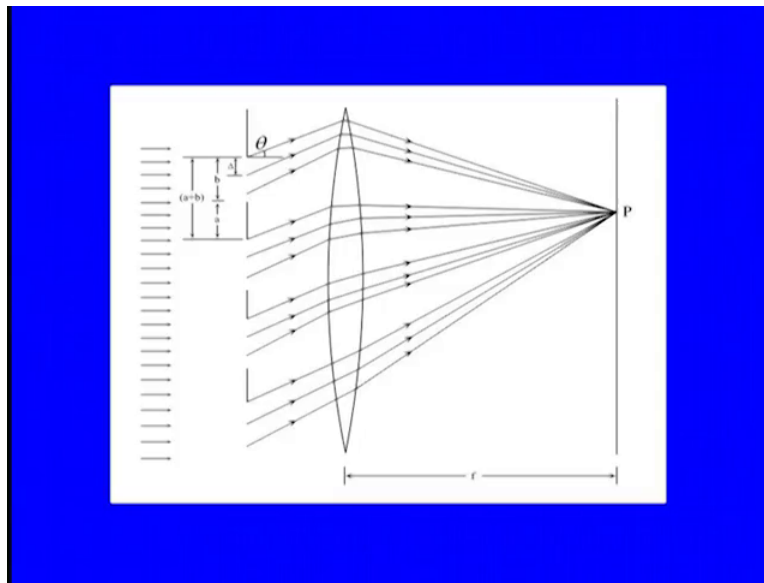


**Engineering Physics 1**  
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**Module-04**  
**Lecture-02**  
**Diffraction Part - 02**

In the previous lecture I discussed single slit and double slit Fraunhofer diffraction. Now I am going to discuss in this lecture from how diffraction pattern of  $n$  parallel slits.

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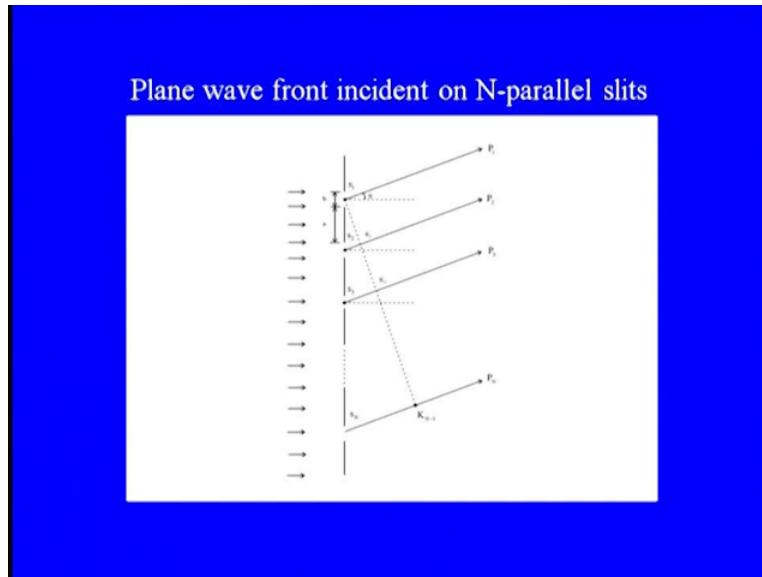


Each of width  $b$  and separated by equal optic spaces  $a$  as shown in this figure. This type of arrangement is known as diffraction grating. We will see that the diffraction pattern gets modified by the number of slits is increased beyond 2. Let a plane wave front of light of wave length  $\lambda$  incident normally on the slit as shown in this figure.

According to the principle of Huygens Reynolds had destined the incident wave front occupies the plane of the slits every point in each slit he is regarded as the origin of secondary spherical wavelets which is spread out in all directions. Therefore the rays are deflected from each slit in all directions if capital  $A$  is the resultant amplitude of light from a single slit along the normal to the plane of the slits.

Then resulting amplitude in a direction making an angle theta with the normal will be capital A sine beta by beta where beta =  $\pi b \sin \theta$ . Therefore in the case of n parallel slits of equal width we can replace all the secondary wavelets in each slit.

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By a single wave of amplitude capital A sine beta by beta starting from its middle point like S1, S2, S3 so on and travelling at angle theta with the normal. Let us now find the resultant effect of these n by raisins when brought to focus by a convergent lens. To calculate the power difference between the reflected waves from successive slits from S1 we draw perpendicular S1 kn - 1 on the parallel paths of these vibrations.

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To calculate the path difference between the diffracted waves from successive slits, from S<sub>1</sub> we draw perpendicular S<sub>1</sub>K<sub>n-1</sub> on the parallel paths of these vibrations. Then the difference of paths S<sub>1</sub>P<sub>1</sub> and S<sub>2</sub>P<sub>2</sub> is S<sub>2</sub>K<sub>1</sub> = (a+b)sinθ and hence phase difference  $\Phi = (2\pi/\lambda)(a+b)\sin\theta$ . The difference of paths S<sub>1</sub>P<sub>1</sub> and S<sub>3</sub>P<sub>3</sub> is S<sub>3</sub>K<sub>2</sub> = 2(a+b)sinθ and the phase difference is 2Φ and so on.

Then the difference of paths S1 P1 and S2 P2 is S2 K1 which is  $= a + b \sin \theta$  and hence phase difference  $\Phi$  would be  $2\pi$  by  $\lambda$  into  $a + b \sin \theta$ . Similarly the difference of paths S1 P1 and S3 P3 is S3 K2 which is  $= 2$  times  $a + b \sin \theta$  and hence the phase difference will be  $2\pi$ . Similarly we can calculate the phase difference between the other reflected waves.

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Thus, if at the instant the waves under consideration arrive in focus, we consider the phase of the wave from  $S_1$  to be zero, then the phase of waves from  $S_2, S_3, S_4$  etc. should be  $\phi, 2\phi, 3\phi, \dots$  etc., i.e. the phases increase in arithmetic progression. Therefore, the problem reduces to find the resultant amplitude of waves of equal amplitude  $A \frac{\sin \beta}{\beta}$  and equal period but the phase increasing in the arithmetic progression, the common phase difference being  $\phi$ .

Thus if at the instant the waves under consideration arrive in focus we consider the phase of the wave from  $S_1$  to be 0 then the phase of waves from  $S_2, S_3, S_4$  so on, should be  $\Phi, 2\Phi, 3\Phi$  etc., that is the phases increase in arithmetic progression therefore the problem reduces to find the resultant amplitude of  $n$  waves of empirical amplitude  $A \sin \beta$  by  $\beta$  and equal period but the phase increasing in the arithmetic progression the common phase difference being  $\Phi$ .

Therefore the resultant amplitude are be  $= A \sin \beta$  by  $\beta$  into  $\sin N\Phi$  by 2 divided by  $\sin \Phi$  by 2 where  $A \sin \beta$  by  $\beta$  is the amplitude of single slit and  $\beta = \pi b \sin \theta$  by  $\lambda$ . If you write  $\Phi$  by 2  $= \gamma$  which is  $= \pi$  by  $\lambda$  into  $a + b \sin \theta$  then the resultant amplitude  $R$  will be  $= A \sin \beta$  by  $\beta \sin N\gamma$  divided by  $\sin \gamma$ .

Therefore the intensity of the resultant wave will be  $= A^2 \sin^2 \beta$  by  $\beta^2 \sin^2 N\gamma$  divided by  $\sin^2 \gamma$ .

