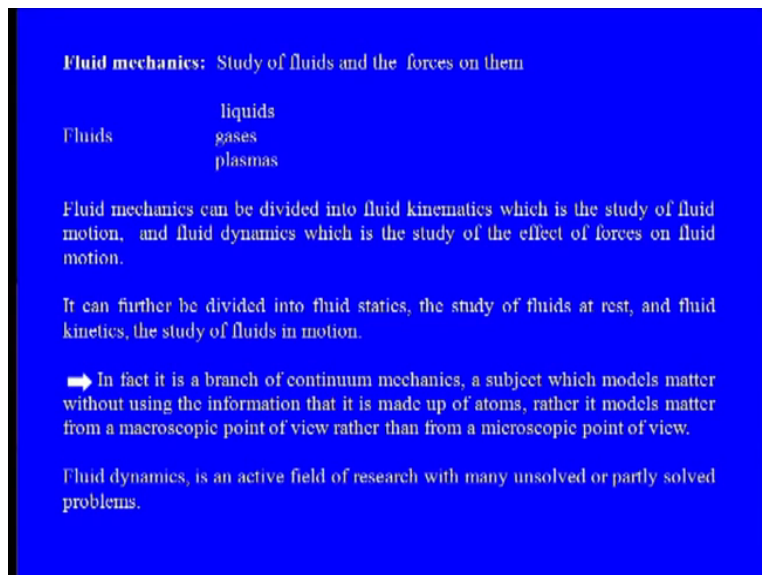


Engineering Physics 1
Dr. B. K. Patra
Department of Physics
Indian Institute of Technology-Roorkee

Module-06
Lecture-01
Fluid Mechanics - Part 01

Today I will talk about the fluid mechanics, in fluid mechanics let me start what does it mean my fluids. And then I will go on explaining each and every each and everything in the flow in the subject. Fluid mechanics is basically it is just it is the study of fluids and they are anti forces on them.

(Refer Slide Time: 00:55)



Fluid mechanics: Study of fluids and the forces on them

Fluids	liquids gases plasmas
--------	-----------------------------

Fluid mechanics can be divided into fluid kinematics which is the study of fluid motion, and fluid dynamics which is the study of the effect of forces on fluid motion.

It can further be divided into fluid statics, the study of fluids at rest, and fluid kinetics, the study of fluids in motion.

➡ In fact it is a branch of continuum mechanics, a subject which models matter without using the information that it is made up of atoms, rather it models matter from a macroscopic point of view rather than from a microscopic point of view.

Fluid dynamics, is an active field of research with many unsolved or partly solved problems.

It includes liquids, gases, plasmas fluid mechanics can be divided into fluid kinematics which is the subject of fluid motion and the fluid dynamics which is the study of the effect of forces on fluid motions. It can further be divided into fluid statics means this is the study of fluids at rest and fluid kinetics which is the study of fluids in motion. So, in fact in fact in terms of mechanics problem it is in fact it is the branch of continuum mechanics which is a sub sect.

It models matter without using the information that it is made up of atoms rather it models macroscopic point of view rather than from the microscopic point of view. Fluid mechanics is an active field of research with many solved or unsolved problems.

(Refer Slide Time: 02:00)

- Sometimes it becomes mathematically complex, so in those circumstances it can best be solved by numerical methods, typically using computers.
- A modern discipline in computer simulation studies is devoted to solve the fluid mechanics problems, is known as computational fluid dynamics (CFD)
- An experimental method for visualizing and analyzing fluid flow is known as Particle Image-Velocimetry by taking advantage of highly visual nature of fluid flow.

Sometimes it becomes mathematically complex, so in those circumstances it can best be handled using some numerical methods typically using computers. Nowadays a modern discipline in computer simulation studies become popular which is devoted to solve the fluid mechanics problem which is known as computational fluid dynamics, CFD. There are some experimental methods also for visualize and analyzing the fluid flow which is known as particle image Velocimetry by taking advantage of highly visual nature of fluid flow.

(Refer Slide Time: 02:45)

History of fluid mechanics

Study of fluid mechanics goes back to the days of ancient Greece, when Archimedes studied fluid statics and buoyancy and formulated Archimedes' principle.

Medieval Persian & Arab natural philosophers, including Abu Rayhan al-Biruni and Al-Khazini, combined that earlier work with dynamics to foretell presage the later development of fluid dynamics.

Rapid advancement in fluid mechanics began with Leonardo da Vinci (observation and experiment), Evangelista Torricelli (barometer), Isaac Newton (viscosity) and Blaise Pascal (hydrostatics), and was continued by Daniel Bernoulli with the introduction of mathematical fluid dynamics in *Hydrodynamica* (1738).

Non-viscous fluid flow was further analyzed by various mathematicians (Leonhard Euler, d'Alembert, Lagrange, Laplace, Poisson) and viscous flow was explored by a multitude of engineers including Poiseuille and Götthelf Heinrich Ludwig Hagen.

Further mathematical justification was provided by Claude-Louis Navier and George Gabriel Stokes in the Navier–Stokes equations, and boundary layers were investigated (Ludwig Prandtl), while various scientists (Osborne Reynolds, Andrey Kolmogorov, Geoffrey Ingram Taylor) advanced the understanding of fluid viscosity and turbulence.

Historically fluid mechanics has a long story, study of fluid mechanics goes back to the days of ancient Greece who in Archimedes studies fluid statics and buoyancy and formulated a theory

which is known as Archimedes principle. Medieval Persian and Arab natural philosophers including Abu Rayhan, Al Biruni and Al Khazini combined that earlier work with dynamics to foretell precedes the later development of fluid dynamics. Rapid development in fluid mechanics began with Leonardo da Vinci with the observation.

As well as he has performed many simple experiments to demonstrate different aspects of fluid mechanics, Torricelli who is who invents the barometer, Isaac Newton which invented the viscosity concept and Pascal who deals the hydrostatic spots and was fire finally continued to by Bernoulli with the introduction of mathematical fluid dynamics in Hydrodynamica in 1738. There are basically two kinds of fluid flow we can call one is viscous flow and another is non viscous flow.

Non viscous flow is known as the ideal flow and viscous flow is known as non-ideal flow. So, non viscous fluid flow was further analyzed by various mathematicians just Leonardo, Euler D' Alembert, Lagrange's, Laplace, Poisons etcetera. And viscous flow was explored by multitude of engineers including Portcullis and Ludwig Hagen. Further mathematical justification was provided by Navier and Stroke which is known as famous Navier-Stokes equations and we will talk about on latter.

And boundary layers who had investigated Ludwig Prantle while various scientists Renauld, Kolmogarov, Taylor they advanced the understanding of fluid viscosity and turbulences.

(Refer Slide Time: 05:07)

Fluid mechanics is a sub discipline of continuum mechanics, as illustrated in the following table.

