Quantum Physics
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Lecture No. #21

So let's start today with problem set five which is on Pauli matrices and spin 1/2 and so on and so forth. I mentioned already that the smallest non-zero quantum number for the angular momentum is a ½. .

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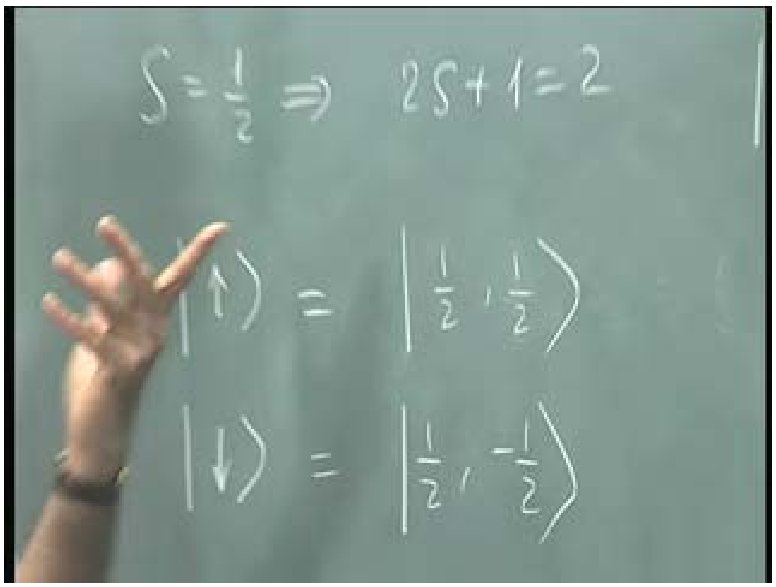

And spin 1/2 or j = 1/2 implies 2S + 1 = 2. So everything is in terms of 2 by 2 matrices. and physically, the first example that we have of 2 dimensional linear vector space is the spin states of the electron which are described by the 2 ket vectors. We have pointed out the up corresponds to S = ½, S_z = ½; all in units of h cross and similarly the down state is |1/2, - ½>. So recall, our notation was j and m where j is always the quantum number associated with the J squared. This has always got eigenvalue h cross squared j times j + 1 J squared rather and J_z has eigenvalue h cross m.

So these were 2 mutually commuting observables and my angular momentum states are labeled by labeling angular momentum quantum number and projection quantum number n. for the spin 1/2 particle like an electron, this number is always a ½. So I don't really have to write this we just have to write what S_z is. This corresponds to eigenvalues + 1/2 h cross - 1/2 h cross and pictorially one would like to say that this corresponds to spin up or a spin down. so i use this kind of notation for it.

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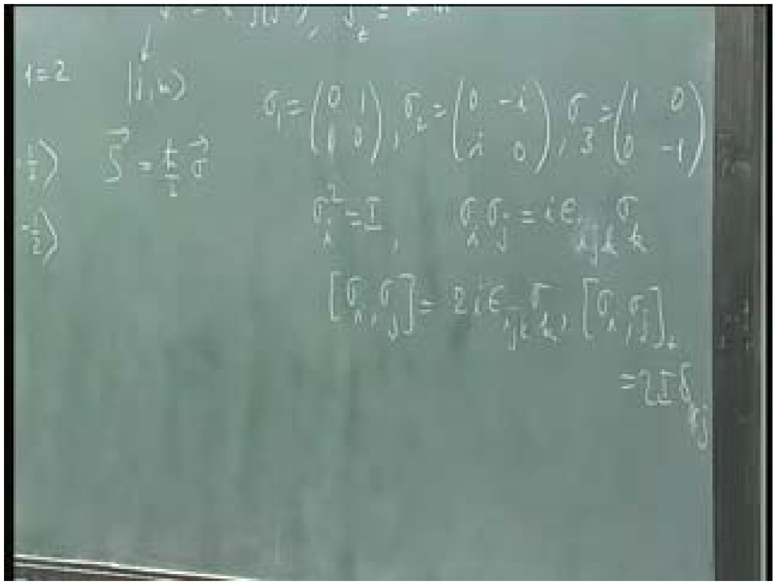

Now we already know that the spin operator of the electron can be written as h cross over 2 times the 3 Pauli matrices, sigma and these matrices have standard representation. Sigma 1 = 0 1 1 0, sigma 2 0 - i i 0, sigma 3 is 1 0 0 – 1. That's a very useful representation for spin 1/2 because each of these sigma matrices, the square is = the identity matrix. So sigma i squared = the identity matrix. And there will be interesting commutation relation. In fact sigma i sigma j = i epsilon $_{ijk}$ sigma k. so sigma 1 sigma 2 is = i times sigma 3 and so on. It's easy to see that [sigma i, sigma j] = 2 i epsilon $_{ijk}$ sigma k and the anti-commutator, sigma i sigma j $_{+}$ 0 if i is not = j and if i = j, then its = 2 I delta $_{ij}$. What are the eigenvalues of the sigma S? It's 1 and – 1. What about sigma 1 and sigma 2? What are their eigenvalues? They are also 1 and – 1. They all have the same eigenvalues 1 and – 1. The square of each of them is the identity matrix. They are linearly independent of each other. The 3 sigmas are linearly independent and no sigma can be written as a linear combination of the other 2. And you could take any arbitrary matrix A.

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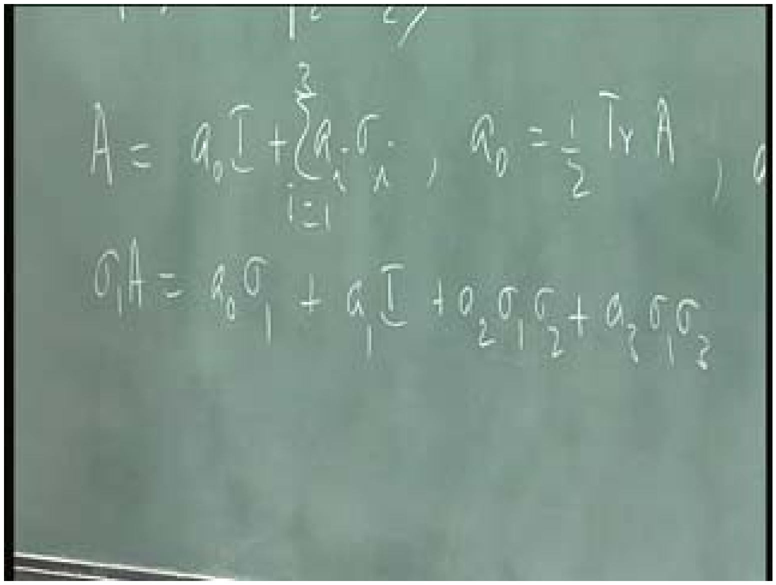

Any 2 by 2 matrix A can be uniquely written as a combination of the sigma's and the unit matrix. So you can always write this as a $_0$ I+ a $_i$ sigma $_i$ summation i = 1 to 3. This is unique. Any 2 by 2 matrix can be expanded in this form. What's a $_0$? What property of A does a_0 reflect? It's the trace. What about the trace of the sigma matrices? It is 0. So it's clear if you take trace on either sides, the trace of A is twice is a_0. So a $_0$ = 1/2 A. what about ai? If i formally want to invert this and write it, what should I write?

