Engineering Physics 1
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Module-04
Lecture-03
Diffraction Part 03

In the previous lecture, we had discussed how the diffraction effects limits the resolving power of optical instruments and explain the relay criterion of just resolution. Using this criterion, I derived the expression for resolving power of rating. Now, in this lecture I am going to describe the resolving power of microscope and telescope. So, let us first discuss about the resolving power of microscope.

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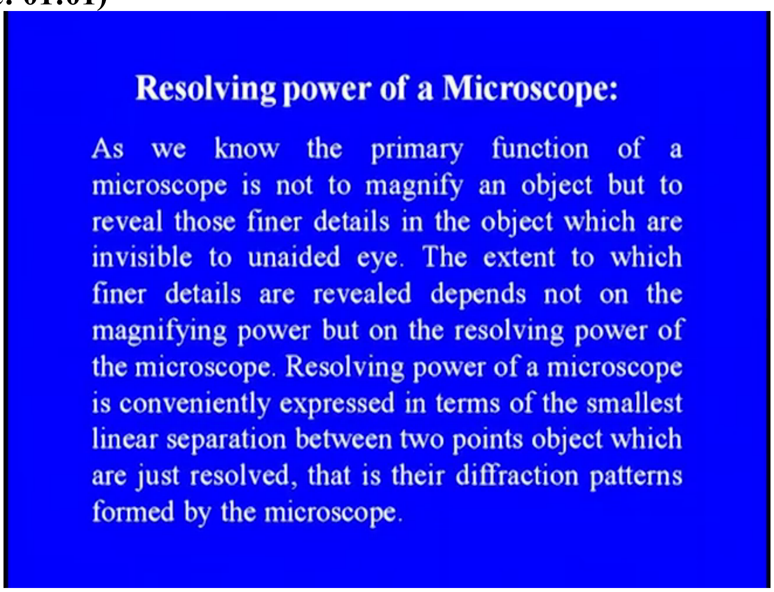

As we know primary function of a microscope is not to magnify an object but to reveal those final details in the object which are invisible to unaided eye. The extent to which final details are revealed depends not on the magnifying power, but on the resolving power of microscope. Resolving power of a microscope is conveniently expressed in terms of the smallest linear separation between two points object which are just resolved. That is the difference and patterns formed by the microscope.

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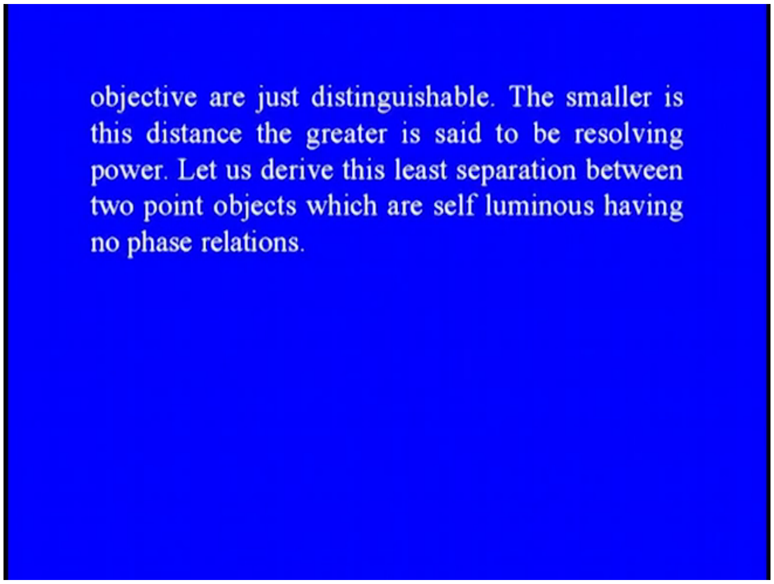

Objective are just distinguishable, the smaller is this distance, the greater is said to be resolving power. Let us derive this list separation between two point objects which are self luminous having no phase relationship. The objective of the microscope is a simple convex lens as shown in this figure.

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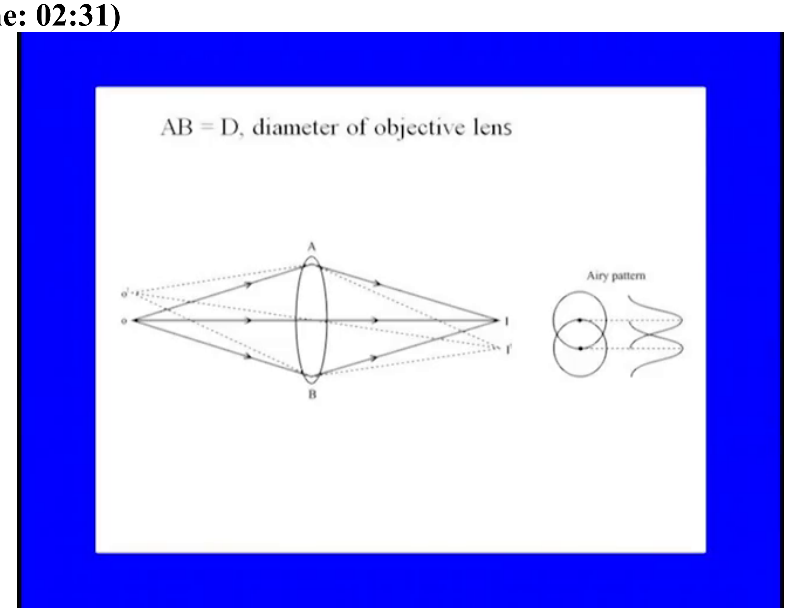

And the waves of light assumed to be of the same wavelength Lambda starting from self luminous point objects O and O prime are diffracted by the circular periphery of the lens. Therefore, a diffraction patterns corresponding to each object is formed in the focal plane of the objective. Each image is therefore a diffraction pattern consisting of a central bright desk bordered by alternate concentric circular dark and bright rings.

The centre of the bright disc in the diffraction pattern of O is at I while for O Prime it is at I prime. If these objects are to be resolved in their images then according to Rayleigh's criterion of resolution the distance I, I prime between the centre of 2 Central Maxima should be at least = the radius of either one. This simply means that the waves of light from O Prime after refraction by the lens should fall first dark ring passing through I.

