Engineering Physics 1
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Module-03
Lecture-06
Coherence and Application of Interference

Coherence and applications of interference by M K Srivastav, Department of Physics, Indian Institute of Technology, Roorkee, Uttarkhand. In this lecture, the last of the present series, we shall first discuss concepts like Temporal coherence, coherence time, coherence length, spatial coherence etcetera and then consider some applications.

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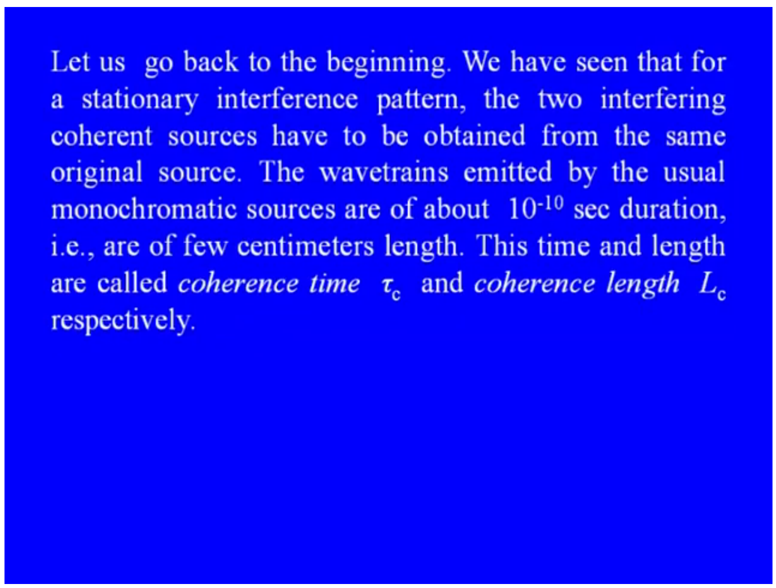

Let us go back to the beginning of the series. You see we have seen that for a stationary interference pattern, the two interfering coherent sources have to be obtained from the same original source. Now, the wavelengths emitted by the usual monochromatic sources are of about 10 to the power -10, second duration. In terms of the length, they are a few centimeters. Now this time and length they are called the coherence time Tau c and coherence length Lc respectively.

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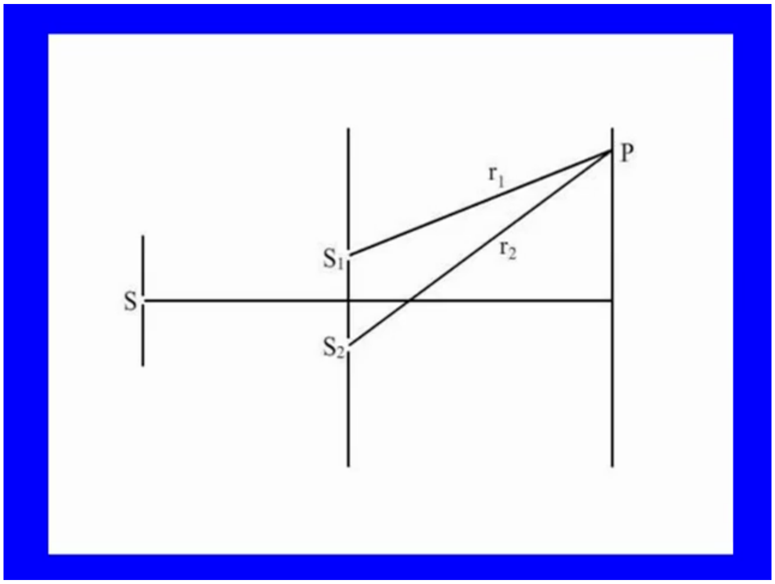

You see, let us consider Young's two hole experiment. Just to see what these things really mean as in the basic source S1 and S2 are the two holes forming a pair of coherent sources. Then the light is reaching the screen, where the point P is there. R1 and R2 are the distances S1 P and S2 P.

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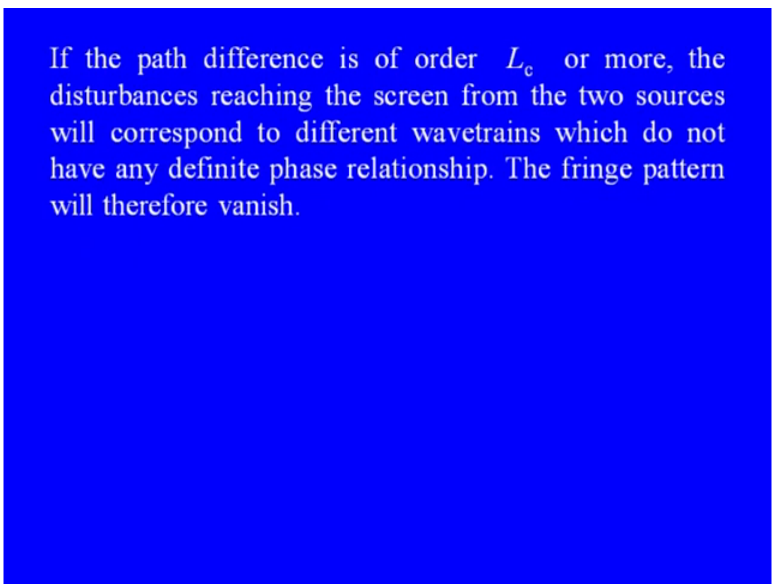

Now, if the path difference R1 - R2 this difference between these sources and the point P on the screen is of order Lc, the coherence length or more. The disturbances reaching the screen from the two sources will then correspond to, different wave trends. We do not have any definite phase relationship we know that. And the result will be the fringe pattern will vanish.

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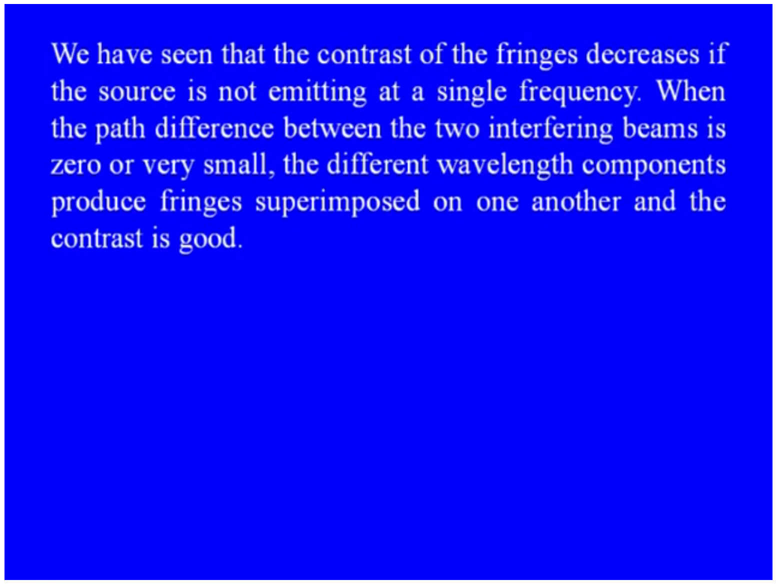

Now, we have seen that the contrast of the fringes decreases if the sources is not emitting at a single frequency. I mean, we are trying to look at this problem from a different point of view. Remember, just now we have seen that even the path difference is more than the coherence length. The phase relationship gets disturbed and the pattern vanishes. Now, here we are talking about the fact that if this source is not emitting at a single frequency then, again the contrast of the fringes decreases.