Suppose you want to find a1, what would you do? I'd multiply both sides by sigma 1 and then it becomes sigma 1 A on this side = a $_0$ sigma1 + a1 I+ a 2 times sigma 1 sigma 2 + a 3 times sigma 1 and sigma 3 and then take trace. So it is clear that you end up with a 1 = 1/2 trace A sigma 1 and so on. So formally that's the inversion. Similarly for a 2, a 3, etc. so its evident immediately that any 2 by 2 matrix can be uniquely expanded in terms of the sigma matrices. The great advantage of this expansion is that the unit matrix on the sigma matrices are all Hermitian matrices whereas the natural basis is not Hermitian.

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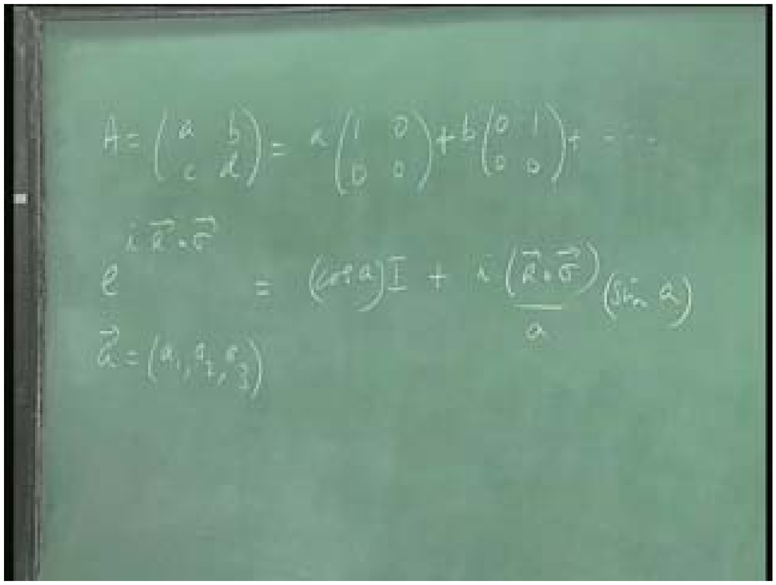

When i write a normal 2 by 2 matrix the natural basis for a 2 by 2 matrix is given by this (Refer Slide Time: 09:56). This basis is not Hermitian. This poses a lot of difficulty. On the other hand, the sigma matrices are Hermitian. That's the great advantage. So that's one of the reasons for why one uses the sigma matrices. The other thing is that exponentiation becomes very simple because e to the power i a dot sigma, where a is an ordinary vector; (a_1, a_2, a_3). this has a very simple expansion in terms of the cosine and sin of the modulus of a. it's easy to verify because you expand this and in the first term for example, it is the identity + i times a1 sigma 1 + a2 sigma 2 + a3 sigma 3 and then the same thing squared. but the squaring of a dot sigma will result in terms of the form a1 a2 times sigma 1 sigma 2 + sigma 2 sigma 1 and that will vanish because it anti-commutes. So only the terms which are squares of the sigma matrices would contribute when you exponentiate.

And the result is this 2 is a 2 by 2 matrix. This (Refer Slide Time: 11:16) is also a 2 by 2 matrix. So, it should also be expandable in terms of the unit matrix and the sigma matrices and the expansion is cos a times unit matrix + i a dot sigma over a sin a, where a stands for the square root of a 1 squared + a 2 squared + a 3 squared. It's a very useful representation. In fact you can now see what a rotation is going to do. When you rotate the coordinate system, you could ask what happens to any operator in this space. It will be the representative of the rotation operator times the operator times the inverse of the rotation operator and this will be easily calculable using this identity (Refer Slide Time: 12:20). Now what i have given here in this problem set is a whole lot of vector identities using sigma matrices. Among them, let me write down a couple of them because they are useful for what we are going to do later.

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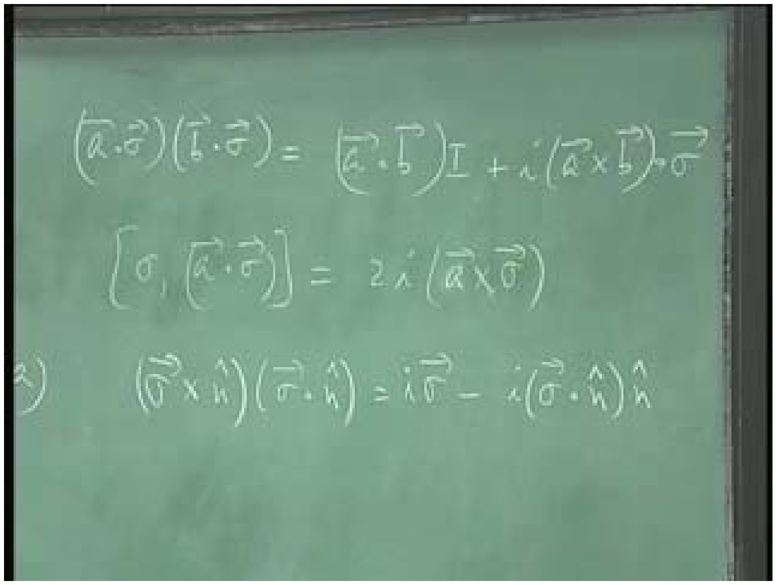

We have a dot sigma times b dot sigma = a dot b times the unit matrix + i times a cross p dot sigma. This is a scalar quantity and that's a scalar quantity but each of them is matrix valued. that is the unit matrix + something which depends on the sigma matrices, the coefficient is a cross b. so the commutator of a dot sigma with b dot sigma is twice i times a cross b dot c because it would just be the inverted thing and b cross a will be - a cross b and that cancels the - sign of the commutator. Then the other one that you need is sigma with a dot sigma = 2 i a cross sigma.

And similarly sigma cross any unit vector sigma dot the same unit vector is - i sigma - i times sigma dot n n. all these identities are easily proved by using the commutation relations between the sigma's. So much for some mathematical aspects of the sigma matrices. I have already explained where spin of a particle comes from. in fact the particles are really the wave functions must transform in a given manner under transformations of the inhomogeneous Lorentz group namely under rotations, velocity transformations, translations of the space time axes. This implies that all these wave functions are labeled by a certain set of quantum numbers among which are the rest mass of a particle and the spin of the particle or the intrinsic angular momentum of the particle. And in that classification, particles like the electron, the neutron, proton etc have spin quantum number ½. Particles like the photon have spin quantum number 1.