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If we put  $N=1$  and  $N=2$  in this expression it can be easily seen that this reduces to the corresponding expressions for the single slit and the double slits diffractions, respectively.

Furthermore, the first factor  $A^2 \left( \frac{\sin^2 \beta}{\beta^2} \right)$  simply gives the intensity distribution in the diffraction by a single slit. The second factor  $\frac{\sin^2 N\gamma}{\sin^2 \gamma}$  may be said to give the interference pattern for  $N$  slits. Thus we can say that each of the  $N$  slits gives rise to diffracted beam in which the intensity distribution depends on the width of the slit and the diffracted beams then interfere with one another to produce the final diffraction pattern.

Now here if we put  $N = 1$  and  $N = 2$  in this expression we will find the result which we have already obtained for single slit and double slit diffraction pattern respectively. Further the first factor in this expression that is  $A^2 \sin^2 \beta / \beta^2$  simply gives the intensity distribution in the diffraction by a single slit. The second factor  $\sin^2 n \gamma / \sin^2 \gamma$  may be said to give the interference pattern for  $N$  slit.

Thus we can say that each of the  $N$  slit give rise to diffracted beam in which the intensity distribution depends on the width of the slit and the diffracted beam then interfere with one another to produce the final diffraction pattern. Now we can find out from this expression the position of maximum and minimum intensity on the screen.

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### Positions of maxima and minima

Now let us find the positions of maximum and minimum intensity on the screen, i.e. at the focal plane of the lens.

Principal maxima

We have  $I = I_0 \left( \frac{\sin^2 \beta}{\beta^2} \right) \frac{\sin^2 N\gamma}{\sin^2 \gamma}$ , where  $I_0 = A^2$

The condition for the principal maxima is

$$\sin \gamma = 0 \quad \text{i.e. } \gamma = n\pi$$

where  $n=0, 1, 2, \dots$

That is at the focal plane of the lens, now from the intensity equation  $I = I_0 \sin^2 \beta / \beta^2 \times \sin^2 N\gamma / \sin^2 \gamma$ . We can write the condition for principal Maxima that is where the intensity is maximum, so intensity will be maximum when  $\sin \gamma = 0$  that is  $\gamma = n\pi$  where  $n = 0, 1, 2, 3$  etcetera.

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For these values of  $\gamma$  both numerator and denominator of  $\frac{\sin N\gamma}{\sin \gamma}$  vanish, that is, term becomes indeterminate. But it may be evaluated by finding its limit as  $\gamma$  tends to  $\pm n\pi$ , by rules of calculus

$$\begin{aligned} \lim_{\gamma \rightarrow \pm n\pi} \frac{\sin N\gamma}{\sin \gamma} &= \lim_{\gamma \rightarrow \pm n\pi} \frac{N \cos N\gamma}{\cos \gamma} \\ &= \frac{N \cos Nn\pi}{\cos n\pi} = \pm N \end{aligned}$$

For this value of  $\gamma$  both numerator and denominator of  $\sin N\gamma / \sin \gamma$  when that is term becomes indeterminate but it may be evaluated by finding its limit as  $\gamma$  tends to  $\pm n\pi$ . So, by method of calculus we can find the limit  $\gamma$  tending to  $\pm n\pi$   $\sin N\gamma / \sin \gamma = \pm N$  where capital  $N$  is the number of slits.

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Therefore, the intensity of the principal maximum is proportional to  $N^2$ , which increases with the increase in the number of slits. The intensities of principal maxima are, however, modulated by the single slit diffraction pattern. The actual intensity is obtained by multiplying  $N^2$  with single slit pattern. Therefore, intensity of principal maxima is given by

$$I = N^2 I_0 \frac{\sin^2 \beta}{\beta^2}$$

where,

$$\beta = \frac{\pi}{\lambda} b \sin \theta$$

Therefore the intensity of the principal maximum is proportional to capital N square which increases with the increase in the number of slits. The intensities of principal Maxima are however modulated by the single slit diffraction pattern. The actual intensity is obtained by multiplying capital N square with single slit pattern. Therefore intensity of principal Maxima is given by N square into I naught into sine square beta by beta square where beta = PI by lambda into b sine theta.

Actually at the principal maxima the field produced by each of the slits are in phase and therefore they add up and the resultant field is N times the field produced by each of the slits.

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Actually, at the principal maxima the field produced by each of the slits are in phase and, therefore, they add up and the resultant field is N times the field produced by each of the slits.

For principal maxima  $\gamma = \pm n\pi$

$$\frac{\pi}{\lambda}(a + b)\sin\theta = \pm n\pi \quad \sin\theta = \pm \frac{n\lambda}{(a + b)}$$

Since  $\sin\theta \leq 1$

$$n\lambda \leq (a + b)$$

values of n should be such that

$$n\lambda \leq (a + b)$$

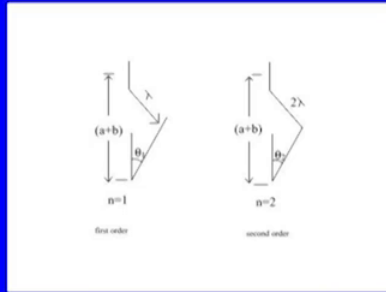
Therefore, for principal maxima:  $(a + b)\sin\theta = \pm n\lambda$

So, for principal Maxima we have  $\gamma = + - n\pi$ , so this condition we can write in terms of for a and b that is  $\pi$  by  $\lambda$  into  $a + b \sin\theta = + - n\pi$   $\sin\theta = + - n\lambda$  upon  $a + b$ . Since  $\sin\theta$  is less than or  $= 1$  so we should have  $n\lambda$  less than or  $= a + b$ . So, value values of n should be such that  $n\lambda$  should be less than or  $= a + b$ .

Therefore for principal Maxima we have  $a + b \sin\theta = + - n\lambda$ . This equation simply expresses the condition that if the difference of parts of the parallel deflector rays from any pair of corresponding points of grating is integral multiple of  $\lambda$  then all the parallel reflected rays in that direction reinforce so as to form the principal maxima in the focal plane of the lens.

The whole number n in the ever equation represents the order of the interference maximum and physically it gives the number of wavelengths in the difference of paths of the deflected rays from any pair of corresponding points of the grating as shown in this figure here.

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The distance  $(a + b)$  is called the grating element. Above equation does not contain  $N$  so that, for a given slit separation the position of the principal maxima are independent of the number of slits.

The distance  $a + b$  is called the grating element every equation does not contain capital  $N$ , so that for a given slit separation the position of the principal Maxima are independent of the number of slits. Now we can find out the position of minimum intensity from the expression of intensity.

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### Positions of Minimum intensity

The value of  $\gamma$  for which only the numerator of the term  $\sin N\gamma / \sin \gamma$  vanishes give points of zero intensity in the diffraction pattern. That is for minima

$$\sin N\gamma = 0 \text{ but } \sin \gamma \neq 0$$

$$\text{or } N\gamma = \pm m\pi$$

where  $m$  has any integral value with the exception of 0,  $N$ ,  $2N$ ,  $3N$  etc.

So, the value of  $\gamma$  for which only the numerator of the term  $\sin N\gamma$  vanishes give point of 0 intensity in the diffraction pattern that is for minima we should have  $\sin N\gamma = 0$ . But  $\sin \gamma$  should not be  $= 0$  or we should have  $N\gamma = + - m\pi$  where  $m$  has any integral value with the exception of 0 capital  $N$  2 time capital  $N$  3 time capital  $N$  so on.

