Engineering Physics 1
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Module-04
Lecture-01
Diffraction Part-01

In this lecture I will start diffraction of light; here first I will explain what is diffraction? And then I will describe single slit and double slit diffraction pattern. Diffraction is defined as the bending of waves around an obstacle are the spreading of waves when they pass through a narrow opening. To explain the diffraction of light wave let us consider a plane wave incident on a long narrow slit of width B.

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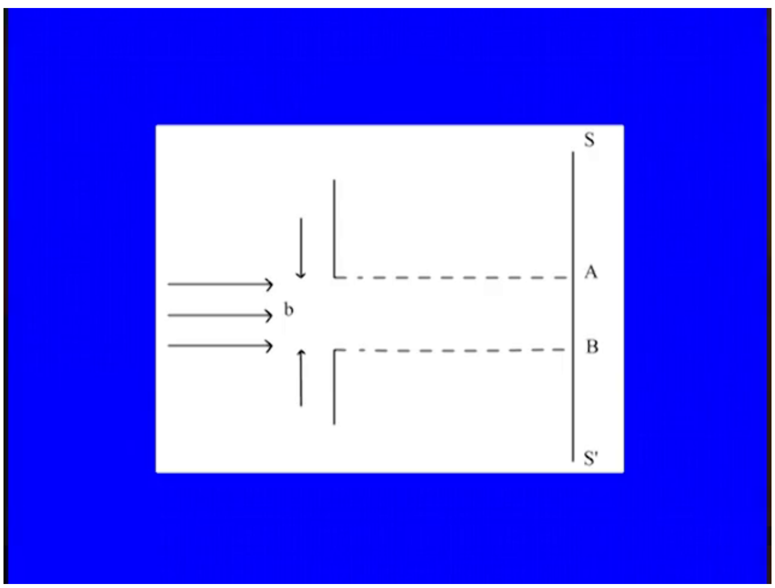

According to geometrical optics we expect the region AV of the screen S, S prime to be illuminated and the remaining position known as geometrical shadow to be absolutely dark. However if the observations are made carefully then we find that if the width of the slit is not very large compared to the wavelength of light. Then the light intensity in the region AB is not uniform and there is also some intensity inside the geometrical shadow further if the width of the slit is made a smaller larger amount of light reach in the geometrical shadow.

This is spreading out of wave when it passed through a narrow opening is known as the phenomenon of diffraction and the intensity distribution on the screen is known as diffraction pattern. Like interference this phenomenon can be explained satisfactorily by assuming a wave character for light. We will see that there spreading out decreases with decrease in the wavelength since the light wave lengths are very small the effects.

Due to diffraction are not easily observed on the other hand sound waves have wavelengths comparable to ordinary objects and our deflected by them. Due to this we hear sound even though we may not be in a direct line to their sources.

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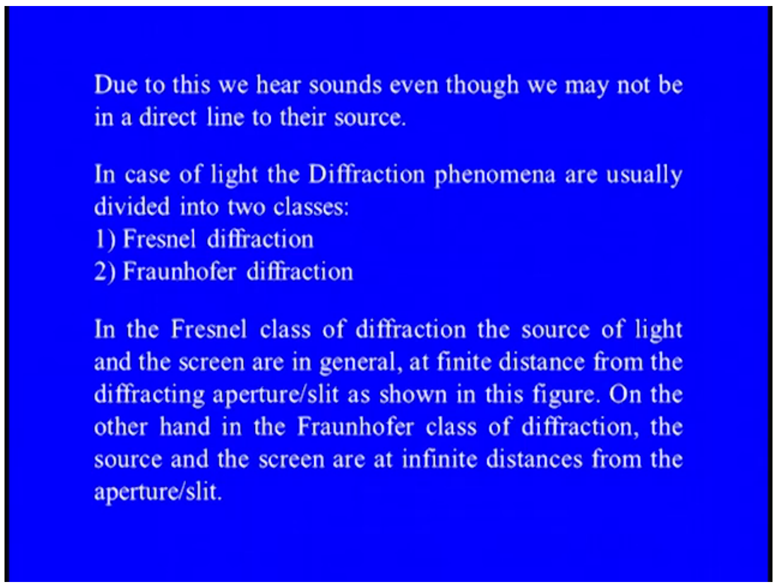

In case of light the diffraction phenomena are usually divided into two classes Fresnel diffraction and Fraunhofer diffraction. In the Fresnel class of diffraction the source of light and the screen or in general had finite distance from the deflecting aperture or slit as shown in this figure. On the other hand in the front of her class of diffraction the source and the screen are at infinite distances from the aperture or slit.

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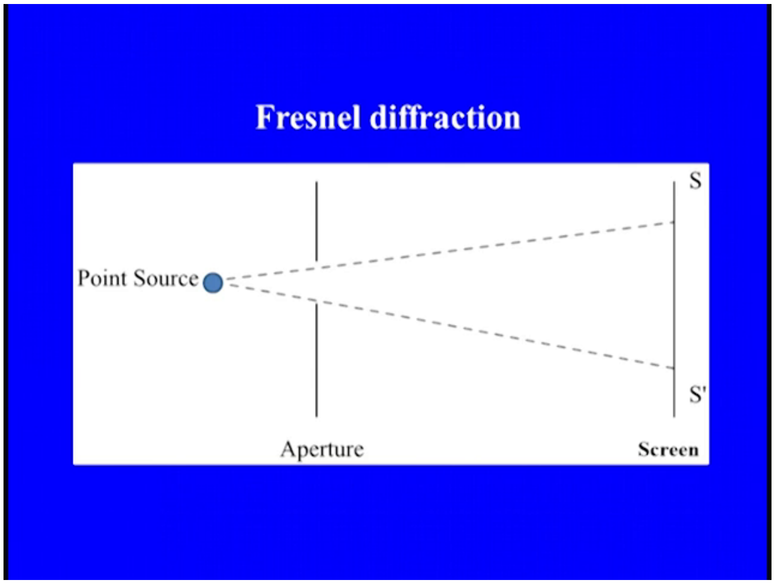

In the laboratory this is easily achieved by placing the source on the focal plane of a convex lens and placing the screen on the focal plane of another convex lens has shown in this figure.

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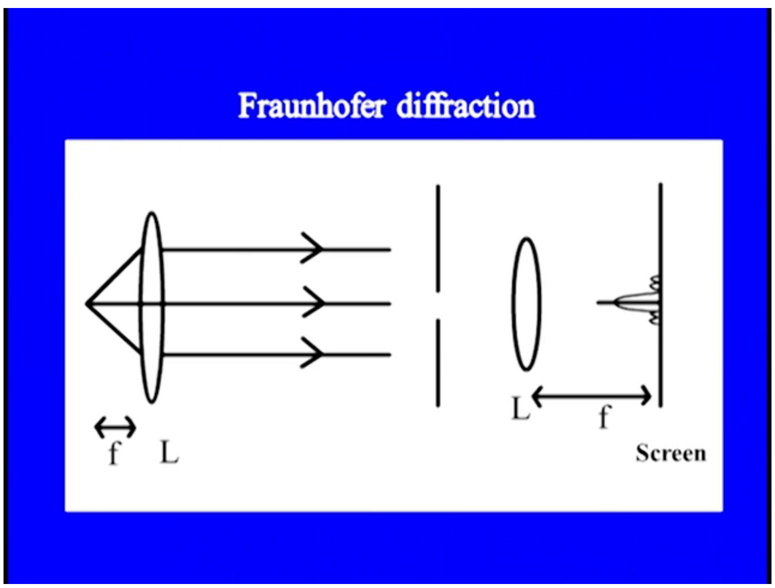

The two lenses effectively move the source and the screen to infinity because the first lens makes the light beam parallel and the second lens effectively makes the screen receive a parallel beam of light.

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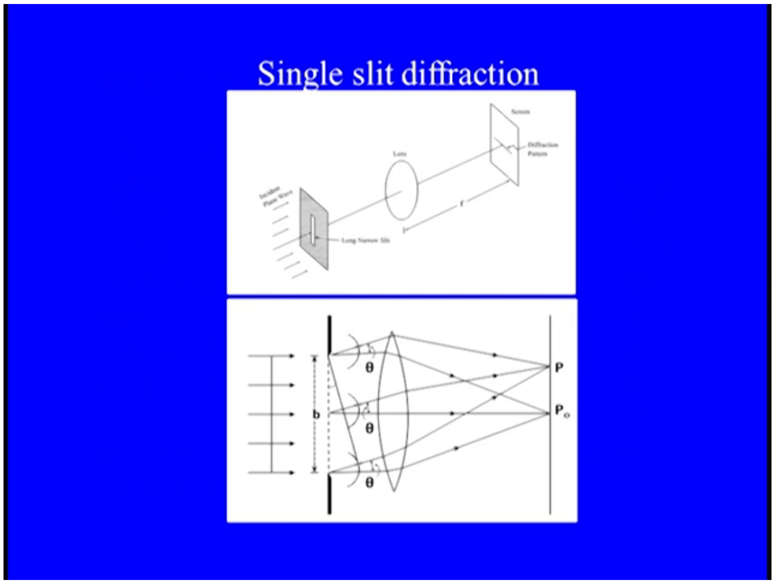

Now I will describe the single slit diffraction patterns due to an infinitely long slit of width b. Let us assume a plane monochromatic wave is incident normally on the slit. Now we want to calculate the intensity distribution on the focal plane of the lens l. When a plane wave front of monochromatic light of wavelength lambda is incident normally on the plane of the slit its size is limited at the instant it occupies the plane of the slit.

Now according to principle of Huygens Fresnel at this instant every point of the wave front in the plane of the slit is to be regarded as the origin of secondary spherical reference which is spread out to the right in all directions. The parts of each wavelet traveling normally to the slit are brought to focus at P naught by the lens while the parts traveling at an angle theta with the normal are brought to focus at P on the screen.

So, we can assume that the slit consists of a large number of equally spaced points sources and that each point on the slit is a source of Huygens secondary wavelets which interfere with the waves emanating from other points.

