Engineering Physics 1
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Module-06
Lecture-02
Fluid Mechanics - Part 02

So, far I have discussed about the foundational axioms of fluid dynamics namely the conservation laws. Particularly conservation of mass is conservation of linear momentum and conservation of energy these are based on classical mechanics however these things can be incorporated into quantum mechanics as well as general theory of relativity also.

In addition to the above we have also discussed about the quantum as some continuum assumptions of the fluid mechanics where we assume that fluid fluids are assumed to obey the continuum assumptions, where we assume that fluids are composed of molecules that collide with one another and solid objects. How are the continuum assumptions considers fluids to be continuous rather than discrete.

Consequently properties such as density, pressure, temperature and velocity are taken to be well defined at infinite similarly small point known as fluid element and are assumed to vary continuously from one point to another. The fact that the fluid is made up of discrete molecules have been ignored. Second point we discussed that the mechanics part that was for fluids which are sufficiently dense to be continuum.

And have velocities small compared to the speed of light in that case the momentum equations for Newtonian fluids are the Navier Stokes theorem and which is non linear set of differential equations that describes the flow of a fluid whose stress depends linearly on velocity gradient and pressure that we have already discussed. Second point we have discussed we told a little bit that the; those who are complicated problem that means the un simplified equations do not have a general engine and do not have a general closed form solution.

So, in that case, they are primarily handle in computational fluid dynamics. In addition to that we have also discussed the different terminologies in the fluid mechanics namely compressible versus incompressible fluid, viscous forces non viscous fluid, what is the meaning of viscous? What is the meaning of non-viscous fluid? Etcetera, Newtonian fluid, non-eternal fluid these are the things we have discussed till now.

Now we are going to start the a little bit more mathematical the continuity equation in fluid mechanics okay. Let me start it. Before going to fluid mechanics what does it mean by continuity equation? A continuity equation in physics is a differential equation that describe the transport of some kind of conserved quantity and so since example since mass, energy, momentum, electric charge and other natural quantities are conserved a vast variety of physics may be described with common continuity equations.

So, continuity equations are local form of conservation laws all the examples of continuity equation expressed the same idea which states that the total amount of the conserved quantity inside any region can only change by the amount that passes in or out of the region through the boundary. A conserved quantity cannot increase or decrease it can only move from one place to another.

Any continuity equation can be expressed in term of integral form closed form in terms of a flux integral which applies to any finite region or in a different cell form in terms of divergence operator which applies at a point that means we have to use the divergence theorem is if you take any closed surface of any vector that means a dot ds closed surface integral which can be converted to a differential form through the divergence theorem which is nothing but the closed surface integral is related to the volume integral using the divergence theorem.

So, that means any continuity equation can either be expressed in terms of closed integral form or in terms of the in a different cell form using the divergence operator. So, the general form for a continuity equation for any dependent for any physical problem is del Phi by del T plus divergence of a = S where Phi is some quantity where Phi is some quantity A is a vector function depending on the flux of Phi and del is a del operator.

For an example in current continuity equation this is del Rho by del T + divergence of E = 0 in that case there is if you will demand the conservation of electric charge. So, no electric charge can be created or annihilated in that case it will be that. So, an S is a function describing the generation and removal of far terms that generate S greater than 0 or removed S less than 0 are referred to as the source and sinks respectively.

In the case that is a conserved quantity that cannot be created or destroyed such as energy in that case above continuity equation will be reduces to S=0 that means del Phi by del T + divergence of F =0. Since there is no source or sink term. These general equation may be used to derive any continuity equation ranging from as simple as the volume continuity equation to as complicated as the Navier Stokes equation.

That we have already discussed which is another form continuity equation okay. Now let me discuss the continuity equation in the fluid dynamics.

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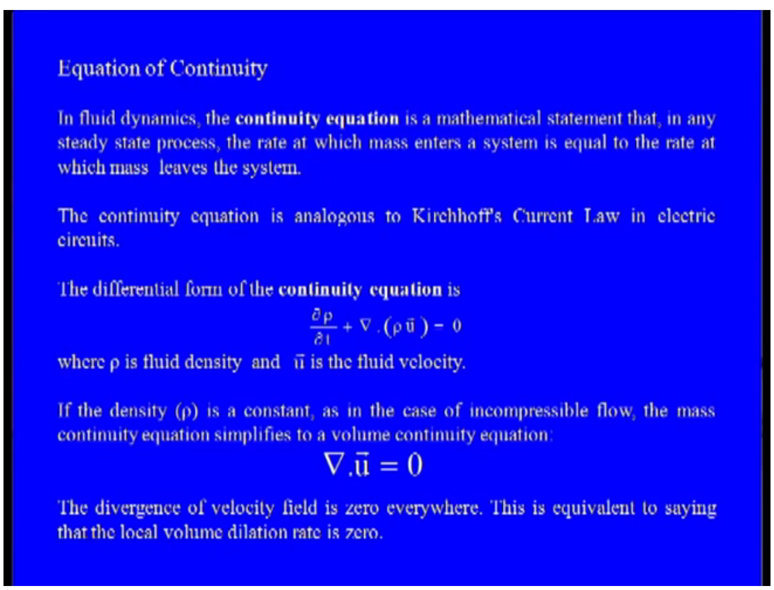