Continuum Mechanics The study of the physics of continuous materials	Mechanics of Rigid-body	Elasticity Describes materials that return to their rest shape after an applied stress
	The study of the physics of continuous materials with a defined rest shape.	Plasticity Describes materials that permanently deform after a sufficient applied stress.
	Fluid mechanics The study of the physics of continuous materials which take the shape of their container.	Non-Newtonian fluids Newtonian fluids

Fluid Mechanics basically it is a sub discipline of continuum mechanics as illustrated in the following table. Basically it is the continuum mechanics which is the study of the physics of continuous materials. It can further divide it into two things one is known as mechanics of rigid body which tells you the study of physics of continuous materials with a defined rest shape. Second one is the fluid mechanics it is the study of physics of continuous materials which take the shape of their container.

So, again rigid body dynamics can be divided can be thought of in two ways one is elasticity which describes the material that return to their rest shape after an applied stress. Second one is plasticity which describe materials and that permanently deform after a sufficient applied stress. Fluid dynamics again can be divided into two fluids one is known as non-Newtonian fluids another is known as Newtonian fluids.

However we will discuss these parts non-Newtonian fluids and Newtonian fluids in the latter part of my talks.

(Refer Slide Time: 06:31)

Assumptions

Like any mathematical model of the real world, fluid mechanics makes some basic assumptions about the materials being studied.

These assumptions are turned into equations that must be satisfied if the assumptions are to be held true. For example, consider an incompressible fluid in three dimensions.

The assumption that mass is conserved means that for any fixed closed surface (such as a sphere) the rate of mass passing from *outside* to *inside* the surface must be the same as rate of mass passing the other way.

➡ mass *inside* remains constant, as does the mass *outside* which can be turned into an integral equation over the surface.

As you know like any mathematical model of the real world you need some assumptions on which your model relies on. So, same way fluid mechanics make some basic assumptions about the materials being studied. These assumptions are turns to into equations that must be satisfied if the assumptions are to be held true. For example consider an incompressible fluid in three dimensions ok.

Incompressible means where the density does not change this is known as incompressible fluid. Though the assumptions that mass is conserved means that for any fixed flow surface such as a sphere the rate of mass passing from outside to inside the surface must be the same as rate of mass passing the other way. That means mass inside remains constant as does the mass outside which can be turned into an integral equation over the surface.

(Refer Slide Time: 07:41)

Fluid mechanics assumes that every fluid obeys the following:

- Conservation of mass
- Conservation of energy
- Conservation of momentum
- The *continuum hypothesis*

In fact if we will summarize the assumptions fluid mechanics assumes that every fluid obeys the following rules conservation of mass, conservation of energy, conservation of momentum and the continuum hypothesis.

(Refer Slide Time: 07:53)

■ Further, it is often useful (at subsonic conditions) to assume a fluid is incompressible i.e., the density of the fluid does not change.

Liquids can often be modeled as incompressible fluids, whereas gases cannot.

Similarly, it can sometimes be assumed that the viscosity of the fluid is zero (the fluid is *non-viscous*). Gases can often be assumed to be non-viscous.

If a fluid is viscous, and its flow contained in some way (e.g. in a pipe), then the flow at the boundary must have zero velocity.

For a viscous fluid, if the boundary is not porous, the shear forces between the fluid and the boundary results also in a zero velocity for the fluid at the boundary.

This is called the no-slip condition.

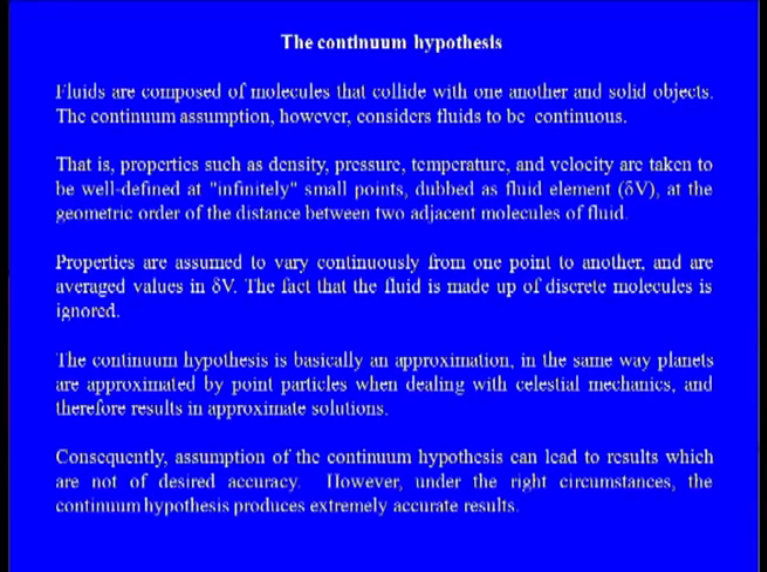
For a porous media otherwise, in the frontier of the containing vessel, the slip condition is not zero velocity, and the fluid has a discontinuous velocity field between the free fluid and the fluid in the porous media.

Further it is often useful at subsonic conditions to assume a fluid is incompressible that means the density of the fluid does not change. Liquid can often be modeled as incompressible fluid whereas gases cannot. Similarly it can sometimes we assumed that the viscosity of the fluid is 0 that means flow is known to be non viscous. Some gases can often be assumed to be non viscous. If the fluid is viscous and it's flow content in some way example in a pipe then the flow at the boundary must have 0 velocity.

For a viscous fluid if the boundary is not porous the shear forces between the fluid and the boundary results also in 0 velocity for the fluid at the boundary. This is called the no slip condition. For a porous media otherwise in the frontier of the containing vessel the slip condition is not zero velocity and the fluid has a discontinuous velocity field between the free fluid and the fluid in the porous media.

Now what does it mean by Continuum Hypothesis we need to understand very well depending on that whether the fluid can be dealt in the conventional fluid mechanics or using this statistical mechanics. So, let me try to understand what does it mean by Continuum Hypothesis in the fluid mechanics.

(Refer Slide Time: 09:25)



The continuum hypothesis

Fluids are composed of molecules that collide with one another and solid objects. The continuum assumption, however, considers fluids to be continuous.

That is, properties such as density, pressure, temperature, and velocity are taken to be well-defined at "infinitely" small points, dubbed as fluid element (δV), at the geometric order of the distance between two adjacent molecules of fluid

Properties are assumed to vary continuously from one point to another, and are averaged values in δV . The fact that the fluid is made up of discrete molecules is ignored.

The continuum hypothesis is basically an approximation, in the same way planets are approximated by point particles when dealing with celestial mechanics, and therefore results in approximate solutions.

Consequently, assumption of the continuum hypothesis can lead to results which are not of desired accuracy. However, under the right circumstances, the continuum hypothesis produces extremely accurate results.

