Quantum Physics

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Lecture No. #16

Let us see a charged particle in a constant uniform magnetic field.

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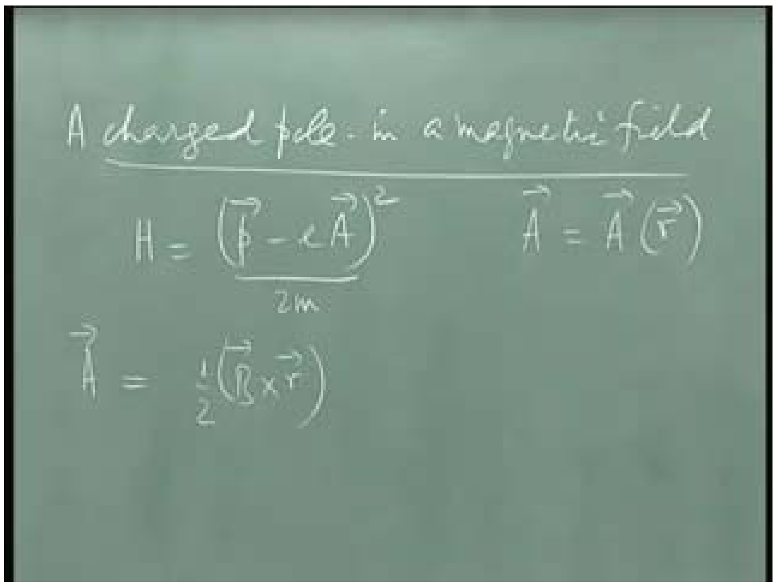

If you recall, we wrote the Hamiltonian down and this was H = p - eA whole squared over 2 m. I have no electric fields and it's just a magnetic field described by a vector potential A and this was the Hamiltonian. And what we would like to do today is to find the eigen values and eigenstates of this Hamiltonian in the position basis. Let's go over several comments that we made earlier just to refresh your memories, A is a function of the position coordinates, A(r).

Therefore in general it doesn't commute with p. the Cartesian components of p and the corresponding components of r don't commute with each other. so when you square this (Refer Slide Time: 02:13) bracket here, you are going to get a term which is of form p dot A and then a term A dot p and these 2 need not be the same. However, we pointed out that p dot A - A dot p, if i write it in terms of the full vectors themselves, this quantity is -ih cross del dot A. you can derive this relation by looking at it component by component. It's very straight forward. If you choose a gauge in which del dot A is zero which is called the Coulomb gauge, then this problem of non commutativity of p and A disappears and you could write either of them. You could treat them as if they commuted with each other. i will come back afterwards to the question of gauge invariance and what happens to the wave function when you make a gauge transformation. But at the moment let's simply say that if del dot A is zero, this implies that [p, A] = 0.

Now the specific problem we have in mind is that of a constant uniform magnetic field. so the vector potential in such an instance has many representations. A convenient choice is 1/2 (B cross r) for a constant magnetic field B. this is easily verified and the curl of A should give you B once again. So let's find the curl of A.

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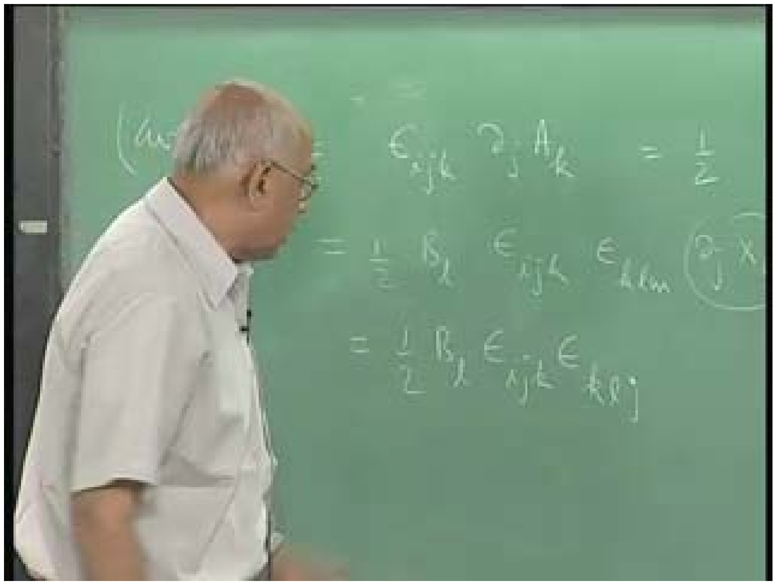

The i component of it = epsilon $_{ijk}$ del $_j$ A_k, where this stands for delta over delta x_j, but this = if i put in that A_k, this is = 1/2 epsilon $_{ijk}$ del $_j$ and then A_k is 1/2 (B cross r). so that's = epsilon $_{klm}$ and then B_l X_m. that's just expending the formula for B cross r. so i write cross products in this particular form and this is = 1/2 B_l because that's not differentiated at all. it comes out and then you have epsilon $_{ijk}$ epsilon $_{klm}$ del $_j$ X_m. Del $_j$ X_m is the partial derivative of X_m with respect to X_j. that's just Kronecker delta. So this quantity here is delta X_m over delta X_j = Kronecker $_{jm.}$ So this is easily written down. Its 1/2 B_l epsilon $_{ijk}$. epsilon $_{k\,lj}$. And now i would like to use this formula for contracting the epsilon symbol. So lets move this k across and this = 1/2 B_l epsilon $_{ijk}$ epsilon $_{ljk}$ because i exchange k and l, i get a - sign and i exchange subsequently k and j and i get another – sign. So taking it across 2 indices gives you a + sign again. And that is 1/2 B_l and then twice delta $_{il}$ = B_i. So indeed B is curl A. we just verified that B is curl A.

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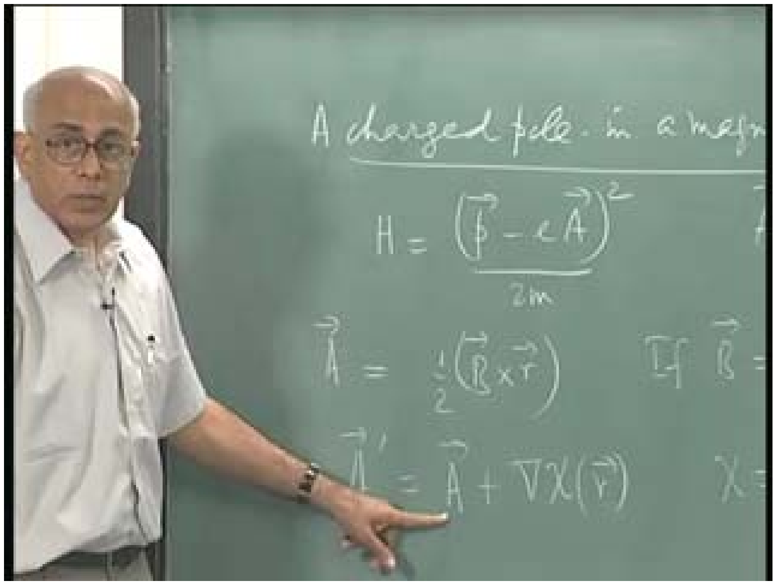

Now incidentally if $B = B\, e_z$, as we are going to do in a short while, I am going to choose the magnetic field along the z direction without loss of generality. Then what does A become?