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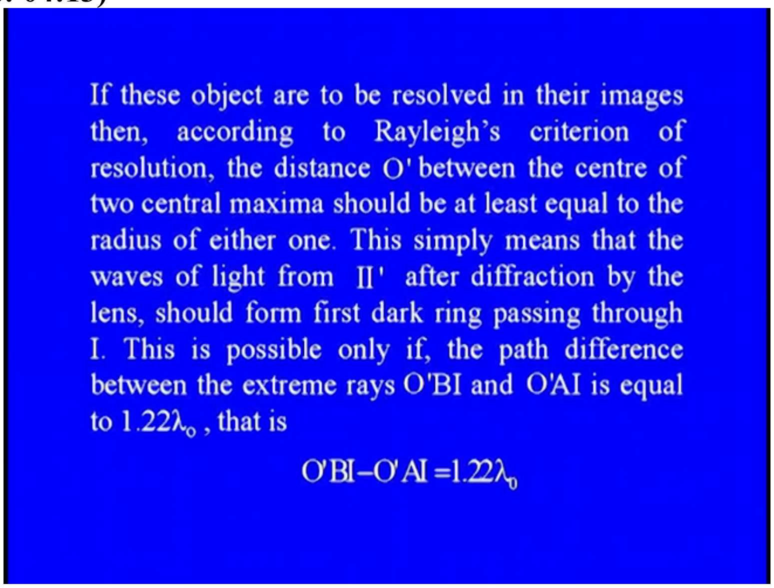

This is possible only if the path difference between the extreme rage O Prime BI and O prime AI is = 1.22 lambda. That is O Prime BI - O prime AI is = 1.22 Lambda.

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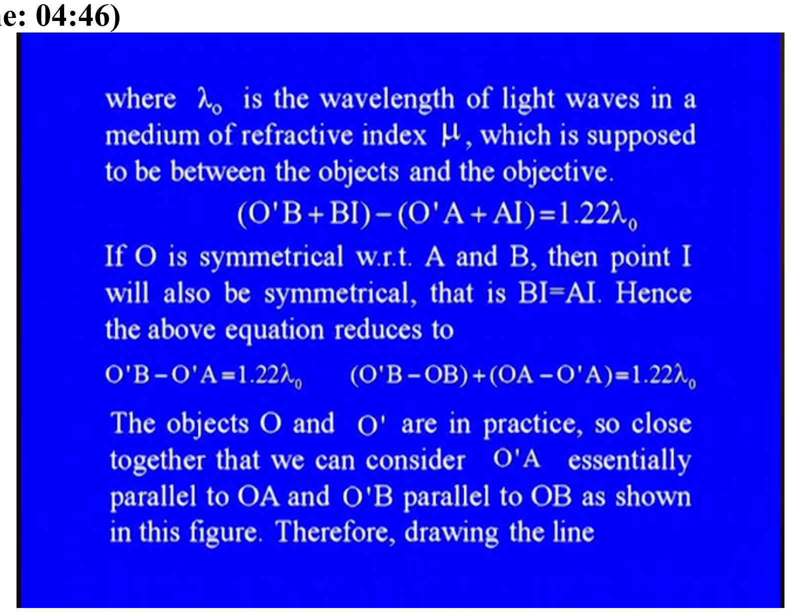

,where Lambda is the wavelength of light waves in a medium of refractive Index Mu, which is supposed to be between the objects and the objective. Now this condition we can also write O prime B + BI - O Prime A + AI = 1.22 lambda. If O is symmetrical with respect to A and B then, point I will also be symmetrical. That is BI will be = AI. Hence, the above equation reduces to O prime B - O Prime A = 1.22 Lambda. This equation we can also write O prime B - OB + OA - O Prime A = 1.22 Lambda.

The objects O and O prime are in practice so close together that we can consider O prime A is essentially parallel to OA and O Prime B parallel to OB as found in this figure;

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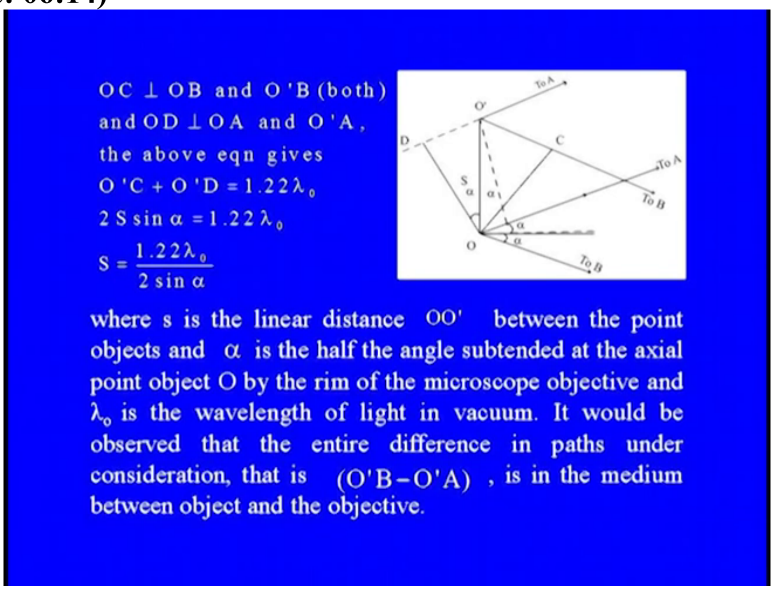

Therefore, drawing the line OC perpendicular to OB and O prime and OD perpendicular to both OA and O prime A, the above equation gives O Prime C + O Prime D = 1.22 Lambda. Therefore, we can we have 2S sin alpha = 1.22 lambda, so S will be = 1.22 lambda upon 2 sin alpha, where S is the linear distance O, O prime between the point objects and Alpha is the half the angle subtended at the axial point object O by the rim of the microscope objective and Lambda is the wavelength of light in vacuum.

It would be observed that the entire difference and path under consideration that is O prime B - O prime A is in the medium between objects and the objective.

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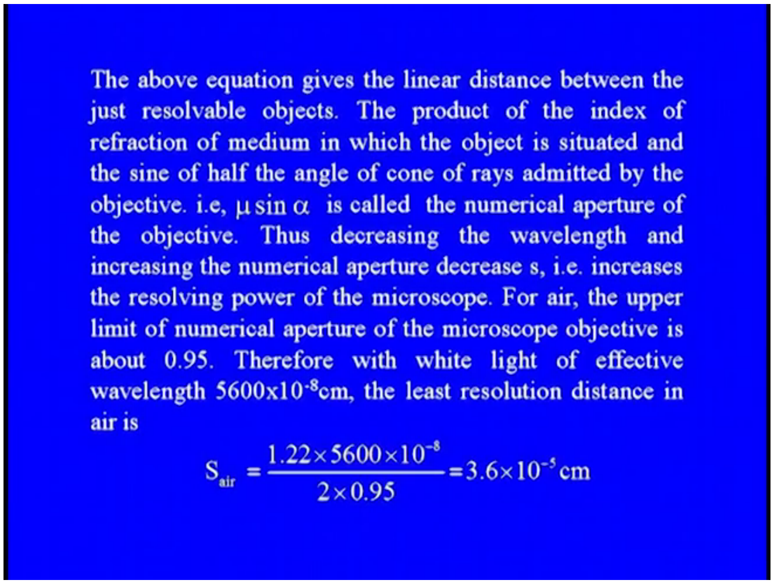

The above equation gives the linear distance S between the just resolvable objects. The product of the index of refraction of medium in which the object is situated and the sign of half the angle of cone of rays admitted by the objective that is Mu sine alpha is called the numerical aperture of the objective, thus decreasing the wavelength and increasing the numerical aperture decrease S that is increases the resolving power of the microscope. For air, the upper limit of numerical aperture of the microscope objective is about 0.95.

