Quantum Physics

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Lecture No. # 19

We are going to spend some more time now on the properties of spinning particles specifically with regard to the spin of the electron because this is an extremely valuable probe not just in particle physics but also in condensed matter physics and many other areas of physics. So its worthwhile spending some time on this and telling you how spin of electron behaves under applied electric & magnetic fields.

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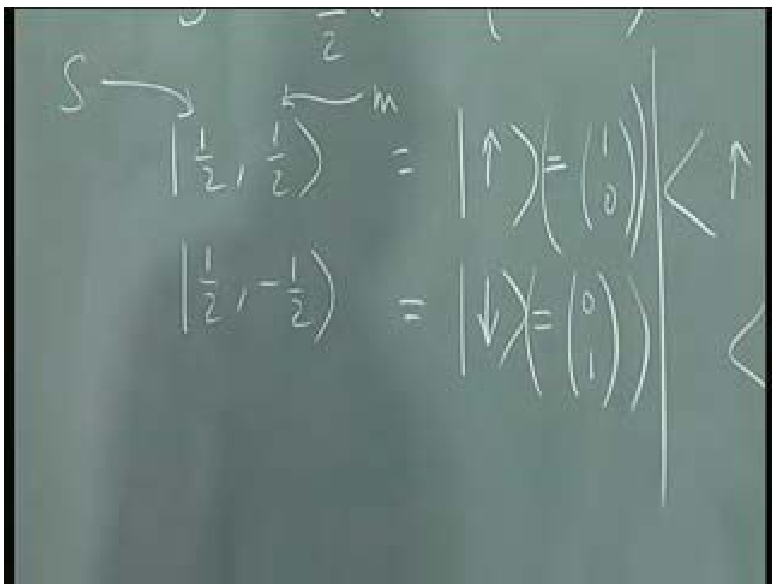

If you recall, the spin operator for an electron S has 3 components, S1,S2, S3 or Sx, S y,Sz, which obey the angular momentum algebra and they are represented by h cross over 2 sigma, where sigma are the Pauli matrices. Each Pauli matrix has eigenvalue + or – 1. So this guarantees that any component of the spin of an electron, the intrinsic angular momentum has eigenvalues + or - 1/2 h cross. So the spin quantum number is a 1/2 for the electron. As you know, if you have a quantum number j for the angular momentum, then the subspace in which the states of this angular momentum live is 2 j +1dimension.

There are 2 j +1 values for the eigenvalues of any component of S or the angular momentum. So in this case, it becomes 2 dimensional and therefore the states that you could talk about are | 1/2 , 1/2 > and |1/2, - ½>. This corresponds to the quantum number S which is always 1/2 for an electron and this (Refer Slide Time: 2:49) corresponds to m where m times h cross is the eigenvalue of any one component of the angular momentum

And we will conventionally take it to be the z component. Then a very convenient representation for the spin operator is this thing here (Refer Slide Time: 03:09). And recall the definition of the Pauli matrices. We have written this down several times. They are sigma 1= (0 110), sigma 2 = (0 - i i 0) and sigma 3 is already diagonal and it's (10 0 - 1). There are numerous properties of the sigma matrices which i have written out and which we have come across earlier when we studied linear vector spaces. The square of every sigma matrix 2 by 2 unit matrix and the eigenvalue of any sigma matrix is + or -1. They don't commute with each other. Sigma 3 is already diagonal. So once you diagonalize that, the other 2 cannot be diagonalized simultaneously but any of them could be diagonalized and the eigenvalues would be 1 and – 1. this state of the electron corresponds to the total angular momentum quantum number 1/2 which we have already known and Sz eigenvalue is +1/2 h cross and this corresponds to - 1/2 h cross (Refer Slide Time: 04:20). Since we will we have in the back of our minds, what happens when you apply a magnetic field, in which case you have either an up state or a down state for the magnetic moment. its conventional to call this in short hand, a spin up and this a spin down (Refer Slide Time: 04:45). the understanding being that the basis that you choose corresponds to states in which Sz is diagonalized. So eigenstates of Sz. then the orthonormality relations are of course up up =1=down down and the scalar product down top is 0. So this is going to be very a convenient notation for us. We use this notation i am going to talk about several spins then this would become very helpful. Now what is the actual representative of this term (Refer Slide Time: 05:31) here? Remember in the basis natural basis, you could also write this as 1 0 and this could be written as 0 1(Refer Slide Time: 05:41).

In any 2 dimensional linear vector space, you could choose 10 and 0 1 as the orthogonal independent vectors which form a basis in this space. Now why are we making such a big first about it? the answer is even though the quantum number 1/2 is extremely small compared to classical quantum numbers, for instance, we know that the angular momentum component has value + or - 1/2 h cross. That's actually true and angular momentum is quantized. So if you if you took a stone weighing 1kg and you whirled it around in a circle of 1m radius, mr squared is 1 in standard international units, that's the moment of inertia. If you chose the time period to be 1 second for example, then the angular frequency is 2 pi.

Therefore the angular momentum is i omega which is 2 pi joule second which is enormous. But that 2 pi joule second must also be = orbital angular momentum multiplied by h cross. This must be of the order 1or 2 pi but this is of the order 10 to the -34 in these units and therefore this quantum number is 10 to the +34. That is so enormous that the discreteness of the angular momentum completely gets washed out absolutely. Whether you add ten or hundred or whether you add million or billion it's still doesn't matter. 10 to the 34 is an enormous number that's the reason why you don't see a angular momentum quantization in daily life in macroscopic objects. But it's very different story when you have a quantum number like a ½. This indeed tiny so this whole thing this has eigenvalues + or – 1. So you can see numerically this is extremely small in the normal daily units that we are used to. So how do you find it? How do we detect this spin at all? the answer is that this spin also implies a magnetic moment for the electron and that magnetic moment would couple to applied magnetic fields and then you can manipulate the spin or probe the spin of the electron using the magnetic field or using the various protocols for the magnetic fields like switching them on and off in different directions etc. Let’s see how this is done.