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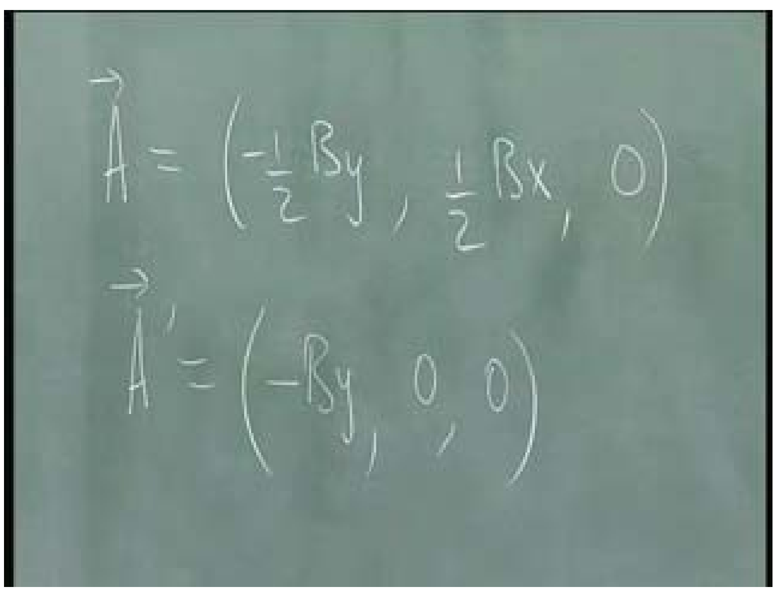

What's the x component of it? It's B_y z - B_z y(Refer Slide Time: 07:17). But then B is only along the z direction. So it's - B_z times y. so A = - 1/2 B y. that's the x component. What's the y component? The z component here and the x component there (Refer Slide Time: 07:42). So it just 1/2 B $_x$ and the z component of course is 0 because B has not a component along the x or y directions.

So that's a general rule. If you have magnetic fields pointing along the z direction, you can always choose a vector potential which doesn't have a z component. moreover the x and y components of the vector potential can't depend on z because if they did, you would differentiate it and get non zero values for the magnetic field along the x and y directions.

So the general rule is if you got a magnetic field in a particular direction, you can choose the vector potential to be transverse to it and the transverse components don't involve the coordinate along the magnetic field. So that's exactly what has happened here. Is this (Refer Slide Time: 08:37) unique by the way? No, you can add the gradient of any scalar field to it and you would get exactly the same magnetic field. For instance, is it possible for me to get rid of one of these components? Can I get read of this(Refer Slide Time: 08:54) component and have just an x component? So let's choose A prime as A + the gradient of some function of r.

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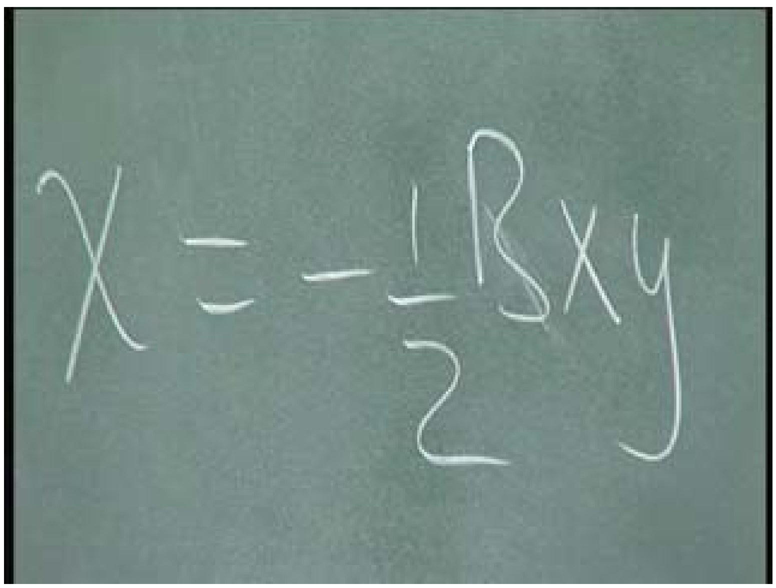

Now look at what happens if i choose chi = - 1/2 B $_{xy}$. That's a nice scalar function of x, y and z and what happens to A prime? it's the original A $_x$, the x component + the gradient which means differentiation with respect to x gives me - 1/2 B $_y$ and i add that here and get - B_y and what about the y component? It vanishes. So in fact if you have a constant magnetic field, it's possible to choose the vector potential in one of the transverse directions and it's linearly dependent on the coordinate. That's very important. It's linearly dependent on y in this case. i could use any of an infinite number of choices and The physics of the problem shouldn't change. The eigen values of the Hamiltonian and the energy levels shouldn't change. The wave function shouldn't get affected. No physical quantity should get affected. We will come back to it as an exercise to see if this is indeed true but keep in mind that if you have a constant magnetic field along this z direction, you can choose the vector potential such that there is no z component to the vector potential.

And moreover, the x and y components of the vector potential don't have a z dependence. With that information, let's look at what happens to this Hamiltonian itself. We could now write down this position space Schrodinger equation because i know that p is represented by a - i h cross gradient in the position presentation and i could put in one of these terms.

