Engineering Physics 1

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Module-01
Lecture 03
Double Refraction

Okay, this is the third lecture of the five lecture series on polarization. In this last lecture which was the second one, we studied Malus law.

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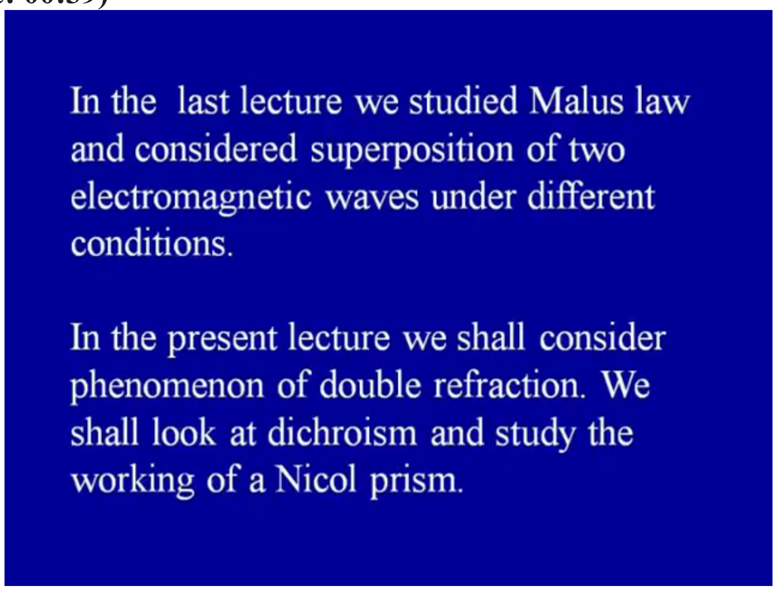

Which defines the intensity of the transmitted beam when a plane polarized light passes through a polarizer, this intensity is proportional to the square of the cos of the angle between the direction of vibration in the incident light and the past direction of the analyzer. We also considered superposition of two electromagnetic waves under different conditions.

That it waves having electric field in the same direction, or electric field in perpendicular direction and with varying phase difference between them. In the present lecture, which is the third one in the series, we shall consider phenomena of double refraction; we shall look at dichroism and study the working of Nicol prism. Let us start with the double refraction. Now, the superposition of two plane polarized waves under different conditions.

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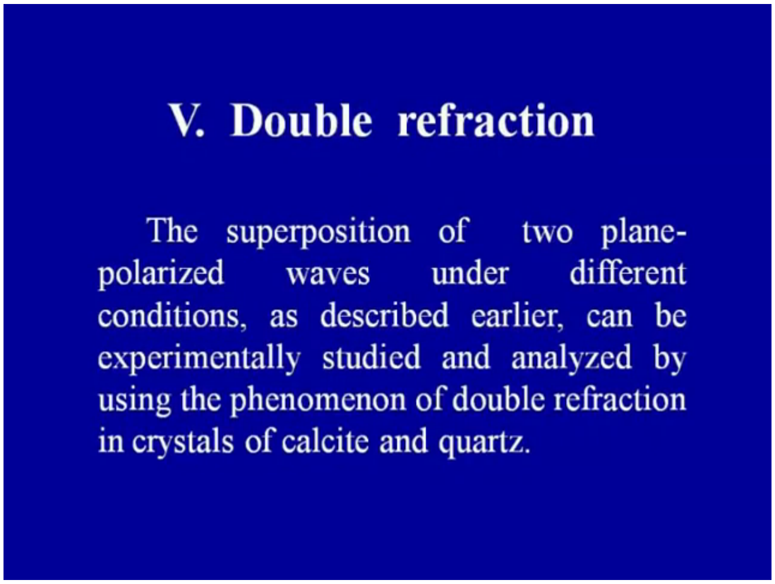

As described earlier in the last lecture can be experimentally studied and analyzed by using the phenomena of double refraction and crystals of calcite and quartz.

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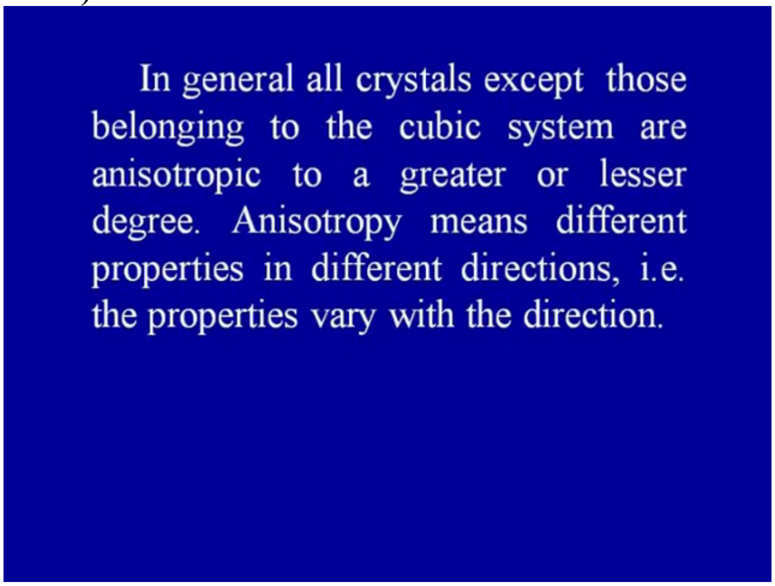

In general, all crystals except those belonging to the cubic system or anisotropic to a greater or lesser degree. Anisotropic means different properties in different directions that is, the properties vary with the direction. Calcite and quartz are very common examples of anisotropic crystals.

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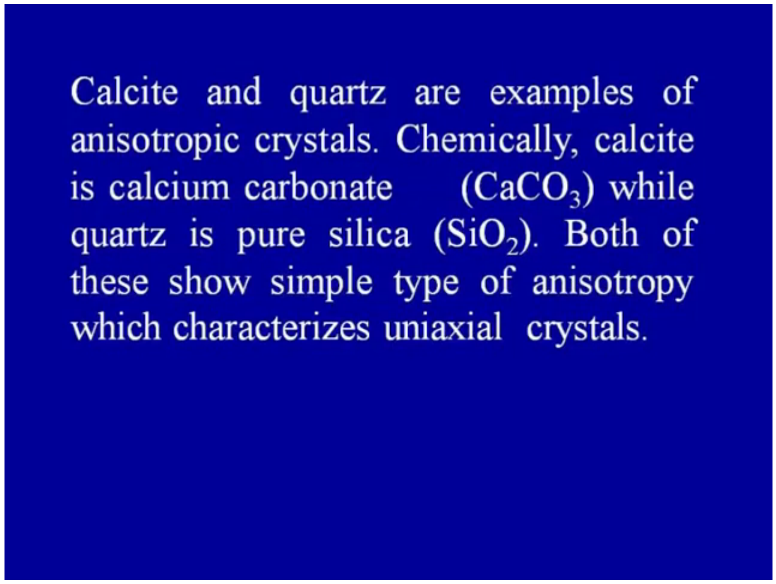

Chemically calcite is calcium carbonate CaCo3, while quartz is a pure silica, silicon dioxide; both of these show simple type of anisotropy which characterizes uniaxial crystals.

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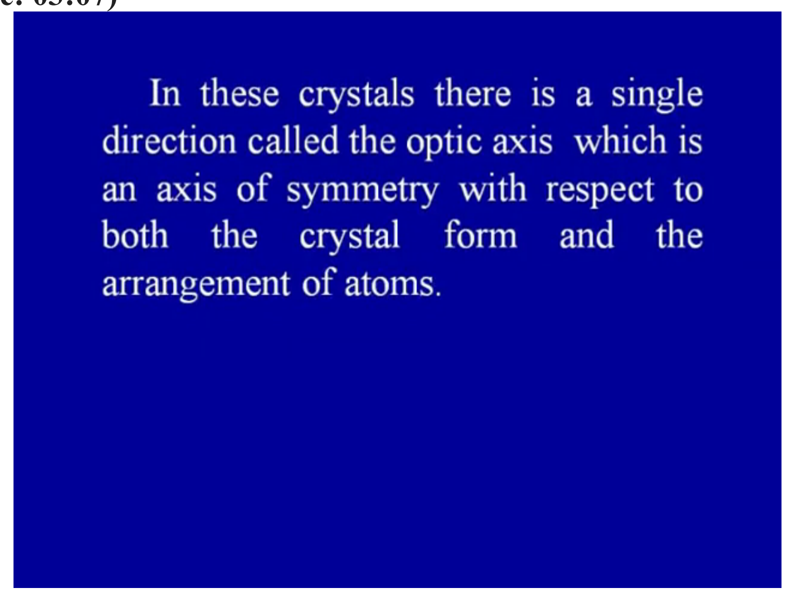

In these crystals, there is a single direction called the optic axis which is an axis of symmetry with respect to both the crystal form and the arrangement of atoms. If any property such as for example heat conductivity is measured for different directions.

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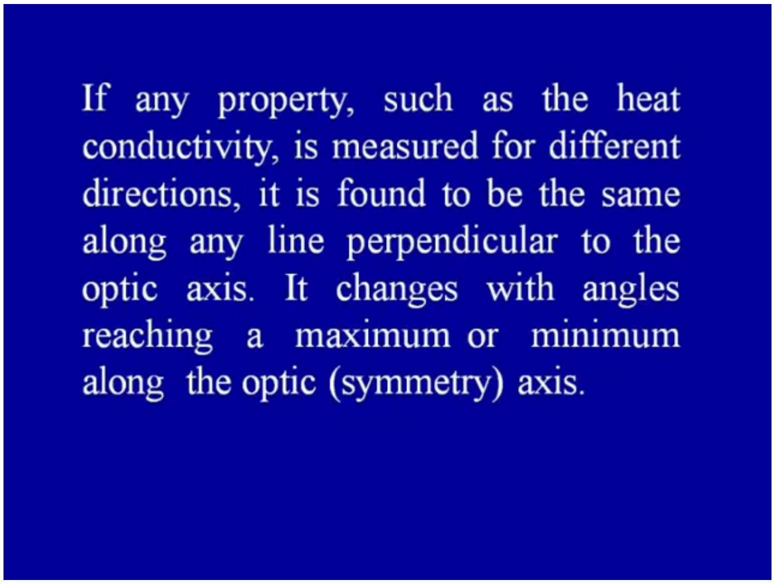

In such a crystal it is found to be the same along any line perpendicular to the optic axis which is the symmetry axis. It changes with angles reaching a maximum or minimum along the optic axis. Now, both of these crystals the calcite and quartz are transparent to visible as well as ultraviolet light. That is why they are of interest to us in this study. When it beam of ordinary unpolarized light is incident on a calcite or quartz crystal.

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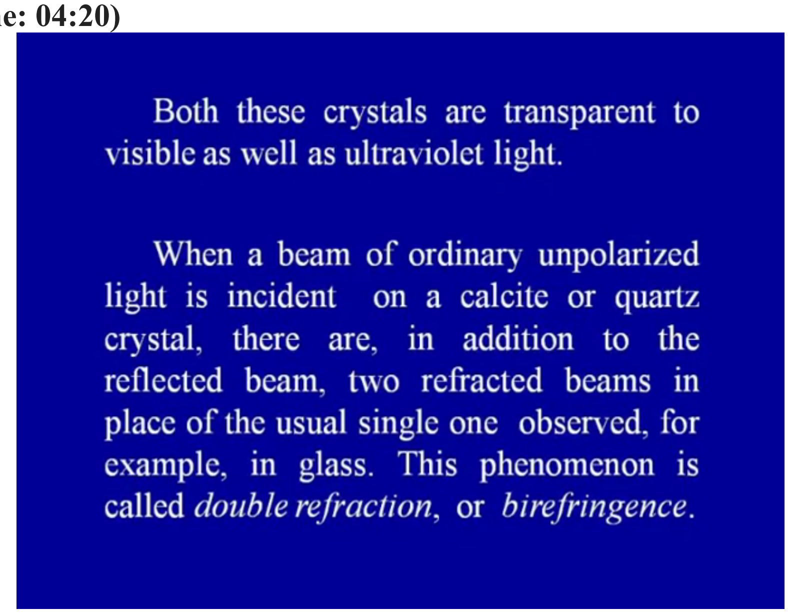

There are in addition to the reflected beam, two refracted beams. Remember, two refracted beams in place of the usual single one. For example, which is observed for example, in glass or any other isotropic material, this phenomena is called double refraction or birefringence.

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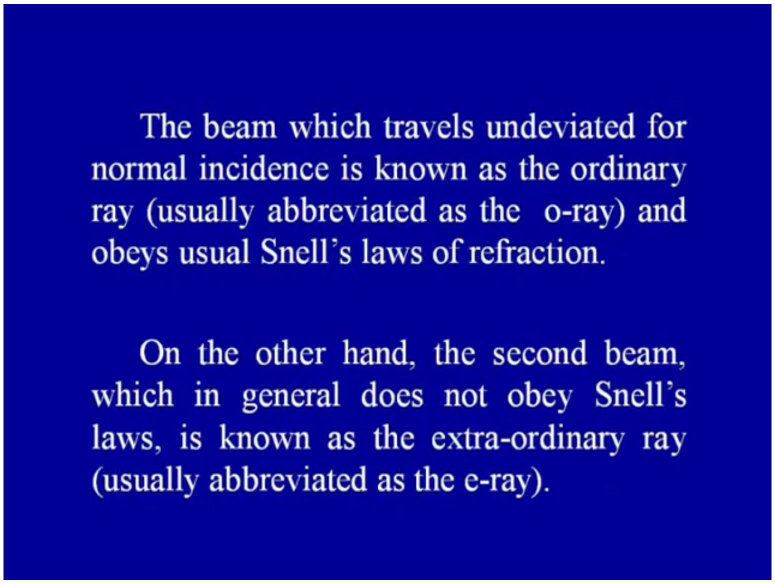

The beam which travels undeviated for normal incidence is known as the ordinary ray usually abbreviated as the o-ray and obeys Snell's law of refraction. Snell's law is the ratio of the sine of the angle of incidence to the sine of the angle of refraction, is constant; does not depend on the angle of incidence. It also does not depend on the direction of propagation. And the incident ray, the refracted ray and normal to the surface, they are all in the same plane.

On the other hand the second beam second refracted beam which in general does not obey Snell's laws is known as the extraordinary ray, usually abbreviated as the e-ray.