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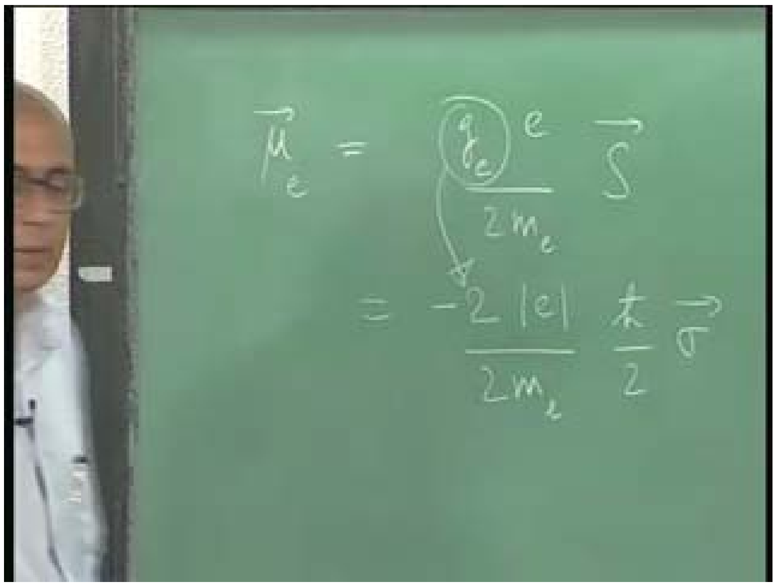

The magnetic moment of the electron mu e as i pointed out last time is = the g factor of the electron ge multiplied by the charge of the electron divided by twice the mass of the electron times the spin operator, the intrinsic angular momentum operator for the electron. But this is =ge which is 2, and i will explain why this is 2. This turns out to be 2 for the electron which is modulus e with the - sign because electron has a negative charge. It’s divided by twice mass for the electron and the spin of the electron is h cross over 2 times sigma. The reason it is convenient to do this is because this sigma is dimensionless and has a eigenvalues + or – 1 for every component. One of the 2 is cancelled and you end up with - mod e h cross over twice m e sigma. And of course you are familiar with this (Refer Slide Time: 09:40) object. This is the Bohr magneton.

So you could write this as = - mu bohr times sigma and mu_b here is the Bohr magneton. It’s the natural unit of the magnetic moment which you get for a particle of charge e and mass m_e. When you put this thing in a magnetic field, there is an extra contribution to the energy of the system or the Hamiltonian of the system. The electron would have some Kinetic energy of some kind. So there is a Hamiltonian but when you switch on a magnetic field, there is an extra contribution which is the magnetic Hamiltonian if you like. So let me call this H magnetic which is = - mu dot B.

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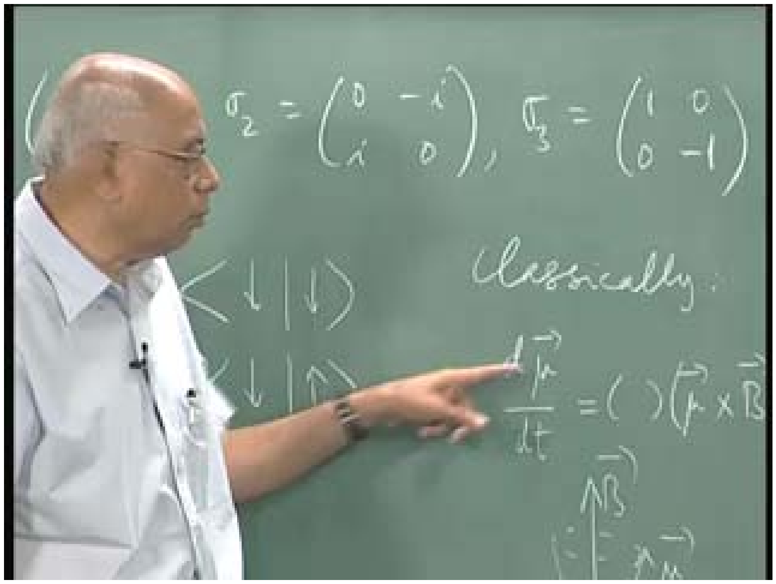

The magnetic dipole moment dotted with B is the potential energy of a magnetic dipole moment in an applied magnetic field B. and if you put this in, this becomes = mu B which is a number and then sigma dot B. that's is the Hamiltonian. And now if you start with the system in some state, you could change the state by switching on this Hamiltonian and then letting the system evolve as time goes along and you would have to see what happens. Classically I expect that if i have a magnetic dipole moment placed in an external magnetic field pointing upwards, then this dipole moment precissess around the component of the magnetic moment along the field. It doesn't change in the magnitude. and the other 2 components undergo harmonic motion. So, you have a precessional motion and the tip of this magnetic moment vector precess a circle and forms a 1/2 cone about the direction of the magnetic field. And the magnetic precession frequency is easily computed. It is the Larmor frequency of precession which is trivially computed once you write down the equation of motion for the dipole moment. so classically, this mu is different from all these other mu's that I have talked about, if you have a magnetic moment dipole moment mu, then d mu over dt is = the torque on this term.

The torque is mu cross B and it's a constant. Actually it's an angular momentum because mu is proportional to the angular momentum. The rate of change of angular momentum is = the torque. Since the magnetic moment and the angular momentum are connected through some gyromagnetic ratio, it's essentially d mu over dt times some constant. so there is some constant sitting here (Refer Slide Time: 12:57) and then a mu cross B. the moment you have an equation of motion of this kind, you know immediately that if this is the direction of B and that is the initial direction of mu then, all that this does is to trace a path of this kind (Refer Slide Time: 13:06 to 13:18). The tip of this vector traces a cone whose direction along the magnetic field doesn't change. and that's easily found because all you have to do is to dot both sides with B. the magnitude of this doesn’t change either because all you have to do is to dot both sides with mu.

So the magnitude doesn’t change and this component doesn’t change so the only thing it can do so move around a circle. And the precession frequency is determined by this constant here which will involve the gyromagnetic ratio and so on and so forth. Quantum mechanically what do you think will happen? Well by Ehrenfest's theorem, I expect that the expectation value of the magnetic moment would obey a classical equation of this kind. so with the expectation values put around this mu, I still expect the same sort of equation. But what does a system actually do? That depends on what this Hamiltonian is, what the direction of the magnetic field is, etc. and we really should write down now is the state of the system at any given time. So lets try to compute it and see what happens but before i do that, I also point out that if the field is sufficiently non-uniform then you know that a magnetic dipole moment not only undergoes a torque but also a force. so its possible if the field is sufficiently non-uniform and strong enough to actually separate out different magnetic moment states, and this was what was done in Stern-Gerlach experiment which is a classic experiment which i will describe a little later because it’s at the root of all that’s being said today about quantum computing and quantum information and so on.