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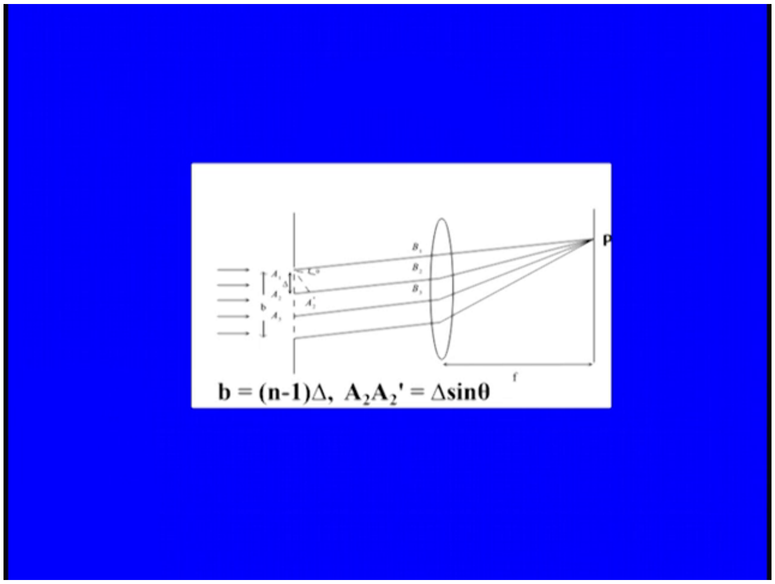

Let the point sources P at A1, A2, A3 so on and let the distance between two consecutive points be Delta as shown in this figure. Thus if there are n point sources on the slit then b will be = n - 1 Delta. We will now calculate the resultant field produced by these n sources at the point P which receive parallel rays making an angle theta with the normal to the slit. Since the slit actually consists of a continuous distribution of sources.

We will in the final expression let n goes to infinity and Delta goes to 0 such that Delta into n tends to b. Now at the point P the amplitude of the disturbance reaching from A1, A2, A3 so on will be very nearly the same because the point P is at a distance which is very large in comparison to b. However because of even a slightly different path lens to the point P the field produced by A1 will differ in phase from the field produced by A2.

For an incident plane wave the points A1, A2 etc are in phase and therefore the additional path traversed by the disturbance emanating from the point A2 will be A2, A2 prime where A2 prime is the foot of the perpendicular drawn from A1 on A2 B2.

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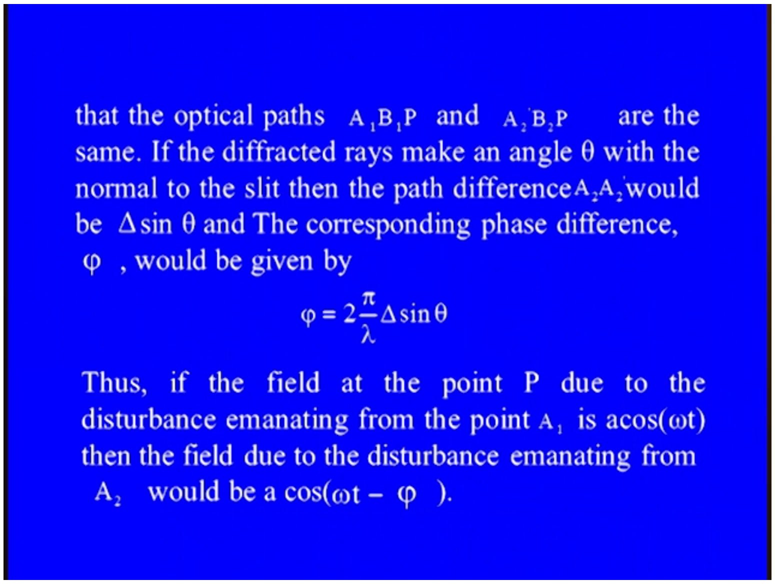

This follows from the fact that the optical paths A1 B1 P and A2 prime B2 P are the same. If the diffracted rage makes an angle theta with the normal to the slit then the path difference A2 A2 prime would be Delta sine theta. And the corresponding phase difference Phi would be 2Pi by lambda into Delta sine theta, thus if the field at the point P due to the disturbance emanating from the point A1 is a cos omega t.Then the field due to disturbance emanating from A2 would be a cos Omega t – Phi.

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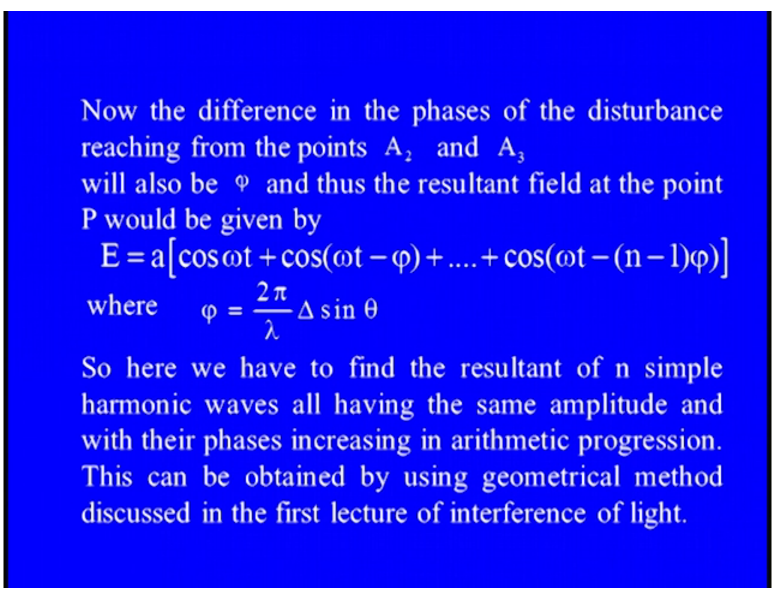

Now the difference in the phases of the disturbance reaching from the point A2and A3 will also be Phi and thus the resultant field at the point P would be given by a cos Omega t + a cos Omega t - Phi + a cos Omega t - 2Phi 2 + a cos Omega t - n - 1 Phi where Phi = 2pi by lambda into Delta sine theta. So, here we have to find the resultant of n simple harmonic waves all having the same amplitude and with their faces increasing in arithmetic progression.

This can be obtained by using geometrical method discussed in the first lecture of interference of light.

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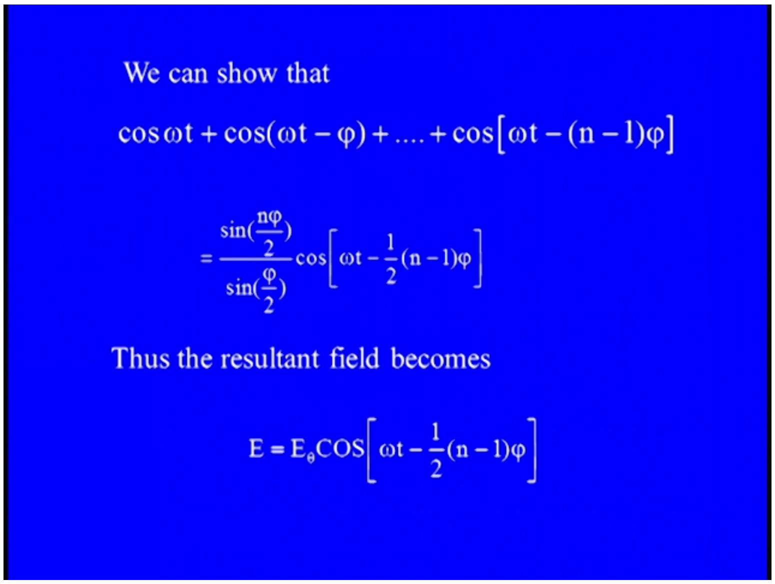

We can show that cos Omega t + cos Omega t - Phi up to + cos Omega t - n - Phi = sine n Phi by 2 divided by sine Phi by 2 into cos Omega t - 1 by 2 n - 1 Phi thus the resultant field at Point P becomes = e theta cos Omega t - 1 by 2 n - 1 Phi.

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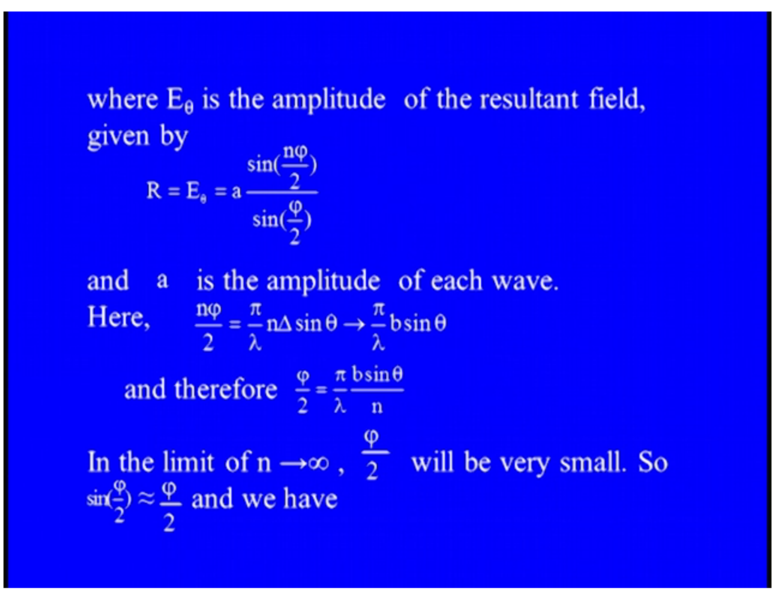

Where E theta = a into sine n Phi by 2 divided by sine Phi by 2 and a is the amplitude of each wave here we can show that n Phi by 2 will be = Phi by lambda into n into Delta sine theta which we can write = Pi by lambda into b sine theta and therefore Phi by 2 will be = Pi b sine theta divided by lambda into n. In the limit of n tending to infinity Phi by 2 will be very small. So, we can write sine Phi by 2 approximately = Phi by 2.

