

Engineering Physics 1
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Module-03
Lecture-02
Interference of Light - Part 02

In the previous lecture I described the basic theory of interference, what is difference between constructive and destructive interference, superposition principle, methods to find the resultant of two or more than two sinusoidal waves of same frequency and acting in the same direction at a point in a space, what is condition for constructive and interval destructive interference, in terms of path difference and phase difference;

How the intensity of result and wave vary with the phase difference of the interfering waves what are the conditions to observe a Stationary interference pattern and how to get two coherent sources for interference, by using division of wave front and division of amplitude. And I also discuss some famous experimental setup, which are used in laboratory to observe interference pattern based on division of wave front such as Young's Double Slits, Fresnel Biprism and Fresnel Double Mirror Experiment.

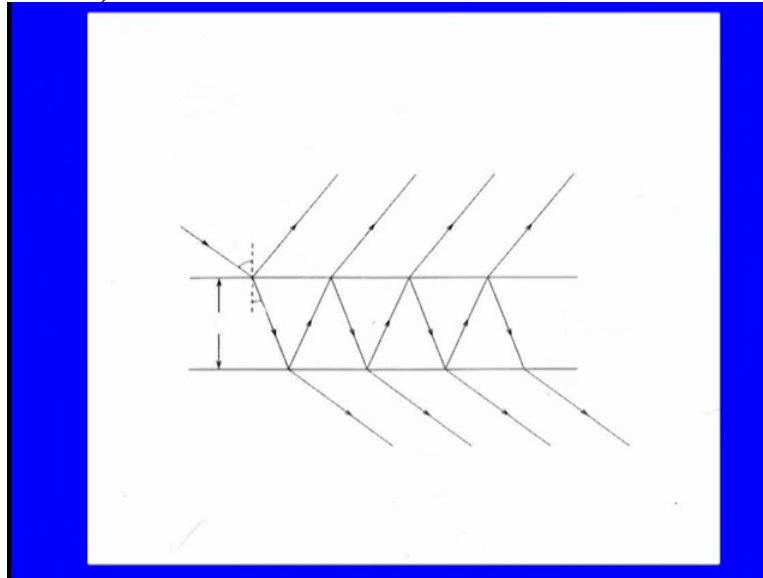
Mostly in all undergraduate optics labs, Fresnel Biprism experiment is performed with the help of this experiment. Wavelength of monochromatic light can be determined by measuring the fringe width with the help of a travelling microscope. I have given the expression of fringe width in the previous lecture, by measuring the required parameters experimentally, with the help of this equation wavelength can be determined.

With the help of this experiment, it is also possible to determine thickness of a transparent plate. For this, the plate is introduced in the path of one of the interfering beams and resulting shift in the position of Central bright fringe is measured with the help of a travelling microscope. The required expression I have already obtained in the previous lecture. Here, it should be noted that with monochromatic light it is not possible to recognise the central bright fringe.

But if we use white light, then, central bright fringe will be wide and few fringes adjacent to this will be coloured. So, shift in central bright fringe on introducing the plate can be measured easily

by using white light source. Now I am going to describe the formation of interference pattern by division of amplitude.

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In this case, the Incident beam is divided into two portions by division of its amplitude by partial reflection and refraction by some optical devices. The two portions afterwards recombined to produce interference effect. For example, if a plane wave falls on a thin film, then, the wave reflected from the upper surface interferes with the wave reflected from the lower surface. The colours when thrown by thin films of oil on water by soap Bubbles or bike racks in a piece of glass can be explained by the phenomenon of interference of light by division of amplitude.

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Let us first consider the interference by a plane parallel film of refractive index μ and thickness t illuminated by a plane monochromatic light of wavelength λ as shown in this figure. Suppose the light wave travelling along AB is incident on the upper surface GH of the film. Part of this wave will be reflected from the upper surface of film in the direction BR and remaining will be refracted in the direction BC. Upon arrival at C, part of this refracted wave will be reflected from surface IH in the direction CB_1 and part

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Upon arrival at C, part of this refracted wave will be reflected from surface IH, in the direction CB₁ and part refracted in direction CT.

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refracted in direction CT. At B₁ the ray CB₁ will be again divided. A continuation of this process yields two sets of parallel rays, one on each side of the film. In each of these sets, the intensity decreases rapidly from one ray to next. If the set of parallel reflected rays: BR, B₁R₁, B₂R₂,... etc., is collected by lens and focus at some point, constructive or destructive interference will be formed at this point depending on the phase difference between the consecutive waves. It is such interference that produces the colours of thin films.

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In order to find the phase difference between these rays, we must first evaluate the difference in the optical path traversed by a pair of successive rays, such as BR and B₁R₁. Suppose B₁D is perpendicular drawn from B₁ on BR. Therefore, optical path difference between the waves BR and B₁R₁ is given by

$$\Delta = (BC + CB_1)_{\text{film}} - BD_{\text{air}}$$

With the help of geometry we can show that

$$\Delta = 2\mu t \cos\theta$$

In order to find the phase difference between these rays, we must first evaluate the difference in the optical path traversed by a pair of successive rays, such as BR and B₁R₁. Suppose B₁D is perpendicular drawn from B₁ on BR. Therefore, optical path difference between the waves BR and B₁R₁ is given by BC + CB₁ in film - BD in air. With the help of geometry, we can show that this path difference will be = 2 $\mu t \cos \theta$. Therefore, phase difference Δ will be given by $\Delta = 2\pi \times \text{path difference} / \lambda = 2\pi \times 2\mu t \cos \theta / \lambda$.
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Therefore, phase difference δ will be equal to $(2\pi/\lambda) \times \text{Path difference}$, i.e.

$$\delta = (2\pi/\lambda)(2\mu t \cos\theta)$$

If the film is optically denser than the surrounding media, then the wave BR originating by reflection from B, a point on the surface backed by a denser medium, i.e. film, will differ in phase by π from the incident wave. But the wave B₁R₁ which originates by reflection at C, a point on the surface backed by a rarer medium, will experience no sudden phase change. Thus, the total phase difference between waves BR and B₁R₁ becomes

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Thus the total phase difference between waves B R and B₁ R₁ becomes $2\pi \frac{\mu t \cos \theta}{\lambda} + \pi$.

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$$\delta = (2\pi/\lambda)(2\mu t \cos \theta) + \pi$$

Therefore, if $2\mu t \cos \theta = (n + 1/2)\lambda$ ($n=0, 1, 2, \dots$), the waves BR and B₁R₁ interfere constructively and they will interfere destructively if $2\mu t \cos \theta = n\lambda$ ($n=0, 1, 2, 3, \dots$).

Like reflected waves, the transmitted waves CT, C₁T₁, C₂T₂, ..., also satisfy the conditions for interference because they originate from the same incident wave. Here, CT and C₁T₁ are obtained without any sudden phase change due to reflection. The phase difference between CT and C₁T₁ is, therefore, only due to optical path difference. Similar to reflected waves, here also we can determine the path difference between

Therefore if $2\mu t \cos \theta = n + \frac{1}{2}\lambda$ where n is integer, the wave BR and B₁R₁ interfere constructively and they will interfere destructively if $2\mu t \cos \theta = n\lambda$ where n is again integer. Like reflected waves, the transmitted waves CT, C₁T₁, C₂T₂ etcetera also satisfy the conditions for interference because they originate from the same incident wave. Here CT and C₁T₁ are obtained without any sudden phase change due to reflection.