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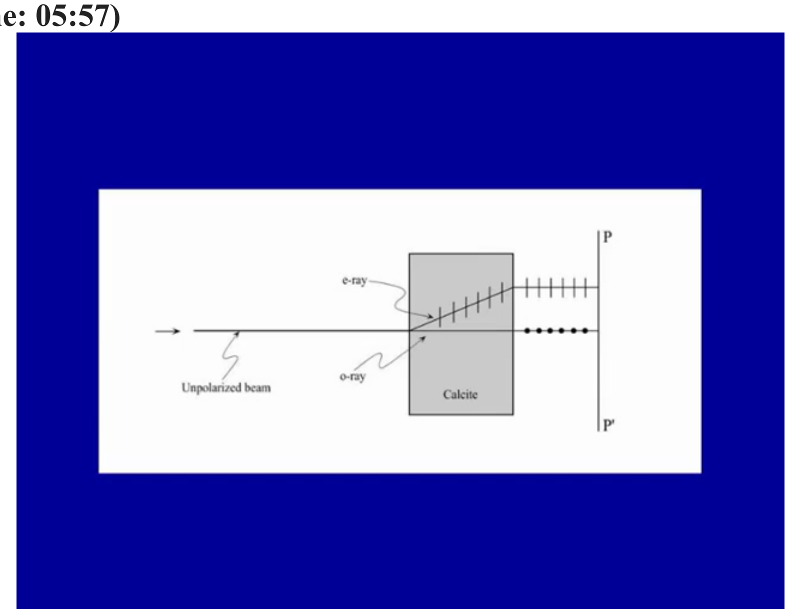

Here the incident beam is getting spitted into two. One is called the ordinary; the other is called the extraordinary. If the two opposite phases of this crystal or parallel, I have shown in the figure.

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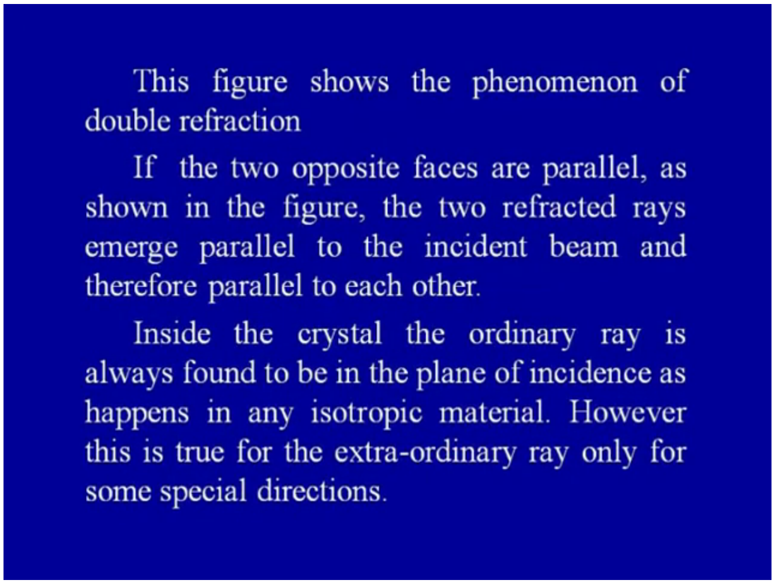

The two refracted rays emerged parallel to the incident beam and therefore parallel to each other. Inside the crystal however, the ordinary ray is always found to be in the plane of incidence, as happens in any isotropic material. That is a normal situation which is very common. However, this is true for the extraordinary ray only for some special directions. In general, this is not true for the extraordinary ray.

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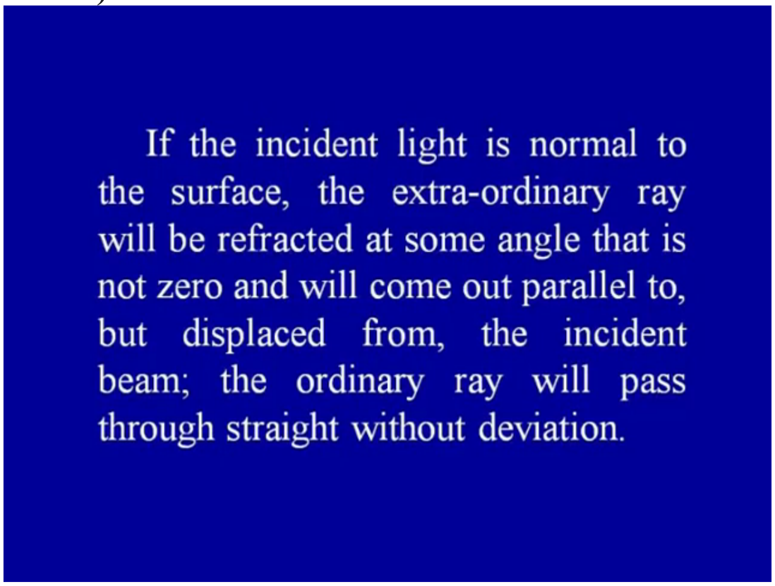

If the incident light is normal to the surface of the crystal the extraordinary ray will be reflected at some angles, that is not zero. Note this is a very special feature. The incident beam is normal the extraordinary ray will be reflected at some angle that is not zero. And will come out parallel to but displaced from the incident beam. The ordinary ray will pass through straight underweighted which is usually the case, which it really should be the case because the angle of incidence is zero. The angle of reflection cannot be anything but zero.

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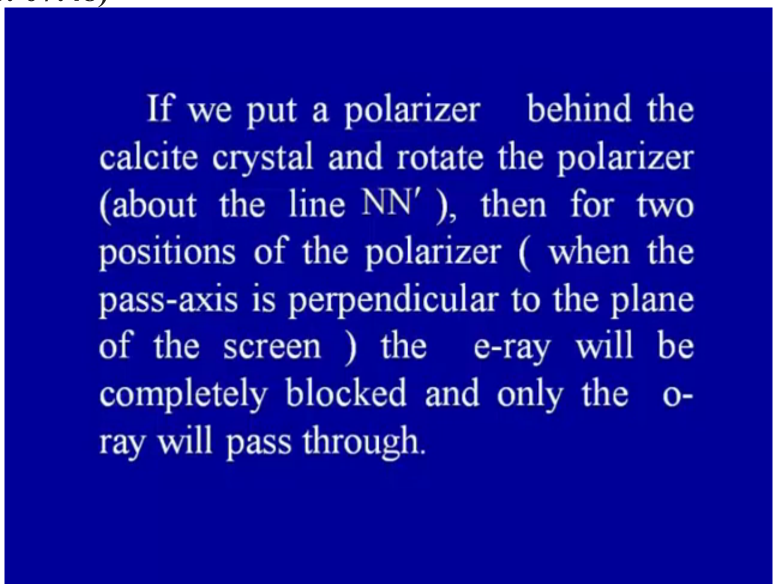

Now, if we put a polarizer behind the calcite crystal and rotate the polarizer about the line NN prime, then, for two positions of the polarizer, when the passed axis is perpendicular to the plane of the screen the e-ray will be completely blocked. And only the o-ray will pass through because the direction of vibrations in the o-ray is along the pass direction of the polarizer that is getting through.

For the extraordinary ray, the direction of vibrations is perpendicular to the pass direction of the polarizer it is getting blocked. Remember, Malus law goes of 90 degrees is zero.

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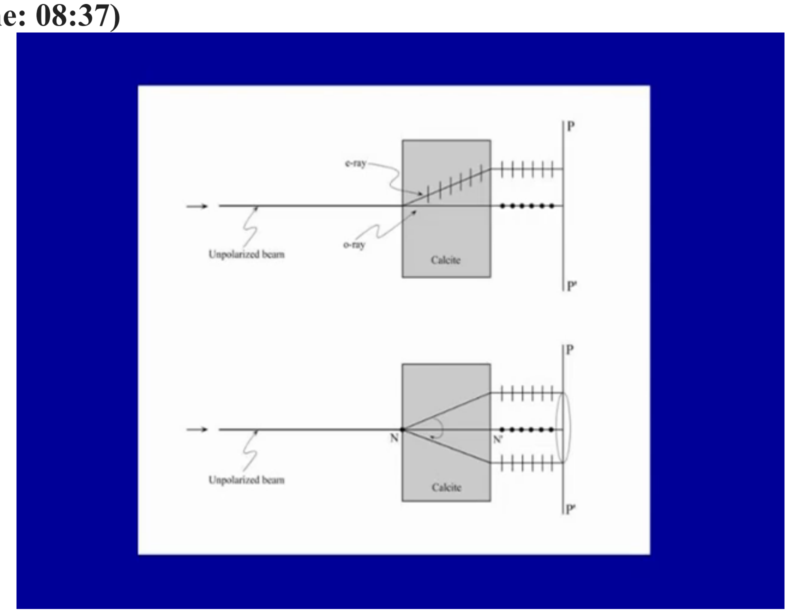

That is the situation in this figure, on the other hand, when the pass axis of the polarizer is in the plane of the screen along the line PP prime.

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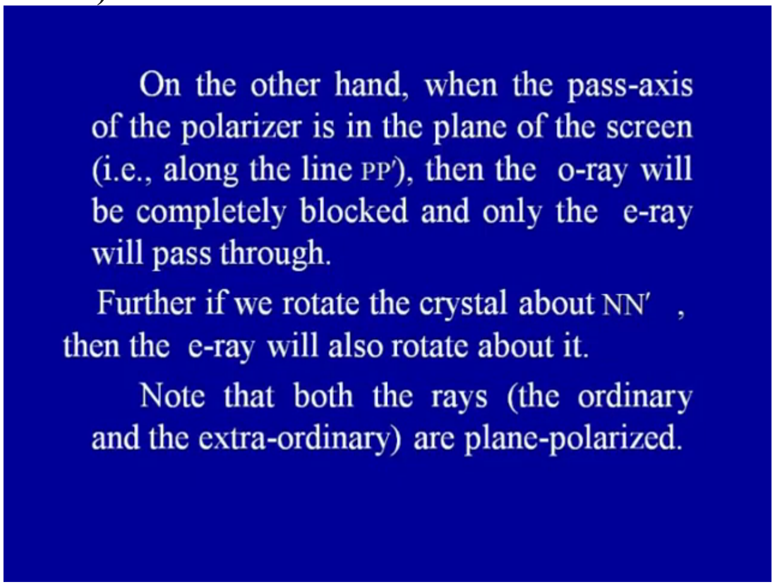

Then, the o-ray will be completely blocked and only the e-ray will pass through. Situation has just interchanged for them if we rotate the crystal about NN prime, then, the e-ray will also rotate about it. Actually the tip of the extraordinary ray will describe a circle whose center will be given by the ordinary ray. Another thing to be noted, both the rays, the ordinary and the extraordinary are plane polarized. Remember, the incident beam was unpolarised.

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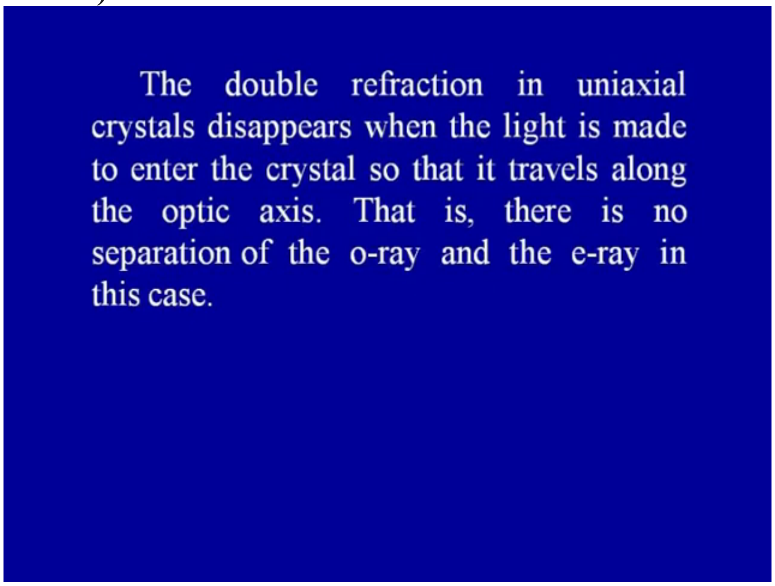

The double refraction in uniaxial crystals disappears, when the light is made to enter the crystals, so that it travels along the optic axis which is the symmetry axis. That is the reason, no separation of the ordinary and extraordinary ray in this case. This is the case when the propagation is along the optic axis.

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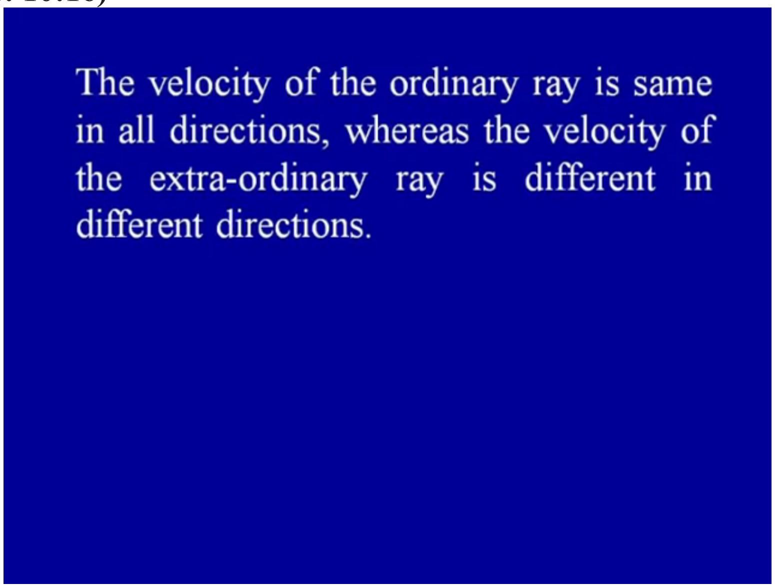

The velocity of the ordinary ray is same in all directions. This is a very usual case that does not depend on the directions as it happens in all isotropic materials; whereas the velocity of the extraordinary ray is different in different directions. These are given by the following equations:

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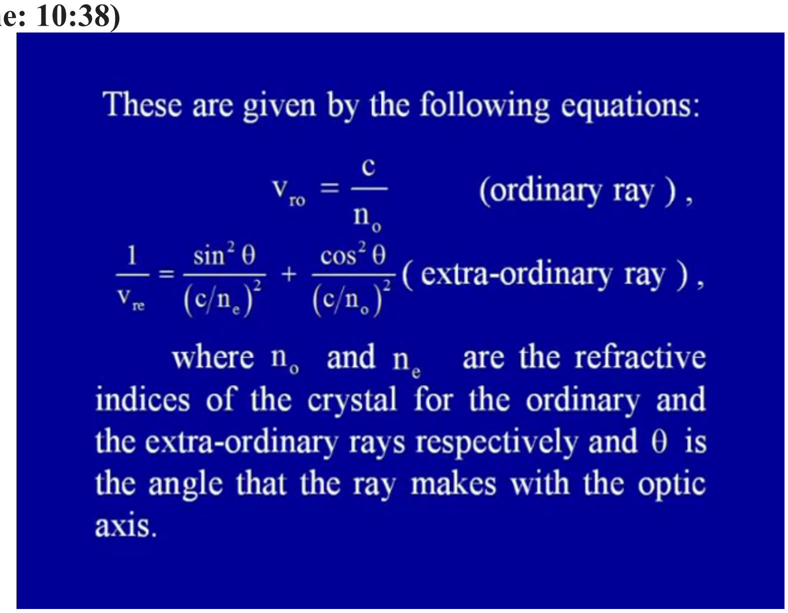

v of ro = c upon no, this is for the ordinary ray 1 upon v of re that is the inverse of the velocity for the extraordinary ray is given by sine square theta divided by c by ne square + cos square theta divided by c of no square this for the extraordinary ray here. And no and ne are the refractive indices of the crystal for the ordinary and for the extraordinary respectively. And theta is the angle that the ray makes with the optic axis. We have assumed the optic axis to be parallel to the z axis here.