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Because for these values of  $m$ ,  $\gamma = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$  and therefore the denominator of the term  $\left(\frac{\sin N\gamma}{\sin \gamma}\right)$  also vanishes.

This corresponds to the position of principal maxima.

So Condition for minima is

$$N \frac{\pi}{\lambda} (a + b) \sin \theta = \pm m \pi$$

$$\text{Or, } (a + b) \sin \theta = \pm \frac{m}{N} \lambda$$

But  $m$  not equal to  $nN$

Because for these values of  $m$   $\gamma = 0 + - \pi + - 2\pi + -3\pi$  so on. And therefore the denominator of the term  $\frac{\sin N\gamma}{\sin \gamma}$  also vanishes. These correspond to the position of principal maxima. So, condition for minima is capital  $N$   $\pi$  by  $\lambda$  into  $a + b \sin \theta = + - m \pi$  or  $a + b \sin \theta = + + m$  upon capital  $N$  times  $\lambda$  but  $m$  should not be  $= n$  time capital  $N$ .

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where  $n=0, 1, 2, \dots$  being the order of principal maxima.

Thus for minima, we have

$$(a + b) \sin \theta = \pm \frac{\lambda}{N}, \pm \frac{2\lambda}{N}, \pm \frac{3\lambda}{N}, \dots, \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}$$

Omitting the values

$$(a + b) \sin \theta = 0, \pm \frac{\lambda}{N}, \pm \frac{2\lambda}{N}, \pm \frac{3\lambda}{N}, \dots \text{ for principal maxima}$$

Where  $n$  small  $n$  has integral value  $0, 1, 2, 3$  so on and this  $n$  give the order of principle Maxima, thus for minima we have  $a + b \sin \theta = + - \lambda$  upon capital  $N + - 2 \lambda$  upon capital  $N + - 3 \lambda$  upon capital  $N$  so on. Omitting the values  $a + b \sin \theta = 0 + - \lambda$  upon capital  $N$

$\lambda \sin \theta = n\lambda$  upon capital  $N + - 2$  times capital  $N$   $\lambda \sin \theta = n\lambda$  upon capital  $N$  so on which are the condition for principle maximum.

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### Secondary maxima

We have found the conditions for principal maxima and zero intensity in the diffraction pattern. From this we can say that between the principal maxima of zero order and the principal maxima of order one  $(N - 1)$  there is equally spaced points of zero intensity i.e. minima. Hence there are  $(N - 2)$  other maxima, known as secondary maxima between any two adjacent principal maxima. Thus for 3 slits there are is only one, for four slits there are two and for five slits there are three secondary maxima.

Now so we have found the condition for principal maxima and 0, intensity in the diffraction pattern. From this we can say that between the principal Maxima of 0 order and the principal Maxima of order 1 there is  $N - 1$  there is equally spaced points of 0 intensity that is minimum. Hence there are  $N - 2$  other Maxima known as secondary maxima between any two adjacent principal maxima.

Thus for 3 slits there are only 1, for 4 slit there are 2 and for 5 slit there are 3 secondary maximum.

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To find the position of secondary maxima, we first differentiate  $\frac{\sin N\gamma}{\sin \gamma}$  with respect to  $\gamma$  and then equate to zero.

Thus,

$$\frac{d}{d\gamma} \left[ \frac{\sin N\gamma}{\sin \gamma} \right] = \frac{d}{d\gamma} [\sin N\gamma \operatorname{cosec} \gamma] = 0$$

This gives  $N \tan \gamma = \tan N\gamma$

To find the position of secondary maxima we first differentiate sine N gamma upon sine gamma with respect to gamma and then equate it to 0. And thus if you differentiate this sine N gamma by sine gamma with respect to gamma and after equating it to 0, we will find capital N tan gamma = tan capital N gamma.

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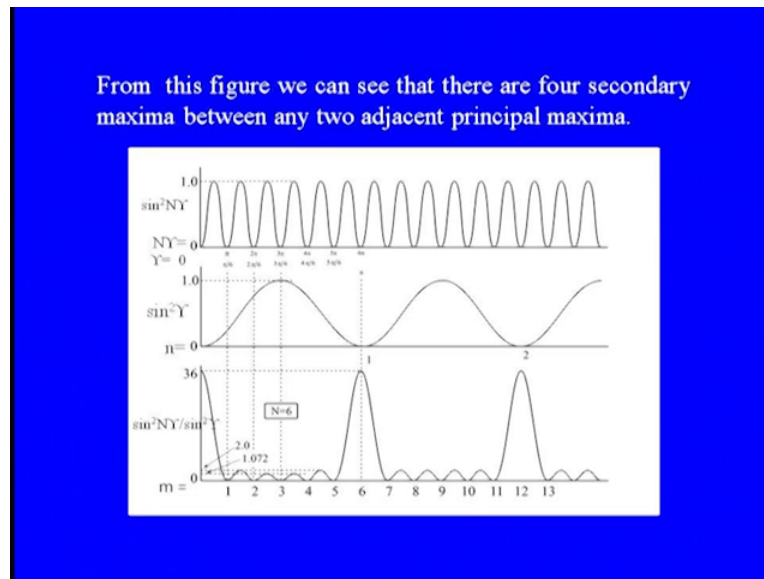
The roots of this equation other than  $\gamma = \pm n\pi$  give the position of secondary maxima. The secondary maxima are not of equal intensity, and intensity decreases gradually as we go out on either side to any principal maxima. Moreover they are slightly shifted, as in the case of single slit pattern, towards the adjacent principal maxima.

Plot of  $\frac{\sin^2 N\gamma}{\sin^2 \gamma}$  for  $N=6$  is shown here.

The root of this equation other than gamma is  $= + - n \pi$  give the position of secondary maxima the secondary Maxima are not of equal intensity and intensity decreases gradually as we go out on either side to any principal maxima moreover they are is slightly shifted as in the case of single slit pattern towards the adjacent. Now the plot of sine square N gamma upon sine square gamma is shown here for  $N = 6$ .



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From this figure we can see that there are 4 secondary maxima between any two adjacent principal maxima which we have already discussed. Here it should be noted that the intensity of the principal Maxima are modulated by the single slit diffraction that is the plot of sine square N gamma upon sine square gamma is multiplied by I naught sine square beta by beta square the single slit diffraction pattern.

When n is very large that is when the number of slit is very large the principal Maxima will be much more intense in comparison to the secondary maximum. A particular principal Maxima may be absent if it corresponds to the angle which also determine the minimum of the single slit diffraction pattern.

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A particular principal maxima may be absent if it corresponds to the angle which also determines the minimum of the single slit diffraction pattern. This will happen when

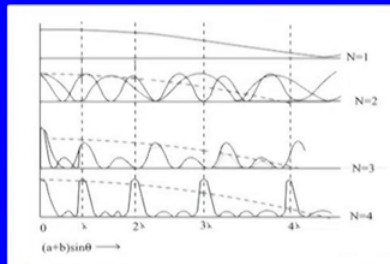
$$(a + b)\sin \theta = n\lambda$$

$$b\sin \theta = \lambda, 2\lambda, 3\lambda, \dots = m'\lambda \quad (m' = 1, 2, 3 \dots m' \neq 0)$$

are satisfied simultaneously and is usually referred to as a missing order.

This will happen when  $a + b \sin \theta = n \lambda$  and  $b \sin \theta = m' \lambda$  where  $m'$  is  $= 1, 2, 3$  so on but  $m'$  is not  $= 0$  are satisfied simultaneously and is usually referred to as a missing order.