And we again let us repeat this. This is an important point. When the path difference between the two interfering beams is 0 or it is very small, the different wavelength components produce fringes, superimposed on one another and the contrast is good. So, there are two approaches to look at this problem, under what situation the fringe pattern gets, I mean, its clarity decreases and it vanishes.

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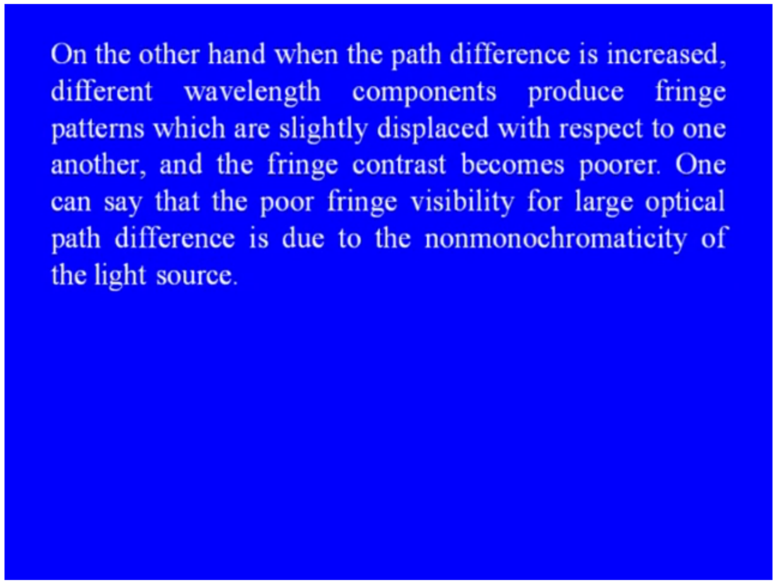

You see, on the other hand, when the path difference sizing is increased, the different wavelength components produce fringe patterns, which are slightly displaced with respect to one another. We have seen that earlier. And the fringe contrast becomes poorer. One can say that the two are invisibility for large optical path difference is due to non monochromaticity of the light source.

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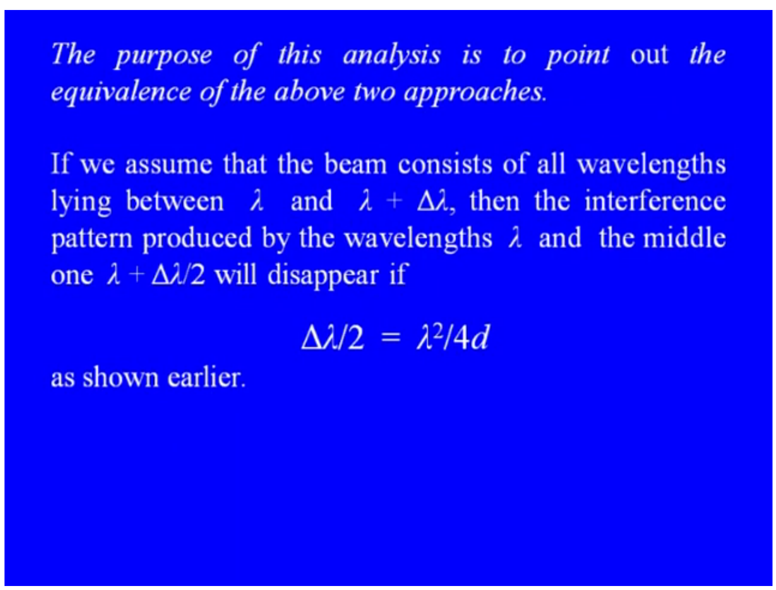

The purpose of this analysis is to point out the equivalence of the above two approaches: The vanishing of the pattern due to the non monochromaticity of the sources or the path difference of the order of or exceeding the coherence length. We want to study the equivalence of these two approaches. If we assume that the beam consists of all wavelengths lying between lambda and lambda + Delta Lambda.

Delta Lambda is the range of wavelengths then the interference pattern produced by the wavelength lambda and the middle one of the range that is lambda + Delta Lambda by 2 will disappear, if this Delta Lambda by 2 is = lambda square by 4d as shown earlier. We talked about that in the last lecture.

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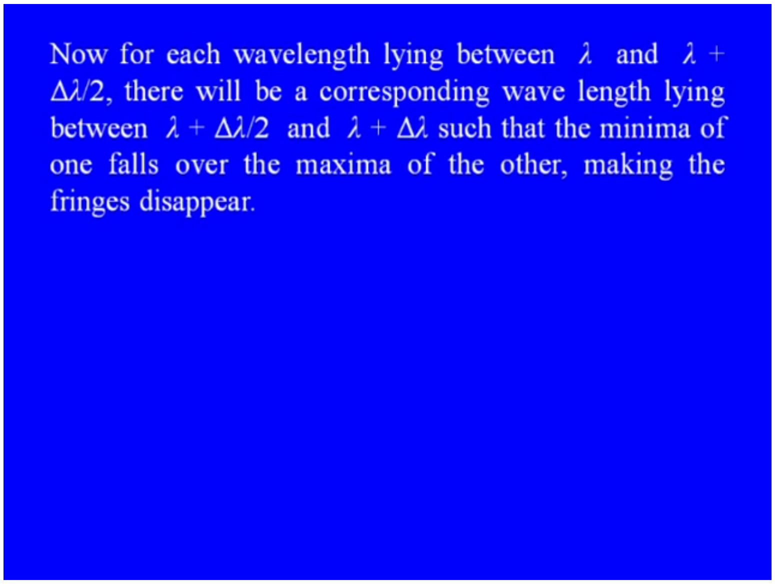

Now, for each wavelength lying between the first interval, lambda + lambda Delta Lambda by 2, there will be a corresponding wavelength, lying between the second interval, lambda + Delta Lambda by 2 and lambda + Delta Lambda such that the minimum of one falls over the Maxima of the other and making the fringes disappear.

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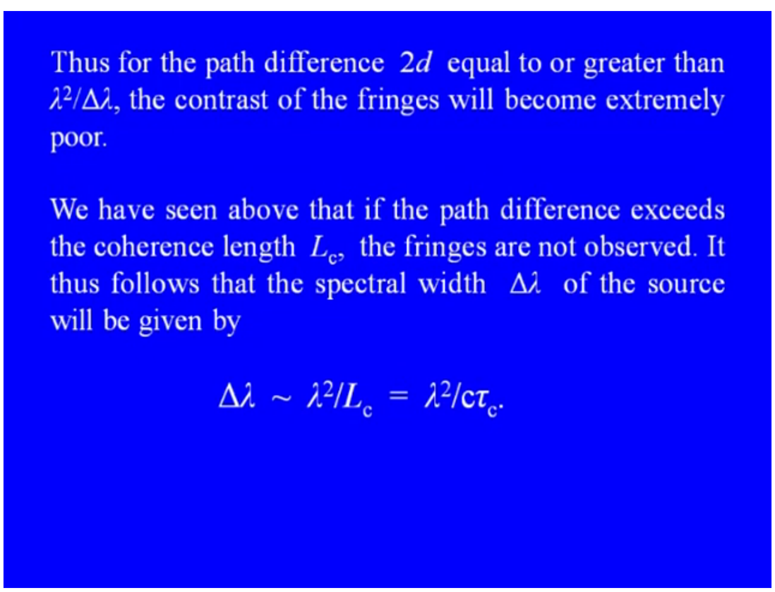

Thus for the path difference 2d = a greater than lambda squared divided by Delta. The contrast of the fringes will become extremely poor. So, this is one way of looking at the problem. Now, we have seen that lemma if the path difference exceeds the coherence length Lc r2 - r1 in that figure the fringes are not observed.