Therefore, with white light of effective wavelength 5600 angstroms, the least resolution distance in air is of the order of 3.6 into 10 to the power -5 cm. But by filling the space between the object and the objective by air, the numerical aperture may be increased to 1.6.

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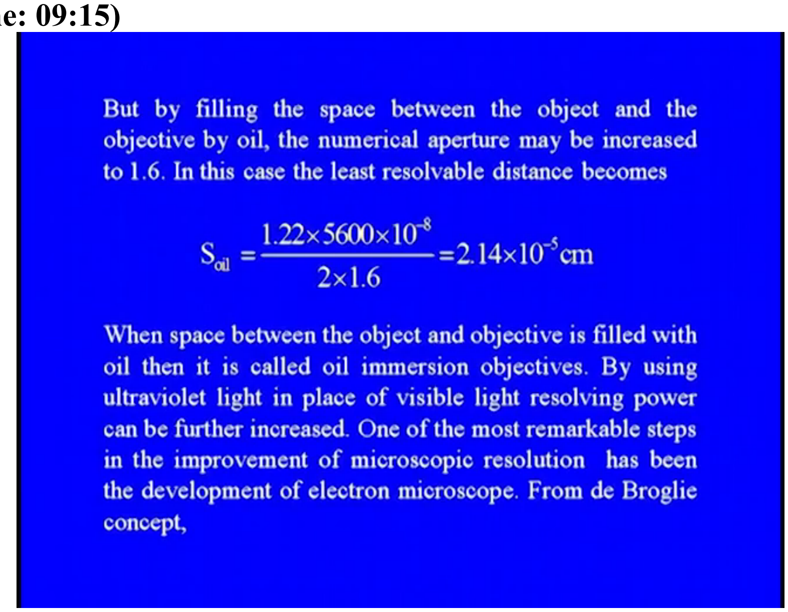

In this case, the list resolvable distance S becomes = 2.14 into 10 to the power - 5cm when space between the object and objective is filled with oil then, it is called oil immersion objectives. So, by using oil immersion objective, we can increase the resolving power of microscope and by using ultraviolet light, in place of visible light, resolving power can be further increased.

One of the most remarkable steps in the improvement of microscopic resolution has been the development of electron microscope. From de Broglie concept, we know that electrons we have as wave, when wavelength where waves wavelength depends on the voltage through which they have been escalated. A voltage where is between 102 to 10000 volt wavelength will vary from 0.122 to 0.0122 nanometre.

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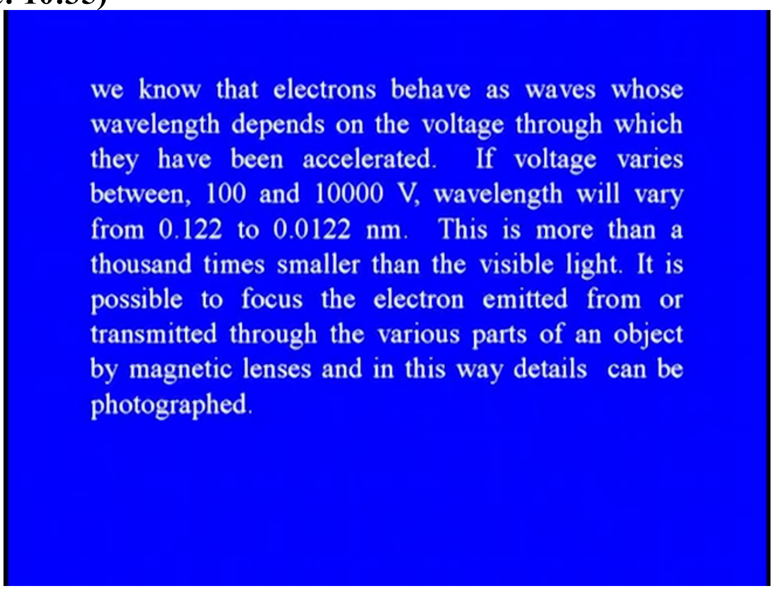

This is more than a 1000 times smaller than the visible light. It is possible to focus the electron emitted from are transmitted through the various parts of an object, by magnetic lenses and in this way, details can be photographed. And in case of electron microscope since the wavelength of electron beam is much smaller than the visible light, so, we expect very high value of resolving power for electron microscope. Now let us discuss the resolving power of a telescope.

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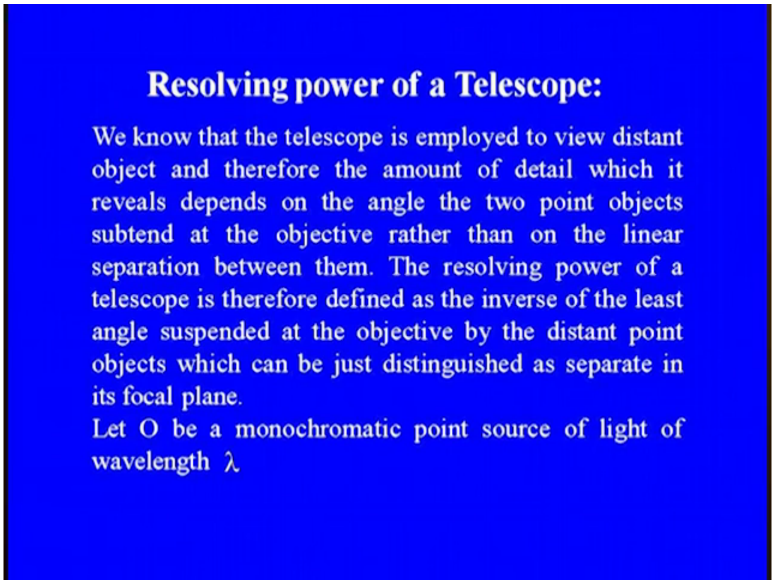

The telescope is employed to view distant object and therefore the amount of detail which it reveals depends on the angle the two point objects subtend at the objective rather than on the linear separation between them. The resolving power of a telescope is therefore defined as the inverse of the least angle subtended at the objective, by the distant point objects which can be just distinguished as separate in its focal plane.

Let O be a monochromatic point source of light of wavelength Lambda situated at a great distance from the telescope objective AV as shown in this figure.