Fluids are composed of molecules that collide with one another and solid objects that means container of the fluid. The continuum assumptions however considers fluid to be continuous that is properties such as density, pressure, temperature and velocity are taken to be well defined at infinitely small point known as the fluid element. At the geometric order of the distance between two adjacent molecules of the fluid.

Properties are assumed to vary continuously from one point to another and are averaged over in a fluid element that means averaged over in ΔV the fact that that fluid is made up of discrete

molecule is ignored. The continuum hypothesis it is basically an approximation in the analogy that the planets are approximated by point masses, point center of a point particle where, when dealing with celestial mechanics and therefore results in an approximate solutions.

Consequently some source of the Continuum Hypothesis can lead to results which are of not desired accuracy. However under these under the right circumstances the continuum hypothesis produces extremely accurate tracer. Now in a couple of minutes we will try to understand at what circumstances Continuum Hypothesis hold good? At what circumstances it does not hold good the continuum hypothesis?

In that case we need to resort to the more fundamental theory which is known as statistical mechanics.

(Refer Slide Time: 11:13)

Those problems for which the continuum hypothesis does not allow solutions of desired accuracy are solved using statistical mechanics.

To determine whether or not to use conventional fluid dynamics or statistical mechanics, the Knudsen number is evaluated for the problem.

The Knudsen number (Kn) is a dimensionless number defined as the ratio of the molecular mean free path to a representative physical length scale.

This length scale could be, for example, the radius of a body in a fluid. The number is named after Danish physicist Martin Knudsen (1871–1949).

The Knudsen number is a dimensionless number defined as:

$$Kn = \frac{\lambda}{L}$$

Where

λ – mean free path

L – representative physical length scale.

These problems for which the continuum hypothesis does not allow solution of desired accuracy are solved using statistical mechanics. To determine whether or not to use conventional fluid dynamics or statistical mechanics we need to evaluate some number for the given problem that number is known as Knudsen number. The Knudsen number which is denoted as Kn is a dimensionless number defined as the ratio of the molecular mean free path.

To a representative physical length scale for an example this length scale could be the radius of a body in a fluid. This number is named after Danish physicist Martin Knudsen. Now let us try to understand how to calculate the Knudsen number and from the Knudsen number how to understand that when I allow, when I should apply the conventional fluid mechanics, when I should apply the statistical mechanics to understand the fluid dynamics.

So, the Knudsen number is a dimensional number which is defined as $Kn = \lambda / L$ where λ is the molecular mean free path and L is the representative physical length scale. So, that justifies that it is a dimensionless number.

(Refer Slide Time: 12:44)

For an ideal gas, the mean free path may be readily calculated so that:

$$Kn = \frac{k_B T}{\sqrt{2} \pi \sigma^2 p L}$$

Where

- k_B is the Boltzmann constant
- T is the thermodynamic temperature
- σ is the particle hard shell diameter
- p is the total pressure

For particle dynamics in the atmosphere, and assuming standard temperature and pressure, i.e. 25 °C and 1 atm,

we have $\lambda \approx 8 \times 10^{-8}$ m, or approximately 2.6×10^{-9} ft

For an ideal gas the mean free path as you know can be calculated easily where $Kn = k_B T / (\sqrt{2} \pi \sigma^2 p L)$ where k_B is the Boltzmann constant T is the temperature of the system, σ is the particle Hertz cell diameter and p is the total pressure. For an example for particle dynamics in the atmosphere and assuming standard temperature and pressure that is 25 degree centigrade and one atmosphere pressure.

We can calculate the value of the mean free path which is of which comes about 8 into 10 to the power - 8 meter or approximately 2.6 into 10 to the -9 feet. Mean free path that means this is the distance during which particle you will not collide to each other okay. Now let us calculate the; let us try to get the relationship between Mac and Reynold number in the gaseous in the fluids

itself because it is a very interesting relation who is tells you under which circumstances you have to deal the fluid dynamics originally conventional fluid mechanics.

And under what circumstances you have to deal with the statistical mechanics. Let me try to understand what is the relation between them?

(Refer Slide Time: 14:15)

Relationship to Mach and Reynolds numbers in Gases

The Knudsen number can be related to the Mach number and the Reynolds number:

Dynamic viscosity,

$$\mu = \frac{1}{2} \rho \bar{c} \lambda$$

Average molecule speed (from Maxwell-Boltzmann distribution)

$$\bar{c} = \sqrt{\frac{8 k_B T}{\pi m}}$$

Thus the mean free path,

$$\lambda = \frac{\mu}{\rho} \sqrt{\frac{\pi m}{2 k_B T}}$$

The Knudsen number can be related to the Mach number and the Reynolds number which you are going to calculate now. Dynamic viscosity is defined as μ is half $\rho \bar{c} \lambda$ where $\bar{c}$ is nothing but the average molecular speed which can be obtained from the Maxwell Boltzmann distribution which I have already talked in my earlier talks where $\bar{c}$ is defined as root over 8 KBT by πm .

Thus the mean free path λ is μ by ρ μ upon ρ root over πm by 2 KBT so that means mean free path can be defined in terms of the dynamic viscosity which is different by the relations that μ upon ρ root over πm by 2 KBT.

(Refer Slide Time: 15:07)

Dividing through by L (some characteristic length) the Knudsen number is obtained:

$$\frac{\lambda}{L} = \frac{\mu}{\rho} \sqrt{\frac{\pi m}{2 k_B T}}$$

Where

$\bar{c}$ is the average molecular speed from the Maxwell-Boltzmann distribution,
 T is the thermodynamic temperature
 μ is the dynamic viscosity
 m is the molecular mass
 k_B is the Boltzmann constant
 ρ is the density

Dividing through L the Knudsen number is defined as λ upon L which is nothing but μ upon ρ root over πm by $2 k_B T$ where $\bar{c}$ is the average molecular speed which I have already told it can be obtained from the Maxwell Boltzmann distribution, T is the thermodynamic temperature if the system is maintained at thermal equilibrium then in that case T is known as the temperature.

μ is the dynamic viscosity m is the molecular mass k_B is the Boltzmann constant and ρ is the density of the fluid, before going to the relations between the Knudsen number and Mach number etcetera. Let us try to understand what does it mean by Mach number.

(Refer Slide Time: 15:57)

Mach number is the speed of an object moving through air, or any other fluid substance, divided by the speed of sound as it is in that substance for its particular physical conditions, including those of temperature and pressure. It is commonly used to represent the speed of an object when it is traveling close to or above the speed of sound.

$$M = \frac{U_\infty}{c_s}$$

where the speed of sound is given by

$$c_s = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{\gamma k_B T}{m}}$$

where

U_∞ is the free stream speed,
 R is the Universal gas constant
 M is the molar mass
 γ is the ratio of specific heats, and is dimensionless.