The phase difference between CT and C₁T₁ is therefore only due to optical path difference. Similar to reflected waves, here also we can determine the path difference between CT and C₁T₁ using basic geometry.

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CT and C_1T_1 using basic geometry and here also it comes out to be $2\mu t \cos\theta$

So, the transmitted waves CT and C_1T_1 will interfere constructively when $2\mu t \cos\theta = n\lambda$ and destructively when $2\mu t \cos\theta = (n+1/2)\lambda$ ($n=0,1,2,3,\dots$).

Thus, the condition for constructive and destructive interference for transmitted waves CT and C_1T_1 are just reverse of that of the reflected waves BR and B_1R_1 .

And here also it comes out to be $2\mu t \cos\theta$. So the transmitted waves CT and C_1T_1 will interfere constructively when $2\mu t \cos\theta = n\lambda$ and destructively when $2\mu t \cos\theta = n + \text{half } \lambda$. Thus, the condition for constructive and destructive interference for transmitted waves CT and C_1T_1 are just reverse of that of the reflected waves BR and B_1R_1 .

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Since the geometry is same, the path difference between waves B_1R_1 and B_2R_2 will be same as between waves BR and B_1R_1 . Since B_1R_1 and B_2R_2 originate only from the internal reflection, so there would not be phase difference of π due to reflection. Now, if $2\mu t \cos\theta = n\lambda$, waves B_1R_1 and B_2R_2 will be in phase and same holds for all succeeding pairs. Therefore, under this condition waves BR and B_1R_1 will be out of phase, but waves B_1R_1 , B_2R_2 , B_3R_3 ,, will be in phase with each other. On the other hand, if $2\mu t \cos\theta = (n+1/2)\lambda$

Since the geometry is same, the path difference between waves B_1R_1 and B_2R_2 will be same as between waves BR and B_1R_1 since B_1R_1 and B_2R_2 originate only from the internal reflection so there would not be phase difference of π due to reflection. Now if $2\mu t \cos\theta = n\lambda$ waves B_1R_1 and B_2R_2 will be in phase and same holds for all succeeding pairs. Therefore, under this condition, waves BR and B_1R_1 will be out of phase.

But waves B_1R_1 and B_2R_2 , B_3R_3 and so on, will be in phase with each other. On the other hand, if $2 \mu t \cos \theta = n + \frac{1}{2} \lambda$

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, waves B_1R_1 will be in phase with BR , but B_2R_2 , B_4R_4 , B_6R_6 , ..., will be out of phase with B_1R_1 , B_3R_3 , B_5R_5 , Since B_1R_1 is more intense than B_2R_2 and B_3R_3 is more intense than B_4R_4 and so on, these pairs can not cancel each other, and hence there will maximum intensity.

For the minima of intensity, wave B_1R_1 is out of phase with wave BR , but BR has a considerably greater amplitude than B_1R_1 , so that these two will not completely annul each other. But if we add the amplitudes of the waves B_1R_2 , B_2R_2 , B_3R_3 , ..., we find that the resultant amplitude is equal to the amplitude of the wave BR . Therefore, in case of minima there will be complete darkness.

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Thus, if the film is of such a thickness that the condition $\Delta = n\lambda$ is satisfied, the film will appear perfectly dark when seen by reflected light. On the other hand, if the thickness of the film is such that the condition $\Delta = (n+1/2)\lambda$ is satisfied, then the intensity of the film will be maximum when seen by reflected light.

For the transmitted light, when film thickness is such that the condition $\Delta = (n+1/2)\lambda$ is satisfied, the pair of transmitted wave CT and C_1T_1 , C_2T_2 and C_3T_3 , etc are in opposite phase and therefore interfere destructively in pair.

Thus, if the film is of such a thickness that the condition path difference is $= n \text{ Lambda}$ is satisfied, the film appears perfectly dark when seen by reflected light. On the other hand, if the thickness of the film is such that path difference is $= n + \text{half Lambda}$ then the intensity of the film will be maximum, when seen by reflected light. For the transmitted light, when film thickness is such that the condition delta of path difference is $= n + \text{half Lambda}$ is satisfied.

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So the resultant intensity will be minimum. If the thickness of the film is such that $\Delta = n\lambda$ is satisfied, the transmitted waves CT, C_1T_1 , C_2T_2 , etc. are in phase with each other and therefore they interfere constructively leading to maximum intensity in the transmitted light.

So, when there is maximum intensity in the reflected light there would be minimum intensity in the transmitted light and vice versa. Thus based on the above discussions it is concluded that when a plane parallel film is illuminated with parallel monochromatic light

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light. So, when there is maximum intensity in the reflected light, there would be minimum intensity in the transmitted light and vice versa. Thus based on the above discussions, it is concluded that when a plane parallel film is illuminated with parallel monochromatic light, (Refer Slide Time: 12:54)

, i.e. θ is constant all over the film, the film will have maxima or minima of brightness all over according as the optical path difference between the directly and internally reflected waves is odd or even multiple of $\lambda/2$.

Thin film illuminated with a parallel beam of white light:

Now suppose the plane-parallel thin film is illuminated with a parallel beam of white light. If we neglect the small variation in the angle of refraction with wavelength, then light waves of different colours follow approximately the same path within the film. Therefore, the optical path

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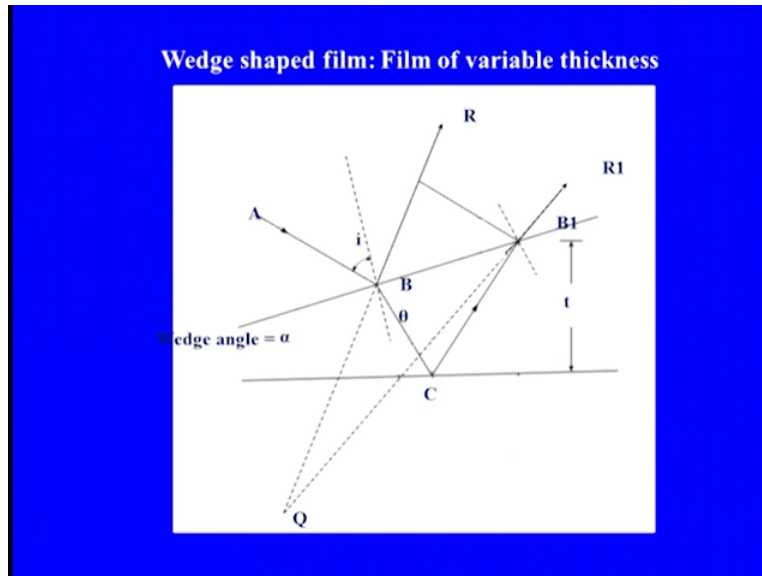
difference $2\mu t \cos\theta$ will be approximately same for all the colours. Hence the colours whose wavelengths satisfy the condition $2\mu t \cos\theta = n\lambda$ will be absent in the reflected beam, but they will be present in the transmitted beam. On the other hand, colors whose wavelengths satisfy the condition $2\mu t \cos\theta = (n+1/2)\lambda$ will be present in the reflected beam but practically absent in the transmitted beam. Consequently, the film will have a uniform colouration all over but the resultant colour of the film by reflected light will be exactly complementary to the resultant colour by transmitted light.