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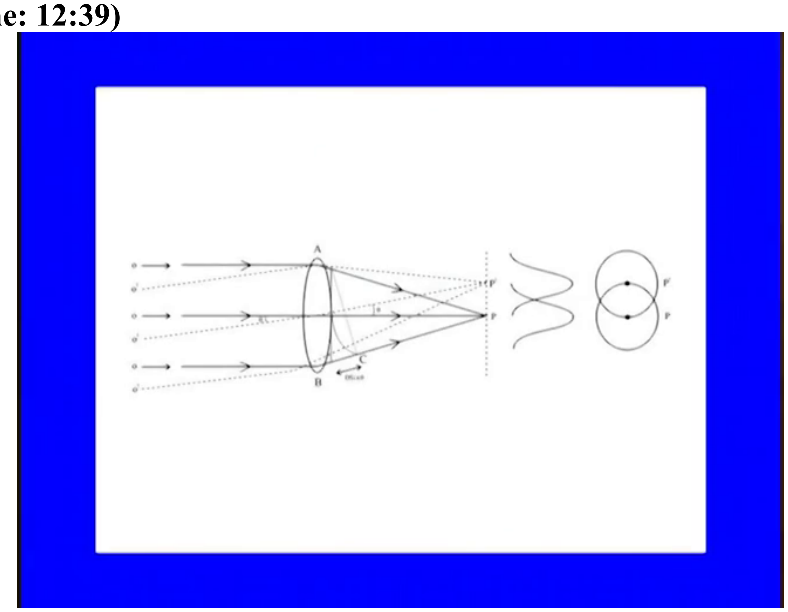

A beam of parallel rays from this object is incident normally on the objective. The ring supporting the lens as well as lens itself serves as a circular aperture and therefore from Fraunhofer diffraction pattern is produced in the focal plane of the lens. The refraction pattern of

O consists of a central bride is surrounded by concentric dark and bright Rings, the common Centre being at the point P, where the point image would have occurred in the absence of diffraction phenomenon.

The image of ended point object O prime situated close to A is also a similar diffraction pattern with its centre at the point P prime. These diffraction pattern overlap and two point objects can be just resolved when according to Rayleigh's criterion the separation PP Prime, between the centre of 2 Central Maxima is at least = the radius of either one. Under this condition, the angle subtended by 2 point objects and the objective is taken as the measure of resolving power of a telescope.

Actually, resolving power is inverse of this angle. Now, with P prime as centre and radius P prime A, we draw an arc cutting C prime B at the point C as shown in the figure. This arc AC therefore, represents the transmitted wave front which gives rise to the central maximum at P Prime, in the diffraction pattern of O prime. The second point object near to the first fastest resolution of O and O prime, the dark ring in the diffraction pattern of O should pass through the point P prime.

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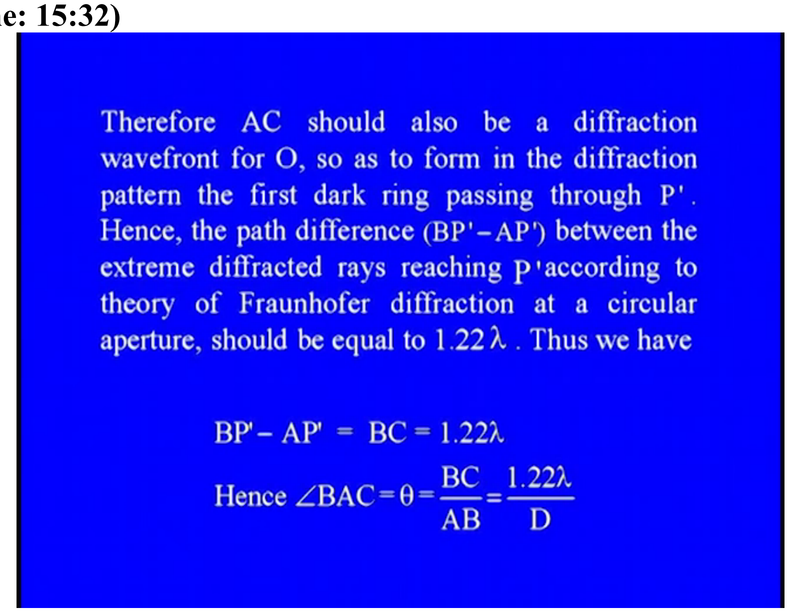

Therefore, AC should also be a diffraction wavefront for O, so as to form in the diffraction pattern, the first dark Line passing through P prime. Hence the path difference BP Prime - AP prime between the extreme different rays reaching p prime according to theory of Fraunhofer diffraction at a circular aperture should be = 1.22 Lambda. Thus, we have BP prime - AP prime =

BC, which is = 1.22 Lambda. Hence angle BAC = theta will be = BC upon AB is = 1.22 lambda upon capital D.

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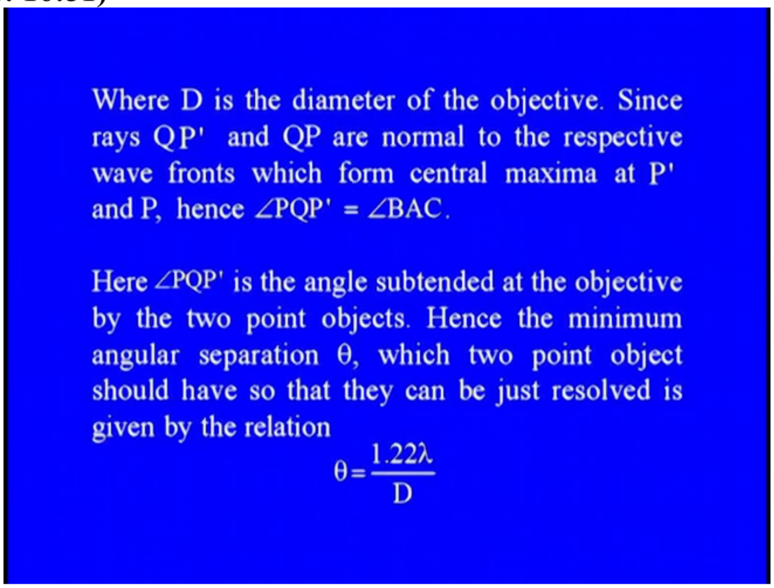

Where capital D is the diameter of the objective; Since rays QP prime and QP are normal to the respective wave fronts which form central maxima at P prime and P hence, angle PQP prime will be = Angle BAC. If angle PQP prime is the angle subtended at the objective by the two point objects. Hence the minimum angular separation theatre which two point object should have so that they can be just resolved is given by the relation theta = 1.22 lambda upon capital D. This angle is therefore the measure of angular resolving power of the telescope objective.