Mach number is the speed of an object moving through air or any other fluid substance divided by the speed of sound as it is in that substance for its particular physical conditions including those of temperature and pressure. It is commonly used to represent the speed of an object when it is traveling close to or above the speed of sound. So, Mach number is defined as the ratio u infinity upon C_s .

Where C_s is the speed of sound which is defined as root over γRT upon M or root over in terms of Boltzmann's constant root over γK_v upon T where γ is the ratio of specific heat C_p upon C_v . This relation we have already derived in my earlier lectures in the kinetic theory of gases. And U infinity is the free stream speed and R is the universal gas constant, M is the molar mass and as usually γ is the sphere II ratio of specific heats, C_p upon C_v .

Second one which is more interesting in the fluid mechanics many people have already heard this name which is known as the Reynold number.

(Refer Slide Time: 17:16)

The dimensionless Reynolds number Re is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842–1912), who popularized its use in 1883.

$$R_e = \frac{\rho U_{\infty} L}{\mu}$$

Dividing the Mach number by the Reynolds number,

$$\frac{\mu}{\rho L} \sqrt{\frac{m}{\gamma k_B T}} \sqrt{\frac{\gamma \pi}{2}} = \frac{\mu}{\rho L} \sqrt{\frac{\pi m}{2 k_B T}}$$

and by multiplying by $\sqrt{\frac{\gamma \pi}{2}}$

$$\frac{M_{\infty}}{R_e} = \frac{U_{\infty} + c_s}{\rho U_{\infty} L} \frac{\mu}{\mu} = \frac{\mu}{\rho L c_s} = \frac{\mu}{\rho L \sqrt{\frac{\gamma k_B T}{m}}} = \frac{\mu}{\rho L} \sqrt{\frac{m}{\gamma k_B T}}$$

the Knudsen number is obtained.

The dimensionless Reynolds number is a dimensionless number that gives a measure of the ratio of the internal forces to the viscous forces and consequently quantifies the relative importance of the two types of forces for given flow condition. The concept was first introduced by Gabrielle Stokes in 1851 but the Reynolds number is named after Reynolds who popularized and it is used in 1883. It is defined as $RE = \text{Rho into } U \text{ infinity } L \text{ upon } \mu$.

Dividing the Mach number by the Reynolds number you can get the ratio $\frac{\mu}{\rho L \sqrt{m \gamma K_B T}}$ into $\frac{\mu}{\rho L \sqrt{\pi}}$ which is nothing but $\frac{\mu}{\rho L \sqrt{\pi}}$ by $2 K_B T$ and if you will multiply this equation by $\sqrt{\pi}$ by 2. You will get $Ma_{RE} = \frac{U}{c_s}$ and $\frac{\mu}{\rho L c_s}$ which is nothing but $\frac{\mu}{\rho L c_s}$.

And if you will substitute the values of c_s you will get $\frac{\mu}{\rho L}$ into $\frac{\mu}{\rho L \sqrt{m \gamma K_B T}}$ which turns out to be $\frac{\mu}{\rho L \sqrt{m \gamma K_B T}}$ and the Knudsen number is obtained.

(Refer Slide Time: 18:51)

The Mach, Reynolds and Knudsen numbers are therefore related by:

$$Kn = \frac{M}{Re} \sqrt{\frac{\gamma \pi}{2}}$$

The Knudsen number is useful for determining whether statistical mechanics or the continuum mechanics formulation of fluid dynamics should be used.

If the Knudsen number is near or greater than one, the mean free path of a molecule is comparable to a length scale of the problem, and the continuum assumption of fluid mechanics is no longer a good approximation. In this case statistical methods must be used.

Problems with high Knudsen numbers include the calculation of the motion of a dust particle through the lower atmosphere, or the motion of a satellite through the exosphere. One of the most widely used applications for the Knudsen number is in microfluidics and MEMS device design. The solution of the flow around an aircraft has a low Knudsen number, making it firmly in the realm of continuum mechanics.

So, finally you can get the relation of Mach, Rayland and Knudsen number are related as $Kn = \frac{Ma_{RE}}{\sqrt{\pi}}$. The Knudsen number is useful for determining whether statistical mechanics or continuum mechanics formulation of fluid dynamics should be used. For an example if the Knudsen number is near or greater than 1 the mean free path of a molecule is comparable to the length scale of the problem.

And the continuum assumption of fluid mechanics is no longer a good approximation. In that case statistical mechanics must be used. Similarly other way around if it is less than 1 then continuum assumptions of fluid mechanics will be valid approximation. In that case the usual fluid mechanics formulation is good enough to calculate any quantities in a fluid dynamics.

Problem for an example, problems with high Knudsen number include the calculation of motions of a dust particle through the lower atmosphere. Or the motion of a satellite through the exosphere one of the most widely used application of the Knudsen real number is in micro fluids and MEMS, device, design the solution of the fluid around an aircraft has a low Knudsen number making it firmly in the realm of continuum mechanics.

However in that case in a aircraft it goes from low Knudsen number to high Knudsen number. So, that makes the complicated nature of the fluid flow across an aircraft. So now let me talk in a small summary what does it mean by Navier Stokes theorem which is one of the most beautiful equation in fluid dynamics.

(Refer Slide Time: 20:53)

Navier–Stokes equations

The Navier Stokes equations (named after Claude-Louis Navier and George Gabriel Stokes) are the set of equations that describe the motion of fluid substances such as liquids and gases.

These equations state that changes in momentum (force) of fluid particles depend only on the external pressure and internal viscous forces (similar to friction) acting on the fluid.

Thus, the Navier–Stokes equations describe the balance of forces acting at any given region of the fluid.

The Navier Stokes equations are differential equations which describe the motion of a fluid. Such equations establish relations among the rates of change of the variables of interest.

The Navier-Stokes equation named after Claudia Lewis Navier and George Gabriel Stokes are the set of equations that describe the motion of fluid substances such as liquids and gases. How are these equations is in the non relativistic fluid equation. These equations state that change in momentum of fluid particles depend only on the external pressure and internal viscous forces similar to friction acting on the fluid.

Thus the Navier Stokes equation described the balance of forces acting any given region of the fluid. The Navier Stokes theorem are differential equation who is describe the motion of a fluid such equation established relation among the rates of change of the variable of interest.

(Refer Slide Time: 21:51)

For example, the Navier Stokes equations for an ideal fluid with zero viscosity states that acceleration is proportional to the derivative of internal pressure.

This means that solutions of the Navier-Stokes equations for a given physical problem must be sought with the help of calculus.

In practical terms only the simplest cases can be solved exactly in this way. These cases generally involve non-turbulent, steady flow (flow does not change with time) in which the Reynolds number is small.