So very often in books on quantum computation, you would see an sg apparatus which means the Stern-Gerlach apparatus. In different directions, you have magnetic fields and so on. we will come back to that we keep that are the backup our mind but right now we will take this B to be a constant magnetic field and ask what happens to an arbitrary state of the system. You start with an arbitrary spin state of this electron, what will happen as a function of time? Well, the first thing to note is that since these 2 states; the up and the down actually form a basis in this space, it’s clear that any spin state of the system denoted by psi(t), can actually be written uniquely as a superposition of these 2 basis states. So this can always be written as a(t) up +b(t) down, where a and b are 2 coefficients which will depend on time. And if you normalize this state, since these are already normalized, we are going to assume that a(t) whole squared +b(t) whole squared is 1for all time.

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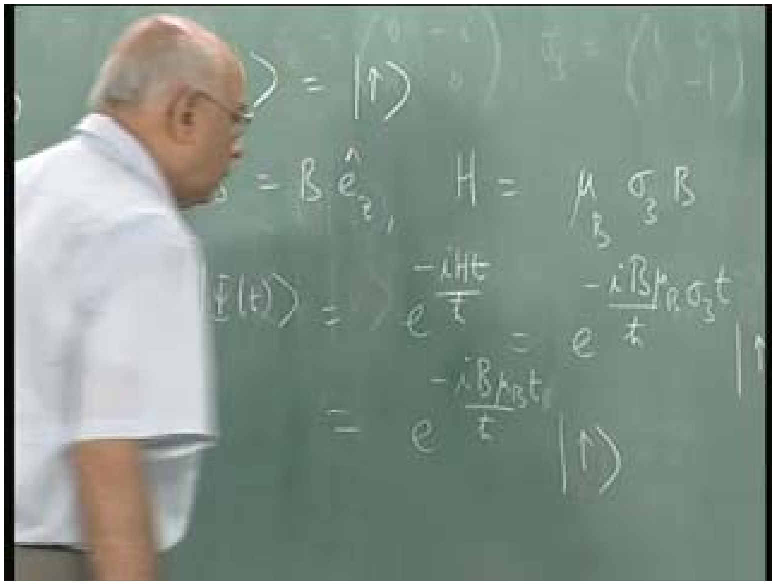

So we will talk about normalized states of the system. Now lets do the simplest problem and that is i prepare the system in the up state say, so i tell you to start with that psi(0) is just up that means a(0) is unity and b(0) is 0 to start with, and then I switch on this Hamiltonian. Now if the magnetic field is in the z direction too, then nothing is going to happen. because if B is B times e_z for example, then the H magnetic the Hamiltonian becomes mu and then sigma 3 B. sigma dot B is just sigma 3 B. this quantity(Refer Slide Time: 17:21) is an eigenstate of sigma 3 with eigenvalue +1/2 h cross or +1 because its sigma 3 alone. So, nothing is going to happen and psi (t) is e to the - i Ht over h cross. This is = e to the - i mu $_B$ over h cross sigma 3 t acting on the up state. So what happens? It's just phase factor here because sigma 3 is a diagonal matrix. So we can compute what this quantity is and it remains something which is proportional to the unit matrix and sigma 3. So what happens finally? This is an eigenstate of sigma 3 with eigenvalue +1. So you can replace the sigma 3 by 1out there.

So this is = e to the - iB mu B by h cross t on up. So nothing happens. so if it starts in this initial eigenstate and you switch on the Hamiltonian you are already in an eigenstate of the Hamiltonian and it just remains there apart from a phase factor e to the i et over h cross. It doesn't do anything interesting. Similarly if I started with down and applied field in the z direction, nothing is going to happen. But if i start with arbitrary state; a superposition of up and down then of course interesting things are going to happen. Because whenever that Hamiltonian acts on the up state, you are going to get +1 and when it acts on the down state you, are going to get – 1. And you will have a superposition of 2 exponentials. They could be interference and so on. so this is already telling us, how in the simplest of instances, quantum interference would play a role. Now classically, if you got a dipole lined up upwards and you want change its alignment, what would you do? There is no use applying a field in the upward direction. You have to apply a field in the transverse direction or at least have a component in the transverse direction. Let's start with the dipole in this fashion. This is the electron and the upstate and let's apply a traverse field in the x direction. Now let's see what happens. So this H is mu $_B$. so the B, i am going to apply is B times e_x in the x direction. So this is mu B times B sigma 1, it's the x direction, so the first Pauli matrix comes in. and then what does this do? psi(t) = A to the -, let me put this factors and so that's e to the - i over h cross Bt. and then mu B mu was modulus e h cross over twice the mass and then a sigma 1. This acts on the up state; the initial state. So the h cross cancels out. And that is =e to the - i mod e eB over 2 m times sigma 1 acting on this. What is eB over m? If you have a charge e of magnitude e in a magnetic field of magnitude B and the mass of this charge is m, what is this combination eB over m? It's a cyclotron frequency. So this is = e to the - omega $_{cyclotron}$ t over 2 and then there is a sigma 1 acting on the up state in this fashion. Now this is not a pure phase factor by any means because there is a sigma 1 here and that is not an eigenstate state of sigma of 1. This is an eigenstate of sigma 3. So you really have to write down what this quantity is explicitly.

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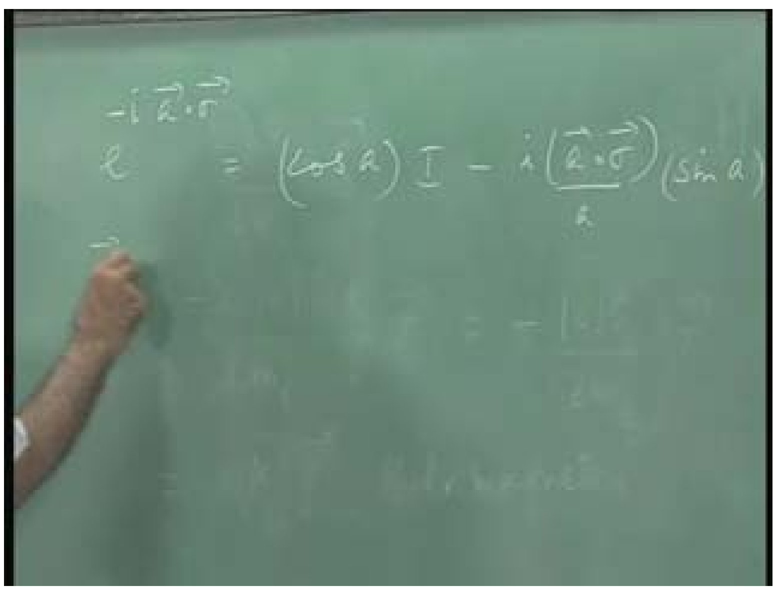

Let's use general formula which we already wrote down. e to the power - i a dot sigma. This is = cos a times the unit vector - i a dot sigma over a. that is the magnitude of this vector (sin a). That was the generalization of this a. Euler's formula e to the i theta is cos theta + i sin theta. How do we get this formula? We use the fact that the square of each sigma matrix is 1 + the commutation properties of the various sigma matrices. So let's use this formula explicitly.