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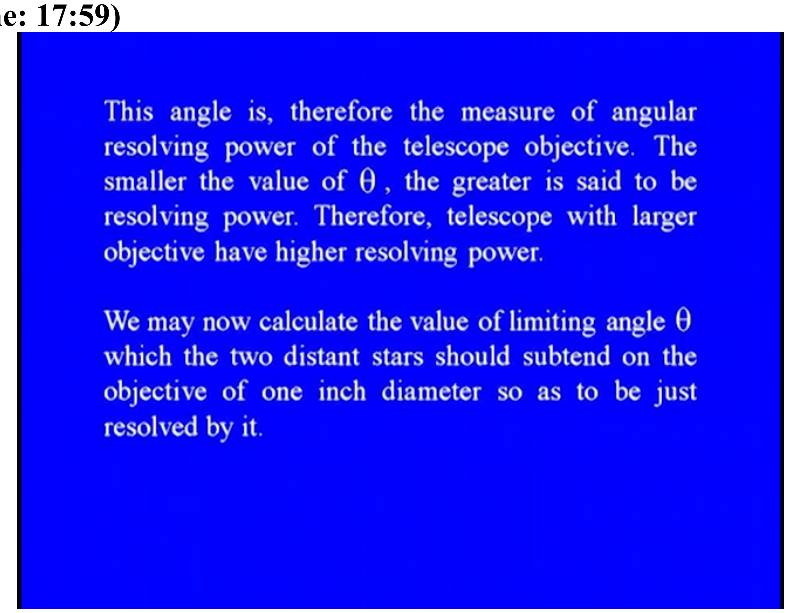

The smaller the value of theta, the greater is said to be resolving power. Therefore, telescope with larger objective and higher resolving power. We may now calculate the value of limiting angle theta with the two distant stars should subtend on the objective of 1 inch diameter so as to be just resolve by it. Now putting the value of Lambda and capital D.

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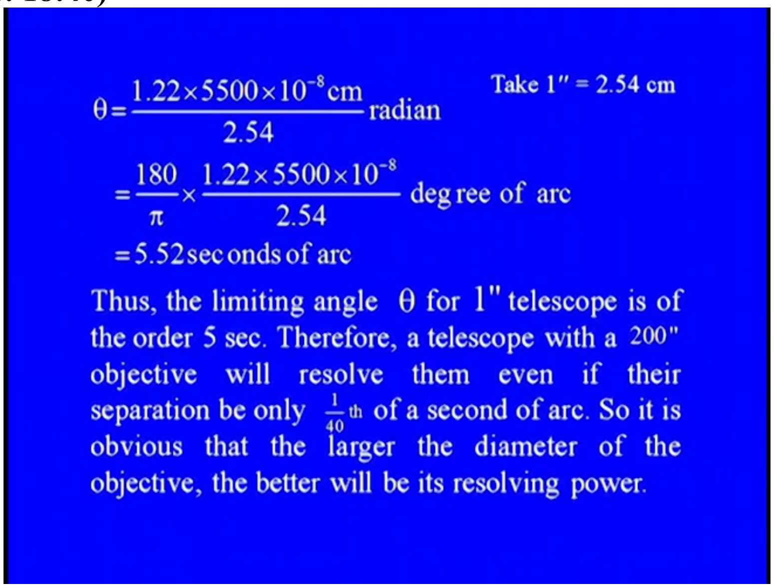

We get theta = 5.52 seconds of arc. Thus, the limiting angle theta for 1 inch telescope is of the order of 5 second. Therefore a telescope with a 200 inch objective will resolve them even if their separation be only one by 4oth of a second of arc. So, it is obvious that the larger the diameter of the objective, the better will be its resolving power. Now, let us take some numerical problems. Problem number 1:

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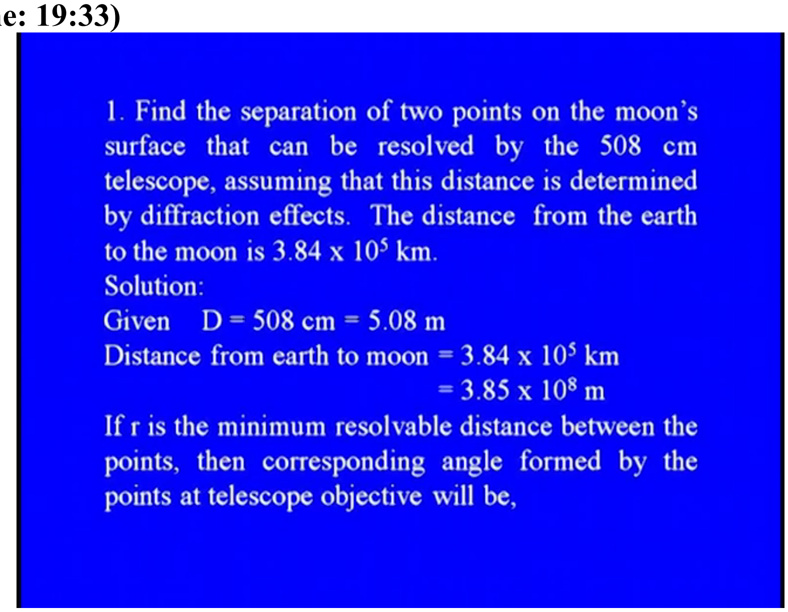

Find the Separation of two points on the moon surface that can be resolved by the 508 cm telescope, assuming that this distance is determined by diffraction effects. The distance from the earth to the moon is about 3.84 into 10 to the power 5 km. Now, in this problem, the diameter of the objective lens capital T is = 508 cm. And the distance from the earth to the moon is 3.84 into 10 to the power 5 km which will be = 3.85 into 10 to the power 8 meter.

Now, if r is the minimum resolvable distance between the points then corresponding angle formed by the points at telescope objective will be

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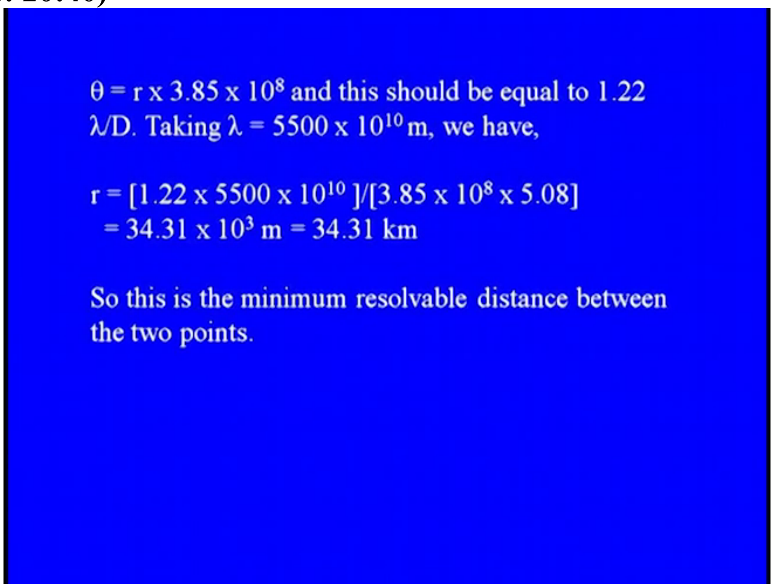

Theta = r into 3.85 into 10 to the power 8 and this should be = 1.22 lambda upon capital D, taking Lambda = 5500 into 10 to the power 10 meter, we have r = 3, 34.31 into 10 to the power 3 meter which will be = 34.31 km. So, this is the minimum resolvable distance between the two points. So, now let us take another problem related to resolving power of satellite.

