Engineering Physics
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Module-01
Lecture 02
Malus Law & Superposition of Ways

I am M.K. Srivastav of the Department of Physics, IIT Roorkee. This is the second lecture of the five lecture series on Polarization. In the last lecture, which was the first one, we considered Maxwell's equations leading to electromagnetic waves. We then considered the meaning of polarized light, polarization by reflection, refraction and scattering. We study Brewster's law and considered working of a Polaroid.

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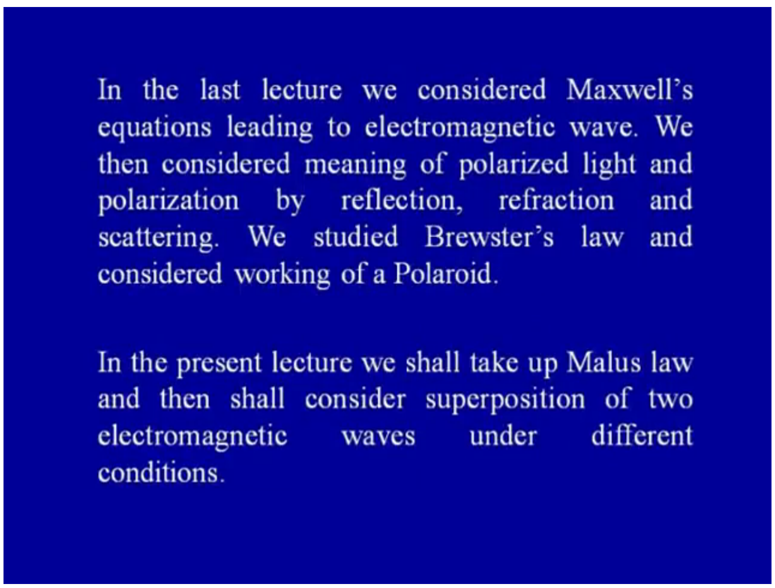

In the present lecture which is the second one of the series, we shall take up Malus law and then shall consider superposition of two electromagnetic waves under different conditions. The different condition means difference in amplitude, different phase differences and things like that. Law of Malus;

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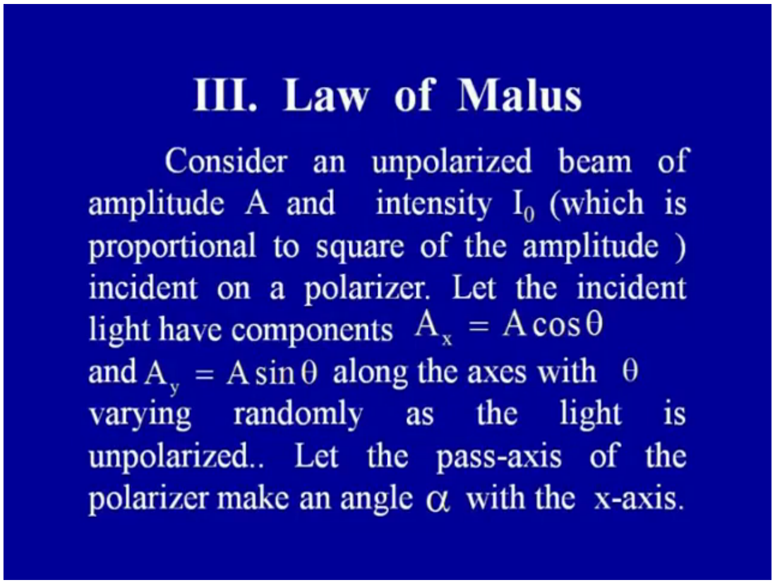

Considered an unpolarized beam of amplitude A and intensity I naught which is proportional to the square of the amplitude, as we know, incident on a polarizer, the incident light have components Ax = A cos theta and Ay = A sine theta along the axis the theta varying randomly as the light is unpolarized.

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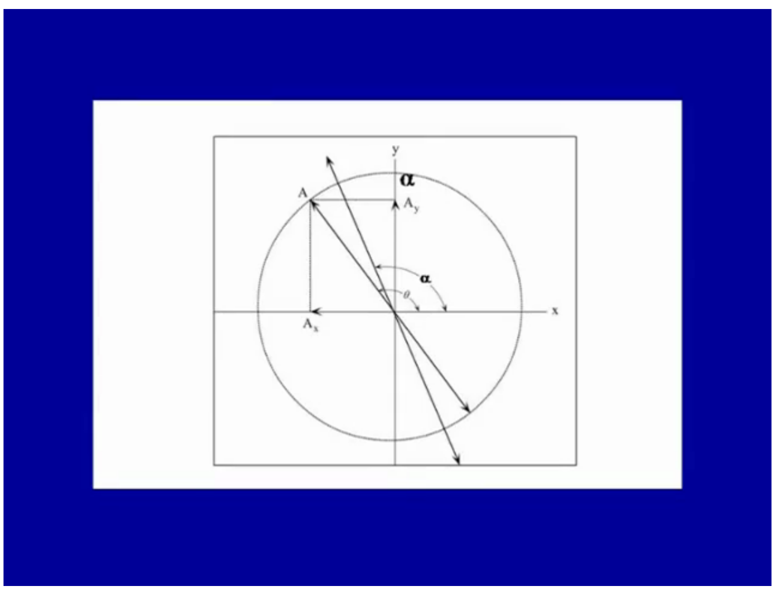

Let the pass axis of the polarizer making an angle alpha with the x axis. The polarizer as we have seen is a device which allows the light wave with electric vibrations parallel to the pass direction to pass through without attenuation essentially and absorb the light wave electric vibrations perpendicular to the pass direction.

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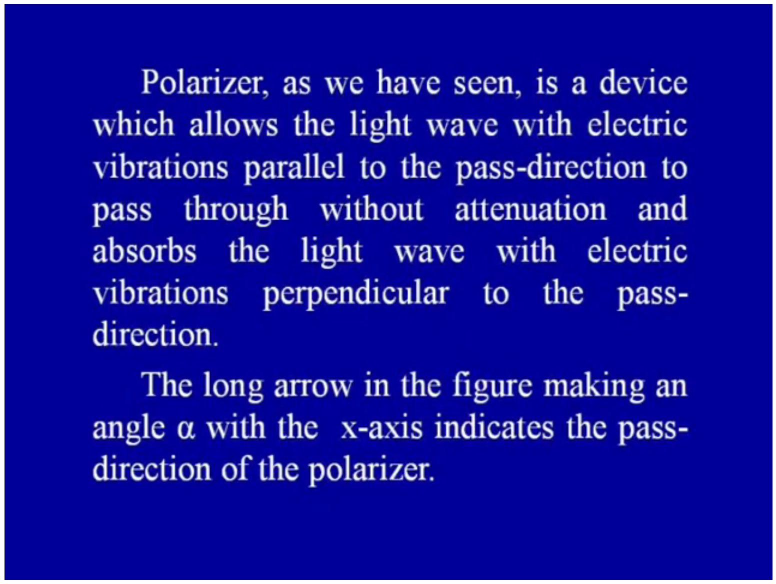

The long arrow in the figure making an angle alpha with the x axis indicates the pass direction of the polarizer.

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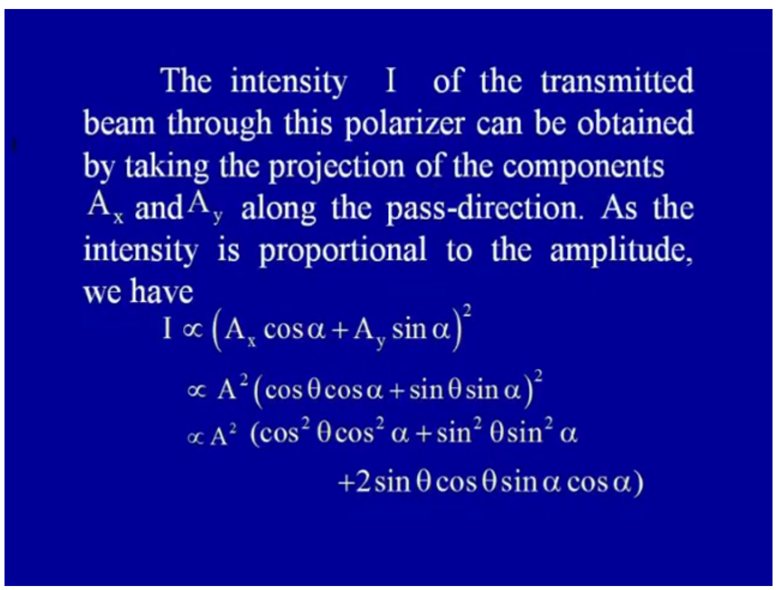

The intensity I of the transmitted beam through the polarizer can be obtained by taking the production of the components Ax and Ay along the pass direction and intensity is proportional to the amplitude we have, Intensity I is proportional to the square of Ax cos alpha + Ay sine alpha it means the proportional to A square times the square of cos theta cos alpha the sine theta sine alpha.