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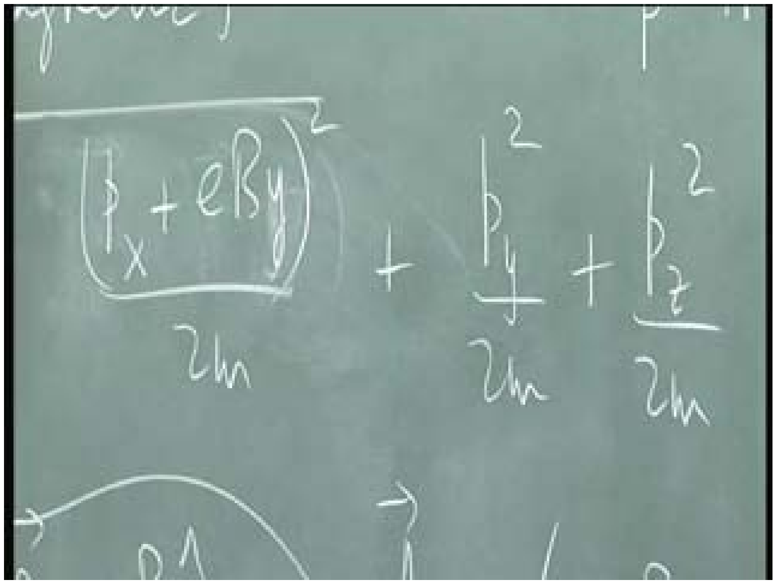

So for instance, if i choose A prime here, this becomes = (px + e By) whole squared over 2 m + and then A has no y or z component at all. So its just p_y squared over 2 m + p_z square over 2 m. notice that y commutes with px. This is why I said that a and p commute with each other because we are working in a gauge in which del dot A is automatically zero. And this problem looks extremely simple. It looks like free motion in the z direction because there is no z dependence in the Hamiltonian except the kinetic energy. so it should move like a plane wave in the z direction and in the xy plane, there is a little bit of complication but the fact is this term commutes with that (Refer Slide Time: 12:35).

So even p_x squared over 2 m looks like free motion although it really isn't. But the y is different. There is a y squared, pxy term and then a py squared term. So this is little more intricate. x and y are mixed up in some strange sense but in the z direction. Looks like its free motion. Now how do we analyze this? We could write down the Schrodinger equation by writing - i h cross gradient with respect to x, y and the remaining terms of del squared here. We will do that but there is an easier way and that's the following.

Let's ask what happens to the velocity of this particle? We call that our definition of the canonical momentum p = mv + eA. That's how we started. We started with a Lagrangian we went to a canonical momentum which is not a mechanical momentum in this problem And you are guaranteed that it's the p and r which have Poisson brackets. But we can ask what about the velocity? So let's write that out. We see that v is = p - eA over m. so i can ask what's the commutator of V_x with V_y for example? What about the commutator of the 2 velocities components?

If there were no magnetic fields, there is no question. These would commute with each other just like the momenta commute with each other. But in the presence of a magnetic field something very strange happens in quantum mechanics. The commutator of the x and y, 2 orthogonal Cartesian components of the velocity may not be zero.

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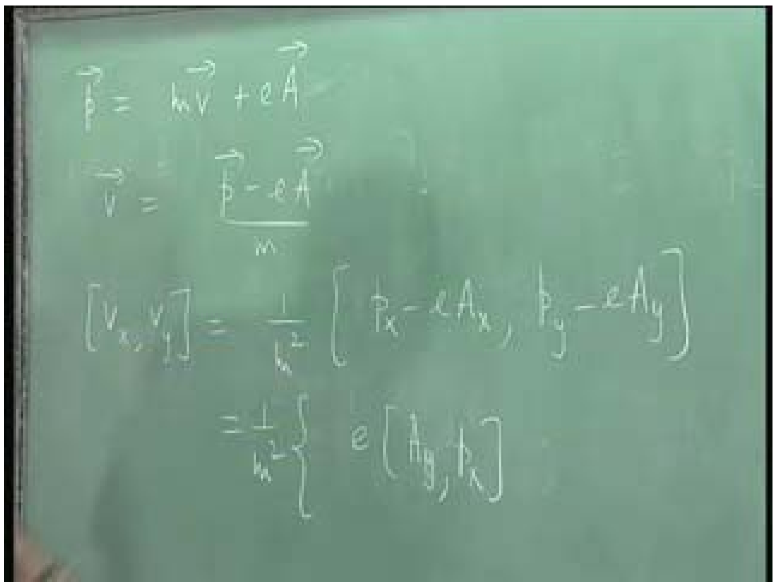

It's 1 over m square a commutator of px - eA x py – eAy. That's = 1 over m squared. Now px and py certainly commute with each other. They are 2 different Cartesian components of canonical momentum. But there is a term which is e, lets call it A $_y$ with p_x. So it's this with that (Refer Slide Time: 15:07) and then similarly it is – e [A_x, p_y].

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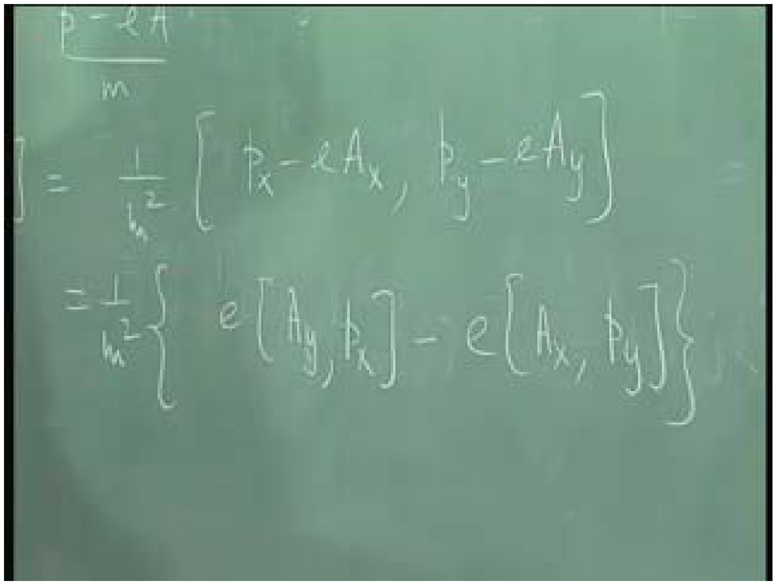

And of course Ax with Ay commutes because both Ax and Ay are functions of position and different components of the position commute with each other. so there is no difficulty. It's only these (Refer Slide Time: 15:40) 2 terms that have any problem at all. But we know from our study of one dimensional motion that if you have any function of x and look at its commutator with px, this is = ih cross f prime (x). We saw that explicitly and we just used that in 3 dimensions. We don't care what Ay and Ax are. They are functions of position. We don't need these representations. In general, they are some functions of position.