In fluid dynamics the continuity equation is a mathematical statement that in any steady state process the rate at which mass enters a system is = the rate at which mass leaves the system. So, the continuity equation is analogous to Kirchhoff's Current Law in elliptic circuit. The differential form of continuity equation in fluid dynamics is as follows del Rho upon del t +

divergence of Rho u = 0. The left hand side is nothing but the total time derivative of Rho, d Rho by dt = del Rho upon del t + divergence of Rho u = 0, since there is no source or sink a Rho is the fluid density and u is the fluid velocity vector.

If the density is a constant as in the case of incompressible flow the mass continuity equation simplifies to a volume continuity equation where we can take outside Rho divergence of Rho u = divergence grad Rho u + u divergence of u. Since Rho does not depend Rho does not change in that case grad Rho will be the 0. So, in that case divergence of u = 0. So, these divergence of velocity field is 0 everywhere this is equivalent to saying that the local volume dilation rate is 0.

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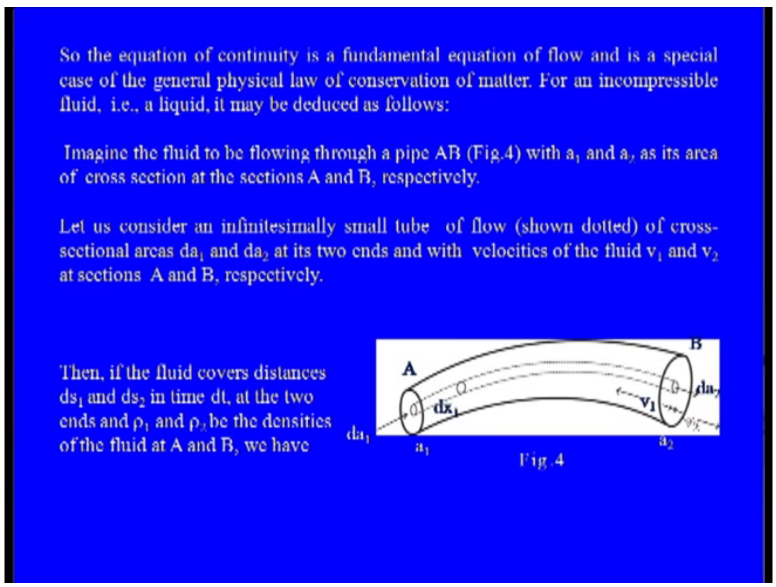

So, the equation of continuity is a fundamental equation of flow and is a special case of general physical law of conservation of matter. For an incompressible fluid that means it is a liquid it may be deduced as follows. Let us take some figure 4, so imagine the fluid to be flowing through A pipe a B where a1 and a2 small a1and small a2 as denotes its area of cross section at the section A and B respectively just see this figure.

Let us consider an infinitesimally small tube of flow which is which is shown on dotted in the figure of cross sectional area da1 and da2 at its two ends and with velocities of the fluid v1 and v2 at this sections say A and B respectively. Then the fluid covers a distance ds1 and ds2 in time dt at the two ends and Rho 1 and Rho 2 are the densities of the fluid at A and B respectively.

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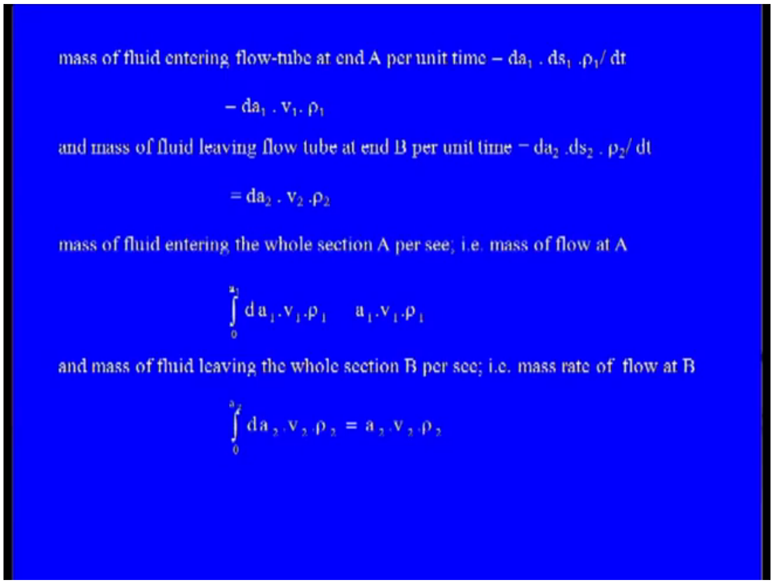

In that case the mass of the fluid entering flow tube at end A per unit time is da1 which is the infinity symbol area times ds1 the distance traverse in infinitesimal time dt into Rho 1 divided by dt because da1, ds1 will give you the volume, multiplied by density will give you the mass which is in dt time. So, that is the; there is an mass of fluid entering flow tube at the end per unit time.

