Quantum Physics
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Lecture No. # 23

The values of the total angular momentum quantum number of the electron are 3/2's and 5/2's.

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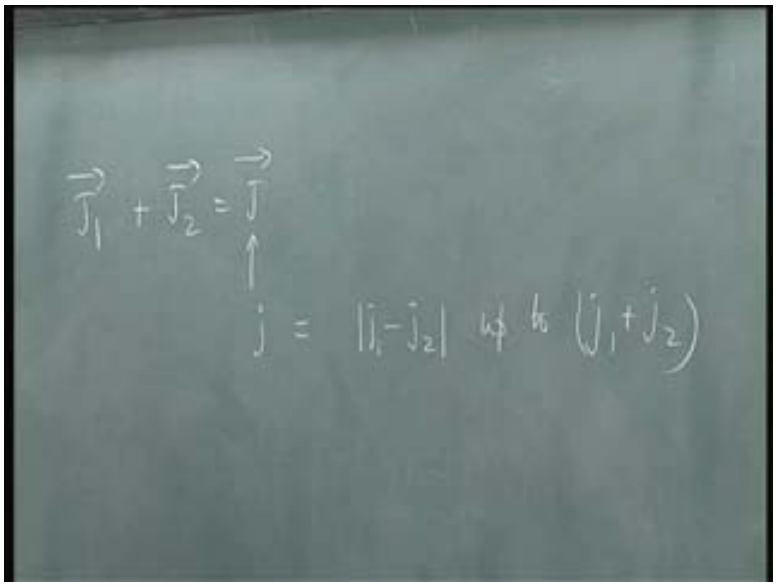

I haven't proved this but I made the statement that if you took 2 angular momentum; J 1 and J 2 and defined that to be the total angular momentum J, then this J runs from modulus j 1 - j 2 up to j 1 + j 2. i didn't prove the statement but its easy to see that its plausible from the rules of what happens when you take the raising operator and act on states and so on.

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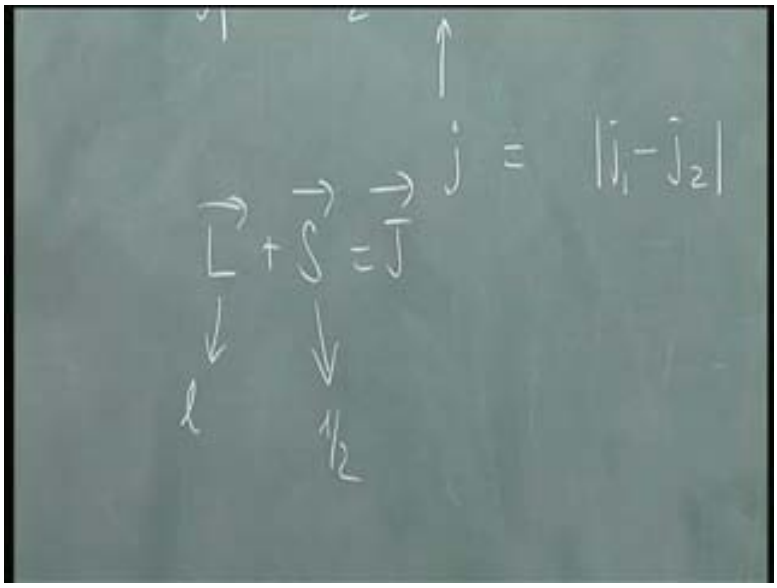

But in the present instance, for an electron L + S is the total angular momentum, J the quantum number of L is l and that of S is always a ½. The spin of the electron is a 1/2 and therefore if this l = 2, then the only allowed values are 3/2's and 5/2's. So this statement is true. What happens is l was 0? If you were in the S state of the electron in an atom, what happens then? The only allowed values are 1/2 because you have to take modulus j1 - j2. So the only allowed value is ½. Then the total angular momentum is in fact just the spin angular momentum of the electron.

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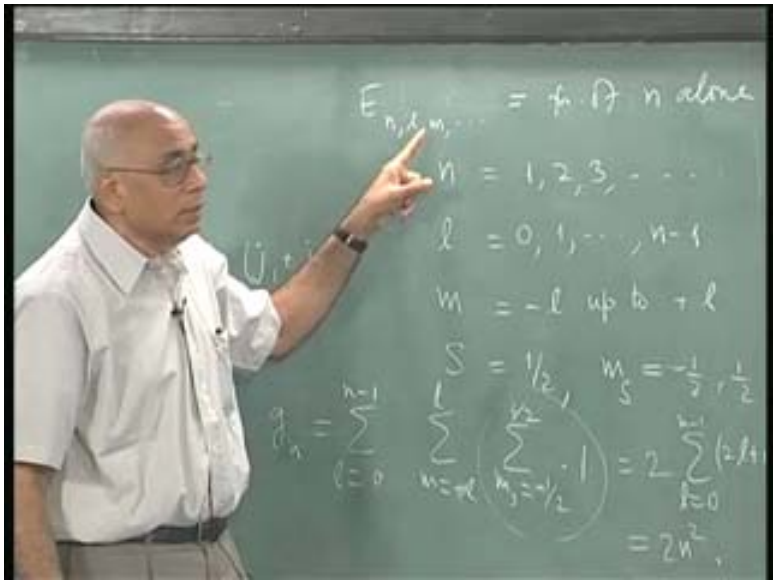

As you know in the hydrogen atom which I have assumed you studied in the chemistry class, the states of the electron for a hydrogen atom are specified by the principle quantum number n which takes on the values 1, 2, 3, etc and then an orbital angular momentum quantum, l and this runs over the values 0, 1 up to n – 1. So when n = 1, you have only the possibility of l = 0. So you have the 1s state and then the 2s 2 p states and so on.

The magnetic quantum number m which is the quantum number corresponding to any component of the orbital angular momentum takes on values from - l up to + l in steps. And then there is a spin quantum number and of course we know that the spin quantum number S is always a 1/2 and the projection, denoted as m_S is = - 1/2 and a ½. These are the only 2 possibilities. and now you could ask what the degeneracy of the electron is when a given state with n, l, m etc. the general state with principle quantum number n and that degeneracy factor g_n so happens that for the 1 over r potential, the Coulomb potential energy levels depend only on n and not on any of these quantum numbers unless you break degeneracy by including some other interaction.

For instance you put a magnetic field, that will couple to the spin of the electron and there is magnetic potential energy - mu dot v and that will distinguish between + 1/2 and - 1/2 for m_s and so on. There are other factors which I will come to which will also tell you that the quantum numbers could depend on l, m etc. But if you don't have any of that then this gn, since En,l,m and the other quantum numbers is a function of n alone for the hydrogen atom, its actually - 1 over n squared in Rydberg units, therefore this gn is = the number of possibilities over m_s = - 1/2 up to + 1/2 times 1 because that's the number of possibilities you are counting, the number of possibilities for m = 0 - l up to + l and then you have to count the number of possible values of l from 0 to n - 1 and this is = gn. This straight away gives you factor 2. So this is = twice this and this gives you 2 l + 1. So you have to sum l = 0 to n – 1, 2 l + 1 and this is = 2 n squared (Refer Slide Time: 06:02 to 06:10). So that was asked as a separate question and that was in fact in one of the fill in the blanks. It said the degeneracy of the electron in the hydrogen atom is g_n taking into account its spin is 2 n squared. If you didn't take the spin into account, if you ignore that internal degree of freedom, then it's just n squared. Now if you switch on a magnetic field, the following is going to happen. there will definitely be a coupling between the intrinsic magnetic moment of the electron and the external field that will break this degeneracy in m_S. And then if you have any potential which is not central then there is also a dependence on m because if it's a central potential, no axis in space is distinguished from anything else and therefore there is degeneracy over m.

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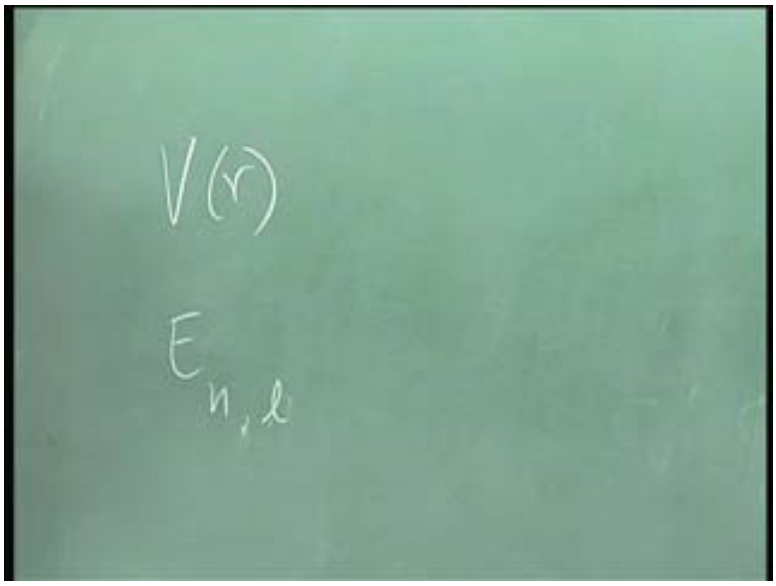

In general, if you had a central potential alone V (r), which was not a coulomb potential or some very special potential like the harmonic oscillator potential but an arbitrary general central potential with no special symmetries whatsoever, then what would the degeneracy be for a particle of spin ½? There is a factor 2 which comes from the spin. But over and above that, there is a degeneracy in m. so in such a case, the energy levels would be a function of n as well as l and what would be the degeneracy of each of these states for a given n and given l? It would be 2 l + 1. all the m values would give you exactly the same energy. If you included spin, its twice 2 l + 1 in that case. Now what happens in practice even in the hydrogen atom is that there is an extra contribution called the spin orbit coupling.