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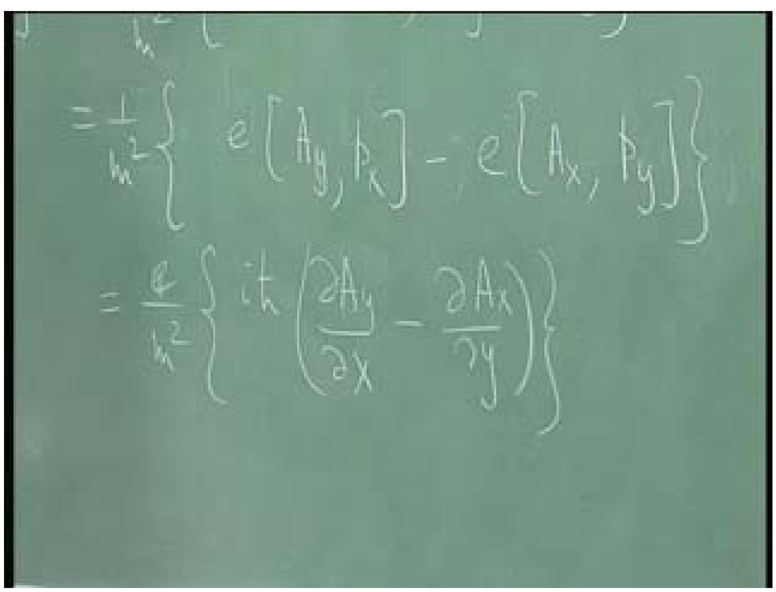

So this is reduces to 1 over m squared, e times Ay as a function of position with px. so this is = e over m squared ih cross delta A_y over delta x because when you have a commutator with px, you differentiate with respect to x. and then similarly, here you have - delta A_x with respect to y. and that's the z component of B. so independent of the representation, this is ih cross e over m squared B_z. and of course you can do this in cyclic permutation. You then end up with this very interesting result.

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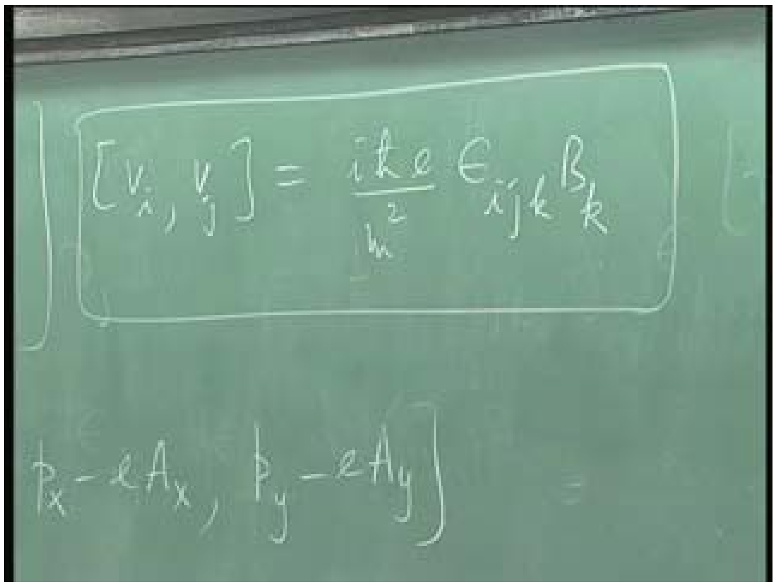

The commutator of the i^th and j^th Cartesian components of the velocity of the charged particle, $[v_i, v_j]$ = ih cross e over m squared epsilon $_{ijk}$ B_k. so we have this very interesting result which says that for a charge particle in a magnetic field, you can't simultaneously measure 2 different Cartesian components of the velocity. They are not commuting variables and for a given constant value of the magnetic field, it's like x with p itself because there is an ih cross and then some constant here (Refer Slide Time: 18:16). So they don't commute and there is a unit operator sitting on the right hand side. So this result here (Refer Slide Time: 18:32) tells us a very interesting thing is going to happen when you put a charged particle in a magnetic field. In fact it gives us a hint to how to solve this problem. So let's go back here to this representation. Let's give this mv a name.

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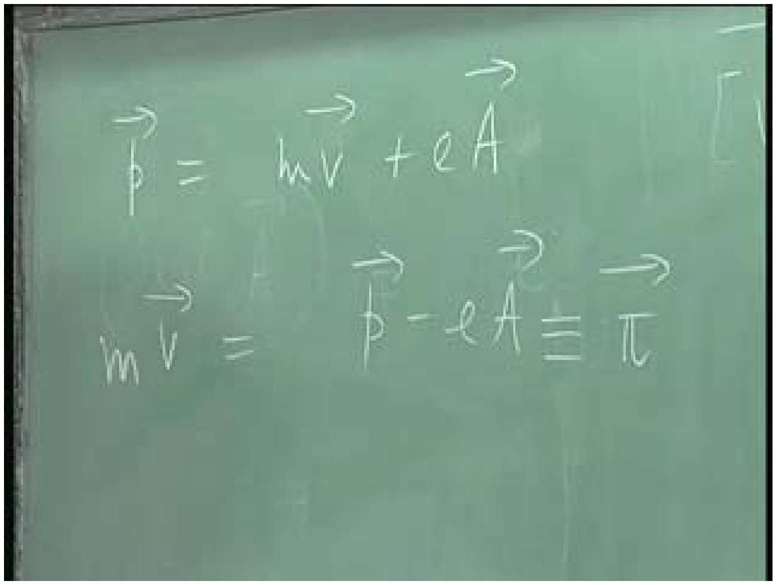

Let's call it vector pi. That's the standard notation. i am sorry we use as a vector as a variable but it's a standard notation. This vector pi is mass times velocity of the particle and it obeys the following computational relations. You have $[p_i, p_j]$ = ih cross e epsilon$_{ijk}$ B_k.

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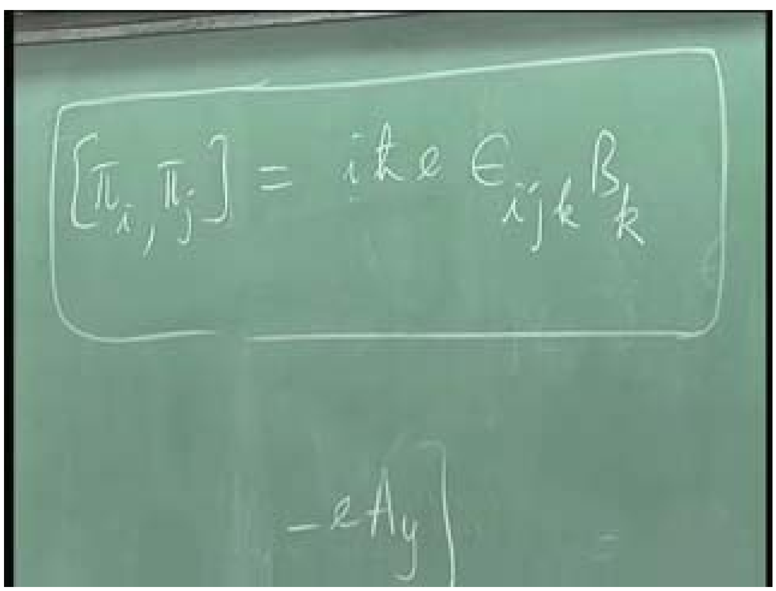