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Although the principal maxima occur at the same position with 3 and 4 slits as with two but they become brighter and narrower while the secondary maxima become weaker and faint with the increase in number of slits. This is clear from this figure.

Although the principal Maxima occurs at the same position with 3 and 4 slits as with the two but they become brighter and narrower while the secondary Maxima become weaker and faint with the increase in number of slit this is clear from this figure. The extension to a very large number of slit as in actual grating say 15,000 lines per inch the Maxima become very narrow and observe intensity while the secondary Maxima are of negligible intensity indivisible in the pattern.

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### Half width of principal maxima

We can easily deduce the angular separation between the first minima on either side of the principal maxima in the diffraction pattern of any order say  $n$ . This angular separation is called the angular width of the principal maximum in the  $n^{\text{th}}$  order. Let  $\theta_n$  be the angle of diffraction corresponding to the principal maximum of wavelength  $\lambda$  in the  $n^{\text{th}}$  order.

Now we can easily deduce the angular separation between the first minima on either side of the principal Maxima in the diffraction pattern of any order say and this angular separation is called the angular width of the principal Maxima in the  $n^{\text{th}}$  order. Let  $\theta_n$  be the angle of diffraction corresponding to the principal maximum of wavelength  $\lambda$  in the  $n^{\text{th}}$  order.

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This direction is given by

$$(a + b) \sin \theta_n = n\lambda$$

$$\text{Or } N(a + b) \sin \theta_n = Nn\lambda \dots\dots\dots (1)$$

The position of minimum of intensity in the diffraction pattern due to  $N$  slits are given by the condition

$$(a + b) \sin \theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \dots, \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \dots$$

Or

$$N(a + b) \sin \theta = \lambda, 2\lambda, 3\lambda, \dots, (N-1)\lambda, (N+1)\lambda, \dots$$

This direction is given by  $a + b \sin \theta_n = n\lambda$  or if we multiply capital  $N$  both sides we will get capital  $N$  times  $a + b \sin \theta_n = \text{capital } N \text{ into small } n \text{ into } \lambda$  the position of minimum of intensity in the diffraction pattern due to an slit are given by the condition  $a + b \sin \theta = \lambda$  by capital  $N$   $2\lambda$  by capital  $N$  so on or capital  $N$  into  $a + b \sin \theta = \lambda$  or  $2\lambda$  or  $3\lambda$ .

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But for the values

$$N(a + b) \sin \theta = 0, N\lambda, 2N\lambda, 3N\lambda \dots$$

We get respectively 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> .....order principle maximum.

Thus, the first minimum on either side of the principal maximum of first order is given by equation.

$$N(a + b)\sin(\theta_1 \pm d\theta_1) = (N\lambda \pm \lambda)$$

But for the values capital N into  $a + b \sin \theta = 0$  capital N lambda or 2 time capital N lambda we get respectively first second so on order of principle maximum. Thus the first minimum on either side of the principle maximum of first order is given by equation capital N  $a + b \sin \theta_1 \pm d\theta_1 = \text{capital N lambda} \pm \lambda$  where the angle of deflection  $\theta_1$  corresponds to the principle maximum of first order.

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Where the angle of diffraction  $\theta_1$  corresponds to principal maximum of first order and  $\theta_1 \pm d\theta_1$  corresponds to first minimum on either side of this maximum. Now it should be obvious that the first minimum on either side of the principal maximum of  $n^{\text{th}}$  order should be given by,

$$N(a + b)\sin(\theta_n \pm d\theta_n) = (nN\lambda \pm \lambda) \quad - (2)$$

And  $\theta_1 \pm d\theta_1$  corresponds to the first minimum on either side of this maximum. Now it should be obvious that the first minimum on either side of the principle maximum of nth order

is given by capital N into a + b sine theta n + - d theta n = small n into capital N into lambda + - lambda.

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Where  $(\theta_n \pm d\theta_n)$  is the angle of diffraction corresponding to first minimum.

Dividing (2) by (1), we get

$$\frac{\sin(\theta_n \pm d\theta_n)}{\sin \theta_n} = \frac{nN\lambda \pm \lambda}{nN\lambda}$$

$$\text{(or)} \quad \frac{\sin \theta_n \pm d\theta_n \cos \theta_n}{\sin \theta_n} = 1 \pm \frac{1}{nN}$$

So, where theta 1 + - d theta n is the angle of diffraction corresponding to first minimum, so, by using the previous two equation we can write sine theta n + - d theta n divided by sine theta n = small n into capital N into lambda + - lambda divided by small n into capital N into lambda or sine theta n + - d theta and cos theta n divided by sine theta n = 1 + - 1 over is small n into capital N.

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$$\text{or } d\theta_n = \frac{1}{nN \cot \theta_n} \quad \begin{array}{l} \text{The principal maxima} \\ \text{becomes sharper as } N \\ \text{increases} \end{array}$$

This equation expresses the half angular width of the principal maximum of order n.

$$\text{we have } (a + b) \sin \theta_n = n\lambda$$

$$n = \frac{(a \pm b) \sin \theta_n}{\lambda}$$

Or we can also write  $d\theta_n = \frac{\lambda}{N(a+b)\sin\theta_n}$  into  $\frac{\lambda}{N(a+b)\cos\theta_n}$  this equation expresses the half angular width of the principal maximum of order  $n$  and we have  $a + b \sin\theta_n = n\lambda$ . So, we can write order of diffraction  $n = \frac{a + b \sin\theta_n}{\lambda}$ .

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$$d\theta_n = \frac{\lambda}{N(a+b)\sin\theta_n \cdot \frac{\cos\theta_n}{\sin\theta_n}}$$

$$d\theta_n = \frac{\lambda}{N(a+b)\cos\theta_n}$$

$N(a+b) \rightarrow$  total width of the ruled surface of the grating

Or  $d\theta_n$  will be  $= \frac{\lambda}{N(a+b)\cos\theta_n}$  which we can simplify and we will get  $d\theta_n = \frac{\lambda}{N(a+b)\cos\theta_n}$ . Now here  $N(a+b)$  is the total width of the ruled surface of grating.

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Thus, the sharpness of the principal maxima depends upon the total width,  $N(a+b)$ , of the ruled surface of grating and not upon the number of lines per centimeter.

Thus the surface of the principal Maxima that is a smaller value of  $d\theta_n$  depends upon the total width  $N(a+b)$  of the ruled surface of grating and not upon the number of lines per

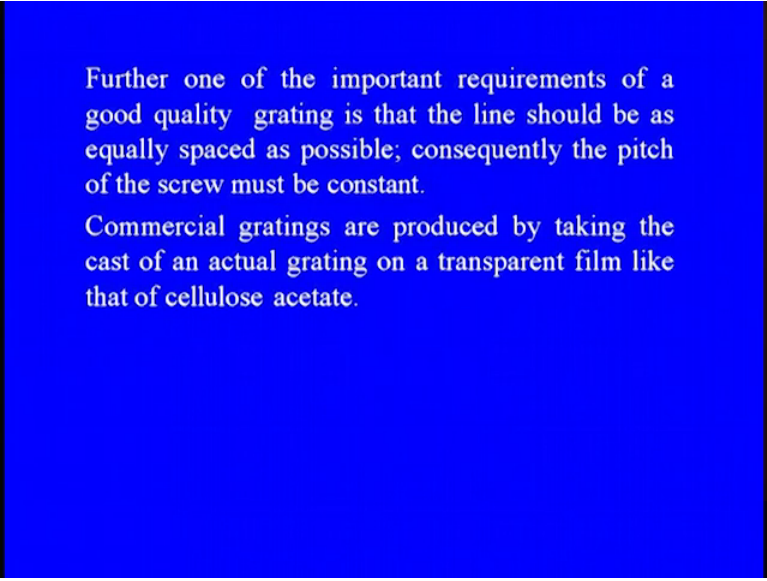


centimeter. Now I am going to describe how gratings are fabricated for laboratory use as we know an arrangement of a large number of equidistant slits is known as a diffraction grating. For narrow principal maximum that is surfer spectral lines a large value of capital N is required.