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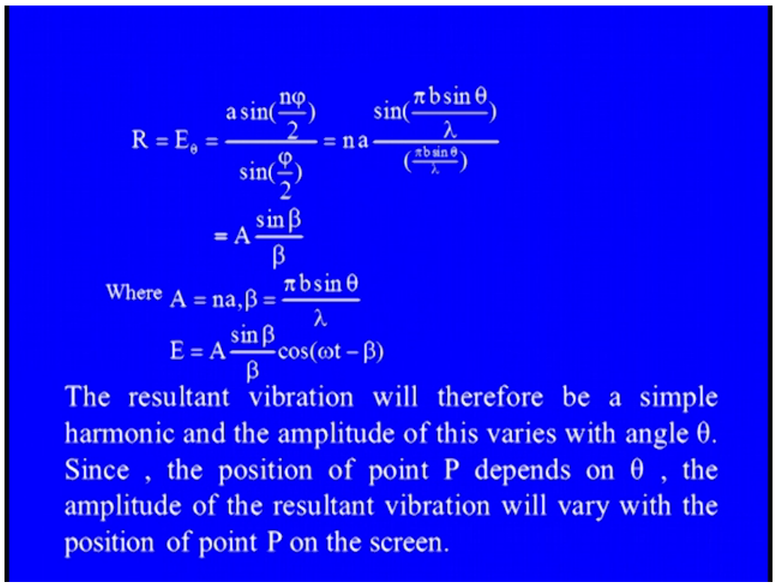

And we have resultant amplitude E theta = n times a sine Phi b sine theta by lambda divided by Pi b sine theta divided by lambda this we can write = capital A into sine beta by beta where capital A = n times a and beta = Pi b sine theta by lambda. So, the resultant field at the point P becomes = a sine beta by beta cos Omega t – beta. Thus the resultant vibration is a simple harmonic and the amplitude of this varies with angle theta.

Since the position of point P depends on theta the amplitude of the resultant vibration will vary with the position of point P on the screen.

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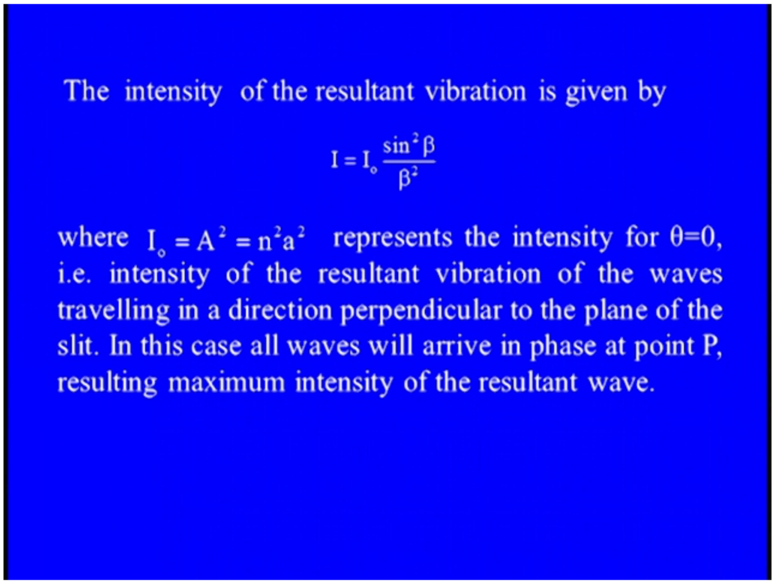

The intensity of the resultant vibration is given by I = I naught sine square beta by beta square where I naught =capital A square which we which is = n square into a square and this represent the intensity for theta = 0 that is intensity of the resultant vibration of the waves traveling in a direction perpendicular to the plane of the slit. In this case all waves will arrive in phase at Point P resulting maximum intensity of the resultant wave. Now I will discuss positions of maxima and minima on the screen.

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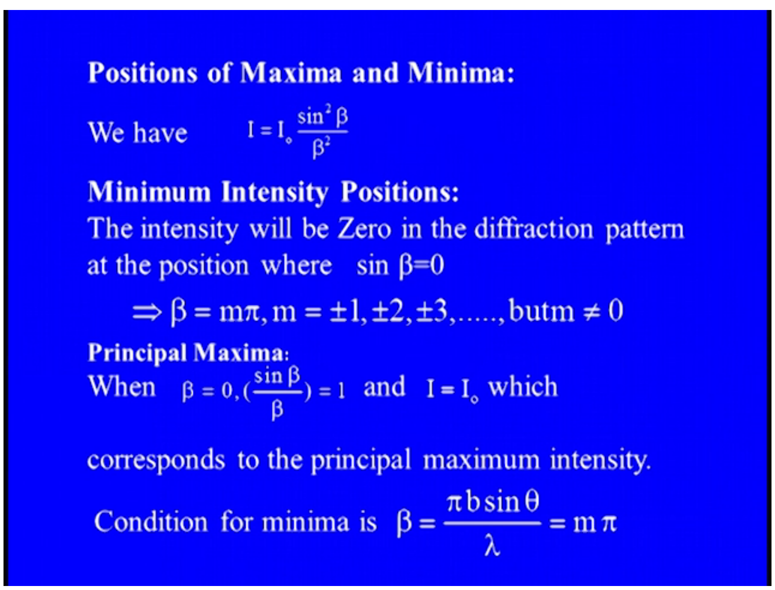

We have a intensity of the resultant wave I = I naught sine square beta by beta square. So, the intensity will be 0 in the diffraction pattern at the position where sine beta = 0 that is beta is = m Pi where m = + - 1 + - 2 +-3 so on. But m should not be = 0 and when beta = 0 we have sine beta by beta = 1 and I = I naught which correspond to principle maximum intensity. And the condition for so condition for minimize beta = Pi b sine theta where lambda = m Pi and the condition for principle Maxima is beta = 0.

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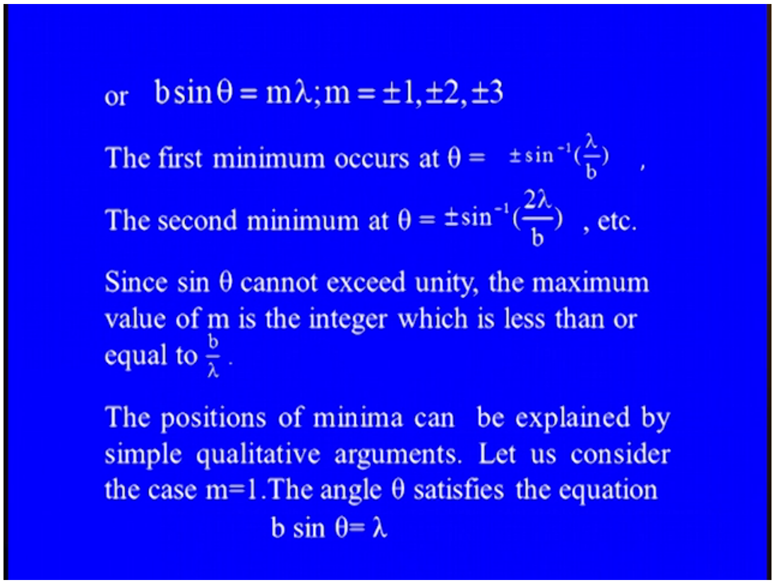

So, for minima we have b sine theta = m lambda where m = + - 1 + -2 + -3 so on. So, the first minimum occurs at theta = + -sine inverse lambda by b and the second minimum will occur at theta = + -sine inverse 2lambda by b etc. Since sine theta cannot exceed unity the maximum value of m is the integer which is less than or = b by lambda. The position of minima can be explained by simple qualitative argument. Let us consider the case m = 1 the angle theta satisfy the equation b sine theta = lambda.

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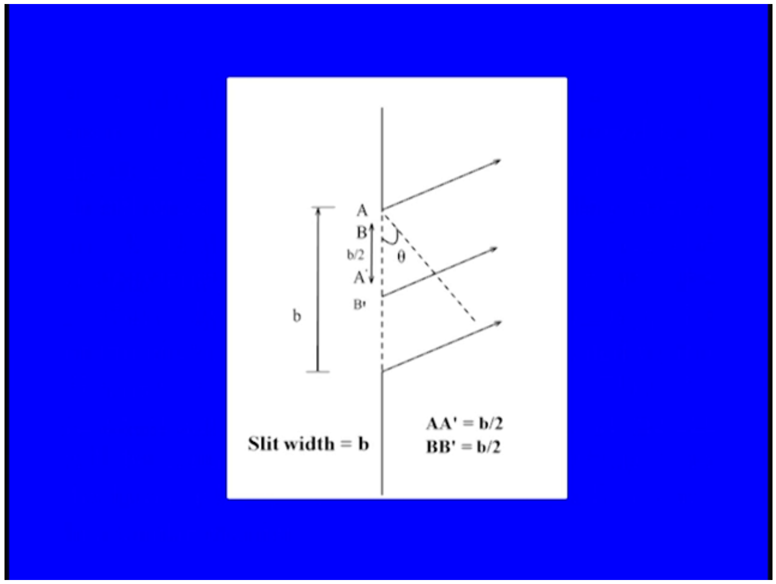

We now we divide the slit into two halves as shown in this figure consider two points A and A prime separated by a distance B by 2. Clearly the power difference between the disturbance emanating from A and A prime is B by 2 sine theta which in this case is lambda by 2. The corresponding phase difference will be Pi and the resultant disturbance will be 0. Similarly the disturbance from the point B will be cancelled by the disturbance reaching from the point B prime.

Thus the resultant disturbance due to the upper half of the slit will be canceled by the disturbance reaching from the lower half and the resultant intensity at Point P will be 0. In a similar manner when b sine theta = 2 lambda, we divide the slit into four parts. The first and the second quarter cancelling each other and the third and fourth quarter cancelling each other, similarly when m is = 3 facility divided into 6 equal parts and so on. Now let us find out the position of secondary maxima.