So, it is as follows: that the spectral width delta lambda of the source will be given by Delta Lambda is of the order of lambda square divided by Lc which is = lambda square divided by c into Tau c. Tau c is naturally = Lc.

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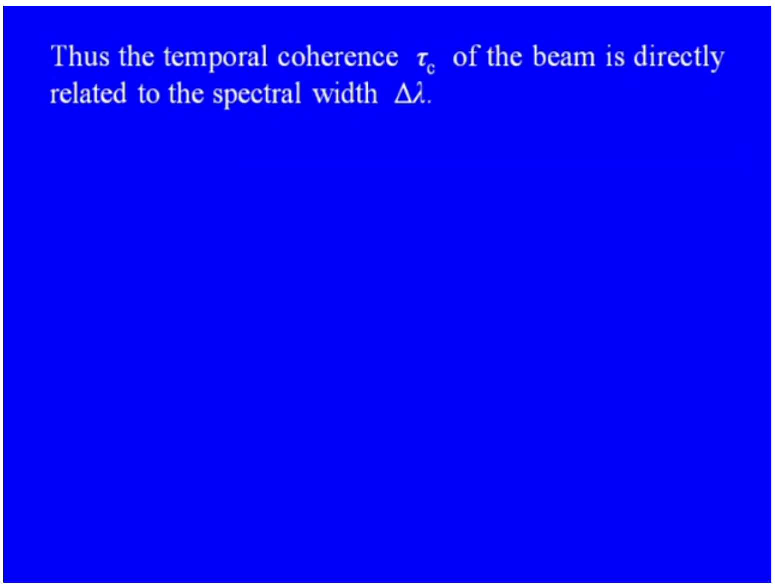

Thus, the temporal coherence Tau c of the beam is directly related to the spectral width delta lambda of the light source. This is very interesting and important result.

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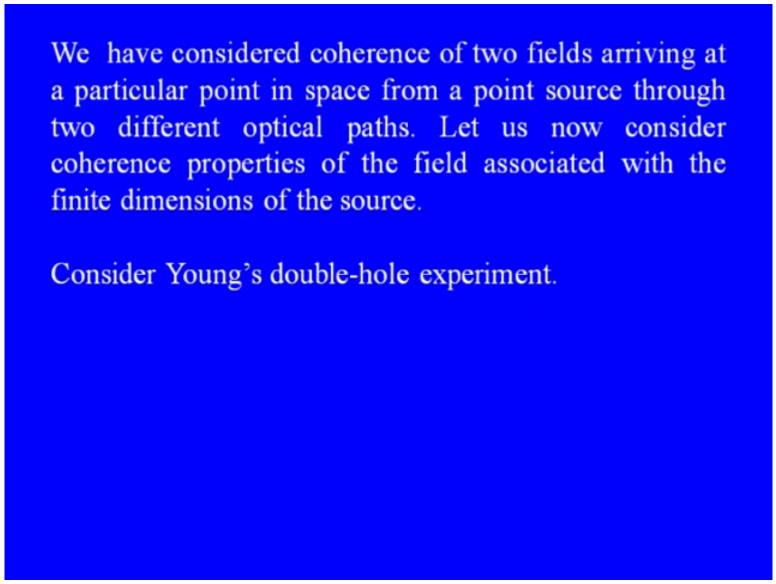

Now, you see, we have considered coherence of two fields arriving at a particular point in space from a point source, through two different optical paths. That is why we were talking about the path difference. We were talking about the, the coherence length light coming from a point source but traveling to different optical paths.

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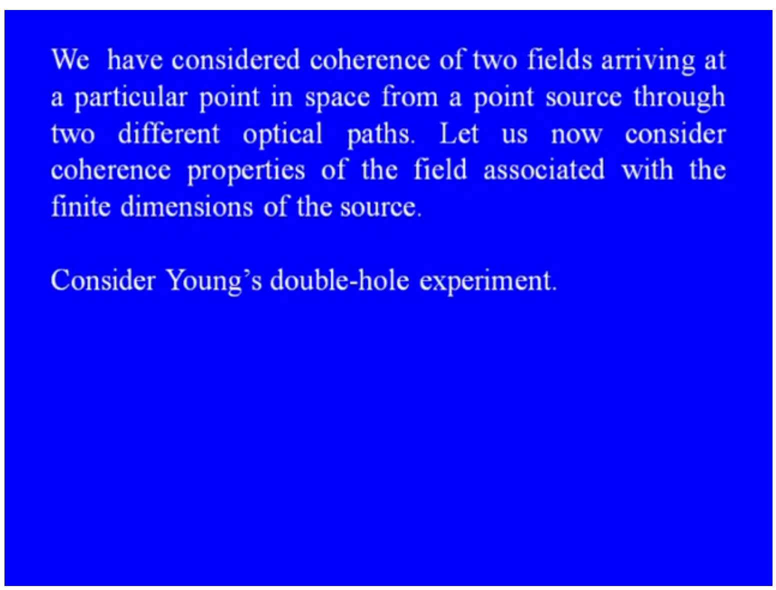

Now, let us consider coherence properties of the field associated with any finite dimensions of the source. So, the source is no longer a, a point source. Again consider Young's double-hole experiment, you see, this is a very interesting experiment. You see in this figure.

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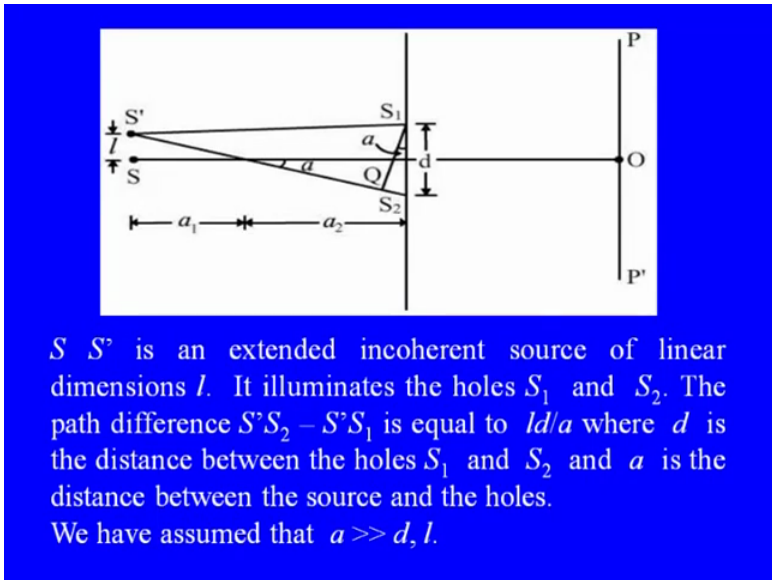

SS prime is an extended incoherence source of linear dimensions l, extended incoherent source means the various points of the extended sources; they have no phase relationship among themselves means they are independent point source. Now, this extended source SS prime it illuminates the holes S1 and S2 these will form a pair of coherent sources. Let us consider this path difference S prime, S2 and S prime, S1.

This is the point s is situated symmetrically. S prime is not symmetrically situated with respect to S1 and S2. So, the path difference S prime, S2 - S prime S1 is = ld by a, d is the distance between the two holes S1 and S2, a the distance between the source and the holes and we have assumed that a is naturally much larger than d or l. And we have a screen P, P Prime. O is the central point on the screen.