There is a coupling between the spin of the electron and its orbital angular momentum in the following sense. It's actually a more complicated effect. But it's as if, if you pretend for a minute that you are on the rest frame of the electron, you still have an intrinsic angular momentum S or a magnetic moment. you see the proton going around you which is a current loop and therefore there is magnetic dipole moment associated with the orbital motion and that will be proportional to the orbital angular momentum the magnetic moment which was l and therefore there is going to be a term in the Hamiltonian proportional to l dot S. that's called lS coupling and the moment you have this or spin orbit coupling and the moment you have an l dot S coupling, you have broken the symmetry which the coulomb potential has. therefore the energy levels become dependent on l. in other words, this (Refer Slide Time: 09:37) accidental degeneracy that these energy levels don't depend on l is gone out and there is in fact a difference in energy between the different l states for a given n.

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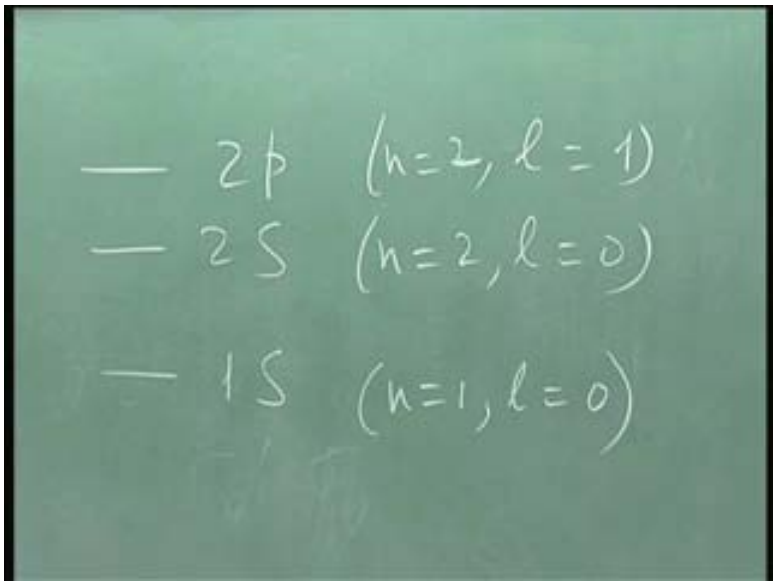

And that's the reason why the spectrum of the hydrogen atom looks like 1S. This corresponds to n = 1, l = 0. I am ignoring now. So there is no external field and therefore no dependence on the spin at all. And then there is a 2S state and 2p state. This corresponds to n = 2, l = 0, n = 2, l = 1. These states are split. They would not be split if you didn't have a spin orbit coupling. Both these would have exactly the same energy. And then of course there is a 3S, 3p and a 3d state and so on. They are all split because of spin orbit coupling. As the effective split is extremely small compared to the actual energy value of the eigenvalue of either the 2 S or the 2 p state, there is a small splitting. But as you go up in the periodic table and as the atomic number z value of the nucleus increases, it turns out that the spin orbit coupling becomes more and more prominent and there comes a stage when it's quite significant.

In fact, it is so significant and there are other effects as well, there is an effect called the hyperfine splitting which arises due to the coupling between the magnetic moment of the electron and the magnetic moment of the nucleus itself. And that's an even smaller effect but as z increases, this becomes sufficiently large so that you have significant amount of splitting. And then this neat ordering of these energy levels; the S state, the p state and then the d state and so on is gone. There could be the f state, maybe lower in energy, then the d state and so on. This happens when you reach the transition metals and is responsible for a lot of interesting effects. but the important point I want to make is that both the spin orbit coupling as well as the effect of an external magnetic field serve to break the symmetry that the original hydrogen atom has the coulomb problem has. And so the general lesson is the application of a field breaks symmetry in some sense. After all, this space in this room is isotropic but all the directions are not equivalent because there is a vertical direction in which there is a gravitational field that breaks the translational symmetry. And that's the reason if you have a particle moving around in this room under the effect of this gravity, the x and y components of its momentum are conserved in the classical picture because there is no force in that direction but the z component certainly is not.

So linear momentum consolation is broken in one direction because of the symmetry breaking field.

The next question was the particle mass moving in 1 dimension is incident upon a delta function barrier, not an attractive delta function but a barrier and the statement was the reflection coefficient of the particle is identically = 0. This is certainly not true. We saw explicitly that there is a definite effect of the barrier even though it's an infinitely thin barrier. The fact is the potential goes to infinity at that point and therefore there is a finite reflection coefficient. we compute at this reflection coefficient as a limiting case of what happens in a rectangular barrier. What you have to do is to take the product of the width of this barrier times the height, retain that to be a finite constant and then you can compute what it is in the limit. The second statement is false. The only normalizable eigen state of the raising operator a dagger of the linear harmonic oscillator is the ground state of the Hamiltonian. Is this true or false? It's false because certainly, the ground state of the Hamiltonian is not an eigenstate of a dagger. And in fact, a dagger has no normalizable eigenstates whatsoever in the space of square integrable functions. What about a? Does that have eigen states? The ground state of the Hamiltonian is also is an eigenstate of a. but then, all coherent states are eigen states of a. now there are several reasons why you call it a coherent state. I will mention some of these a little later. I am going to talk about coherent states specifically. But right now, by coherent state I mean an eigen state of the lowering operator a for the harmonic oscillator. So, one should really call them harmonic oscillator coherent states if you like. So you have been careful to do that every now and then. They are also minimum uncertainty states. As you saw that in suitable units, delta x and delta p are each = 1 over square root of 2. They are Gaussian wave packets in the position representation or in the momentum representation. But they are not the ground state of the harmonic oscillator itself but displaced with respect to the ground state. There are several ways of defining coherent states and they all happen to coincide in this particular instance.

The next question was the angular momentum commutation relations together with a requirement that the eigenstates of the angular momentum be normalizable suffice to determine the possible eigen values of j squared and j dot n where n is a unit vector along any arbitrary direction. This is true. We saw that this was so but the important thing is to put in the normalization condition and that was true even for the harmonic oscillator because i required normalization on the states and that's how I got a discrete spectrum. If you don't require normalizable eigen states, then there is no guarantee that the spectrum is discrete. In a more general family of wave function or eigen states, you could have a continuous spectrum for the same operator. So again in these cases, given an operator its spectrum is decided by what class of eigenstates you would like to look at.

You would like to have for physical reasons. We want normalizability for the conservation of probability and the interpretation of quantum mechanics. But conceivably mathematically they could be more. There could be a continuous spectrum with non-normalizable states. So that statement was true. A particle moves in 1 dimension in a symmetric potential V (x) is V (- x) .all the energy levels of the particle are discrete. So you are given that you only have bound states all the wave functions die down exponentially fast at infinity on both sides. The position space wave function of the particle in the ground state phi $_0$ of x is an even of function of x. True. In this

problem, given that the potential is V (x) = V (- x) in 1 dimension, there is no degeneracy everything is a discrete spectrum and there is nothing no degeneracy. There are no other functionally independent constants of the motion. Nothing other than the Hamiltonian. Because classically you know that saying the Hamiltonian H of qp = constant fixes the phase trajectories. You can't have a second independent constant of the motion functionally.

Quantum mechanically it means you cannot have another operator other then the Hamiltonian or some function of the Hamiltonian which commutes to the Hamiltonian. So the levels are non-degenerative. And then if the potential is symmetric the Hamiltonian commutes with the parity operator and therefore you can find a complete set of common Eigen states of the 2. Now the parity operator has Eigen states all even functions and all odd functions. But the Hamiltonian has special Eigen functions. They form a complete set in the space. So the conclusion is every eigenstate of the Hamiltonian is also a parity Eigen state. Which means every Eigen state of the Hamiltonian is either an even function of x or an odd function of x in the positions space representation. In the ground state there are no nodes. If you write down the Schrodinger equation, every time the function crosses the axis, it has to go up come down again go to 0 on both sides. It costs energy because there is a second derivative term in this Schrodinger equation. So the lowest energy state would just start of from 0 at - infinity it goes up and it comes down and it's normalizable. It has no nodes. by the requirement that it should be either in even or odd state, since it never crosses the axis it is mostly in even state. So that is a true statement.