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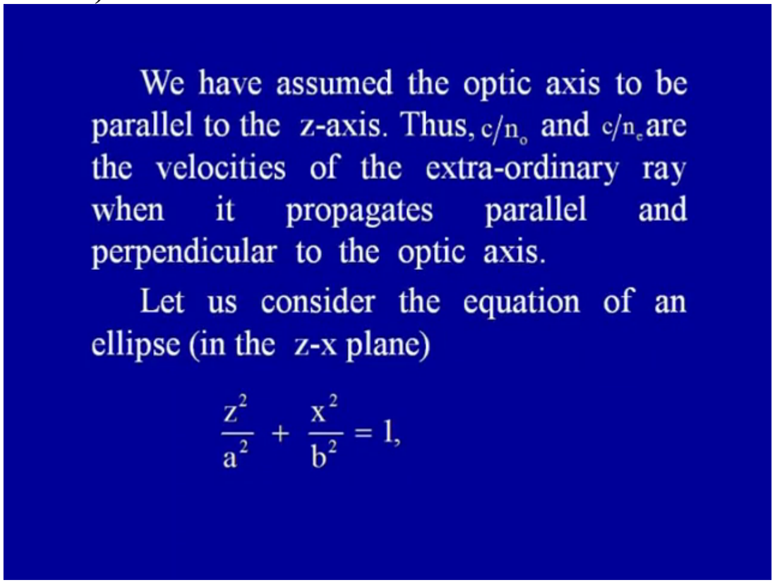

The c by no and c by ne are the velocities of the extraordinary ray when it propagates parallel and perpendicular to the optic axis; parallel to the optic axis means theta =0. Perpendicular to the optic axis means theta equal to Pi by 2. Let us consider the equation of an ellipse in the zx plane. That is z square upon a square + x square upon b square = 1 where a and b are proportional to the velocities of the ordinary and the extraordinary rays, c upon no and c upon ne respectively.

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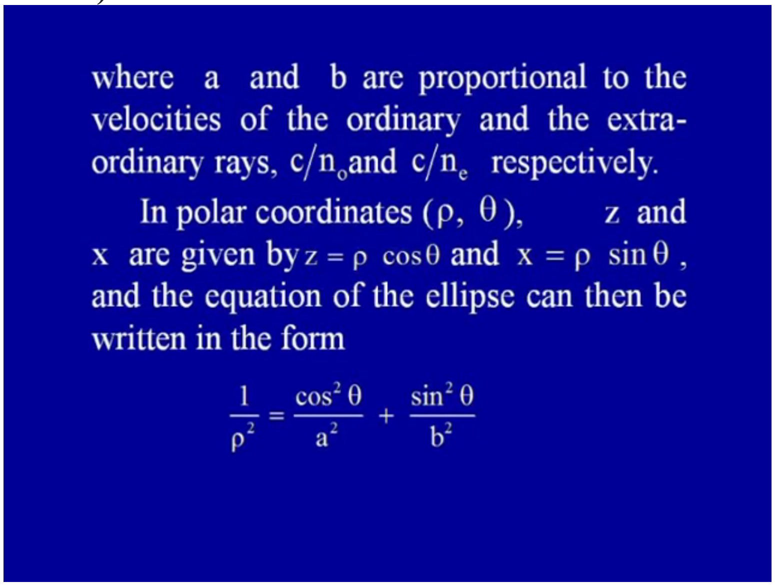

In polar coordinates Rho theta z and x are given by z = Rho cos theta and x = Rho sine theta and the equation of the ellipse which is given above can then be written in the form, 1 upon Rho square = cos is square theta upon a square + sine square theta upon b square.

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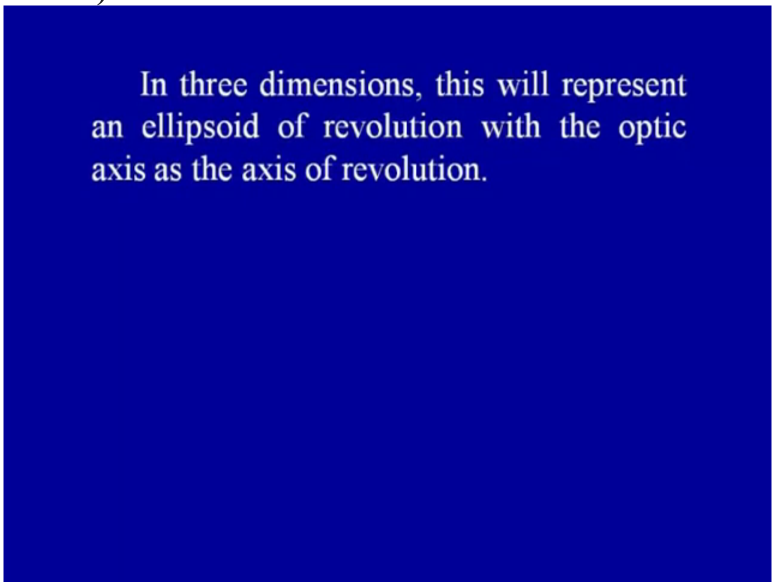

In three dimensions this will represent an ellipsoid of revolution with the optic axis as the axis of revolution. Remember, optic axis is the symmetry axis.

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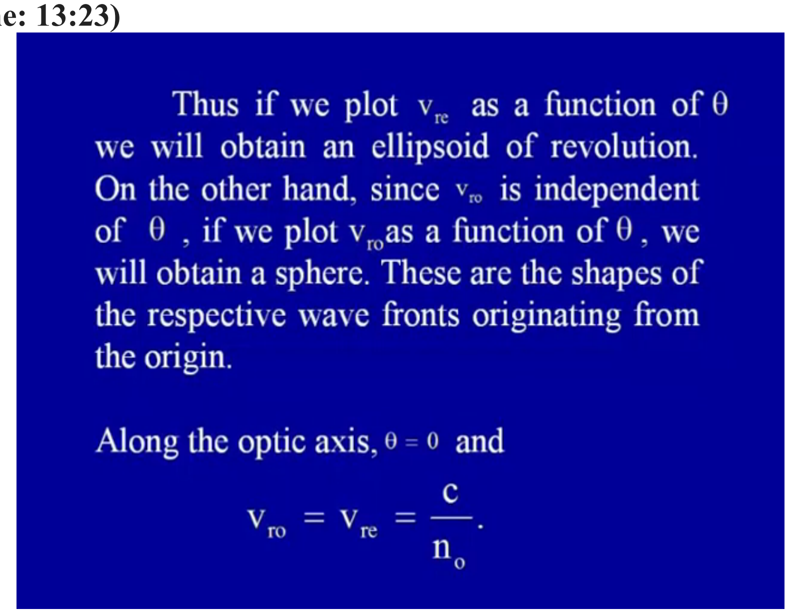

Thus, if we plot v of r e as a function of theta, we will obtain an ellipsoid of revolution because the velocity is different in different direction. On the other hand, since v of ro is independent of theta if we plot v of ro as a function of theta will obtain a sphere. These are the shapes of the respective wave fronts originating from the origin. Along the optic axis theta = 0 and v of ro = v of re which = c upon no.

The two wave fronts touch each other along this direction that is the propagation beam along the optic axis for the angle theta = 0. Let us now consider the case v re perpendicular to the optic axis, the progression perpendicular to the optic axis. In this case, theta equal to PI by 2.

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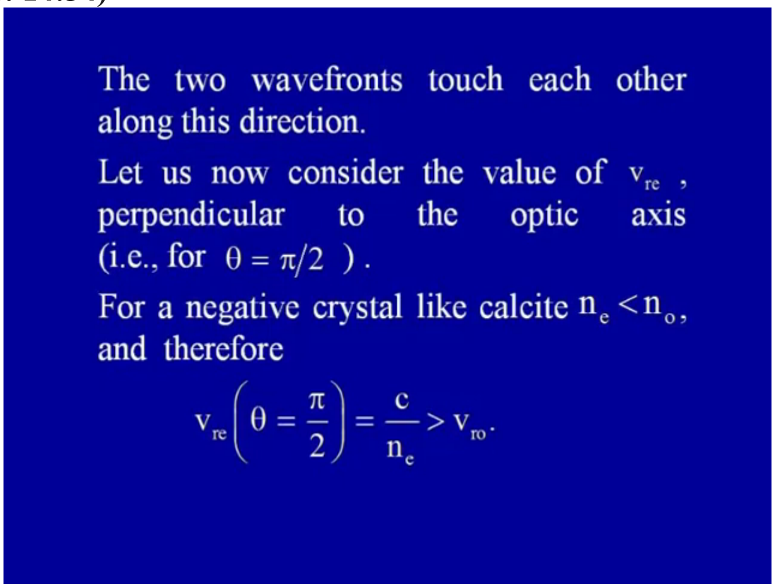

For a negative crystal like calcite n of e less than n of o and therefore v of re for this direction theta equal to PI by 2 = c of ne which is greater than v of ro. In this case, the minor axis of the ellipsoid will be along the optic axis and lie side of revolution will be outside the sphere.

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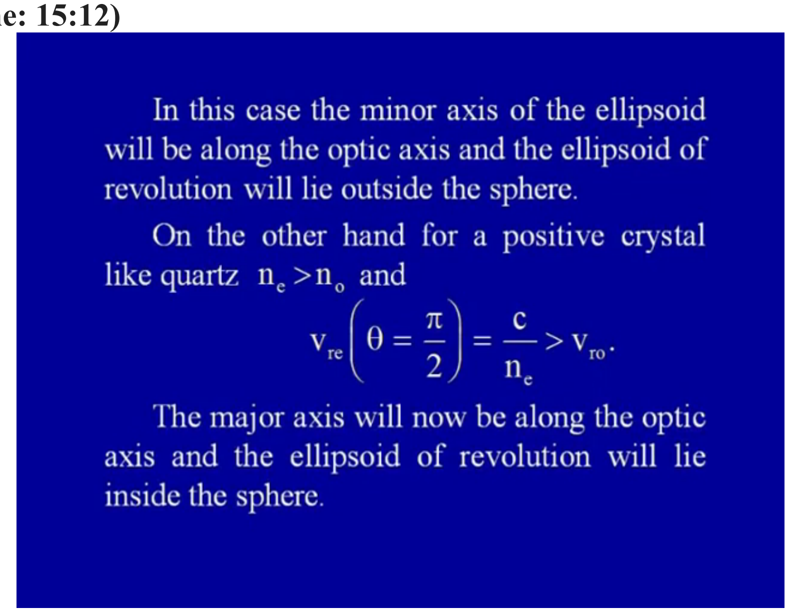

On the other hand, for a positive crystal like quartz ne is greater than no and v of re therefore for theta equal to PI by 2 is greater than v of ro. The major axis will now be along the optic axis and ellipsoid of revolution will lie inside the sphere.

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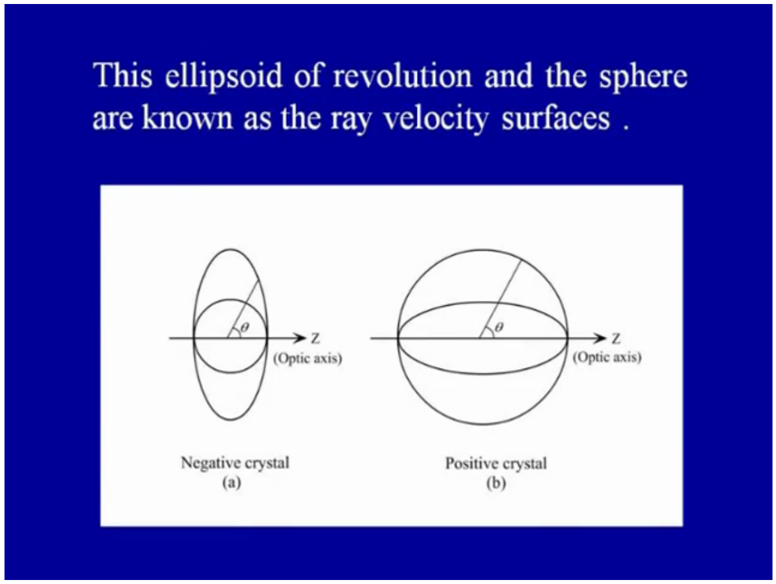

These are given in these pictures. The ellipsoid of revolution and the sphere, they are known as velocity ray surfaces. For both waves, now that is important.

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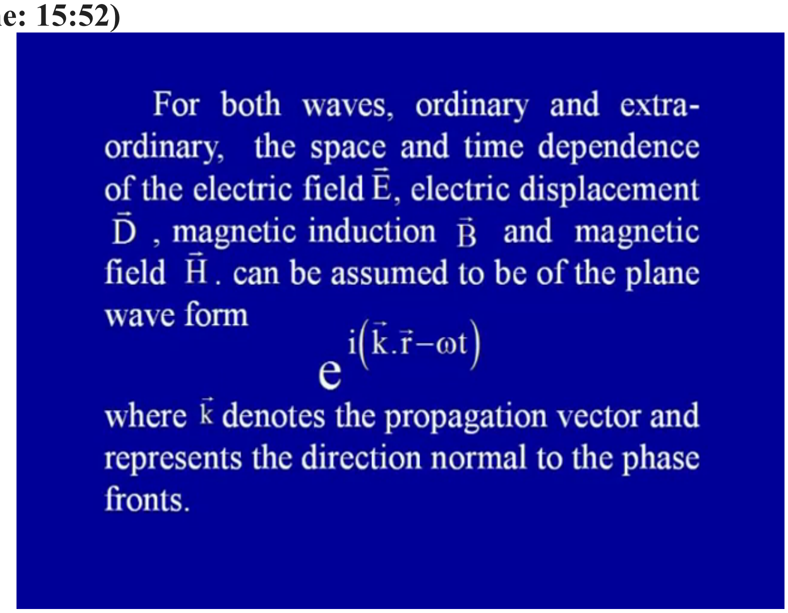

For both waves ordinary and extraordinary the space and time dependence of the electric field E, electric displacement D, magnetic induction B, magnetic field H, see all these quantities appeared in the Maxwell's equations. So, all of these can be assumed to be of the plane wave form e raised to the power ik dot r - Omega t for a plain monochromatic wave. Here k denotes the propagation vector and represents the direction normal to the phase fronts.