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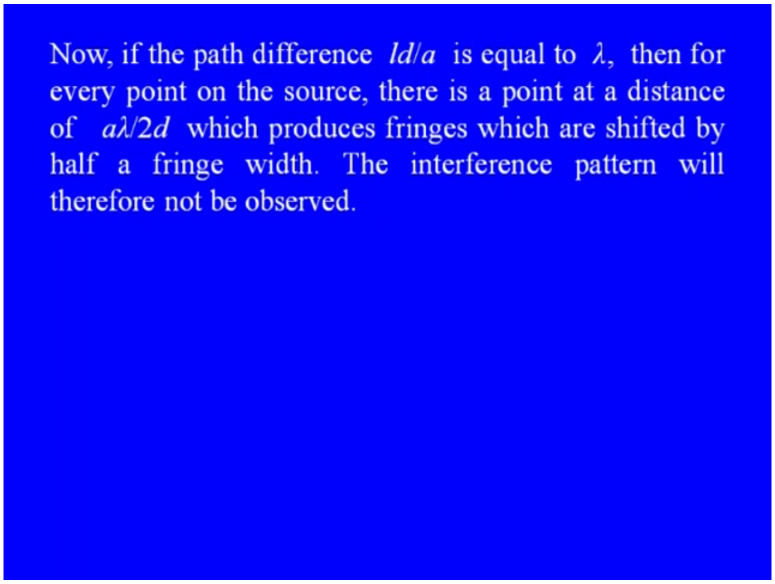

Now, if the path difference this ld by a of the light reaching S1 and S2 from the point S prime of the extended source is = lambda. Then, for every point on the source there is a point at a distance of a lambda upon 2d which produces fringes which are shifted by half a frame width which means maximum falling over a minimum. The interference pattern will therefore not be observed.

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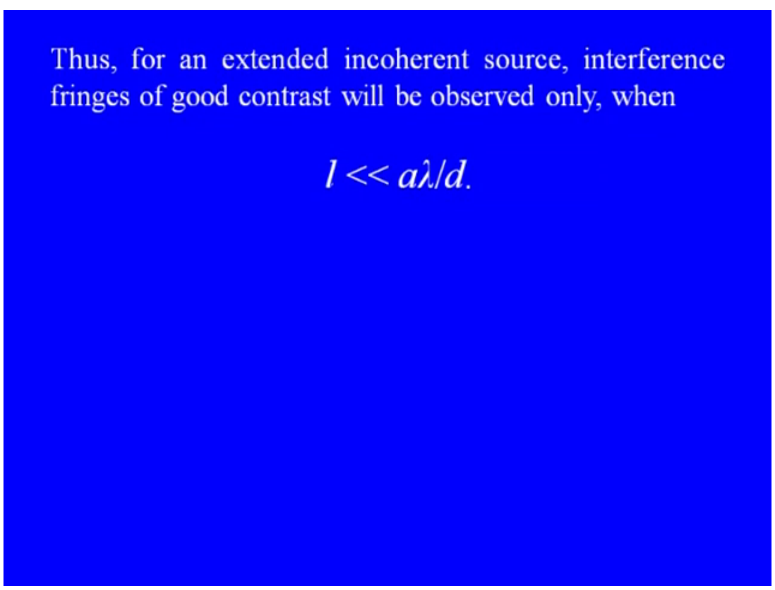

Thus for an extended incoherent source, interference changes a good contrast will be observed only if l which is the dimensions of the extended source is very, very small, compared to a lambda by d in terms of the angle.

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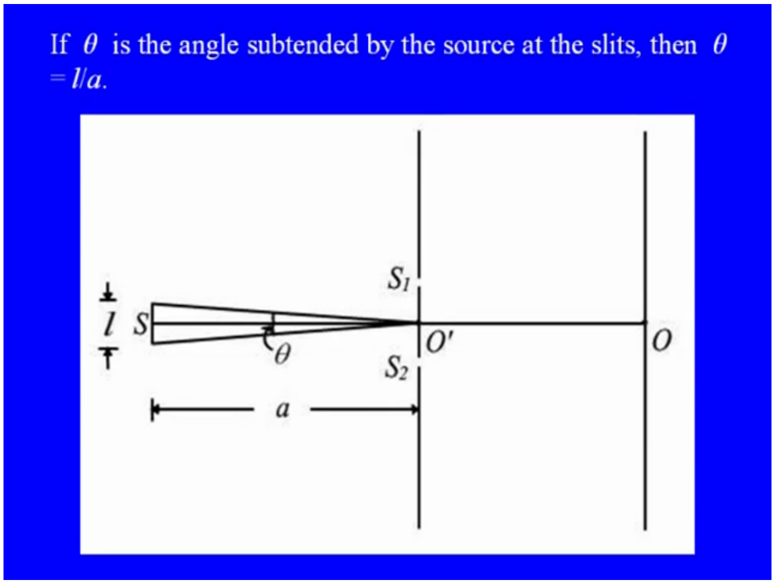

If theta is the angle, subtended by the source extended source of dimension l, at the slits theta is = l by a.

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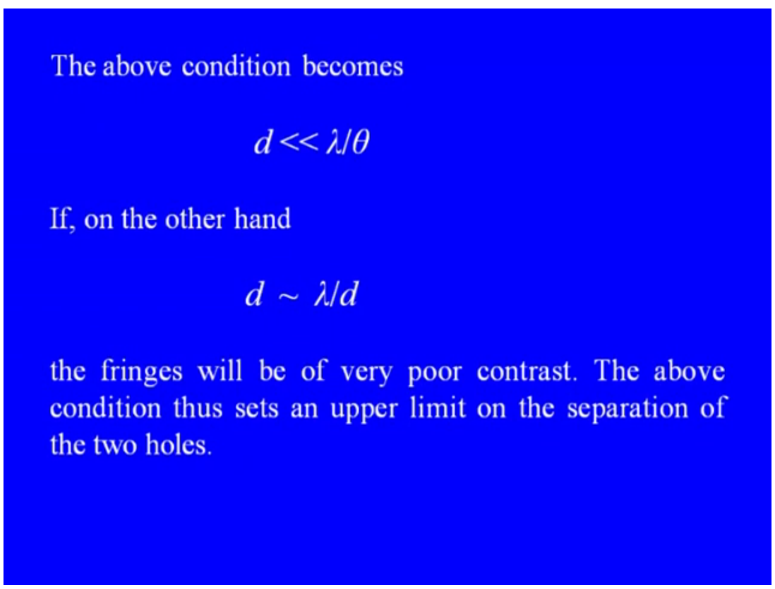

So, the above condition now becomes the d should be very, very small compared to lambda by theta. If on the other hand, d use of this d of the same order the fringes will be a very poor contrast d is of the order of lambda by theta. The above condition thus sets an upper limit on the separation of the two holes.