Therefore the optical path difference $2 \mu t \cos \theta$ will be approximately same for all the colours. Hence, the colours whose wavelength satisfy the condition $2 \mu t \cos \theta = n \lambda$ will be absent in the reflected beam. But they will be present in the transmitted beam. On the other hand, colours whose wavelength satisfies the condition $2 \mu t \cos \theta = n + \frac{1}{2} \lambda$ will be present in the reflected beam. But practically, absent in the transmitted beam.

Consequently, the film will have a uniform coloration all over. But the resultant colour of the film by reflected light will be exactly complementary to the resultant colour by transmitted light. **(Refer Slide Time: 14:36)**

Furthermore, when film thickness t is much smaller than λ , then $2\mu t \cos \theta$ is negligible in comparison to λ . In this case phase difference δ , which is equal to $(2\pi/\lambda)(2\mu t \cos \theta)$ will be zero for all wavelengths. Therefore, the film will appear perfectly dark by reflected light even when it is illuminated with white light. On the other hand it will appear white when seen by the transmitted light.

Furthermore, when film thickness t is much smaller than λ then, $2 \mu t \cos \theta$ is negligible, in comparison to λ ; in this case, phase difference Δ which is $= 2 \pi$ by λ into $2 \mu t \cos \theta$ will be 0 for all wavelength. Therefore, the film will appear perfectly dark by reflected light even when it is illuminated with white light. On the other hand, it will appear white when seen by transmitted light. Now, let us consider a film in the shape of a thin wedge whose side form a small angle α as shown in this figure. **(Refer Slide Time: 15:15)**



Now, suppose this film is illuminated with plain monochromatic light waves. Here also, the directly reflected wave BR and internally reflected wave B1R1 originate from the same incident wave propagating along AB. And hence, they are capable of producing observable interference effects. Here, the two interfering waves do not reach there, along parallel path but they appear to diverge from a point Q in the rear of the film. So, destructive and constructive interference occurs at the point Q which is however virtual.

If the two interfering waves BR and B1R1 fall on lens, they will cross each other at a real point Q prime, the focus conjugate of Q. As a consequence, really reinforcement are destructive interference would occur at 2 prime. Thus Q will be the position where the interference fringe appears to be formed. From geometry, we can find the optical path difference Delta between the waves B R and B1 R1 and this comes out to be:

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$$\Delta = 2\mu t \cos(\theta + \alpha)$$

where, t is the thickness of film at the point B_1 . The path difference, thus varies both on account of changing thickness as well as changing angle of incidence.

Now, taking into account the abrupt phase change of π due to reflection at B, the conditions for constructive and destructive interference between waves BR and B_1R_1 become

$$2\mu t \cos(\theta + \alpha) = (n + 1/2)\lambda \quad [\text{maxima}]$$

$$2\mu t \cos(\theta + \alpha) = n\lambda \quad [\text{minima}]$$

$2\mu t \cos \theta + \alpha$, where t is the thickness of film at the point B_1 ; The path difference thus varies both on account of changing thickness as well as changing angle of incidence θ . Now, taking into account the abrupt phase change of π , due to reflection at B, the condition for constructive and destructive interference between waves BR and B_1R_1 becomes $2\mu t \cos \theta + \alpha = n + \frac{1}{2}\lambda$ for maxima and $2\mu t \cos \theta + \alpha = n\lambda$ or minima.

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If the wedge-shaped film is illuminated by a parallel beam of monochromatic light of wavelength λ , the angle of incidence θ will be same at every point of the film and so also angle of refraction θ . So, in this case variation in the optical path difference will take place only due to variation in the thickness of the film ' t ' from point to point of the film. At the edge of the wedge film since $t = 0$, the film appears perfectly dark because the two interfering waves are π out of phase. At a distance from the edge such that $\Delta = \lambda/2, 3\lambda/2, 5\lambda/2, \dots$, the film will be bright, while at a distance so that $\Delta = \lambda, 2\lambda, 3\lambda, 4\lambda, \dots$, the film will appear dark. Thus

If the wedge shaped film is illuminated by a parallel beam of monochromatic light of wavelength λ , the angle of incidence θ will be same at every point of the film and show also angle of refraction. So, in this case, variation in the optical path difference will take place only due to variation in the thickness of the film t from point to point of the film. At the edge of the waves,

film wedge films since t is = zero, the film appears perfectly dark because the two interfering waves are π out of phase.

At a distance from the edge such that the path difference is $= \lambda$ by 2, 3λ by 2, 5λ by 2 and so on. The film will be bright while at a distance so that path difference is $= \lambda$, 2λ , 3λ , 4λ and so on. The film will appear dark.

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As we proceed along the film in the direction of increasing film thickness, we shall encounter, alternately dark and bright bands parallel to the edge of the film. Deviation from perfect plainness of the surface will be immediately obvious through the curvature of these bands, because any one band is the locus of constant thickness of the film. That is why such fringes are called fringes of constant thickness. It can be shown that for small angle of incidence, i.e. $\cos\theta = 1$, the fringe width β , i.e. the separation between two consecutive dark or bright fringes is given by $\beta = \lambda/(2\alpha\mu)$, where α is the wedge angle and μ is the refractive index of the film.

Thus as we proceed along the film, in the direction of increasing film thickness, we shall encounter alternatively dark and bright bands parallel to the edge of the film. Deviation from perfect plainness of the surface will be immediately obvious through the curvature of these bands. Because any one band is the locus of constant thickness of the film. That is why such fringes are called fringes of constant thickness.

It can be shown that for a small angle of incidence that is $\cos\theta$ nearly $= 1$ the fringe width β that is the separation between the two consecutive dark or bright fringes is given by $\lambda/(2\alpha\mu)$ where α is the wedge angle and μ is the refractive index of the film. Now, let us consider that the wedge shaped thin film is illuminated with parallel beam of white light.

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Now let us consider that the wedge shaped thin film is illuminated with parallel beam of white light. If the thickness at the thin edge is very small in comparison of the wavelength of violet light, then at the edge a truly achromatic black fringe parallel to the edge will be seen when it is viewed by reflected light. Now if we move along the film in the direction of increasing thickness, we will first reach a point where the thickness of the film is such that the condition of constructive interference is satisfied for violet colour, because wavelength of this colour is least. Therefore, a violet fringe is seen at this point. Proceeding still further successively blue, green, yellow and red fringes will be observed.

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Therefore, a violet fringe is seen at this point. Proceeding still further successively blue, green, yellow and red fringes will be observed.