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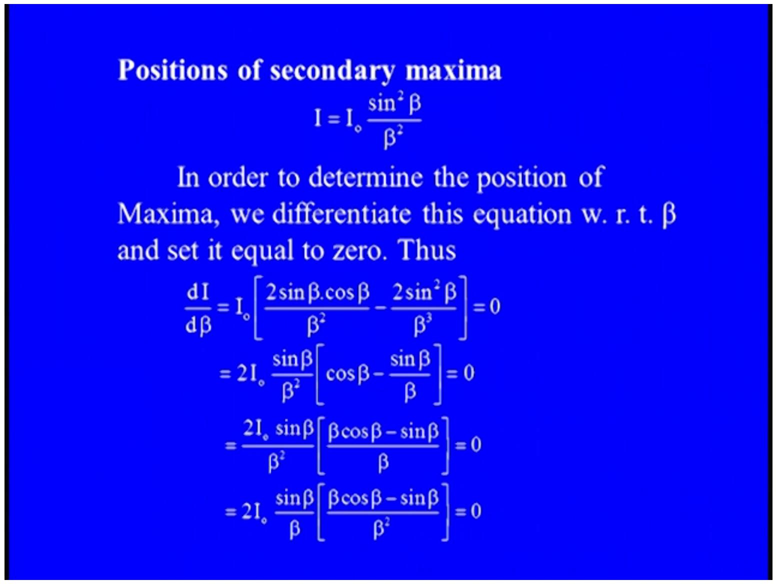

So, we have our intensity I = I naught sine square beta by beta square. So, in order to determine the position of maxima we differentiate this equation with respect to beta and set it =0. thus dI by d beta = I naught 2 sine beta + 2 cos beta divided by beta square - 2 sine square beta by beta square and this we have to equate to 0 and after simplification we will get di by d beta = 2 I naught sine beta by beta into beta cos beta - sine beta upon beta square =0.

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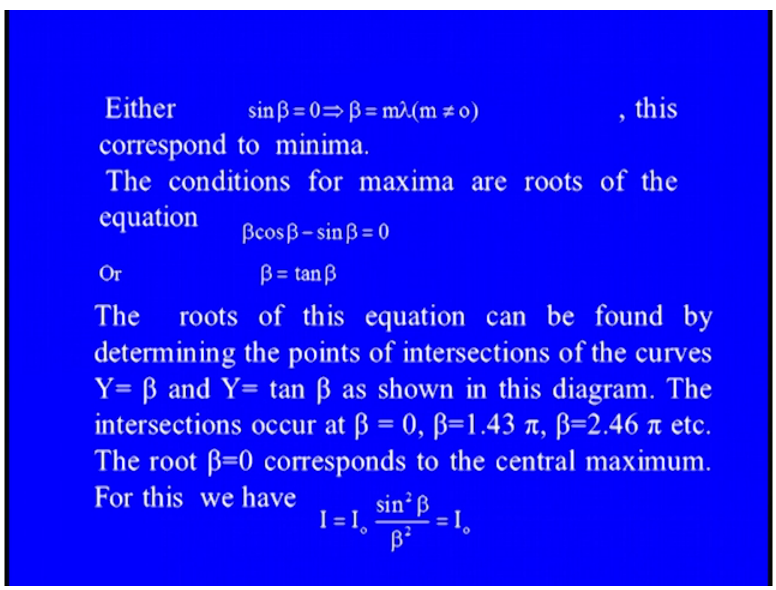

So, here either sine beta =0 this will gives beta = m lambda where m is not is = 0 this corresponds to minimum intensity. The condition for maxima are roots of the equation beta cos beta - sine beta = 0 or beta = tan beta.

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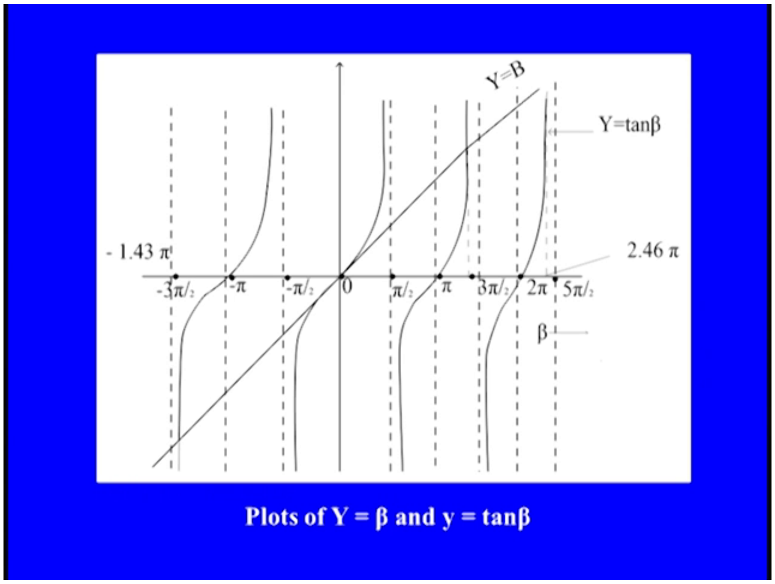

Plots of Y = β and y = tanβ

The roots of this equation can be found by determining the position of intersections of the curves Y = beta and y = tan beta as shown in this diagram. The intersections occur and beta = 0 and beta = 1.43 Pi and beta = 2.46 by etc. The root beta = 0 correspond to the central maxima and for this we have I = I naught. The other roots like beta = 1.43 Pi beta = 2.46 Pi etc give the position of first Maxima, second Maxima so on.

So intensity of principal Maxima as we have seen is given by I = I naught. So, for the first Maxima we have beta = 1.43 Pi, so putting this value of beta in the intensity equation.

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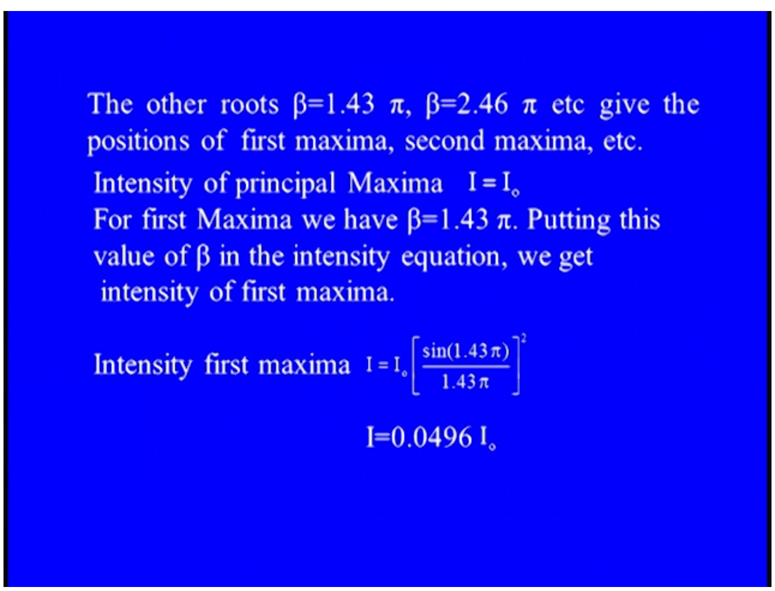

We get the intensity of first Maxima = I naught into sine 1.43 Pi divided by 1.43 Pi whole square. So, this comes out to be 0.0496 I naught.

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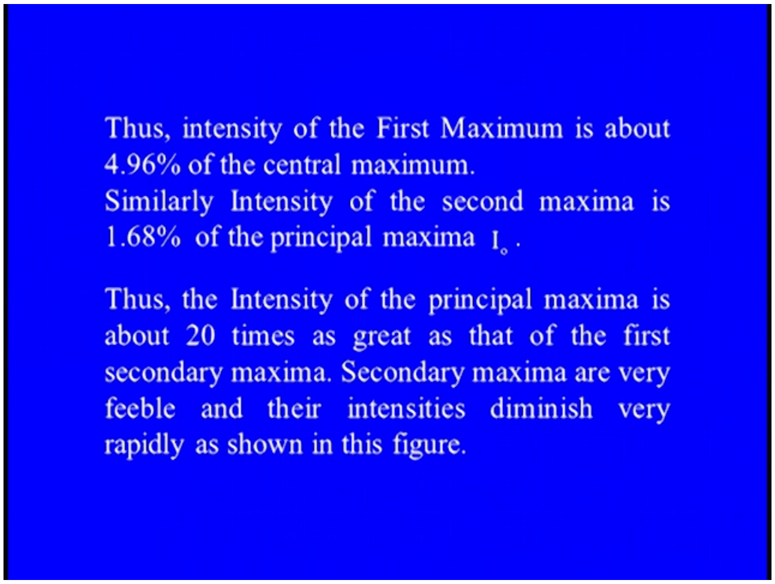

Thus the intensity of the first Maxima is about 4.96% of the central maximum.

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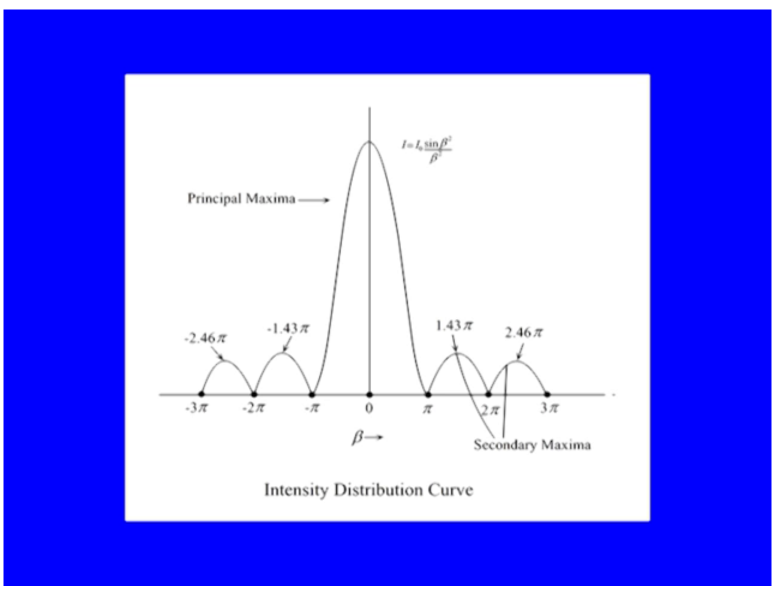

Similarly intensity of second Maxima is 1.68% of the principle Maxima I naught. Thus the intensity of the principal Maxima is about 20 times as great as that of the first secondary maximum. Secondary Maxima are very feeble and their intensities given is very rapidly as shown in this figure.

