$$\ell_o \sin \theta_o = \ell \sin \theta$$


So, now if you subtract x_2 prime from x_1 prime and y_2 prime you subtract from y_1 prime of the other way round. So, what do you get you get this you get x_2 prime - x_1 Prime that gives you $x_2 - x_1$ divided by root over of $1 - v^2$ by c^2 okay and y_2 prime - y_1 prime is $y_2 - y_1$, so therefore you get $\ell \cos \theta$ which you know to be $\ell_o \cos \theta_o$ which you know to be the length along the x prime axis that is x_2 prime - x_1 prime is again = $\ell \cos \theta$ divided by root over of $1 - v^2$ by c^2 .

It is very interesting of course and then the in the direction perpendicular to the direction of motion that is in the y direction $\ell_o \sin \theta_o$ is same as $\ell \sin \theta_o$ okay. So, what is the length of this rod in S frame? Okay. So, we come back to our old question and what angle does it make with the x axis in the S frame okay. So, to do that all we have to do is divide those angles which we found.

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$$\tan \theta_o = \tan \theta \sqrt{1 - \frac{v^2}{c^2}}$$

$$\begin{aligned} \ell^2 &= (\ell \cos \theta)^2 + (\ell \sin \theta)^2 \\ &= \ell_o^2 \left(1 - \frac{v^2}{c^2} \cos^2 \theta_o\right) \end{aligned}$$

$$\ell = \ell_o \sqrt{1 - \frac{v^2}{c^2} \cos^2 \theta_o}$$

And we found that we will find that it is a $\tan \theta_o = \tan \theta \sqrt{1 - \frac{v^2}{c^2}}$, how do you do that that is simple, you have this relations $\ell_o \cos \theta_o$ is $\ell \cos \theta$ divided by $\sqrt{1 - \frac{v^2}{c^2}}$ and then this $\ell_o \sin \theta_o$ is $\ell \sin \theta$. So, we divide the equation with the sine with this equation with the cost and then you get this tan that is all.

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$$\ell = \ell_o \sqrt{1 - \frac{v^2}{c^2} \cos^2 \theta_o}$$

Check

$$1) \theta_o = 0$$

$$\ell = \ell_o \sqrt{1 - \frac{v^2}{c^2}}$$

$$2) \theta_o = \frac{\pi}{2}$$

$$\ell = \ell_o$$

So, this gives you first the angle you know so the angle is not the same the angle with which the angle this rod makes with the x axis in the S frame okay. It is data and then the angle this rod makes with the S Prime with the x prime axis in the S prime frame that is θ_o okay. So, just check that they are not the same and then this is the relation you can use to find these angles.

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$$\tan \theta_0 = \tan \theta \sqrt{1 - \frac{v^2}{c^2}}$$

$$\begin{aligned} \ell^2 &= (\ell \cos \theta)^2 + (\ell \sin \theta)^2 \\ &= \ell_0^2 \left(1 - \frac{v^2}{c^2} \cos^2 \theta_0\right) \end{aligned}$$

$$\ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2} \cos^2 \theta_0}$$

What about the length remember ℓ was the length in the S frame, so ℓ square you know this very you so identity of $\ell \cos \theta$ square + $\ell \sin \theta$ squared and we know these what is $\ell \cos \theta$ and $\ell \sin \theta$ in terms of the proper length in the S prime coordinates, in the coordinates of the S prime frame. So, we get the thing everything in terms of the proper length in terms of all the quantities in the primed frame okay.

So, interestingly this comes this ℓ that is the length of the rod comes out to be ℓ right, so, ℓ so what is it so in the S frame its mesh it is moving with a certain velocity uniform velocity v . So, that is and the length is measured to be ℓ and that $\ell = \ell_0$ that is the proper length of this rod to root over of $1 - v$ square by c square and now see that you have a \cos , it is $\cos^2 \theta_0$ okay. So, what is θ_0 that is the angle the rod makes with the x prime axis? That is the frame in which it is at rest okay ok.

So, let us spend a couple of minutes with this equation just to see whether it is right or not and whether remember when we were doing some length contraction formula earlier.

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$$\ell = \ell_o \sqrt{1 - \frac{v^2}{c^2} \cos^2 \theta_o}$$

Check

$$1) \theta_o = 0$$

$$\ell = \ell_o \sqrt{1 - \frac{v^2}{c^2}}$$

$$2) \theta_o = \frac{\pi}{2}$$

$$\ell = \ell_o$$

It was $\ell = \ell_o$ into root over $1 - v^2$ by c^2 and then the rod was moving along its length so if we put $\theta_o = 0$, so that that simulates this the system or the rod to move along its length we get back our old line contraction formula okay, $\ell = \ell_o$ into root over $1 - v^2$ by c^2 okay. But what happens we put $\theta_o = 0$ is $= \pi/2$ okay that is perpendicular to its direction.

Well $\ell = \ell_o$ then ok, so if it is moving perpendicular to its direction so the perpendicular component that is not the contracted.

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$$1) \theta_o = 0$$

$$\ell = \ell_o \sqrt{1 - \frac{v^2}{c^2}}$$

$$2) \theta_o = \frac{\pi}{2}$$

$$\ell = \ell_o$$

• Component of length **parallel** to direction of motion is **length contracted** by $\sqrt{1 - v^2/c^2}$

• Component **perpendicular** to the direction of motion is **unaltered**

So, let us be more specific so the component of the length parallel to the direction of motion is length contracted by a certain amount dependent on the velocity ok that go and the quantity is

root over of $1 - v^2/c^2$. And the component perpendicular to the direction of motion is unaltered okay, right, so, that being our idea of length contraction in the next segment in the next lecture.

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Consequences of Special Relativity - II

- 'Time dilation'

We will talk of another consequence of special relativity and that is called time dilatation till then goodbye.