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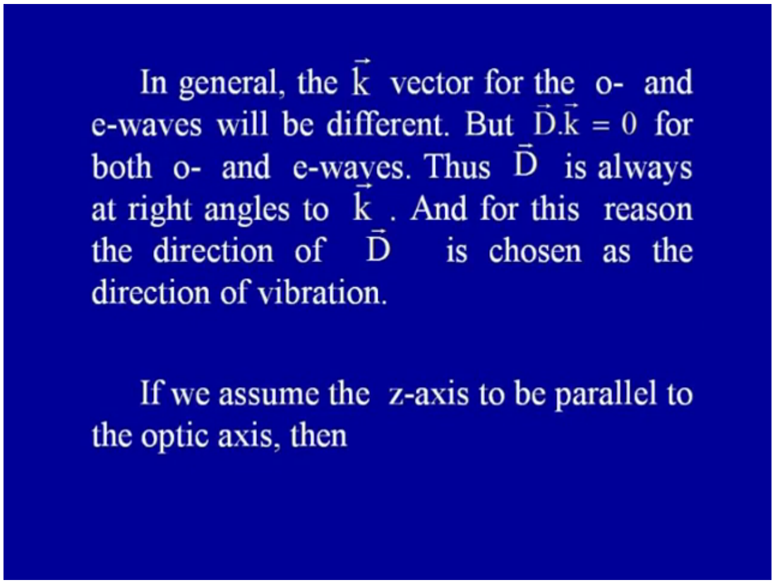

In general, the k vector for the o and the e waves will be different. But D dot k is 0 for both o and the e rays. This means the electric displacement D is always perpendicular to the k vector and for this reason the direction of D is chosen as the direction of vibration. If we assume that z axis to be parallel to the optic axis then,

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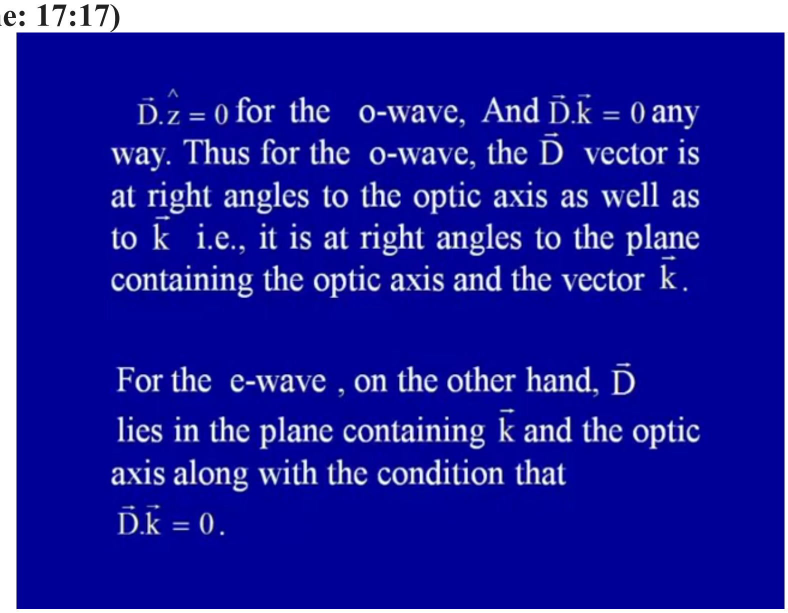

D dot z will be equal to zero for the o wave. On top of this, D dot k is 0 anyway. Thus for the ordinary wave, the D vector that is the direction of vibration is at right angles to the optic axis as well as to k. This means the direction of vibrations for the ordinary ray it has right angles to the plane containing the optic axis and the vector k. For the e wave on the other hand, directed displacement vector D lies in the plane containing k and the optic axis along with the condition that D dot k is 0.

Let us consider the general propagation in a doubly refracting crystal. Let us consider incident wave, incident normally on a uniaxial crystal like calcite.

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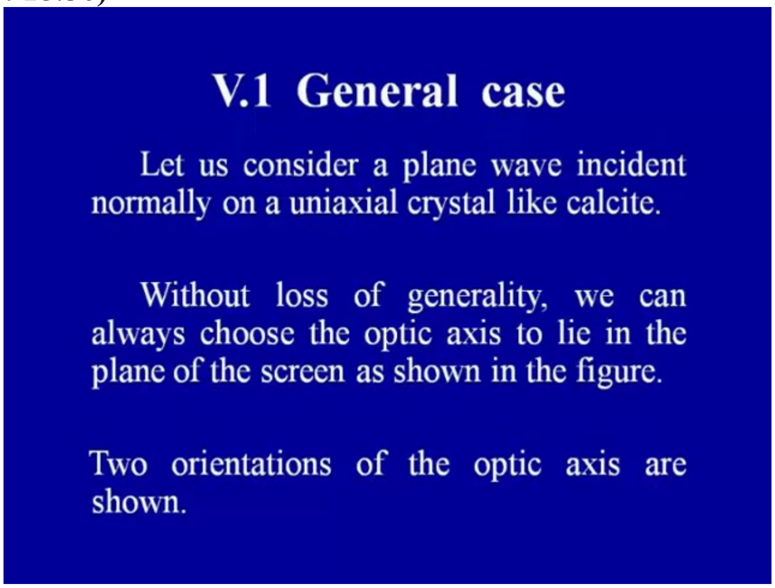

Without loss of generality we can always choose the optic axis to lie in the plane of the screen as shown in the figure. Two orientations of the optic axis are shown.

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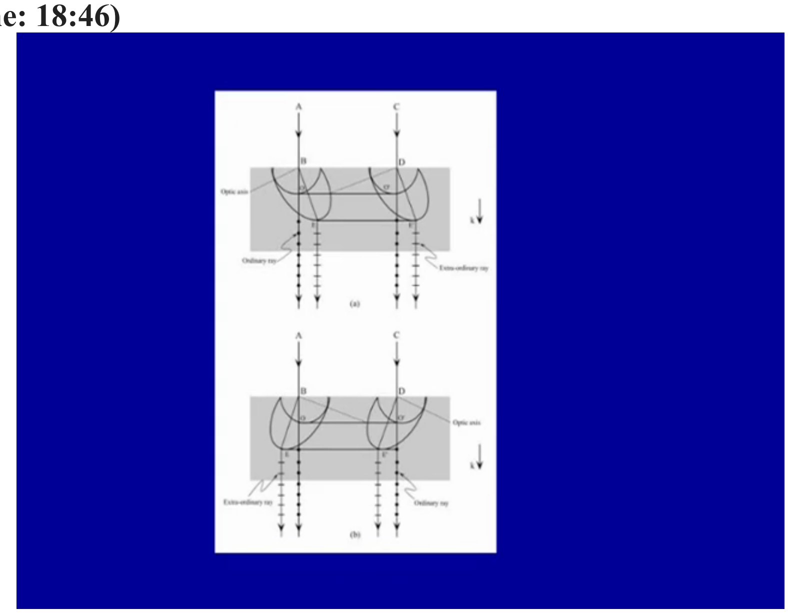

A, B and C, D or the incident rays; BD line is the line of intersection of the incident wave front and the plane of the screen. Once the light enters the material, we find that there is splitting. The points B and D are touched at the same time. Remember, the beam is incident normally.

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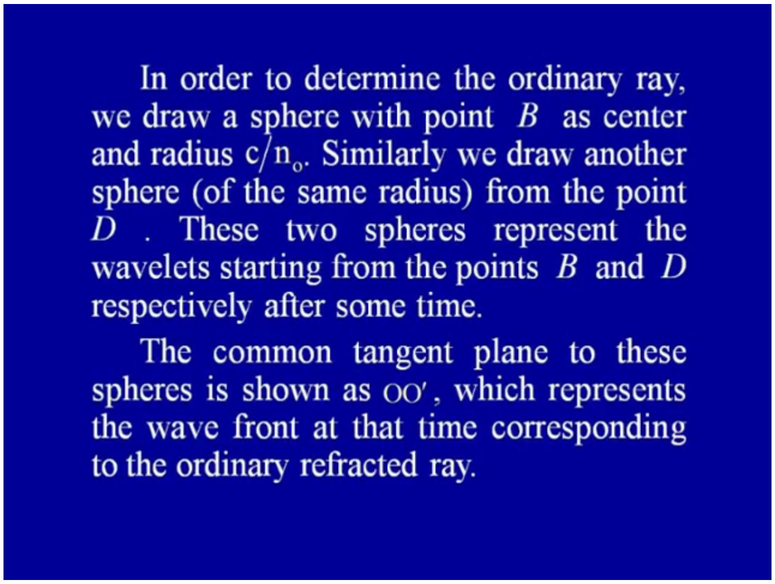

Now, in order to determine the ordinary ray, we draw A sphere with point B at center and radius c upon no. Remember, c upon no is the speed of the ordinary ray. Similarly, we draw another sphere of the same radius because both these spheres are originating at the same instant. The points B and D become, they become active at the same time. So that is why the radius is same for this field is starting from the point D.

These two spheres represent the wavelet is starting from the points B and D respectively after some time. The common tangent plane to these spheres is shown as O O prime which represents the wave front at that time corresponding to the ordinary refracted ray. Note that the dots show the direction of vibrations.

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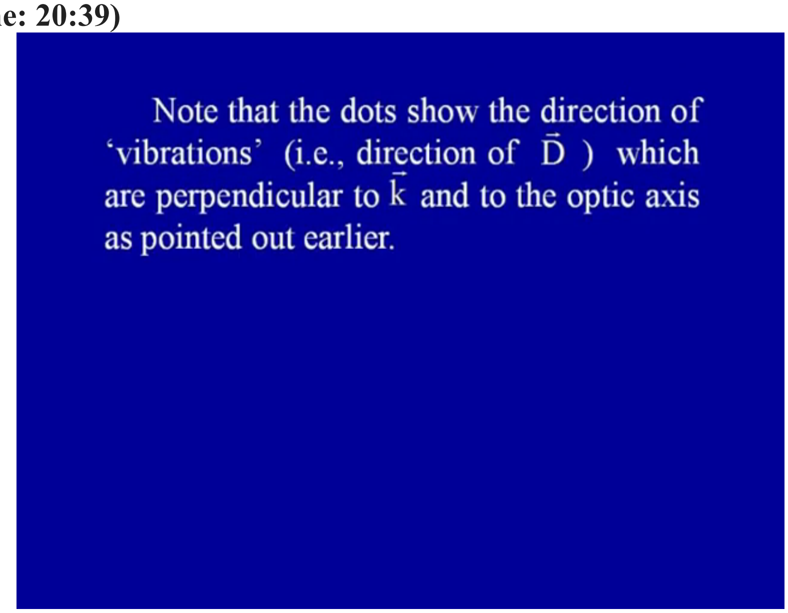

The direction of the electric displacement vector D which are perpendicular to k and to the optic axis as pointed out earlier. Remember, we are talking about the ordinary ray.

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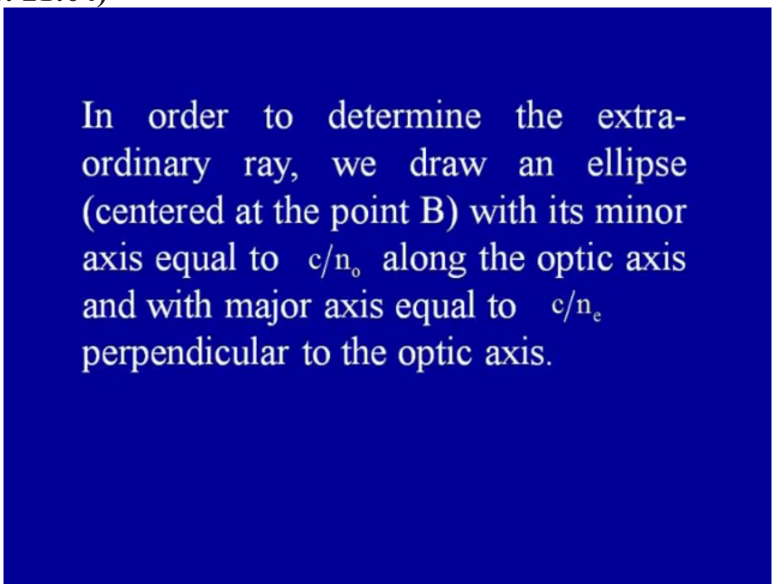

In order to determine the extraordinary ray, we draw an ellipse centered at the point B with this minor ax= c upon no along the optic axis and with the major ax= c upon ne perpendicular to the optic axis. The lips wider revolution is obtained by rotating the ellipse about the optic axis which is the symmetry axis. Similarly, we draw another ellipsoid of revolution from the point D identical in shape, identical in size. Remember, the points B and D become active at the same time.

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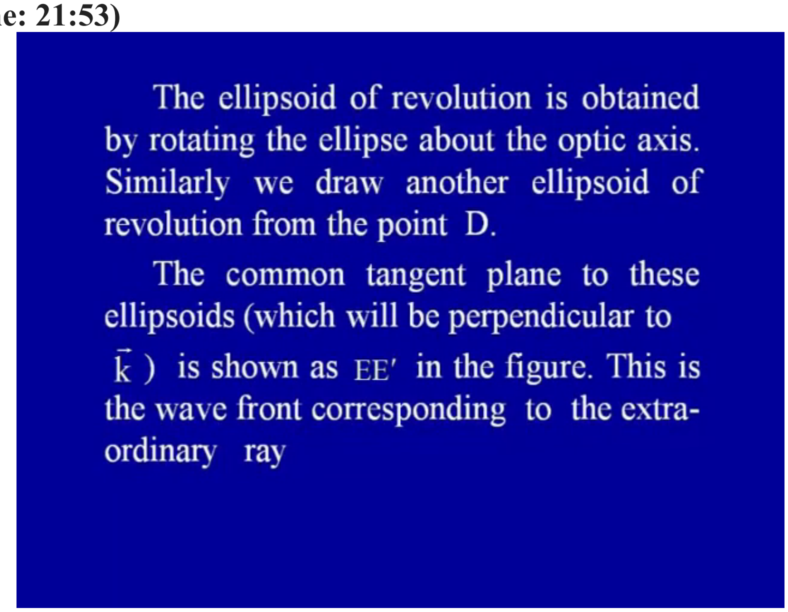

The common tangent plane to these ellipsoids which will be perpendicular to k is shown as E E prime in the figure. This is the wave front corresponding to the extraordinary ray. If you join the point B to the point of contact O then, corresponding to the incident ray AB the direction of the ordinary ray in the crystal will be along BO.