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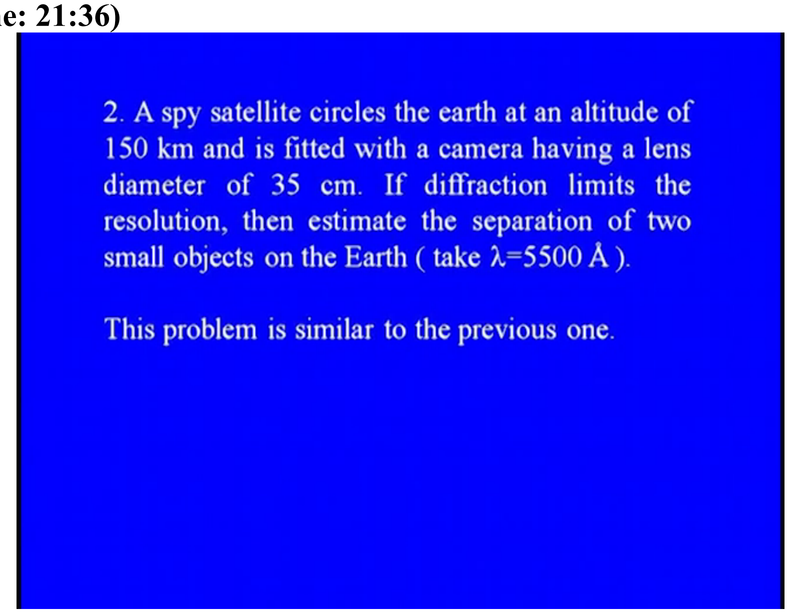

Suppose a spy satellite circles the earth at an altitude of 150 km and is fitted with a camera having a lens diameter of 35cm. If diffraction limit the resolution, then estimate the least separation of two small object on the earth here we can take Lambda is = 5500 angstrom. In this problem, the distance between telescope objective and object is given; diameter of objective is also given.

So, like previous problem this in this problem also we can find the separation between the two objects. So, thus we have finished the discussion related to resolving power of microscope and satellite and we have taken, we have discussed some numerical example also. Now, I will discuss some problem related to refraction of single slit, double slit and grating.

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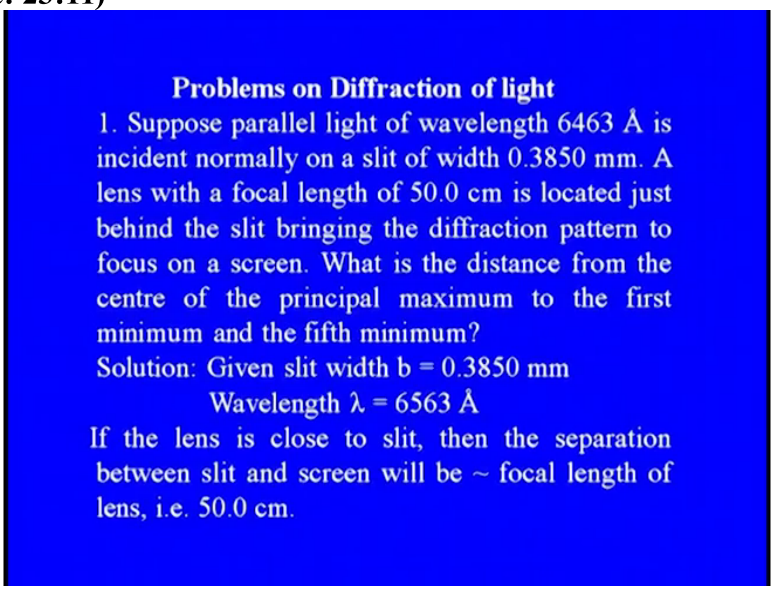

Problem number 1: Suppose parallel light of wavelength 6463 angstrom is incident normally on a slit of width 0.3850 millimetres with a focal length of 50 cm is located just behind the slit bringing the diffraction pattern to the focus on a screen. What is the distance from the centre of the principal maximum to the first minimum and 5th minimum? In this problem, slit width b is = zero point 3850 millimetre and the wavelength of light Lambda is = 6463 angstrom.

If the lens is close to slit then, the separation between the slits and the screen will be = focal length of the lens that is 50 cm.

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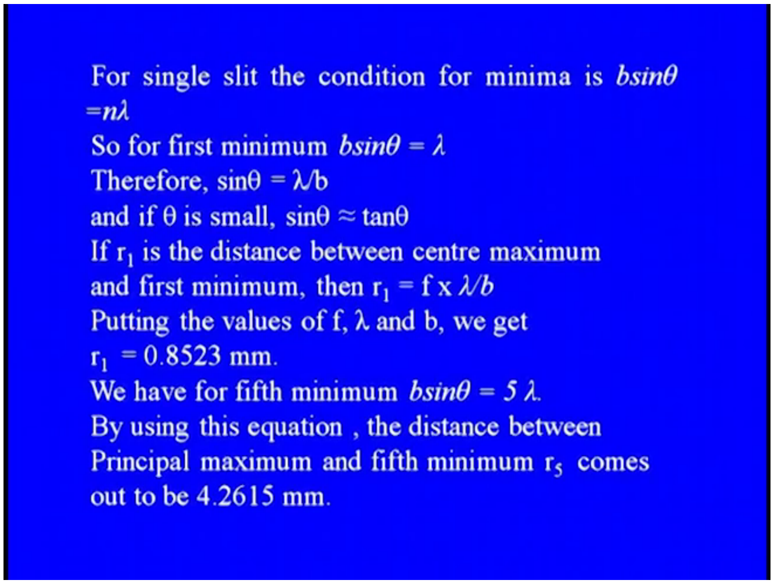

For single slit the condition for minima is b sin theta = and Lambda. So, for first minimum b sin theta will be = Lambda therefore sin theta will be = Lambda upon b. Now, if theta is small then, sin theta will be nearly = tan theta. If r1 is the distance between Central maximum and first minimum then r1will be = focal length into Lambda upon b. Putting the value of focal length Lambda and b in this equation, we get, r1 is = 0.85 to 3 millimetre.

For 5th minimum this be sin theta should be = 5 times Lambda and by using this equation the distance between principal maximum and 5th minimum, r5 comes out to be 4.2615 millimetre.