It means the proportional to A square times cos quare theta cos square alpha + sine square theta sine square alpha + twice sine theta cos theta sine alpha cos alpha. Now and the angle theta varies randomly in the unpolarized beam. The average values of term appearing there are cos is square theta, the average value is half sine squared theta again.

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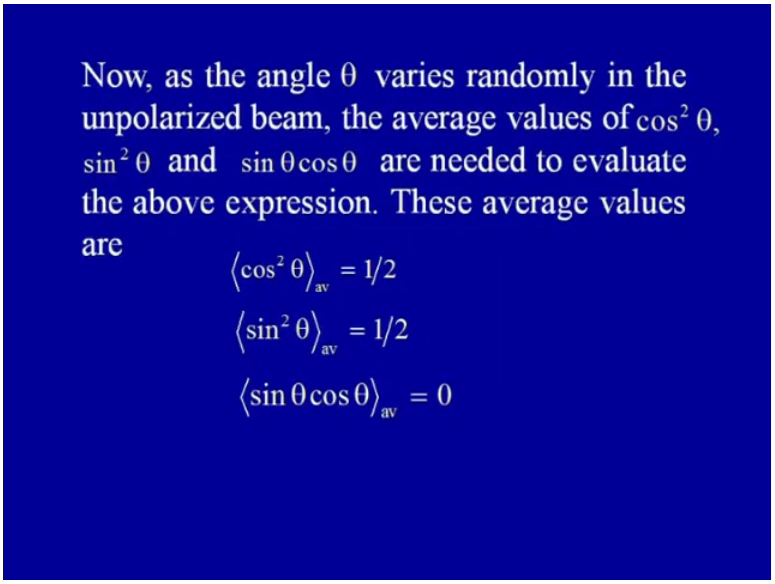

The average value are half you see the averages are taken over a complete cycle then the average value of sine theta into cos theta again over a complete cycle is 0.

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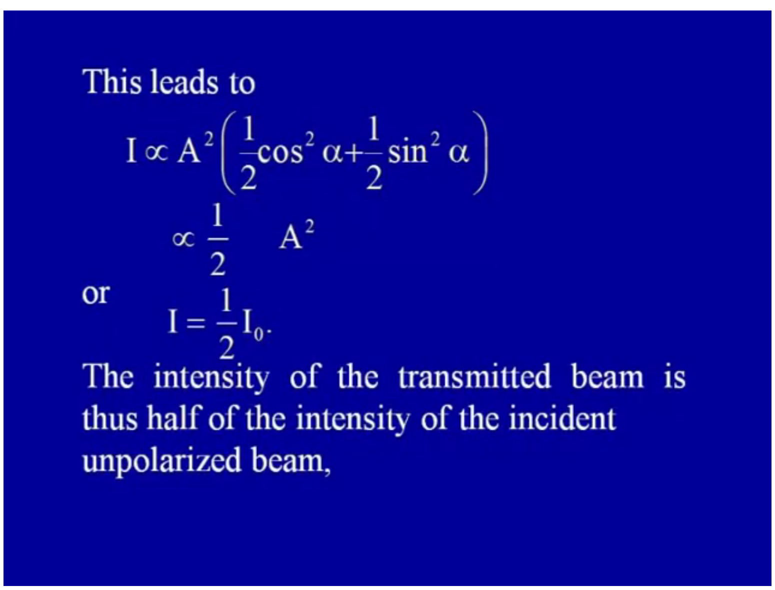

This leads to intensity is not proportional to A square times half of cos is square alpha + half of sine square alpha, which means it is proportional to half of A naught square. And we get the final result that I is = half of I naught. The intensity of the transmitted wave mean, remember, the incident beam is unpolarized; so, the intensity of the transmitted beam is thus half of the intensity of the incident unpolarized beam.

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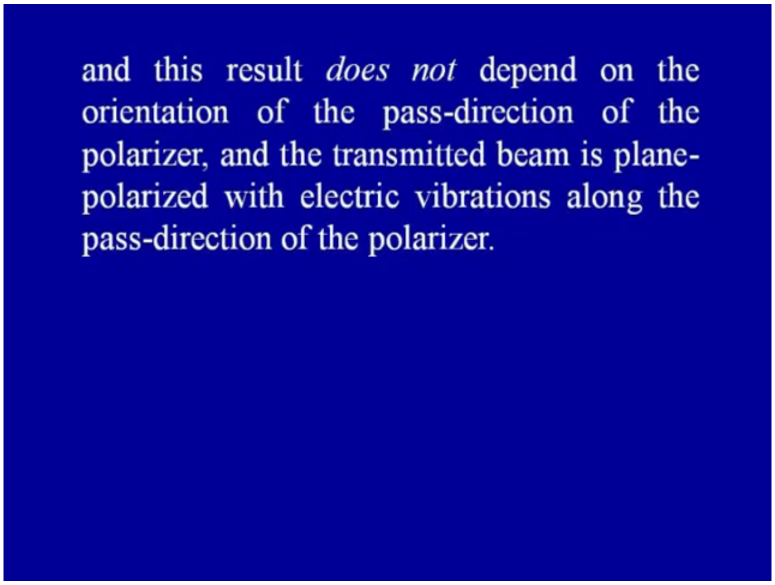

And another thing this result does not depend on the orientation of the pass direction of the polarizer. That is it is independent of the angle alpha. And the transmitted beam is plane polarized with electric vibrations along the pass direction of the polarizer.

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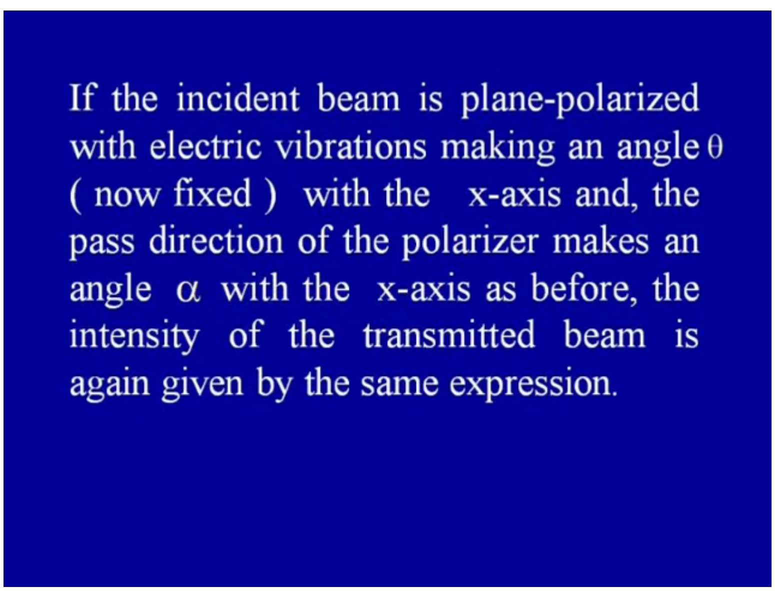

If the incident beam is plane polarized with electric vibrations making an angle theta as before but now this angle is fixed with the x axis and the pass direction of the polarizer makes an angle alpha with the x axis as before the intensity of the transmitted beam is again given by the same expression.

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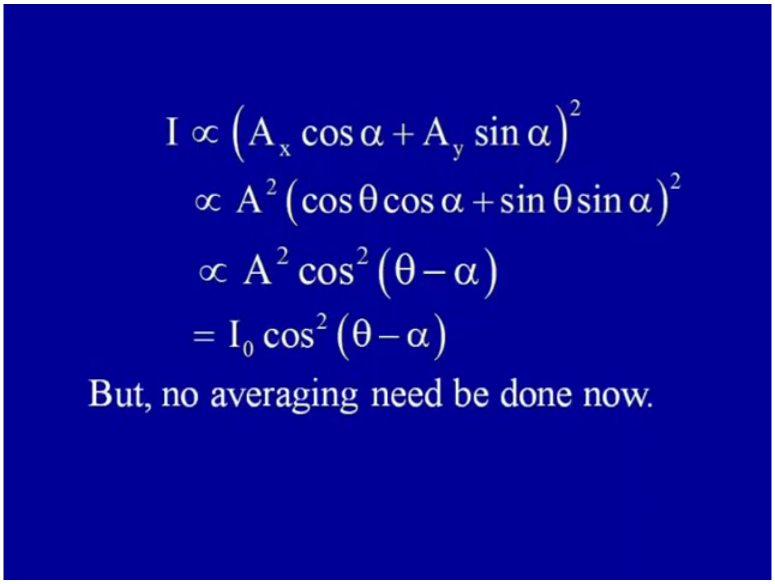

That is, the intensity proportional to the square of Ax cos alpha + Ay sine alpha which means it is a proportional to A square times cos theta cos alpha + sine theta sine alpha squared. So, it is proportional to A square times cos square of theta - alpha. Therefore the intensity I = I naught times cos square of theta - alpha. You see, no averaging need be done now because the angle theta is fixed; the incident light beam is polarized.