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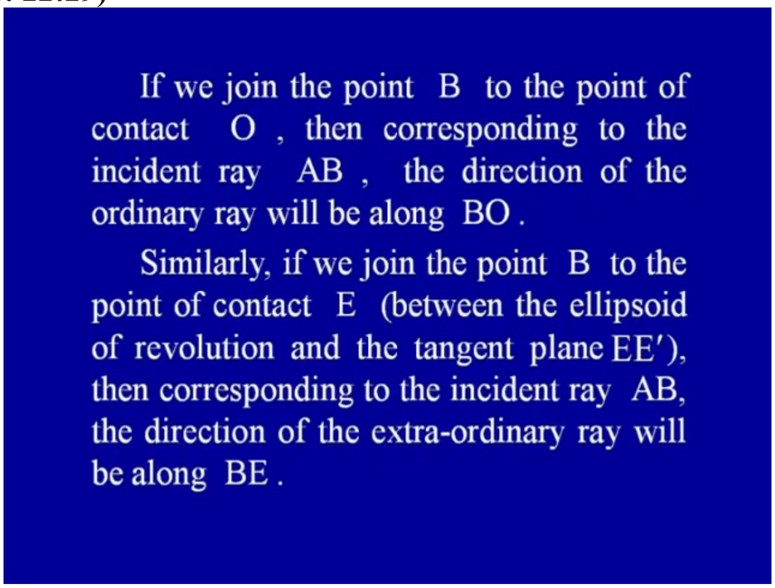

Similarly, if we go in the point B to the point of contact E between the ellipsoid of revolution and the tangent plane EE prime, then, the corresponding to the incident ray AB the direction of the extraordinary ray will be along BE.

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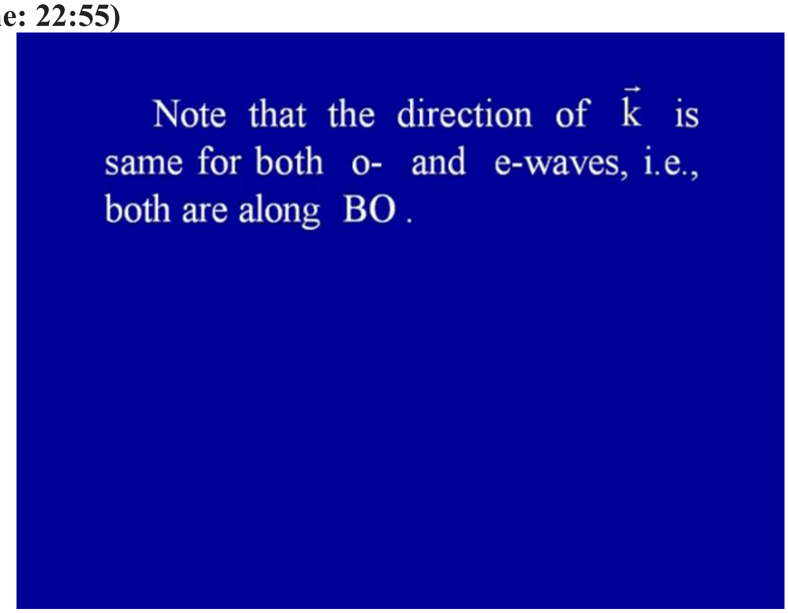

Note that the direction of k is same here for both o and e waves. Both are along BO.

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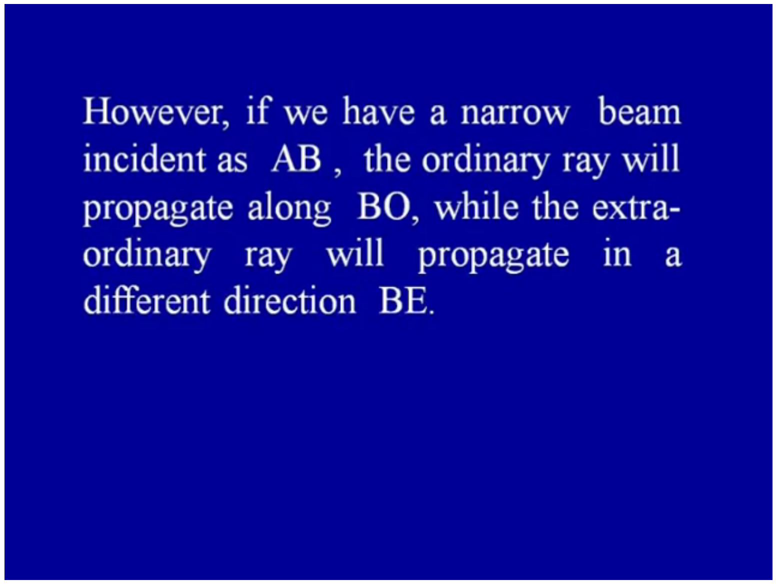

However, if we have a narrow beam of light incident as AB, the ordinary ray will propagate along BO while the extraordinary ray will propagate in a different direction BE, the distance will between them will go on increasing.

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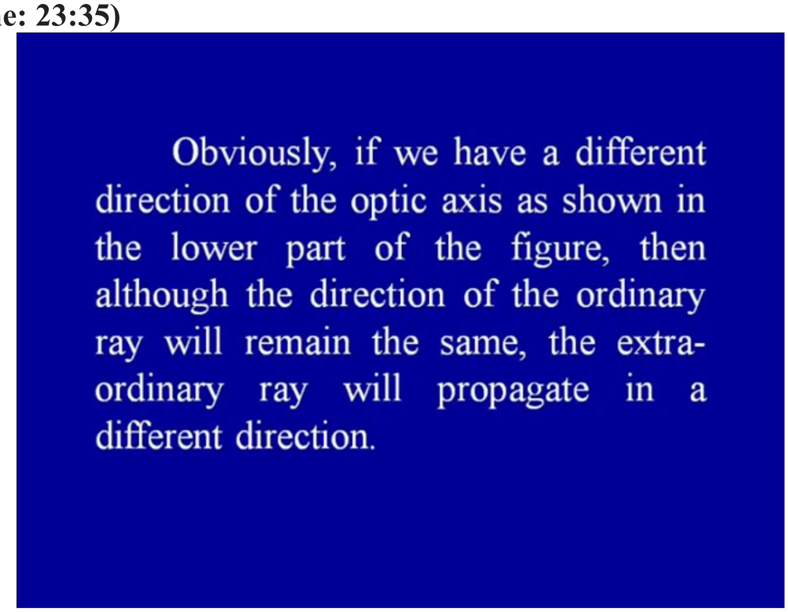

Obviously, if we have a different direction of the optic axis as shown in the lower part of the figure, then, although the direction of the ordinary ray will remain the same as it should. Remember, the ordinary will satisfy the Snell's laws of refraction the extraordinary ray will propagate in a different direction now.

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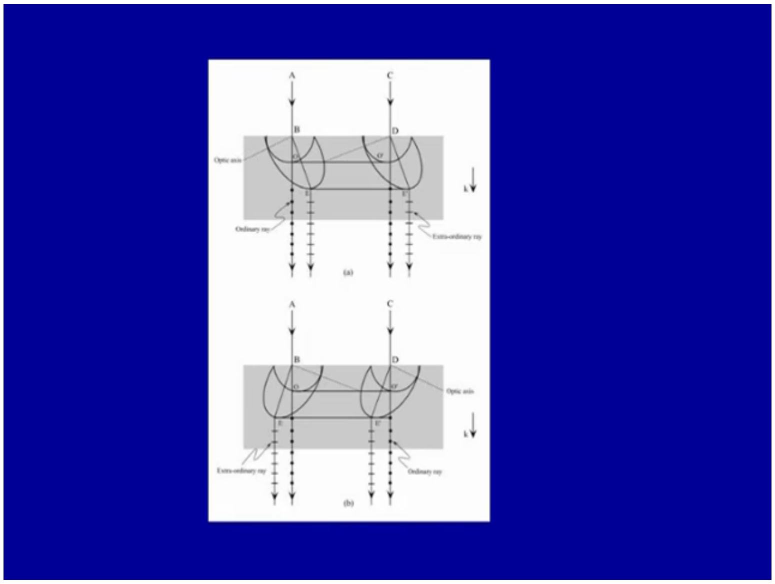

This is because of erection optic axis a change and they have, it have changed the orientation of the ellipsoid of revolution. Now, if we ray is incident normally on a calcite crystal and if the crystal is rotated about the normal then, the optic axis.

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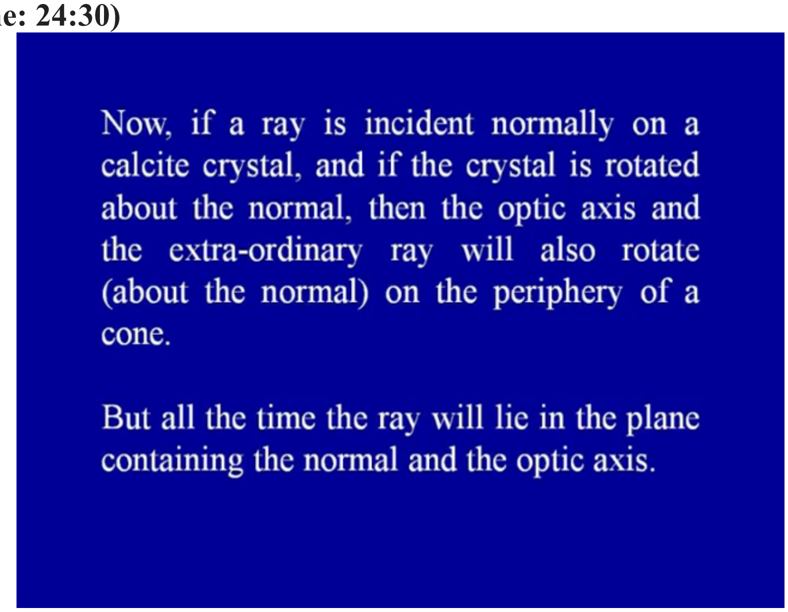

And extraordinary ray will also rotate about the normal on the periphery of a cone. The tip of the extraordinary ray will be described by a circle. So, for the ordinary ray is concerned, it remains unchanged all the time. The effective index and n of re corresponding to the extraordinary ray propagating along a direction making an angle theta with the optic axis is given by.

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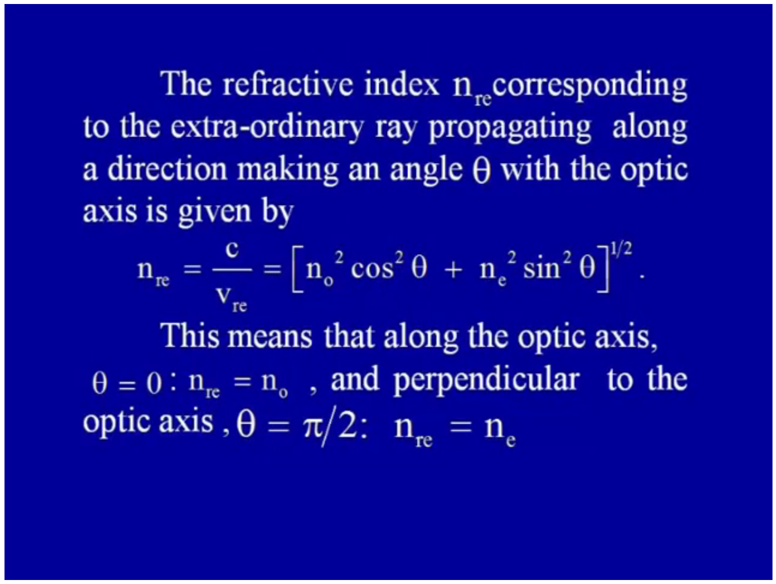

n of re = c upon v of re which = the square root of no square cos s square theta + ne square sine square theta this means that on the optic axis theta = 0, n of re = no. That is for the ordinary and perpendicular to the optic axis theta = PI by 2 n of re = ne.

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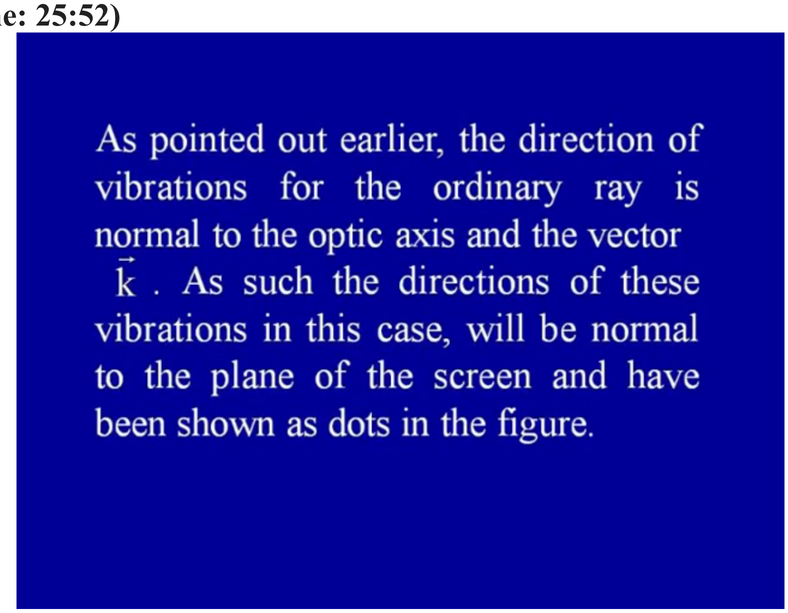

As pointed out earlier, the direction of vibration for the ordinary ray is normal to the optic axis and the vector k. As such the directions of these vibrations in this case, will be normal to the plane of the screen and that is why they have been shown as dots in the figure. For the extraordinary they are in the same plane and they have been shown by short arrows.