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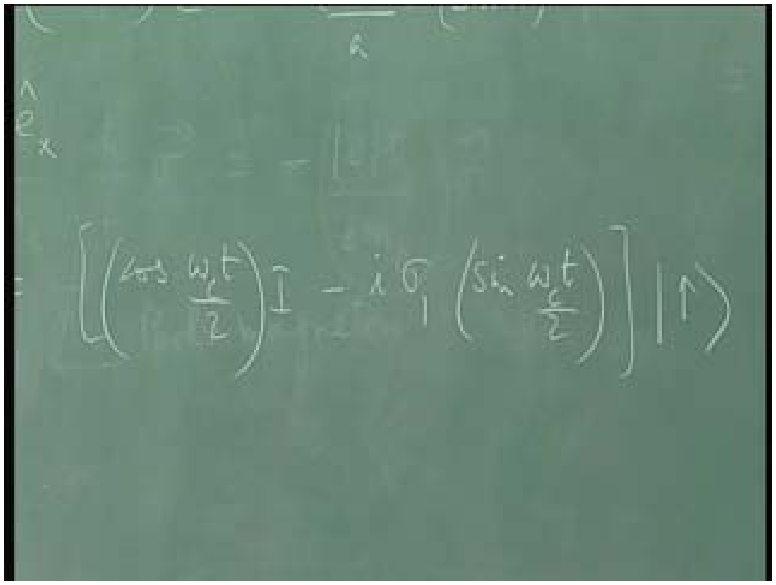

In our present instance a is = omega $_c$ t over 2 a unit vector in the x direction, e_x. so psi(t) is = cos omega $_c$ t over 2 I - i dot sigma $_1$ sin omega $_c$ t over 2. This whole thing acting on the up state. And that’s explicit. Now so this is what it is exactly at later instant of time. But let’s compute what it is.

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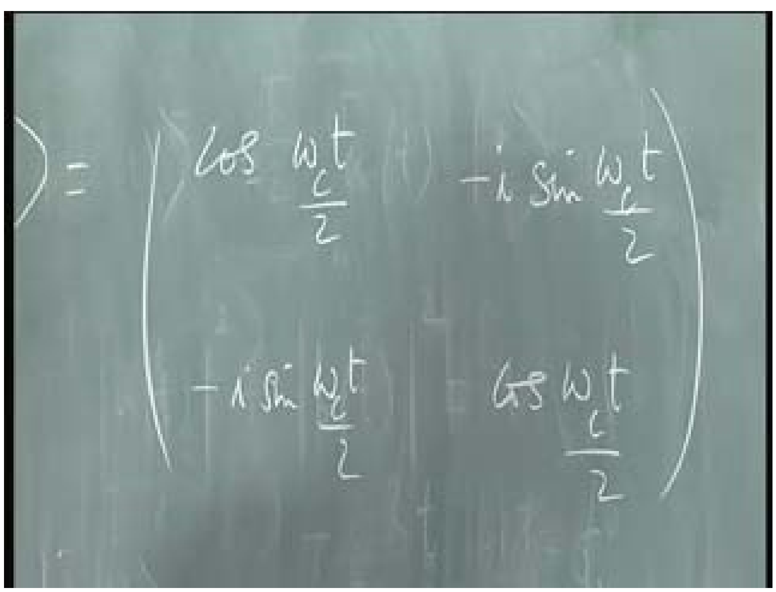

Remember that sigma 1 is 0 110. psi (t) = it is a ket vector of some kind which going to be written as column matrix and this is = cos omega $_c$ t/2, cos omega $_c$ t/2, and then - i sin omega $_c$ t over 2, - i sin omega $_c$ t/2. That's this matrix acting on the up state. But that is 1 0. And that gives you cos omega $_c$ t /2 and - i sin omega $_c$ t/ 2.

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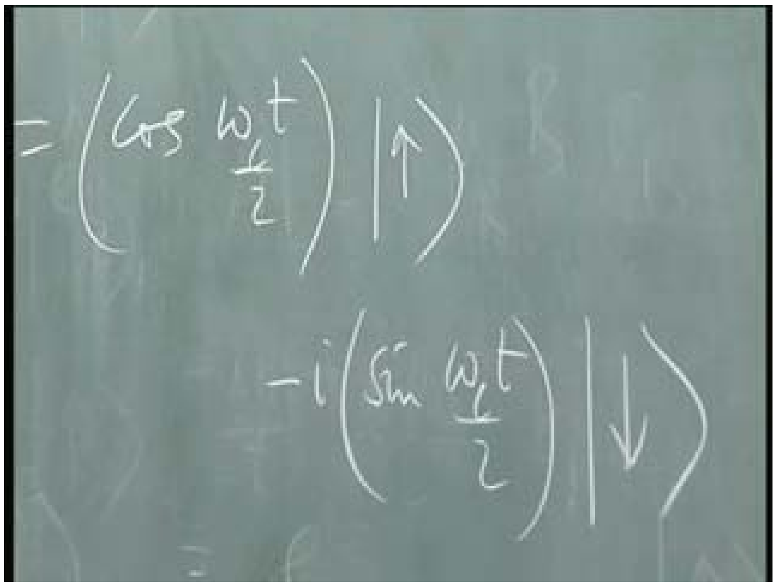

Which is =cos omega $_c$ t, that is the up matrix and - i sin omega $_c$ over 2, that is the down state. So that's my a(t) and my b(t) and its clear that mod a squared + mod b squares is 1 at all times. It's normalized as you would expect. a (0) is 1 as it should be we started with the state of and b(0) is 0. Now what is the significance of these coefficients?