For more complex situations, such as global weather systems like El Niño or lift in a wing, solutions of the Navier Stokes equations can currently only be found with the help of computers.

This is a field of sciences by its own called computational fluid dynamics.

For example the Navier Stokes equation for an ideal fluid with 0 viscosity states that that acceleration is proportional to the derivative of the internal pressure okay. In case of ideal fluid that means there is no viscosity okay this means that the solution of Navier-Stokes equation for a given physical problem must be sort with the help of calculus. In practical terms only the simplest cases can be solved exactly in this way.

These cases generally involved non turbulent steady flow. Flow does not change with time in which the number is small as we know that if the Reynolds number is very small then the flow is the laminar flow, non turbulent flow. If the Reynolds number become large if the velocity will become too large in that case Reynolds numbers will be very large. In that case the flow will no more be the laminar flow it will be the turbulent.

For more complex situation such as global weather systems like El Nino problem or lift in a wing solution of the Navier Stokes equation can currently only be found with the help of computers. This is the field of sciences by its own called computational fluid dynamics. General let us try to we are not going to derive the Navier Stokes equation, let us try to understand the different terms in Navier Stokes equation. What does it tell?

(Refer Slide Time: 23:31)

General form of the equation

The general form of the Navier Stokes equations for the conservation of momentum is

$$\rho \frac{D \mathbf{v}}{D t} = \nabla \cdot \mathbf{P} + \rho \mathbf{f}$$

where

ρ is the fluid density,

$\frac{D}{D t}$ is the substantive derivative (also called the material derivative),

$\mathbf{v}$ is the velocity vector,

$\mathbf{f}$ is the body force vector, and

$\mathbf{P}$ is a tensor that represents the surface forces applied on a fluid particle (the stress tensor).

Unless the fluid is made up of spinning degrees of freedom like vortices, $\mathbf{P}$ is a symmetric tensor.

Easily the general form of Navier Stokes equation for the conservation of momentum is $\rho \frac{D \mathbf{v}}{D t} = \nabla \cdot \mathbf{P} + \rho \mathbf{f}$. Where ρ is the fluid density and $\frac{D}{D t}$ is the substance of substantive derivative also called the material derivative that means this is the rate of change of time in the fluid frame itself.

And $\mathbf{v}$ is the velocity vector $\mathbf{f}$ is the body force vector and $\mathbf{P}$ is the tensor that represents the surface forces applied on fluid particle which, is the stress tensor we are going to give its form now. Unless the fluid is made up of spinning degrees of freedom like vertices $\mathbf{P}$ is a symmetric tensor.

(Refer Slide Time: 24:26)

In general, (in three dimension s) $\mathbf{P}$ has the form:

$$\mathbf{P} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

Where

σ are normal stresses,

τ are tangential stresses (shear stresses).

The above is actually a set of three equations, one per dimension. By themselves, these aren't sufficient to produce a solution.

However, adding conservation of mass and appropriate boundary conditions to the system of equations produces a solvable set of equations.

In that case P has the form of 3 cross 3 matrices where each element σ_{xx} , τ_{xy} , τ_{xz} similarly as you can see its form where σ are the normal stresses and τ are the tangential stresses or shear stresses. The above equation is usually a set of three equations one per dimension if you will open up it for three dimension. So, it gives the three differential equation one part each dimension.

By themselves they are not sufficient to produce a solution. However adding conservation of mass and appropriate boundary conditions to the system of equation produces a solvable set of equation. So, Navier Stokes theorem itself is not a solvable with the help of other conservation equation, conservation of mass and appropriate boundary condition. Then in that case it leads to a solvable set of equations. However we are not going to discuss it now.

Now let us take as we told you that fluid mechanics fluids can be divided in terms of the Newtonian fluid and non Newtonian fluid. Let us try to understand what is Newtonian fluid and what is non-Newtonian fluids.

(Refer Slide Time: 25:47)

Newtonian versus non-Newtonian fluids

A Newtonian fluid (named after Isaac Newton) is defined to be a fluid whose shear stress is linearly proportional to the velocity gradient in the direction perpendicular to the plane of shear.

This definition means regardless of the forces acting on a fluid, it continues to flow. For example, water is a Newtonian fluid, because it continues to display fluid properties no matter how much it is stirred or mixed.

A slightly less rigorous definition is that the drag of a small object being moved slowly through the fluid is proportional to the force applied to the object.

Important fluids, like water as well as most gases, behave up to a good approximation as a Newtonian fluid under normal conditions on Earth.

In Newtonian fluid named after Isaac Newton is defined to the fluid whose shear stress is linearly proportional to the velocity gradient in the direction perpendicular to the plane of shear. So, this definition means regardless of the forces acting on a fluid it continues to flow. For example water

is a Newtonian fluid because it continues to display fluid properties no matter how much it is stirred or mixed.

A slightly less rigorous definition is that the drag of a small object being moved slowly through the fluid is proportional to the force applied to the object. Important fluids like water as well as most gases behave up to a good approximation as in Newtonian fluid under normal conditions on the earth.

(Refer Slide Time: 26:41)

By contrast, stirring a non-Newtonian fluid can leave a "hole" behind. This will gradually fill up over time

this behaviour is seen in materials such as pudding, oobleck, or sand (although sand isn't strictly a fluid)

Alternatively, stirring a non-Newtonian fluid can cause the viscosity to decrease, so the fluid appears "thinner" (this is seen in non-drip paints).

There are many types of non-Newtonian fluids, as they are defined to be something that fails to obey a particular property

for example, most fluids with long molecular chains can react in a non-Newtonian manner.

By contrast stirring a non-Newtonian fluid can leave a hole behind this will gradually fill up over time. These behavior is seen in materials such as pudding, oobleck or sand all those sand is not a strictly a fluid. Alternatively starring a non-Newtonian fluid can cause the viscosity to decrease. So, the fluid appears thinner this is seen in non drip paints.

There are many types of non-Newtonian fluids as they are defined to be something that fails to obey a particular property. For example most fluid with long molecular chains can react in a non-Newtonian manner.

(Refer Slide Time: 27:31)

Equations for a Newtonian fluid

The constant of proportionality between the shear stress and the velocity gradient is known as the viscosity. A simple equation to describe Newtonian fluid behaviour is

$$\tau = -\mu \frac{dv}{dy}$$

where

τ is the shear stress exerted by the fluid ("drag")

μ is the fluid viscosity, a constant of proportionality

$\frac{dv}{dy}$ is the velocity gradient perpendicular to the direction of shear.

For a Newtonian fluid, the viscosity, by definition, depends only on temperature and pressure, not on the forces acting upon it.