This is a whole host of particles with other spins. 1/2 integer spin particles are called fermions because when you put a collection of them together, they obey Fermi Dirac statistics where as integer spin particles are called bosons because a collection of them obeys Bose Einstein statistics. And a little later, we will talk about the differences between the 2 statistics. Right now i would like to point out that the way you prove the spin of a single particle like an electron is by using a magnetic field and the reason is as follows. We talked little bit about this earlier. so let me continue on the same lines.

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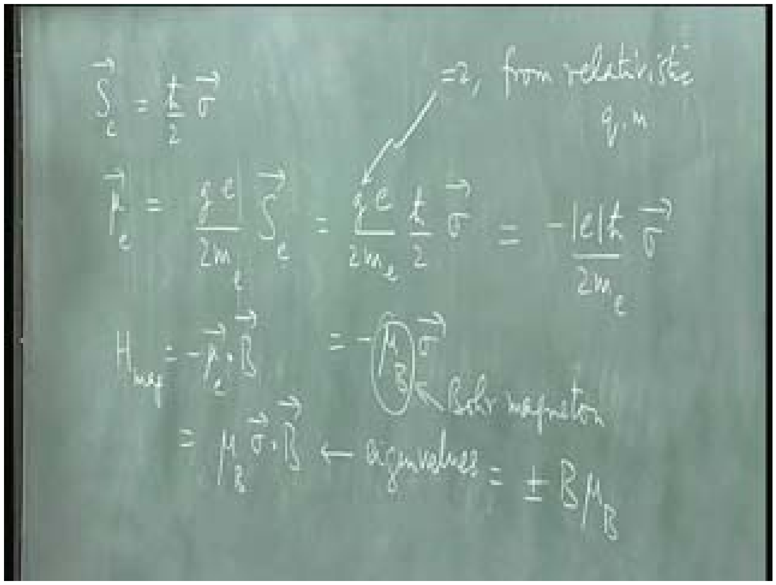

The spin operator of an electron, S_e = h cross over 2 sigma. The eigenvalues of any component are guaranteed to be the eigenvalues of sigma dot n, where n is a unit vector and they are + or - 1 multiplied by h cross over 2. Now what we do know is that the intrinsic magnetic moment operator of an electron mu e this is = a factor called the g factor of the electron, multiplied by the charge of the electron divided by twice the mass of the electron. e over 2 m is a standard gyromagnetic ratio, if you have a classical orbiting particle of charge e and mass m. that's the one proportionality constant between its magnetic moment due to the current loop that it forms when it revolves in an orbit and the orbital angular momentum.

That relation is generalized to g e over 2 m e times the spin operator of the electron. So if i put this (Refer Slide Time: 17:42) in, this = g, the charge of the electron twice the mass of the electron h cross over 2 sigma. Therefore the eigenvalues of any component of the intrinsic magnetic dipole moment of the electron are given by + or - this quantity because any component of it has eigenvalues + or – 1. But the g factor of the electron itself turns out to be 2 from relativistic quantum mechanics. So this is input information. There is no way of deriving this g factor in classical physics. It's not a classical concept or even in non-relativistic quantum mechanics.

It has to come from relativistic quantum mechanics which predicts that g is equal to 2. Actually the g, if you measure it exactly, is not 2. It has a small correction and this correction is known to a very large number of places and has been verified. The correction comes due to what are called radiative corrections due to quantum field theory. We are not concerned with that right now. We will simply put in the fact that g is = 2 and therefore this becomes = - modulus of charge of the electron e, h cross over 2 m e times sigma. And this is = - the Bohr Magneton times sigma. so if you are in S_z = + 1/2 state, which corresponds to saying sigma 3 = 1, then the magnetic moment in the z direction has values - the Bohr magneton. Otherwise it has a value + the Bohr magnetic in the other state. And this difference in the sign between the magnetic moment and the spin comes about obviously because a charge of the electron is negative. So this is really all one needs to know.

The moment you put this electron in the magnetic field, if it's a uniform magnetic field, then it experiences a torque. The dipole moment experiences a torque. And the magnetic part of the Hamiltonian is = - mu e dot B. that's the potential energy and if i put this in, this is = Bohr Magneton and sigma dot B, the - sign goes away and this is your magnetic Hamiltonian. What are the eigenvalues of this Hamiltonian? Well, sigma dot B is a component of sigma in some direction multiplied by the magnitude of B. so it's clear you take this to be the unit vector and you divide it by its magnitude. You would get the unit vector and that portion has eigenvalues + or – 1. So it is clear that the eigenvalues are = + or - B times mu $_B$, where B is the magnitude of the magnetic field. Since we know that sigma dot any unit vector has eigenvalues + or – 1. It's + B mu if the spin points along the direction of B and - if it points in the opposite direction. So at this level it's completely trivial. Now what we would like to do is the following.

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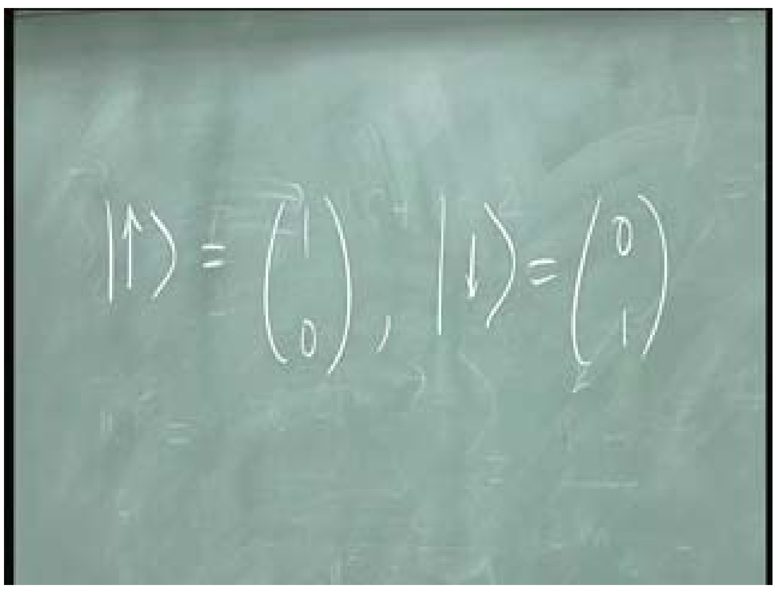

Here is my Hamiltonian and i start in an arbitrary state of the electron and remember that this up state here can be represented by 1 0 and the down state is conveniently represented by 0 1. I choose the z axis as my axis of quantization for the moment. Then the complete set of spin states of the electron is comprised of just 2 orthonormal states which are the up state and the down states and they are each normalized.