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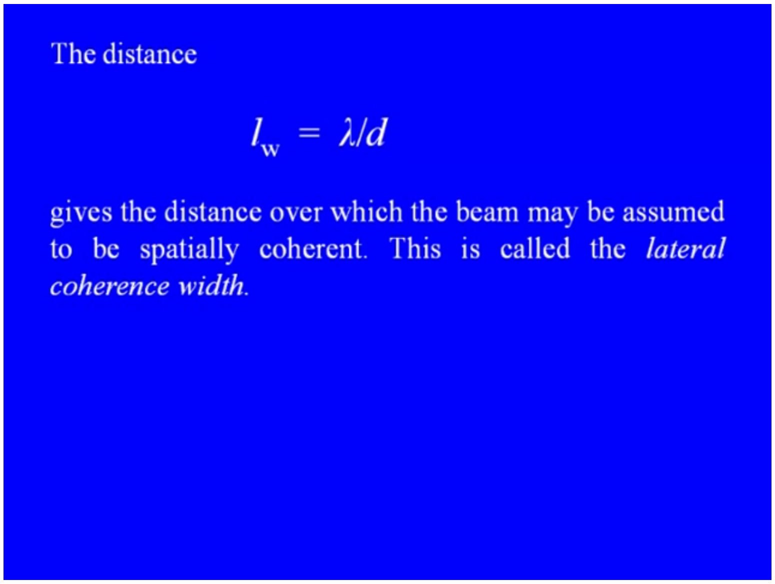

This distance lambda by d gives the distance over which the beam may be assumed to be especially coherent. So, for a given small d or a given lambda over the lw gives the maximum extent of the source this is called the lateral coherence width. Let us now come to the applications.

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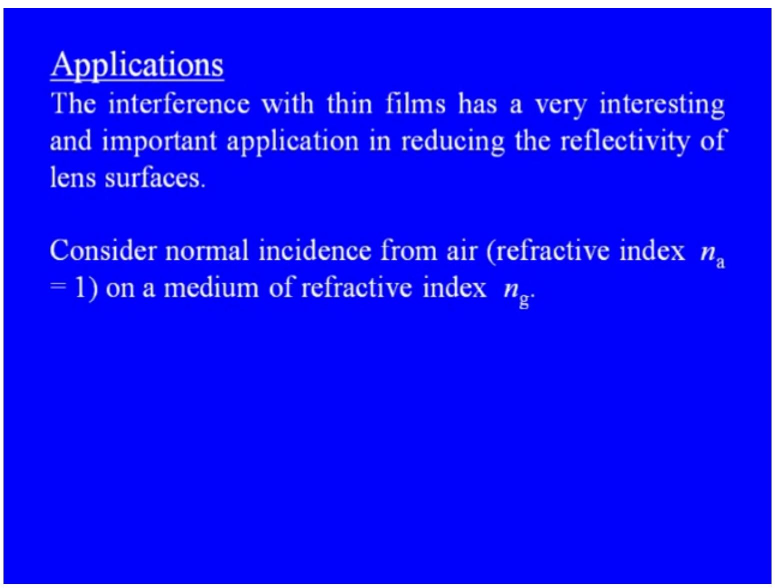

The interference with thin films has very interesting and important application in reducing the reflectivity of lens surfaces. This is a very interesting application and very important because all these optical instrument consists of is several lens components, several surfaces and naturally the reflectivity at each of these surfaces must be reduced to the minimum possible to a wide day overall loss of light. So, consider normal incidence from a refractive index n of a is = 1, on a medium of refractive index ng.

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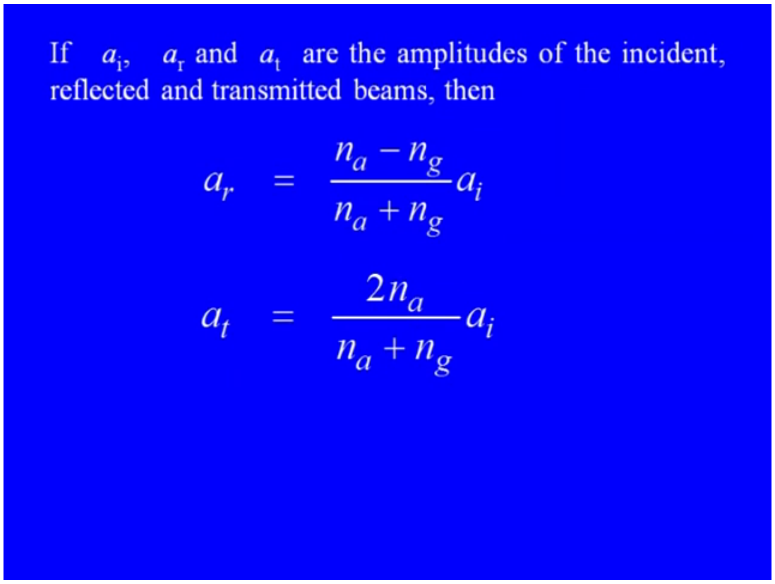

Now, if ai, ar and at are the amplitudes of the incident reflected and transmitted beams then they are related as follows: ar is = na - ng upon na + ng times ai. And a t is = twice of na upon na + ng times ai.

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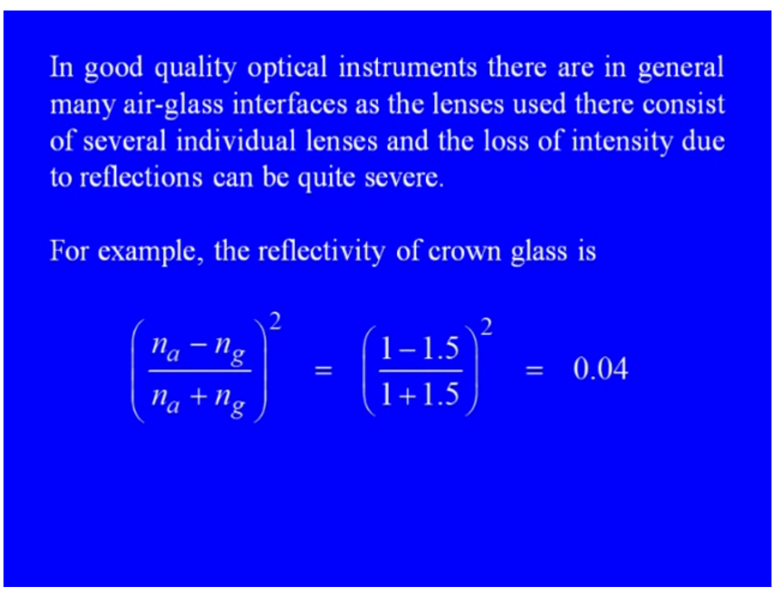

So, in good quality optical instruments, as I said earlier, there are in general, many glass interfaces, as the lenses used their consists of several individual lenses and the loss of intensity loss of overall loss of intensity due to these reflections can be quite severe. It must be avoided, minimized. For example, the reflectivity of crown glasses using the above expression na - ng upon na + ng squared, 1 - 1.5 upon 1 + 1.5 is squared 0.304 that is 4 % of the incident light;

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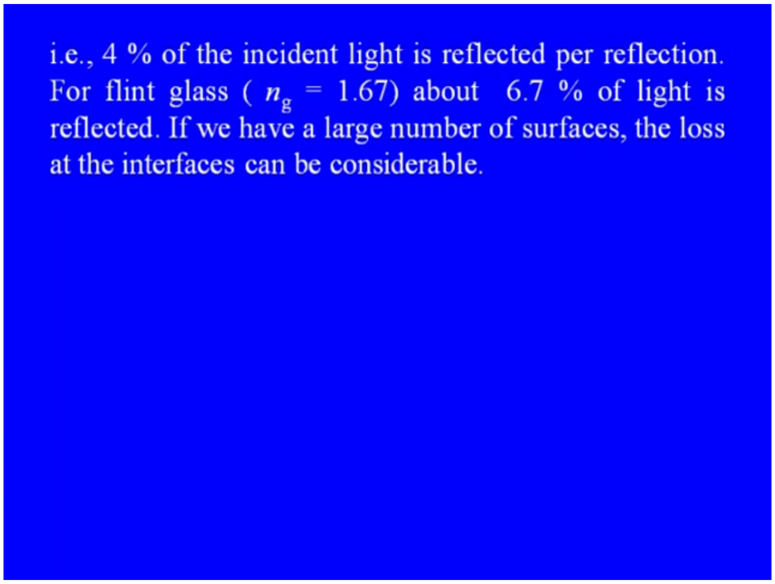

is reflected per reflection or Flint glass where the refractive index is 1.67 about 6.7 % of the light is reflected at each surface. If we have a large number of surfaces the loss at interfaces can be quite considerable.