A good quality grating therefore requires a large number of slits typically of the order of 15,000 per inch this is achieved by ruling groups with a diamond point on an optically transparent sheet of material. The group's act as opaque spaces after each group is ruled the machine lifts the diamond points and move the seat forward for the ruling of the next group. Since the distance between the two consecutive groups is extremely small.

The moment of the seat is obtained with the help of the rotation of a screw which drives the carrier's carrying it.

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Further one of the important requirements of a good quality grating is that the line should be as equally spaced as possible; consequently the pitch of the screw must be constant.

Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate.

Further one of the important requirements of a good quality grating is that the line should be as equally spaced as possible. Consequently the pitch of the screw must be constant. Commercial gratings are produced by making the cost of an actual grating on a transparent film like that of cellulose acetate.

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## The grating spectrum

The condition for principal maxima in the diffraction pattern of plane grating, when light is incident normally on it, is given by,

$$(a + b)\sin \theta = \pm n\lambda$$

Now we will discuss about the grating spectrum. So, the condition for principal Maxima in the diffraction pattern of plane grating when light is incident normally on it is given by  $a + b \sin \theta = \pm n \lambda$ .

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When the number of ruling is very large say 14000 to an inch and the primary source of light is a narrow slit, the principal maxima are observed as sharp bright lines in the focal plane of the eyepiece of the telescope adjusted for the parallel rays. Consequently principal maxima are commonly spoken as spectrum lines. If the slit is parallel to the rulings of grating, the spectrum lines will be also parallel to the rulings.

When the number of ruling is very large say 14,000 to an inch and the primary source of light is a narrow slit the principal Maxima are observed as or bright lines in the focal plane of the eyepiece of the telescope adjusted for the parallel rays. Consequently the principal Maxima are commonly spoken as a spectrum. If the slit is parallel to the rulings of the grating the spectrum lines will be also parallel to the rulings.

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These lines are simply the images of the slit.  
It is clear from the above equation that in a given order  $n$ , the angle of diffraction  $\theta$  depends upon  $\lambda$ .  
Therefore, if the source gives light of number of wavelengths.

These lines are simply the images of the slit; it is clear from the above equation that in a given order  $n$  the angle of deflection  $\theta$  depends upon wavelength  $\lambda$  therefore if the source gives light of number of wavelengths.

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e.g.  $\lambda_1, \lambda_2, \lambda_3$  then in the given order (except  $n=0$ ) we will find as many lines as there are different wave-lengths in the light source. Thus a line spectrum is formed in each orders. But for the central image ( $n=0$ ) the path difference is zero for every wavelength. Hence the central maxima of different wave-length coincide forming the central image, obviously of the same color as that of the primary source of light.

That is  $\lambda_1, \lambda_2, \lambda_3$  so on then in the given order except  $n = 0$ , we will find as many lines and there are different wavelengths in the light source. Thus a line spectrum is form in each order but for the central image that is for  $n = 0$  the path difference is 0 for every wavelength. Hence the central maxima of different wavelength coincide forming the central image obviously of the same color as that of the primary source of light.

On the other side of the central image a similar set of spectra are formed in each other. The spectrum lines' corresponding to shortest wavelength is formed on the side towards the central image.

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### Determination of wave-lengths

$$(a + b)\sin \theta_n = \pm n\lambda$$

$n \rightarrow$  Order of the spectrum

$(a+b) \rightarrow$  grating element

If  $N$  is the number of lines per cm in the grating.

Now let us see how to determine the wavelength of light by using the grating spectrum so we have  $a + b \sin \theta_n = \pm n\lambda$ , so here  $n$  is the order of the spectrum and  $a + b$  in the grating element and if capital  $N$  is the number of lines per centimeter in the grating.

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$$(a + b) = \frac{1}{N}$$

Thus, by noting the value of  $n$  and measuring the angle of diffraction for a definite spectral line, we can easily calculate its wave-length.

Then we can calculate  $a + b = 1$  over capital  $N$  thus by noting the value of  $n$  that is order of the respect and measuring the angle of diffraction for a definite spectral line, we can easily calculate

its wavelength. So, this angle  $\theta$  is measured by using a spectrometer in the laboratory. So, to determine the value of wavelength by using grating we have to measure experimentally the deflection angle  $\theta$ .

And then by knowing the order of the deflections and then using this equation  $a + b \sin \theta = n\lambda$  we can calculate the value of wavelength.

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### Overlapping of orders

When we examine a large range of wavelengths with the help of diffraction grating, then some overlapping of spectral lines occurs in the higher order of spectrum. This is due to the reason that for a given value of  $\theta$ , the fundamental equation of grating viz.,

Now let us discuss about the overlapping of orders, so when we examine a large range of wavelengths with the help of diffraction grating then some overlapping of spectral lines occurs in the higher order of the spectrum. This is due to the reason that for a given value of  $\theta$  the fundamental equation of grating.

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$(a + b)\sin \theta = n\lambda$  , may be satisfied by the several values of  $\lambda$  with appropriate values of  $n$ .

For example if we have

$$(a + b)\sin \theta = \lambda_1 = 2\lambda_2 = 3\lambda_3 \text{ etc.}$$

then the spectral line of wavelength

$\lambda_1, \lambda_2, \lambda_3 \dots$  occurring in the first, the second and the third orders, respectively will coincide in position in the spectrum.

That is  $a + b \sin \theta = n \lambda$  may be satisfied by the several values of  $\lambda$  with appropriate values of  $n$ . For example if we have  $a + b \sin \theta = \lambda_1 = 2 \text{ times } \lambda_2 = 3 \text{ times } \lambda_3$  so on. Then the spectral line of wavelengths  $\lambda_1, \lambda_2, \lambda_3$  so on occurring in the first, the second and the third orders respectively will coincide in position in the spectrum that is they will overlap.

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### Angular Dispersive Power of the Grating

The angular dispersive power of the grating is defined as the rate of change of the angle of diffraction with the change in wave-length. Mathematically , it is expressed by  $\left(\frac{d\theta}{d\lambda}\right)$  and its value can be very easily obtained by differentiating the equation

$$(a + b)\sin \theta = n\lambda \text{ (Principal maxima)}$$

Now let us calculate the angular dispersive power of the grating the angular dispersive power of the grating is defined as the rate of change of the angle of diffraction with the change in wavelength. Mathematically it is expressed by  $d\theta$  by  $\lambda$  and its value can be very easily obtained by differentiating equation  $a + b \sin \theta = n \lambda$ .

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with respect to  $\lambda$ , keeping  $n$  constant.

$$\text{Angular dispersive power: } \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

1) This equation shows that the angular dispersion is directly proportional to  $n$ , the order of the spectrum. This means that the angular separation  $d\theta$  between the principal maxima corresponding to wave-length  $\lambda$  and  $\lambda + d\lambda$  increases with the order of the spectrum.