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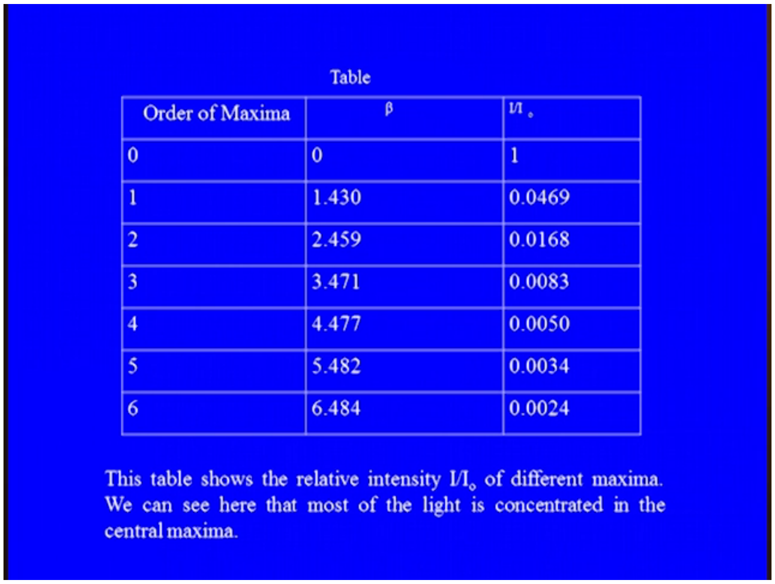

Table

Order of Maxima	β	I/I_o
0	0	1
1	1.430	0.0469
2	2.459	0.0168
3	3.471	0.0083
4	4.477	0.0050
5	5.482	0.0034
6	6.484	0.0024

This table shows the relative intensity I/I_o of different maxima. We can see here that most of the light is concentrated in the central maxima.

This table describes the intensity of various secondary maxima and from here we can again see that most of the light is concentrated only in the principal Maxima. Now let us find the width of the principal Maxima.

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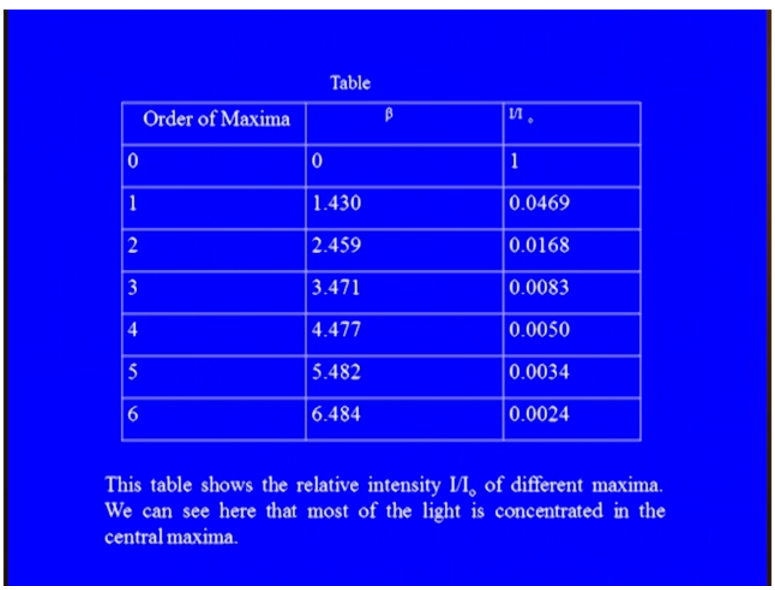

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This table shows the relative intensity I/I_o of different maxima. We can see here that most of the light is concentrated in the central maxima.

So, we have beta = Pi b sine theta by lambda and for first minima betas should be = + - Pi or Pi b sine theta by lambda = +- Pi from here we have sine theta = +- lambda by b where b is the width of the slit.

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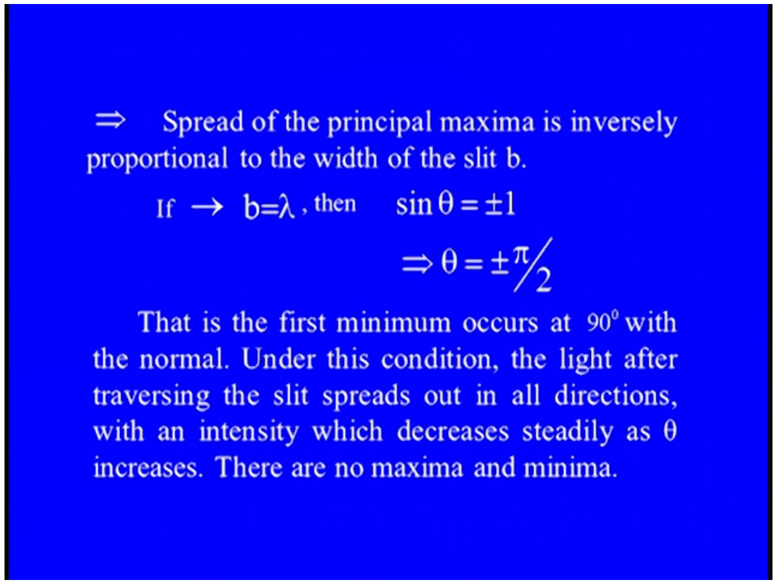

So, from this equation we see that the spread of the principal Maxima is inversely proportional to the width of the slit b. If we take b = lambda then sine theta will be = +- 1 R theta = + - Pi by 2 that is the first minimum occur at 90 degree with the normal. Under this condition the slit the light after traversing the slit is spread out in all directions with an intensity which decreases steadily as theta increases there are no maxima and minima.

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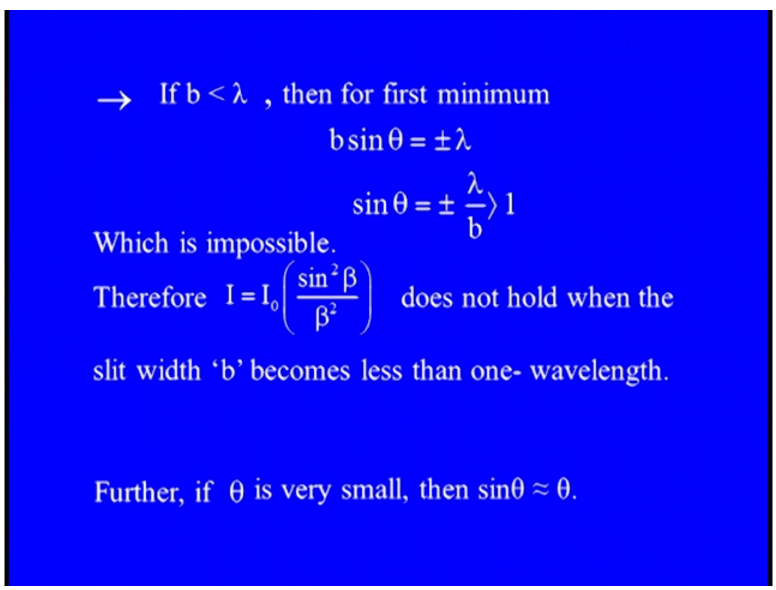

If we take beta less than lambda then first then for first minimum b sine theta should be = + - lambda and sine theta = + - lambda by b which will be greater than 1 if b is less than lambda which is impossible. Therefore I = I naught sine square beta by beta square does not hold when the slit width b becomes less than 1 wavelength. Further if theta is very small we can write sine theta approximately = theta.

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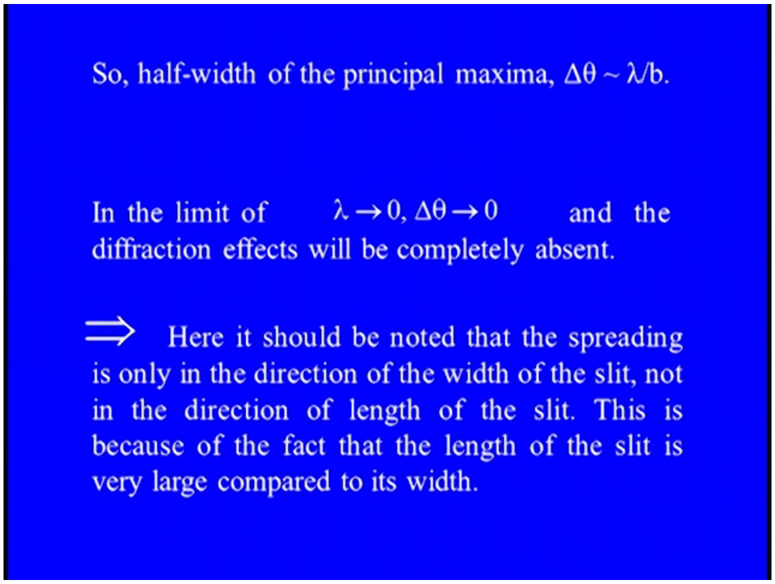

So, half width of the principal maximum Delta theta will be approximately = lambda by b in the limit of lambda tending to 0 Delta theta will be tending to 0 and the deflection effect will be completely absent. Here it should be noted that the spreading is only in the direction of the width of the slit not in the direction of length of the silt. This is because of the fact that length of the slit is very large compared to its width.

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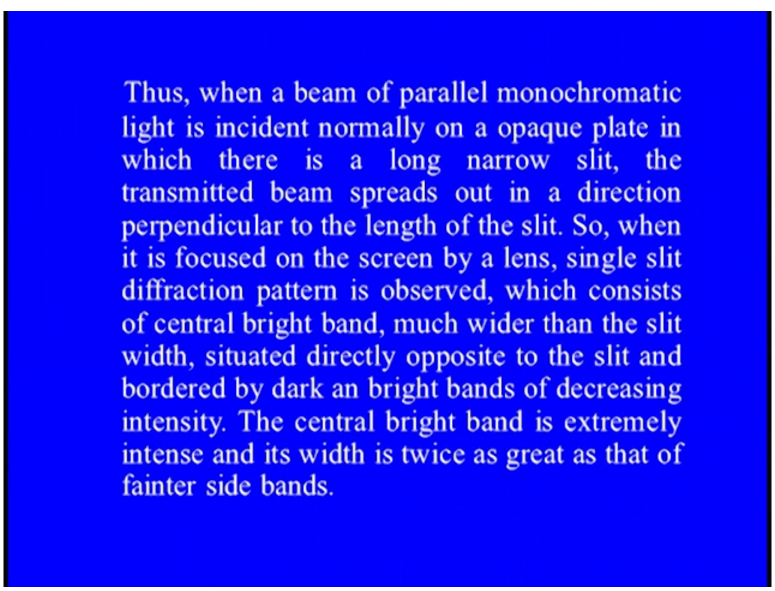

Thus when a beam of parallel monochromatic light is incident normally on a opaque plate in which there is a long narrow slit the transmitted beam is spread out in a direction perpendicular to the length of the slit. So, when it is focused on this screen by a lens single slit diffraction pattern is observed which consists of central bright band much wider than the slit width situated directly opposite to the slit and bordered by dark and bright bands of decreasing intensity.