The intensity of the transmitted beam now you find thus varies as the square of the cosine of the angle theta - alpha, the angle between the plane of vibration, of the incident beam and the pass direction of the polarizer, working here as an analyzer.

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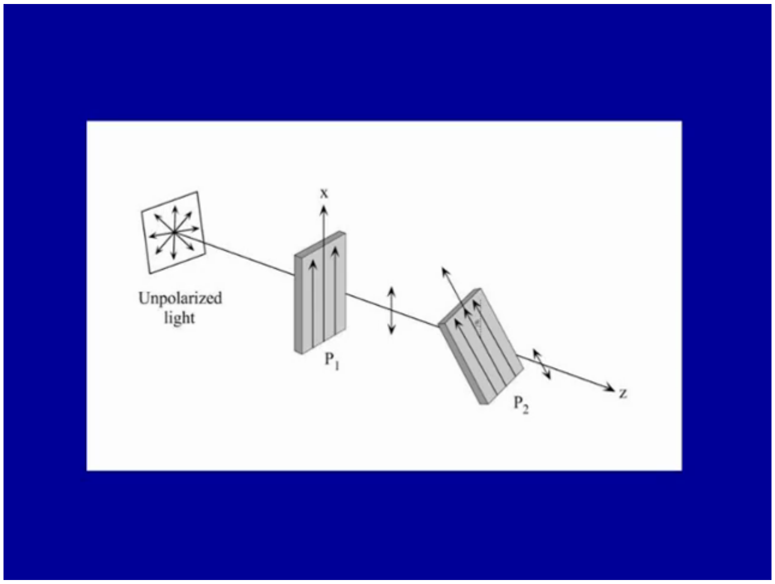

This is called Law of Malus, Now if a plane polarized beam, polarized along the x axis by the polarizer P1, the incident on another polarizer P2 here.

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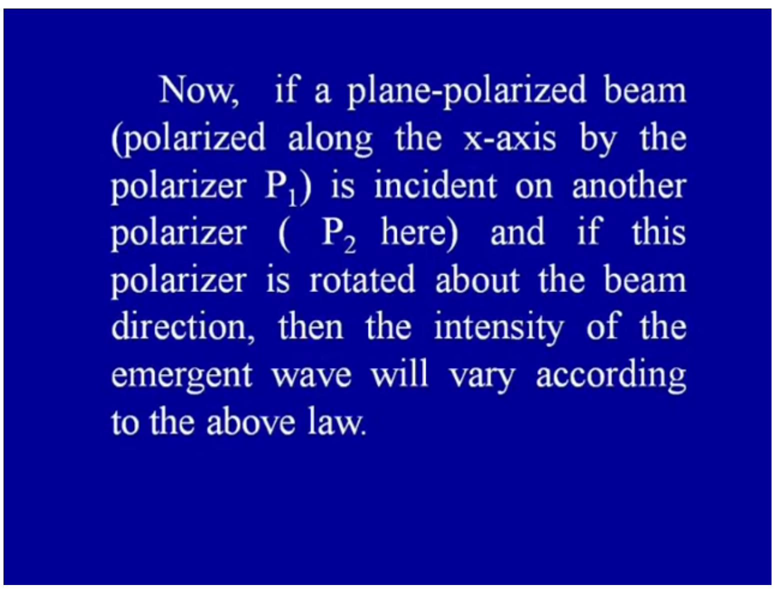

And if this polarizer is rotated about the beam direction then, the intensity of the emergent wave will vary according to the above law.

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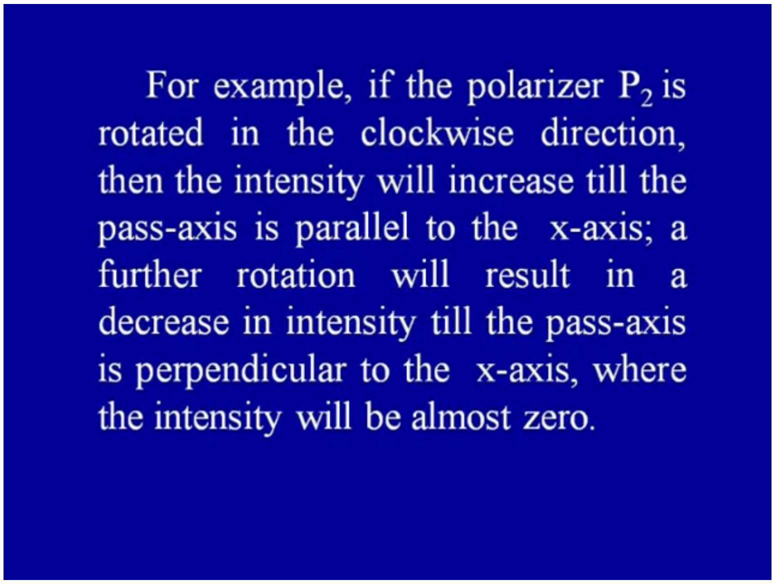

For example, the polarizer P2 is rotated in the clockwise direction, then, the intensity will increase, till the pass axis is parallel to the x axis. Further rotation will result in a decrease in intensity till the pass axis is perpendicular to the x axis where the intensity will be almost 0. If we further rotate it will pass through maximum and again a minimum before it reaches its original position.

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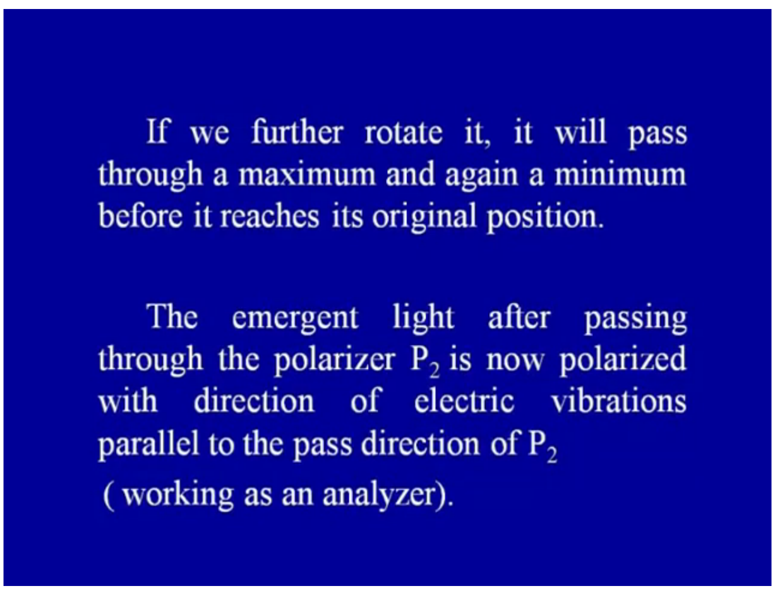

The emergent light after passing through the polarizer P2 is now polarized with the direction of electric vibrations parallel to the pass direction of P2 working here as an analyzer. Let us now consider superposition of two waves.

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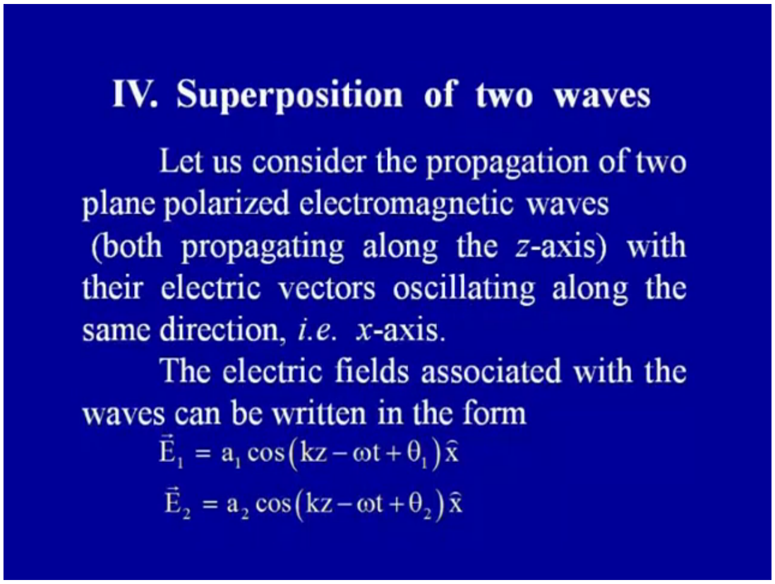

Consider the propagation of two plane polarized electromagnetic waves both propagating along the z axis that is the same direction with their electric vectors oscillating along the same direction that is the x axis. For the electric fields associated with the waves can be written in the form, E1 = a1 cos of kz - Omega t + theta 1 and E2 which is a2 times cos of kz - Omega t + theta 2.