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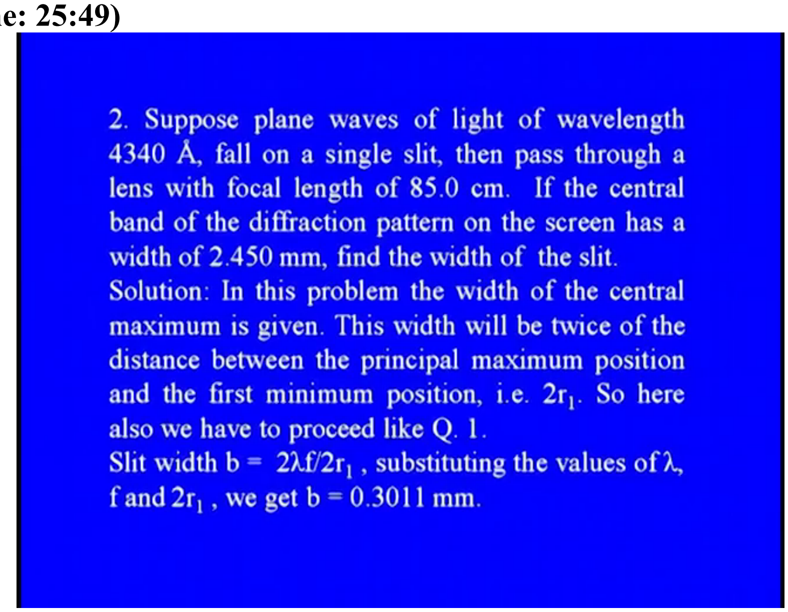

Problem number 2: Suppose plane waves of light A wavelength 4340 angstrom fall on a single slit then passed through a lens with focal length of 85 CM if the central brand of the diffraction patterns on the screen has a width of 2.450 mm, find the width of the slit. Now in this problem, the width of the central maximum is given. This width will be twice of the distance between the principal maximum position and the first minimum position that is 2 times r1.

So, here also we have to proceed like question number 1. So, slit width b is = 2 Lambda f divided by 2 r1 where f is the focal length of the lens substituting the value of Lambda f and 2 r1 we get b = 0.3011 millimetre. That is the width of the slit is 0.3011millimeter.

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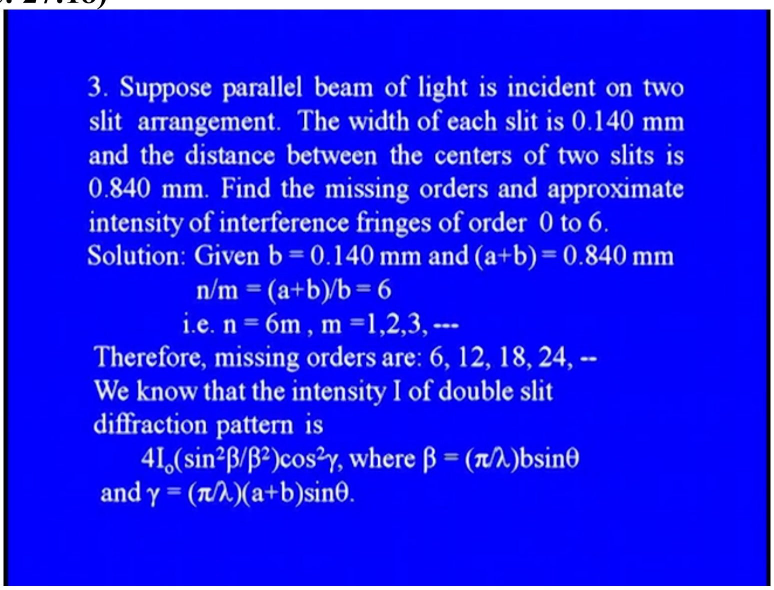

Problem number 3: Suppose, parallel beam of light is incident on two slit arrangement. The width of a slit is 0.140 millimetre and the distance between the centres of two slit is 0.840 millimetre. Find the missing Orders and their approximate intensity of interference fringes of order 0 to 6. In this problem, slit width b is = 0.140 millimetre and a + b = 0.840 millimetre, so we can find out n by m which is = a + by b = 6.

That is n is = 6 time m where m is = 1, 2, 3 and so on. Therefore missing order are: 6, 12, 18, 24 and so on. We know that the intensity I of the double slit diffraction pattern is 4I naught sine square beta by beta square into cos square gamma, where beta is = Pi by lambda into b sin theta and gamma is = Pi by lambda into a + b sin theta. We have already discussed about these expressions while discussing the diffraction pattern of double slit.

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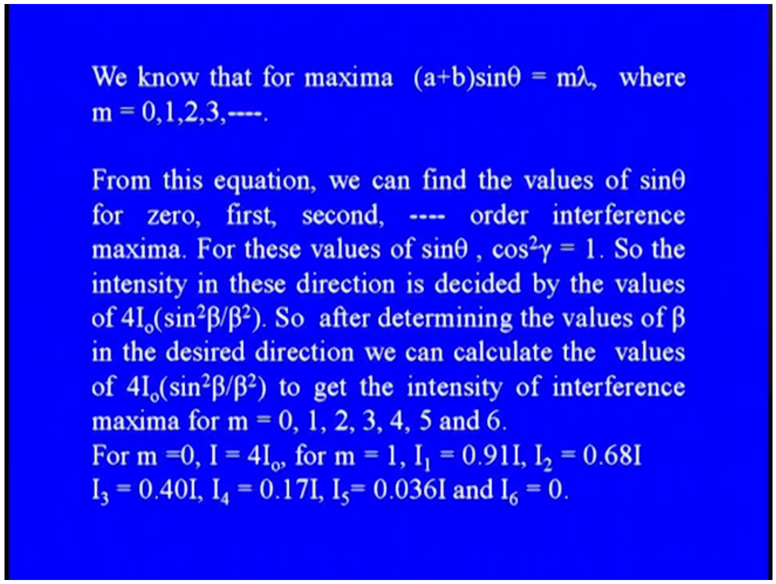

And we know that for maxima a + b sin theta = m Lambda where m is = 0, 1,2, 3 and so on, from this equation, we can find the values of sin theta for zero, first, second, third, so on, order in reference maxima. For these values of sin theta, cos square gamma will be 1, so, intensity in this direction is decided by the values of 4I naught sin square beta by beta square. So, after determining the values of Beta in the desired direction, we can calculate the values of 4I naught sin square beta by beta square, to get the intensity of interference maxima for m is = 0, 1, 2, 3, 4, 5 and 6.

For m is = 0 I will be = 4I naught, for m is = 1, I1 will be = 0.91 times I, that is intensity will be about 90% off Central Maxima. Similarly, I2 will be = 0.68I, I3 will be = 0.40I, I4 will be = 0.17I, I5 is = 0.036 I and I6 will be 0. Thus, we see that the intensity of interference maxima is gradually decreasing as order increases.