The central bright band is extremely intense and it width is approximately twice as great as that of inter site band. So, this finishes the description of single slit diffraction patterns. And now I will discuss the diffraction at a circular aperture.

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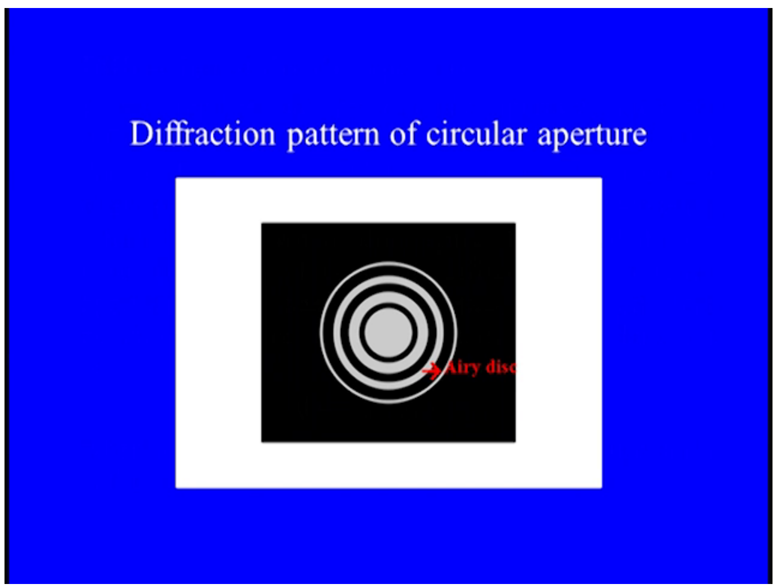

So, if we replace single slit as we have discussed in previous discussion by a circular aperture then the diffraction pattern is consisted of a bright disc called Harris ring, Harris disc surrounded by a number of concentric alternate dark and bright rings of rapidly decreasing intensity as shown in this figure. I will not discuss here the detailed theory of diffraction at circular aperture.

But on the basis of theoretical calculation it assume that the angular radius of the first dark ring is 1.22 lambda by d where d is the diameter of the circular aperture and lambda is the wavelength of the monochromatic light. This may also be taken as the angular radius of the bright central disc about 84% of the transmitted light energy is concentrated in the central bright disc. This result we will use later to calculate the resolving power of telescope and microscopes.

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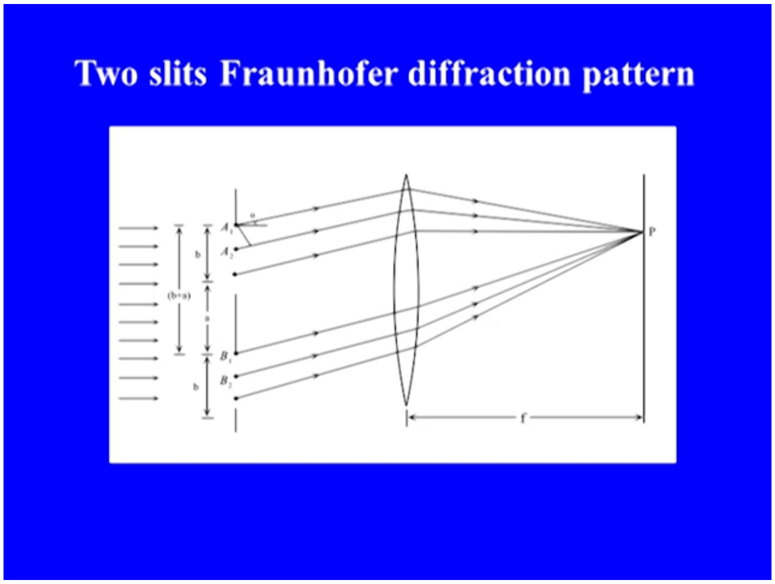

So, now let us discuss the diffraction pattern of double slit so here let us consider two parallel slits each of width b and separated by a opaque region of width a has shown in this figure. Now we will study in the diffraction pattern produced by two slits when a beam of monochromatic light of wavelength lambda is incident normally on the plane of the slits. Like single slit let us consider that both slit consists of a large number of equally spaced point sources.

And that each point on the slit is his source of Huygens secondary wavelets. Let the point sources be at A1, A2, A3 etc in the first slit and V1, V2, V3 etcetera in the second slit. We assume that the distance between the two consecutive points in either of the slit is d = a + b, if the deflected rays make an angle theta when the normal to the plane of the slit then the path difference between the disturbance reaching the point P on the screen from two consecutive points in a slit it will be d sine theta.

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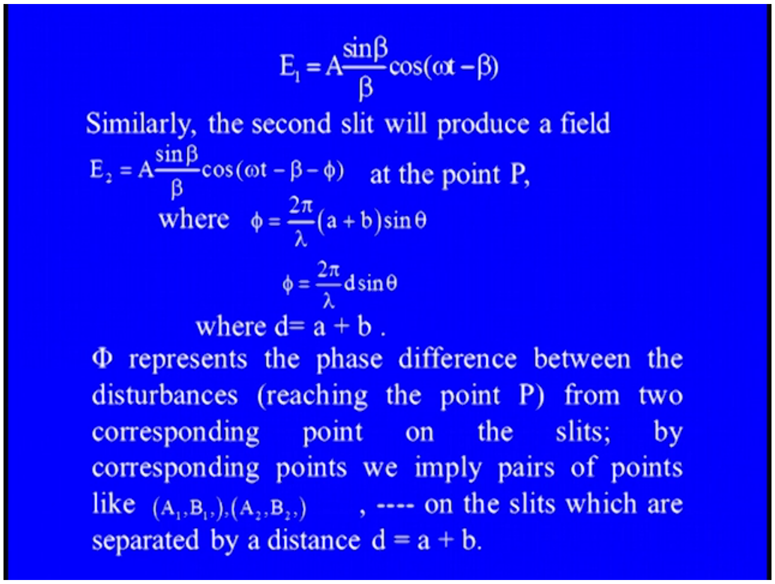

The field produced by the first slit at the point P will therefore be given by E1 = a sine beta by beta cos Omega t – beta, similarly the second slit will produce a field E2 = a sine beta by beta cos Omega t - beta - Phi at the point P where Phi = 2 Pi by lambda into a + b sine theta or Phi = 2 Pi by lambda into d sine theta where d = a + b. Phi represent the phase difference between the disturbance reaching the point P from two corresponding point on the slit.

By corresponding points we imply pair of points like A1, B1, A2, B2 so on. On the slit which are separated by a distance d = a + b.

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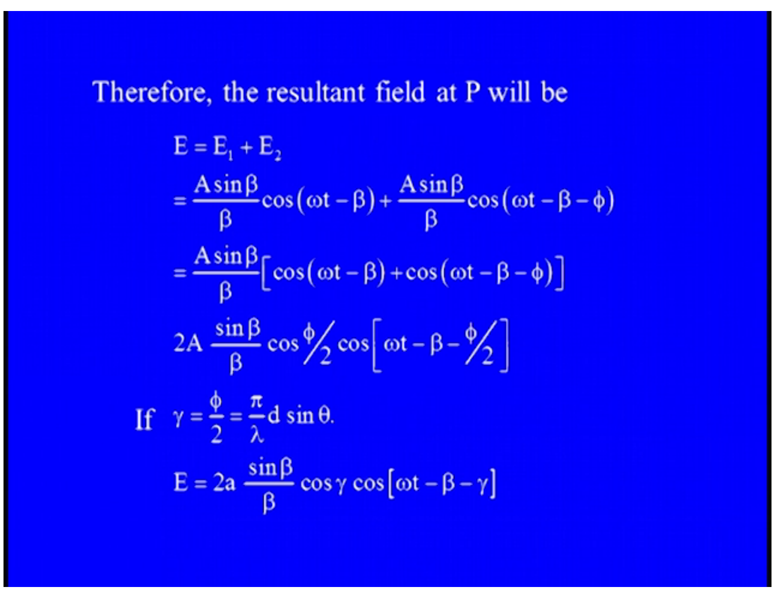

Therefore the resultant field at P will be = E1 + E2 and so now substituting the value of E1 and E2 we get the resultant field at the point P, E = 2a sine beta by beta cos Phi by 2 into cos Omega t - beta - Pi by 2 if we define gamma = Pi by 2 = Pi by lambda into d sine theta then the resultant field E = 2a sine beta by beta cos gamma cos Omega t - beta – gamma.

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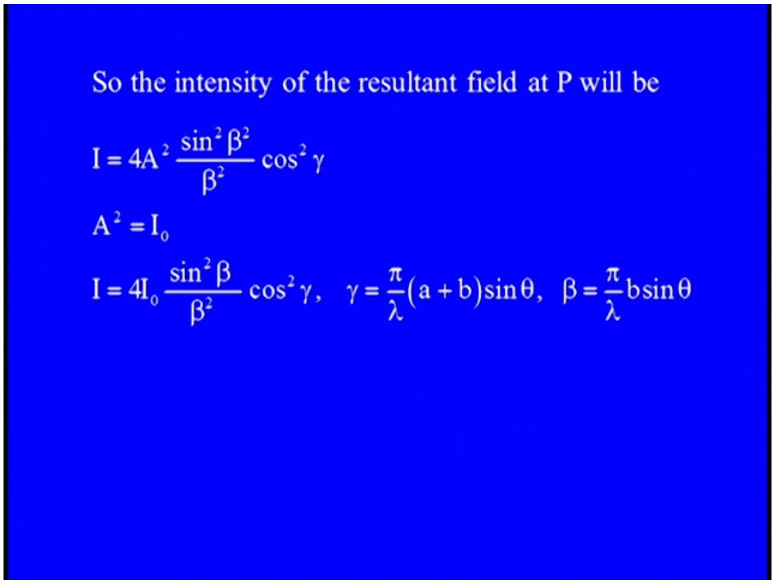

So, the intensity of the resultant field at P = 4A square sine square beta square divided by beta square into cos square gamma. Here A square = I naught then I will be = 4I naught sine square beta by beta square into cos square gamma where gamma = Pi by lambda into a + b sine theta and beta = Pi by lambda b sine theta.

