Quantum Physics
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Lecture No. # 22

Now so let's go back and look at few of the problems in the sheets given earlier. We start with set p which was on the harmonic oscillator. I hope some of you have got copies and after writing down the Schrodinger equation plus the solution and so on, there is a set of problems that are given. So let's go through these systematically. The first one refers to a particle in a box, the orthonormality condition, the completeness relation, etc. You are supposed to verify these. So they are just straight verifications. And then i have to also ask for the momentum space wave function.

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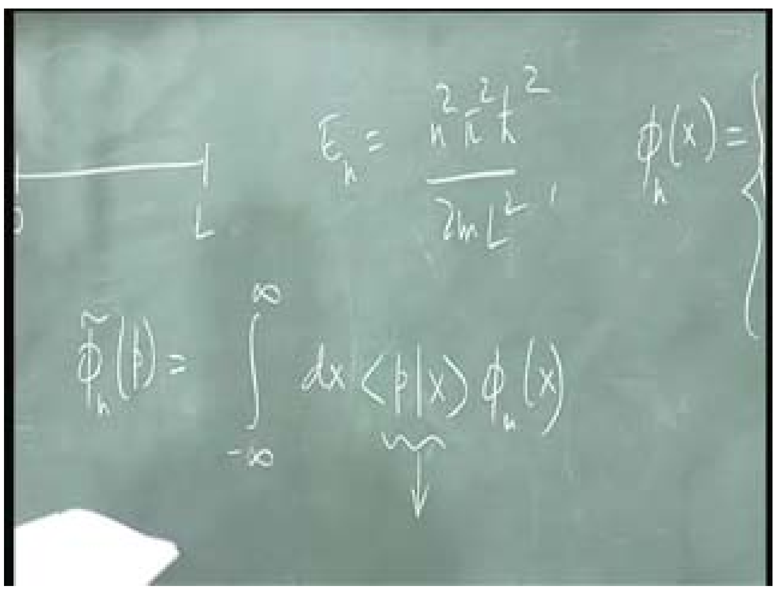

So the problem was the particle in a box, 0 to L, the energy levels E_n squared pi squared h cross squared over 2m L squared and the normalized wave functions where n is 1, 2, etc and the wave function is zero outside the box. you are also asked to find the momentum space wave function corresponding to this and that has also been written out here as phi $_n$ tilde (p) is equal to integral, - inifinty to infinity, dx p x phi $_n$ (x) and this quantity here (Refer Slide Time: 03:03) the normalized value p x is 1 over square root 2 pi h cross e to the ipx over h cross. as you can see, if this thing had been just e to the ipx or something like that or a sin or cosine from - infinity to infinity, then this (Refer Slide Time: 03:35) would be just a super position of a discrete number of momentum states but, because the wave function gets cut off outside the box, it's localized to within this box. So on the x axis, the particle doesn't ever go outside the box.

So this means, the uncertainty in the position can at no stage be bigger than L itself. Definitely it is inside the box which means the uncertainty in the momentum cannot be smaller than the order of h cross over L. so that immediately says that the momentum space wave function is actually spread out unlike the position space wave function. And you can compute this once you put this (Refer Slide Time: 04:22) in and then take mod squared of this to find the momentum density. In the case of the ground state, if you plot phi_n tilde (p) mod squared, you would get something like this (Refer Slide Time: 04:50)

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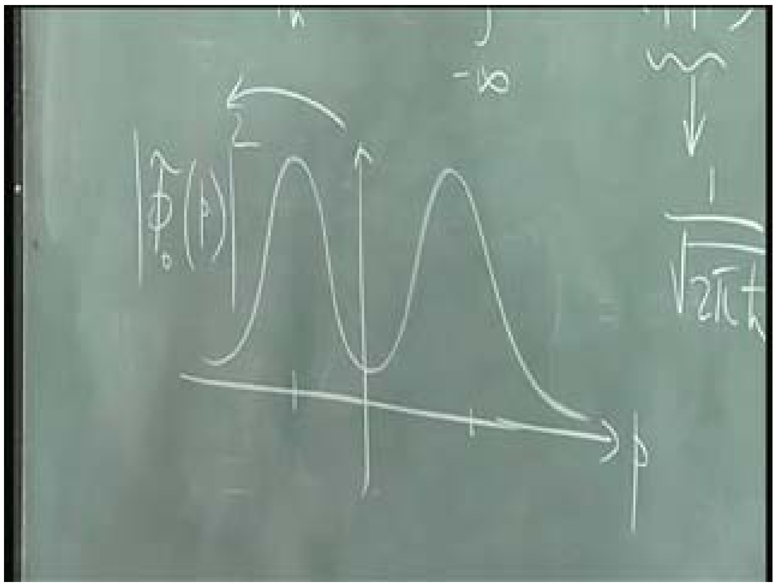

It is a peak here and another peak here at these values and this will be h cross pi by L and - h cross pi over L. n corresponds to 1 here. So it's phi_1. I leave this to you to actually work it out. So it is an extended wave function in any case. It's a kind of a statement if we have function which has compact support, then the Fourier transform has support from - infinity to infinity. The more you try to localize in position, the more it spreads out in the momentum. So it is not a momentum eigenstate for sure. In fact, there is a result given for this. it is cos squared pL over 2h cross and then there is a rational function. it is easy to calculate the position and momentum uncertainty. You have to simply calculate some integrals here and then show that delta x delta p for any eigenstate n is certainly much bigger than h cross over 2 including the ground state.

The next problem was a particle moving freely in 1 dimension but it was given in the position basis by Gaussian wave packet. So the idea is to try to localize the particle near some point x_0. The question is what happens to this as a function of time, and compute the momentum space eigen function etc. so you are supposed to calculate delta x delta p, find the wave function at any instant of time greater than zero, find what the corresponding momentum space wave function is, the uncertainties, etc.

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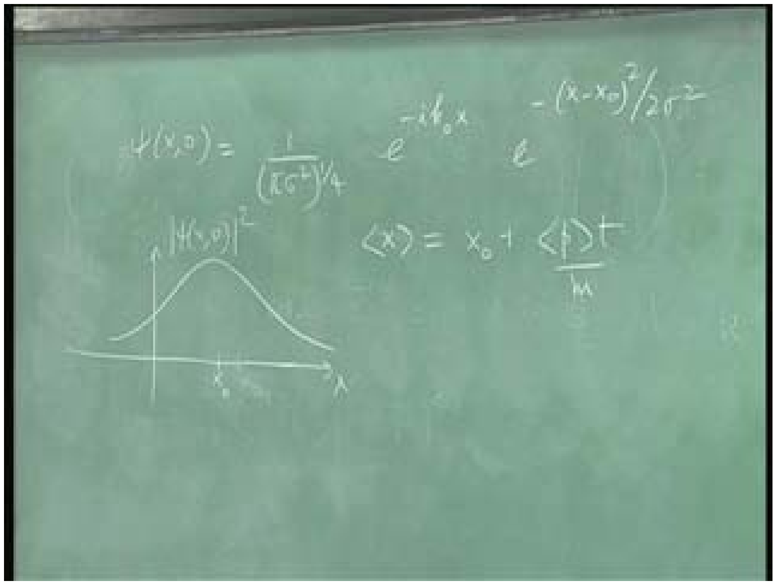