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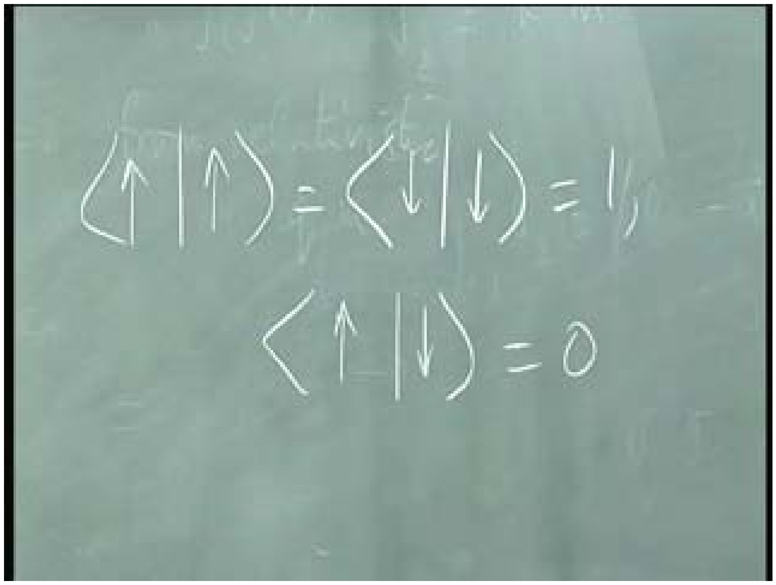

So now i ask, suppose i start in an arbitrary spin state of the electron and here is my arbitrary state.

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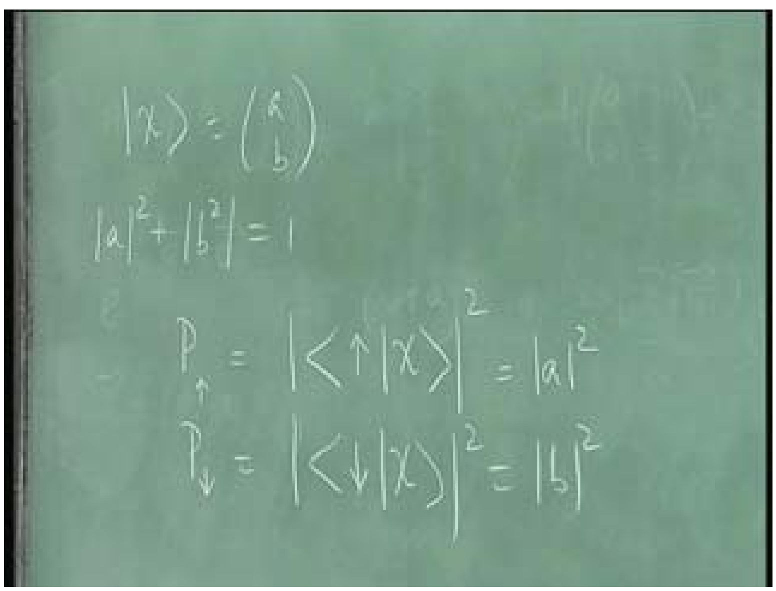

Chi = (a b).That's an arbitrary state of the electron. I would like to start with a normalized state always. a and b are 2 complex numbers which satisfy mod a squared + mod b squared is 1. In this state what is the probability that the S_z component of electron has eigenvalue + 1/2 h cross?

How would you compute that? Well, it's clear that the probability we ask for p up; we want the probability at the spin is up, this is = the modulus squared of this probability amplitude by definition. This (Refer Slide Time: 25:20) is the probability amplitude that when it's in the state chi. it is in state up and the mod squared of it is the probability itself. So what's this (Refer Slide Time: 25:31) = mod a squared and similarly p down = b squared. So that's my initial state. Now I switch on a magnetic field in some arbitrary direction. What would happen to the state? Would these probabilities be the same? First let's ask a very simple question. What if i switch on field in the z direction itself? Will chi of P change? So in all cases here is the Hamiltonian (Refer Slide Time: 26:19). So what would happen to that state? Let's say I start with 0 state t = 0. What would happen to this state?

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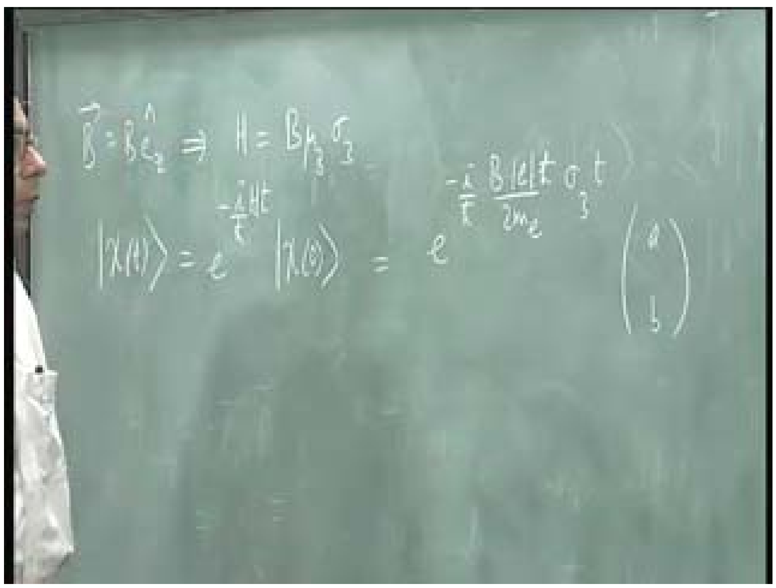

So I put B and this implies the Hamiltonian is B mu B sigma 3. That's the Hamiltonian. So I ask chi(t) = e to the - i by h cross Ht chi(0). this is = e to the - i over h cross, the Hamiltonian is B mu Bohr Magneton, which is modulus e h cross over 2 m e; you just expand what this mu B is, and then sigma 3, this t as well acting on a b. so what does this give us? This is an h cross which cancels out and what does this give us? i need to use that expansion that i wrote down in terms of cosine and sin and so on. Let's write that out just for convenience.

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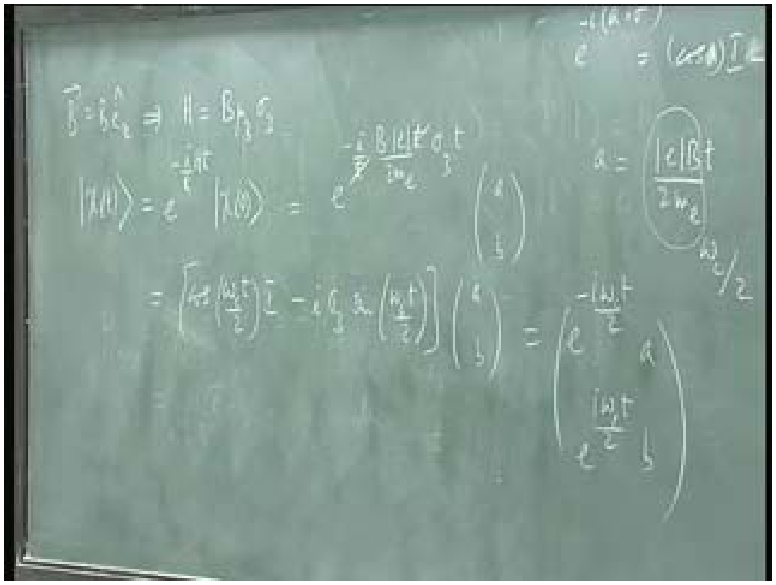

e to the i a dot sigma is = cos a times I - i a dot sigma over a sin a. So let's use that and what's a in this problem? a dot sigma, a along a z direction is just 1 component here. So what does that give you? a in this problem is = modulus e B over 2 m e multiplied by time. So what would happen here? What's eB over 2 m? Its 1/2 the cyclotron frequency because remember there is a difference between a classical orbiting particle and the spinning electron because this has a g factor = 2. So this constantly there is going to be change of factor of 2 everywhere. So we have to be careful. this is = cosine omega$_c$ t over 2 as a unit operator - i a dot sigma and the a parts simply cancels out and then you get sigma 3 sine omega $_c$ t over 2 acting on a b. now it's trivial to find out. So what does this give you? Its e to the - 2 on a and the other 1 is the bottom element because with a - sign so that becomes a +. Do the probabilities change?