With respect to  $\lambda$  keeping  $n$  constant so angular dispersive power  $d\theta$  by  $d\lambda$  will be  $= \frac{n}{(a+b)\cos\theta}$ . So, this equation shows that the angular dispersion is directly proportional to  $n$  and the order of the spectrum. This means that the angular separation  $d\theta$  between the principal maximum corresponding to wavelength  $\lambda$  and  $\lambda + d\lambda$  increases with the order of the spectrum.

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Therefore, the second order spectrum is twice as wide as the first and so on.

The angular separation  $d\theta$ , for a given  $d\lambda$ , is inversely proportional to  $(a+b)$  the width of the grating element and therefore directly proportional to the number of lines per cm on the grating. The smaller is the grating elements, the greater is the spread of spectrum of given order.

Therefore the second order spectrum is twice as wide as the first and so on the angular separation  $d\theta$  for a given  $d\lambda$  is inversely proportional to  $a + b$  the width of the grating element



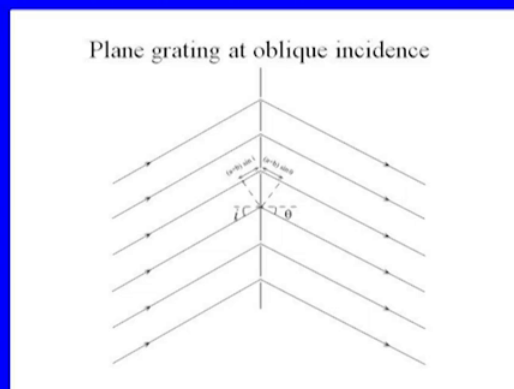
and therefore directly proportional to the number of lines per centimeter on the grating. The smaller is the grating element the greater is the spread of a spectrum of given order.

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The angular dispersion increases  $\theta$  with  $\lambda$  and therefore with the wave length. Since  $\lambda$  increases with increase in  $\theta$ . Therefore, grating spectrum are spread much more at the red end than at the blue end of the spectrum.

The angular dispersion increases theta and therefore with the wavelength since theta increases with increasing lambda therefore creating a spectrum are spread much more at the red end than had the blue in of the spectrum.

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Now let us consider the plane wave front of light of wavelength  $\lambda$  is incident obliquely on the grating had the angle of incidence  $i$  as shown in this figure. For a set of parallel rays deflected at an angle  $\theta$  with the grating normal. It can be shown that the difference of paths of the



deflected rays from the corresponding point of grating is  $a + b \sin i + b \sin \theta$ . Therefore in this case the condition for principle Maxima becomes  $a + b \sin i + b \sin \theta = n \lambda$ . So, this is the condition for principle Maxima for oblique incidence.

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#### Plane grating at oblique incidence

Let us now consider that plane wavefront of light of wavelength  $\lambda$  is incident obliquely on the grating at the angle of incidence  $i$ , as shown in this figure. For a set of parallel rays diffracted at an angle  $\theta$  with the grating normal, it can be shown that the difference of paths of the diffracted rays from the corresponding points of grating is  $(a+b)(\sin i + \sin \theta)$ . Therefore, in this case, the condition for principal maxima becomes

$$(a+b)(\sin i + \sin \theta) = n \lambda$$

For the other side of the direct beam, the condition for principal maxima will be

$$(a+b)(\sin i - \sin \theta) = n \lambda \quad (n = 0, 1, 2, \dots)$$

For the other side of the direct beam the condition for principle Maxima in case of oblique incidence will be  $a + b \sin i - b \sin \theta = n \lambda$  where  $n = 0, 1, 2, 3$  so on.

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#### Concave grating

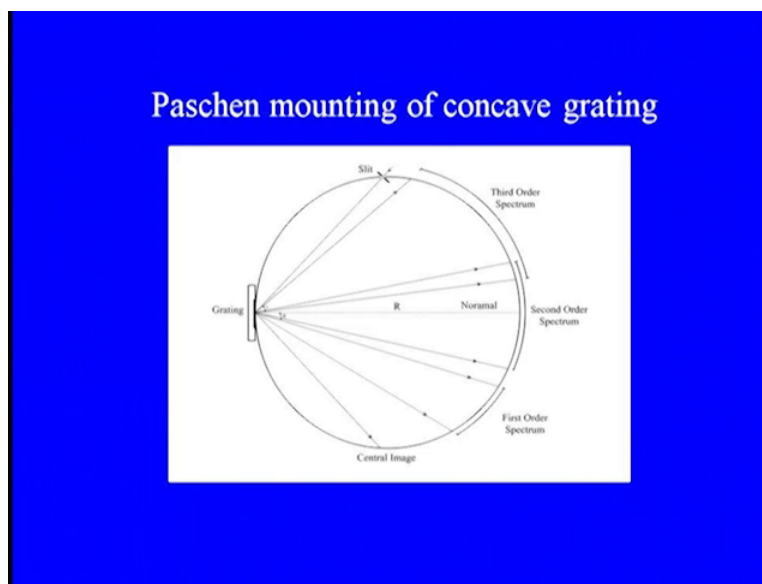
We have seen that the plane grating requires the use of two convergent lenses for the formation of spectra, one for rendering the light incident on the grating parallel and the other for bringing the parallel diffracted rays in focus. But the glass lenses are not suitable for investigation of ultra-violet region, because they are not transparent outside the visible spectrum. Also, the lenses can not be made truly achromatic and hence chromatic aberration comes in the spectra. To remove these problems concave grating has been developed. In this case, the grating, instead of being ruled on a plane surface, is ruled on a concave spherical mirror of metal, so that it diffracts and focus the light at the same time.

Now we will discuss briefly about the concave grating, so we have seen that the plane grating requires the use of two convergent lenses for the formation of a spectra. One for rendering the light incident on the grating parallel and the other for bringing the parallel deflected rays in focus

but the glass lenses are not suitable for investigation of ultraviolet region because they are not transparent outside the visible spectrum.

Also the lenses cannot be made truly achromatic and hence chromatic aberration comes in the spectra. To remove these problems concave grating has been developed. In this case the grating instead of being ruled on the plane surface is ruled on a concave spherical mirror of metal so that it deflects and focus the light at the same time. This will eliminate the chromatic aberration and can be used in ultraviolet region.

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So if  $r$  is the radius of curvature of the spherical surface of the grating a circle of diameter  $R$  can be drawn tangent to the grating at its midpoint which defines the locus of points where the spectrum is in focus provided the source slit also lies on this circle as shown in this figure. This circle is called the Rowland circle and the mounting is known as Paschen mounting. So, till now we have discussed single slit, double slit and multiple slit diffraction pattern.

Now I am going to discuss about the resolving power of certain instrument like grating telescope and microscopes. So, let us first discuss how the diffraction phenomena limit the resolving power of the instruments.

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## Resolving Power

The upper limit to the useful magnification by any instrument is set by the fact that the image of the point source formed by the lens, even in the absence of all aberrations is not a point image. But in reality it is a circular diffraction pattern formed by the limited portion of the wave-front transmitted by the lens system.

So, the upper limit to the useful magnification by an any instrument is set by the fact that the image of the point source formed by the lens even in the absence of all of aberrations is not a point image. But in reality it is a circular diffraction pattern formed by the limited portion of the wave front transmitted by the lens system.