In particular, for this wave packet, which is an interesting wave packet psi (x, 0) = 1 over pi sigma squared to the power 1/ 4, these are normalization factors, e to the power – i k_0 x. e to the – (x - x_0) whole square over 2 sigma squared. so mod psi (x, 0) squared is a Gaussian peaked about the point x_0 and for this Gaussian wave packet, we are supposed to compute the uncertainty in position, momentum, etc and then show that at any instant of time, x average is equal to x_0 + pt over m. so again for this Gaussian wave packet, the mean value of x moves like a classical particle. It is a free particle. So classically the momentum is constant and x would be x_0 + pt over m but now Ehrenfest theorem kicks in and you have expectation values moving according to this rule. You can do this directly by integrating or writing down the Hamiltonian which is just p squared over 2m using Heisenberg's equation of motion. You are also asked to find the spread of the wave packet as a function of time.

This wave packet will spread and the physical reason it spreads is because different components of different wavelengths don't travel at the same velocity. The wave velocity and the group velocity are not the same in this case because the energy is dependent quadratically on the wave number. It is a free particle. So E is p squared over 2m and p is h cross k. E is h cross omega. So omega is proportional to k squared which means d omega by dk is not omega over k. that immediately leads to dispersion. The rest were some identity in the harmonic oscillator for the linear harmonic oscillator what is the expectation value in any of the energy eigenstates? What is the expectation value of the kinetic energy and that of the potential energy?

They happen to be equal in the case of the harmonic oscillator. These 2 are equal because that problem is extremely symmetric under the interchange of x and p. in some sense, its quadratic in x and quadratic in p and the Hamiltonians are very symmetrical and then there were some Baker Campbell Hausdorff type of identities. The only significant thing you must note in this problem set is our definition of coherent state.

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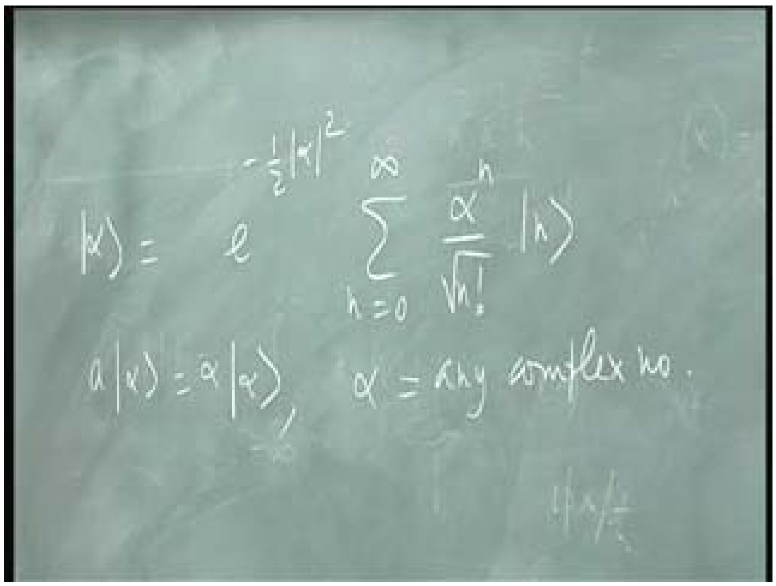

So an oscillator coherent state was defined in this following way where alpha is any complex number summation n = 0 to infinity, alpha to the power n over root n factorial n where these were the oscillator eigenstates. We know that a on alpha is alpha on alpha, alpha is any complex number but you can also rewrite this in another form.

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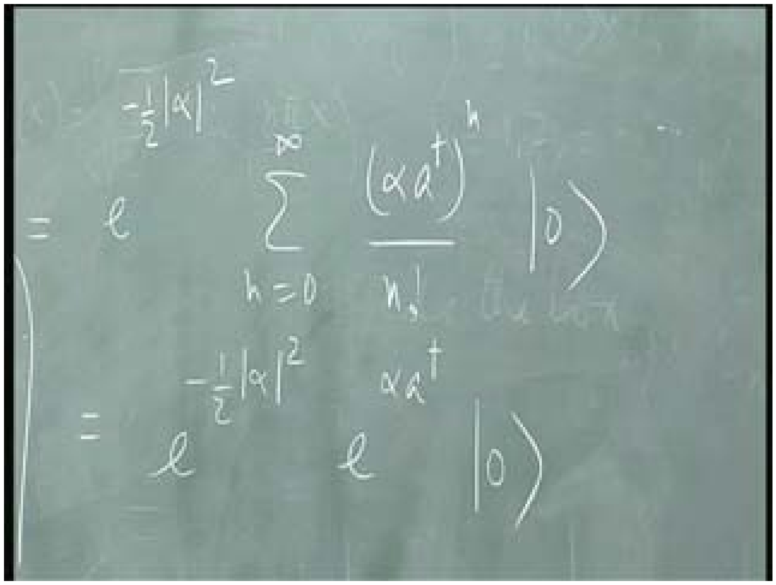

We can also write this as e to the power - half mod alpha squared. We know that n itself can be found by taking a dagger and acting on the ground state. Therefore this becomes summation n = 0 to infinity, alpha a dagger to the power n over n factorial acting on the ground state. because a dagger to the n on 0, divided by square root of n factorial is in fact the excited state n. so this is equal to e to the - half mod alpha squared, e to the power alpha a dagger acting on the ground state. Now we know the commutation relation between a and a dagger, so we know the one between e to the alpha a dagger and e to the alpha star a as well. And if you use that commutation relation or any of these relations which have been given to you earlier, this can also be written as e to the alpha a dagger - alpha star a acting on 0.

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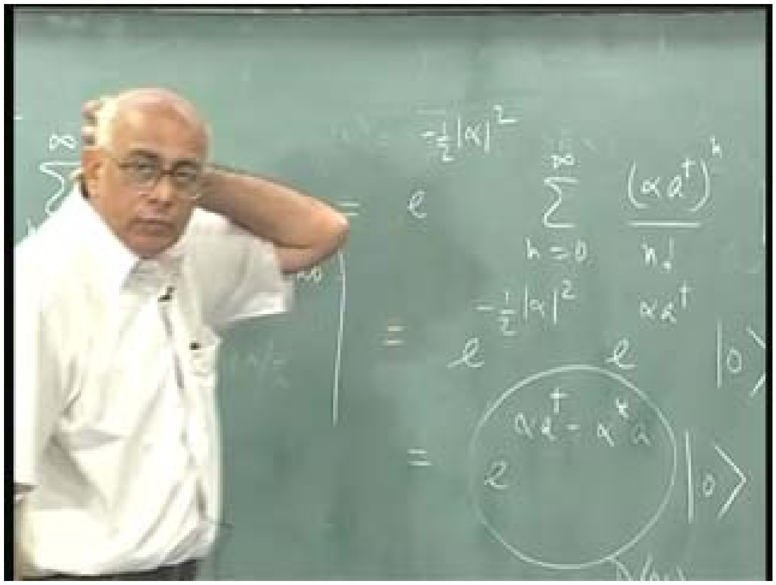

So, there is a certain operator called the displacement operator; D (alpha), it should be D (alpha, alpha star) because it is linearly independent of each, but just for convenience of notation I will call it D of alpha. This operator which is the exponential of this combination here acting on the ground state gives you the coherent state. Now the reason this is called the displacement operator is the following. First of all, this operator is a unitary operator. That is easy to see that D D dagger = D dagger D = the identity operator. So it’s unitary transformation on the ground state that gives you the coherent state. And physically what it corresponds to is the following. The wave function in the position bases corresponding to this alpha would be found in the usual way.