Since I expanded this in the basis up and down, what's the significance of these coefficients? What is the interpretation of their mod squares? It's the probability. I start with the electron in the spin of state. i switch on a transverse magnetic field at t =0. Then mod a square (t) gives me the probability that at time t, the state is up and mod b squared is the probability that the state is down. The sum of the 2 must always be 1. So it's immediately clear that we could write this down and plot as a function of t.

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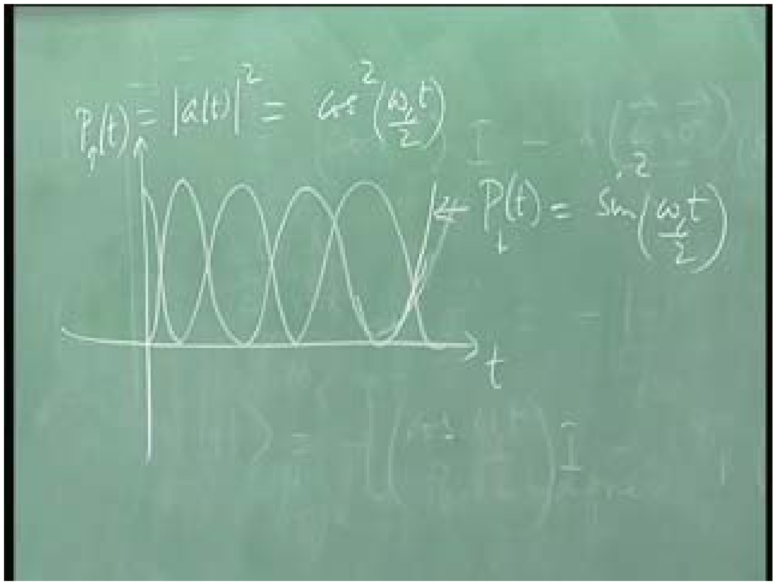

The probability up (t) =mod a (t) whole squared and that's = cos squared omega $_c$ t/ 2 and what does that graph look like? It goes up and down like this (Refer Slide Time: 29:07). On the other hand, the one starts like this (Refer Slide Time: 29:20) and this is = p down time t = sin squared omega $_c$ over 2. And the frequency with which this happens is precisely omega $_c$ because cos squared is 1. You can write this in terms of cos twice the angle. So the periodicity is that of cos or sin omega $_c$ t. so quantum mechanically, this Hamiltonian which is in the transverse direction plays the role of a spin flip operator. It flips the spin from up to down and down to up repeatedly. And at any time t, the system is actually in a superposition of the 2 states expected. So that happens quantum mechanically. You can't say that the system is definitely in the upstate of the downstate unlike what you could do classically. This is because the Hamiltonian doesn't commute. The up and down states are not eigenstates of the Hamiltonian but we interested on using that as a basis.

Quantum mechanics is a way of computing probabilities. Certainly that's true but the actual rule for computing these probabilities; expectation values and so on are quite deterministic. There is nothing arbitrary about it totally. What i want to illustrate by this example is the basic nature of superposition. You start with an eigenstate and you switch on a Hamiltonian. The state you start with is not an eigenstate Hamiltonian. And then the Hamiltonian takes you through all the other states of the system which in this case, is all the other states of the basis. There are only 2 in this particular instance but it causes these spin flips. Therefore one sometimes says that a transverse magnetic field causes spin flips. This the way to flip the quantum mechanical spin. And this is with a constant field in the z direction. I am going to leave it to you as a problem to work out happens if i start with an arbitrary state a(0) and b(0) normalized to unity and the magnetic field in an arbitrary direction.

It's not very much harder. The algebra gets a little more complicated but you can see easily that in this 2 dimensional space, you always have a superposition of the upstate and downstate. Suppose I have an eigenstate of sigma x or sigma 1, I could pictorially write it like this (Refer Slide Time: 33:06). This can also be written as a superposition of up and down. Any spin state in this Hilbert space can be written as a superposition of an up and down. That's important to remember.

Now all this was with a constant time independent Hamiltonian. If I start now switching the Hamiltonian on and off, switching a field on and off as a function of time, matters will get considerably more complicated. Then we need a new formulism. We need to know what to do when you have time dependent Hamiltonian because e to the power - iht over h cross is no longer the time development operator. We have to be a little careful. We will do that too. He what we have at hand is a 2 level system in some sense. Just 2 states possible; an up and a down.

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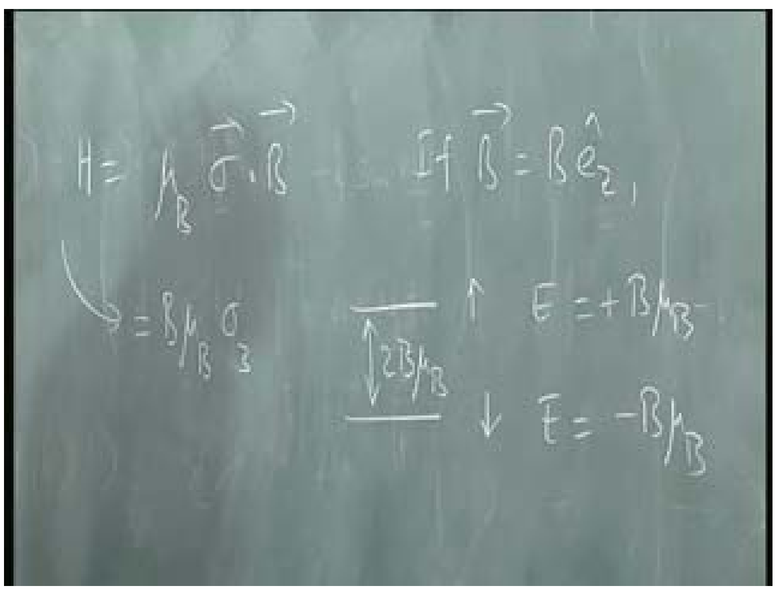

So if you assign energies, remember that we wrote H is =mu $_B$ sigma dot B here. And if I now say that the magnetic field is along the z direction for example, then this Hamiltonian becomes mu $_B$ times sigma 3. And this has eigenvalues +/- 1. So the energy levels of the system would correspond to the following for the down state and the upstate state (Refer Slide Time: 35:02). And there is a gap between them. This energy is - B mu $_B$ and this energy is +B mu $_B$ and there is a gap of 2 B mu $_B$. this corresponds to 2 level system but it's not an ordinary 2 level system. This is a quantum mechanical 2 level system. This is a general state that you have and the superposition gives you enormous freedom because you have 2 coefficients a and b such that mod a squared + mod b squared is = 1. Therefore an infinite number of superpositions are possible.