Consider the set of functions phi n of x = the oscillator Eigen functions. This is the normalization constant e to power - f squared Hn of x where Hn is the Hermite polynomial. Any arbitrary square integral function of x in - infinity infinity is any l2 function can be written as a linear combination of the elements of the set in a unique manner. True or false? This is true because this set forms an orthonormal basis. Once it's an orthonormal basis and any function in that function space can be written uniquely. So the coefficients determine the function completely. What happens if you didn't have, suppose for a minute I looked at l, 2 from 0 to infinity instead of - infinity to infinity. Let's say we looked at the solutions of the 3 dimensional Schrodinger equations and I want the radial part of the wave function. The radial coordinate r and from 0 to infinity you would still require square integrability. So you would now want from 0 to infinity the function mod square etcetera is finite. What would happen in that case? Would be Hermite polynomials form a complete set still? Would they be orthogonal? Do you think so? Would they be a good basis to expand things in? They may not be unique. Do you think so?

First of all we have this weight factor e to the power - 1/2 x squared. Hn of x is a polynomial. That grows at infinity. But e to the power - 1/2 x squared dies down faster than any power and therefore you guaranteed that all these polynomial would be integrable when you square it. But if you function if you are going to variable is going to run from 0 to infinity in x for example then you don't need e to the power - x squared e to the power - x would do. You put the x squared because it should be it - infinity too. And I don't want a mod x because that would mean there is cusp at the origin and so on. So that's one reason why we have e to the power - x squared appearing there naturally. But from 0 to infinity e to the power - x would do. And what do you call that set of functions with the weight factor e to the power - x integrable? They are the Laguerre polynomials. That's why that appears in the hydrogen atom problem in the natural way.

Consider a particle with spin quantum number ½. The particle can never be in a spin state in which the uncertainty product delta Sx delta Sy is 0. There were mixed results from this question. So let me explain what's happening.

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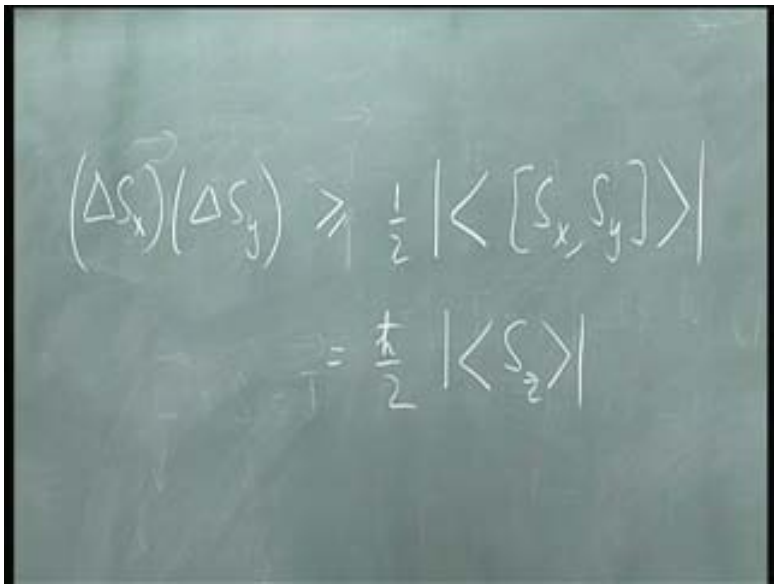

We would like to find out what's delta Sx delta Sy in any state. In any arbitrary state, remember that the uncertainty principle says that this is greater than or = 1/2 the modulus of the commutator of Sx with Sy. But this is = h cross over 2. This is ih cross S z times the expectation value of Sz in an arbitrary state. How big can Sz be? No matter what state you are in, that can't be bigger than ½. Because the Eigen values Sz would be either + 1/2 or - 1/2 in units of h cross. So this is finite. Can it be 0? Yes of course. Classically, you can see that if this is your axes of quantization and you put your spin along the x axis, it's an Eigen state of Sx for example. Then the expectation value of Sz is 0. Just as when you put it in an Eigen state of Sz, The expectation value of Sx and Sy are 0. In exactly the same way, you could have it in a state where the expectation value of this is 0. Classically, this would mean the spin quote and quote lies along the x in the xy plane. There is no z projection. So the least value of this quantity in fact is 0. So the statement is actually false. It can definitely be in such a state. So you see the significance of the generalized uncertainty principles. Really you should go back to this algebra and see look at here and this could be 0. It can't be negative because the product of uncertainties can be never being negative. It could be 0 in some situations. Again the harmonic oscillator functions but I will put a tilde and put a p there. Otherwise it's exactly the same function as on the position basis. Are Eigen functions of the Fourier transform operator?

The answer is yes. They definitely are because this is the momentum space Eigen function of the harmonic oscillator. The position space Eigen function had exactly the same functional form but we know the momentum space Eigen function is a Fourier transform of the position space function. Therefore here is a set of functions whose Fourier transforms are essentially constants

time the original functions. What can those Eigen value is be? There are the fourth roots of unity. Because we know that the fourth power of the Fourier transform operator is the identity operator in this function space. Those Eigen values can only be 1 i - 1 and – i. the ground state of the oscillator corresponds to Eigen value 1. That we know follows also from the fact that the Fourier transform of Gaussian is a Gaussian and with suitable normalization coefficients, it's exactly the same function.

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Consider the 1 dimensional potential barriers V (x). Let the reflection and transmission coefficients be R and T. so I have in arbitrary potential barrier and i would like to find out for incidence from the left, there is reflection R and there is transmission T.

and now you are asked suppose you started from the right, there would be a reflection R prime and a transmission T prime and the question is: is R = R prime and T = T prime? Now the way we derived this expression was simply to say e to the power ikx was incidence from the right e to the power - ikx should been incident) from the right and e to the power - + ikx should mean going rightwards from the left. The way we derived this entire expression had nothing to do with the directions in which we moved. It's completely arbitrary all we did was to say that it's a plane wave asymptotically on either side. So in fact the reflection and transmission coefficients are identical. it doesn't matter which direction you have been incidence. It doesn't require the symmetry property of the potential because we never use that. So do it for a non-symmetric potential and you will discover that this is exactly the same answer.

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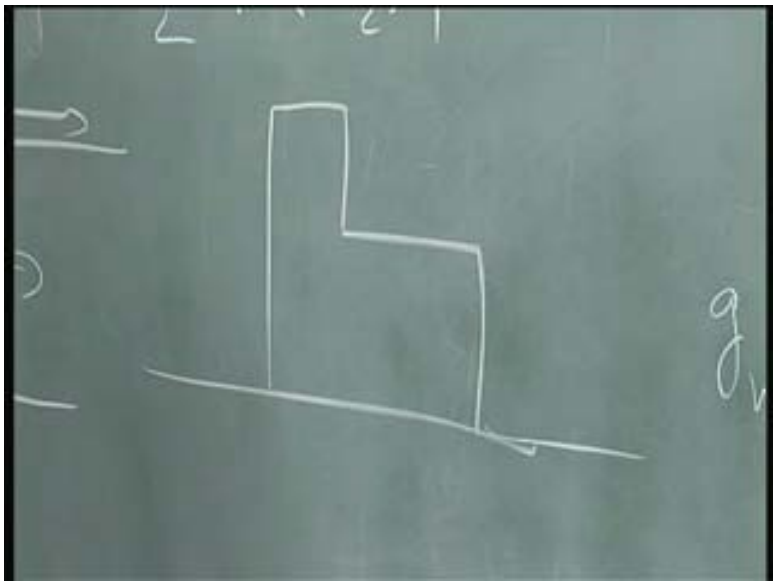

So here is a simple potential which would look like this. Do it for a potential of this shape. It would remain exactly the same thing. We want to make sure the potential vanishes at infinity. So i said there is a finite barrier. This is just a barrier at some point and it definitely doesn't mean that the potential extends. If it does, then the energy has to be greater than that value. Otherwise you can't even have such a state.