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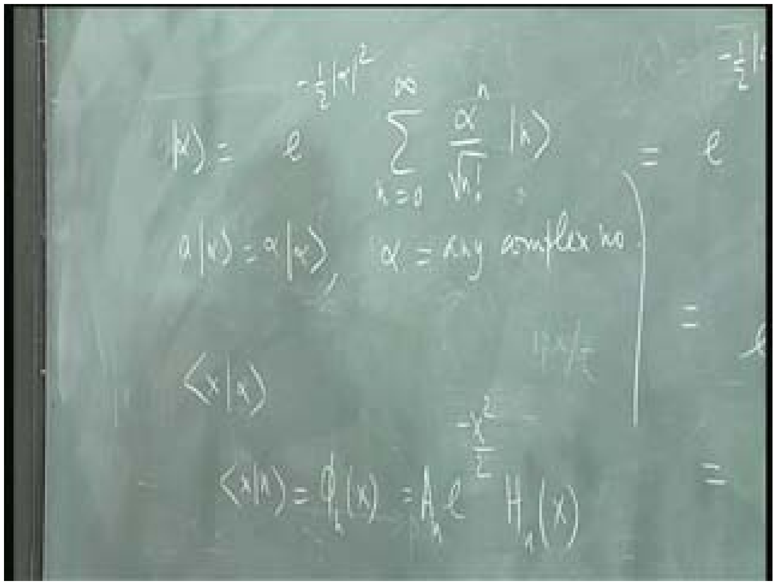

It is x on alpha and that would be x on n and what is x on n? By definition it is phi $_{n}$ (x), it is the normalized eigen function in the position basis. And what is phi $_{n}$ (x) equal to for the harmonic oscillator? There is a Gaussian, so its = e to the - x squared over 2 in units of h cross over m omega or whatever. so really you should have m omega over 2h cross and so on sitting here multiplied by the Hermite polynomials Hn (x) again in those units and then there some normalization constant here, 1 over 2 n factorial square root and so on. so for you to find this, you have to put in here and compute it which doesn't look like a very trivial exercise because you have to remember that if i put this in, you have to find alpha to the n and then H_n but now you use the expression for the generating function of Hermite polynomials and then this sum collapses and you can't compute this sum and the answer is again a Gaussian but centered not at the origin like this (Refer Slide Time: 14:59) one is but centered at the real part of alpha.

What about the imaginary part of alpha? What would that correspond to? Would that play a role because I am claiming this is a Gaussian of the form e to the – (x - alpha $_{1}$) whole squared over 2 and other factor and there are phase vectors and so on but alpha $_{1}$ is the real part of alpha. What you think is a role played by the imaginary part of alpha? it would be the center of the Gaussian wave packet in the momentum space wave function because you know that the momentum space wave function is a Fourier transform of the position space wave function and the Fourier transform of a Gaussian is a Gaussian once again. So alpha $_{2}$ would appear if you did p with alpha. You would get p - alpha $_{2}$ and so on. What does that imply physically?

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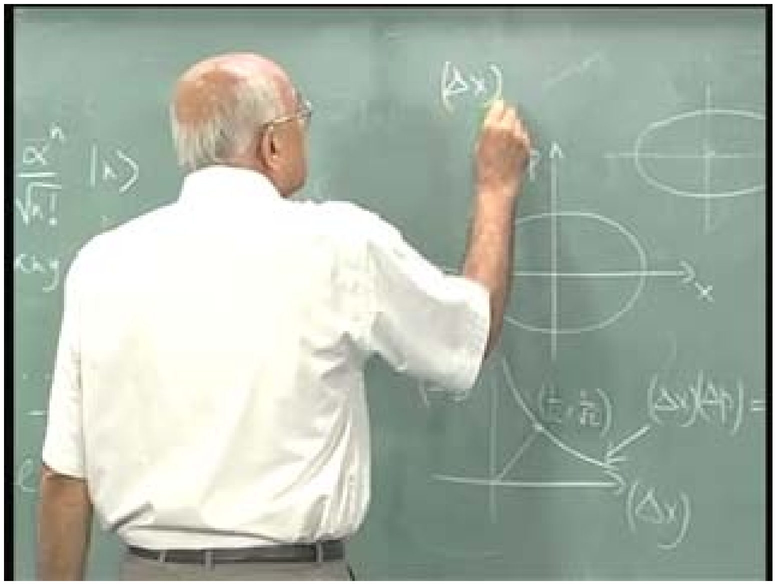

It says a Gaussian wave packet has been essentially shifted in the sense that the harmonic oscillator, classically not quantum mechanically would correspond to an elliptic orbit in x and p. this is an oscillator oscillating about the point (0,0) in x and p. the equilibrium point at (0,0) once you apply this displacement operator e to the alpha a dagger - alpha star a, you get a new state which is also again an oscillator but would correspond to something like this (Refer Slide Time: 16:58). It is simply displaced from this origin to a new equilibrium point. This is alpha $_1$ that is alpha $_2$ about there. That is why i use the symbol D and that is why it is called the displacement operator. It displaces this oscillator. What is the uncertainty product delta x delta p for the ground state of the harmonic oscillator?

It is exactly h cross over 2. It's a Gaussian wave packet. so it is the minimum uncertainty state but so is every coherent state for arbitrary alpha. it is again Gaussians. so once again the uncertainty product is a minimum value. in fact, if you plot delta x verses delta p, delta x is in units of square root of m omega over h cross and delta p is in units of 1 over square root of m omega h cross. Then, both these are dimensionless quantities. Delta p delta x must be greater than or equal to ½. It's equal to ½ for this hyperbola. And symmetrically at this point 1 over square root of 2e. For each of the two. This (Refer Slide Time: 18:37) is where the ground state of the harmonic oscillator is and this is where all coherent states are for all of them. You have this minimum uncertainty with this symmetry between x and p. the excited states in the harmonic oscillator would be (delta x_n) (delta p_n) = (n + ½) h cross. so the ground state n = 0 is 1/2 h cross and the excited states, I have set units such that h cross disappears on both sides. Those would correspond to the states here (Refer Slide Time: 19:25) here. They are square root of 3 over 2 squared, five over 2 and so on.