Both of these fields are along the x axis. a1and a2 reporting the amplitudes of the waves and x hat represent the unit vector along the x axis, theta 1 theta 2 or arbitrary for its constants. The resultant of these two waves would be given by

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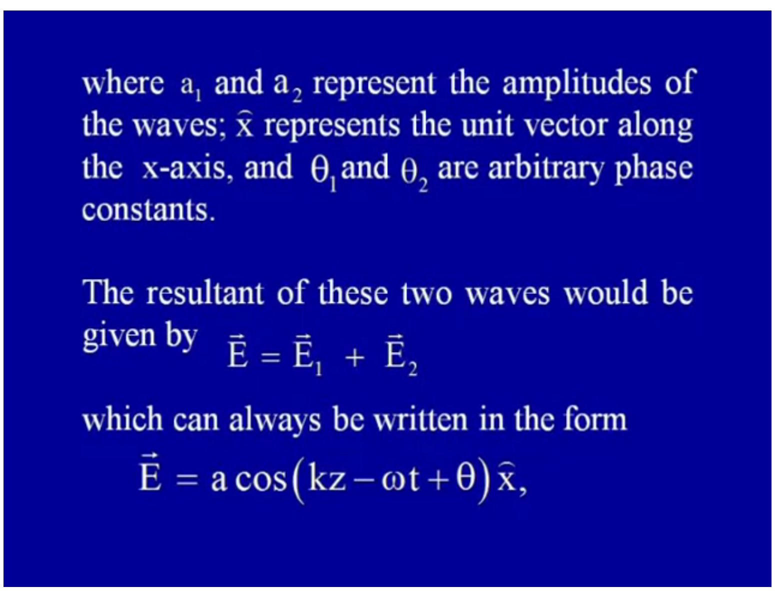

Just the vector addition, so E is = E1 + E2. Now, this can always be written in a form like this: E is = a times cos kz minus Omega t + theta along the x axis naturally,

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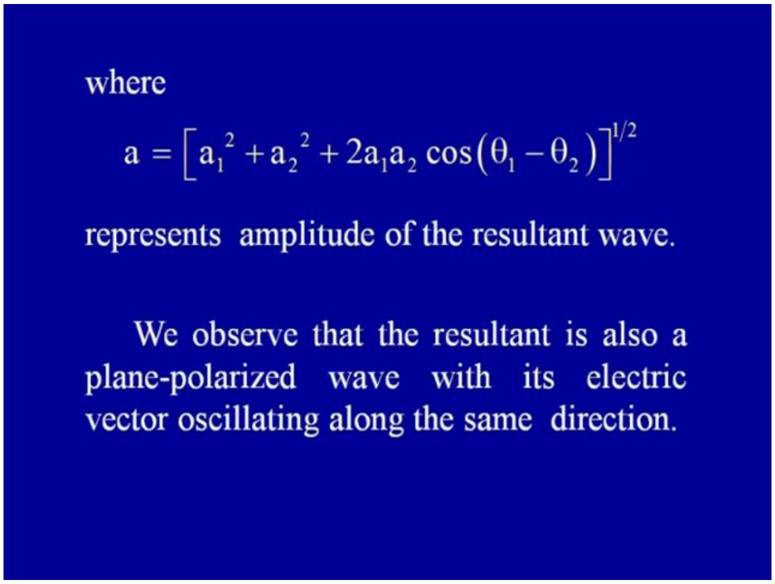

Where a is given by the square root of a1 square + a2 square + twice a1 a2 times cos of theta 1 - theta 2, a represent the amplitude of the resultant wave; resultant of the superposition of the original two waves. We observe that the resultant is also a plane polarized wave with its electric vector oscillating along the same direction, along the x axis.

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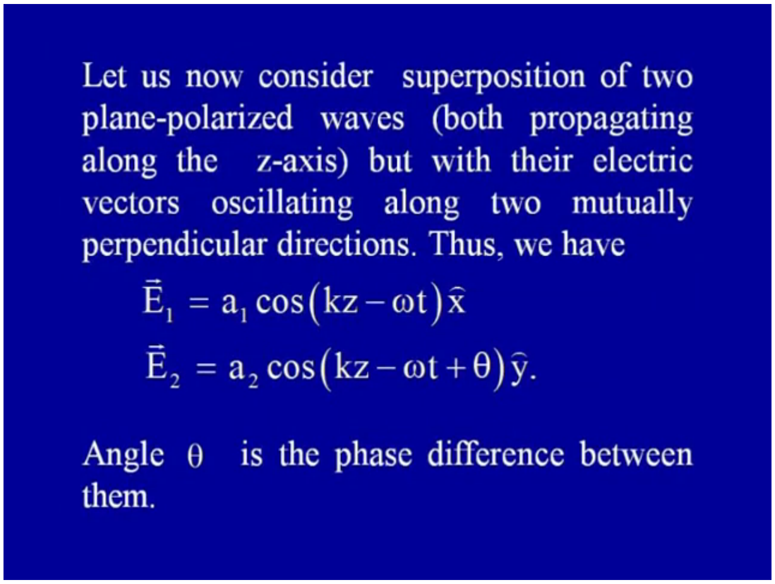

Let us now consider superposition of two plane polarized waves both propagating along the z axis but with their electric vectors oscillating along two mutually perpendicular directions.

Earlier they were along the same direction; now, they are mutually perpendicular directions, say along the x axis and along the y axis. Remember, both the waves are propagating along the z axis.

So, we can write E1 = a1 cos of kz - Omega t this is along the x axis. E2 = a2 cos of kz - Omega t + theta along the y direction. The angle theta is the phase difference between these two waves.

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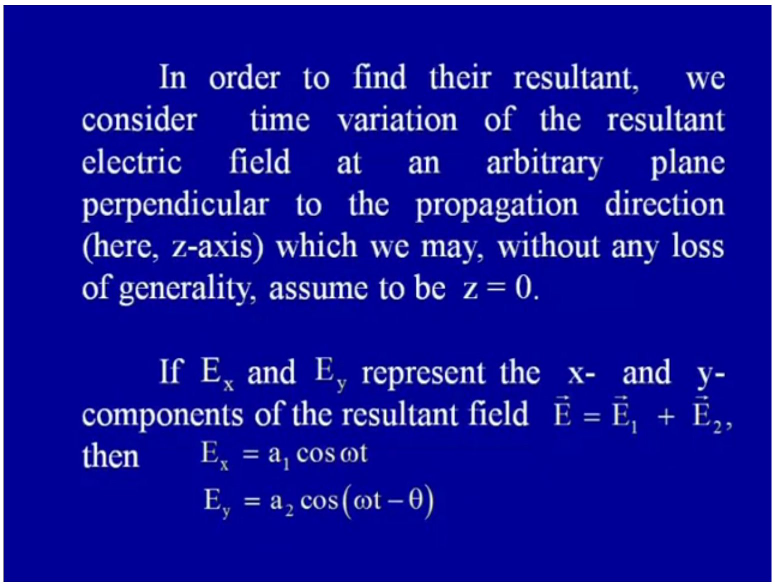

Now, in order to find the resultant, we consider the time variation of the resultant electric field at an arbitrary plane, just for simplicity, at an arbitrary plane perpendicular to the propagation direction which is the z axis. This arbitrary plane without any loss of generality we can assume it to be z = 0 just for simplicity. Ex and Ey represent the x and y components of the resultant field E1+ E2, then Ex will be = a1 cos Omega t and Ey is = a2 cos Omega t - theta.

Now eliminate t between these two equations. And for that we rewrite the above expression for Ey has;

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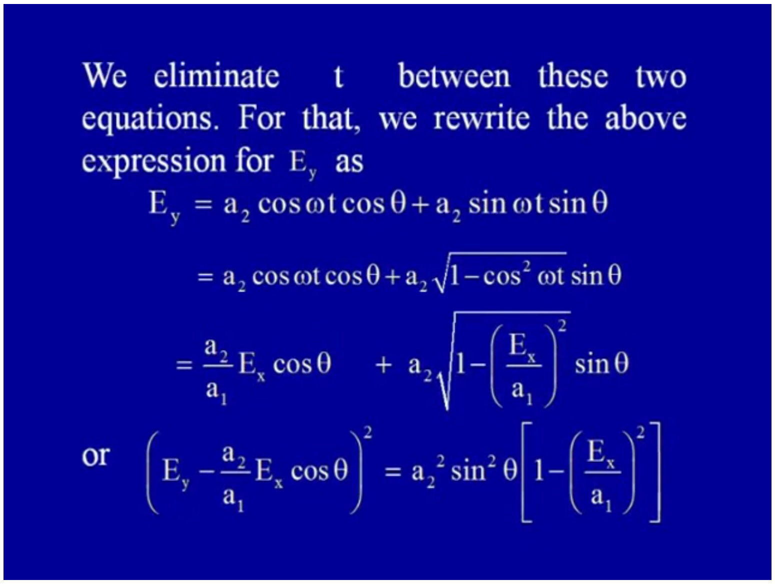

Ey = a2 cos Omega t cos theta + a2 sine Omega t sine theta. We can write it as first term remaining unchanged, a2 cos Omega t cos theta + a2 times the square root of 1 - cos square Omega t times sine theta. This can be written as = a2 upon a1 times, Ex cos theta + a2 times the square root of 1 - Ex upon a1 square times sine theta, or this means Ey - a2 upon a1 Ex cos theta whole squared is = a2 squared sine square theta times 1 - Ex upon a1 square.