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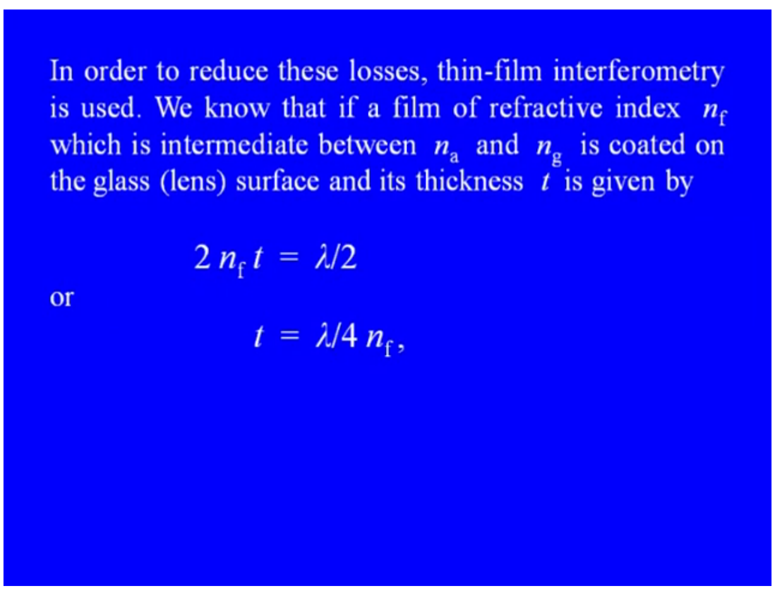

Now, in order to reduce these losses, thin film interferometry is used. We know that, if a film of refractive index n f which is intermediate between na and ng, na the refractive index of air and ng with that of the glass. Such a film of a material of refractory index nf is coated on the glass surface on the lens surface and its thickness t is given by 2 nf times t is = lambda by 2 that is t is = lambda by 4 times n f.

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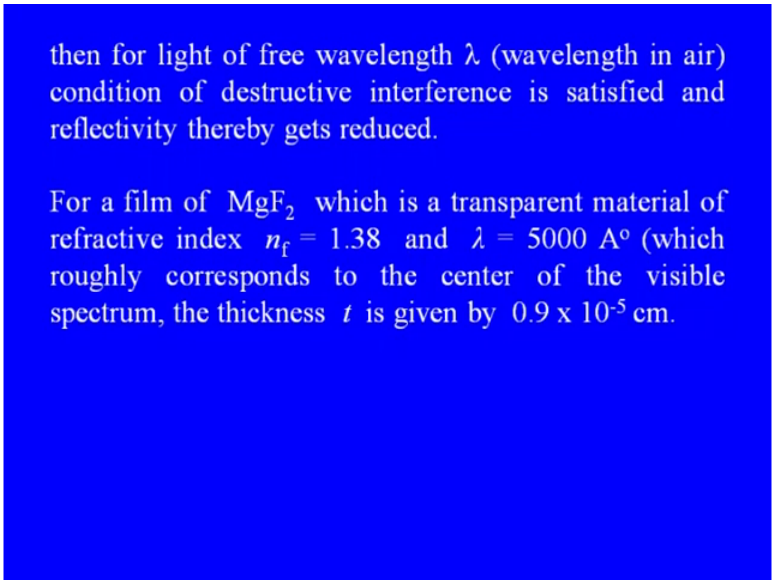

Then for light of free wavelength lambda, free wavelength means lambda wavelength in air condition of destructive interference is satisfied. And reflectivity thereby gets reduced. Say for a film of magnesium chloride which is the transparent material of refractive index 1.38 and for the wavelength of 5000 angstrom which roughly corresponds to the center of the visible spectrum. The thickness t is given by 0.9 into 10 raise to power - 5 centimeters.

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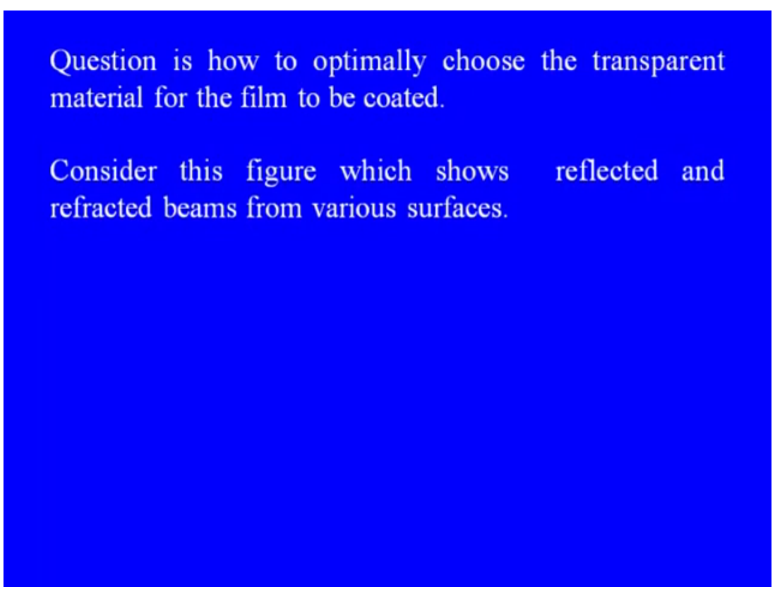

So, if the lens surface is coated by a film of the thickness it will reduce that reflectivity. But now, the important question is, how to optimally choose the transparent material. I mean how to choose its refractive index n f for the film to be just to be, just to be coated. What is the best value for and how to choose the material and see? That is, in this figure, which shows the reflected and the refracted beams from various surfaces?

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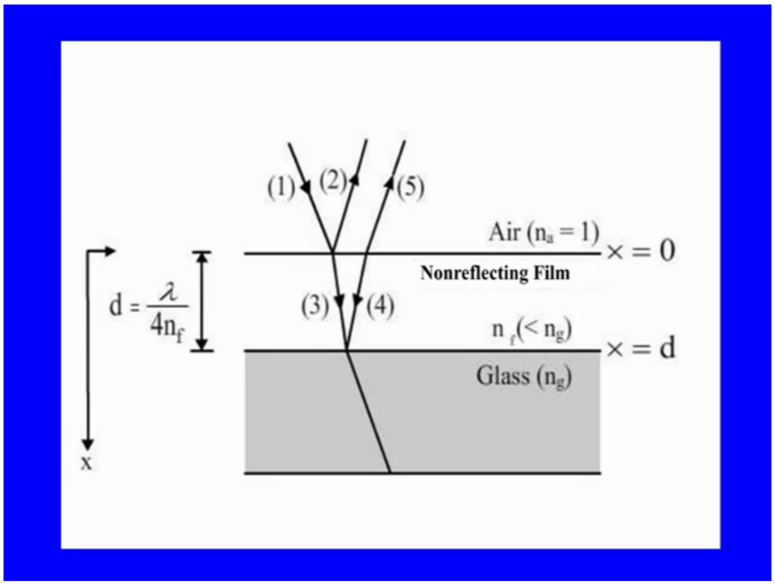

Bottom is the glass, it is coated by a non reflecting film of reflective index nf which is less than ng and the top is naturally air, refractive index is 1, nf is intermediate between na and ng. The thickness of the non reflecting film is d is = lambda upon 4 and it is called lambda by 4n for film and thickness of this film is d which is = lambda by 4 and m.