So, now let us define what is Newtonian fluid in a mathematical form the constant of proportionality between the shear stress and the velocity gradient as we told that if the relation is linear then it is known as the Newtonian fluid. So, if the proportionality constant is viscosity in that case a simple relation to describe, to define the Newtonian fluid is $\tau = -\mu \frac{dv}{dy}$ where τ is the shear stress exerted by the fluid or drag force.

And μ is the fluid viscosity which is a constant of proportionality and $\frac{dv}{dy}$ is the velocity gradient perpendicular to the direction of this shear. For a Newtonian fluid the viscosity by definition depends only on temperature and pressure not on the forces acting upon it.

(Refer Slide Time: 28:30)

If the fluid is incompressible and viscosity is constant across the fluid, the equation governing the shear stress in Cartesian coordinates is

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

Where

τ_{ij} is the shear stress on the i^{th} face of a fluid element in the j^{th} direction

v_i is the velocity in the i^{th} direction

x_j is the j^{th} direction coordinate.

If a fluid does not obey this relation, it is termed a non-Newtonian fluid, of which there are several types.

Among fluids, two rough broad divisions can be made: ideal and non-ideal fluids. An ideal fluid really does not exist, but in some calculations, the assumption is justifiable. An ideal fluid is non viscous offers no resistance whatsoever to a shearing force.

One can group real fluids into Newtonian and non-Newtonian. Newtonian fluids agree with Newton's law of viscosity. Non-Newtonian fluids can be either plastic, dilatant etc.

If the fluid is incompressible and viscosity is constant across the fluid the equation governing the shear stress in Cartesian coordinates is $\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$ where τ_{ij} is the shear stress on the ij face of a fluid element in the j th direction, i stands for phase of the fluid j stands for the direction of the fluid. And v_i is the velocity of the i th direction x_j is the j th direction coordinate.

If a fluid does not obey this relation it is termed as non-Newtonian fluid of which there are several types. Among fluids to rough broad divisions can be made ideal non-ideal fluids and ideal fluid really does not exist but in some calculations the assumption is made such a way so that it will behave like an ideal fluid and ideal that means an ideal fluid is a non viscous which offers no resistance whatsoever to a shearing force .

One can group real fluid into Newtonian and non-Newtonian, Newtonian fluids agree with Newton's law of viscosity non-Newtonian fluids can be either plastic, dilatant etcetera. Now let us try to understand from the, what does it mean by fluids, liquid, gases from a very grassroots level. Now let us try to understand.

(Refer Slide Time: 30:17)

Fluids—Liquids and Gases

Hydrostatics deals with the mechanics of fluids in equilibrium and our first step, therefore, is to understand clearly as to what exactly do we mean by a fluid.

Unlike a solid, in which, the strain set up under a shearing stress lasts throughout the period of application of the stress, a fluid may be defined as that state of matter which cannot indefinitely or permanently oppose or resist a shearing stress.

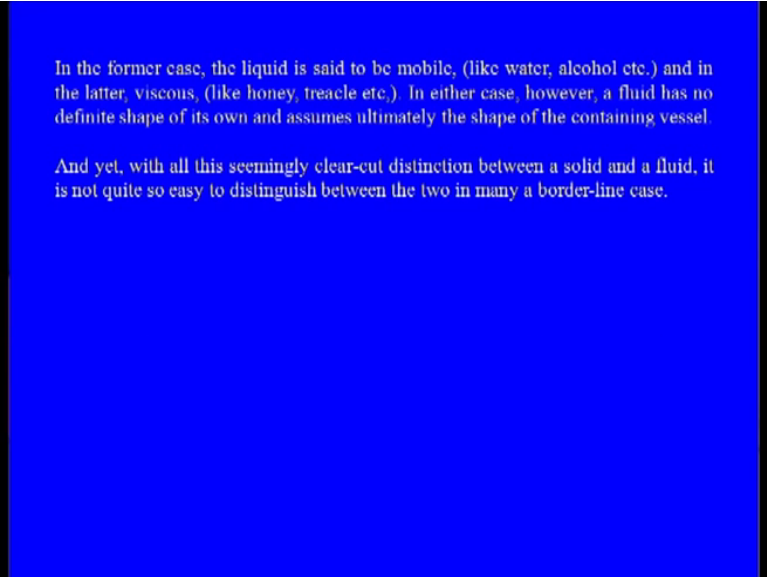
In fact, it constantly and continuously yields to it, though the yield may be rapid in some cases and slow in others.

So, hydrostatics deals with the mechanics of fluids in equilibrium and our first step therefore is to understand clearly as to what exactly do, we mean by a fluid. Unlike a solid in which the strain set up under a shearing stress lasts throughout the period of application of this stress

whereas if fluid may be defined as that state of matter which cannot indefinitely or permanently oppose or register shearing stress.

That is the basic difference between solid and in fact it constantly and continuously yields to it. Though the yield may be rapid in some cases and slow in others.

(Refer Slide Time: 31:07)



In the former case, the liquid is said to be mobile, (like water, alcohol etc.) and in the latter, viscous, (like honey, treacle etc.) In either case, however, a fluid has no definite shape of its own and assumes ultimately the shape of the containing vessel.

And yet, with all this seemingly clear-cut distinction between a solid and a fluid, it is not quite so easy to distinguish between the two in many a border-line case.

In the former case the liquid is said to be mobile like water, alcohol etcetera. In the latter, viscous like honey, trickle etcetera in either case however a fluid has no definite shape of its own and assumes ultimately the shape of the containing vessel. And yet with all this seemingly clear cut distinctions between a solid and a fluid it is not quite so easy to distinguish between the two in many a borderline cases.

There are many borderline cases where you cannot exactly distinguish which one is solid and which one is fluid.

(Refer Slide Time: 31:52)

The fundamental distinction between the two nevertheless remains, and we declare a substance to be a fluid or a solid according as it does or does not yield to a shearing stress applied to it over a long enough period.

Fluids too are further divided into two classes : (i) liquids and (ii) gases

A liquid is a fluid which, although it has no shape of its own, occupies a definite volume which cannot be altered, however great the force applied to it!!

➡ a liquid is a fluid which is quite incompressible and has a free surface of its own, as, for example, water, alcohol, ether, honey, treacle etc.

A gas, on the other hand, is a fluid, which cannot only be easily compressed when subjected to pressure but, with a progressive reduction of the pressure on it, can also be made to expand indefinitely, occupying all the space made available to it. Thus, the whole of the gas will escape out from a vessel, if there be the tiniest aperture in it somewhere.

Summarizing then, a gas is a fluid which has neither a shape nor a free surface of its own, e.g., oxygen, hydrogen, carbon dioxide, air (a mixture of gases) etc.

The fundamental distinctions between two nevertheless remains and we declare a substance to be a fluid or a solid according as the does or does not yield to a shearing stress applied to it over a long enough period. Fluids two are further divided into two classes' liquids and gases. A liquid is a fluid which although it has no safe of its own occupies a definite volume which cannot be altered however great the force applied to it.