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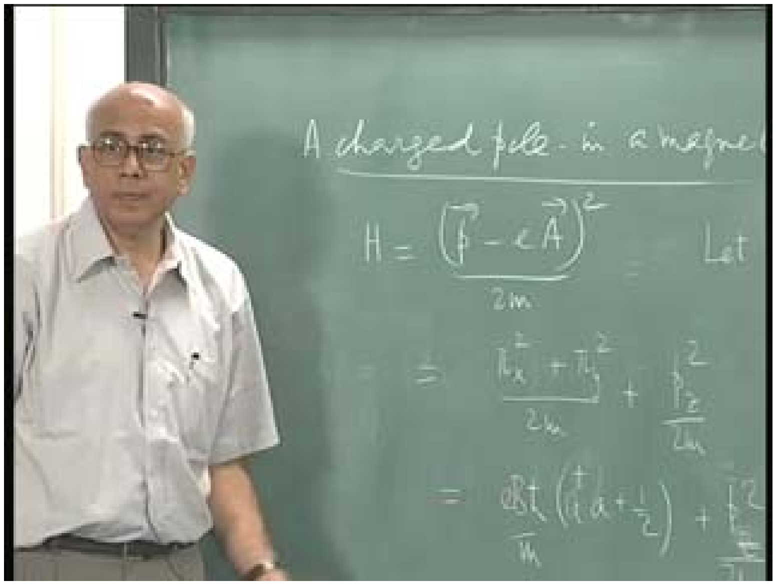

And what happens to the Hamiltonian? This (Refer Slide Time: 19:37) is = pi x squared + pi y squared over 2 m + p z squared over 2 m. let B = B e_z. So that A_z=0. So this suggests something exactly similar to the simple harmonic oscillator.

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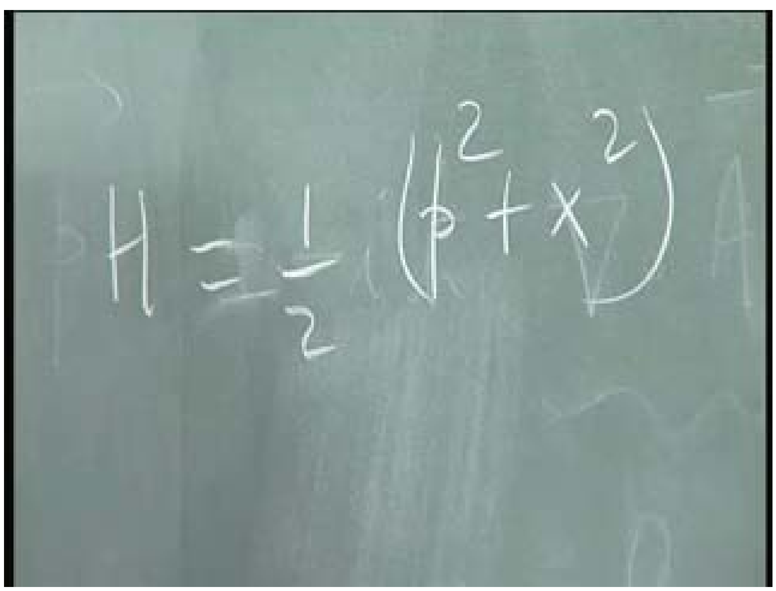

The simple harmonic oscillator if you recall had a Hamiltonian which was = 1/2 p squared + x squared in suitable units. So essentially p squared + x squared by the commutator of p with x or x with p was i times ih cross times unit operator. And then we defined a and a dagger. a was essentially x + ip and a dagger was x – ip. So that this x squared + p squared became a dagger a and then we could diagonalize.

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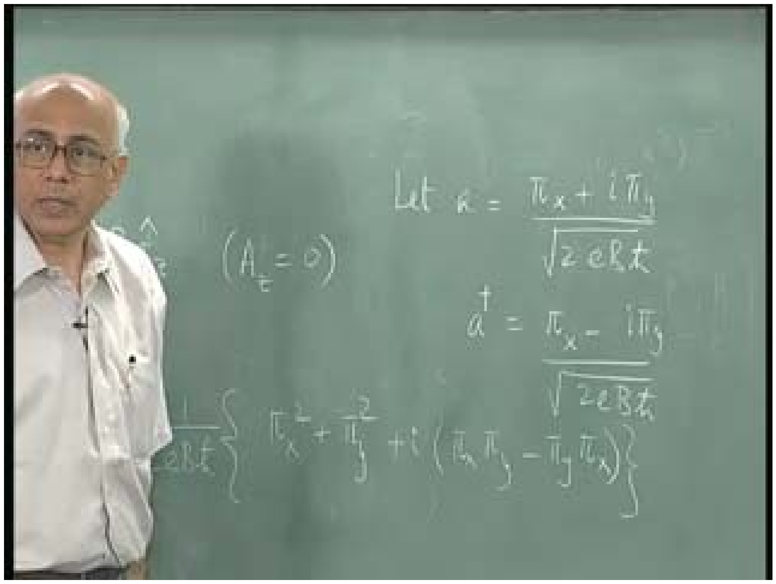

So let's now put a = pi $_x$ + i pi $_y$ divided by square root of 2 e times B h cross. a dagger is then pi $_x$ – I pi y over square root of 2 e B h cross. And a dagger a is = 1 over 2 e B h cross {pi $_x$ squared + pi $_y$ squared +i(pi $_x$ pi $_y$ – pi $_y$ pi $_x$)}. But what's this (pi $_x$ pi $_y$ – pi $_y$ pi $_x$)? This is a commutator. It's ih cross e B, where B is along this z direction.