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This diffraction pattern is characterised by the central bright disc bordered by the dark and the bright concentric circular rings of rapidly decreasing intensity. An optical system is said to be able to resolve two point objects if the corresponding diffraction patterns are sufficiently small or sufficiently separated to be distinguished as separate image patterns. With the analysing instruments we are mainly concerned with its ability to just separate in the diffraction spectrum two spectral lines corresponding to light of slightly different wave-length  $\lambda$  and  $\lambda + d\lambda$ .

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With the analyzing instruments we are mainly concerned with its ability to just separate in the deflection spectrum to spectral lines corresponding to light of is slightly different wavelengths  $\lambda$  and  $\lambda + d\lambda$ .

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### The Rayleigh Criterion for Limit of Resolution

From a detailed study of the resultant intensity distribution in the diffraction pattern of closely spaced point sources lord Rayleigh arrived at the conclusion that two equally bright point sources could be just resolved by any optical system, when their distance apart is such that the central maximum in the diffraction pattern due to one source coincided exactly with the first minimum in the diffraction pattern due to the other.

Now let us discuss about the Rayleigh Criteria for the limit of resolutions. From a detailed study of the resultant intensity distribution in the diffraction pattern of closely spaced point sources Lord Rayleigh arrived at the conclusion that two equally bright point sources could be just resolved by any optical system when the earth distance if apart is such that the central maximum in the diffraction pattern due to one source coincided exactly with the first minimum in the diffraction pattern due to the other.

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This is known as Rayleigh Criterion of resolution and it is also applicable to the resolution of spectral lines of equal intensity, formed by grating on prism spectrographs. When applied to spectral lines, it is equivalent to the condition that in the diffraction pattern, the angular separation between the principal maxima of two spectral lines in a given order,

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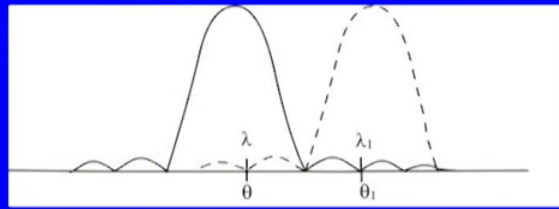
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should be equal to half angular width of either principal maximum. This means that if the principal maximum of wavelength  $\lambda$  is formed by an optical instrument at an emergent angle  $\theta$ , and the principal maximum of  $\lambda + d\lambda$ , in the same order, is formed at an emergent angle  $\theta + d\theta$ , then the first minimum in the diffraction pattern due to  $\lambda$  should also be formed at an angle  $(\theta + d\theta)$ .

Should be = half angular width of either principal maximum, this means that if the principal maximum of wavelength  $\lambda$  is formed by an optical instrument at and emergent angle  $\theta$  and the principal maximum of  $\lambda + d\lambda$  in the same order is formed an emergent angle  $\theta + d\theta$ . Then the first minimum in the diffraction pattern due to  $\lambda$  should also be formed at an angle  $\theta + d\theta$ .



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The physical significance of Rayleigh's criterion for resolution can be explained by considering the resultant intensity distribution in the diffraction patterns due to two wavelengths  $\lambda$  and  $\lambda_1$ .

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Let the wavelength difference be such that the principal maximum of one falls exactly over the second point of minimum intensity in the diffraction pattern of the other. The two wavelength must have been emitted by different sources-atoms so that the disturbance are incoherent.

Let the wavelength difference be such that the principal Maxima of one falls exactly over the second point of minimum intensity in the diffraction pattern of other. The two wavelengths must have been emitted by different sources so that the disturbances are incoherent.

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Consequently, the total intensity at any point in the spectrum is simply the sum of the intensities for each wavelength. In this case there is a distinct point of zero intensity in the middle of the resultant intensity curve which has been obtained summing the intensities due to separate patterns. Thus the two spectral lines are distinctly separated.

Consequently the total intensity at any point in the spectrum is simply the sum of the intensities for each wavelength. In this case there is a distinct point of 0 intensity in the middle of the resultant intensity curve which has been obtained summing the intensities due to separate pattern thus the two spectral lines are distinctly separated.

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Hence if the wavelength difference is smaller, the two spectral lines with their principal maxima are closer. There is one limiting value of  $\lambda_1 = (\lambda + d\lambda)$  for which the angular separation between their principal maxima is such that the principal maximum of one falls exactly over the first point of minimum intensity in the diffraction pattern due to the other. Therefore, the intensities of maxima in the resultant intensity patterns are equal to the those of original maxima.

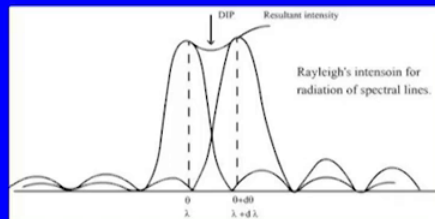
Hence if the wavelength difference is smaller the two spectral lines with their principal Maxima are closer. There is one limiting value of  $\lambda_1 = \lambda + d\lambda$  for which the angular separation between their principal Maxima is such that the principal maximum of one falls exactly over the first point of minimum intensity in the diffraction pattern due to the other.



Therefore the intensities of Maxima in the resultant intensity patterns are = the those of original Maxima.

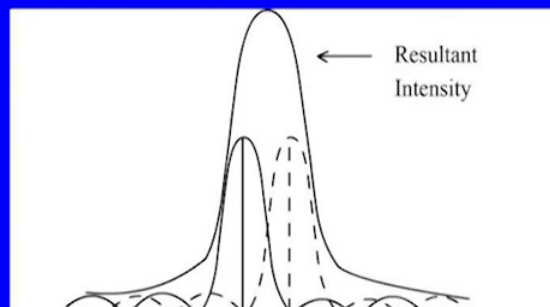
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Thus the resultant intensity Curve is a double humped curve with a distinct dip at a point half way between the two principal maxima. This change of intensity is easily visible to the eye and enables us to recognize that in the resultant pattern there are two spectral lines Corresponding to  $\lambda$  and  $\lambda + d\lambda$ . The two spectral lines, under this condition are said to be resolved.



hus the resultant intensity curve is a double humped curve with a distinct dip at a point half way between the two principal Maxima. This change of intensity is easily visible to the eye and enables us to recognize that in the resultant pattern there are two spectral lines corresponding to  $\lambda$  and  $\lambda + d\lambda$ . The two spectral lines under this condition are said to be resolved.

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But if  $\lambda_1$  is less than  $\lambda + d\lambda$  the two spectral lines come little closer and the intensity curves of the principal maxima so considerable overlapping. The resultant intensity curve exhibits no dip but only one maximum in the center. Thereby indicating as if it is a diffraction pattern of only one spectral line thus under this condition the two spectral lines are not resolved in their diffraction pattern.

Hence for two spectral lines to be resolved it is absolutely essential that the least angular separation between their principal Maxima should be such that the principal maximum of one should fall on the first point of minimum intensity of the other. Now let us use this Rayleigh Criteria to derive the expression for resolving power of a grating.

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### Resolving power of a grating

The function of an analyzing instrument like grating is to resolve two images of the same slit, formed by light waves of slightly different wavelength say  $\lambda$  and  $\lambda + d\lambda$ .

The images of the slit are known as the spectral lines corresponding to wavelength  $\lambda$  and  $\lambda + d\lambda$ . Accordingly the limit of resolution of a grating is defined as the smallest wavelength difference for which the spectral line can be just resolved at the wavelength  $\lambda$ . This limit of resolution is mathematically expressed by the ratio  $\frac{\lambda}{d\lambda}$ .