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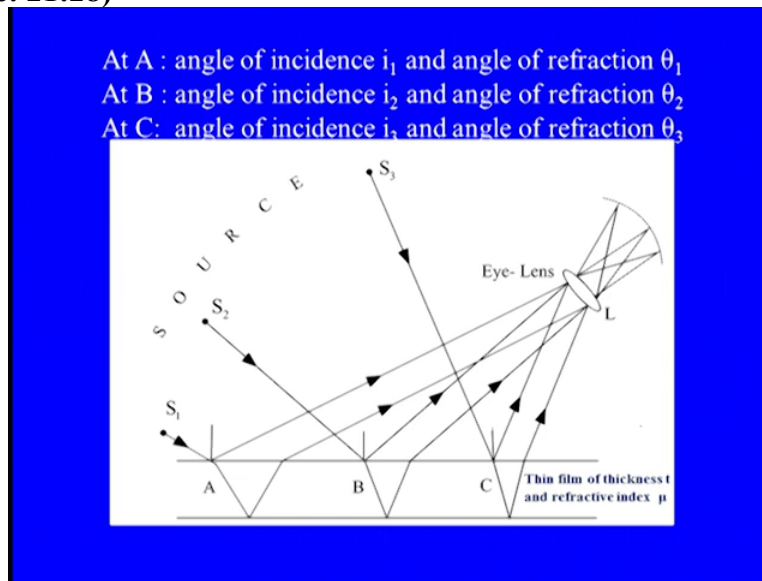
These fringes are called “Fringes of equal chromatic order”. Beyond certain point thickness of the film becomes so large that the condition of constructive interference is satisfied for two or more colours simultaneously and therefore a coloured band due to overlapping of bright fringes of more than one colours will be seen. If we still move in the same direction, we shall arrive at the point where such overlapping of different colours produces uniform illumination.

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colours simultaneously and therefore a coloured band due to overlapping of bright fringes of more than one colour will be seen. If we still move in the same direction, we shall arrive at a point, where such overlapping of different colours produces uniform illumination.

Now let us consider that a thin parallel film is illuminated by using extended white source as shown in this figure.

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In this case, light from each point of the source gives rise to a pair of coherent waves on reflection from the film and $\cos \theta$ is not same for all of them. Each point of the film is also seen by light from different points of the extended source. If at points A, B, C, the conditions $2\mu t \cos \theta_1 = n_1 + \frac{1}{2}\lambda_1$, $2\mu t \cos \theta_2 = n_2 + \frac{1}{2}\lambda_2$ and $2\mu t \cos \theta_3 = n_3 + \frac{1}{2}\lambda_3$ are satisfied respectively.

We shall observe a bright band of λ_1 at A, bright band of λ_2 at B, the bright band of λ_3 at C and so on. Thus we shall observe a set of equal inclination colour bands, if the film is of variable thickness, we shall again observe a set of colour bands each being a locus of equal thickness of the film. The colour of any particular region of the film changes, if the I is shifted to a new position.

Now, light from other points of extended source is reflected from that particular region of the film, at different angles to I. In effect, $2\mu t \cos \theta$ alters throughout the film leading to change in colours exhibited by thin film. This clearly explains the origin of colour exhibited by a

thin film of oil; while on the surface of water or colour of soap bubbles by light reflected from the sky. It should be noted that bright colours will be seen only when film is extremely thin.

That is up to few wavelengths thick. The colours become fit for thicker films and for such cases the direction of incidence should be kept nearly normal. Otherwise at other angles, the directly and indirectly reflected coherent waves may get separated to such an extent that their wave front may not interfere each other and they may get so far apart that only one wave enters the eye at a time. And so, all colours effectively may vanish when viewed by naked eye. Now, let us discuss the classification of fringes.

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Classification of fringes

As we have discussed, when a thin film is illuminated by light, the resultant transmitted or reflected intensities will depend upon the phase difference δ between two consecutive transmitted or reflected waves $\delta = (2\pi/\lambda) 2\mu t \cos\theta$. Since, the value of phase difference δ may be varied by varying θ or t or λ , accordingly we can divide the interference fringes of thin films into three classes:

(a) : If phase difference varies mainly due to variation of θ , while t remains constant, then fringes are termed as fringes of equal inclination or Haidinger fringes.

As we have discussed, when a thin film is illuminated by light the resultant transmitted or reflected intensities, will depend upon the phase difference Δ between two consecutive transmitted or reflected waves. And phase difference is given by $2\pi/\lambda$ into $2\mu t \cos\theta$. Since the value of the phase difference Δ may be varied by varying θ , or t , or λ , accordingly, we can divide the interference fringes of thin films into three classes.

If phase difference varies mainly due to variation of θ , while t remains constant then, fringes are termed as fringes of equal inclination or Haidinger fringes.

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(b) If phase difference varies with variation in thickness t of the film, then fringes are called fringes of equal thickness or Fizeau fringes.

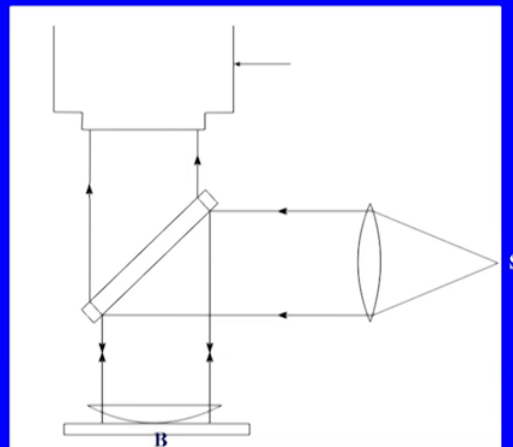
(c) Now if the phase difference varies with variation in λ , then the fringes are known as fringes of equal chromatic order or FECO fringes. The term FECO was used by Prof. S. Tolansky.

If phase difference varies with variation in thickness t of the film then, fringes are called fringes of equal thickness or Fizeau fringes. Now, if the phase difference varies with variation in λ then, the fringes are known as fringes of equal chromatic order or FECO fringes. The term FECO was used by Professor S Tolansky. Now, in this lecture, I will describe some of the experimental setup used in laboratory to observe the fringes of equal thickness and fringes of equal inclinations one.

One of the most important experiments which is based on the division of amplitude is Newton ring experiment.

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Experimental set up to observe Newton's rings



In this experiment a thin air film are progressively increasing thickness in all direction from one point can be formed by placing a plane convex lens of large radius of curvature on a plane glass plate such that its convex surface faces the plate. The air film does form possesses a radial symmetry about the point of contact and when it is illuminated normally with monochromatic light, the observed interference fringes are circular ring concentrate with the point of contact.

These rings are known as Newton's ring. The experimental arrangement to observe the Newton rings in laboratory is shown in the figure. Here, A is a Plano convex lens placed on the glass plate B. The glass plate G reflect the light down so that it is incident normally on the plates, after reflection, it is transmitted through G and observed in the travelling microscope. If the film in close between the lens and the plane glass plate is extremely thin, then, wave single alpha can be neglected as compared to angle of incidence theta.