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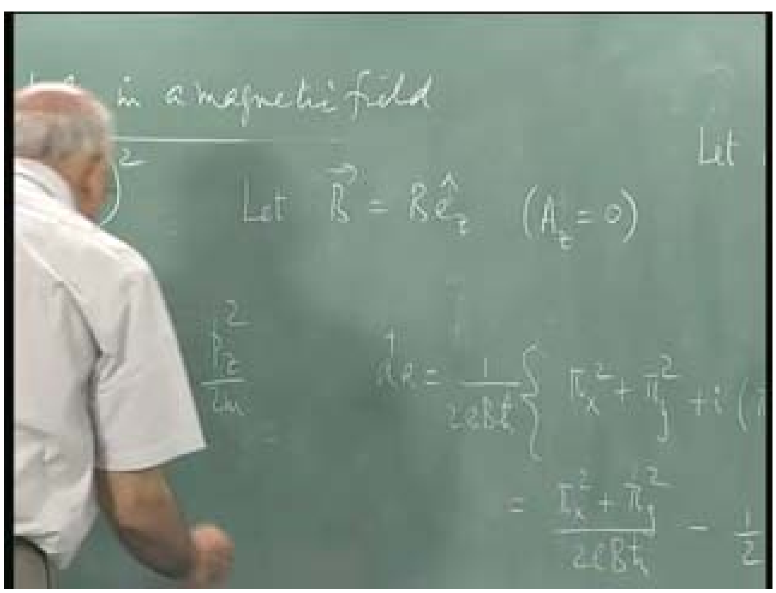

So this is = pi $_x$ squared + pi $_y$ squared over 2 eB h cross and then - because of the i and i, and then there was h cross eB which cancelled out and essentially we got a 1/2 here. Therefore this Hamiltonian becomes = a dagger a + ½.

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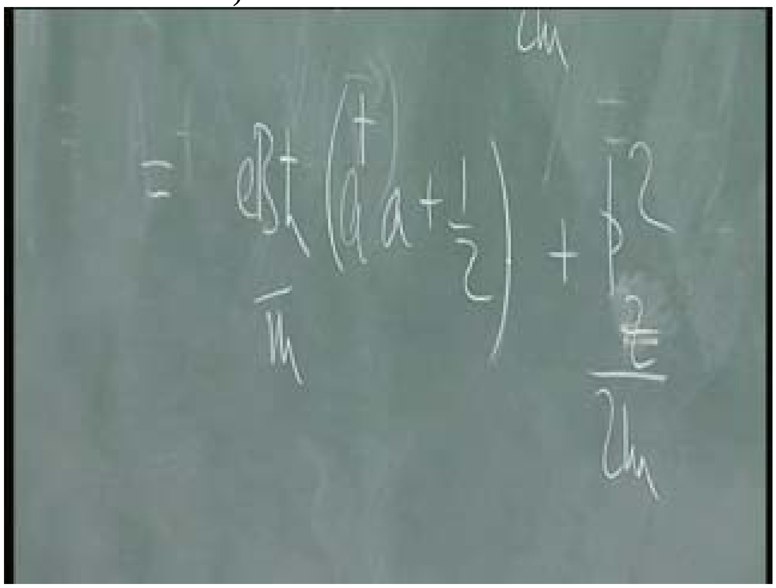

When I substitute for pi $_x$ squared + pi $_y$ squared over 2, it becomes a dagger a + 1/2 multiplied by e B h cross + p_z squared. now we are on business because i know that the eigen values of this (Refer Slide Time: 25:06) part of the Hamiltonian is exactly the same as the eigen values of a harmonic oscillator because a with a dagger is 1, then the operator a dagger a has eigen values n = 0,1,2, 3. So we've actually reduced the problem to a harmonic oscillator + a free particle motion in one direction. These levels are quantized completely. So it's a strange thing that's happening. We can write this a little better. Notice the quantity e B over m has the dimensions of frequency. Classically what do you call a B over m? I have a magnetic field and i put a charge in it, what happens to the charged particle motion? It goes around in circles and we call this frequency as the cyclotron frequency.

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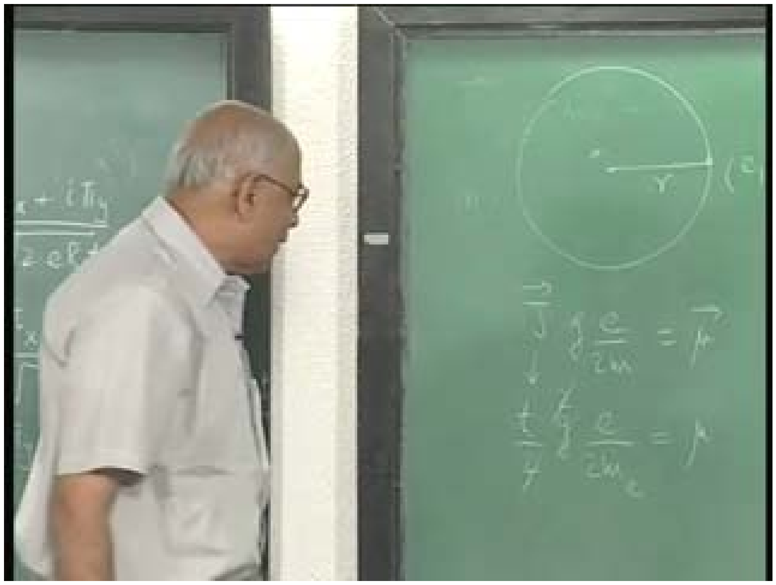

I have a particle in an orbit of radius r and this is a charged particle of charge e and mass m for example. What’s the orbital angular momentum of this particle?

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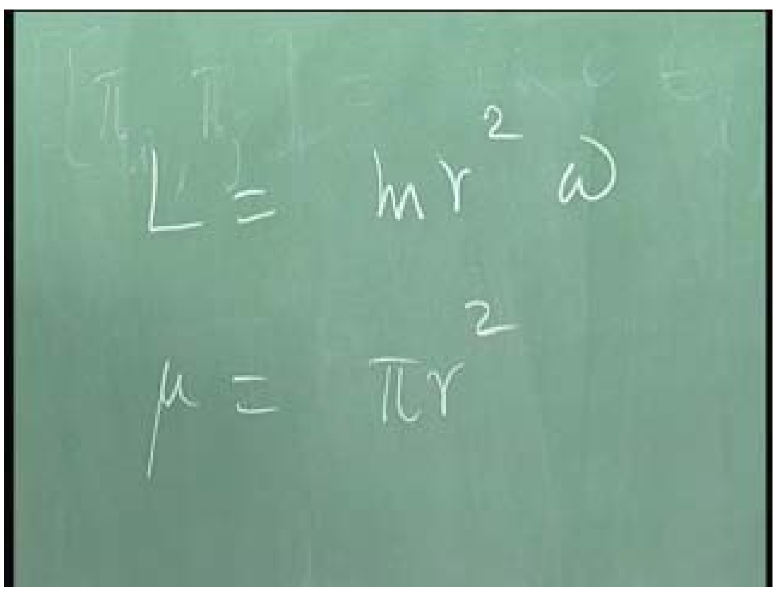

So angular momentum is moment of inertia, which is mr squared times omega. Omega is 2 pi over T, the time period of a rotation on this orbit. And what’s the magnetic moment by Ampere's theorem? This is the area times the current. Now the area is pi r squared. And what’s the current? It’s e over T.