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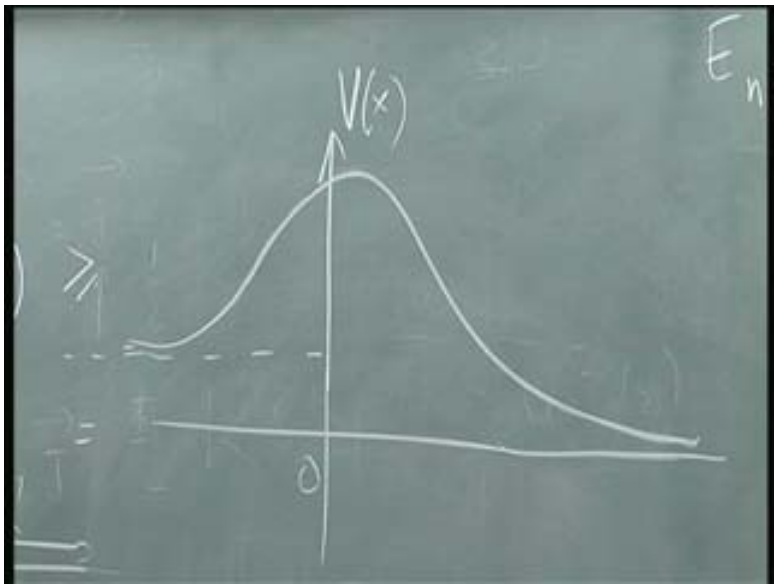

So his point is if the potential were to be a barrier like this and this was 0 of energy you can't have an energy state below this value. To start with it, it has to be above that (Refer Slide Time: 28:24). And of course it's easy to see what would happen if you had a lack of symmetry. Suppose you have V (x) looking like this. Now we computed things because they are analytically computable. We computed things where the potential barrier was actually compact. Beyond a certain stage on either side there was no potential at all. But that's not absolutely essential. You could have a situation like this. This asymptomatically goes to 0 and these asymptotes to some finite value here. One could look at this barrier as well. It's just that it's only at - infinity and + infinity that you apply these plane wave conditions otherwise it's not necessary a plane wave at all the function Eigen function can be quite complicated. But for computing R and T you don't need to know the exact potential except the formal expressions that you have when you compute the numerical values of the coefficients which would depend on the potential. You need to solve the Schrodinger equation here.

Let j be the total angular momentum quantum number of a system. Then in any arbitrary state psi of the system the operator J squared must necessarily have the expectation value h cross squared J times J + 1. What we are actually saying is if it has definite value then it's an eigen state of J squared. So we are really saying that an arbitrary state of the system is an Eigen state of the total angular momentum. True or false? It's true. You haven't given any other conditions. So it's certainly true. So the idea is that you have maybe an orbital part. You have other in degrees of freedom. So the total wave function is a product of the spin wave function and the spatial wave function for instance and it's always an Eigen state. Just like an electron no matter what you do to this electron, its spin quantum number is ½. Therefore you are guaranteed that if you measure S squared, the answer is going to be 3 quarters h cross squared. So that is certainly true always. (Conversation between student and professor :). That's a good point he has raised.

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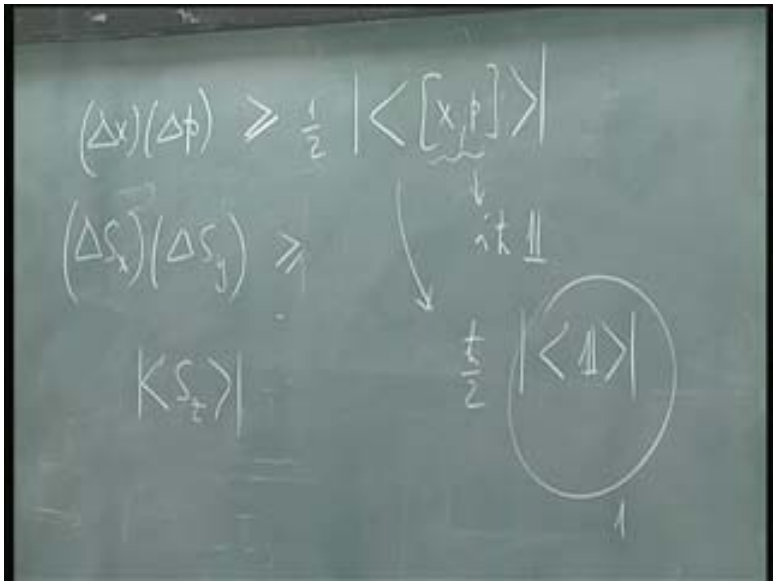

In the normal course of events, if you took a Cartesian coordinate delta p, this is certainly greater than or = 1 1/2 the modulus of the expectation value of x with p. but it so happens that this is = ih cross the times unit operator and therefore, this thing here becomes h cross over 2 the modulus of the expectation value of the unit operator. But the expectation value of the unit operator is by definition, 1 no matter what you do. That's the reason you get the usual commutation relation saying it can never be 0. its greater than or = 1/2 h cross. But in the spin case, this commutator is Sz. so what you have to deal with is this quantity <Sz> and this could be 0. So that's the reason its different from what it is for the usual pair of Cartesian coordinates. So I will give a problem where you work this out explicitly and you will see this. The next 1 was fill in the blanks and the first question was a simple question but was a little bit of trick.

It said a particle moves in a 1/2 oscillator potential.V(x) is given to you in any case- 1/2 m omega squared x square and infinity. For x greater than 0, it's the oscillator. For x less than 0, there is a wall. so its like attaching a spring to 1 end to the origin, the other end to this particle and letting this particle vibrate but not fully along the x axis but whenever it hits the x = 0, it bounces back. So in this 1/2 oscillator potential the question asked is: what's the ground state of this problem.

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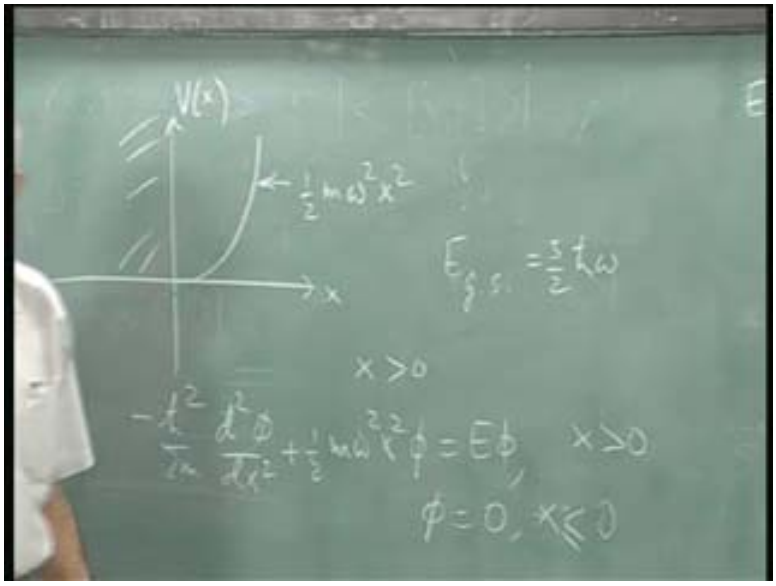

So the potential is infinite here. This is 1/2 m omega squared and squared and its infinite for x less than 0 (Refer Slide Time: 33:54). Now, you are asked what the ground state energy Eigen value is and the eigen functions. Now you see the earlier problem which you solved the oscillator problem at the Hermite polynomials etcetera and you used to boundary conditions at - and + infinity the wave function went to 0. Now it's exactly the same differential equation but it applies in the region x greater than 0. so in this region, you have the equation - h cross squared over 2 m d 2 phi over dx 2 + 1/2 m omega squared x squared phi = E phi and this is for x greater than 0. Since the potential is infinite, the wave function is 0 in this region including at this point

(Refer Slide Time: 34:54). So when you solve this and you impose the condition of square integrability that the function be finite at infinity the energy levels turn out to be discrete as before the solutions of a Hermite polynomials but they would be those Hermite polynomials which vanish at the origin. They are the odd ones that vanish at the origin. The even ones don't vanish at the origin. Therefore they can't be solutions to this problem. What's the lowest energy value? Its 3 1/2 h cross omega. So this spectrum has all the odd eigenvalues only. So the lowest energy value E ground state = 3 1/2's h cross omega purely from the boundary condition. So the even ones are not allowed in this case and it picks out a way other one. So by the way that now is the set that forms a complete set. So only the odd ones are sufficient they form a complete set once again.

So, if you insist on the weight function e to the power - x squared instead of e to the power – x, then you don't get the full set of Hermite polynomials provided you impose the boundary condition. You look at a class of function which vanishes at the origin. Then e to the power - x squared times the odd Hermite polynomials form a complete set. What happens if the spring constant becomes negative? What happens if the potential is - 1/2 constant times x squared?