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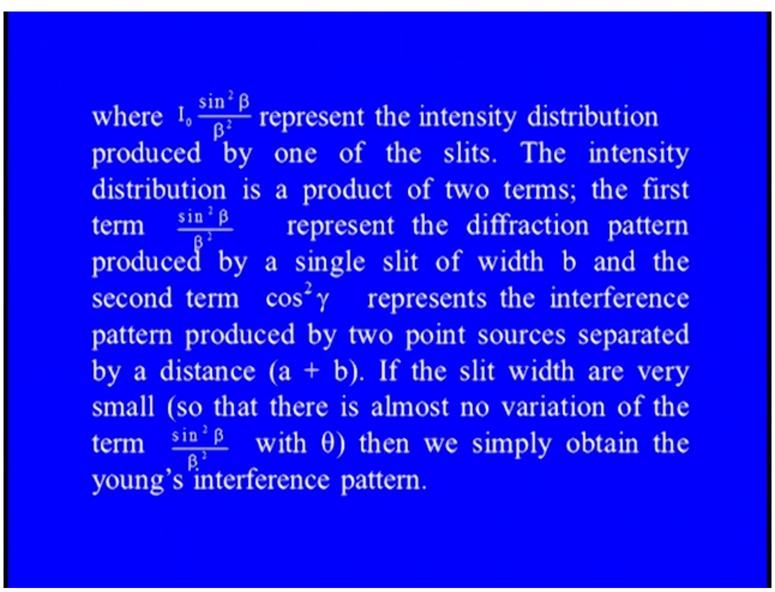

So, this equation described the intensity of double slit diffraction pattern on the screen in this expression I naught sine square beta by beta square represent the intensity distribution produced by one of the slit. The intensity distribution is a product of two terms the first term sine square beta by beta square represent the diffraction pattern produced by a single slit of width b and the second term cos square gamma represent the interference pattern produced by two point sources separated by a distance a + b.

If the slit width are very small, so that there is almost no variation of terms sine square by beta square with theta then we simply obtain the Young's interference pattern.

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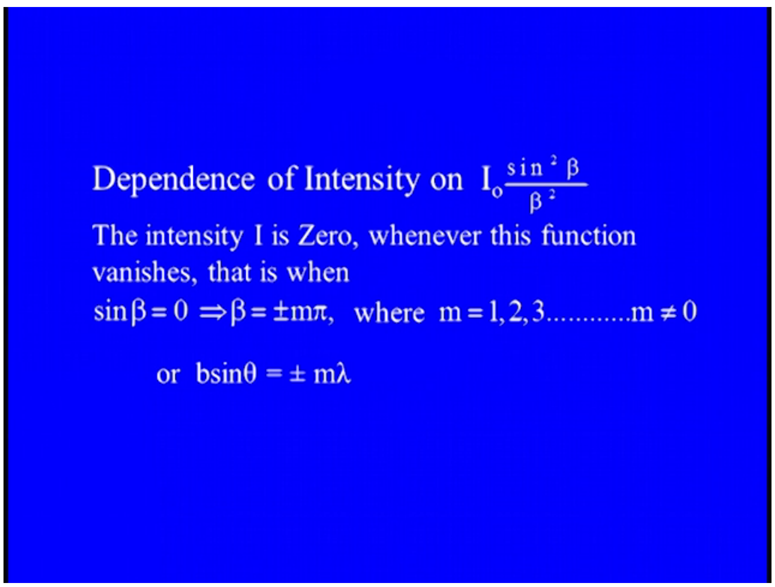

Now let us discuss the dependence of intensity on I naught sine square beta by beta square term that is single slit intensity pattern. The intensity I is 0 whenever this function vanishes that is when sine beta = 0 that is beta = +- m Pi where m = 1, 2, 3 but m is not is = 0 or b sine theta = + - m lambda. So, whenever this condition is satisfied the intensity of intensity in the diffraction pattern or double slit will be 0.

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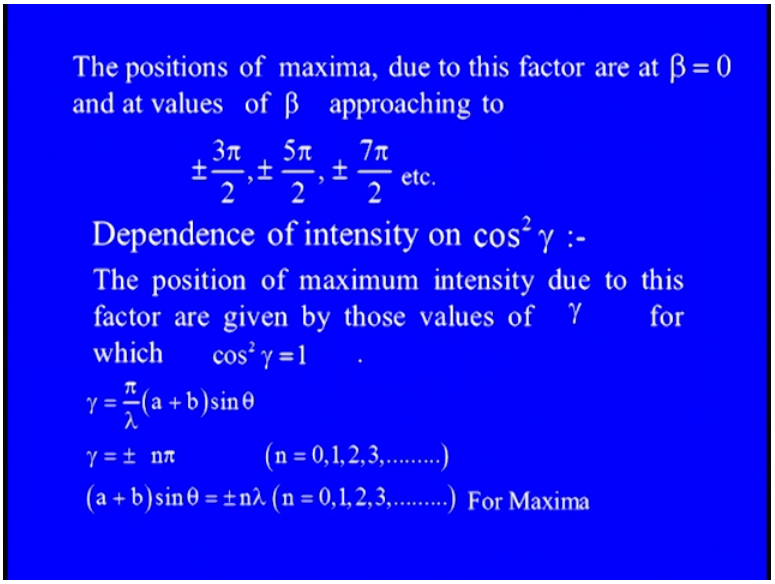

Now the position of Maxima due to this factor are at beta = 0 and at value of beta approaching to + - 3 Pi by 2 + - 5PI by 2 + - 7 Pi by 2 etc. Now let us discuss the dependence of intensity on cos square gamma term. The position of maximum intensity due to this factor are given by those value of gamma for which cos square gamma = 1 or where gamma = Pi by lambda into a + b sine theta. So, cos square gamma will be 1 when gamma = + - n Pi where n = 0, 1, 2, 3 so on or a + b sine theta = +- and lambda.

Again n = 0, 1, 2, 3 this is the condition for maximum intensity in double slit diffraction pattern.

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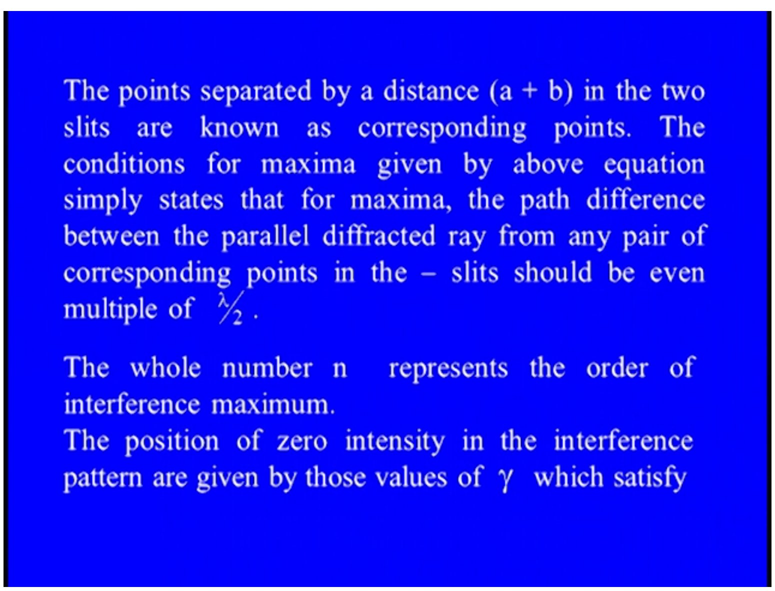

So, the point separated by distance a + b in the 2 slits are known as corresponding points. So, the condition for Maxima given by the above equation simply states that for Maxima the power difference between the parallel refracted ray from any pair of corresponding point in the slit should be even multiple of lambda by 2. The whole number n represents the order of interference maximum. The positions of 0 intensity in the interference pattern are given by those values of gamma.

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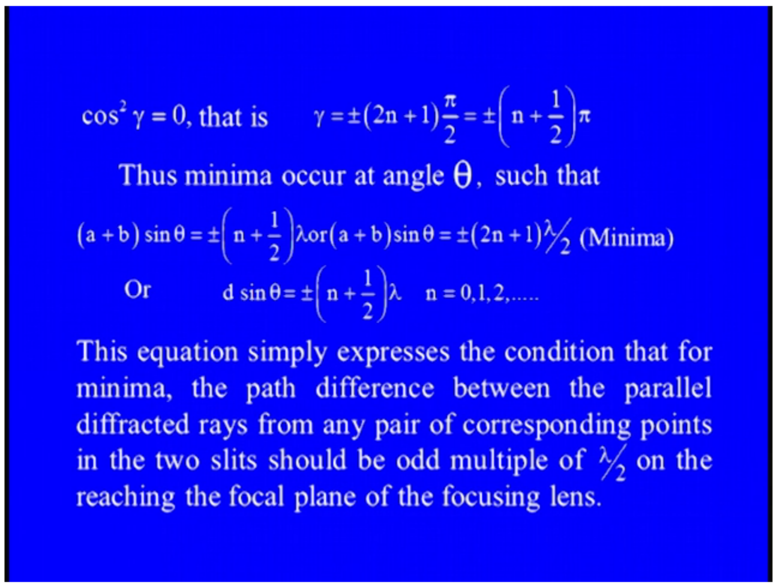

Which satisfy cos square gamma = 0 that is in gamma = + - n + half Pi, thus the minimum occur at angle theta such that a + b sine theta = + - n + half lambda or d sine theta = + - n + half lambda where n = 0, 1, 2, 3. This equation simply expresses the condition that for minima the power difference between the parallel deflected rays from any pair of corresponding point in two slits should be all odd multiple of lambda by 2 on reaching the focal plane of the focusing lens.