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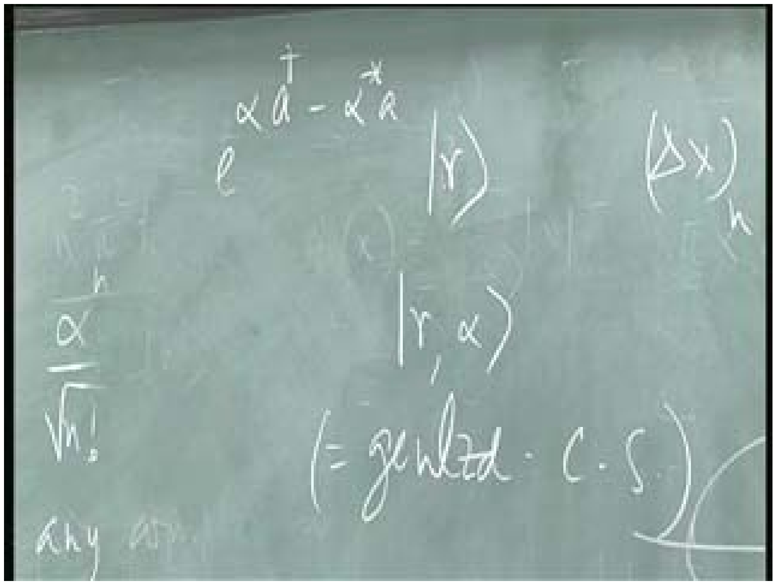

Let's take e to the alpha a dagger - alpha star a and apply it on 0. i get alpha which is a coherent state. What happens if i apply this on some excited state r and not on 0? It's the same operator acting on an excited state of the oscillator. This is called the generalized coherent state. It has interesting properties and they are not minimum uncertainty states including the ground state. This is denoted by |r, alpha> and it is called a generalized coherent state. They again have a lot of applications in quantum optics which we will not get into here.

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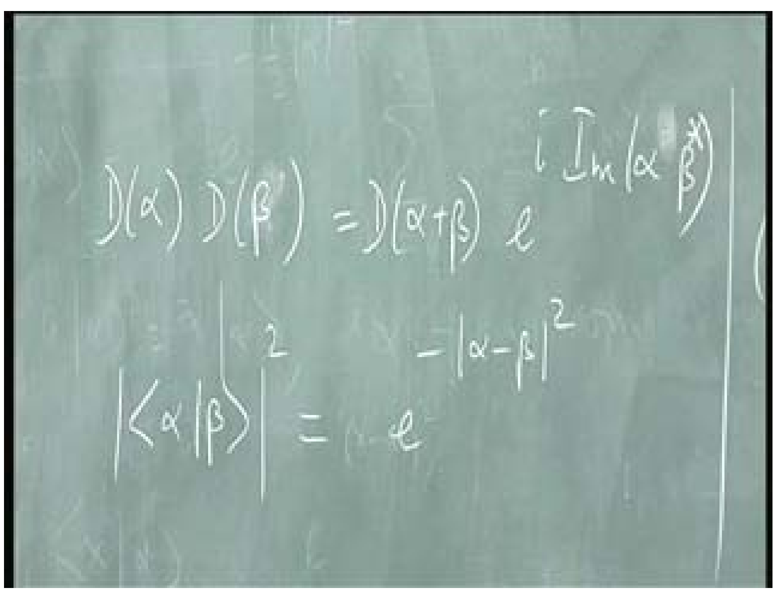

What is interesting to note is that these operators D(alpha) D(alpha prime) have interesting multiplication properties among themselves and that has been given here as an exercise. i have given D (alpha) D(beta) = D(alpha + beta) e to the I, imaginary product of (alpha beta star). Then it is fairly straight forward to verify this relation here. This is again another way of writing the Weyl commutation relation between x and p. recall that earlier when we said x commutator p is ih cross unit operator, what is e to the i ax e to the i bp? , where a and b ordinary numbers and that again had a form similar to this. a and a dagger if you like are linear combinations of x and p and are non Hermitian. so this is essentially the same Heisenberg algebra being rewritten in different ways. The important point about the coherent states is that they are not orthogonal to each other and this is crucial because |<alpha| beta>| mod squared is e to the - mod |alpha – beta| squared. so certainly if it is normalized; if alpha is equal to beta you get 1. Otherwise it is not zero since they are not orthogonal states. They are an over complete set. it is not a complete set and are not orthogonal to each other. The next problem in problem set 3 was on finding the time dependent propagator for the simple harmonics oscillator. So let's go back to the actual time dependent Schrodinger equation and try to solve this equation completely by using a Green's function for it and that is what i call the propagator. So let's write it down and you are just asked to verify this because it requires a little bit of mathematics to work it out.

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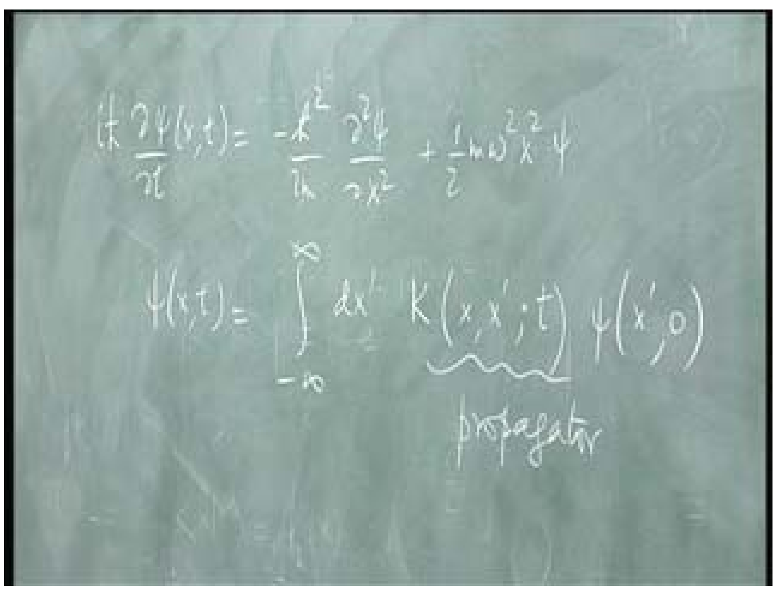

So the equation was i h cross delta psi /delta t (x, t) = - h cross squared over 2m d 2 psi over dx 2 + 1/2 m omega squared x squared psi. And now the point is, if you specify adequate boundary conditions, and in this case we want the wave function to be normalized. so that means psi (x, t) must go to zero as x goes to +/ - infinity together with an initial condition. then it is possible to write the solution at any time as integral dx prime and then a kernel which is the function of x, x prime and t; psi (x prime, 0).