So, for normal incidence the optical path difference between the two consecutive reflected waves become $2\mu t$. Thus, for this case, the condition for constructive and destructive interference for two consecutive reflected waves are given by

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G reflects the light down so that it is incident normally on the plates. After reflection, it is transmitted through G and observed in the travelling microscope. If the film enclosed between the lens and plane glass plate is extremely thin, then wedge angle α can be neglected as compared to angle of refraction θ . So for normal incidence the optical path difference between two consecutive reflected waves becomes $2\mu t$. Thus, for this case the conditions for constructive and destructive interference for two consecutive reflected waves are given by

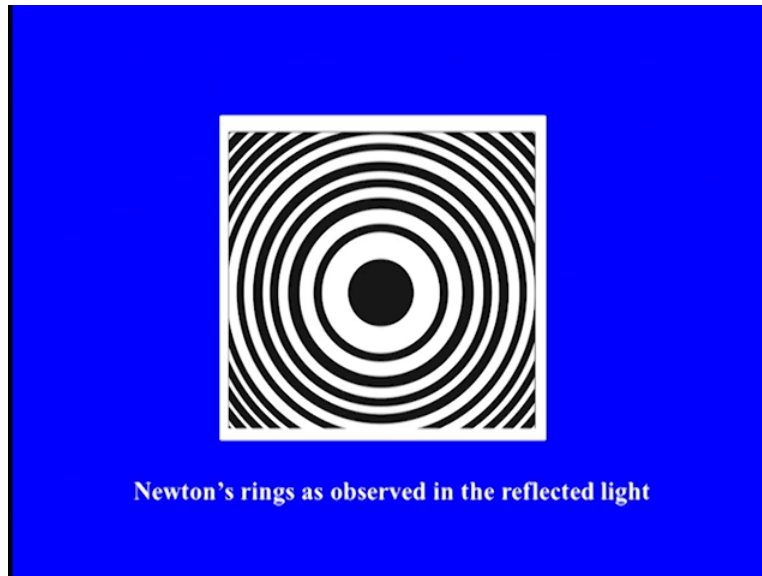
$$2\mu t = n\lambda \quad (\text{Minima})$$

$$2\mu t = (n+1/2)\lambda \quad (\text{Maxima})$$

where, μ is the refractive index of the film and t the thickness of the film. For air film μ will be equal to 1.

$2\mu t = n\lambda$ for minimum and $2\mu t = n + \text{half } \lambda$ for maximum where μ is the refractive index of the film and t the thickness of the film. For air film, μ will be = one. As we know an interference fringe of a given order n is the locus of the points of constant optical path difference.

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A dark fringe of the order n is the locus of those points where the film thickness t is $n\lambda/2\mu$. Since the film enclosed between the convex surface and the plane surface possesses radial symmetry about the point of contact, the dark fringe will be circular in shape. So, here we will observe concentric bright and dark circular colour fringes as shown in this figure. It can be shown that the square of radius of dark ring of n^{th} order is $= n\lambda R/\mu$

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As we know an interference fringe of given order n is the locus of the points of constant optical path difference, a dark fringe of the order n is the locus of those points where the film thickness t is $n\lambda/2\mu$. Since the film enclosed between the convex surface and plane surface possesses radial symmetry about the point of contact, the dark fringe will be circular in shape. So here we will observe concentric bright and dark circular fringes as shown in this figure. It can be shown that the square of radius of dark ring of n^{th} order is given by

$$r_n^2 = n\lambda R/\mu$$

and square of radius of bright ring of n^{th} order is given by

$$r_n^2 = (n+1/2)\lambda R/\mu$$

where, R is radius of curvature of the convex surface.

R divided by μ and square of radius of bright ring of n^{th} order is $= n + \text{half } \lambda R/\mu$, where R is the radius of curvature of the convex surface.

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Thus, when Newton's rings are observed by reflected light, the diameter of the bright rings are proportional to the square roots of the odd natural number 1, 3, 5, 7, etc., whereas the diameters of dark rings are proportional to the square roots of natural numbers 1, 2, 3, 4, 5 etc. It can be shown that the rings gradually become closer as their radii increase as shown in this figure. Since $t = 0$ at the point of contact of the lens and the glass plate, at this point phase difference between the waves reflected from the upper and lower surfaces will be π . Therefore, for the reflected light, in the interference pattern the central spot appears dark.

Thus, when Newton rings are observed by reflected light, the diameter of the Bright rings are proportional to the square root of the odd natural numbers like 1,3,5,7, so on. whereas the diameters of dark rings are proportional to the square root of natural number 1, 2, 3, 4 and so on. It can be shown that the rings gradually become closer as their radii increase as shown in this figure. Since t is $= 0$ at the point of contact of the length and the glass plate at this point phase difference between the waves reflected from the upper and lower surfaces will be π .

Therefore, for the reflected light in the interference pattern, the central spot appears dark. But if it is seen in the transmitted light, the central spot will be bright. Now, I will discuss some important application of Newton's ring experiment. One of the, one example of, is the determination of the wavelength of the monochromatic light used.

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Determination of wavelength

By measuring the diameters of Newton's rings with the travelling microscope, the wavelength of the monochromatic light employed to illuminate the film can be determined by using the relation

$$\lambda = (D_{n+p}^2 - D_n^2) / 4pR$$

where, $(D_{n+p}^2 - D_n^2)$ is the difference of the squares of the diameters of $(n+p)^{\text{th}}$ and n^{th} dark rings and R is the radius of curvature of the lens.

Determination of refractive index of liquid

By measuring the diameter of Newton's rings of air film and repeating the same by forming a liquid film between the lens and glass plate, the refractive index

So, for this purpose we have to measure the diameter of the Newton rings with the help of the travelling microscope and by using the relations $\lambda = (D_{n+p}^2 - D_n^2) / 4pR$ where $D_{n+p}^2 - D_n^2$ is the difference of the squares of the diameter of the $n + p^{\text{th}}$ and n^{th} dark rings and R is the radius of curvature of the lens. With the help of Newton's ring experiment it is also possible to determine the refractive index of a given liquid.

For this purpose also we have to measure the diameter of the Newton rings with air film as well as that as well as by introducing the liquid in the glass plate and convex lens.

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of the liquid (μ) w.r.t. air can be determined by using the relation

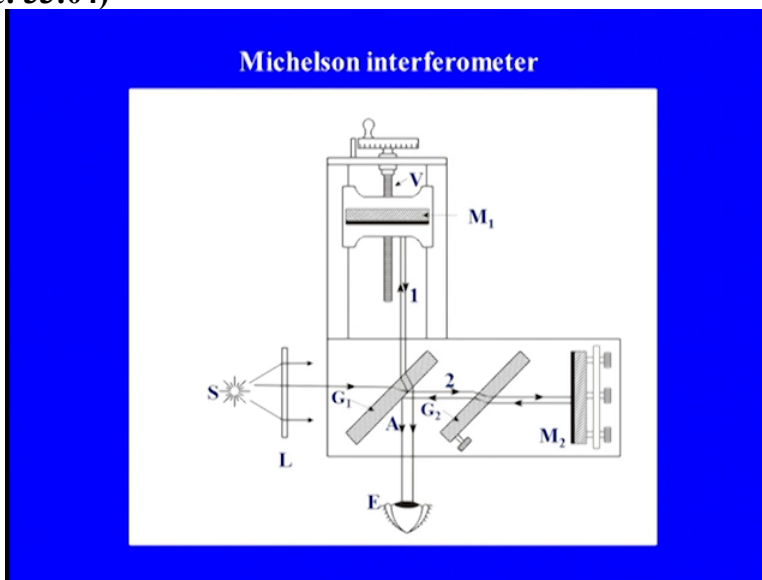
$$\mu = (D_{n+p}^2 - D_n^2)_{\text{air}} / (D_{n+p}^2 - D_n^2)_{\text{film}}$$

And by using the expression $D_{n+p}^2 - D_n^2$ of air divided by $D_{n+p}^2 - D_n^2$ for film, we can determine the refractive index of the liquid. We can show with the help of Newton's ring experiment, we can determine the wavelength of the monochromatic light used.