It can't because your Hamiltonian is also diagonal in sigma 3. So the probabilities don't change. There is not flip at all. Mod a squared and mod b squared are exactly the same at mod a(t) whole squared and mod b (t) whole squared. If i had the field which had any component other than the 3 component, then of course there would be an off diagonal element here. For example, there is a portion sigma 1 and a sigma 2, they would have off diagonal elements here and they would mix up things between these two (Refer Slide Time: 32:24) and then you would get oscillations. Then the probabilities would indeed change. This means that the spin up and spin down, the total probability always remains 1 but there are transitions between the 2 spin states caused by a transverse speed. What we have to ask for here is somewhat more interesting than and that's the following.

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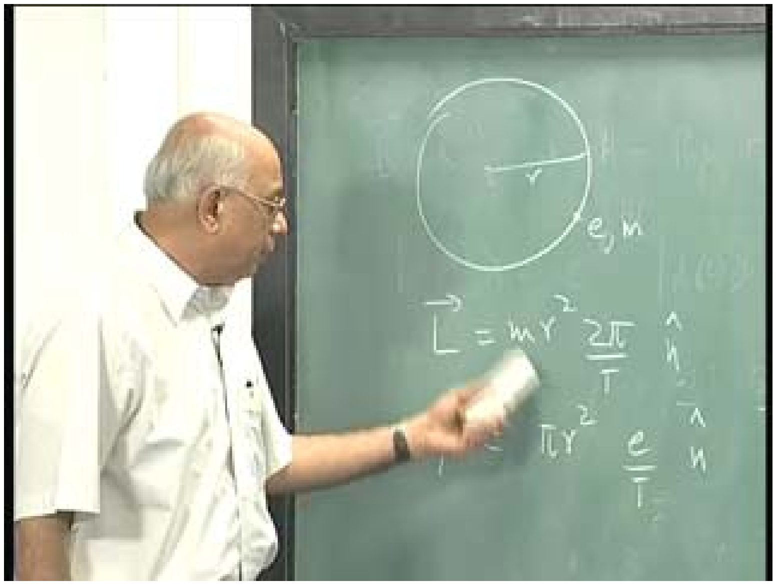

The idea was to find out how this magnetic dipole behaves in a magnetic field and what kind of precessional motion it undergoes. Let me point this out carefully. Consider a classical particle orbiting in a radius r. the charge of this particle is e and the mass of the particle is m. then what’s the relation between the magnetic moment and the orbital angular moment in this case? The orbital angular momentum is m r squared times the angular velocity of this particle which is 2 pi over the time period, T in the direction of r cross p. So let’s say some unit vector n. the magnetic dipole moment of this particle by Ampere's law is the area of the current loop multiplied by the current. The area of the current loop is pi r square and the current is e over T and it’s also in this direction. So this gives you a relation between L and mu. the gyromagnetic ratio not surprisingly is e over 2 m. this is a 2 m factor here and there is an e here so e over 2 m is a gyromagnetic ratio (Refer Slide Time: 34:30). Mod mu = over 2 m mod L.

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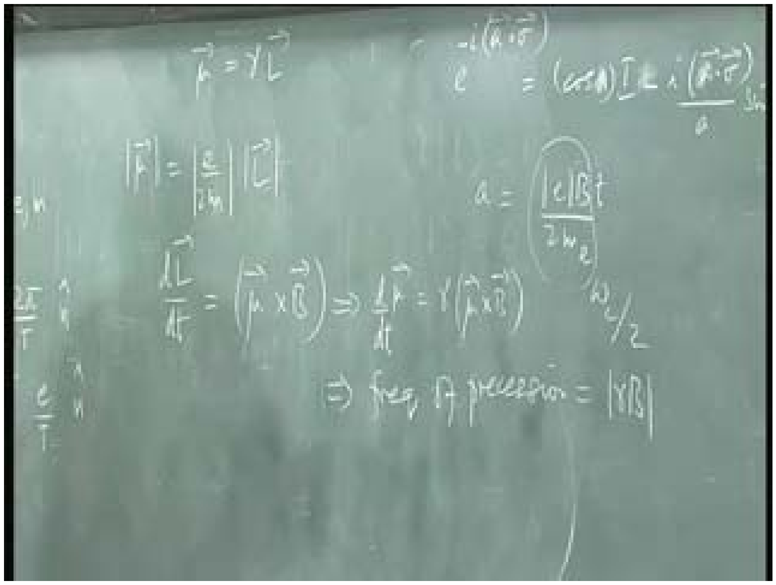

What’s dL over dt, the rate of change of angular momentum? It’s the torque. It’s mu cross B. what’s the Larmor frequency of precision? What we have to do is to convert this L (Refer Slide Time: 35:17). So if we call this constant gamma, so we write mu = gamma L in this fashion, then L is mu by gamma. So this would imply d over dt mu = gamma times mu cross B. This implies frequency of precession modulus gamma B. that’s trivial to see from this equation. Once you get an equation like this, you know its precessional motion and the frequency of motion is just the magnitude of gamma times magnitude of B. so in the classical case this would become modulus eB over 2 m. what happens if we put a quantum mechanical particle?