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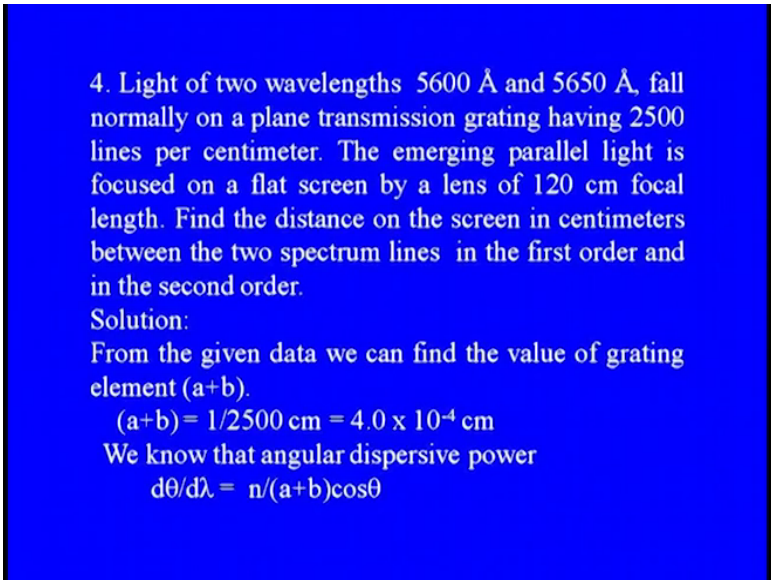

Question number 4: light of two wavelengths 5600 angstrom and 5650 angstrom fall normally on a plane transmission grating having 2500 lines per centimetre. The emerging parallel light is focused on a flat screen by a lens of 120 CM focal length. Find the distance on the screen in cm between the two spectrum lines in the first order and in the second order. In this problem, first we have to find out the grating element a + b which will be = 1 upon 2500 CM which will be = 4.0 into 10 to the power - 4 cm. We know that angular dispersive power d theta upon d lambda is = n divided by a + b cos theta.

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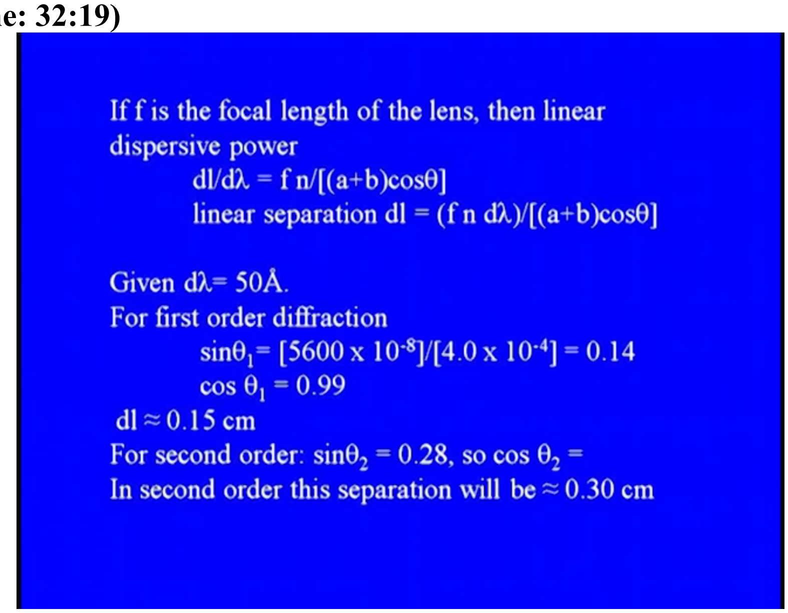

If f is Focal length of the lens then linear dispersive power dl by d Lambda will be = f n divided by a + b cos theta. From this equation, we can calculate the linear separation dl = f into nd lambda divided by a + b cos theta. In the problem, d Lambda is = 50 angstrom. So, for first order diffraction, we can calculate what is the value of sin theta 1 and after knowing the value of sin theta 1, we can calculate what is the cos theta 1.

And then, we can find out the value of dl. So, in the first order value of dl is approximately 0.15 cm. Similarly, for the second order, we can calculate what is the value of sin theta2, then, we can find out the corresponding value of cos theta2. And after substituting the value in the above equation we can find the value of dl here. So, dl comes out to be 0.30 cm.

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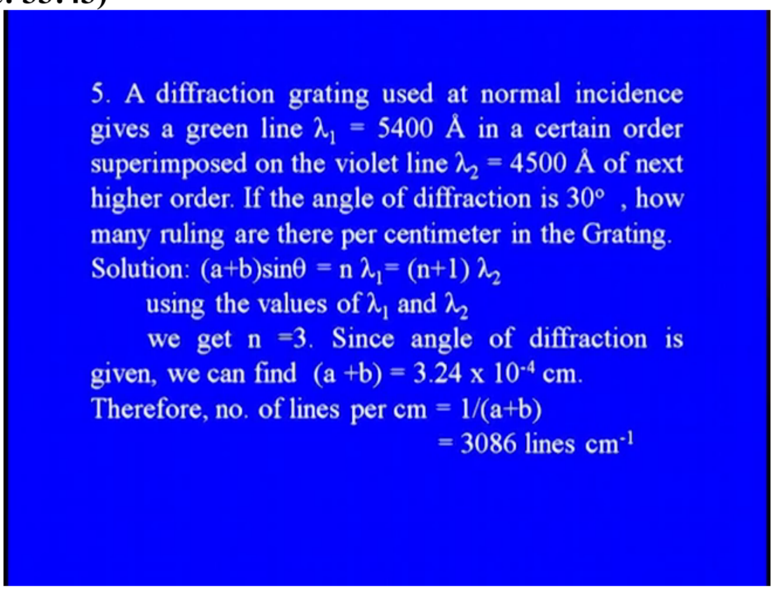

Problem Number 5: A diffraction grating used at normal incidence gives a green line Lambda 1 is = 5400 angstrom in a certain order superimposed on the violet line lambda 2 is = 4500 angstrom of next higher order. If the angle of diffraction is 30 degree, how many ruling are there per centimetre in the grating? So, according to the problem here, a + b sin theta will be = n lambda 1 which will be = n + 1times lambda 2.

Now putting the value of Lambda 1 and Lambda 2 we get, n is = 3 here, since the angle of diffraction is given, we can find a + b = 3.24 into 10 to the power - 4 cm. With the value of a + b, we can find the number of lines per centimetre is comes out to be 3086 lines per centimetre. So, here we have solved some problem related to single slit, double slit and getting.

Some more problems can be found in any standard textbook of optics. Thus, we have finished the discussion related to refraction of light and here we have mainly discussed the Fraunhofer diffraction pattern of single plate, double plate and grating. I also discussed how by using grating we can find out the wavelength of light in the laboratory. And I also discussed how the diffraction effect, limit in the resolving power of certain institute, optical instruments.

And explained the Rayleigh criteria for just a resolution and by using this criteria I derive the expression for resolving power of grating microscope and telescope. Thank you.