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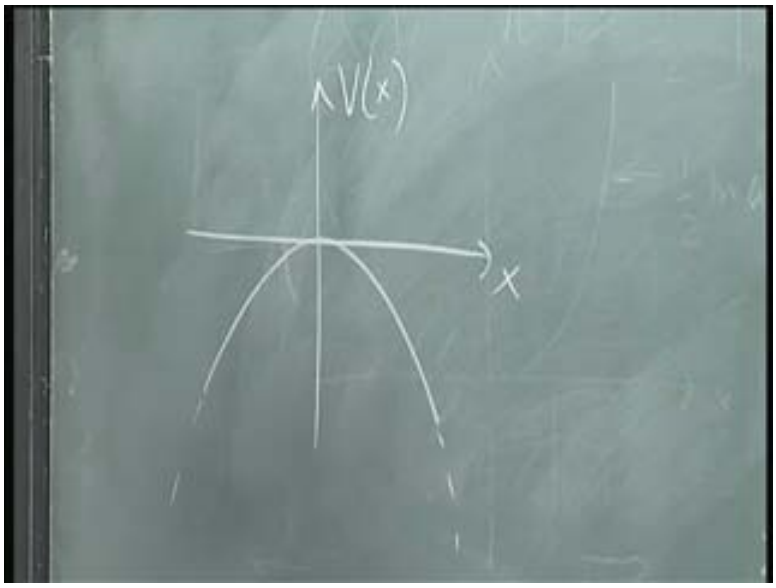

This is your potential. Can you have bound states in this problem? There are normalizable Eigen functions of the Hamiltonian. No bound states at all. Classically the origin is an unstable equilibrium point. It just falls off from the origin. So corresponding to that in quantum mechanics there are no bound states. The general lesson is that bound states would correspond to periodic solutions in the classical case. Anything that is periodic becomes a bound states in quantum mechanics. No periodic orbits are allowed here. The next was the Hamiltonian of a perturbed oscillator is given by the usual h cross omega a dagger a + 1/2 + lambda times a + a dagger and the question is: what's the exact value of the ground state? This is just the shifted oscillator. All you have to do is to recognize that you have shifted the oscillator because a + a

dagger is x. what you have d1 is to take p square + x square + put an x term so complete squares. It's like shifting the oscillator and changing the 0 of the energy. All you are asked to find is: what's the change in the 0 of the energy.

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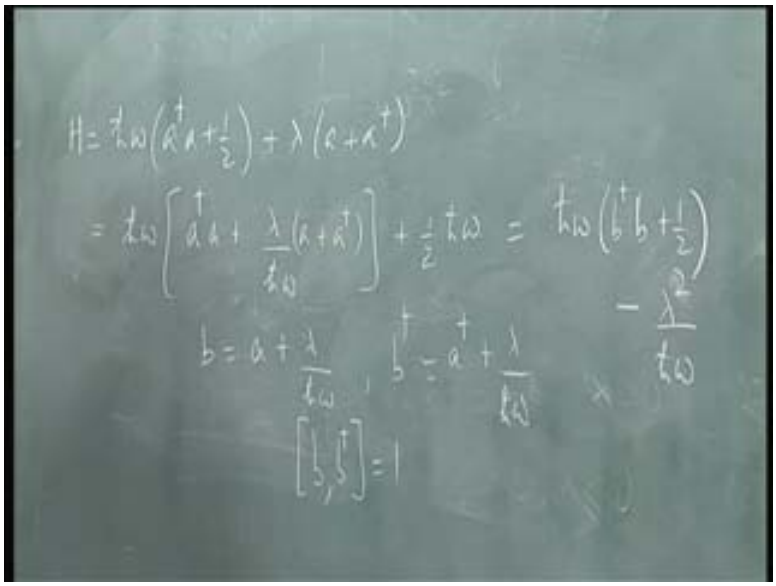

So that's easily found because you have h cross omega a dagger + 1/2 + lambda a + a dagger. Notice lambda is real and this is a Hermitian quantity. So the Hamiltonian is Hermitian and the Eigen values are guaranteed to be real. So you could write this as h cross omega times a dagger a + lambda over h cross omega a + a dagger + ½ h cross omega. This is the 0 point energy level. So all you have to do is to define b = a + lambda over h cross omega. So b dagger is a dagger + lambda by h cross omega. The important thing is the commutator of b with b dagger is still = 1. That's all that decides the spectrum. So this whole thing becomes = h cross omega b dagger b + a 1/2 - lambda squared over h cross omega. If I complete squares, there is an extra term i have added which is lambda squared over h cross square omega squared but there is 1 sitting outside here and that's it. So what's the ground state energy?

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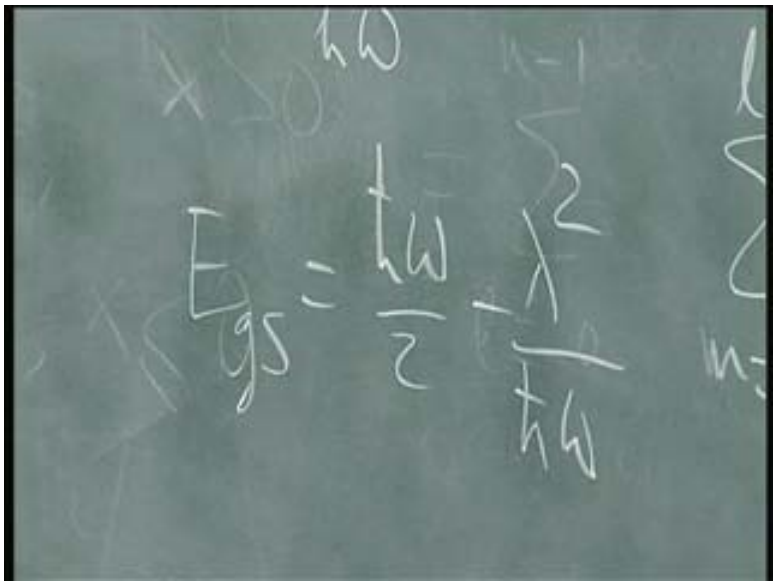

This portion has a ground state energy which is 1/2 h cross omega. So E ground state = h cross omega by 2 - lambda square over h cross omega. That's the exact answer. You just subtract that portion out. That's it.

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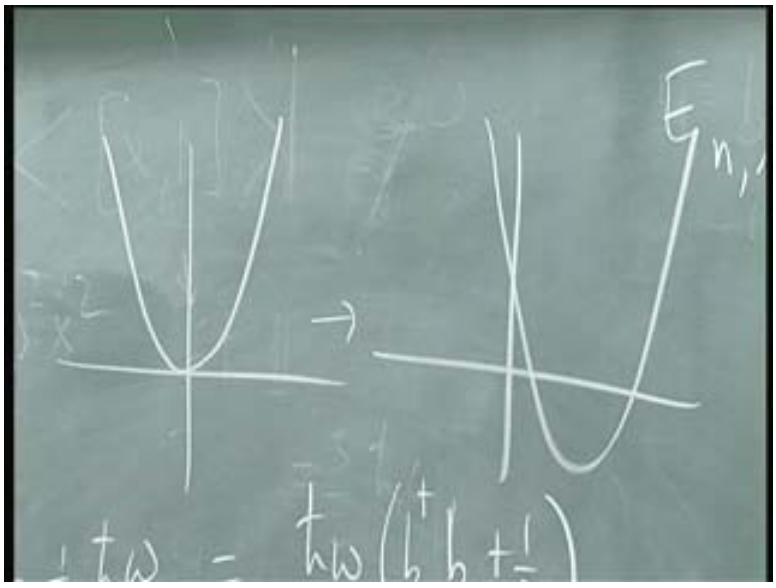

So what's been done by adding this thing is to start with this potential and move to a potential which looks like this (Refer Slide Time: 40:39). That's all that's happened. It's still a discrete set of eigenvalues. All we are saying is that what was 0 here has been shifted to - lambda squared over h cross. The next one said: let S be the spin operator for spin 1/2 particle and n and n prime 2 arbitrary directions and you are asked to find the commutator S dot n S dot n prime.

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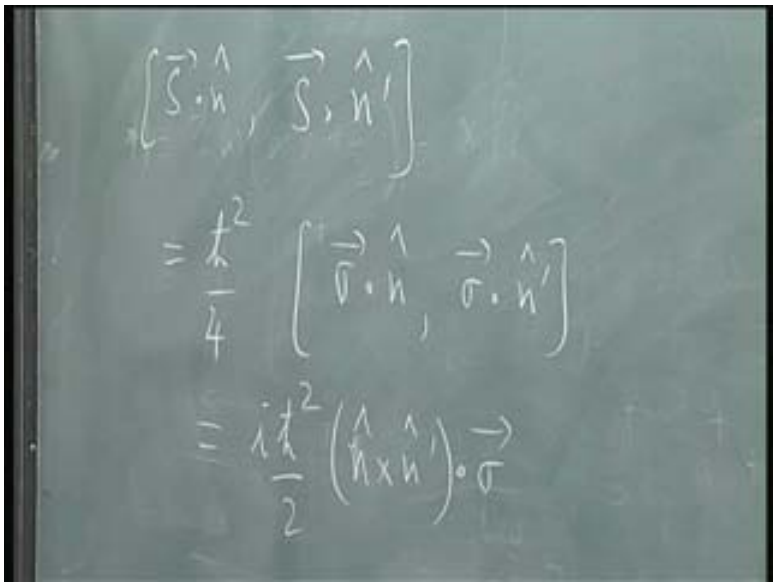

Remember that we have relations for the Pauli matrices. So S dot n S dot n prime = h cross over four because S is h cross over 2 times a Pauli matrix. Then the commutator of sigma dot n with sigma dot n prime. So this is = h cross square over 2 i n cross n prime dot sigma. It's got to be operator then at the end. So there has to be a sigma. You can re-express it in terms of S if you like. Now up and down are the usual Eigen states and you are asked: what's the operator Sx Sy in this basis. It's just a simple trivial exercise, understanding what's meant by the basis.