So in some sense, the fact that you start with the 2 level system but really you have an infinite number of possibilities. This is why it's called the cubit, as opposed to classical bit which would be a 0 or 1.but you starting with a 0 and 1 but all possible superposition's are allowed. Therefore things are little more complicated. It is the superposition that's going to lead to all the interesting features.

So this tells you what happens in the simplest 2 level system when i switch on a transverse magnetic field. Let me also introduce another concept here then we will come back to this. And this is the concept of entanglement which is something which is very common these days.

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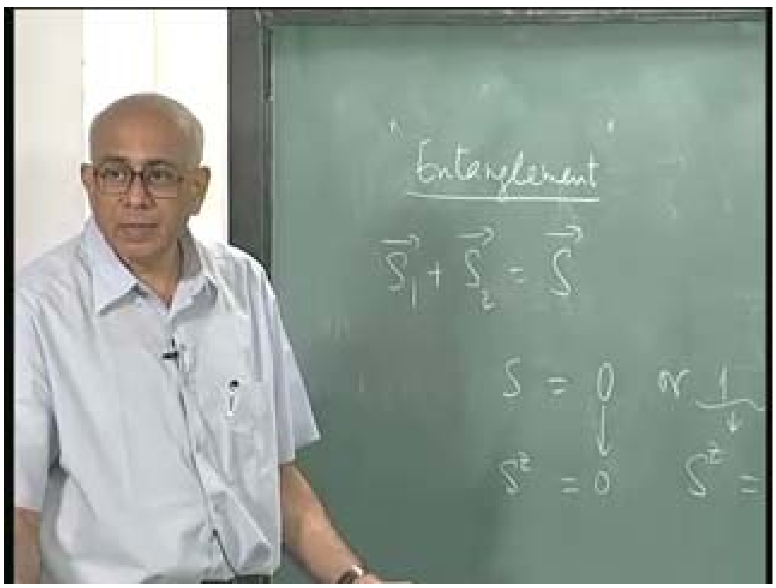

Suppose you have two 2 level systems, 2 electrons and look at the spin states of the 2 electrons together. I am not interested in the other degrees of freedom. I am not interested in the column interaction, the potential energy due to the coulomb interaction, the kinetic energy due to that motion and so on. I am looking only at the spin states of the system. What does the Hilbert space look like? For the first electron, you have a Hilbert space H 1 and it has 2 states which would correspond to either up or down for the first electron. That's this Hilbert space.

For the second one similarly, I have an up 2 and a down 2. I have these 2 states. Now I put these 2 together and i look at the possible states of the full system. How many states would you say there are? There are four possible states to start with. A naive guess would be say (up, up), (up, down) or (down, up) and (down, down). But say there are these four states but now the matter gets a little more complicated. We need to know how to add angular momenta in quantum mechanics. We are going to do this but let me state the result which you have already learnt in spectroscopy. if you have 2 angular momenta j_1

and j_2 which are independent of each other, the quantum mechanical rule for adding these angular momenta is that the components of j 1commute with the components of j but among themselves the components of j 1satisfy the angular momentum algebra and similarly for j 2. then the question is if i add these 2 and i call this is the total angular momentum here, what are the possible values of this capital j or j square what are the eigenvalues of j square given the eigenvalues of j 1square and j 2 squared. This is the theory of the addition of angular momenta and i will make short excursion to that a little later.

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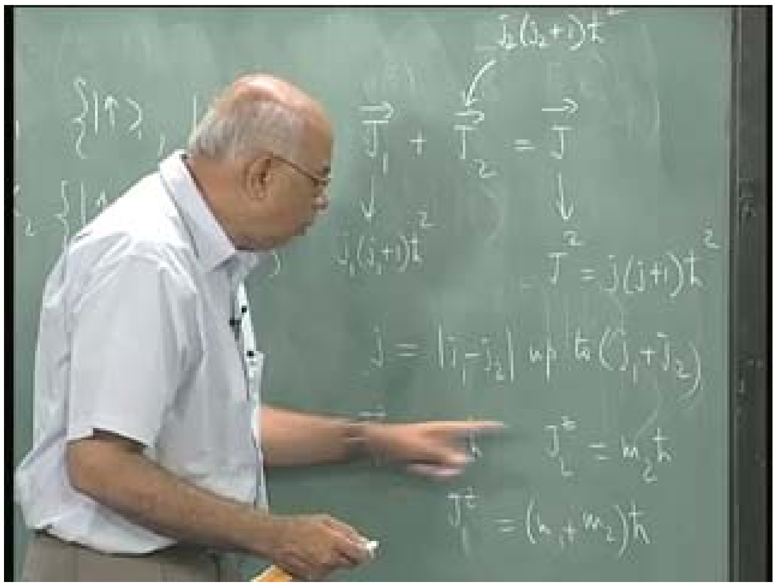

And answer is that if j squared has eigenvalues j times(j +1) h cross squared and J_1 has a spin quantum number j_1. so its j_1 times (j_1+1)h cross squared and J_2 is j_2 into $(j_2 +1)$h cross squared. Then the rule of addition of angular momenta tells you that little j can take on a large number of possible values ranging from $|j_1- j_2|$ up to $|j_1+j_2|$. We will establish this is a very well known rule in addition of angular momenta. But you see already that quantum mechanical angular momenta are not going to add up like classical vectors. Not surprising because these are operators the vector valued operators.

So this is an operator and that's an operator and therefore they have their own peculiarities and this quantum number can take on all values in the range. any component adds up linearly for example if this a $j_1{}^z$ component of it, then this is = m_1 h cross because it must have eigenvalues which are some multiple of h cross and this is magnetic quantum number m_1 and this m_1 can run from - j_1 to $+j_1$in steps of unity. Similarly $j_2{}^z$ is m_2 h cross, then j^z is (m_1+m_2)h cross. They add up completely linearly because these 2 operators commute with each other. and if this there is 1 eigenstate of $j_1{}^z$ and $j_2{}^z$ with these eigenvalues, then you are also in an eigenstate of capital J^z with eigenvalue but the allowed values for the total quantum number run from here up to there (Refer Slide Time:

42:37). Take this as a given rule for the moment then the question is what happens if i add the spins of 2 electrons?