So, which is nothing but the da1 if ds1 by dt you can write v1, so da1 into v1 into Rho1. Similarly you can write down the mass of fluid leaving flow tube at end B per unit time is da2 times ds2 times Rho 2 which is nothing but the mass because da2 is the infinitesimal volume element ds2 is the infinite single length traverse in time dt. So, da2 time ds2 will give you the volume times Rho 2 will give you the mass.

This mass flow occurs in time dt, so mass flow per unit time is by dt, so which you can write down in terms of velocity is da2 times v2, v2 is nothing but the velocity at the section B because ds2 by dt is nothing but the v2 times Rho 2. So, total now let me calculate in total mass, mass of fluid entering the whole section A. So, the mass of the fluid entering the whole section A per second that is mass of flow at A you just integrate it of our da1.

So, what you will get 0 to a1 da1 v1 times Rho 1, so if you will integrate 0 to a1 da1 you will get a1, so result is a1 times v1 times Rho 1. Similarly you can calculate mass of fluid leaving the hole section B per second that is mass rate of flow at B again you integrate over da2 what you will get 0 to a2 da2 v2 times Rho 2 integration over a2 will gives you da2 gives you the a2 because b2 and Rho 2 will not depend on a2. So, the result is a2 times v2 times Rho2.

So, this is the mass of fluid mass flow at a which is a1 v1 into Rho 1, similarly mass flow at mass rate of flow at B is a2 b2 times Rho2. Since, the fluid is incompressible so velocity will not change from A to B, so Rho 1 is the density of fluid entering at A, Rho 2 is the density of fluid at b2 at B.

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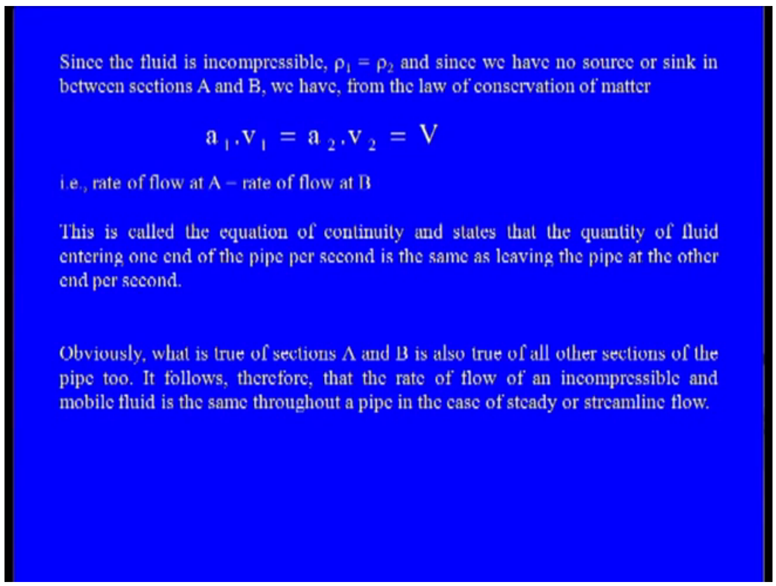

So, since fluid is incompressible, so density will not change, so Rho 1 must be = Rho 2 and since we have no source or sink in between sections A and B. So, we have from the law of conservation of matter = a1 v1 = a2 v2 = let us say B because since Rho1= Rho 2, so it will cancel from left hand side and right hand side. So, conclusion is that rate of flow at A should is = rate of flow at B this is called the equation of continuity.

And states that the quantity of fluid entering one end of the pipe per second is the same as leaving the pipe at the other end per second obviously what is true of sections A and B is also true for all other sections of the pipe so it follows therefore that the rate of flow of an incompressible and mobile fluid is the same throughout a pipe in the case of steady and streamline flow.

So, be careful this is true only for a steady or streamline flow, if the flow becomes turbulent so in that case this equation does not hold good.

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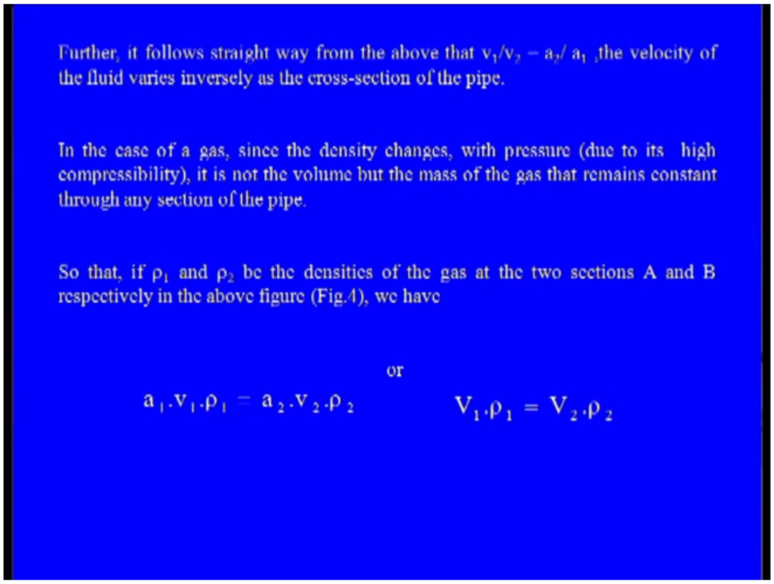