Now, consider the wave 1, it is incident on the reflecting surface, partly it is reflected which is the ray 2, partly goes through the reflect the coated film ray 3, reflected at the glass surface which is the reflected ray 4 and then ultimately comes out in air, to ray 5. So, this is a situation ray 1, top reflected ray 2, bottom reflected and comes out as ray 5. Now, what we would like here basically is that the density of this reflected rays; this should be the minimum possible first.

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If a_i is the amplitude of the incident wave, then the amplitudes of the reflected and refracted waves corresponding to the rays (2) and (3) would be

$$\frac{n_a - n_f}{n_a + n_f} a_a \quad \text{and} \quad \frac{2n_a}{n_a + n_f} a_i$$

respectively. We have assumed normal incidence.

That is the interest. If ai now is the amplitude of the incident wave then the amplitudes of the reflected and refracted waves corresponding to the rays 2 and 3 in the figure earlier, do you join with a transmitter theory, it is the one which is reflected. So, na - nf upon na + nf that is the reflected one and 2 n a divided by na + nf times a i is the transmitted, I mean, the refracted goes in the coated material. We have assumed normal incidence because these relations are really valid for normal incidence.

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The amplitude of the waves corresponding to rays (4) and (5) would be

$$\frac{2n_a}{n_a + n_f} \frac{n_f - n_g}{n_f + n_g} a_i$$

and

$$\frac{2n_a}{n_a + n_f} \frac{n_f - n_g}{n_f + n_g} \frac{2n_f}{n_f + n_a} a_i$$

respectively.

Now, the amplitude of the waves corresponding to the rays 4 and 5, they are 2 n a upon na + nf times nf - ng upon nf + ng times ai. And further ray 5, 2 na upon na + nf times and the factor nf - ng upon nf + ng times 2n f upon nf + na times ai.

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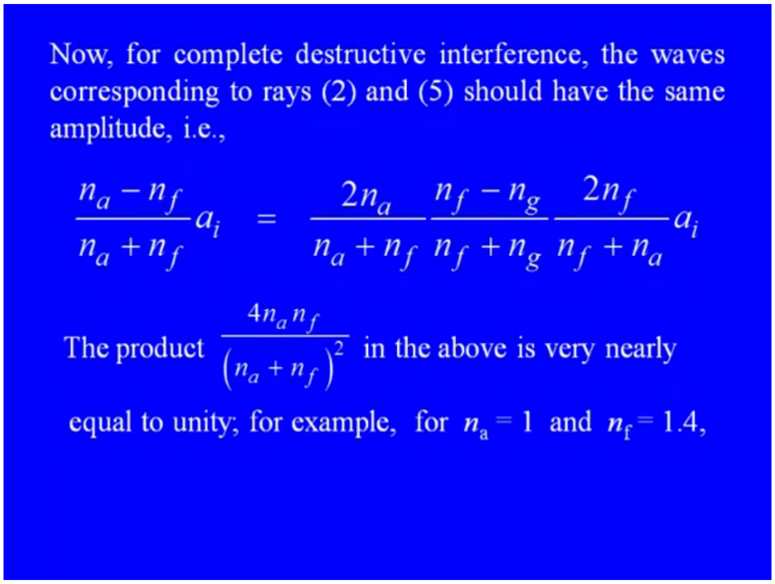

Now, for a complete destructive interference, that in the ideal situation, the waves corresponding to raise 2 and 5 should have the same amplitude. You see, the ray two here which is reflected first time from the top surface and reflected in the Ray 5 have come after reflection from the bottom. So, these 2 are in opposite phase. We would like their amplitude should really be equal that is the ideal situation.

That is na - nf upon n a + n f times a i. That is amplitude of the Ray 2 and 2 na upon na + nf and the factor nf - n g upon n f + n g multiplied behind the factor 2 nf upon nf + na times ai. Now, in this expression the product first and the third factors, 4 times na nf upon na + nf is squared is very nearly = unity. For example, for na is = 1 and nf = 1.4, it is factor is =

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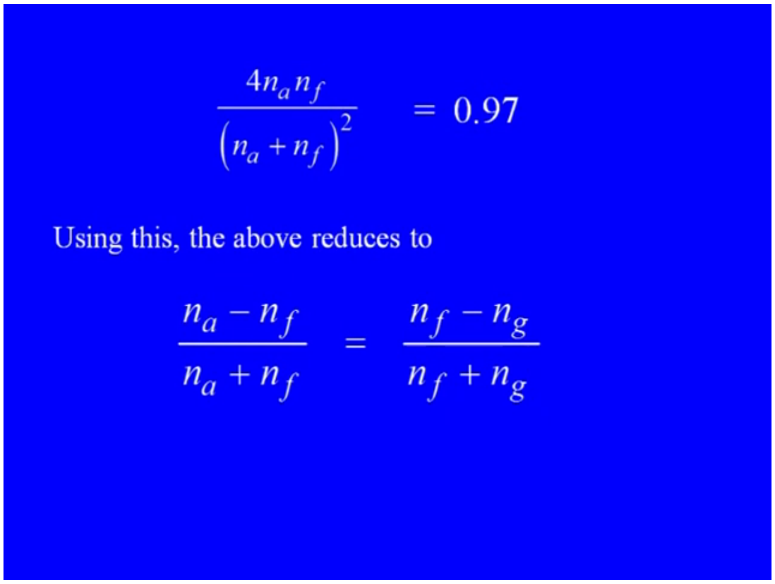

, 0.97 almost = 1, so let us take it = 1. So, if we use this, then the above condition simplifies to na - n f divided by na + nf is = nf - ng divided by nf + ng. And this some simplification leads to a very simple condition:

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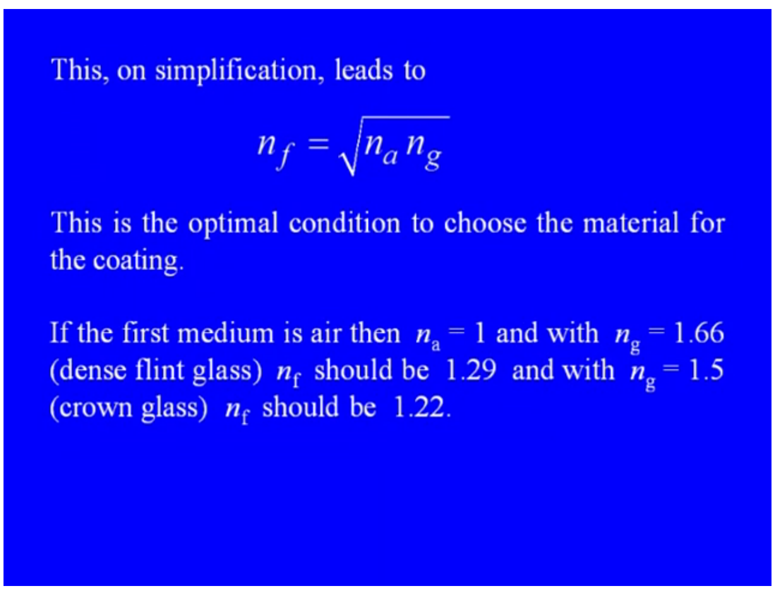

That nf should be the geometric mean between na and ng. This is the optimal condition to choose the material for the coating. If the first medium is air than na is = 1 and with ng = 1.66 which this is for the dense filling glass then nf should be 1.299 and with ng = 1.5 which is for the crown glass, nf should be 1.22.