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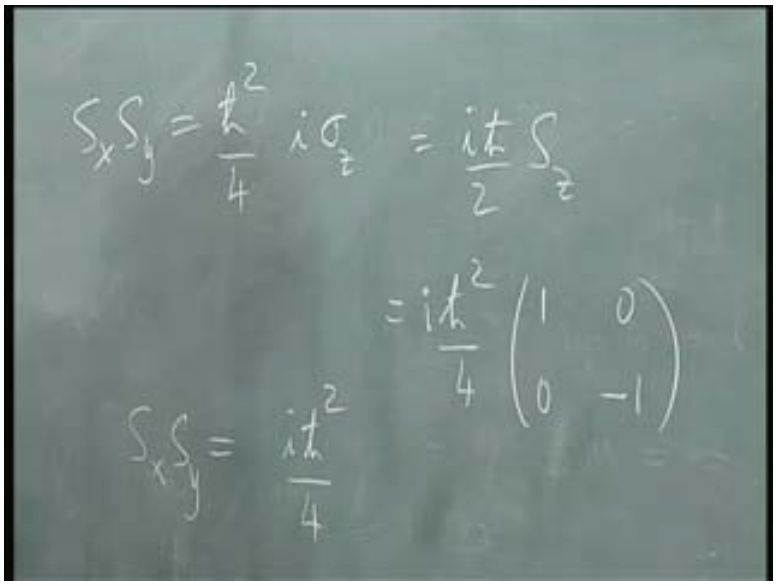

So we know that Sx Sy = h cross squared over 4 sigma x sigma y but sigma x sigma y is i sigma z which is = ih cross over 2 Sz. so we can retain it in either of the forms. It doesn't matter really. Then, what is this matrix? As a matrix this is = h cross squared over 4 i and then a 1 0 0 – 1. That's it. So as an operator, remember this is in the basis in which you diagonalized along Sz itself.

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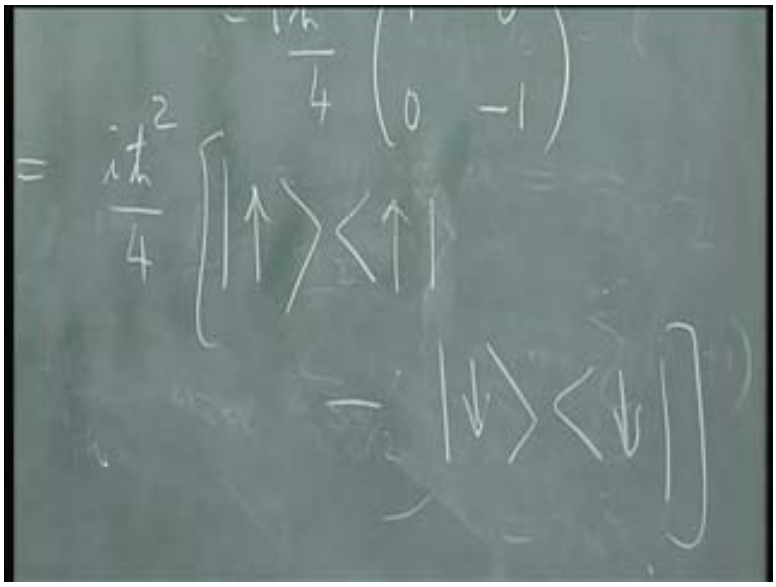

So as an operator it says Sx Sy = ih cross squared over 4 [|up> <up| - |down> <up|]. In this basis, it's a diagonal operator. So the 1 with the bra, the ket up bra down ket down and the bra up doesn't appear at all. These elements are 0 and this - sign takes care of this – sign. That's it. Then in the state cos theta e to the power - phi sin 1/2 theta the expectation value of Sx + iSy.

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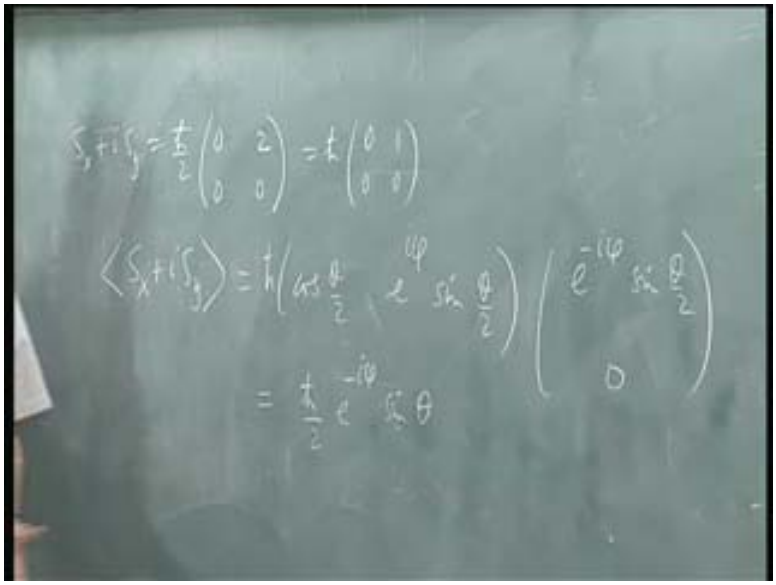

Now once again you just have to write the representation for these. You have to compute a simple matrix element. By the way, what is the Sx + i Sy? It is h cross over 2 and its = sigma x + i sigma y. this is 0 here sigma y has - i on top. So it becomes 2 and then it has 0 and has 0 which is h cross times 0 1 0 0. And then the expectation value of this quantity is = (given this state) cos theta over 2 e to the i phi complex conjugate is needed when you take the bra. That's the only place you have to be a little careful. t so its h cross 0 1 0 0 on cos theta over 2. That's it. So you just have to carry out this trivial multiplication. So this is = h cross over 2 e to the power - i phi sin theta because cos theta over 2 sin theta over 2 multiplied by 2 and divide and that's sin. That's it. so that's the answer. Now of course, you should check if theta is 0, where are you then? you are in eigen state of the Sz itself and then i know the expectation value of Sx and Sy are both 0. So you should check if theta is 0. That is certainly true. It vanishes at theta = 0. Now the degeneracy of an electron we have already talked about and then the formal solution of the Schrodinger equation for a free particle for the propagator here, what do you think it is? Recall I had already given the answer for a harmonic oscillator and if you said omega = 0, then you get the answer for the free propagator. That has combinations like an omega on top and a sin omega t below and as omega goes to 0 sin x over x goes to 1 or sin ax over x goes to a so we need to use that fact.

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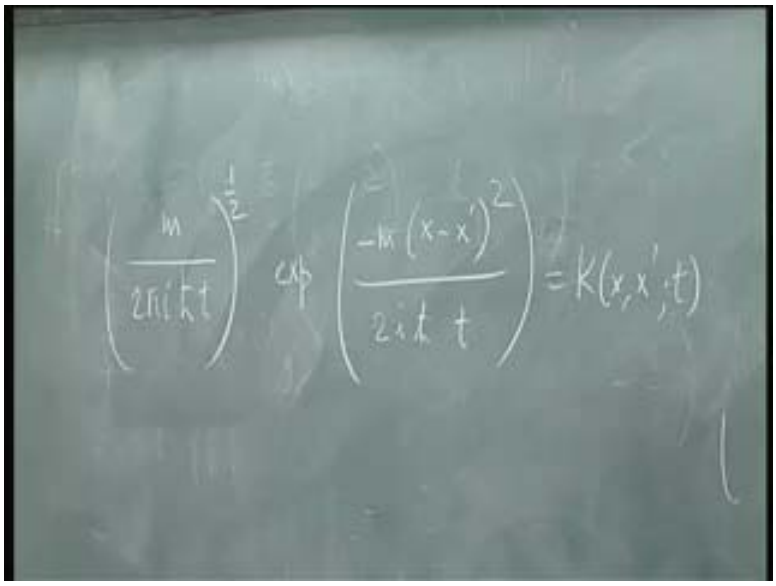

Even otherwise you could write this down for the free particle propagator. It would be something like m over 2 phi h cross t. i don't swear to this but its something of this kind exponential of a - x - x prime whole square - m over 2 ih cross t and this is = k of x x prime t. I just wrote this down because i remember the solution to the diffusion equation and i know that ih cross over 2 m is a diffusion coefficient for the Schrodinger equation. How are you going to exponentiate it you found this by actually solving this equation?