Further it follows straight away from the above that v1 if you take from the continuity equation v1 a1 = v2 a2 you will get it the ratio of velocities at the two sections A and B which is v1 upon v2 =a2 upon a1 which is inversely proportional to the area of cross section. So, the velocity of the fluid varies inversely as the cross section of the pipe. Suppose in one end of the pipe if the cross section is small the velocity of the fluid passing through that cross section will be much larger compared to the velocity of a bigger cross section.

So, in the case of a gas since the density change with pressure due to its high compressibility it is not the volume but the mass of the gas that remains constant through any section of the pipe. So, in that means in the case of a gas you cannot say Rho 1 = Rho 2 in that case Rho cannot be canceled from the light hand side to the left hand side. In that case main equations means a1 v1 Rho 1 must be = a2 v2 Rho 2 and we know a1 v1 into Rho 1 is nothing but the mass at the section A.

And a2 v2 Rho 2 is the mass of the fluid at section B. So, instead of a1 v1 the total equation should hold so that means mass should remain the constant in both ends. So, let us see if Rho a

Rho 1 and Rho 2 be the densities of the gas at the two sections A and B respectively. In that case a1 v1 Rho 1 = a2 v2 Rho 2 or a a1 and v1 let us call capital V1 which is the mass flow. So, V1 times Rho 1 = V2 times Rho 2.

So, now let us see, now let us calculate the energy of a liquid or in a flow when it is in motion. So, let us calculate what will be the energy of a liquid in a flow.

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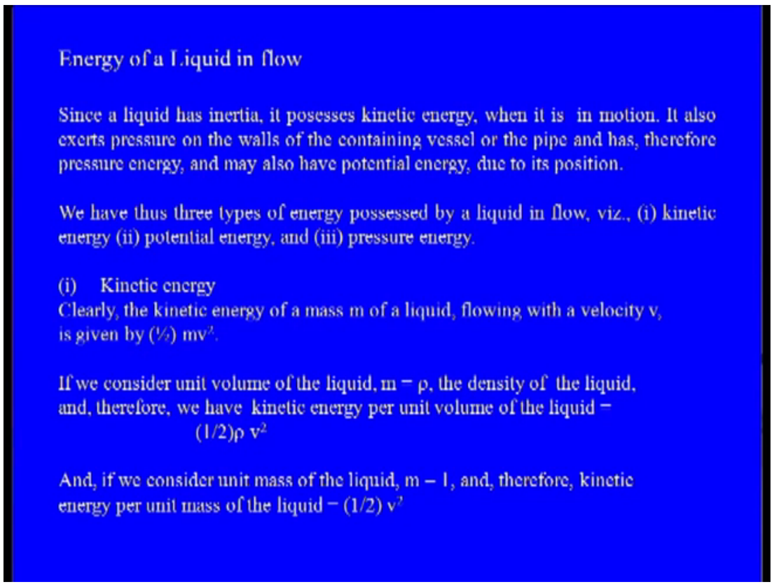

Since a liquid has inertia so it must voyages kinetic energy when it is in motion. It also exerts pressure on the walls of the containing vessel or the pipe and has therefore pressure energy and may also have potential energy due to its position. So, we have thus three types of energy possessed by liquid in a flow namely the kinetic energy due to motion, the potential energy and the pressure energy.

Let us calculate one by one let us calculate first kinetic energy. Clearly the kinetic energy of a mass m of a liquid flowing with a velocity v is given by half mv square. If we consider unit volume of the liquid in that case m = Rho the density of the liquid and therefore we have kinetic energy per unit volume of the liquid is half Rho v square okay.

So, and if we consider unit mass of the liquid means if you put m = 1 therefore the kinetic energy per unit mass of the liquid will be simply half v square because in that case Rho will be 1. So, kinetic energy will be simply half v square.

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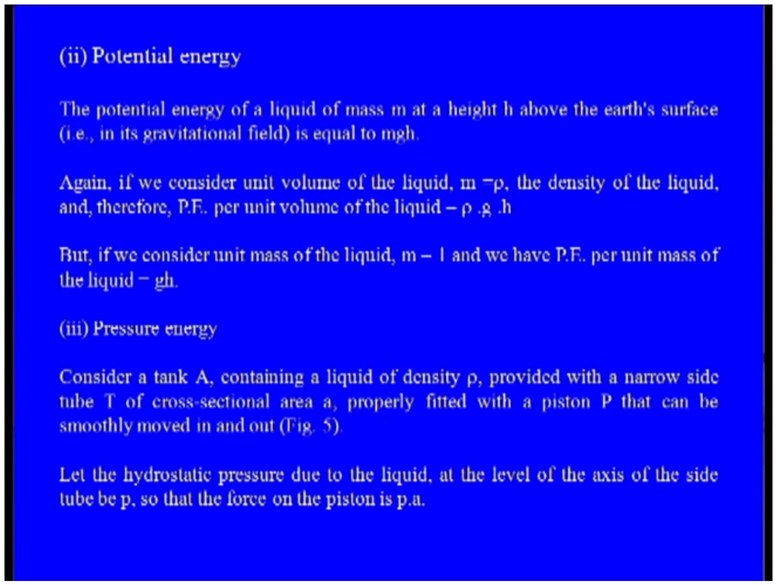