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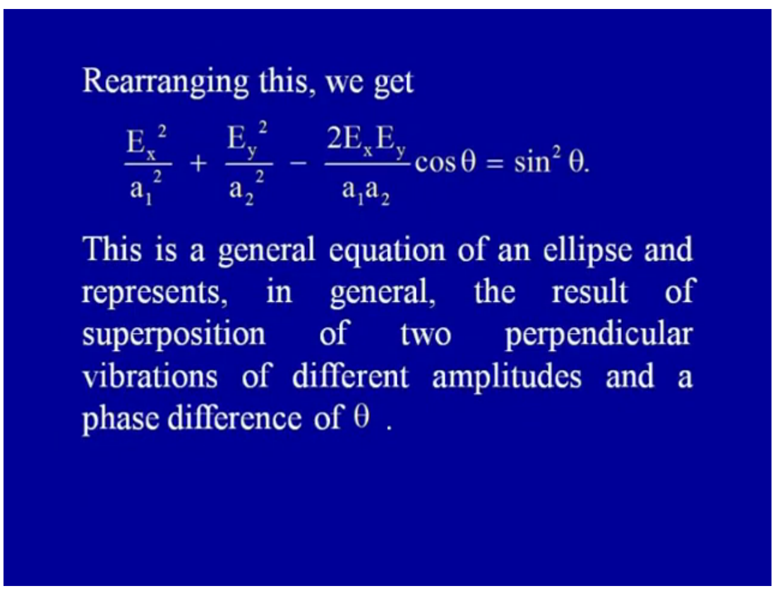

We can rearrange this and when we do this, we get, Ex is squared upon a1 square + Ey square + a2 square - twice Ex Ey upon a1 a2 times cos theta. And all of this is = sine square theta. This is the general equation of an ellipse. The general equation of an ellipses, is an ellipse is at the center of the origin which is the major minor axis making some angle with the coordinate axis.

So, this represents in general the result of superposition of two perpendicular vibrations of different amplitudes and the phase difference of theta. For the phase difference theta = n Pi where n is 0, 1, 2. This means an integral multiple of Pi 1, Pi 2, Pi 3, Pi 4.

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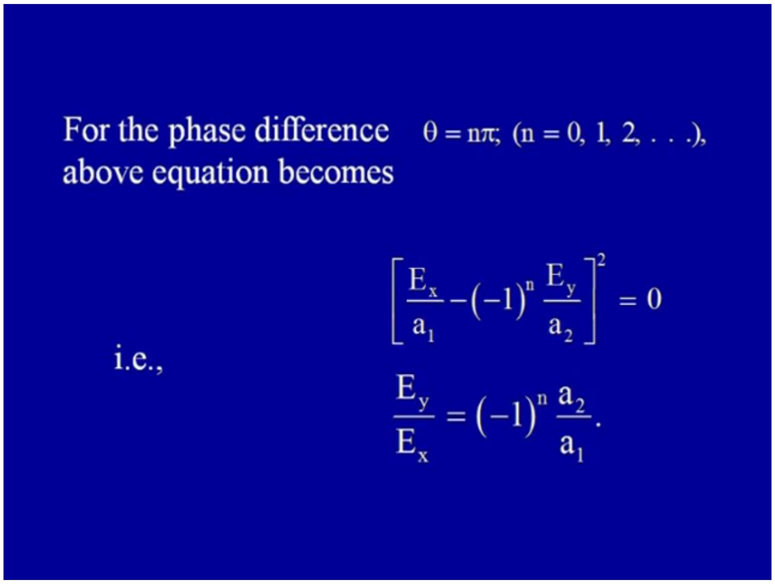

The above equation becomes Ex upon a1 - 1 raised to the power n, Ey upon a2 and the whole square = 0 which really means Ey upon Ex. The ratio of the Y component to the X component is = -1 raised to the power n, a2 upon a1. In the Ex, Ey plane the above expression represents a straight line. The angle Phi that this line makes with the x axis depends naturally on the ratio a2 upon a1.

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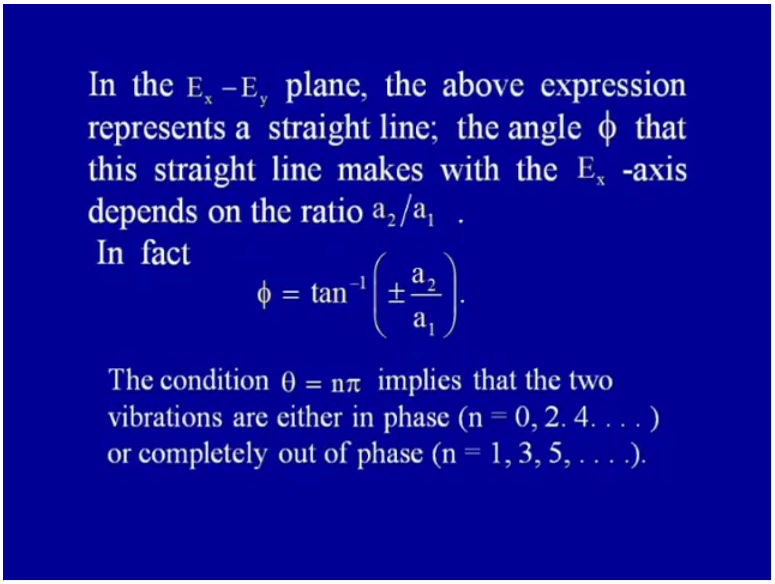

In fact, angle Phi is tangent inverse + - a2 upon a1. If a1 = a2, this angle will be 45 degrees. The condition theta = n Pi implies that the two vibrations are, either in phase, if n is even 0, 2, 4 which means theta is = 0 or the phase difference of 2Pi, 4Pi, 6Pi or completely out of phase which means n is = 1, 3, 5; the phase difference theta is = 1 pi, 3 pi, 5 pi, like this. This means the phase difference of half a vibration, 3 half vibrations or 5 half vibrations;

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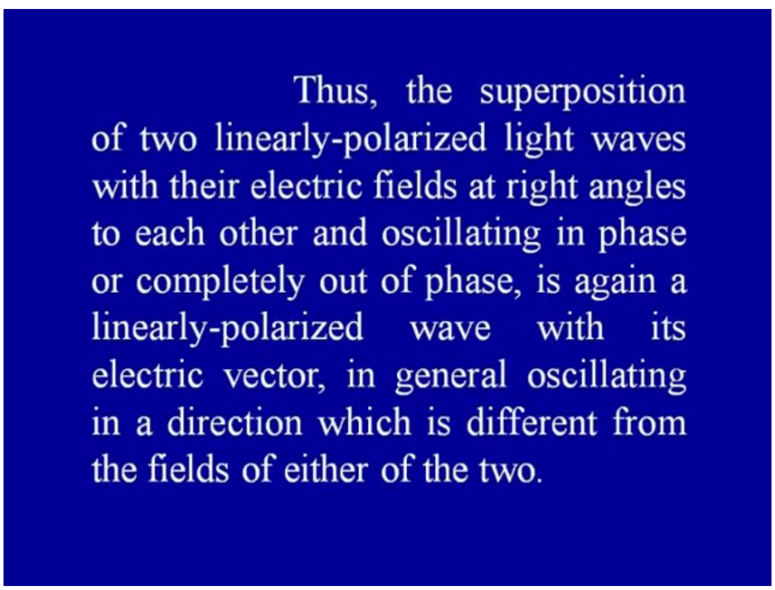

Thus the super position of two linearly polarized light waves with their electric fields at right angles to each other. And oscillating in phase are completely out of phase, ok. These just these two conditions: digital vibrations are perpendicular to each other; but they are in phase, or completely out of phase. Then, the resultant is again the linearly polarized wave with a selective vector in general oscillating in a direction which is different from the fields wider of the two making an angle of Phi.

We would say the first vibrations and this angle Phi depends on the ratio of the two amplitudes a2 to a1.