This quantity is called the propagator because it helps you go from the wave function at time zero to the wave function at any later instant of time. That's called the Feynman propagator and i given an expression for the propagator here. It's a fairly complicated expression and the idea is to verify whether it's true or not. By the way, what is the propagator for the free particle? Suppose you had no oscillator at all, what does the propagator look like? As a free particle propagator, there is no potential. Certainly you can still write the wave function at time t in this (Refer Slide Time: 25:29) form. It is an initial value problem. You are interested in psi of x comma t for all t greater than equal to zero.

You are given psi (x, t) at t=0 and you are trying to find what it is at later instance of time. So you are interested only in the half line in t namely t greater than 0. It's a first order differential equation in t. so what is the automatic thing to do? You would use Laplace transforms. And for the x variable that is defined from - infinity to infinity, we would use Fourier transforms. So you do a Fourier transform in x, Laplace transform in t and the job is done and you compute the solution. What do you think is going to happen? that is the free particle propagator and we could do this very painfully but you can see very easily that if you took a Laplace transform in t, if s is the conjugate variable to it, this (Refer Slide Time: 26:52) is going to be s times a transform - the initial value and if you do a Fourier transform in x and k is the conjugate variable, differentiation with respect to x corresponds to multiplication by k.

So it is clear that the transform is going to have a k squared multiplying it (Refer Slide Time: 27:11) and it is going to have an s multiplying this (Refer Slide Time: 27:13) and there is an in homogeneous term. So eventually this transform is going to be 1 over some (s + k) squared or (s – k) squared acting on the initial value and the inverse transform of that is an exponential. So it will have an e to the - k squared which is a Gaussian in k and if you take the inverse transform of a Gaussian, you would get an e to the - x squared. So I would expect in this problem that i am going to get something like e to the - x squared over t acting on the initial state. That would be my transform. Are you familiar with the diffusion equation? We know the solution to that.

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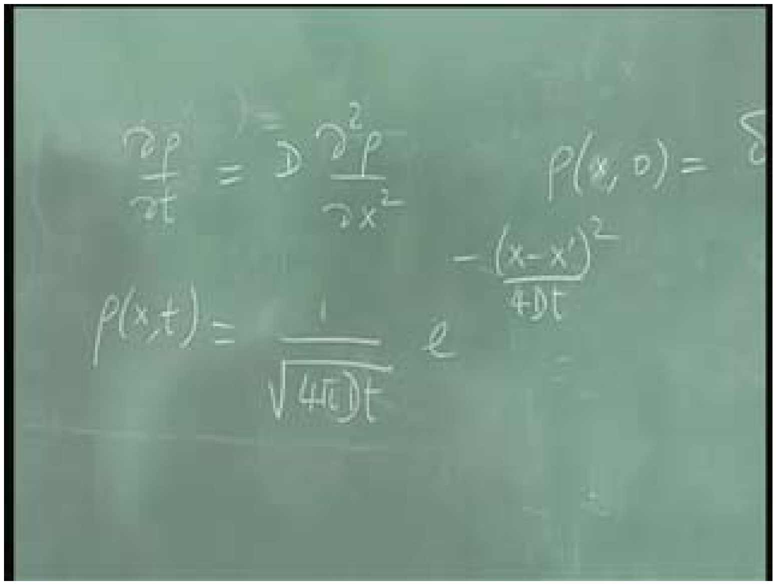

I will write this down by inspection because remember, the diffusion equation says delta rho / delta t for any concentration rho is D times d 2 rho/ dx 2. That is the diffusion equation and what is the solution to this? So if you say rho (x, 0) = delta (x - x prime), some special point x prime where every all the entire molecular species is concentrate at some point x prime and then you let it diffuse as a function of time, the answer is some kind of Gaussian. Then rho (x, t) = 1 over square root of 4 pi Dt e to the – (x - x prime) squared over 4 Dt. that is the standard solution to the diffusion equation.

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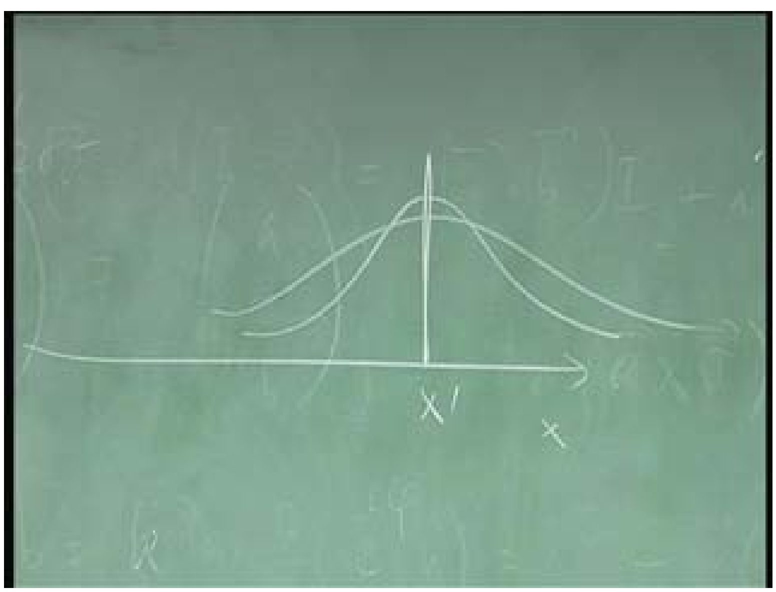

So it says if on the x axis, you start with an initial concentration at the point x prime as a function of t, after some time we are going to get this and then it is going to get widened out and so on. So it is a Gaussian concentrated about that point. Well this equation looks pretty much like that because this - h cross squared could be written as ih cross squared. So it is essentially the same equation on both sides. so let us write this as - h cross squared and bring the i h cross here. So this goes away and there is 1 over i here (Refer Slide Time: 30:15) which gives you ih cross. It's the same solution. So this means that if the initial particle was concentrated at the time x prime, that Gaussian is a solution but if it's given by distribution by itself; psi(x prime, 0), all you have to do is to put this corresponding solution in here (Refer Slide Time: 30:39). So let's put that in.