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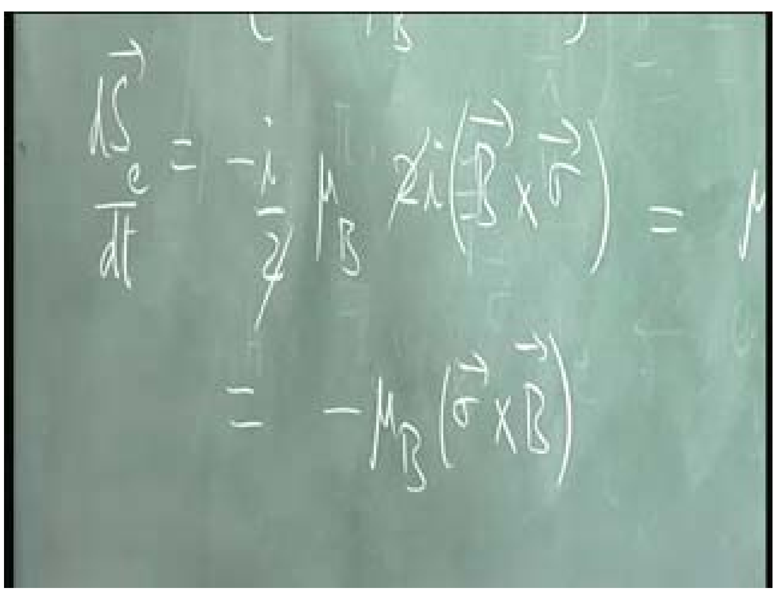

Then for the spinning particle, we have to compute what ds over dt. We can't just write it down as torque because that's a classical equation of motion. How do i compute it? This is the Heisenberg equation of motion. So I have ih cross dS $_e$ over dt on the side is = the commutator of the spin operator with the Hamiltonian. This is = h cross over 2 sigma that's S e with the Hamiltonian. But for the Hamiltonian we had a very simple expression in an arbitrary magnetic field. This is a Bohr magneton multiplied by sigma dot B. this is what we discovered. So this says dS $_e$ over dt is = - i, i am going to bring this i to this side. - i the h cross cancels, the factor 2 remains, mu$_B$ remains and then the commutator of sigma with sigma dot B. but we have an convenient expression for it.

Sigma with a dot sigma is 2 i a cross sigma. These (Refer Slide Time: 38:42) are vectors. These stand for all 3 components together and they don't commute with each other. so that's the reason you get non 0 answers here but you could write this as a dot sigma or sigma dot a. that doesn't matter because a is an ordinary vector just as the magnetic field B is an ordinary vector. So you got a 2 i B cross sigma. This is multiplied by 2 i B cross sigma. The 2 is cancelled. This = + mu$_B$ B cross sigma. That is = - mu $_B$ sigma cross B. so this was my classical equation and now i would like to write the quantum equation. The quantum equation says dS e over dt is = - mu $_B$.

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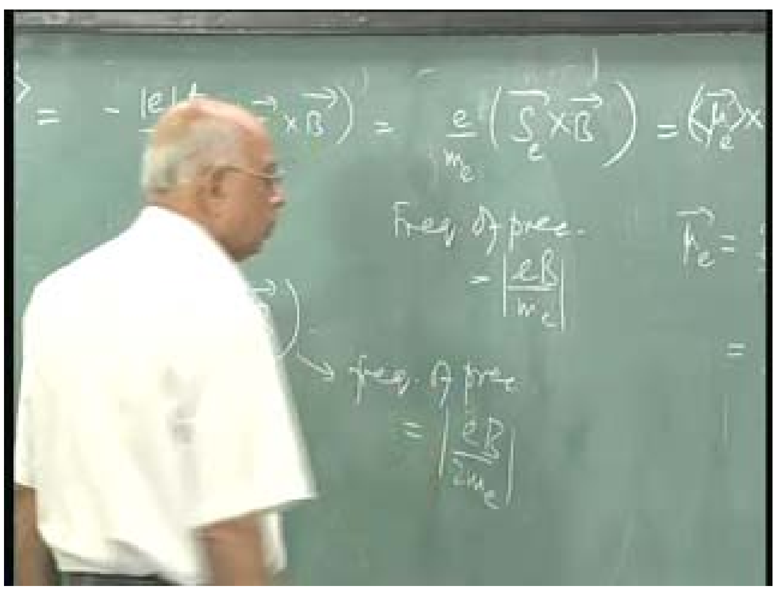

So this is - modulus e h cross over 2 m e times sigma cross B which is = e over m e, the spin operator; S cross B. therefore again you have precessional motion but what' the frequency of precession? Its e over m and not e over 2 m and the reason for this difference was because of the g factor of the electron. So the precessional motion still appears. It's exactly the same as before. In fact what was mu e in terms of the spin operator? It was g e over 2 m 2 m e into the spin. So this was exactly equal other e over m e S_e. so in fact you could write this as mu e cross B. so the point is the following. The classical equation of motion says rate of change of angular momentum is = the torque. the torque on a dipole of magnetic moment mu is mu cross B. so it says dl over dt is mu cross

B. quantum mechanically the exact equation of motion for the spin operator which is an angular momentum operator is rate of change of the angular momentum operator again = mu cross B. therefore the equation of motion doesn't change at all. This is another example of Ehrenfest's theorem because if i took expectation values on both sides, then it says this = that (Refer Slide Time: 43:16) because those are the operators.

And Ehrenfest's theorem says that quantum mechanical expectation values obey classical equations of motion for such classes of Hamiltonians and that is exactly what has happened. But when you want to actually compute what the precession frequency is, you have to convert this mu into an L or you have to convert this L into a mu because that's how you get the precession of mu. So you have to convert this S into a mu and the factor that converts is in fact the gyromagnetic ratio and that's e over m here. So the frequency of precession is = modulus e B over m_e whereas as the frequency of the precession here = modulus e over twice m_e (Refer Slide Time: 44:16). if you have the way classically you have a magnetic dipole moment is by imagining there is motion of a current in the form a loop. so i said let's take the simplest instance where you have a particle moving in a circular orbit of radius r and so on and found that in the classical case the gyromagnetic ratio is charged divided by twice the mass.

Here for the internal motion for the spin degree for freedom, the relations between the magnetic moment and the intrinsic angular momentum has an extra factor g which has to be 2 for the electron. This implies that event though the expectation value of this (Refer Slide Time: 45:19) angular momentum undergoes precessional motion exactly as the classical counter part would, the frequency of precession is eB over m here but its eB over 2 m e here that's because there was a extra g factor sitting there. But otherwise the equations of motion are exactly the same in both cases. Rate of change of angular momentum is magnetic dipole movement cross the magnetic field. It's exactly the same equation except in classical mechanics you write that down from the rules of classical electromagnetism but in quantum mechanics you have to compute it. given the Hamiltonian you actual you have to calculate this commutator here and you discover exactly the same equation of motion.

Now what i would like to do is to ask suppose i have a general ket vector of this kind (Refer Slide Time: 46:13), how could i interpret this ket vector? We know that if it was 1 0 i would say that corresponds to an eigenstate of Sz. if it was 0 1, it would be eigenstate of Sz with the opposite eigenvalue. What happens if it was some a b could i interpreted as the eigenstates corresponding to + 1/2 h cross of some spin pointing in some arbitrary direction? This is the question i would like to ask. Suppose i chose my axis of quantization along some arbitrary direction, then could i interpret this b as the eigenstate corresponding to spin up in that new direction? So let's see if that works out. That will be very useful for what we are going to do next.