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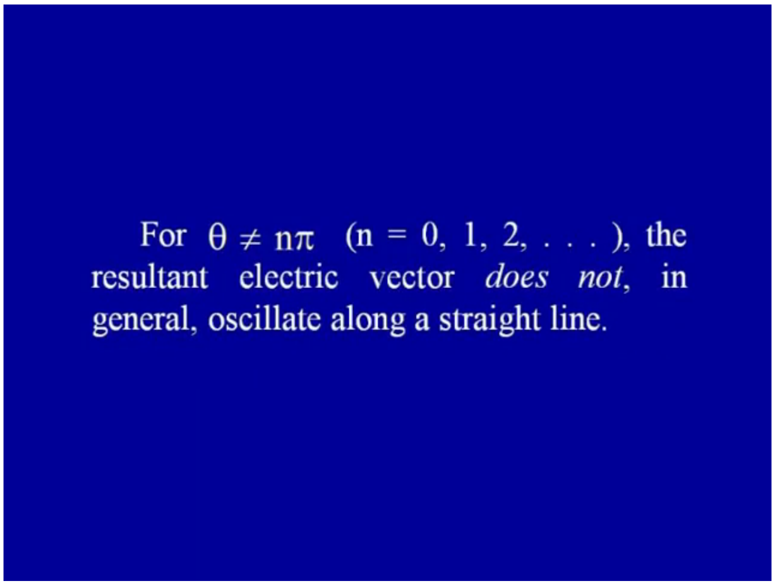

For theta not = n Pi, it is not if the phase difference is not a integer multiple of Pi is not = 1Pi or 2Pi or 3Pi, the resultant electric vector in this case does not oscillate along a straight line.

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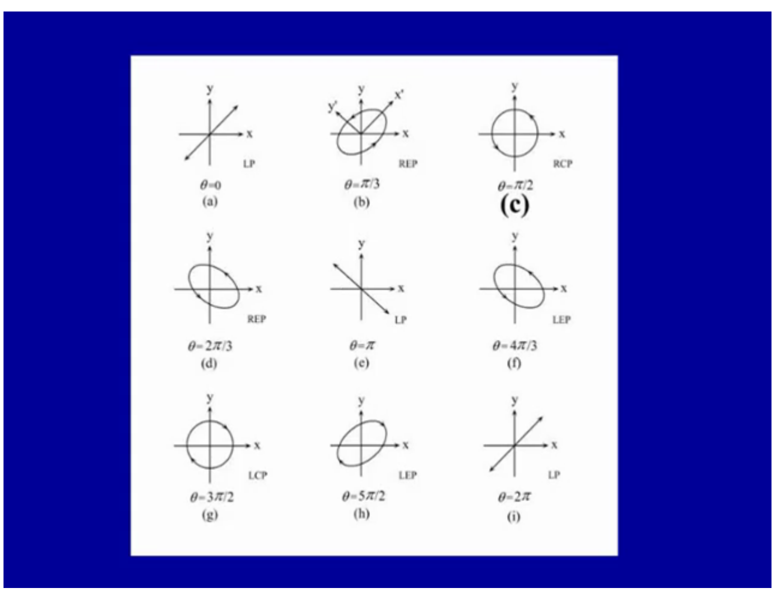

This is this figure shows the resultant for different values of theta; theta = 0, Pi by 3, Pi by 2, 2Pi by 3, theta = Pi, 4Pi by 3, different values up to a value of 2Pi is a complete range of theta varying from 0 to 2 Pi.

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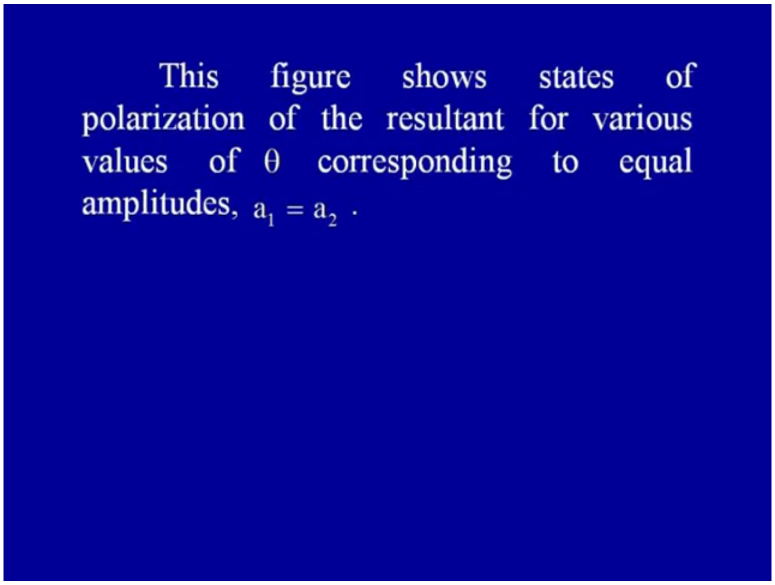

Corresponding to the simpler case when the amplitudes are equal a1 is = a2. We shall consider these guesses in a little more detail.

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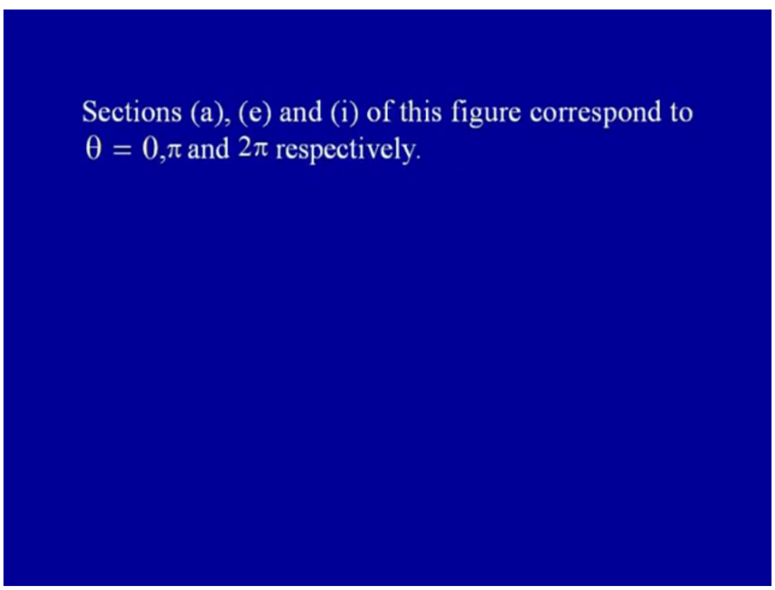

The sections a, e and i of this figure correspond to theta = 0, Pi and 2Pi respectively, a, corresponds to theta = 0. That is the two vibrations are in the same phase. Remember here the amplitudes are equal a1 is = a2. The E corresponds to theta = Pi they are completely out of phase. The phase difference of half a vibration and i is again when the phase difference is 2Pi which is essentially means phase difference is 0.

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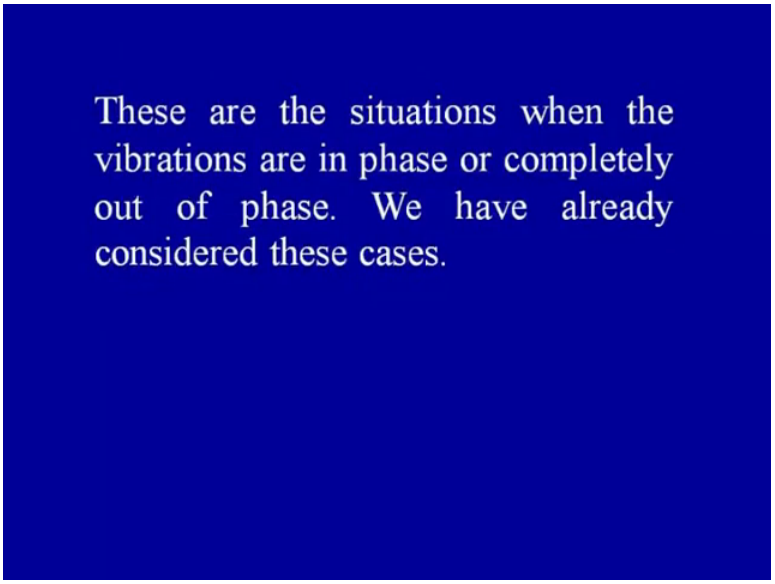

These are the situations as I pointed out earlier, when the vibrations are in phase or completely out of phase. We have already considered these cases. Let us consider now, a simple case corresponding to theta = Pi by 2. That is very interesting. The difference is of a quarter vibrations.

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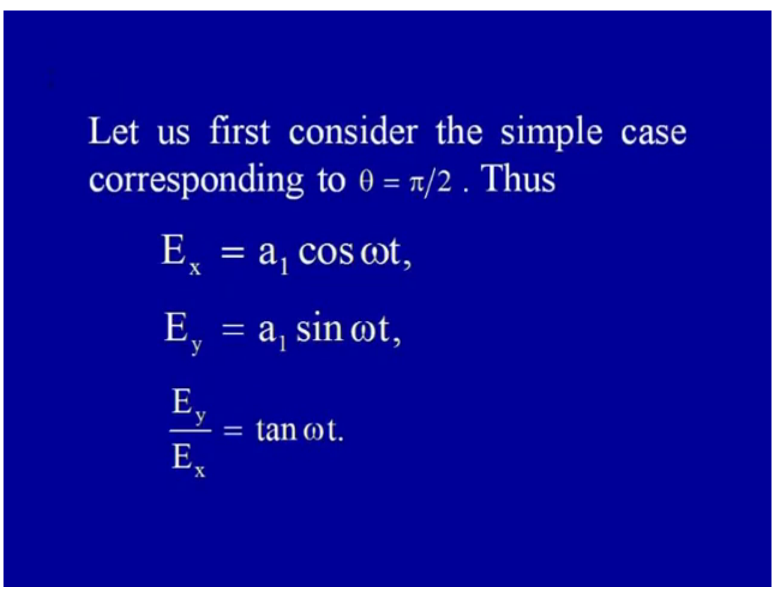

Ex, a1 cos Omega t; Ey a1 sine Omega t. Remember a1 is = a2. Ex is a cos Omega t and Ey is the sine Omega t. The phase difference of Pi by 2, Ey upon Ex, comes out to be tan Omega t. If we plot the time variation of the resultant of these two, we would find that the tip of the electric vector rotates from the circumference of a circle of radius a1, Ex square plus Ey square is = a1 square in the anti clockwise direction.