So, now let us calculate the potential energy the potential energy of a liquid of mass m at a height h above the art surface that is in its gravitational field is = mgh as you know already. And again if you will consider unit volume of the liquid in that case mass will be the Rho the density of the liquid therefore the potential energy per unit volume of the liquid will be simply Rho gh because of the unit volume mass will be replaced by the density of the liquid.

But if you will consider unit mass of the liquid, so m = 1 in that case or potential energy per unit mass of the liquid will be simply gh in that case Rho will be replaced by 1. So, last calculate the pressure energy, now let consider a tank A see in the figure in figure 5 consider a tank A containing a liquid of density Rho provided with a narrow side tube T of cross sectional area small a properly fitted with the piston P that can be smoothly moved in and out okay.

Let the hydrostatic pressure due to the liquid at the level of the axis of the side tube P. So, that the pressure, so that the force on the piston is p times a, because force per unit area is nothing but the pressure since the cross sectional area is a, and pressure is p. so, the force on the piston is p times a.

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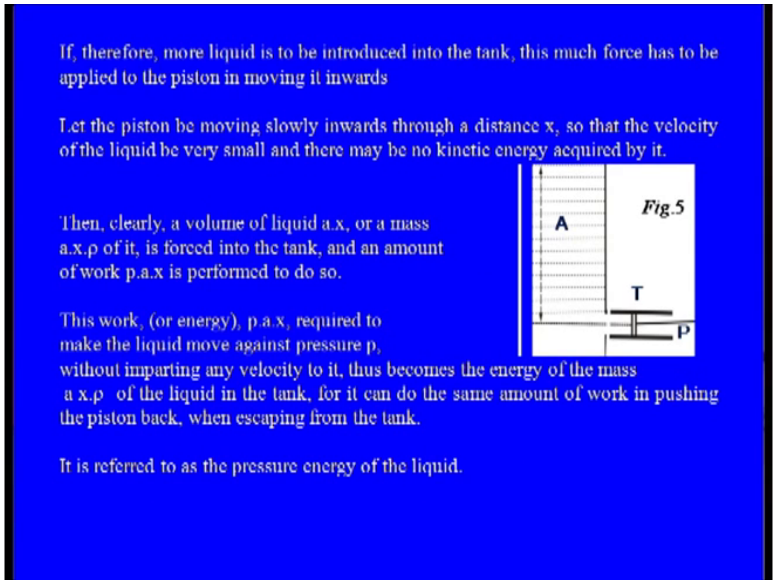

If therefore more liquids is to be introduced into the tank this much force has to be applied to the piston in moving it inwards. So, let the piston be moving slowly through a distance dx, so that the velocity of the liquid be very small and there may be no kinetic energy acquired by it. Then clearly a volume of liquid a times xa is the cross sectional area or a mass air times x times Rho because Rho is the density of the liquid of it is forced into the tank.

And an amount of work p times a times x is performed to do so because pa is the force times x force into distance will give you the amount of work. So, this work which is p times a times x required to make the liquid move against pressure p. Without imparting any velocity to it thus become the energy of the mass a times x times Rho of the liquid in the tank, for it can do so the same amount of work in pushing the piston back. When escaping from the tank it is referred to as the pressure energy of the liquid.

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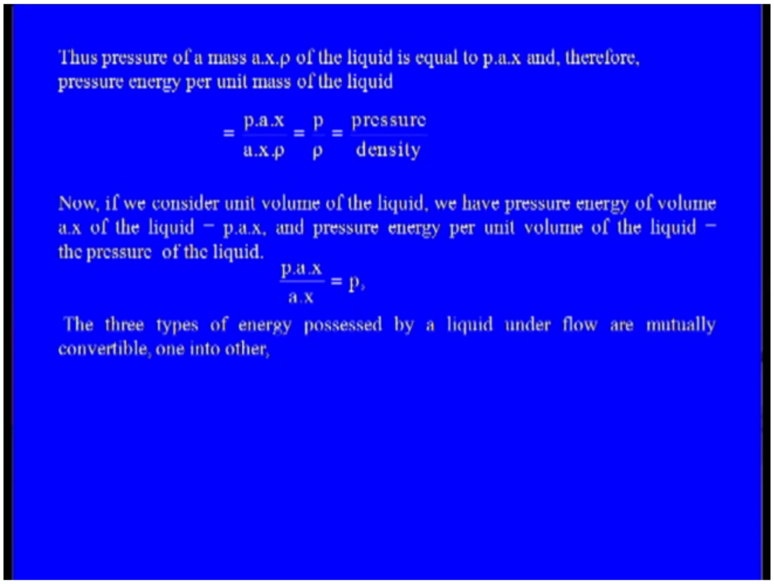

Thus the pressure of a mass a times x times Rho of the liquid is = p times a times x and therefore pressure energy per unit mass of the liquid is p times a times x divided by the mass which is a times x times Rho which is nothing but the p by Rho that means pressure by density. Now if you will consider unit volume of the liquid we have pressure energy of the volume ax of the liquid is p times a times x.