And we can also measure the refractive index of a given liquid. Only thing is that we have to determine the diameter of the Newton rings precisely by using the travelling microscope.

Now, I will discuss another important experimental setup which is based on division of amplitude is known as the Michelson interferometer.

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The main optical part of this interferometer consists of two highly polished plane mirrors, M_1 and M_2 . Two plane parallel glass plates G_1 and G_2 of equal thickness as shown here in the figure. The mirror M_2 is fixed while M_1 can be moved such that during the motion it remains exactly parallel to its proceeding position. Normally the rear side of the plate, G_1 is lightly silver, so that the light coming from source S is divided into reflected beam one and transmitted beam 2 of equal intensity.

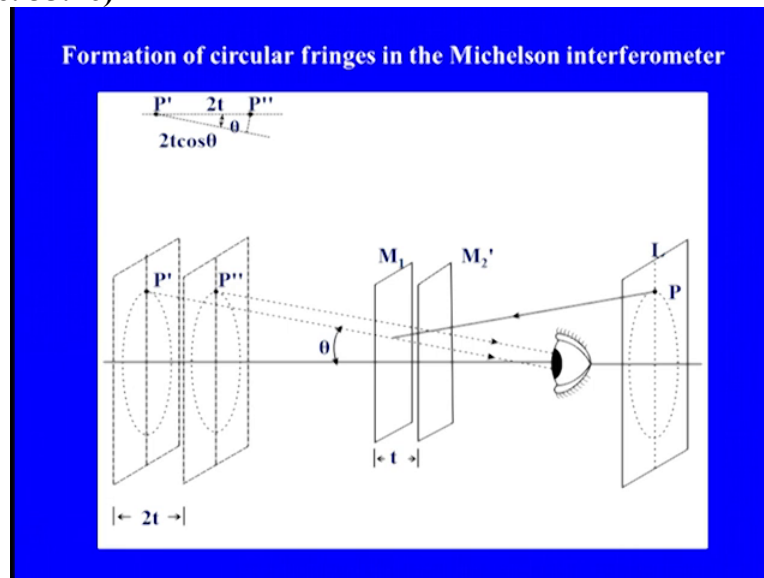
The beams one and two are reflected back from mirror M_1 and M_2 as shown in this figure respectively. The reflected beam 1 is transmitted through G_1 along A and reflected beam 2 is reflected from G_1 along AE . These two waves travelling along AE are derived from the same source by division of amplitude. So, they will satisfy the condition of interference and hence interference fringes can be observed by looking into mirror M_1 through G_1 from position E provided, proper adjustments are made.

Here the glass plate G_2 is called compensating plate. Its purpose is to equalise the total optical path of the beam being reflected from M_1 and the total optical path of the corresponding beam reflected from M_2 . This is essential when white light is used, but not for monochromatic light. In the Michelson

interferometer, fringes of various forms such as straight line, circle, parabola, ellipse, hyperbola, can be observed depending upon the angle between the Mirrors M1 and M2.

But in the laboratory, the interferometer is adjusted for circular fringes for various applications. Therefore, here I will discuss the formation of circular fringes in the Michelson interferometer.

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To obtain the circular fringes, the Mirrors M1 and M2 are made exactly perpendicular to each other with the help of screws attached with the mirror and to see good quality fringes, extended live shows is used. An extended source may be obtained by placing a ground glass screen in front of the source of monochromatic light. The origin of circular fringes can be understood with the help of this diagram.

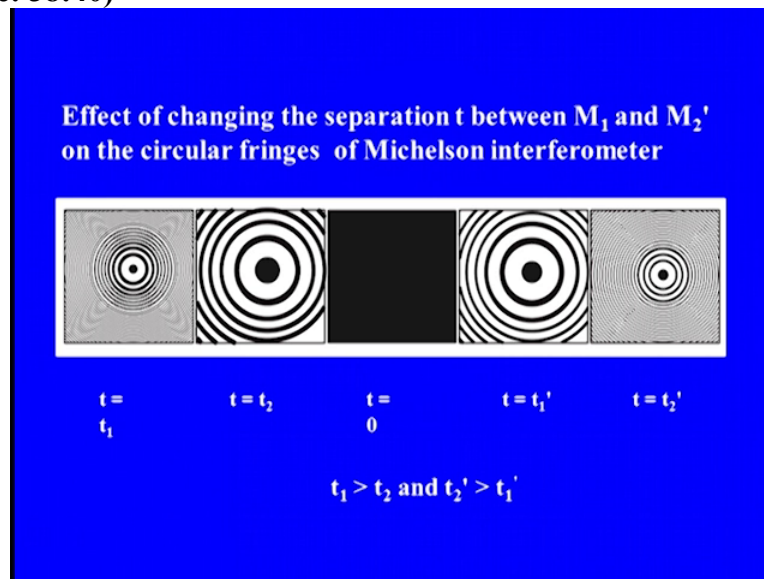
Here M2 prime at the virtual image of M2 formed by reflection in G1. We can also think the extended source as being held behind the observer and forming to virtual images L1 and L2 in M1 and M2 Prime respectively. These virtual sources are coherent, if t is the separation between M1 and M2 prime. The virtual sources will be separated by 2t when t is exactly and integral number of half wavelength that is the path difference 2t is = an integral number of whole wavelength, all the range of light reflected normal to the Mirrors will be in rage.

Rays of light reflected at an angle power bill in general not be in rage the path difference between the two rays coming to the I from corresponding points P prime and P double prime is 2t cos theta as shown here in the figure. The angle theta is necessary the same for the two rays when M1 is parallel to M2 prime so that the rage are parallel. Hence when the I, is focused to

receive parallel rays the rays will reinforce each other to produce maximum for those angle θ satisfying the condition $2t \cos \theta = n \lambda$.

Since for a given $n \lambda$ and t , the angle θ is constant. The maximum will lie in the form of circle about the foot of the perpendicular from the I to the mirrors. Fringes of this kind when parallel beam are brought to interfere with the phase difference determined by angle of inclination θ are called fringes of equal inclination.

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In this figure, effect of changing t on the fringe pattern is shown. When M_1 is moved slowly toward M_2 Prime so that t is decreased; then, the radius of a given rings given ring characterized by a given value of order n must decrease, because the product $2t \cos \theta$ must remain constant. The Rings therefore, shrink and vanish at the centre. The ring disappearing each time t decreases by $\lambda/2$.

This follows from the fact that at the centre $\cos \theta = 1$ then $2t = n \lambda$ so the change n by unity must change by $\lambda/2$. Now, as M_1 approaches M_2 prime, the ring, rings become more widely spaced until finally we reach a critical position where the central fringe has spread out to cover the whole field of view. This happens when M_1 and M_2 Prime are exactly coincidence that is $t = 0$.