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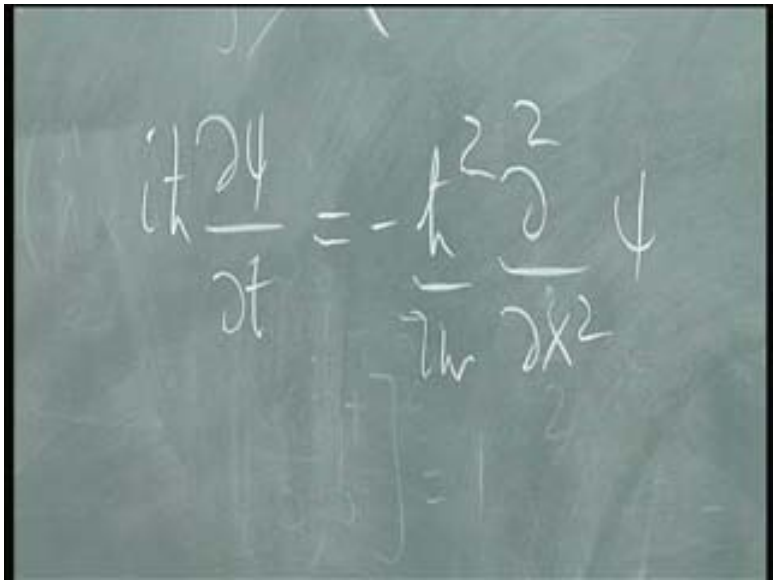

So the statement is: if you write delta psi over delta t ih cross is - h cross squared over 2 m d 2 over dx 2 psi, then regarded as a function of t alone, this is a first order differential equation and this quantity is an operator in x but not in t. so in a sense, psi of x at time t is this exponential of this operator acting on psi of x at time 0. But the exponential of that operator is an integral operator. so when you explicitly write it out, although formally you can write it in this form, you get an integral operator with some kernel.

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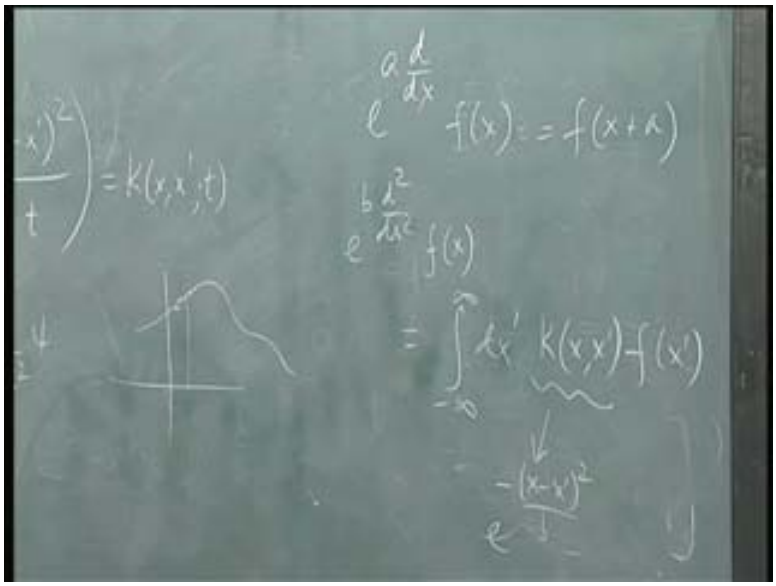

Just as if I write e to the power a d over dx on f (x), this is not a local operator. Even though d over dx is a local operator, each point you find the derivative. This is not a local operator because all powers of this derivative are involved here when you do the expansion of the exponential. Therefore, this as we know is actually = f (x) + a. so it's a translation operator. in exactly the same way, it turns out e to the power sum b d 2 over dx 2 on f(x) cannot be written as something which depends on x alone. It's not a local operator just as this depends on x + a set of finite distance away. in this case, it turns out that this can only be written as an integral for - infinity to infinity dx prime acting on f (x) prime but what acts on it is a k(x , x prime) and this kernel here is of the form e to the - x - x prime whole squared over something which involves b and so on. so it's a nonlocal quantity. It shouldn't be too surprising because if I have a function at some point here (Refer Slide Time: 51:48), if I specify the value of this x axis, you know what the function's value is. Now if I differentiate this function, then I need to know the value of the function at 2 neighboring points so that i can take the slope. If I would like to find the curvature at that point, the slope is not enough. I need to take the second difference. That means, I go a little further and take the difference of the differences and so on. So in a sense, to go a finite distance, you need an infinite number of derivatives. That's what is happening here to the infinite number of all powers. all derivatives are involved. so it is not surprising that the value of this quantity depends on the function everywhere else with that weight factor with that kernel. So the answer is

definitely a function of x finally, but it would be a function of x depending on everything else. That's what is happening here. So what we found here is this explicit solution to this equation which you can find in many ways but this quantity is a propagator. It has the advantage that if you give an arbitrary initial function and not a delta function or say that the function is 0 everywhere except at 1 point or anything like that, I can still find the solution.

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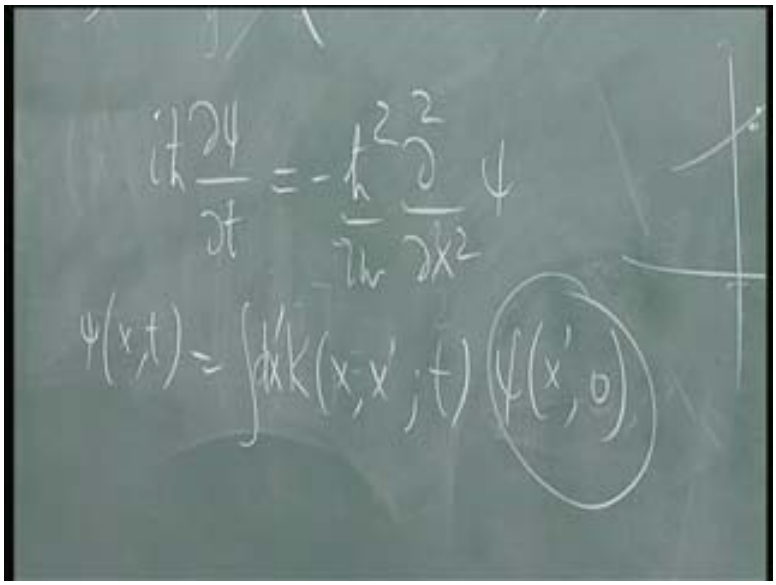

So the formal solution to this once I put that in, then I know that psi(x, t) is an integral k (x, x prime) t dx prime psi (x prime 0). So this is the initial value so you convert it this it's an initial value problem. So given the initial probability amplitude I am able to find the probability amplitude at all latter times with this propagator that takes me from the initial to the final state.

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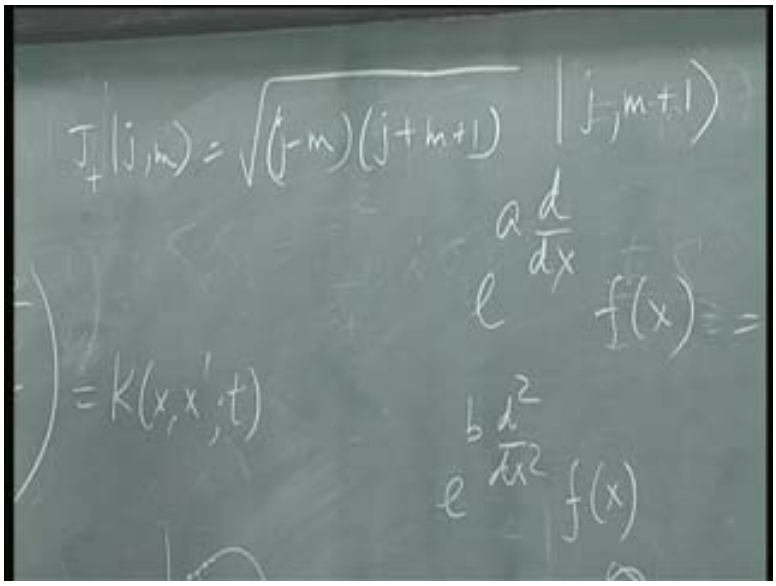

The last one was the action of j + on jm. This is something we have already worked out. It must vanish when m = j. so that's our mnemonic device. It says j - m and then it is j + m + 1 it's the raising operator. So it takes j and converts it and makes it m + 1. Had you had j -, this would be j + m and that would be j - m and this would be m – 1. It lowers it by 1. Now I didn't ask for the solution. I asked for the equation of motion and the equation of motion is a classical equation of motion.