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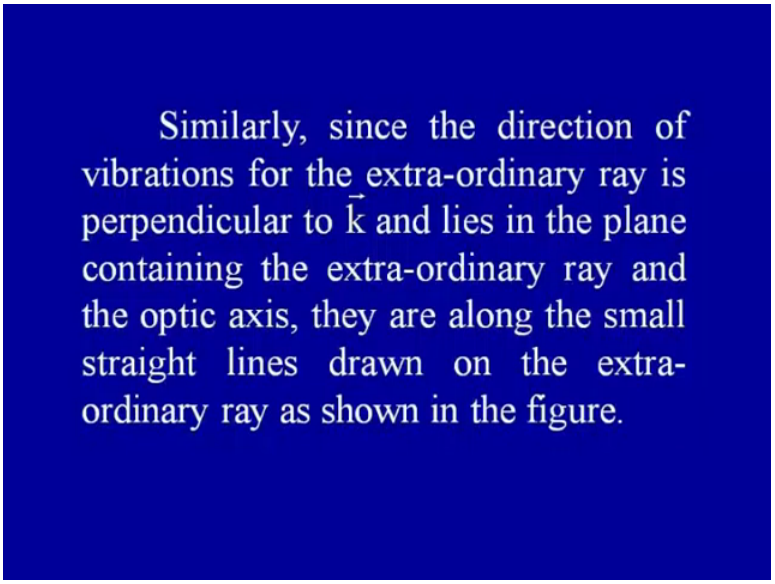

That occurs as x-ray extraordinary rays perpendicular to k and lies as I pointed out earlier just now in the plane containing the extraordinary ray and the optic axis.

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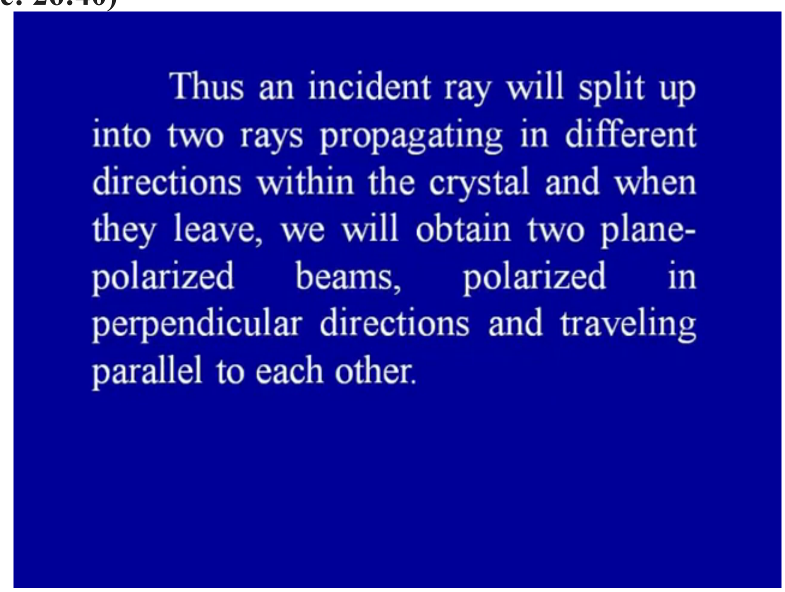

Thus an incident ray will split up into two rays propagating in different directions within the crystal and when they leave, we will obtain two pain polarized beams polarized in perpendicular directions and traveling parallel to each other and also separated from each other. We shall now consider some special cases.

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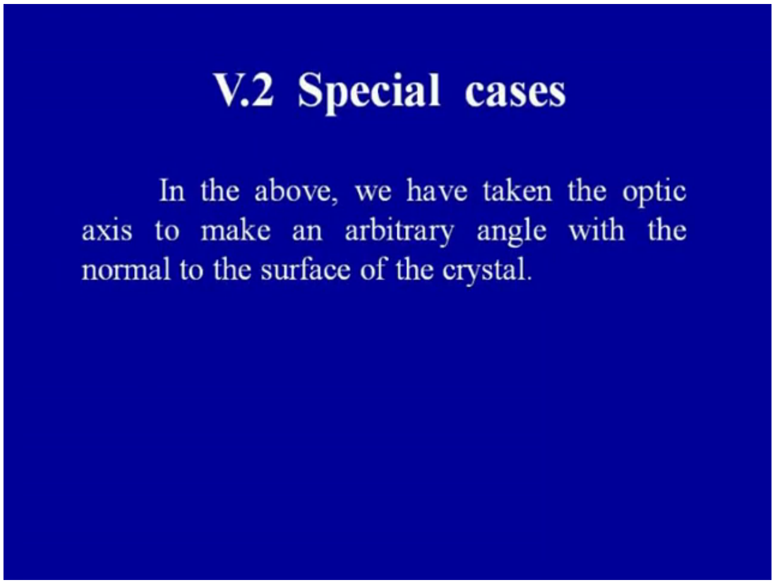

In the above we have taken the optic axis to make an arbitrary angle with the normal to the surface of the crystal.

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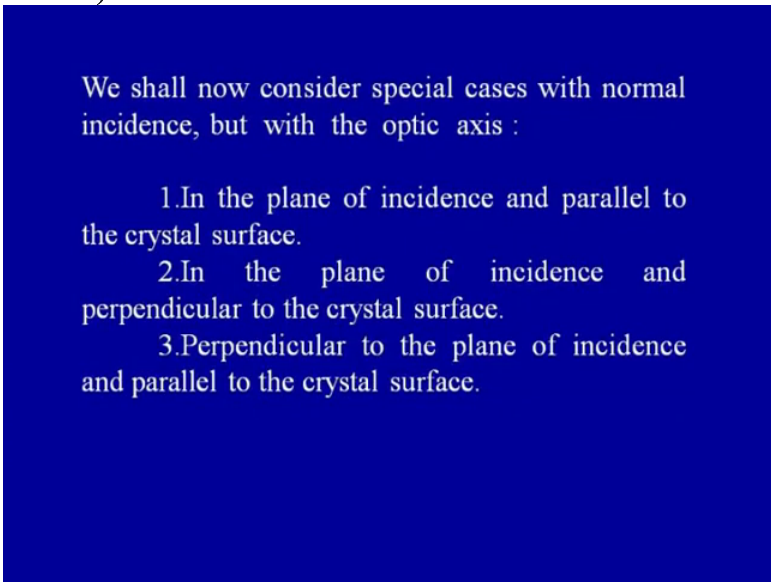

We shall now consider special cases, the normal incidence again but with the optic axis in the plane of incidence and parallel to the crystal surface or in the plane of incidence perpendicular to the crystal surface or perpendicular to the plane of incidence and parallel to the crystal surface. Let us consider these one by one. First we take up optic axis in the plane of incidence and parallel to the crystal surface.

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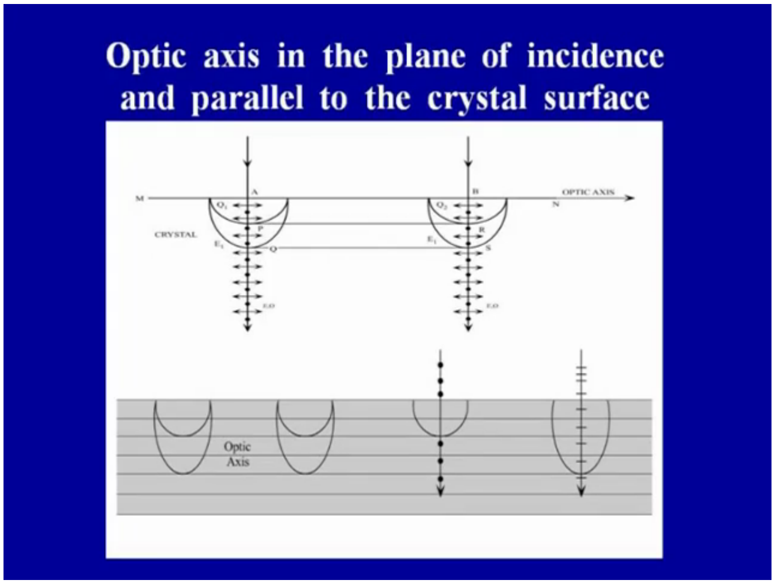

The optic axis here lies along MN. AB is the incident plane wave front of the rays falling normally on the surface MN of calcite.

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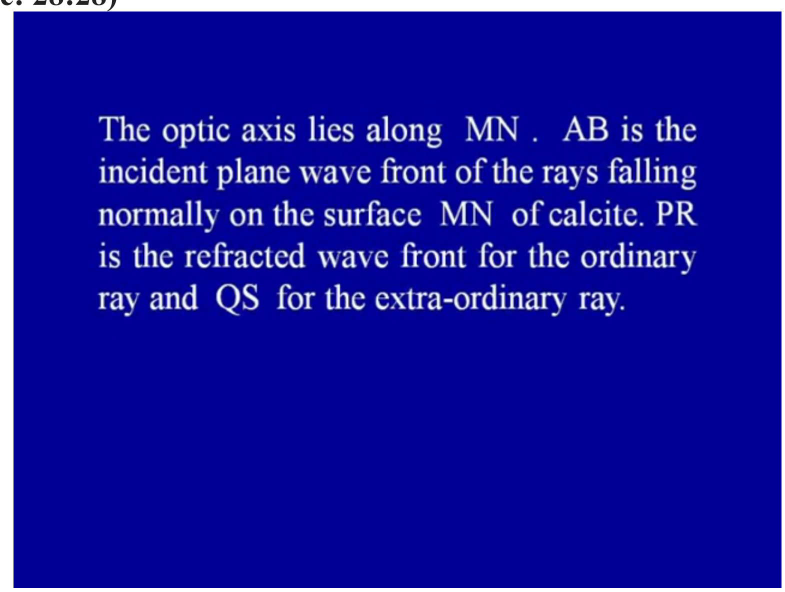

PR is the refracted wave front for the ordinary ray and QS for the extraordinary ray. They have been described earlier; we follow the same construction here.

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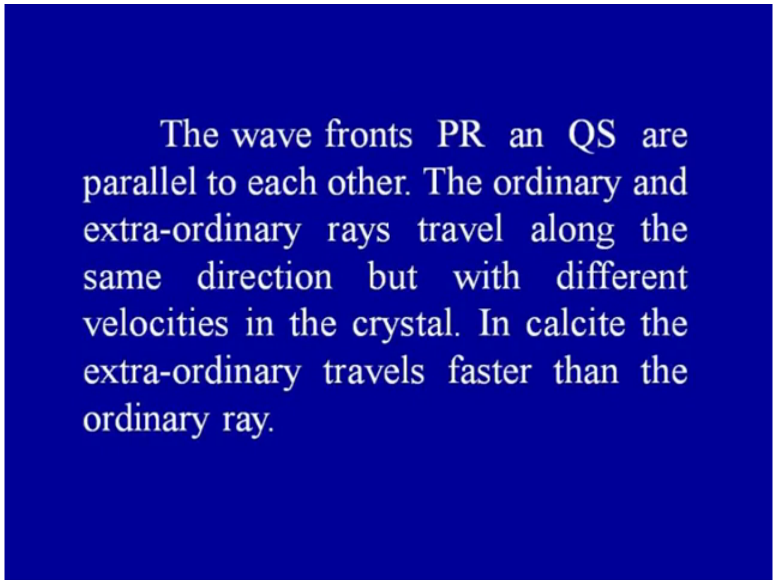

The wave fronts PR and QS are parallel to each other. The ordinary and extraordinary rays travel along the same direction but with different velocities in the crystal. In calcite the extraordinary rays rolls faster than the ordinary ray. A phase difference naturally gets introduced between them as they propagate in the crystal.

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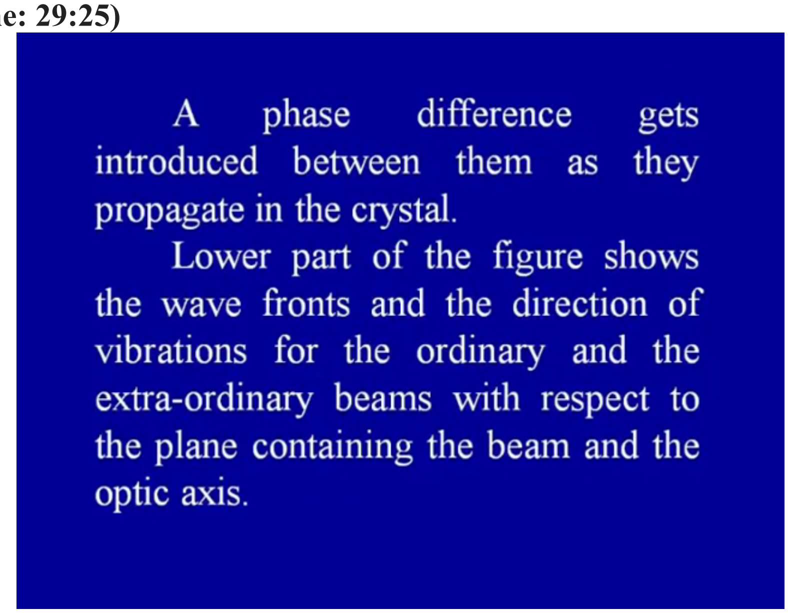

See the lower part of the figure shows the wave fronts and the direction of vibrations for the ordinary and the extraordinary beams respect to the plane containing the beam and the optic axis.

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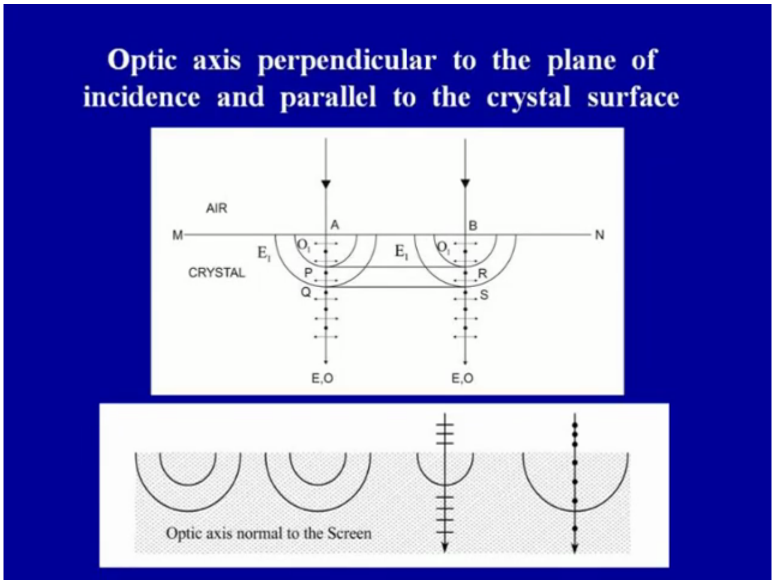

Now, let us consider the second possibility, optic axis perpendicular to the plane of incidence and parallel to the crystal surface.