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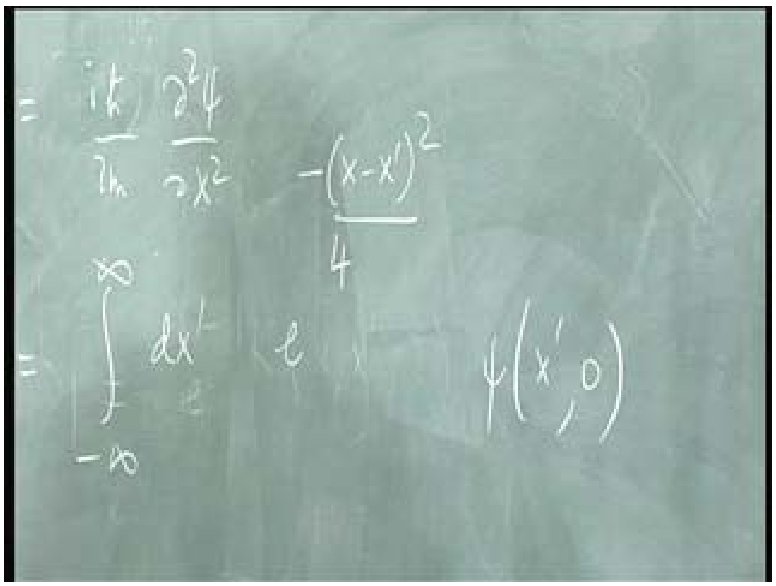

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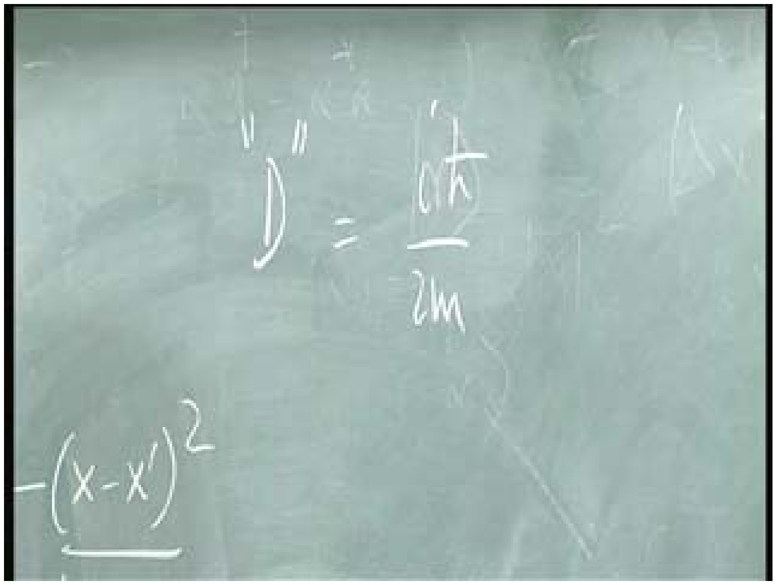

e to the - x - x prime whole squared over 4 times D. Now D in our problem is ih cross over 2m. It is imaginary and it is not real diffusion of course. It is just the Schrödinger equation. The resulting equation is this (Refer Slide Time: 31:24). Had this (Refer Slide Time: 31:30) been some initial point t_0, then this would be just $(t - t_0)$ for all t greater than t_0. And then the normalization, square root of 4 pi, D is ih cross over 2m, root m by 2pi h cross. That's the exact solution to the time dependent Schrodinger equation for an arbitrary initial distribution. You specify any psi (x prime, 0) and that is the solution at any time t. this quantity this thing here together with this (Refer Slide Time: 32:48) factor is called the propagator. In this case the free particle propagator. So what I have done in writing this solution here is exponentiating the second derivative operator. Because it says pretend that the right hand side is just some number, and then it says the following.

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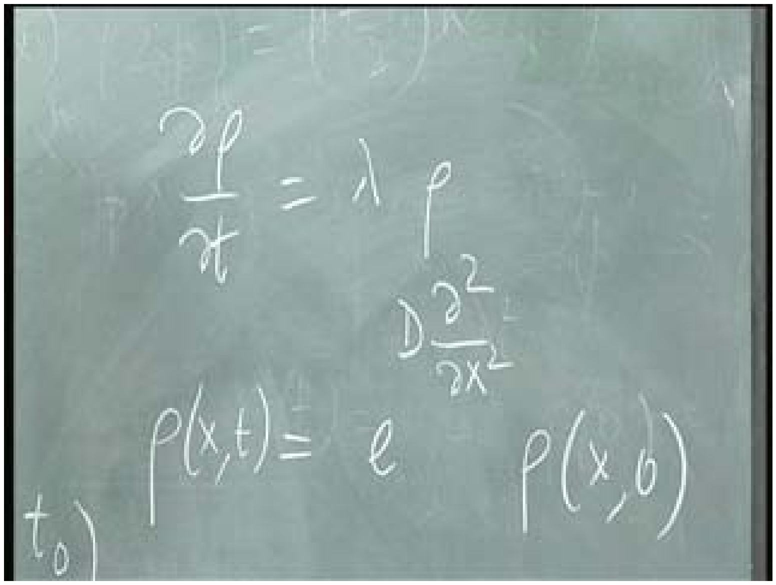

delta rho over delta equal to some lambda times rho, lambda is an operator which acts on the x variable etc but the solution here is rho at time t equal to e to the lambda t rho at time 0. So going back to the original x variable, it says rho x comma t = e to the power Dt d 2over dx 2 rho (x, 0). Dt has dimensions of length squared. So this is dimensionless. So the question is what is this (Refer Slide Time: 34:12) equal to? If this was a d over dx, what would this be? If this were a first derivative, then you can use Taylor's theorem and this would be just rho (x) displaced by D. So this quantity would just be root Dt d over dx, this (Refer Slide Time: 34:48) would be just x + square root of Dt but when you have a second derivative, then the answer is some integral operator because it must depend on all x because all derivatives are acting on it, arbitrarily high orders.

Therefore it must depend on rho not just at one x but at all sorts of places and it becomes an integral operator whose kernel is given by this (Refer Slide Time: 35:14). so that is the reason you get the Gaussian multiplying the psi and then in integrating over x prime, because you are really exponentiating the second derivative operator. And the answer is that it is an integral operator and the kernel is a Gaussian. Now for the harmonic oscillator, you are exponentiating not just d 2over dx 2 but you are exponentiating the operator d 2over d x 2+ x squared and those two parts don't commute with each other. So this is Green function and it is a very non trivial object. That's not always possible to write it down for all problems.

The path integral way of doing quantum mechanics starts at this point. It tells you how to find these propagators by what is called time slicing but we are not going to that in this course. However, this expression for the harmonic oscillators Green function is sufficiently important that you need to know. So that was a reason for giving it. The idea is that you give me the Schrodinger equation and you specify the initial wave function, then the future wave function is obtained by some propagator acting on the initial wave function. This takes you from any point x prime to any other point x.