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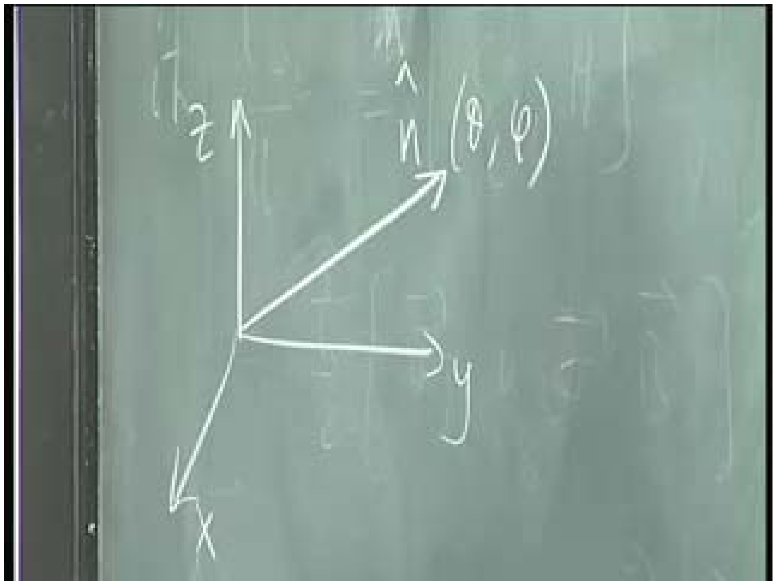

So let's take some arbitrary unit vector n in space with respect to my fixed coordinates system specified by polar angles theta and phi. What are the components of this n?

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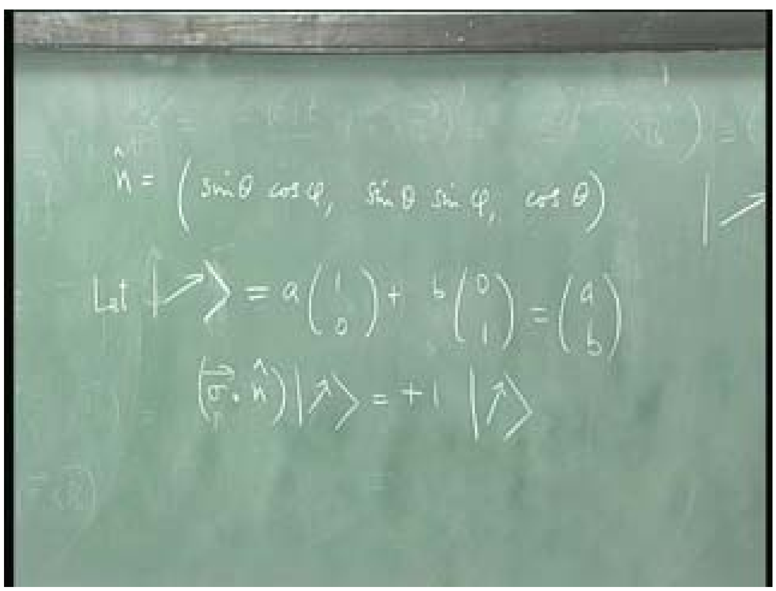

The magnitude is 1. So what is it in spherical polar coordinates? Its = sin theta cos phi, sin theta sin phi cos theta such that mod n squared is n squared is 1. Now we would like to ask i mean make the question precise. What's the up state for this? let me denote this (Refer Slide Time: 48:57) as up state of S dot n or sigma dot n and the opposite arrow wound be the down state of this. So what should i do? I would like to find that up state.

i expand this up state. Any arbitrary state can be expanded in eigenstates of Sz which was my original axis of quantization. So let me simply expand. We can simply write up state here is = some a times 1 0 + b times 0 1. I know that sigma dot n on this state must be = 1 + 1 times the same state. i require it to be an eigenstate of + 1/2 h cross. So when i multiply the sigma by h cross over 2, then i am going to get s dot n and the eigenvalues + 1/2 h cross. So that's why we put a + 1 here. And we have to solve this eigenvalue equation. That's all we have to do. What does this give you? Sigma dot n is sigma 1 times this (Refer Slide Time: 50:54). so it's clear that sigma 1 is off diagonal completely, sigma 2 is fully off diagonal and sigma 3 has diagonal terms. We want sigma 3 times n 3 which is cos theta here.

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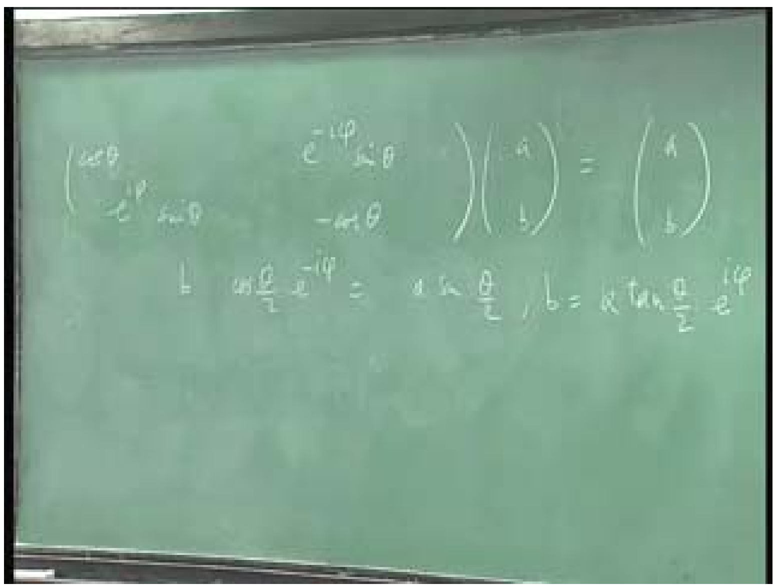
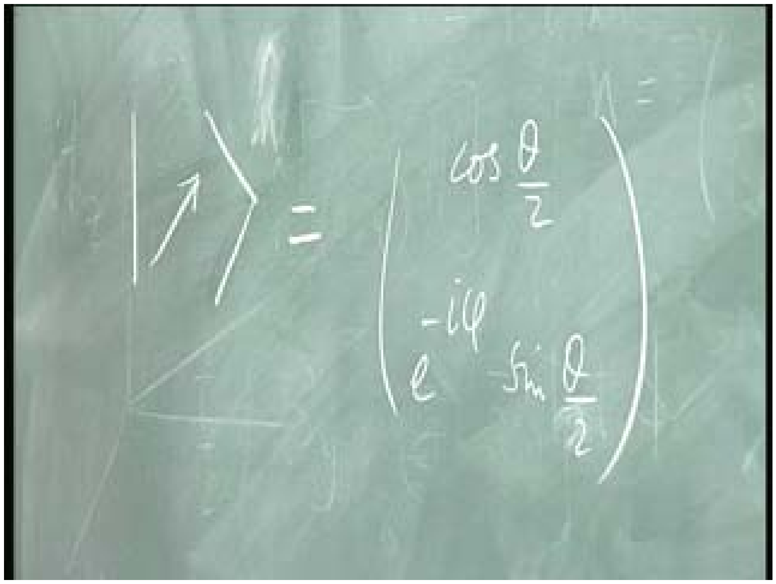