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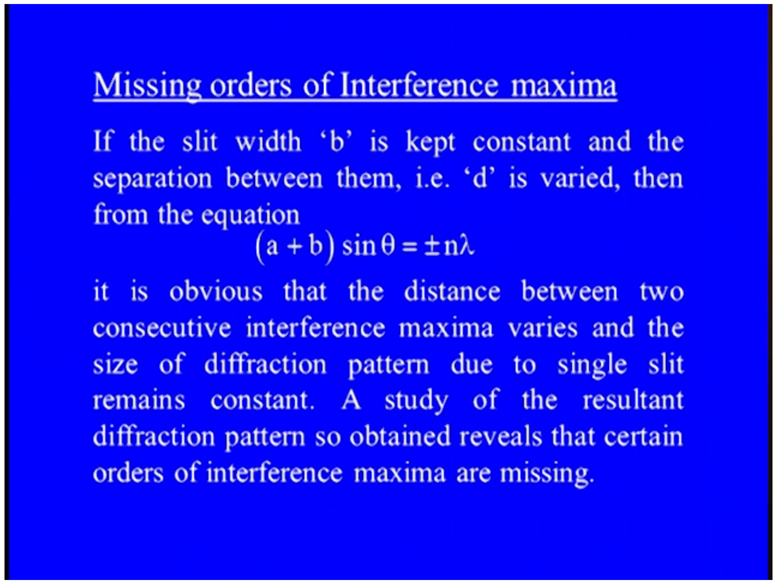

Now let us discuss the missing order of interference Maxima in the double slit diffraction pattern if the slit width b is kept constant and the separation between them that is d is varied then from the equation a + b sine theta = + - 1 n lambda. It is obvious that the distance between two consecutive interference Maxima varies and the size of diffraction pattern due to single slit remains constant.

A study of the resultant diffraction pattern so obtained revealed that certain orders of interference Maxima are missing.

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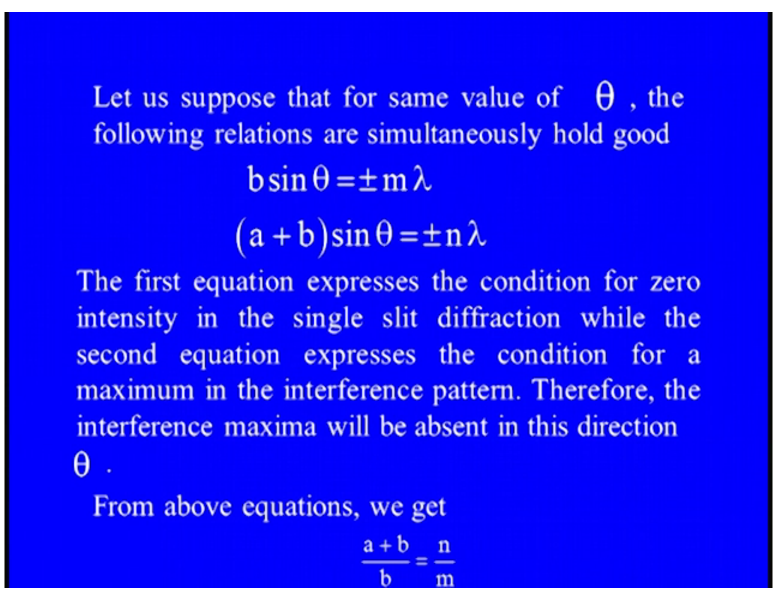

Let us suppose that for the same value of theta the following relations are simultaneously hold good, b sine theta = + - m lambda and a + b sine theta = + - n lambda. The first equation expressed says the condition for 0 intensity in the single slit diffraction while the second equation expresses the condition for a maximum in the interference pattern therefore the interference Maxima will be absent in this direction theta. From the above equations we can write a + b divided by b = n by m.

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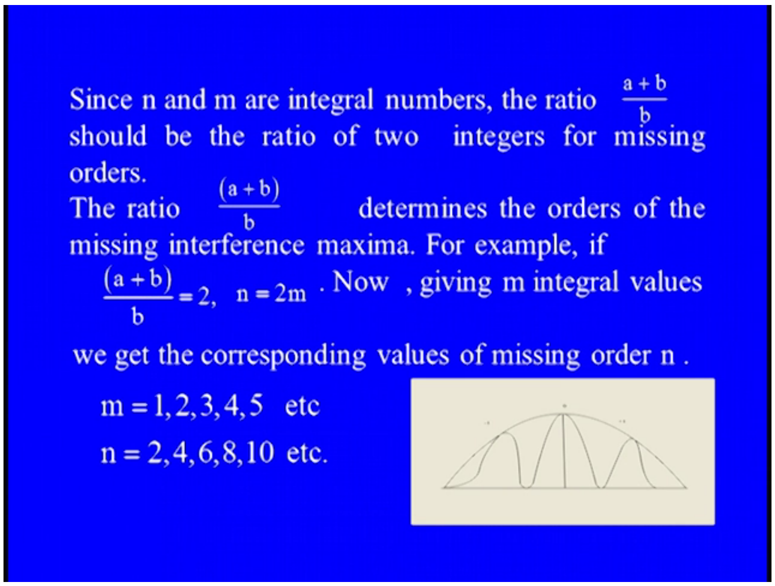

Since n and m are integral numbers that is ratio a + b by b should be the ratio of two integers for missing order. The ratio a + b by b determines the order of the missing interference Maxima. For example if a + b by b = 2 then n will be = 2 times m. Now giving the m integral values we get the corresponding values of missing order n. So, if we have m = 1, 2, 3, 4, 5 etc then n = 2, 4, 6, 8, 10 etcetera.

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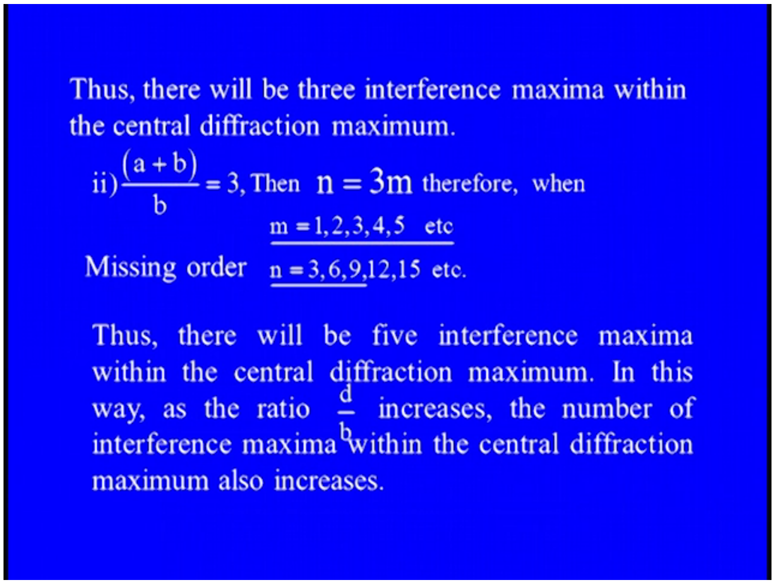

Thus there will be 3 interference Maxima within the central diffraction Maxima. Similarly if we have a + b by b = 3 then n will be = 3 times m. Therefore when m = 1, 2, 3, 4, 5 etc then n will be = 3, 6, 9, 12, 15 etc and this value of n correspond to missing order so, thus there will be 5 interference Maxima within the central diffraction maximum.

In this way either ratio d by b increases the number of interference maxima within the central diffraction maximum also increases. The physical explanation of missing order is as follows.

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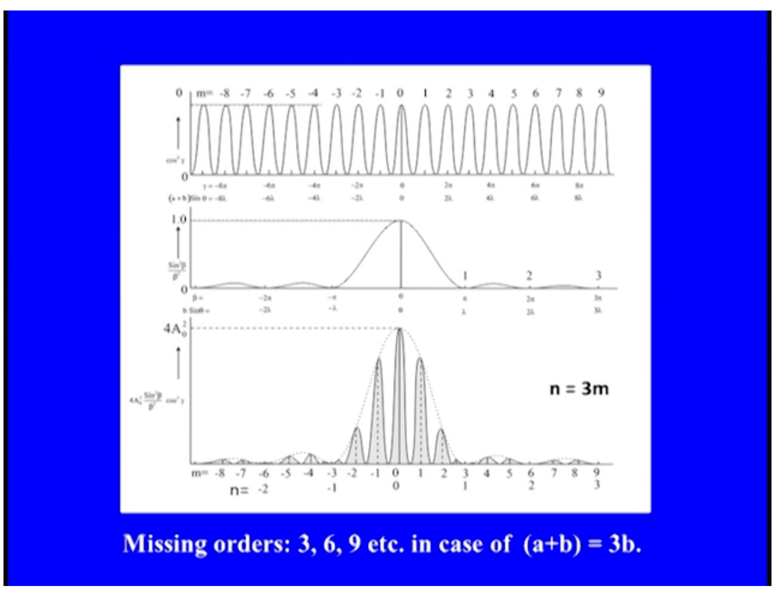

Consider for an example the missing order n = 2 the point on the screen where we expect this order is just 2 lambda further from the center of one slit than from the center of the other. But since slit width is same as opaque region between slit this point is one wavelet further from one age then the other age of the same slit. Consequently the secondary wavelets from this slit produce zero intensity at this point. The same is true for the second slit.

Thus it is obvious that the resultant intensity at this point due to the resultant of each slit will be 0 that is the second order maximum will be absent. This figure shows the intensity curve for double slit diffraction for the case a + b = 3 times b and explain the missing order 3, 6, 9, etc in the diffraction pattern. So, let me summarize what we have discussed till now in this lecture. So, first I explained what is diffraction and then I explained the diffraction pattern of single slit.

So, there I we saw that in the single slit diffraction pattern we have a central maxima where almost all light incident light is confined and this central maxima is surrounded by many secondary Maxima of decreasing intensity. Then I describe the double slit diffraction patterns so in the double slit diffraction pattern is the determined by two terms. First term is the single slit diffraction pattern and then there is a interference term coming in the final expression which describe the intensity pattern of double slit diffraction patterns, thank you.