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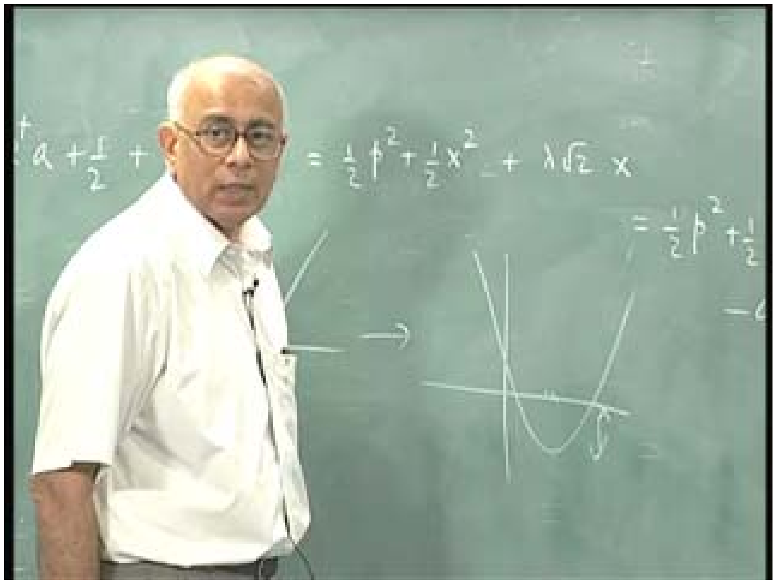

The next problem is had this harmonic oscillator Hamiltonian in suitable units + lambda (a + a) dagger in suitable units, what does this Hamiltonian correspond to? Lambda is a real constant. You are also asked to find the eigenvalues of this Hamiltonian. One way to do it this is we are asking what does this correspond to physically, so we should go back to the physical variables x and p. if you go back to the original variables, the Hamiltonian was equal to 1/2 p squared + 1/2 x squared. We have set h cross as 1, omega as, 1 m as 1 and so on + some constant lambda (a + a dagger). But what is (a + a dagger)? It is root 2 x. what does that (refer Slide Time: 38:27) correspond to? But physically what has been done to the oscillator? You put a constant electric field or just a spring which goes up and down and you add gravity to it, so it has moved the center of oscillation. So the potential is no longer like this (Refer Slide Time: 38:57) but this potential has gone over to something like this (Refer Slide Time: 39:01).

All that's happened is you shifted the oscillator. So would you expect the energy levels to be changed? You wouldn't expect the spacing to be changed but certainly there has in added constant. So you should complete squares in this case. You end up with some x prime - some constant. So all you have done is to shift this some other position and got a potential like this (Refer Slide Time: 39:57). So you can change the energy levels by adding an overall constant. So this problem is trivially solvable. It is just a linear perturbation on a quadratic Hamiltonian and it is trivially solvable in this case. What value should lambda take in order that the ground state energy be exactly 0? Well, you can fix this and you know what this (Refer Slide Time: 40:44) quantity is. You know how much you are going to add. That depends on lambda and that must be exactly equal to half h cross omega in this problem half. So once you fix that, then the ground state energy is exactly at 0. so that is a trivial problem.

The next one was the problem of a free particle in a constant field of force. So the potential was - fx and I worked this problem out explicitly in class. The solutions in the position basis are airy functions but the solutions can be written in terms of the momentum basis. The equation is easy to solve because the Fourier transform has things like e to the power of a p cubed term and the p term. And the normalization is energy normalization. So that was for problem set 3. And then in set 4, there were problems on the harmonic oscillator eigen functions themselves.

One of the points i wanted to point out was the following. The differential equation satisfied by the position space wave functions in the harmonic oscillator and that satisfied by the momentum space wave functions are exactly the same in suitable units. Therefore the solutions are exactly the same in functional form.

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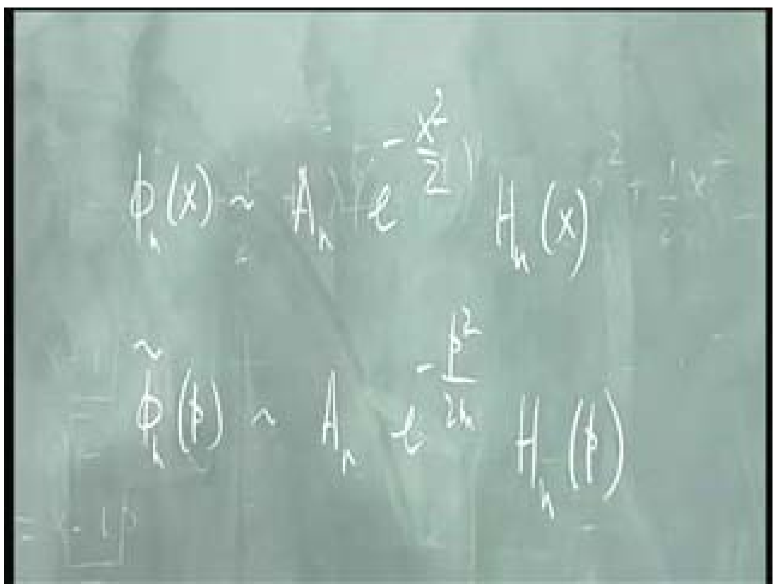

The solutions in x if you put in the appropriate constants and so on, then the solutions in x are of the form phi $_n$ (x) which is some normalization constant, e to the - x squared over 2 and then $H_n(x)$. in the momentum basis, phi$_n$ (p) tilde for the same eigen state would go like some A_n e to the - p squared over 2m H_n (p). So the functional forms are exactly the same. On the other hand, I know that the momentum space wave function is a Fourier transform of the position space wave function.

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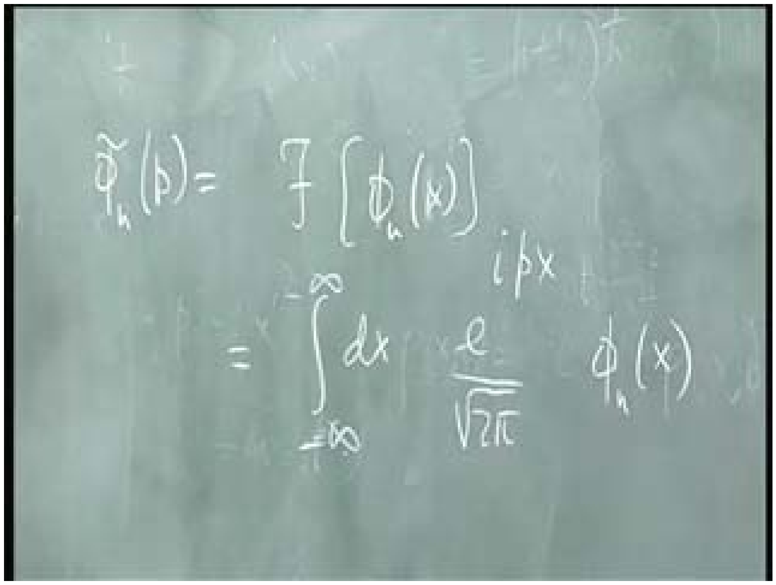

So I know also that phi $_n$ tilde (p) = Fourier transform of phi $_n$ tilde, and this is equal to integral dx e to the ipx over root 2pi setting h cross = 1 times phi $_n$ (x). so i could regard this as an integral operator with kernel e to the ipx acting on a function in L $_2$ to produce another function in L $_2$ because this 2 is square integrable. This is normalized to 1, mod phi $_n$ (p) whole squared dp is 1 and so is mod phi $_n$ (x). We know that the Fourier transform of an L $_2$ function is also L $_2$. In this case, we got something better. It says phi $_n$(p) has the same functional form as phi $_n$ (x). So this is like an eigen value equation and apart from an overall multiplicative constant whose modulus is 1, this and that are exactly the same. This is some other function; modulus of A $_n$ is modulus A $_n$ prime.