So this will give you cos theta, - cos theta on this side. And then sigma 1 is 0 1 1 0. So it's going to give you sin theta, cos phi, and then sin theta, cos phi acting on a b is = a b. that's my eigenvalue equation. This (Refer Slide Time: 52:06) operator here is just a b. then i have sigma 2 which is multiplied by this (Refer Slide Time: 52:14) coefficient but sigma 2 is a – i. so this is = sin theta cos phi - i sin theta sin phi and then + i sin theta sin phi and then - cos theta. That's my eigenvalue equation. My job is to find a and b and normalize it to unity and then i am guaranteed to get this ket vector a b which is the eigenstate of sigma dot n with eigenvalues + 1. So this will be give you cos theta - i sin theta. Sin theta comes out common. So those are the eigenvalue equations (Refer Slide Time: 53:24). So what does it give you?

a cos theta + b sin theta e to the - i phi = a and similarly a e to the i phi sin theta - b cos theta = b. either of them would do because this is something way we going to normalize a and b such that mod a squared + mod b squared is 1.so it says b sin theta is a into 1 - cos theta and then i normalize it to unity. So this (Refer Slide Time: 54:31) can be written as 2 b sin theta over 2 cos theta over 2 which is 1 - cos theta which is 2 a sin squared theta over 2. So the 2 goes away in both sides. a sin theta over 2 goes away and it tells you b =

a and tan theta over 2 e to the i phi. So I have in fact written the answer down. So let me write the answer down after you normalize it.

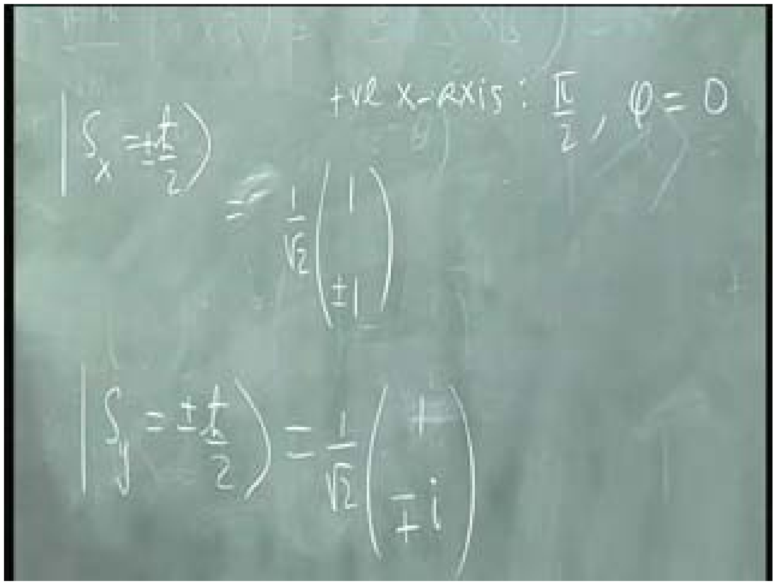
(Refer Slide Time: 00:55:29 min)



This state (Refer Slide Time: 55:36) is = cos theta over 2 and then e to the - i phi sin theta over 2. That's the normalized state. Mod a squared mod + mod b square is 1 in this case. It's worth remembering this expression because you can then write down things very fast. By the way, the overall phase factor is irrelevant because you can always multiply this by some e to i alpha and it will not change any probability. It won't change any physics at all. So you could if you make this (Refer Slide Time: 56:32) look a little more symmetric by multiplying by e to the i phi over 2 in which case you get e to the i phi over 2 cos theta e to the - i phi over 2 sin theta over 2.

This looks a little more symmetric but this expression is worth remembering because it keeps appearing over and over again. so you agree that if you give me any arbitrary ket a b such that mod a squared + mod b squared is 1, i could interpret that ket as the eigenstate of S dot n with an appropriate n. the polar angles of n are given by the a's and b's in this fashion. Their relative phase difference between a and b is the azimuthal angle. And a is like cos theta over 2, b is like sin theta over 2. so the ratio of a to b in fact gives you the cot tangent of theta over 2 of b 2 a mod b over mod a gives you tan theta over 2. So that's the way to interpret an arbitrary state a b. now you can quickly check that all the usual results will come out. For example, if theta 0 that means axis of quantization is z axis itself. i should get 1 0 and indeed i do. What happens if theta is pi? You get 0 1 apart from a phase factor which is irrelevant because the phi angle is undefined. So that works. What about this eigenstate, S_x= +h cross over 2?

(Refer Slide Time: 00:58:24 min)



What kind of state corresponds to + 1/2 h cross by 2 as the eigenvalue for Sx? All we have to do is to substitute values of theta and phi into it. What about the x axis? What does that correspond to? What value of theta does it correspond to? It’s pi over 2. Phi = 0. so what does this become in the z basis? This is cos phi over 4 which is over root 2. This is gone and that’s also 1 over root 2. So it’s clear this is just 1 over root 2, 1 1 and the same thing is -, what would happen? What would that correspond to interms of phi? Phi goes from 0 to pi. It’s the the negative x axis. So that becomes + or – 1. What about Sy = + or - h cross over 2? What would these correspond to? Now you want to know the coordinates of the positive y axis? Again theta is = pi over 2 but phi is either pi over 2 or 3 pi over 2 in the other direction. so this (Refer Slide Time: 10:00:27) is = cos theta over 2 that’s = 1 over root 2 once again comes out always and then this is a 1 and when phi is = pi over 2, its e to the - i phi over 2 which is - i and e to the - 3 i pi over 2 is e to the + i pi over 2 which is + i. so this is 1, - or + i.

so if we see such states, you incidentally recognize that these corresponds to eigenstates of the x and y coordinates but a general eigenstates looks like that. I am going to stop here and then we resume this. to answer this question it says consider an arbitrary initial state chi = a b. suppose you measure not S_z in which case the probability of + 1/2 h cross would have a mod a squared and the other 1 is mod b squared but you measure S_x, and the question is what’s the probability that you obtain values + 1/2 h cross and - 1/2 h cross respectively. What would you do? you change basis. The obvious thing to do is to change basis. so we start with a b and write it as a linear combination of these (Refer Slide Time: 01:02:16) 2 states and then probability they are also orthonormal. Every one of them has been made orthonormal as a basis and then identifies the coefficients. So in that case, the answer will turn out to be mod a + b whole squared over 2 and mod - b squared over 2. Similarly if you measured Sy, the answer would be a + iB mod squared a - iB mod squared and 1/2 of these quantities give you the probabilities. So i hope it’s clear what’s happening. you started with an axis of quantization, by convention z axis and

I took an arbitrary ket vector and identified it with the eigenstate of S dot n, particular n but I use that backwards to write down what the eigenstates are for any axis of quantization directly. and if you want to know what happens if i take an arbitrary initial state and measure in any other arbitrary component, what are the probabilities of getting + or - 1/2 h cross because those are the only 2 answers possible. The answer is take this ket vector and expand in that basis. So it is just change of basis and take the mod squared of the coefficients. So let me stop here.