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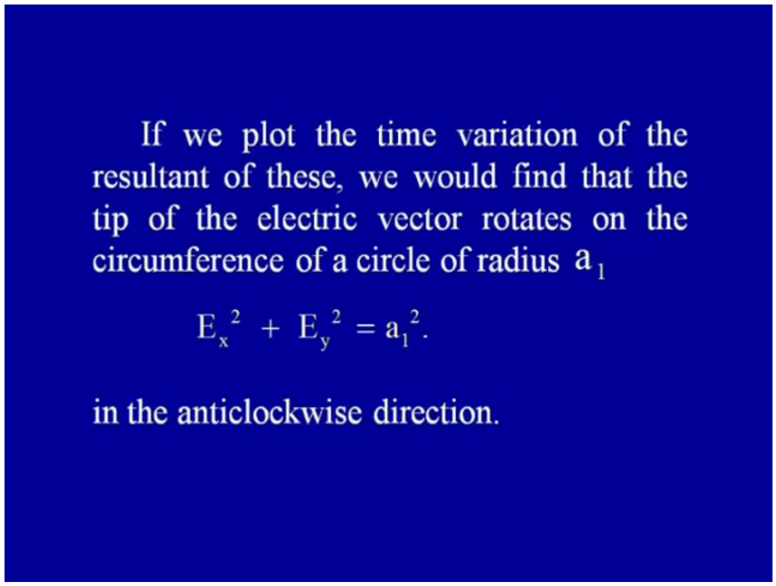

The propagation of light in the pole is in the positive direction of that axis. This had been taken here to be coming out of the screen.

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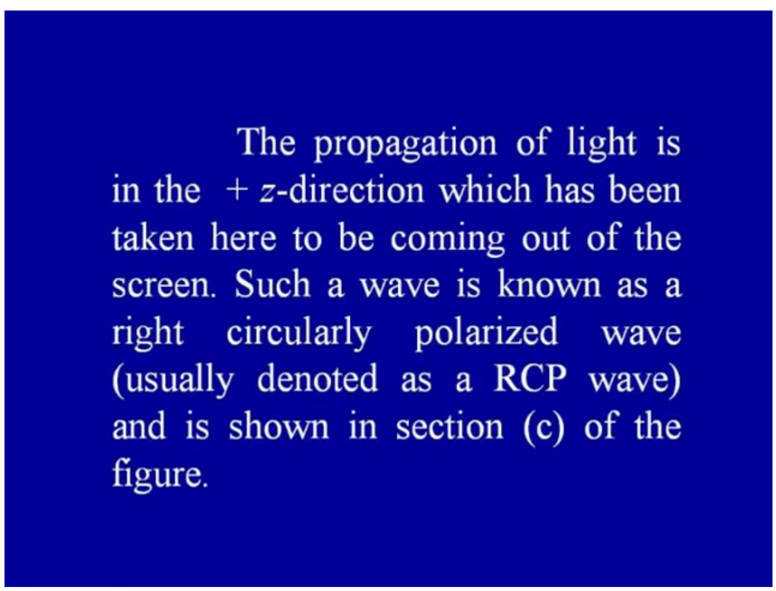

Such a wave is known as right circularly polarized wave usually denoted as RCP wave, right circularly polarized and is shown in Section c of the figure which is a section c. This shows a circular vibration. Circular because a1 is = a2 and it is anti clockwise direction. Remember the beam propagation is coming out of the screen. Directive vector is rotating in the xy plane in the anti clockwise direction. And theta = 3Pi by 2.

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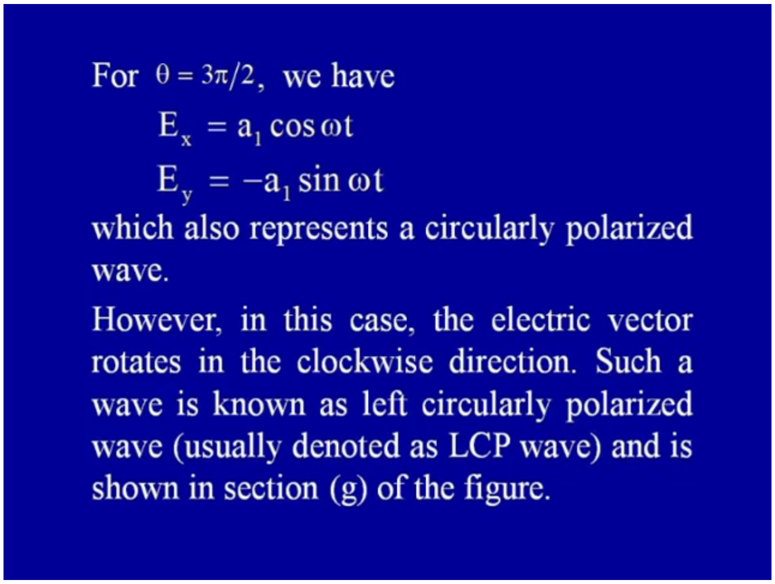

Now we have Ex = a1 cos Omega t as before. But now Ey is = with the minus sign a1 sine Omega t. It again represents a circularly polarized wave as before. However, in this case, the electric vector rotates in the clockwise direction. Such a wave is known as left circularly polarized wave usually denoted as LCP wave and is shown in Section g of the figure.

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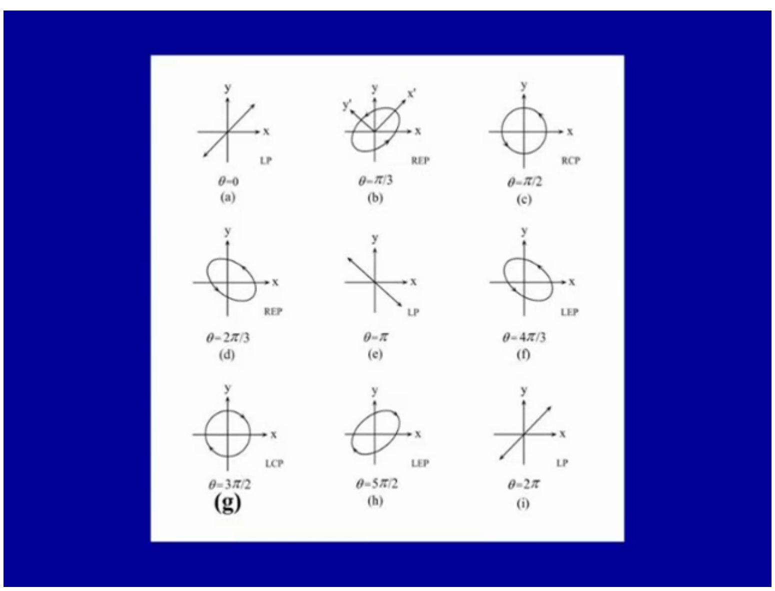

Again a circular vibration, the only difference with the earlier case c is that here the electric vector rotates in the clockwise direction. Remember, the beam is propagation Direction is coming out of the screen. This is called a left circularly polarized.

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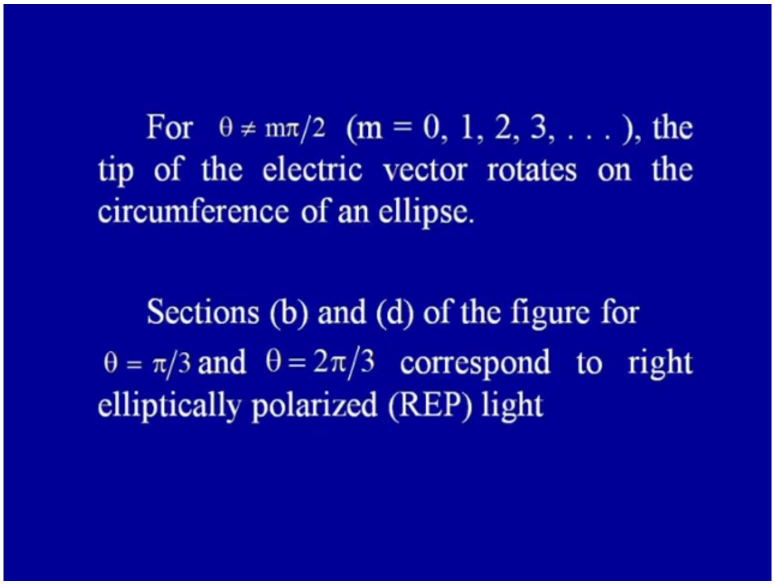

Now for theta not = mPi by 2; so, this means, this is a case where it is not = an integer multiple of Pi by 2. In this case, the tip of the electric vector rotates on the circumference of an ellipse. Sections b and d of the figure for theta = Pi by 3 and theta = 2Pi by 3 correspond to right polarized elliptical light. This is the REP light elliptically polarized.