And pressure energy per unit volume of the liquid is p times a time x by a into x which is simply p so the pressure of the liquid. The three types of energy as we have discussed kinetic energy; potential energy and pressure energy possessed by liquid in motion under in motion or underflow are mutually convertible one into another. So now let us discuss this point one by one.

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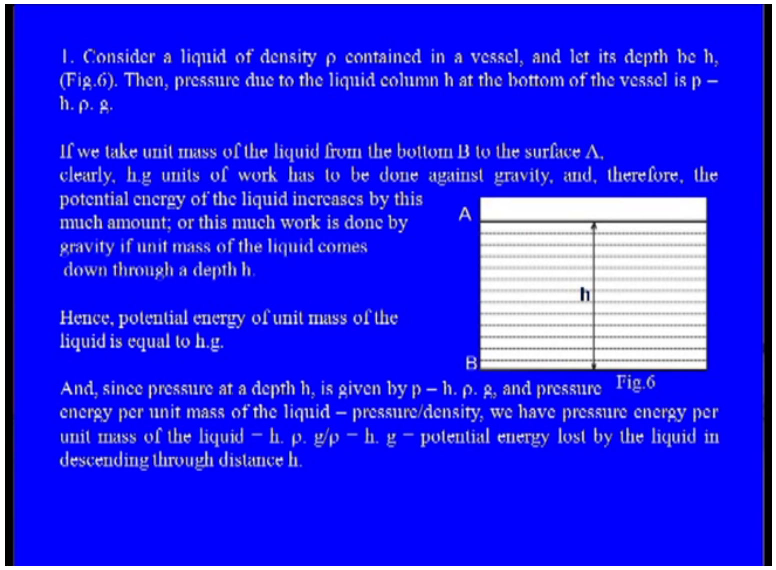

Suppose let us take considered a liquid of density Rho contained in a vessel and let its depth be h. Then pressure due to the liquid column h at the bottom of the vessel will be p = h times Rho times hg where g is the gravitational acceleration, acceleration due to gravity. So, if you take unit mass of the liquid from the bottom B to the surface A. Clearly a unit mass so that means clearly h times g unit of work has to be done against gravity because you want to bring it from the bottom to the surface A.

So, you have to work has to be done against gravity and therefore the potential that work done against gravity will be stored into the liquid. So, therefore the potential energy of the liquid increases by this amount or this much work is done by the by gravity if unit mass of the liquid comes down through a depth h. Hence the potential energy of unit mass of the liquid is = h times g.

And since pressure at a depth p is p = h times Rho times g and pressure energy per unit mass of the fluid is pressure by density as we have already seen. We have pressure energy per unit mass of the liquid = h Rho g by Rho which is nothing but the h times g which is nothing but the potential energy lost by the liquid in decreasing through distance h.

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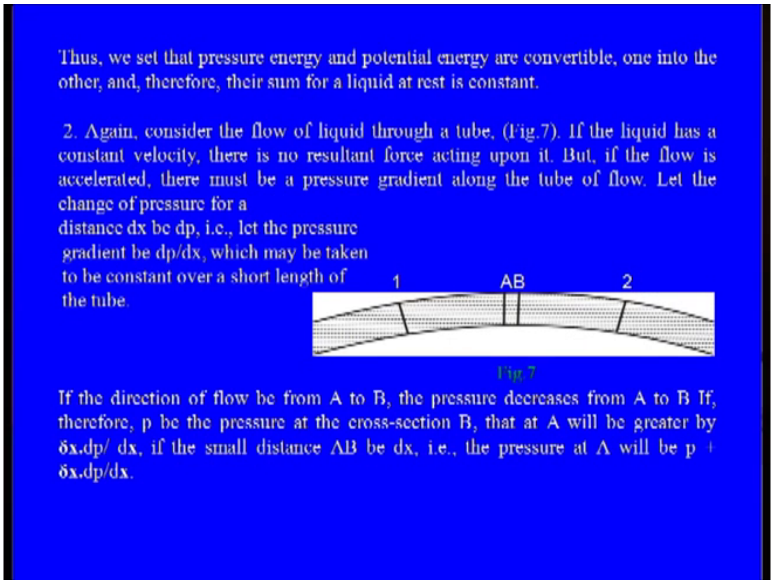

Thus concludes we can conclude it that the pressure energy and potential energy are convertible one into other and therefore they are some for a liquid at rest each constant. Now let us take other example to clarify this statement. Again consider the flow of a liquid through a tube okay. Now let us take a small cross sectional area AB. If the liquid has a constant velocity there is no resultant force acting upon it.