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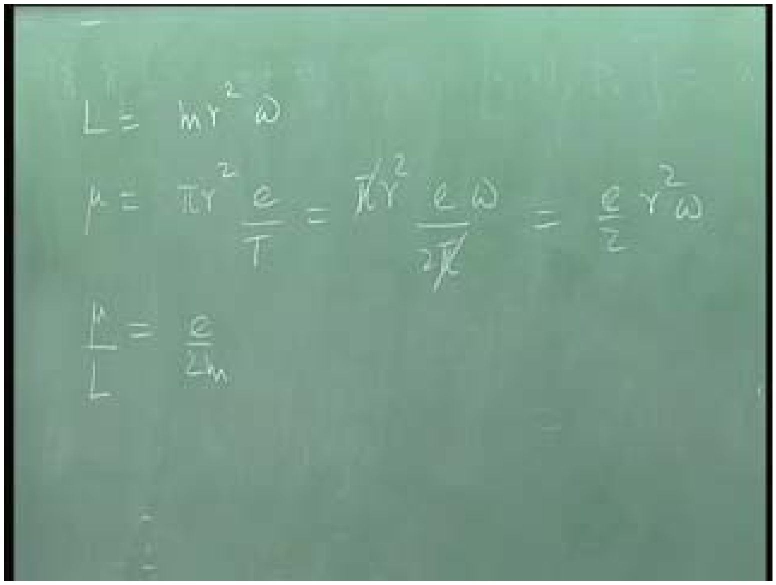

But that's = pi r squared e over 2 pi and then there is an omega on top because T is 2 pi over omega. So this is = 1/2 e over 2 r squared omega. in terms of L, it is e over 2m = mu over L. the ratio of the magnetic moment magnitude to the angular momentum magnitude is e over 2 m and e over 2 m is called the gyromagnetic ratio. Of course, we are going to generalize this very shortly and I am going to argue that every time you have a charge and you have an intrinsic angular momentum like spin, associated with it is a magnetic moment which is the charge divided by twice the mass multiplied by the angular momentum. But of course this is valid classically for an orbiting particle. In more general cases, all you can say that it's a proportionality constant.

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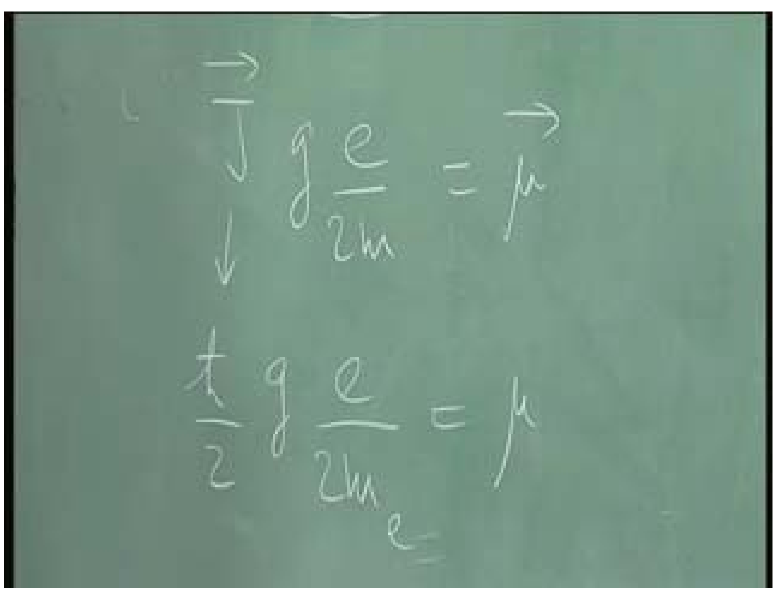

So in general, it will turn out that if you have some angular momentum j which could become a quantum mechanical operator, this multiplied by e over 2 m multiplied by a fudge factor will turn out to be the magnetic moment and this is called the g factor. We will see what this factor is for the case of the electron and so on when we talk about spin but the general case of that formula comes from here. In fact, let me just go a step further and anticipate myself a little bit.

This j for the electron is the spin operator for the electron and we have not introduced that here but this will turn out to be of the form h cross over 2 multiplied by some operator whose eigen values would have unit magnitude. So if we are talking only about magnitudes, there's an h cross over 2 here (Refer Slide Time: 30:59) and then there is a g and there is an e over 2 m e and this would be the magnitude of the intrinsic magnetic movement of the electron. notice all these factors of 2 come from the fact that the spin is ½, another 2 coming from that 2 pi there and one more 2 coming from the fact that the Dirac theory of the electron says that g is = 2 for the electrons. So there is one more 2.

So this 2 and that 2 cancels again and you have h cross e over 2 m which is mu. eh cross over 2 m e is called the physical dimensions of the magnetic dipole movement. This term is known to you already. It's called the Bohr magneton. It turns out that g is not exactly 2. It's 2.00 something or the other and that something of the other has been computed to 8 decimal places and also agrees experimentally to 8 decimal places. So this is the big triumph for quantum electrodynamics. To come back to where we were, this frequency here eB over m is the cyclotron frequency. So let me write the Hamiltonian now.

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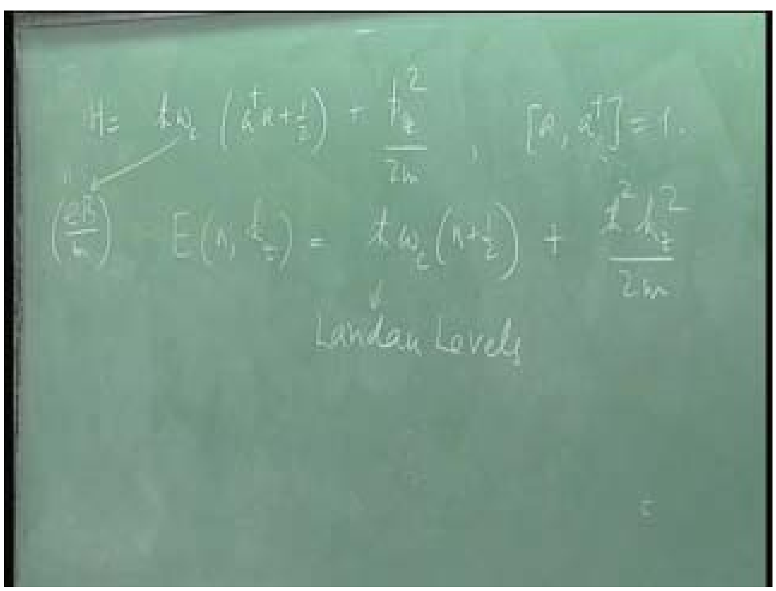

This Hamiltonian is = h cross omega $_c$ where omega $_c$ is eB over m, times a dagger a + 1/2 + p_z squared over 2 m with [a, a dagger] = the unit operator. We can therefore write down the energy eigen values immediately. They would depend on the quantum number n for the simple harmonic oscillator but they also involve the free particle motion along the z direction.