Because under this condition, in the path difference is 0 for all angles of incidence if the mirror M_1 is moved still farther, it effectively passes through M_2 prime and new widely spaced fringes appear, growing out from the centre. These will gradually become more closely spaced as the

path difference increases. Now, I will describe some of the applications of Michelson interferometer.

One important up location of this interferometer is the measurement of wavelength of monochromatic light. For the measurement of wavelength, concentric circular fringes are invariably used.

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(a) Measurement of wavelength

For the measurement of wavelength concentric circular fringes are invariably used. So first M_2 is adjusted perpendicular to M_1 to get concentric circular fringes. Now suppose the separation between M_2' and M_1 is such that a bright fringe of the order m is formed in the centre of the field of view. We, therefore, have for the central bright fringe $2t = n\lambda$, since $\cos\theta = 1$. Now we move M_1 such that t increases by $\lambda/2$. In this case the path difference between the normally reflected waves from the centre of field becomes

So, first M_2 is adjusted perpendicular to M_1 to get concentric circular fringes. Now, suppose the separation between M_2 prime and M_1 is such that a bright fringe of the order M is formed in the centre of the field of view. We, therefore have for the central bride fringe, $2t$ is $= n$ lambda since $\cos \theta$ is $=$ one. Now, we moved M_1 such that it increases by lambda by 2. In this case, the path difference between the normally reflected waves from the centre of field becomes to $2t + \lambda$ by 2.

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$2(t + \lambda/2)$ and therefore now we have $2(t + \lambda/2) = (m+1)\lambda$, i.e. the condition of constructive interference is again satisfied at the centre of field but the new bright fringe is of the order $(m+1)$

instead of m . We, therefore, conclude that each time t is increased by $\lambda/2$, in the field of view where a particular bright fringe originally appeared, the neighboring bright fringe of next higher order shall be visible, i.e. there will be a lateral shift of one fringe in the field of view. Therefore, for the displacement of N bright fringes in the centre of field, M_1 must be moved through $N\lambda/2$. Thus, to determine λ , the mirror M_1 is moved from one

And therefore, now we have $2t + \lambda/2 = m + 1 \lambda$. That is the condition of constructive interference is again satisfied at the centre of field. But the new bright fringes now are of the order one + one instead of M . We therefore, conclude that each time t is increased by $\lambda/2$ in the field of view, where a particular bright fringe originally appears. The neighbouring bright fringe of next higher order shall be visible.

That is, there will be a lateral shift of one fringe in the field of view. Therefore for the displacement of n bright fringes, in the centre of M_1 must be moved through $N \lambda/2$.

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position corresponding to micrometer reading x_1 to another position corresponding to micrometer reading x_2 and the number N of the bright monochromatic fringes which cross the centre of the field is counted. So on the basis of above argument, we can write the relation $N\lambda/2 = x_2 - x_1$. From this relation we can compute the value of λ by measuring $x_2 - x_1$.

Measurement of difference between the wavelengths of sodium D-lines:

Another, important application of Michelson interferometer is in determination of difference

Thus to determine λ the mirror M_1 is moved from one position corresponding to micrometre reading x_1 to another position corresponding to micrometre reading x_2 . And the

number of the bright monochromatic changes which cross the centre of the field is counted. So, on the basis of every argument we can write the relation $\Delta x = x_2 - x_1$. From this relation, we can compute the value of λ by measuring $x_2 - x_1$.

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between the wavelengths of sodium D-lines. Sodium light contains two very close wavelengths, which are known as D_1 ($\lambda_1 = 5896 \text{ \AA}$) and D_2 ($\lambda_2 = 5890 \text{ \AA}$) lines of sodium. The difference in λ_1 and λ_2 can be precisely determined with the help of Michelson interferometer. So now I will discuss how this is done in this interferometer. Suppose for a particular position of M_1 , at the central field of view the conditions $2t = n_1 \lambda_1$ and $2t = n_2 \lambda_2$, are satisfied simultaneously. In this case bright ring of order n_1 of wavelength λ_1 practically coincides

Another important application of Michelson interferometer is the determination of difference between the wavelengths of sodium D-lines. Sodium light contains two very close wavelengths which are known D_1 of wavelength 5896 angstrom and D_2 lines of wavelength 5890 angstrom. That difference in λ_1 and λ_2 can be precisely determined with the help of Michelson interferometer.

So, now I will discuss how this is done in this interferometer. Suppose, for a particular position of M_1 at the centre field of view the condition $2t = n_1 \lambda_1$ and $2t = n_2 \lambda_2$ are satisfied simultaneously, where λ_1 and λ_2 are the wavelengths of D_1 and D_2 line of sodium respectively.

In this case, bright ring of order n_1 of wavelength λ_1 practically coincides with the bright ring of order n_2 of wavelength λ_2 leading to maximum visibility at the centre.

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with the bright ring of order n_2 of wavelength λ_2 leading to maximum visibility at the centre. Due to difference in λ_1 and λ_2 , as the separation between M_1 and M_2' gradually increases the maxima at first fall farther and farther from coinciding, ultimately at a particular separation the maxima of one practically coincides with the minima of other wavelength and vice-versa, thereby producing minimum visibility. This happens when the extra path difference introduced at the centre due to motion of M_1 contains one half wavelength more of radiation λ_2

Due to difference in λ_1 and λ_2 at the separation between M_1 and M_2' , gradually increases the maxima at first fall further and further from coinciding and definitely at a particular separation the maxima of 1 practically coincides with the minima of other wavelength and vice versa thereby reducing minimum visibility. This happens when the extra path difference introduced at the centre due to motion of M_1 contains 1 half wavelength more of radiation λ_2

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than that of the other. Continuing the motion of M_1 in the same direction again gradually brings the maxima of two patterns close together. Ultimately they again coincide at the centre when the extra optical path difference $2t_0$ introduced at the centre contains one more wavelength of one radiation than of the other. The visibility of the fringe is again maximum and the conditions $2(t + t_0) = (n_1 + N)\lambda_1$ and $2(t + t_0) = (n_1 + N + 1)\lambda_2$ will be satisfied. Here, it has been assumed that $\lambda_1 > \lambda_2$ and t_0 is the displacement of M_1 between successive visibility of maxima and N represents the number of bright fringes of λ_1 emerging at the centre due to displacement t_0 of mirror M_1 . Therefore, we can also write $2t_0 = N\lambda_1$ and $2t_0 = (N+1)\lambda_2$. From these

than that of the others. Continuing the motion of M_1 in the same direction again, gradually brings the maxima of two patterns close together. Ultimately, they again coincides at the centre when the extra optical path difference $2t$ naught introduced at the centre, contains one more

wavelength of one radiation than of the other. The visibility of the fringes is again maximum and the conditions $2t + t_{\text{naught}} = n_1 \lambda_1$ and $2t + t_{\text{naught}} = n_1 + 1 \lambda_2$ will be satisfied.

Here, it has been assumed that λ_1 is greater than λ_2 and t_{naught} if the displacement of M_1 between successive visibility of maxima and capital N represent the number of bright fringes of wavelength λ_1 emerging at the centre due to displacement t_{naught} of mirror M_1 . Therefore, we can also write $2t_{\text{naught}} = N \lambda_1$ and $2t_{\text{naught}} = N + 1 \lambda_2$.