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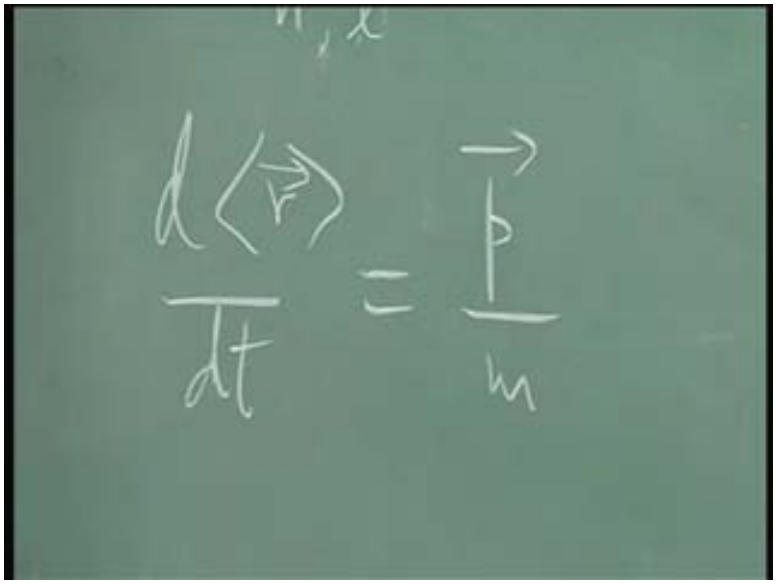

So it just says d over dt < r> = p over m. that's the equation of motion. Of course if you solve it, then you need to know what is the expectation of r at z = 0. So this is the equation of motion. It follows from Heisenberg's equation of motion. for a free particle the Hamiltonian is p squared over 2 m. so dp over dt is 1 over ih cross the commutator of r with H. the expectation value of a dagger a in a coherence state alpha with a linear harmonic oscillator S tells us 1 of the meanings of this complex number alpha which is the eigen state of a and the expectation value is not hard to find.

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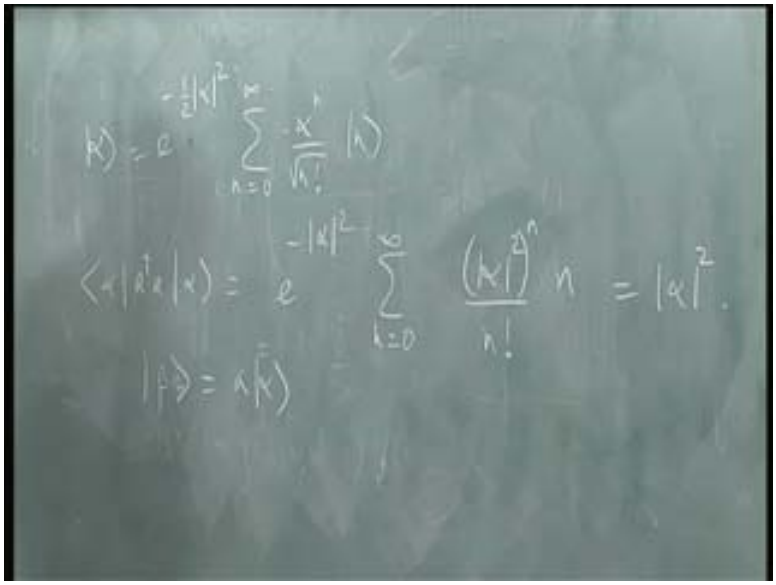

Remember, that the coherent state alpha was defined as e to the - mod alpha squared summation n = 0 to infinity alpha to the n over square root of n factorial acting on n. You could now ask what's a dagger a acting on this and find with an alpha on the left hand side. So alpha a dagger a alpha = e to the - mod alpha squared because you are going multiply 2 of these fellows here. And then a summation n = 0 to infinity alpha to the n alpha star to the n because this would have been m on the left but I am going to use the orthonormality condition.

So it becomes alpha star on this side. Divided by n factorial because you can't do square root of n factorial. You can't sum but n factorial in the denominator is easy. and then put an alpha a dagger a. when it acts on n, it just produces an n and n and of course this is mod alpha squared to the power n. this sum is easy to do. You start from n = 1 onwards. n = 0 is 0 by definition. This series is = mod alpha squared. So if you put beta = a 1 alpha which incidentally is alpha on alpha, then this quantity here (Refer Slide Time: 57:58) is nothing but norm of beta on beta = mod alpha squared. So this is the meaning of mod alpha squared. It's in fact the expectation value of the number of quanta. If we regard this as state of the radiation field, it's a number of protons in a coherent state. The average value of the number photon because you are not in the number state. You are not an Eigen state of a dagger a but in the superposition these states. Therefore the sum average value and that's mod alpha square in this problem. Now what is the

allowed value of mod alpha squared? It is 0 to infinity. Its 0 if and only if alpha is 0. When alpha is 0, then the state that you get is in fact the ground state of the harmonic oscillator in which the expectation value of n is 0 anyway. It's the vacuum state for the radiation field. Otherwise you get a finite number.

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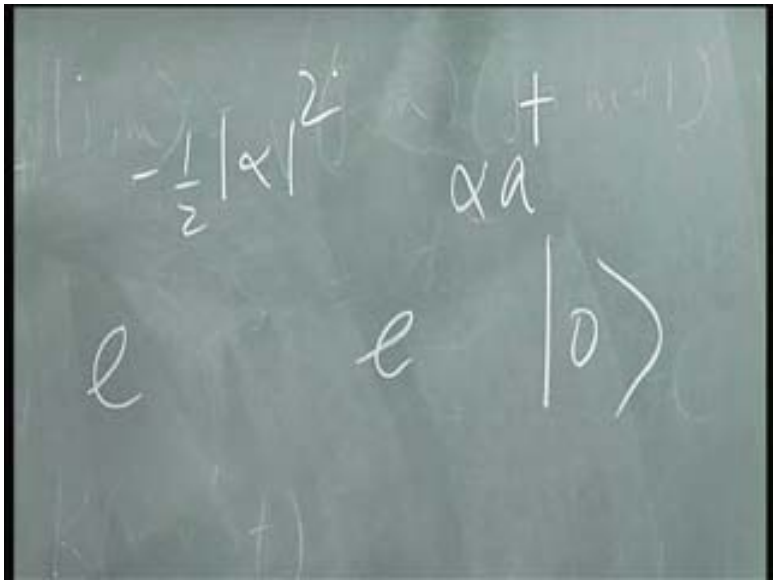

Now of course you can also generate this. You could write this as = a dagger to the n over square root of n factorial on 0 in which case you get e to the power alpha a dagger on 0. The advantage of this is there is no summation. It tells you there is some operator acting on the ground state produces this coherence state but an even better way of writing this is by using Lemma combinations.

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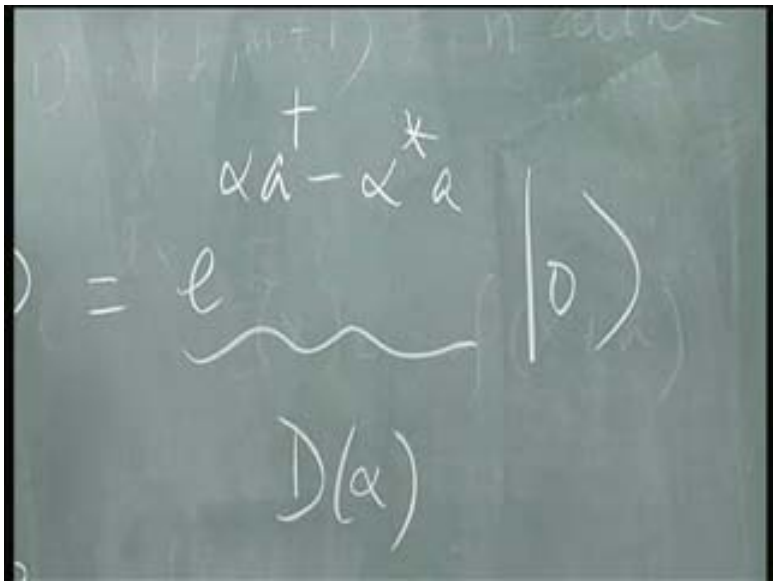

You can also write this as e to the alpha a dagger - alpha star a acting on 0. That's a very important way of writing it. This is called the displacement operator D of alpha. It's a function and alpha star but I will just omit the alpha star there. it's a unitary operator. So in a sense a coherent state is a unitary transformation on the ground state of the harmonic oscillator. And this unitary transformation is parameterized by a complex number here. Therefore, the natural question to ask is: what's this really telling you? Well, this set of unitary transformation forms a group and there is a group multiplication law which says D (alpha) D (beta) is D(alpha + beta) times a phase factor. It's called wild group because after all, a and a dagger form a a dagger and the unit operator form the Heisenberg algebra. We will come back to this because they play a significant role in quantum optics. Now let me stop here today.