The function of an analyzing instrument like grating is to resolve two images of the same slit formed by light waves of slightly different wavelengths say  $\lambda$  and  $\lambda + d\lambda$ . The images of the slit are known as the spectral lines corresponding to wavelength  $\lambda$  and  $\lambda + d\lambda$ . Accordingly the limit of resolution of a grating is defined as the smallest wavelength difference for which the spectral line can be just resolved at the wavelength  $\lambda$ . This limit of resolution is mathematically expressed by the ratio  $\lambda$  to  $d\lambda$ .

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Let a plane wave front of light to be analyzed be incident normally on the grating, which forms the diffraction pattern in the focal plane of the telescope lens, corresponding to each of the wave lengths, say  $\lambda$  and  $\lambda + d\lambda$  present in the incident light. The angular deviation  $\theta_n$  of the centre of principal maximum formed in the  $n^{\text{th}}$  order, by light of wavelength  $\lambda$ , is given by

$$(a + b) \sin \theta_n = n\lambda \quad (a + b \rightarrow \text{grating element})$$

The angular separation between the centers of the principal maxima, corresponding to wavelength  $\lambda$  and  $\lambda + d\lambda$  formed in the order  $n$  is obtained by differentiating above eqn w.r.t.  $\lambda$ . We get

Now let a plane wave front of light to be analyzed be incident normally on the grating which forms the diffraction pattern in the focal plane of the telescope lens corresponding to each of the wavelengths say  $\lambda$  and  $\lambda + d\lambda$  present in the incident light. The angular deviation  $\theta_n$  of the center of principle maximum formed in the  $n^{\text{th}}$  order by light of wavelength  $\lambda$  is given by  $(a + b) \sin \theta_n = n\lambda$  where  $a + b$  is the grating element.

The angular separation between the centers of the principal maxima corresponding to wavelength  $\lambda$  and  $\lambda + d\lambda$  formed in the order  $n$  is obtained by differentiating above equation with respect to  $\lambda$ .

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$$(a + b) \cos \theta_n \frac{d\theta_n}{d\lambda} = n \quad d\theta_n = \frac{n d\lambda}{(a + b) \cos \theta_n}$$

The half angular width of the principal maximum of  $\lambda$  in the  $n^{\text{th}}$  order is  $d\theta_n = \frac{1}{N_n \cot \theta_n}$

where  $N$  is the total number of line in the grating.

According to Rayleigh's criterion for resolution the two spectral lines will be just resolved, when the principal maximum of one falls exactly over the first point of minimum intensity of the other line. This condition will be realized if the angular separation,  $d\theta_n$ , between the two principal maximum is exactly equal to half angular breadth of the principal maximum of  $\lambda$  in the same order.

So, we get  $a + b \cos \theta_n \frac{d\theta_n}{d\lambda} = n$  or  $d\theta_n = \frac{n d\lambda}{a + b \cos \theta_n}$ . The half angular width of the principal maximum of  $\lambda$  in the  $n$ th order is therefore  $d\theta_n = \frac{1}{N} \cot \theta_n$  and where capital  $N$  is the total number of lines in the grating. Now according to the Rayleigh's criterion for resolution the two spectral lines will be just resolved when the principal maximum of one falls exactly over the first point of minimum intensity of the other line.

This condition will be realized if the angular separation  $d\theta_n$  between the two principal maximum is exactly = half angular breadth of the principal maximum of  $\lambda$  in the same order.

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$$\begin{aligned} \therefore \frac{1}{N_n \cot \theta_n} &= \frac{n d\lambda}{(a+b) \cos \theta_n} \\ \text{or } \frac{1}{N_n} &= \frac{n d\lambda}{(a+b) \sin \theta_n} \\ \frac{1}{N_n} &= \frac{n d\lambda}{n\lambda} \\ \frac{\lambda}{d\lambda} &= Nn \quad \rightarrow \text{Chromatic resolving power} \end{aligned}$$

The resolving power is, therefore, directly proportional to the order  $n$ , and in the given order it is proportional to the total number of lines effective in the formation of the diffraction pattern. It would be observed that the resolving power is independent of the spacing  $(a+b)$  of lines.

So,  $\frac{1}{N} \cot \theta_n$  should be  $= \frac{n d\lambda}{a + b \cos \theta_n}$  with the help of this equation we can write  $\lambda \cot \theta_n = n d\lambda$  which is known as the chromatic resolving power of grating. So, the resolving power is directly proportional to the order  $n$  and in the given order it is proportional to the total number of lines effective in the formation of diffraction pattern. It should be observed that the resolving power is independent of the spacing  $a + b$  of lines.

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We have

$$n = \frac{(a+b)\sin\theta_n}{\lambda}$$

$$\frac{d\lambda}{d\theta_n} = \frac{N(a+b)\sin\theta_n}{\lambda} = \frac{w\sin\theta_n}{\lambda}$$

$$\frac{d\lambda}{\lambda} = \frac{w}{\lambda} \sin\theta_n$$

Where  $w = N(a+b) \rightarrow$  total width of the ruled surface of the grating.

Thus, a change in the number of line  $N$  in a given width of ruled surface, will not affect the resolving power. However, a change in  $N$  would change the order  $n$  of spectrum in a given direction. Since  $\sin\theta_n < 1$  the resolving power of a grating cannot exceed  $\frac{w}{\lambda}$ .

We have small  $n = a + b \sin\theta_n$  by  $\lambda$  so we can also write the resolving power  $\lambda$  upon  $d\lambda = \frac{N(a+b)\sin\theta_n}{\lambda}$  which we can write  $= \frac{W \sin\theta_n}{\lambda}$  where  $W = N(a+b)$  which is the total width of the ruled surface of the grating. Thus a change in the number of lines  $N$  in a given width of ruled surface will not affect the resolving power.

However a change in  $N$  would change the order  $n$  of the spectrum in a given direction. Since  $\sin\theta_n < 1$  the resolving power of a grating cannot exceed  $\frac{W}{\lambda}$ .

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So if we want to resolve the  $D_1$  and  $D_2$  lines of wavelength  $5890 \text{ \AA}$  and  $5896 \text{ \AA}$  of sodium by plane diffraction grating, the number of rulings  $N$  should be at least  $\sim 1000$ .

So, if we have to resolve the D1 and D2 lines of wavelength 5890 angstrom and 5896 angstrom of sodium by plane diffraction grating the number of ruling  $N$  should be at least of the order of thousands. Now let us calculate the resolving power of a microscope so the primary function of a microscope is not to magnify an object but to reveal those finer details in the object which are invisible to and had an eye.

They extend to which finer details are revealed depends on depends not on the magnifying power but on the resolving power of the microscope the resolving power of a microscope is conveniently expressed in terms of the smallest linear separation between two points object which are just resolved. That is their diffraction patterns formed by the microscope objective are just distinguishable.

The smaller is this distance the greater is said to be resolving power. Let us derive this list separation between the two point object for two distinct cases. Number one the object or self luminous having no phase relation, number two objects are not non luminous but their illumination is coherent. So, let me summarize what I have discussed in this lectures so in this lecture I discussed that if refraction pattern of  $n$  slit.

And I discuss how to use grating spectrum to calculate the wavelength of light in the laboratory and then I discussed the Rayleigh criteria for resolution and derive the expression for resolving power of grating. In the left next lecture I will discuss about the resolving power of microscope and telescope and I will also solve some numerical problem related to single slit double slit and multiple slit Fraunhofer diffraction, thank you.