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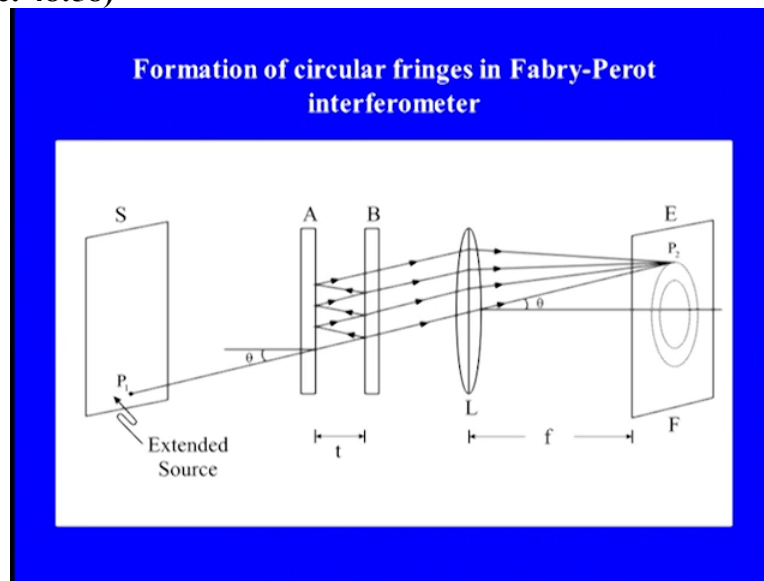
two equations we have $\lambda_1 / \lambda_2 = (N+1)/N$. Here, it is very difficult to count N . But N can be determined from relation $N = 2t_0 / \lambda_1$. Using this we can write $(\lambda_1 - \lambda_2) / \lambda_2 = 1/N = \lambda_1 / 2t_0$. Finally from this we can get $\lambda_1 - \lambda_2 = \lambda_1 \lambda_2 / 2t_0 = \lambda_{\text{av}}^2 / 2t_0$, where λ_{av} is the average of λ_1 and λ_2 . Thus, by measuring the positions of M_1 for two successive visibility maxima in the central field of view $\lambda_1 - \lambda_2$ can be determined. Since visibility minima can be distinguished more accurately than the maxima. The value of t_0 is, therefore, determined by the positions of M_1 for two successive minimum visibility positions.

From these two equations, we can write $\lambda_1 / \lambda_2 = N + 1 / N$. Here, it is very difficult to count N , but N can be determined from the relation capital N is equal to $2t_{\text{naught}}$ divided by λ_1 . Using this, we can write $\lambda_1 - \lambda_2$ divided by $\lambda_2 = 1 / N$ and which is $= \lambda_1$ divided by $2t_{\text{naught}}$. Finally from this, we can write $\lambda_1 - \lambda_2 = \lambda_1 \lambda_2 / 2t_{\text{naught}}$ which will be nearly $= \lambda_{\text{average}}^2$ divided by $2t_{\text{naught}}$;

Where λ_{average} is the average of λ_1 and λ_2 , thus by measuring the positions of M_1 for two successive visibility Maxima in the central field of view, $\lambda_1 - \lambda_2$ can be determined. Since visibility minima can be distinguished more accurately than the Maxima, the value of t_{naught} is therefore determined by the position of M_1 for two successive minimum visibility positions.

Further, in order to minimize the mirrors, normally the position of mirror M1 is noted for 10 successive visibility minima and from this the separation between value t naught can be determined more accurately. Now, I will discuss another important interferometer which is known as the Fabry Perot interferometer.

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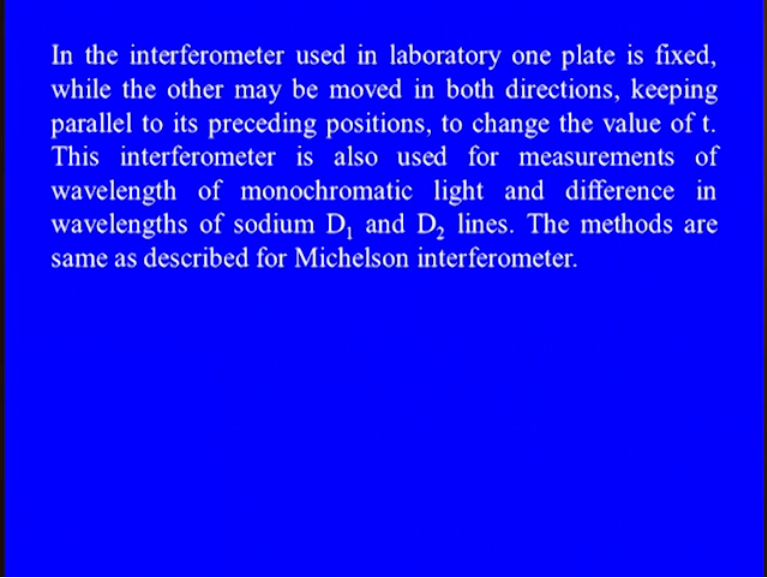


In this interferometer, the interference fringes are produced in the transmitted light after multiple reflections in the air film between two plane glass plates thinly silvered on the inner surfaces as shown in the figure. To observe the fringes a broad source of monochromatic light is used. Suppose a ray from the point P1 on the extended source is incident at the angle θ , this will produce a series of parallel transmitter rays at the same angle which may be brought together at the point P2 on the screen if with the help of a lens.

The condition for reinforcement of the transmitter is $2t \cos \theta = m \lambda$ where m is integer for air film of thickness t . This condition will be fulfilled by all points on a circle through P2 with their centre at O, the intersection of the axis of the lens with the screen EF. When the angle θ is decreased, the cosine will increase until another maxima is reached for which m is greater by 1, 2, 3, and so on so that we have for the Maxima a series of concentric rings on the screen with O, their Centre.

These rings are known as fringes of equal inclination. In this case, effect of changing t on circular fringes is same as for as we have discussed for Michelson interferometer. In this inter, in the interferometer used in the laboratory, one plate is fixed.

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In the interferometer used in laboratory one plate is fixed, while the other may be moved in both directions, keeping parallel to its preceding positions, to change the value of t . This interferometer is also used for measurements of wavelength of monochromatic light and difference in wavelengths of sodium D_1 and D_2 lines. The methods are same as described for Michelson interferometer.

While the other might move in both direction keeping parallel to its preceding position to change the value of t . This interferometer is also used for the measurement of wavelength of monochromatic light and difference in wavelength of sodium D_1 and D_2 lines. The methods are same as described above for the Michelson interferometer. So, let me summarize what I have discussed in this lecture.

So, in this lecture I have discussed how to get interference pattern by division of amplitude by taking example of thin parallel films of constant thickness as well as taking the film of variable thickness. And I also discussed how this colour effect is observed through thin films. And I discuss three important experiments which are usually performed in the laboratory.

Like a Newton ring experiment, Michelson interferometer and Fabry Perot interferometer and how to determine the wavelength of monochromatic light, by using these experimental setups.

Thank you