Each of them is a 2 level system if you like and what if I add the spins of the 2 electrons. What are the possible eigenvalues of S_1+S_2? This is the total spin. What are the possible values of the total spin quantum number S? Let's use this rule. What's j_1 in this case? It's just S_1 and what's that? spin ½. No h cross it just the quantum number. This quantum number is 1/2 for the electron. It can be negative with some number 0, ½, 1, etc. and i have already said that the electron spin quantum number is ½. So it is completely clear that in this case, this is a 1/2 and that is a ½ (Refer Slide Time: 43:53). So this total spin quantum number can be = 0 or 1/2 +1/2 which is 1. So you see you have 2 components each of which has angular momentum quantum number 1/2 but they can add up to 0 and they can also add up to 1. They clearly cannot add up to ½. They can't add up to 1/2 because it runs in steps of unity.

But I have also said that something with a 1/2 odd integer quantum number is a fermion and something with an integer quantum number is a boson. i will explain what these things mean they have to do with statistics. So it implies that you take 2 fermion and you put them together we can create a boson. This has profound implications. This is where superconductivity occurs for instance. So there are many implications to this but the fact right now i want to emphasize that the total spin quantum number can be either 0 or 1. Then the next question is what are the possible states? What are these actual states and what do they look like? We have these four states here and now that actual space is the direct sum of H_1 and H_2.

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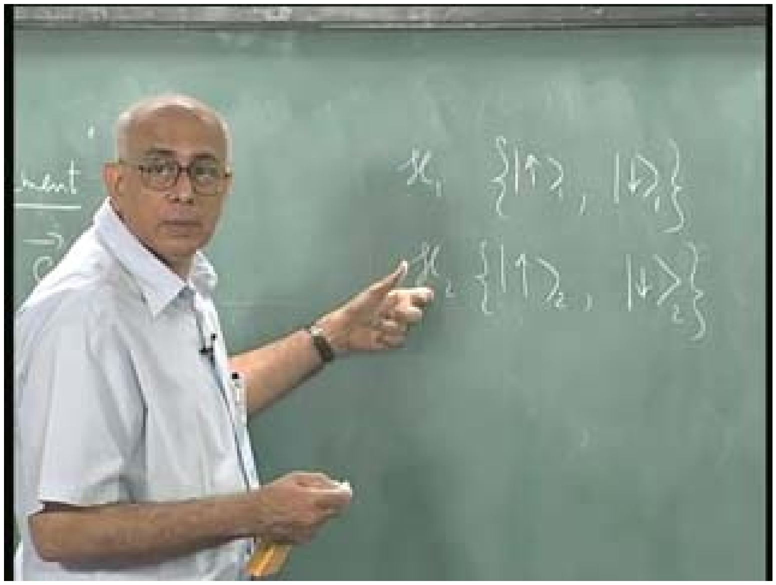
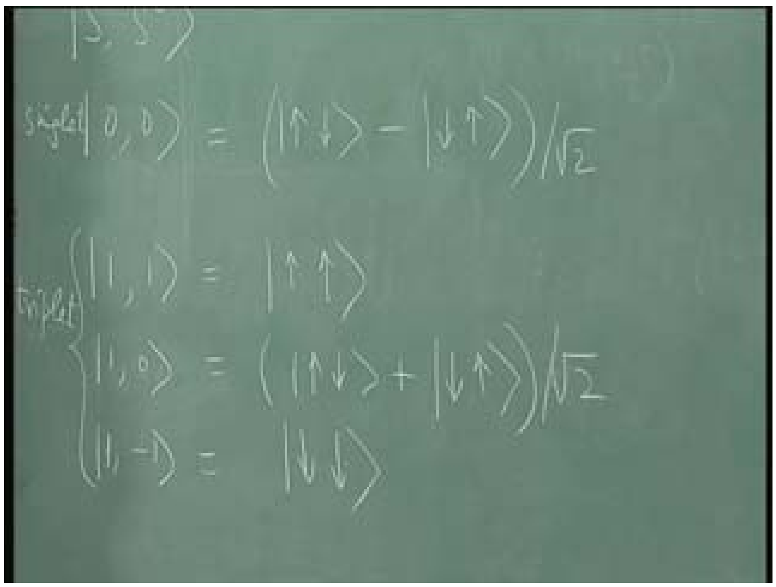


The states that we are talking about are four in number. They could be either 1direct product 2 and so on. There are four of these direct product states and I would like some slightly less complicated notation for this. So let me do exactly as i did when i took 2 simple harmonic oscillators and constructed the angular momentum algebra. Let me just write this as up up, with the understanding that the first arrow always talks about the state of the first electron and the second arrow about the state for the second electron. So I have four of these states up up up, down down, up down & down down. They obviously form a basis in the full Hilbert space. So every spin state of both the electrons can be written as superposition of these four states.

Now the question is what are the these states and what do they correspond to? You see the moment i say S =0, then it also implies that S_z is =0. So these are eigenvalues that i am writing down here on the right hand side because if this is 0 in units of h cross always, this must be 0 because a single component cannot have a non-zero value while the total quantum number is 0. The square of this angular momentum has eigenvalue 0. So every component definitely has a eigenvalue 0 also. Even if they don’t commute with each other, that’s a state of 0 total angular momentum. This is a state in which every component of the angular momentum has eigenvalue 0.

So here is an example where 2 operators which don’t commute with each other can have a common eigenstate. The trivial case of 0 total angular momentum. Then every component of the angular momentum also has 0 as the eigenvalue. This is the only possibility here but the moment i say it is 1in units of h cross, then what are the possible values of Sz? So corresponding to this (Refer Slide Time: 48:12) you have this and corresponding to this, what are the possible values? It can go from - the total quantum number to +the total quantum number in steps of 1. - little j to +little j in steps of unity and now that S is 1, what are the possible values? It can be – 1,0 +1. So we have 3 possible sub states here if you like or states in this space corresponding to S =1 and in this space you have a 0 here (Refer Slide Time: 48:57). There are 4 possible states. They must be linear combinations of these 4 possible states. And it is these states that would like to describe in terms of those direct product states or the basis set. So what would they be and how would you write this down? And now let's write the states down as the total S and S_z. recall this is my j m notation. That's exactly the notation I am going to use. So I would like to write 0 0, that is this (Refer Slide Time: 49:44) state.