A liquid is a fluid which is quite incompressible and has a free surface of its own as for example water, alcohol, ether, honey, trickling etcetera. A gas on the other hand is a fluid which cannot only be easily compressed when subject to pressure but with a progressive reduction of the pressure on it. It can also be made to expand indefinitely occupying all the space made available to it. Thus the whole of the gas will escape out from a vessel if they are the tiniest efforts are in it somewhere.

If you will summarize these distinctions between fluid solid and within a fluid liquids and gases a gas if you will summarize all these concept together then a gas is a fluid which has neither shape nor a free surface of its own example oxygen, hydrogen, carbon dioxide, air which is a mixture of gases etcetera. But liquid is a fluid which is quite incompressible and has a free surface of its own unlike the gases.

(Refer Slide Time: 33:51)

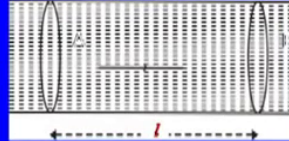
Rate of flow of a fluid

Fluids, as we now know, include both liquids and gases, their chief characteristic being that they cannot permanently withstand any shearing stresses, however small.

We shall concern ourselves here only with ideal liquids and gases, i.e. with liquids which are perfectly mobile (zero viscosity) and incompressible (zero compressibility or infinite bulk modulus) and with gases which perfectly obey Boyle's and Charles' laws.

The rate of flow of a fluid is defined as the volume of it that flows across any section of a pipe in unit time. It is really the volume rate of flow of the fluid or its discharge, usually represented by Q or V .

Considering the fluid to be incompressible, if its velocity of flow be v , in a direction perpendicular to two sections A and B, (Fig. 1) of area a , and distance l apart, and if t be the time taken by the liquid to flow from A to B, we have $vt = l$



Now let us try to understand how a rate of flow of a liquid can be estimated. Fluids as you know include both liquids and gases they are main characteristics being that they cannot permanently withstand any searing stresses however it is small. We shall concern ourselves here only with ideal liquids or gases that is with liquids which are perfectly mobile that means there is no viscosity that means it is ideal and incompressible means 0 compressibility or infinite bulk modulus.

And with gases which perfectly obey the Boyle's or Charles law. So, now let us calculate the rate of flow. So, the rate of flow of a fluid is defined as the volume of it that flows across any section of a pipe in unit time that means volume of it that flows across any section of the power per unit. It is really the volume rate of flow of the fluid or its discharge usually represented by the symbol Q or V .

Now let us see this picture of a pipe considering the fluid to be incompressible if it is velocity of flow v be in a direction perpendicular the two sections A and B as shown in the figure 1 of area a , and distance l apart and if t is the time taken by the liquid to flow from A to B. We have vt should be the length of this tube which is l .

(Refer Slide Time: 35:41)

Obviously, the volume of liquid flowing through the section AB, in this time, is equal to the cylindrical column AB $= l \times a = vt \times a$. This is, therefore, the volume of liquid flowing across the section in time t.

So,
 volume rate of flow of liquid $\times$ discharge Q or $V = vt \times a / t = v \times a$
 $=$ velocity of liquid $\times$ area of cross-section of the tube.

When measured in cubic feet per sec. (F.P.S. system), the unit is the cusec (c.ft/sec).

Sometimes, the rate of flow of a liquid is also expressed in terms of the mass of the liquid flowing across any section in unit time and is referred to as its mass rate of flow.

Thus, mass rate of flow of liquid $=$ mass of liquid flowing across any section per unit time $=$ velocity of liquid $\times$ area of cross-section $\times$ density of liquid $= v \times a \times \rho$.

So, obviously the volume of the liquid flowing through the section AB in this time is equal to the cylindrical column AB which is nothing but the length time area l times a , l is nothing but the vt , so vt times a this is therefore the volume of the liquid flowing across the section in time t . So, which is nothing but the vt times a . So, the volume rate of flow of liquid into discharge Q or V which is nothing but the vt times a by T , TT will cancel which is nothing but the a times v which is nothing but the velocity of the liquid v and a is the area of the cross section of the tube.

If you will measure in a pear system this is known as cubic feet per second. This unit is Cusec which is known as Cusec which is nothing but the cubic feet per second. Sometimes the rate of flow of a liquid is also expressed in terms of the mass of the liquid flowing across any section in unit time and is referred to as mass rate of flow. When you described in terms of mass so it is known as mass rate of flow.

Thus the mass rate of flow of liquid equal to mass of liquid flowing across any section per unit time which is nothing but the velocity of the liquid into area of cross section into density of the liquid to get in terms of mass which is nothing but the v times a times ρ , v is the velocity of the liquid a is the area of cross section and ρ is the density of liquid.

Now let us try to understand the lines and tubes of flow which are very similar to the lines of force in electrostatic and magnetic fields. So, it is very easy to understand the lines of lines and tubes of flow in fluid mechanics.

(Refer Slide Time: 37:53)

Lines and Tubes of Flow

In a simple flow of liquid, i.e., when it is slow and steady, the velocity at every point in the fluid remains constant in magnitude as well as in direction, the energy needed to drive it being used up in overcoming the 'viscous drag' between its layers.

In other words, each particle follows exactly the same path and has the same velocity as its predecessor and the fluid is said to have an orderly or a streamline flow.

In such a case, if we consider a line along which the particles of the fluid move, the direction of the line at any point is the direction of the velocity of the fluid at that point. Such a line is called a streamline.

More correctly, a streamline may be defined as a curve, the tangent to which at any point gives the direction of flow of the fluid at that point.

It may be straight or curved, according as the lateral pressure on it is the same throughout or different.

In a simple flow of liquid that is when it is slow and steady the velocity at every point in the fluid remains constant in magnitude as well as in detail. This is the most important concept in the fluid mechanics. So, let me repeat it again when the velocity of the liquid is very slow and steady. In that case the velocity at every point throughout the fluid remains constant in magnitude as well as indeed x.