But if the flow is accelerated there must be a pressure gradient along the tube of the flow. Let the change of a pressure for a distance dx be dp. So, let the pressure gradient be dp by dx which may be taken to be constant over a short length of the tube. If the direction of flow B from A to B the pressure decreases from A to B, if therefore B, p be the pressure at the cross section B. So, at A it will be greater than by delta x into dp by dx. If this small distance A to B is dx that is the pressure at a will be p + del x into dp by dx.

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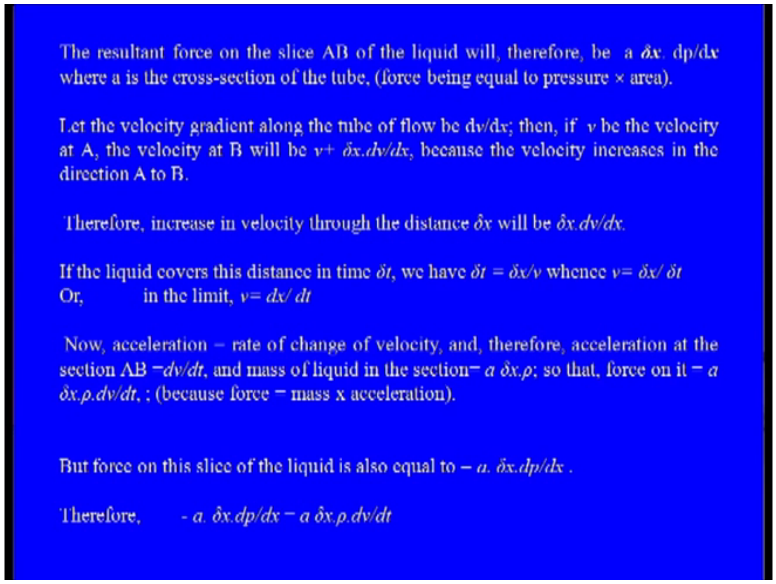

So, the resultant force on this slice AB of the liquid will therefore be a times del x into del p by del x because del x into del p by del x that is the excess pressure, force will be pressure into area so that is the reason a times del x into dp by dx where a is the cross section of the tube. Let the velocity gradient along the tube of flow be dv by dx in that case if v be the velocity at A the velocity at B will be v + del x dv by dx.

Because the velocity increases in the direction A to B, So, therefore increase in velocity through the distance del x will be del x into dv by dx if the liquid covers this distance in time t we have del t = del x upon v where v = del x by del t or in the limit v = dx by dt, dt tends to 0. So, now acceleration = rate of change of velocity and therefore acceleration at the section AB = dv by dt and mass of the liquid in this section is a times del x into Rho.

So, that force on it is a into del x Rho dv by dt because force = mass times acceleration. So, mass is a into del x into Rho this is the mass, acceleration is dv by dt but force on this slice of the liquid is also = a due to pressure gradient which is a del x into del p by del x, so, these two things must equal so by equaling these two equations a del x del p by del x which is due to the pressure gradient.

This force is due to the pressure gradient minus sign is pressure decreases which is = the mass times acceleration a del x Rho dv by dt.

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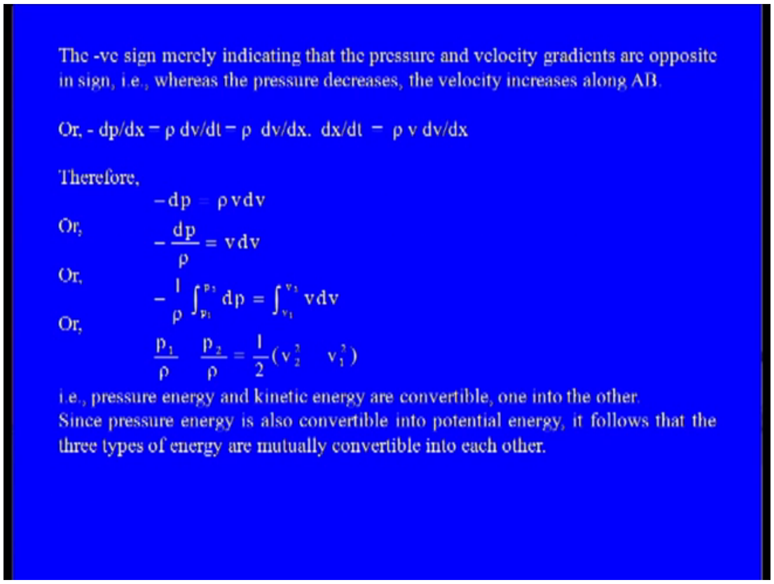

The negatives are merely indicating the pressure and velocity gradient are opposite in sign that is when the pressure decreases the velocity increases along B okay this is the meaning of the negative sign. So, from that equation let us say from that equation, so if you can cancel a from both sides and del x you can cancel also from both sides. So, finally we will get - dp by dx = Rho dv by dt.