(Refer Slide Time: 00:49:45 min)



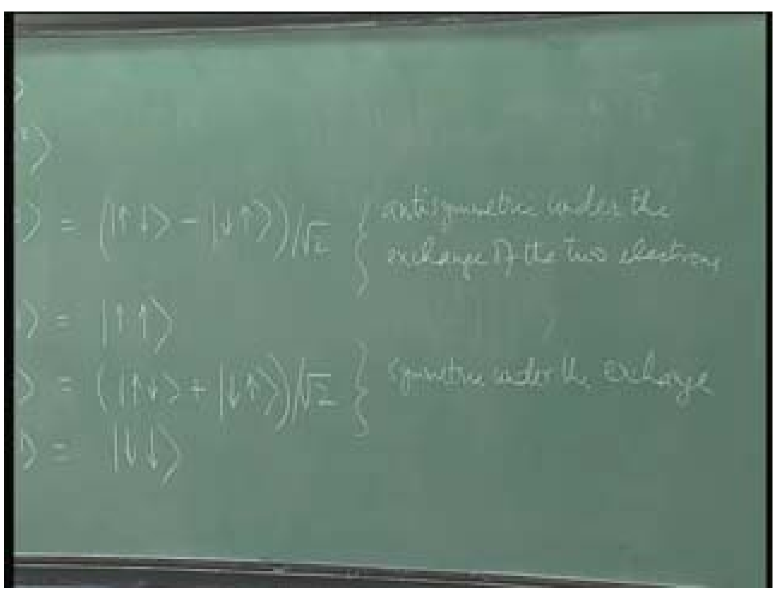
Can it be this (Refer Slide Time: 49:57)? Because remember this corresponds to remember in any component the angular momenta just add up. So this corresponds to S_1^z =+1/2 h cross, S_2^z is +1/2 h cross. So S_z must be = h cross. So that fellow actually corresponds to this (Refer Slide Time: 50:15). Similarly this fellow corresponds to this because both are down and those are easily written down. So |1,1> can be written down and that is just up up. |1,- 1>can also be written down and you leave a gap and write it 1-1can be written down and that's the down down.

What we are left with is to answer what is the 10 and what is the 0 0? We need to know these 2. Which is which this also has Sz =0 because 1is up 1is down. So does this. This is down & that is up. How will you say this corresponds to that or this corresponds to this or any superposition of the 2 how would you how would you decide? You decide by what is the called symmetric property of the states. This thing here (Refer Slide Time: 51:31) is called a singlet state. S =0 is called a singlet state.

The reason is there is only 1 state. There is 1only S_z possible. On the other hand S =1 has 3 possible states associated with it and therefore what would you call this (Refer Slide Time: 51:52)? This is called the triplet state. So |0,0> is the singlet and these 3 together form the triplet. Now you notice that the singlet state sets alone out here and the triplet state here and this state here have certain symmetric property. Suppose I exchange the 2 electrons, I interchange all the degrees of freedom of this electron with that, does this (Refer Slide Time: 52:43) state change at all? It doesn't change at all. It's symmetric under the exchange of the 2 electrons. It's completely symmetric. So really under the exchange of these electrons, this state has remained exactly the same. It is indistinguishable. And so this now turns out that all the members of a multiplet must have exactly the same symmetric. This is the whole idea of putting them together and one bin. Therefore this state must have total S_z =0 that means a 1arrow is up the other arrow must be down but in such a way that when you exchange the 2, the state doesn't change at all.

What would do you? you have this and this as the 2 possibilities but if i exchange the 2, i go from here to here and this goes from here to there (Refer Slide Time: 53:33 to 53:44). So what should i do there? You have to superpose the 2. So this is up down + down up divided by square root of 2 because I have to normalize this. Each of this is normalize to unity. And therefore if i divide by square root of 2, I normalize this state. And then of course the other state is an orthogonal superposition and would this (Refer Slide Time: 54: 20) be? It must be orthogonal. Every state here must be orthogonal to all the states. So what is this got to be? It's this (Refer Slide Time: 54:35). Now you could ask why did i choose up down - down up? Why didn't I choose down up - up down? It just multiplies it by a phase factor. It doesn't do anything to it multiplying this by any complex number of unit modulus doesn't anything to this state.

(Refer Slide Time: 00:55:05 min)



So this is antisymmetric and under the exchange of the 2 electrons. And this is symmetric. This is the spin part of the state but you know it is something called the Pauli Exclusion Principle for these particles which says that if you have 2 electrons they are absolutely identical then and no matter what state these electrons are in, the total state must be anti symmetric under the exchange of the 2 electrons. But this is only the spin part of the wave function.

There is also a spatial part. If these are moving about in the position representation, there is a psi (r 1, r 2, t), a wave function and product of that with the spin wave function is a total wave function that must be anti symmetric. So this immediately says that when the 2 electrons are in the singlet state, the spatial part of the wave function must be symmetric under the exchange so that this antisymmetric gives you an overall - sign and in the triplet state the special part must be antisymmetric and the spin part is symmetric. This plays fundamental role in all of physics. This idea that when you have these 2 electrons and the Pauli Exclusion Principle is implemented in this fashion. So what you have to remember is a that spin singlet state is anti symmetric and spin triplet state is symmetric. And notice that we got these linear combinations by imposing that all the members of this multiplet have the same symmetric.

That's why their part for the same multiplet here. Now the question is what I mean by entanglement. The answer is these states cannot be written as a direct product state. These (Refer Slide Time: 57:15) are direct product states but a superposition of these states. So the fate of 1 electron is inextricably linked with that of the other electron in these states. This is called entanglement and it has very deep consequences. I will subsequently show how this also leads to the phenomenon of magnetism because this is a very simple observation. It is actually going to lead us to a so called exchange interaction. This was Heisenberg's discovery. It explains the origin of ferromagnetism. So we will talk about that but right now, I brought this up in order to tell you what entanglement is but we are going to come to this concept and talk.

You could derive this from the rules of addition of angular momenta but what i gave was a very compelling reason. The free components of an ordinary vector are not 3 random numbers or anything like that. They go into each other under rotations. In an arbitrary rotation, a given vector becomes linear combination of the 3 components here. so together those 3 quantities transform in a specific manner under rotations. In exactly the same way, this is the fact that these members are all symmetric under the exchange, is part and parcel of they being a multiplet. This is essential, it's necessary to this. But I want you to appreciate that these are not writeable. These are not writeable as direct product states. That's the crucial point, although this one happens to be and this one happens to be writeable, the other states are not. We will come back to these concepts. Let me do that tomorrow.