So apart from that, these 2 are exactly the same function. So it says that these functions are something special. They could also be regarded as eigenfunctions of the Fourier transform operator because you take the function and you have to do a Fourier transform on it. You get another function in the same space and is again in L 2 but that function has the same form as the original function. Therefore this is an eigen function of the Fourier transform operator. And the eigenvalue must have unit modulus. So in general, the eigenvalue is with some complex number of unit modulus.

Student -now what about the 4ier transform of Gaussian? Professor - It gives you exactly the Gaussian and in fact, if you work with the ground state H $_0$ is 1, you would discover that it is equal to itself in functional form. So the eigenvalue is 1 in this case. So the ground state in fact has eigen value + 1. So the ground state wave function of a harmonic oscillator is also an eigen function of the Fourier transform operator and has eigen value + 1. What about the excited states? Well, the first excited state you will discover in functional form that phi $_1$ tilde(p) is i times phi $_1$ of x which implies that it is an eigen function of the Fourier transform operator with eigen value + i. the next excited state would have - 1 as eigen value the third excited state has eigen value - I and the forth 1 comes back to + 1 and this is because the forth power of the 4ier transform operator acting on functions in L_2 is the identity operator.

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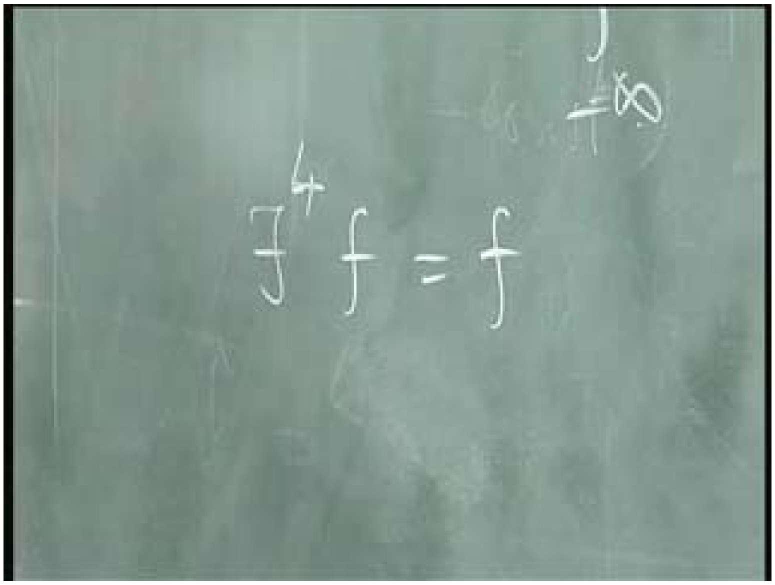

So F to the power 4 f = f itself. You can verify this. In fact you can verify that f squared acting on f (f squared f) of x = f (- x). In other words, the square of the Fourier transform operator is a parity operator. so you take a function of x, you do the Fourier transform of the Fourier transform; not the inverse, once again Fourier transform of the Fourier transform and you get a f (- x). That is easy to verify. Therefore you do it 4 times you get back the original function. this immediately suggest that the Fourier transform operator itself has eigen values equal to the 4th roots of unity; 1, i, - 1 and –i. and the harmonic oscillator energy eigenfunctions are also Fourier transform operator eigenfunctions such that the ground state has eigenvalue + 1. The first excited state i, second excited state – i, third excite - 1 third excited state - i and 4th one back to 1 and that keeps going. So this is an extremely interesting property of these functions.

So they are not arbitrary functions as you can see. It is the symmetry between x and p that gives you some profound inside into what's happening with regard to the Fourier transform operator itself. Well, the obvious question to ask is if these functions of the eigenfunctions of the Fourier transform operator, then the Fourier transform operator is like the square root of the parity operator. The parity operator is the like the square root of the unit operator, then what about the square root of the Fourier transform operator? Would the harmonic oscillator eigenfunctions also be eigenfunctions of this operator with eigenvalues at the 8th roots of unity on the unit circle and so on? Then you can ask for the square root of that operator the 16th roots of unity and so on. I leave you to verify whether this is true or not.

It is an interesting exercise in mathematical physics and you have to verify this is true or not and it has implications in other areas of mathematics but we won't go into that. But up to the level of the Fourier transform operator, I have given here what the expected answers are, so please check that out. Then, last questions I have to do with reflex and transmission. These are things I have already done in class. I have just written them down systematically here. So, that you have the exact formulas with the correct notation and so on. And finally the last question is an extension of the uncertainty principle.

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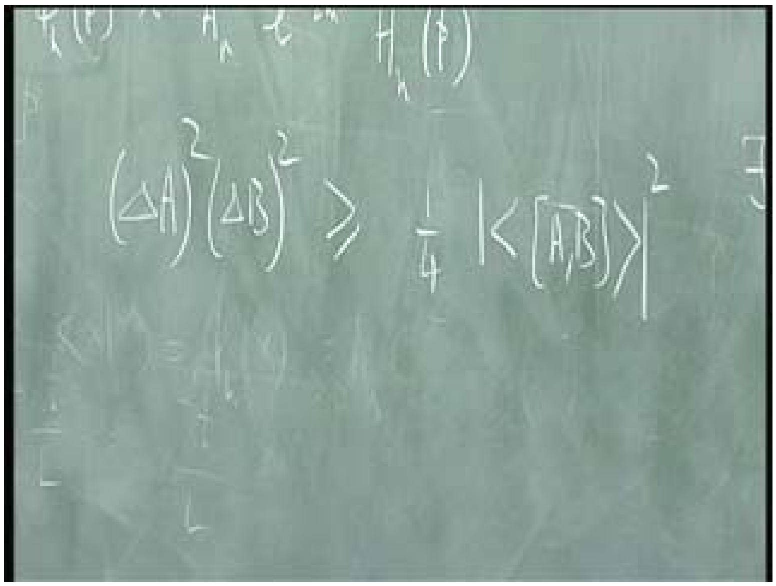

I already mention that delta a delta b, if you had 2 arbitrary operators then delta A squared delta B whole squared is greater than equal to 1/4th the expectation value of the commutator of A with B squared. this was the generalized uncertainty principle I mentioned, where A commutator B is some - i C or whatever, where C is the Hermitian operator but you can also ask what is the exact relation and it turns out to be greater than equal to this commutator squared + a similar expectation value squared of the anti-commutator. And since that is again a non negative quantity, you play it safe by saying this is certainly bigger than this. So there is an extra term in here. It is actually bigger than this + another term here which could to be 0 under circumstances. And that is the actual best result that you can get. That is the generalization of the uncertainty principle. It is an extension of the uncertainty principle. I believe it is called the Schrodinger Robertson uncertainty principle. i can't swear to that right now because i don't remember for sure but that extra term is sometimes used also and plays a role. So that should take care of these problem sets. Please go through this completely and make sure you solve all of them and if there is any problem, just let me know. So let me stop here. Thank you!