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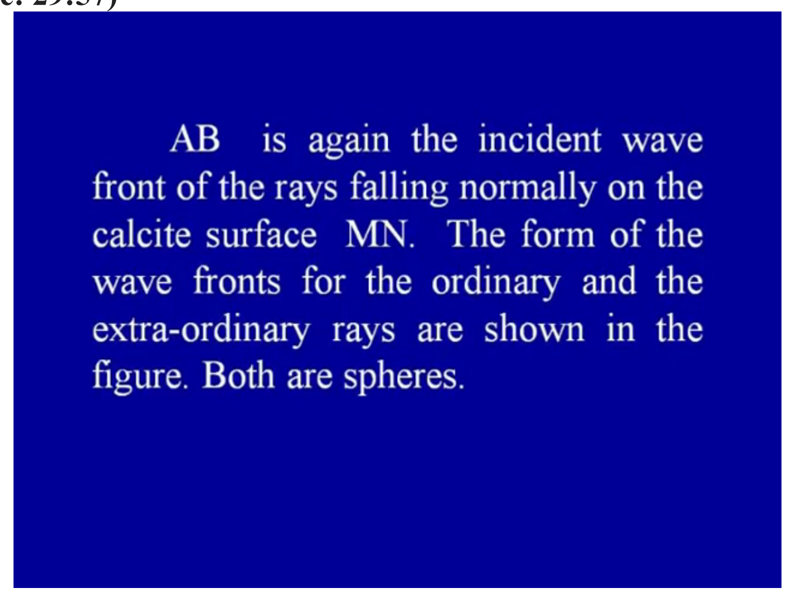

AB is again incident wave front of the rays falling normally on the calcite surface MN. The form of the wave front, for the ordinary and the extraordinary rays are shown in the figure. Both are spheres.

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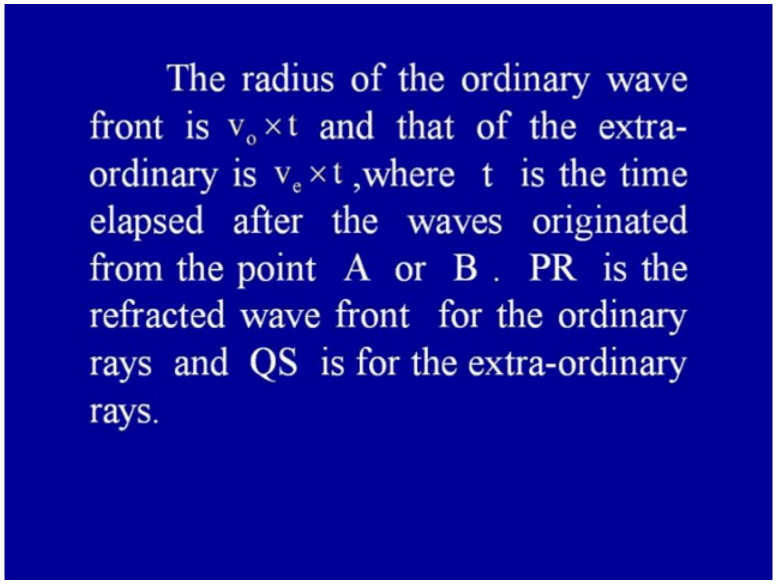

The radius of the ordinary wave front is vo into t; that of the extraordinary is ve into t where t the time elapsed after the waves originated from the points A or B. PR is the refracted wave front for the ordinary rays QS is for extraordinary rays. They have been obtained up in the same way as in the earlier case. The ordinary and extraordinary rays travel with different velocities along the same direction.

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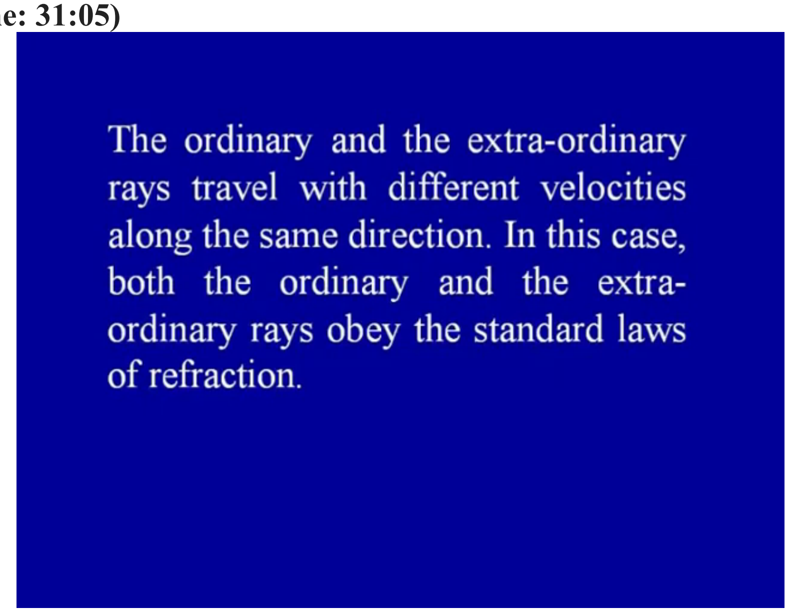

In this case, both the ordinary and extraordinary rays obeys the standard laws of refraction is Snell's laws. Because we finally see the wave fronts are is spherical in this geometry. Again note the direction of vibrations for the ordinary and extraordinary beams given in the lower part of the figure for the extraordinary ray they are in the plane containing the propagation and the optic axis. For the present case this plane is perpendicular to the screen. And therefore vibrations for the extraordinary rays are shown as dots.

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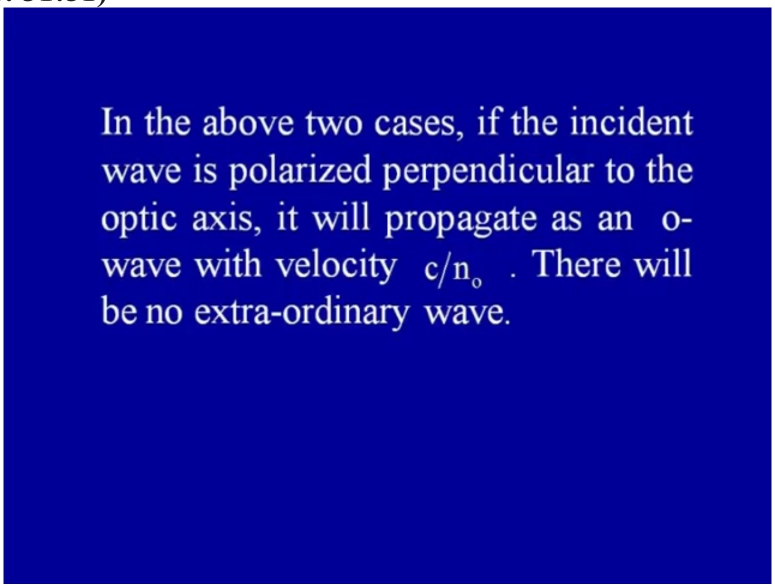

In the above two cases, if the incident wave is polarized perpendicular to the optic axis, it will propagate as an ordinary wave with velocity c upon no. And there will be no extraordinary wave. On the other hand, if the incident wave is polarized and polarized parallel to the optic axis, it will propagate as an e wave with velocity c over ne. And there will be no ordinary wave. Let us now take an the third case.

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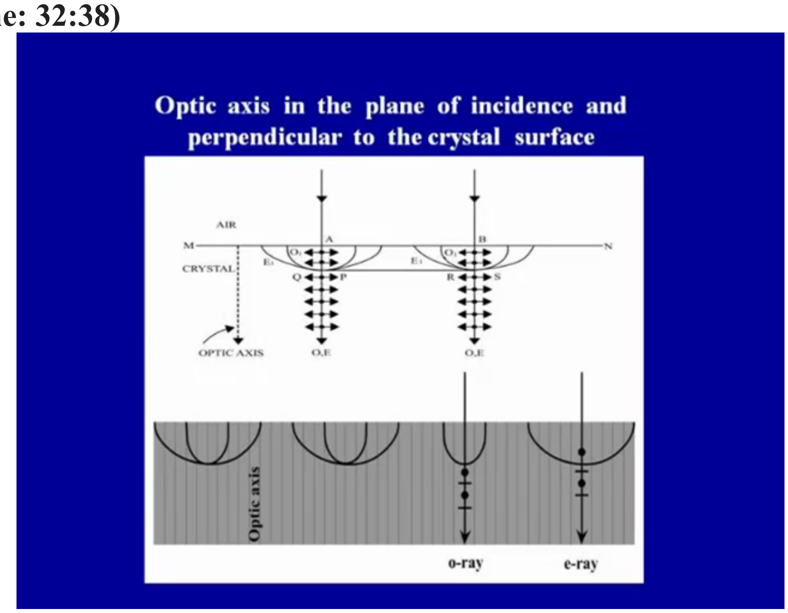

Optic axis in the plane of incidence and perpendicular to the crystal surface: The optic axis has been shown as a dashed line. AB is incident plane wave front as in the earlier cases of the rays falling normally on the surface of the crystal MN as before.

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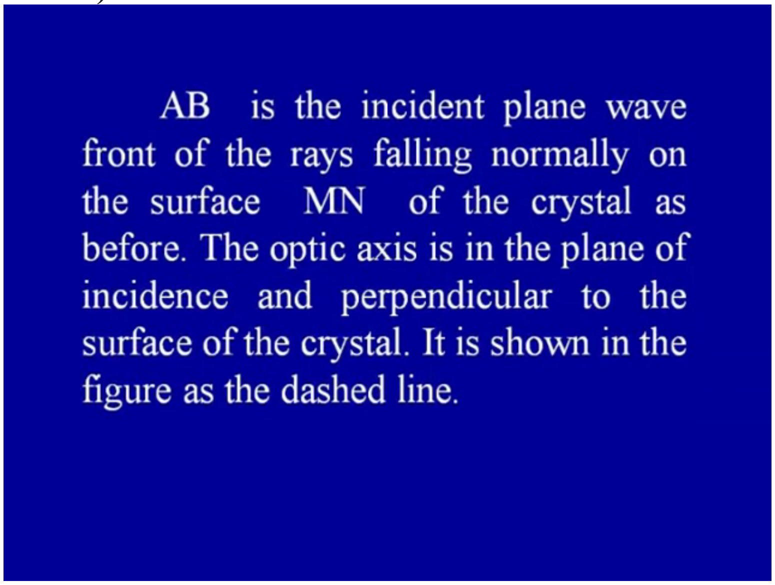

The optic axis is in the plane of incidence and perpendicular to the surface of the crystal. As I said earlier, it is shown in the figure as the dashed line;

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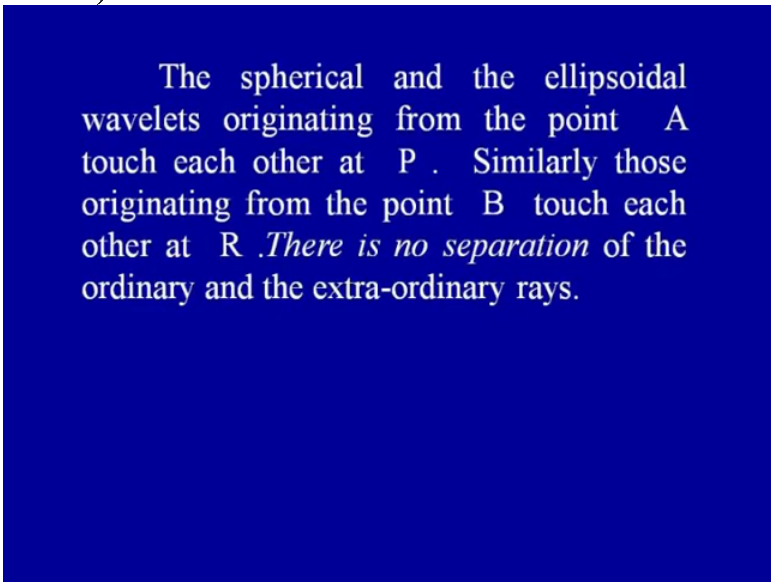

The spherical and ellipsoidal wavelets originating from the point A touch each other at P similarly those originating from the point B touch each other at R. There is no separation of the ordinary and the extraordinary rays.

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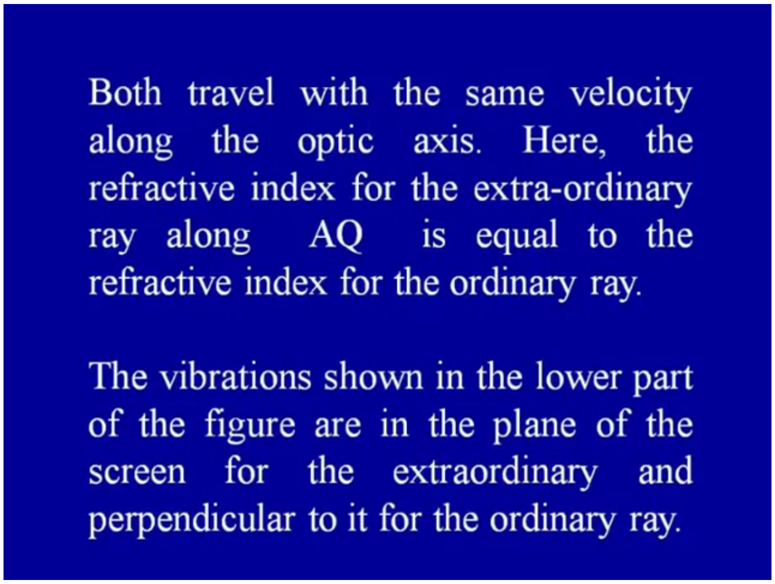

Both travel with the same velocity along the optic axis, remember, which is the symmetry axis. Here, the refractive index for the extraordinary ray along AQ = the refractive index for the ordinary ray. The vibrations shown in the lower part of the figure are in the plane of the screen for the extraordinary ray and perpendicular to it for the ordinary ray just following the same arrangement as in the earlier cases. Let us now consider yet another interesting phenomena.

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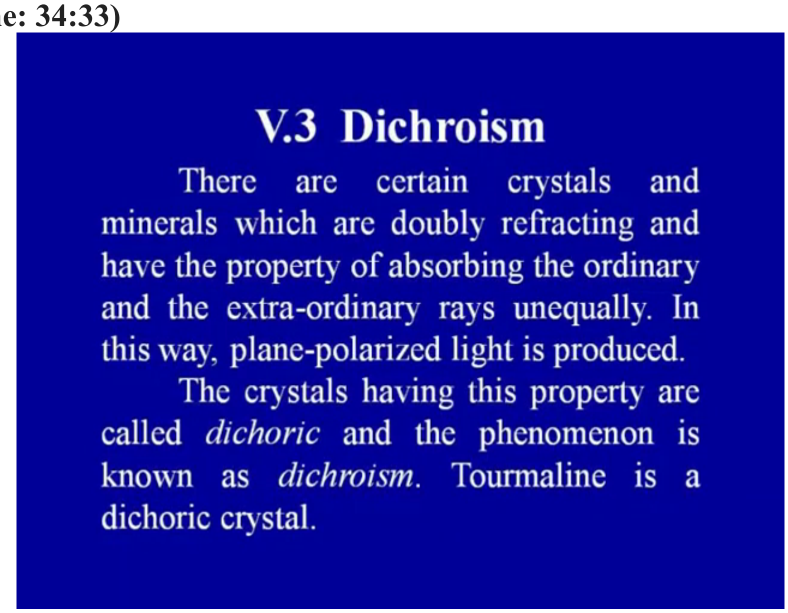

The phenomena of Dichroism: You see, there are certain crystals and minerals which are doubly refracting; but they have another special property and that is they have the property of absorbing the ordinary and the extraordinary rays unequally. And in this way, a plane polarized light is produced, at least in principle. The crystals having this property are called dichoric and the phenomena is known as dichroism. Tourmaline is a dichroic crystal.