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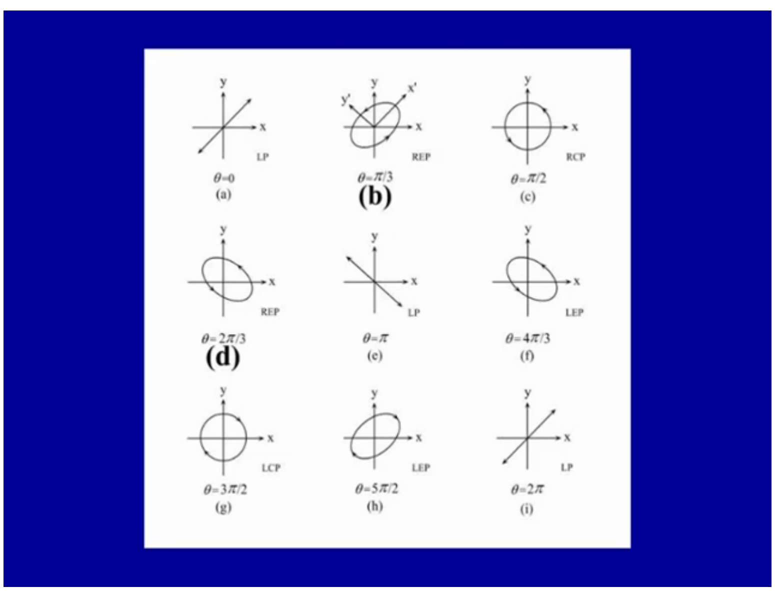

Remember again the a1 is = a2. But the only difference here is the phase difference is not just Pi by 2 or 3Pi by 2, it is Pi by 3 or in the other. In the in the section d, it is 2Pi by 3. And that is why the resultant is, is a general ellipse whose major minor axis makes some angle with the coordinate axis. So, this is the situation when the phase difference is Pi by 3 and 2Pi by 3.

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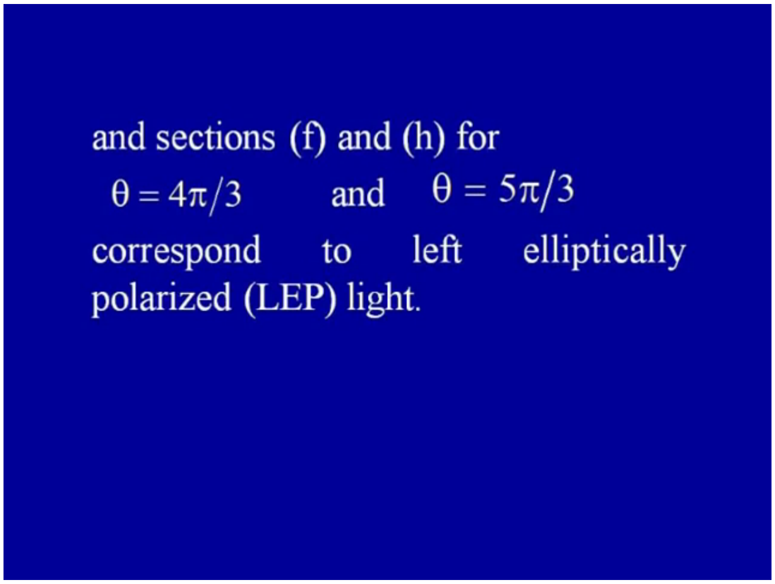

Similarly, now, the sections f and h for theta = 4Pi by 3 and theta = 5Pi by 3, they correspond to left elliptically polarized LEP light.

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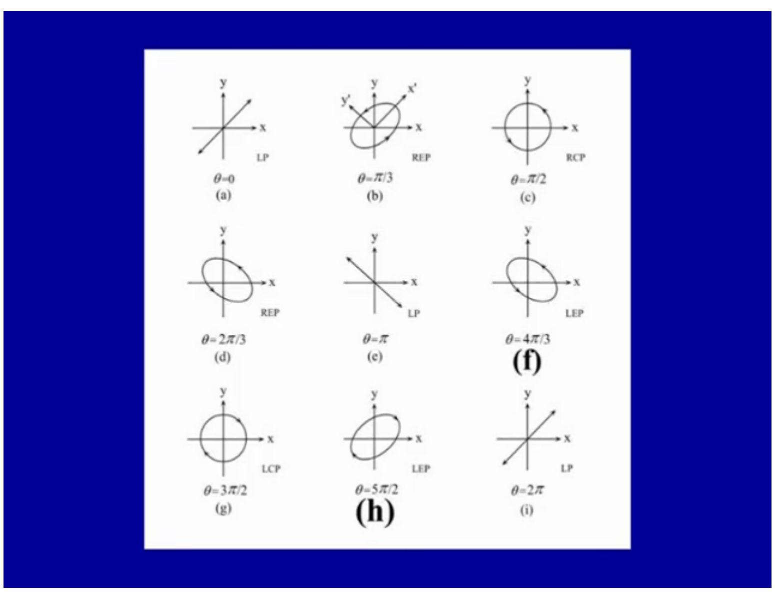

These are all in here section F again we have elliptical the only see the sense of rotation is different compared to the earlier case. And again remember the amplitudes are equal. So it is the wide change in the resultant is found depending on the phase difference between the two vibrations. The two vibrations are perpendicular to each other. Their amplitudes are same, equal. a1 is = a 2. The only difference is the phase difference.

If the phase difference is 0 or Pi, 2Pi, 3Pi or 4Pi completely in phase or out of phase, the resultant is again a plane polarized wave linear vibrations making an angle of 45 degrees or 135 degrees in this case, a1 is = a2. If the phase difference is Pi by 2 or 3Pi by 2, this means half a vibration or three half vibrations the result is a circular vibration this is called a circularly polarized light; right circularly or left circularly depending on where the tip of the electric vector rotates in anti-clockwise direction or clockwise direction and the general case when the phase difference is not that simple.

If it is Pi by 3, in this case, I showed in the figure or 2Pi by 3 or 5Pi by 3, the result is elliptically polarized light, right elliptically or left elliptically depending on whether the electric vector rotates as can be seen from the figure.

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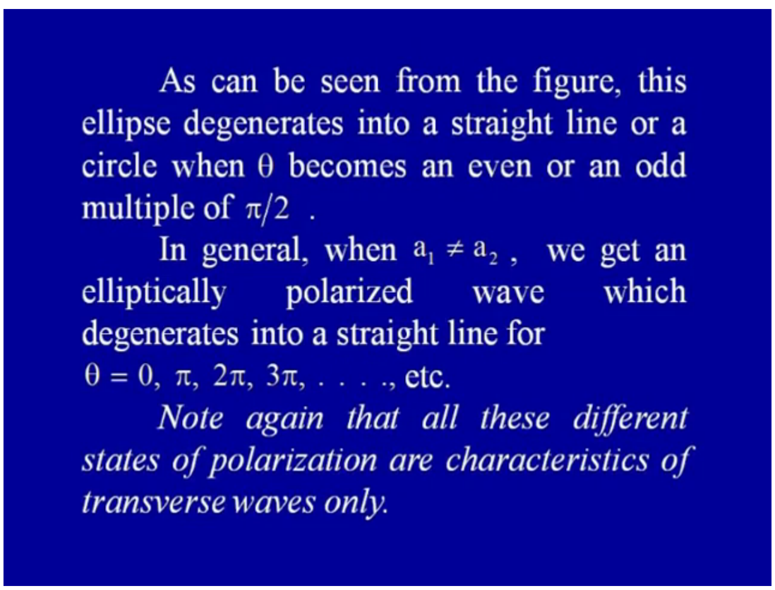

Just let us in general degenerate into a straight line or a circle, as we have seen, whenever theta becomes an even or an odd multiple of Pi by 2 as we have seen. But in general, when a1 is not = a2, we get an elliptically polarized light waves which again degenerates into a straight line for theta = 0 pi, 2Pi, 3Pi, etcetera. Remember, theta = 0, 2 pi, 4Pi, means they are in the same phase.

And theta = Pi, 3 Pi, 5 Pi means they are completely out of phase.

Again note one thing all these different states of polarization which we have considered here depending on the value of theta our characteristics of transverse waves only and electromagnetic waves are transverse waves. So, this is the end of the second lecture. I hope you enjoyed this lecture and also the earlier one. Thank you.