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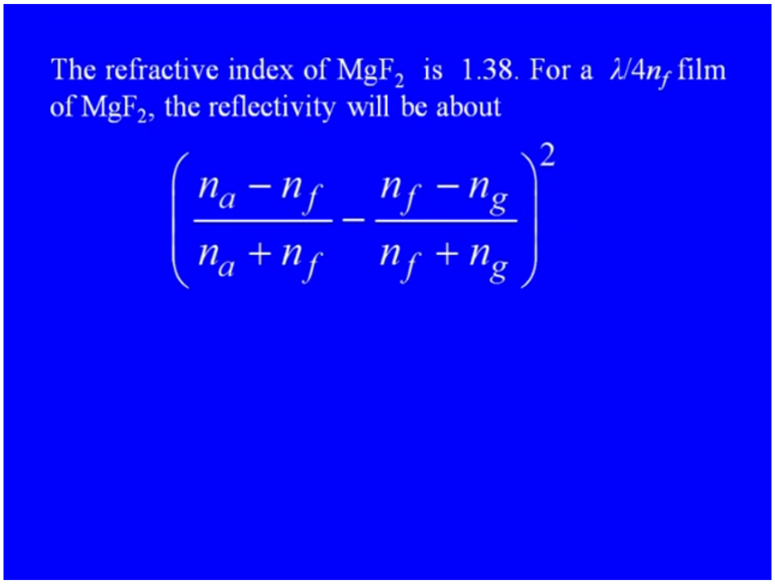

Now, they refractive index of magnesium fluoride is 1.38 not very different from these optimal values. But anyway for a lambda by 4 and a film of magnesium fluoride the reflectivity will be about this factor: na - f upon na + n. This is subtract from this nf - ng upon nf + ng whole squared.

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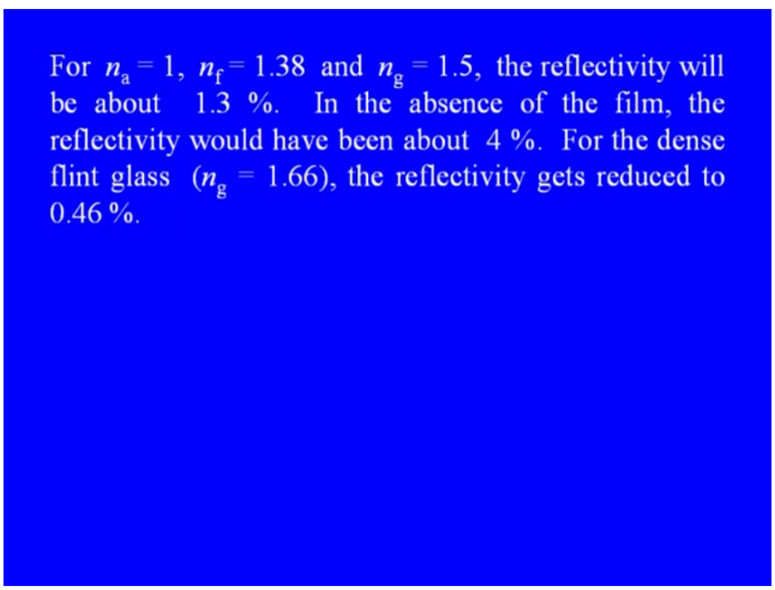

For na = 1 and nf = 1.38 which is for the ng nation fellow right. And ng is = 1.5 which is for the crown glass, the reflectivity will be about 1.3 %. In the absence of the film, remember, we have seen that, the reflectivity was about 4 % for the dense flint glass whose refractive index is 1.66, the reflectivity which was about 6.7 %, now gets reduced to 0.46 %.

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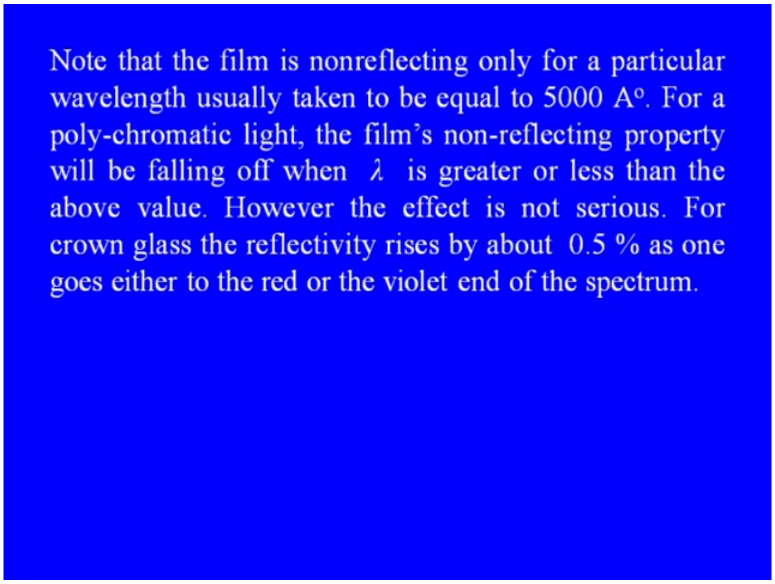

Now, one thing should be noted the film is non reflecting only for the particular wavelength where we have carried out the calculations this wavelength is usually taken to be = 5000 angstroms about the middle of the visible spectrum. For a polychromatic light, the film's non reflecting property will be falling off, when lambda is greater or less than the above value where the calculations have been done.

However, the effect is not serious for crown velocity reflectivity rises by about 0.5 % as one goes either to the red or the violet end of the spectrum.

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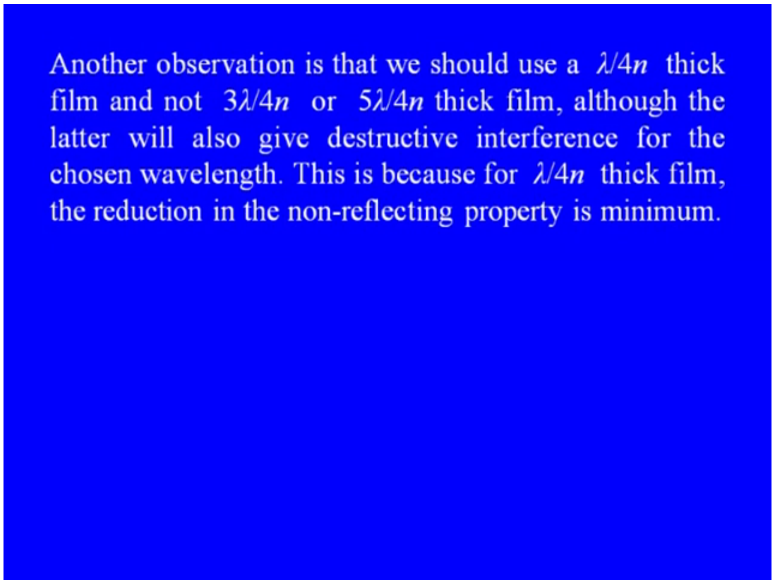

Another observation is that we should use a lambda by 4 thick film and not 3 lambda by 4, or 5 lambda by 4, although these films also give destructive interference for the chosen wavelength. This is because for the lambda by 4 and thick film, the reduction in the non reflecting property is minimal. Therefore that is the best choice to be used ok. So, with this we have come to the end of this series of lectures. Hope you enjoyed them, thank you.