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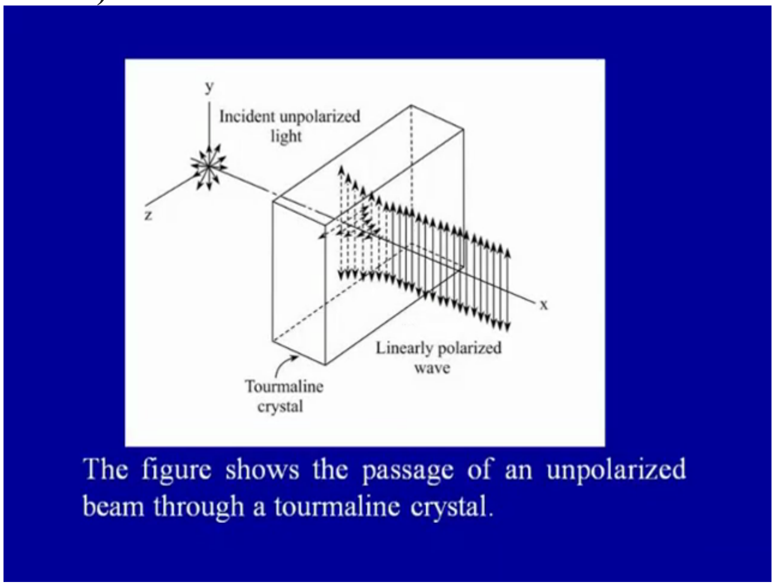

This figure shows the passage of an unpolarized beam to a tourmaline crystal. It progressively absorbs the horizontal vibrations which correspond here to the ordinary ray with a result that by the time the beam comes out,

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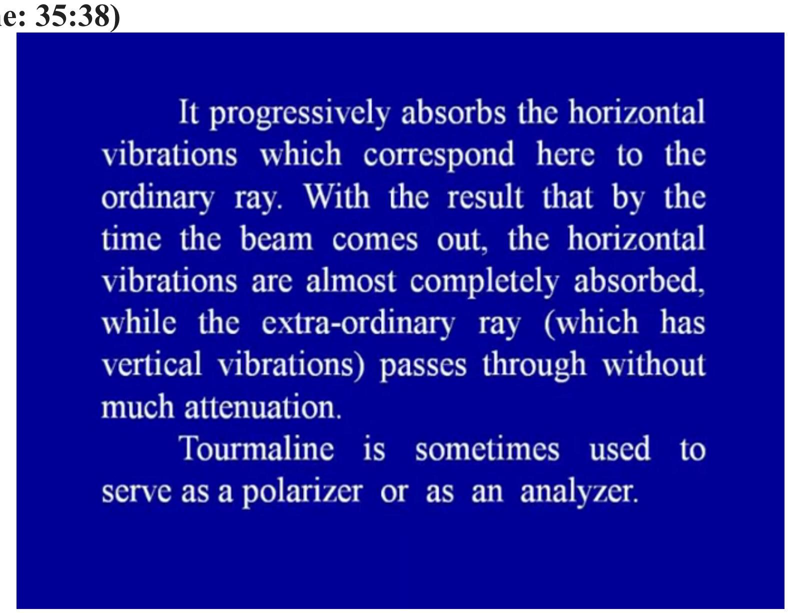

The horizontal vibrations are almost completely absorbed while the extraordinary ray which has vertical vibrations here passes through without much attenuation. The tourmaline crystal is sometimes used to serve as a polarizer or as an analyzer. Now, we come to another interesting device work as a polarizer, Nicol prism.

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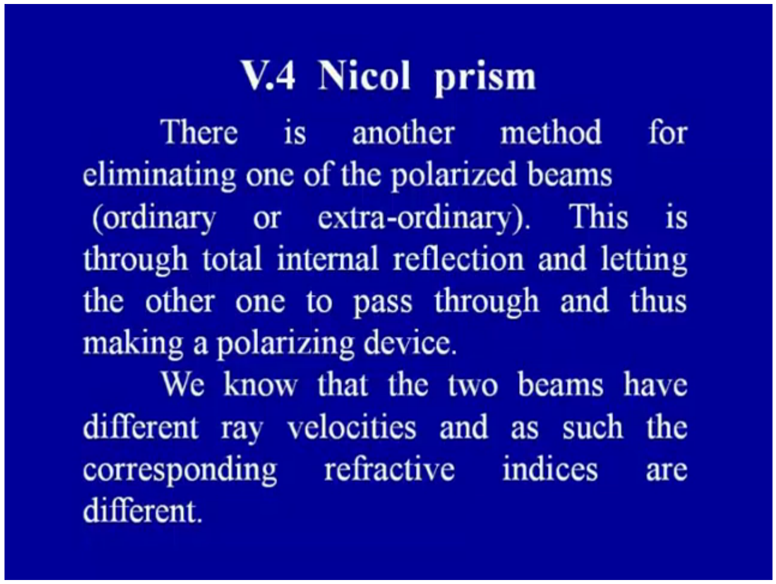

There is another method for eliminating one of the polarized beams ordinary or extraordinary. This is through total internal reflection and letting the other one to pass through and thus making a polarizing device. It is a very ingenious method. We know that the two beams have different ray velocities and as such the corresponding refractive indices are different.

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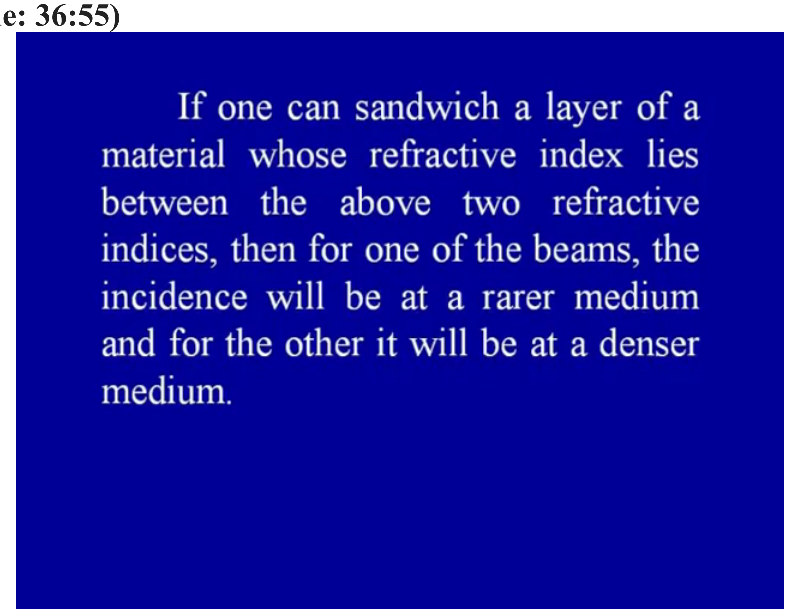

If one can sandwich a layer of a material whose refractive index lies between the above two refractive indices, then, for one of the beams, incidence will be at a rarer medium. And for the other, it will be at a denser medium.

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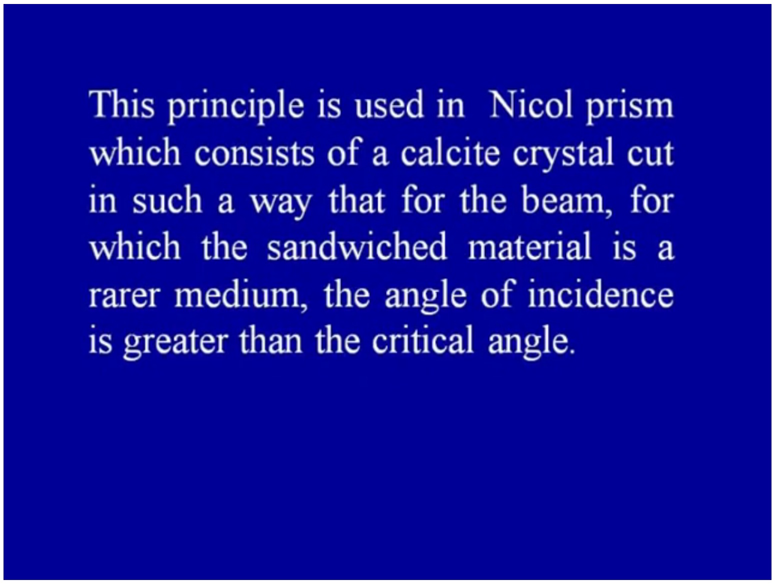

This principle is used in Nicol prism which consists of a calcite crystal cut in such a way that for the beam for which the sandwich material is a rarer medium, the angle of incidence is greater than the critical angle.

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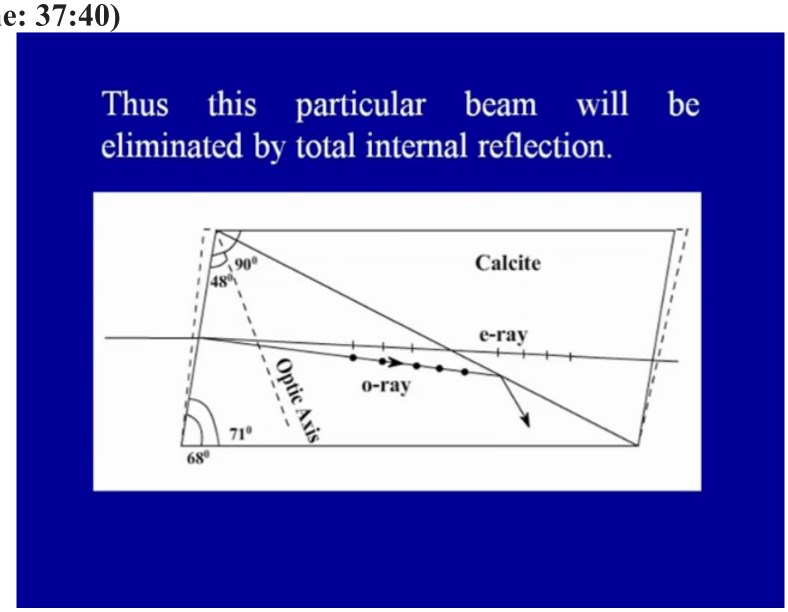

Now, this particular beam will be eliminated by total internal reflection. This is happening here for the ordinary ray. See, the dashed outline in the figure corresponds to the natural crystal.

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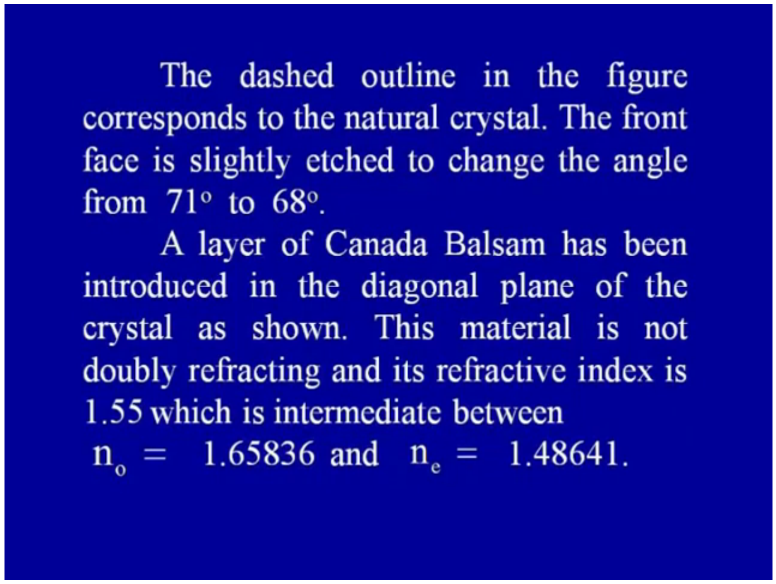

The front face is slightly etched to change the angle from 71 degrees to 68 degrees a layer of Canada balsam has been introduced in the diagonal plane of the crystal as shown in the figure. This material is not doubly refracting and its refractive index is 1.55 which is, intermediate between refractive index for the ordinary which is 1.65836 and ne refractive index for the extraordinary 1.4864.

So, Canada balsam offers a rarer medium for the ordinary ray. The crystal is cut in such a way so that the angle of incidence at the Canada balsam layer is more than the critical angle.

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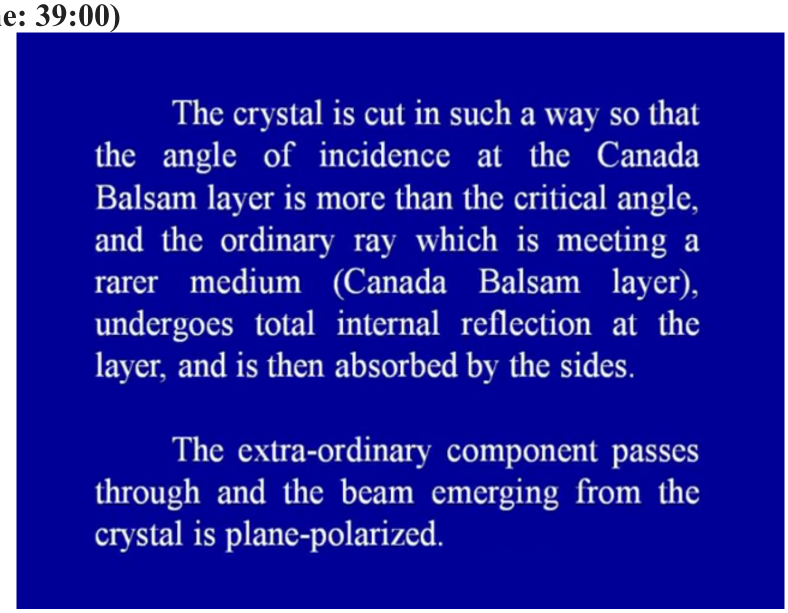

And the ordinary ray which is meeting a rarer medium, Canada Balsam layer, undergoes total internal reflection at the layer and is then absorbed by the sides of the crystal. The extraordinary component passes through and the beam emerging from the crystal is plane polarized. So, we have a simple polarizing device, okay. With this, we have come to the end of this lecture.