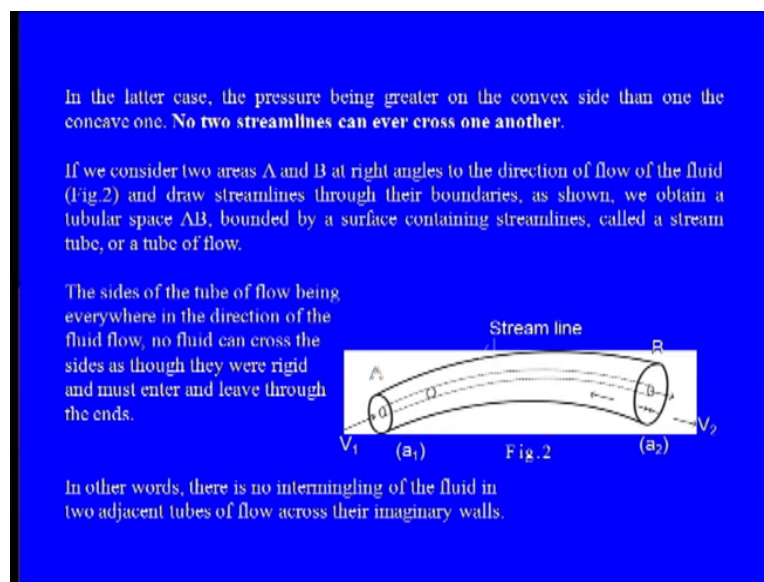
The energy needed to drive it used up in overcoming the viscous drag between its layers. In other words each particle in the fluid follows exactly the same path and has the same velocity as its predecessor and the fluid is said to have an orderly or streamline ok. This is the meaning of streamline. There are two kinds of flow one is streamline flow another is turbulent. In streamline flow if every particle in the fluid, it follows exactly the same path.

And has the same velocity as its predecessor in that case that flow is known as streamline flow. Deviation of this streamline flow is known as turbulent flow which is always very tough to handle. In such a case if we consider a line in the streamline case if you will consider a line along

which the particles of the liquid move, the direction of the line at any point is the direction of the velocity of the fluid at that point.

Such a line is called a streamline, more correctly a streamline may be defined as a curve the tangent to which at any point gives the direction of flow of the fluid at that particular point. It may be straight, curved according as the lateral pressure on it is the same throughout or different. Depending on the lateral pressure whether it is same throughout the fluid or different the size, shape of this streamlined it could be straight or curved.

(Refer Slide Time: 40:23)

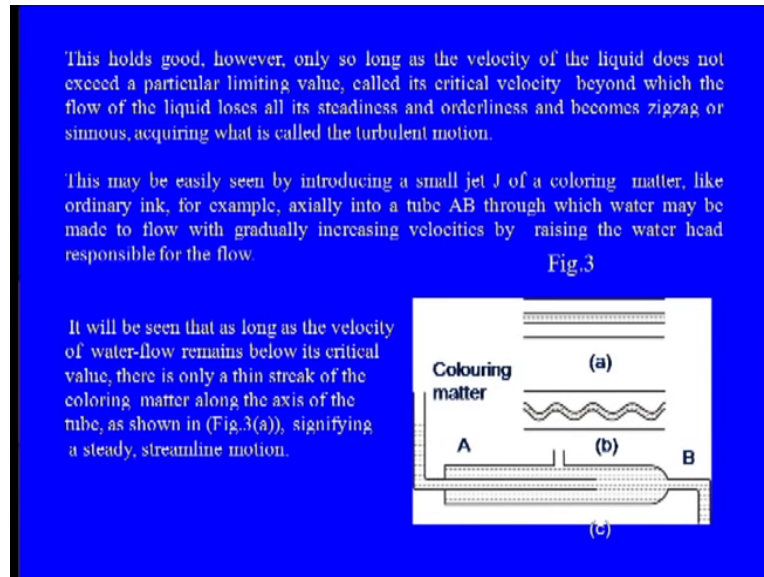


In the latter case when the lateral pressure is different the pressure being greater on the convex side than on the concave one. No two streamlines can ever cross one another. So, if we consider let us try to understand this thing in a given example in given figure, figure 2 if you will consider two areas A and B at right angles to the direction of flow of the fluid just you see this figure 2. And draw streamlines through their boundaries as soon.

We obtain a tubular space AB bounded by a surface containing streamlines called a stream tube or a tube of flow. So, this is the definition of tube of flow. First we have understood what is streamline and using this streamline we can construct a tube, we can construct a stream tube or a tube of flow using two streamlines. The sides of the tube of flow being everywhere in the direction of the fluid flow no fluid can cross the sides as though they are rigid.

And must enter or leave through the ends in other words there is no intermingling of the fluid in the two adjacent tubes of flow across the imaginary walls.

(Refer Slide Time: 41:54)



This holds good however only so long as the velocity of the liquid does not exceed a particular limiting value called its critical velocity. Beyond which the flow of liquid loses all its steadiness and orderliness and becomes zigzag or sinus acquiring what is called as turbulent motion. So, whatever descriptions I have given those descriptions are only valid when the velocity of the liquid does not exceed a particularly limiting value known as the critical.

This thing can also be understood in terms of the Reynolds number, if the critical velocity will be too large the Reynolds number will be too greater than one in that case these concepts will no longer be valid. This may be easily seen by introducing a small jet of a coloring matter. Let us try to do this experiment also in this figure. Like ordinary ink for example axially into a tube AB you just see this figure AB.

Through which water may be made to flow with gradually increasing velocity by raising the water head responsible for the flow. It will be seen that as long as the velocity of water flow remains below its critical value there is only a thin streak of the coloring matter along the axis of the tube as you can see in Figure 3, signifying a steady stream line motion.

(Refer Slide Time: 43:38)

As soon as the velocity attains its critical value, the coloring matter takes a zigzag path (Fig.3(b)) indicating haphazard changes of velocity from point to point and consequent distortion of tubes of flow.

And, later, when this value is exceeded, the coloring matter spreads out in all directions, filling the entire tube, showing that the tubes of flow have now broken down and the motion has become utterly disorderly or turbulent.

Reynold actually used this very method to determine the critical velocity for water. Obviously, the velocity of the water just before the thin streak of coloring matter becomes zigzag or sinuous [Fig. 3(b)], directly gives the value of the critical velocity v_c for it and is equal to rate of flow of the liquid, i.e., volume of liquid flowing out of the tube per second, divided by the area of cross-section of the tube.

Thus I have talked about the meaning of different terminologies used in fluid mechanics, assumptions of fluid mechanics etc.

But as soon as the velocity attains its critical value the coloring matter takes a zigzag path which is shown in Figure 3 indicating haphazard change of velocity from point to point and consequent distortion of the tubes of flow. And later when this value is exceeded far apart the coloring matter spreads out in all directions filling the entire tube showing that the tubes of flow have now broken down and the motion has become utterly disorderly or turbulent.

So, this experiment even you can perform me in your home also, in your small lab also that that shows you the streamline flow and the turbulent flow. Reynolds all actually use this very method to determine the critical velocity for water. Obviously the velocity of the water just before the thin streak of the coloring matter becomes zigzag or a sinus directly gives the value of the critical velocity v_c for it and is equal to the rate of flow of the liquid.

That is the volume of the liquid flowing out of the tube per second divided by the area of the cross section of the tube. So, from there Reynolds calculated the critical velocity from there he calculated the Reynolds number, if the velocity is less than it, it will be the stream line flow if the velocity will be more the critical velocity it will be the turbulent.