So, if you will decompose dv by dt what you will get Rho dv upon dx into dx into dt, so since dx into dt, dx by dt is v, so Rho v dv by dx, so final expression is - dp by dx = Rho v dv by dx, so finally we will get - dp = Rho v dv, if you integrate both sides so ranges from the p1 to p2 to v1 to v2. If the pressure changes from p1 to p2 correspondingly accordingly the velocity changes from v1 to v2.

So, we get p1 by Rho - p2 by Rho you assume that velocity are not changing = half v2 square - v1 square that is the pressure energy and kinetic energy are convertible one into other. Since the pressure energy is also convertible into potential energy it follows that the three types of energy kinetic energy, potential energy and pressure energy. These are all mutually convertible to into each other.

So, from this way we got this argument from this simple example we understood that all three types of energy are mutually convertible into each other. So, now we are in a position to state one of the most remarkable theorems in fluid mechanics which is the Bernoulli's theorem.

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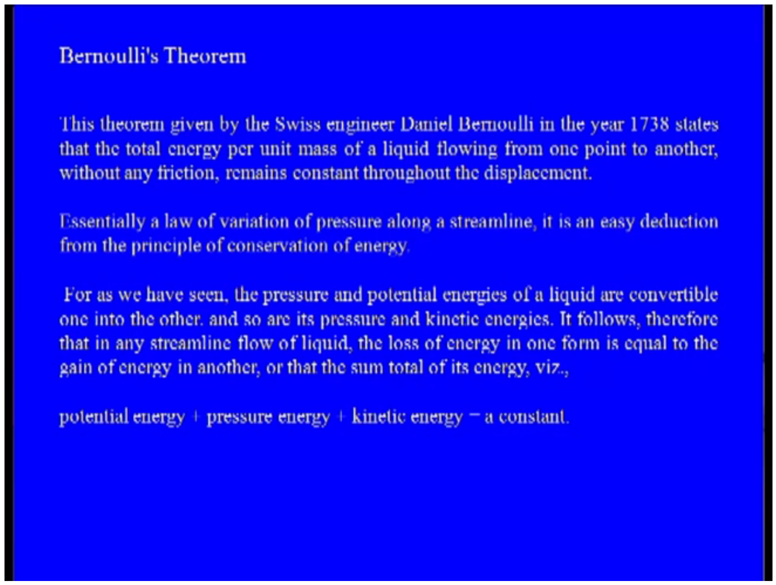

So, this theorem given by the Swiss engineer Daniel Bernoulli in the year 1738 which states that the total energy per unit mass of a liquid flowing from one point to another without any friction remains constant throughout the displacement, essentially a law of variation of pressure along a streamline it is an easy deduction from the principle of conservation of energy or other way around Bernoulli’s theorem is nothing but a conservation of energy in other form in the case of a fluid.

For as we have seen the pressure and potential energy of a liquid are convertible into the other. So, are the pressure and kinetic energy it follows therefore that in any streamlined flow of liquid be careful this statement, the loss of energy in one form is = the gain of energy in other. Or that the sum total of its energy namely potential energy plus pressure energy plus kinetic energy will remain constant.

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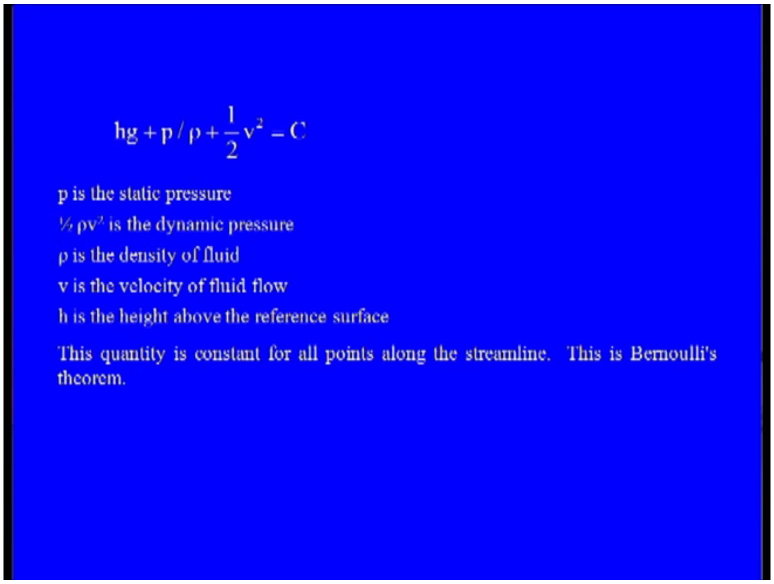

In mathematical form hg +p upon Rho + half v square that means potential energy plus pressure energy plus kinetic energy will be constant. Let us constant c, so p is the static pressure half if you will multiply by Rho, so you will get its more clearly p + half Rho v square + h Rho g = constant. In that case it looks more elegant. So, half Rho v square is the dynamic pressure, Rho is the density of fluid, v is the velocity of the fluid flow, h is the height above the reference surface these quantities constant for all points along the streamline. So, this is Bernoulli's theorem.