Classically, what would happen is you put a particle in such a field along the z direction, then depending on its initial velocity, it wound perform a helical motion about the direction of the magnetic field. It wound be circular if you project it in the plane perpendicular to the magnetic field and it’s helical because there is an initial velocity in the z direction and that continues. So it’s a helix and has a constant pitch. And in the xy plane; the transverse plane, it would perform circular motion with the cyclotron frequency. This is all that would happen. Quantum mechanically, the energy levels have this free motion on the side parameterized by some wave number kz. so E is a function of n and kz and this is = h cross omega $_c$ n + a 1/2 + h cross squared kz squared over 2 m. so if you could confine your particle to the XY plane, then the levels would look quantized like those of a simple harmonic oscillator.

But if not there is a free motion in the z direction, it drifts in the z direction with any real number, kz either up or down. Would you say these levels are degenerate? Because remember the harmonic oscillator doesn’t have any degeneracy. Every level is unique. And similarly for the plane wave, its completely unique but these levels are actually degenerate and there is a physical reason for it. i will come and explain where the degeneracy comes from. We will see that very shortly when we write the wave functions down. These levels are called Landau levels. It was first done by Landau in 1930 when he first solved the problem of a charge particle in a magnetic field. They have acquired enormous importance because with a lot of other complications, it’s possible to take electrons and confine them essentially to 2 dimensions.

So in MOSFET devices, for example, you can confine them to 2 dimensions. You can then look at all the properties that an electron gas would have in a 2 dimensional situation. And they are very different from what happens in 3 dimensions. Let’s do that step by step. Let’s first write down the wave functions. This is free motion.

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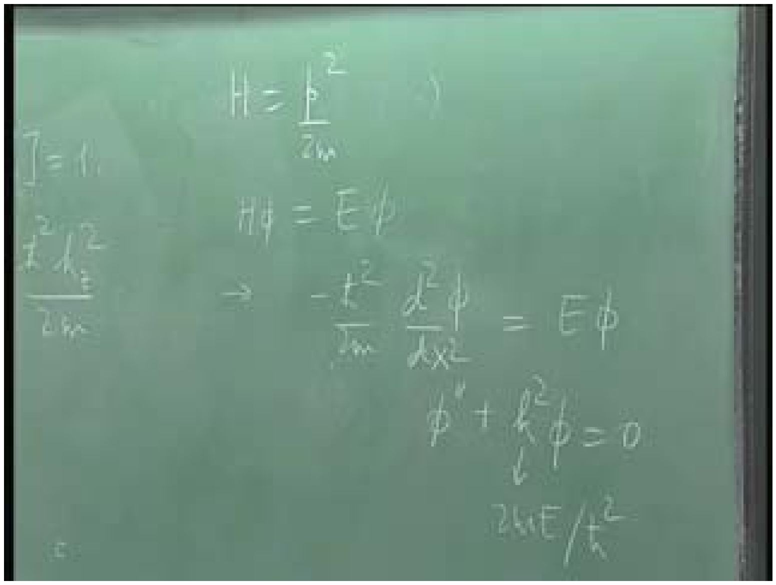

The moment I have a single free particle, i have H = p squared over 2 m. this immediately implies the solution of H psi = E ps and that would be the solution of - h cross squared over 2 m d 2 phi over dx 2 = E phi which i can write as phi double prime + k squared phi = 0, where k squared is 2 mE over h cross squared. Then the solutions are e to the + or - ikx plane waves specified by a continuously varying wave number k.

Physically what does it say? It says if the motion is occurring along this axis for instance, when i tell you its momentum, which is what I am doing, its position is completely uncertain. The uncertainty in position for a plane wave is infinite because you can see the wave function is just a plane wave. It has unit modulus. So the probability density is constant. It doesn't vary at all. It's not even normalizable unless you put the whole thing in a box. So the interesting question to ask is what sort of wave function do you get here?

How do we find out? Well, let's go back and solve this in the position basis, but now to write down the wave functions explicitly, I must choose the vector potential explicitly. And notice that the magnetic field appears through the cyclotron frequency. The vector potential doesn't appear. It should not because the answer should be independent of the gauge that you choose. So it can't explicitly depend on the vector potential but only on the magnetic field and that's we have seen. So we made no assumption here. These commutation relations didn't assume any specific form of the vector potential at all but now, to write the wave function down, I have to tell what the vector potential is. so let's see what happens to the Hamiltonian.

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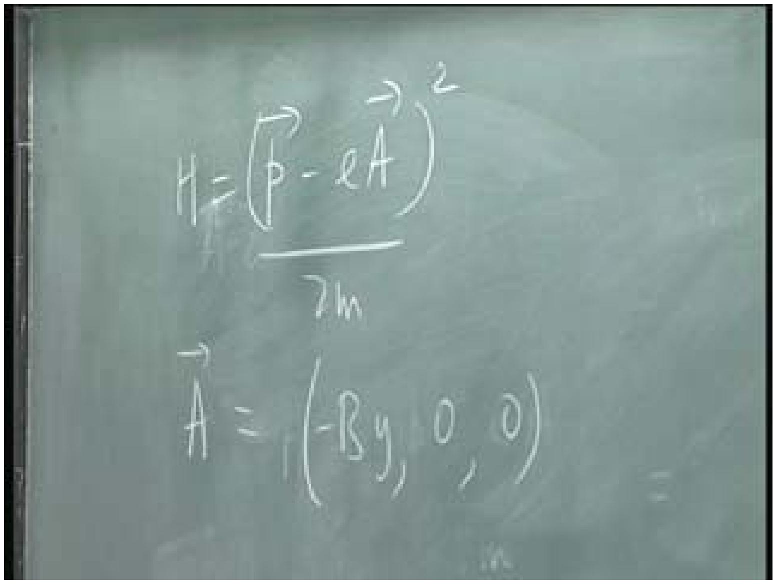

So let's choose H = p - eA whole squared over 2 m and let's choose the vector potential A to be = (- B_y, 0, 0), just to be definite. I don't loose any generality by doing this and you could ask, suppose i choose a different gauge and then i have a different value of A, then i write the wave functions down, how are these 2 related to each other? That's the next question. No physical quantity must depend on this choice. But let's first write it down for this choice. Now what's the time independent Schrodinger equation?

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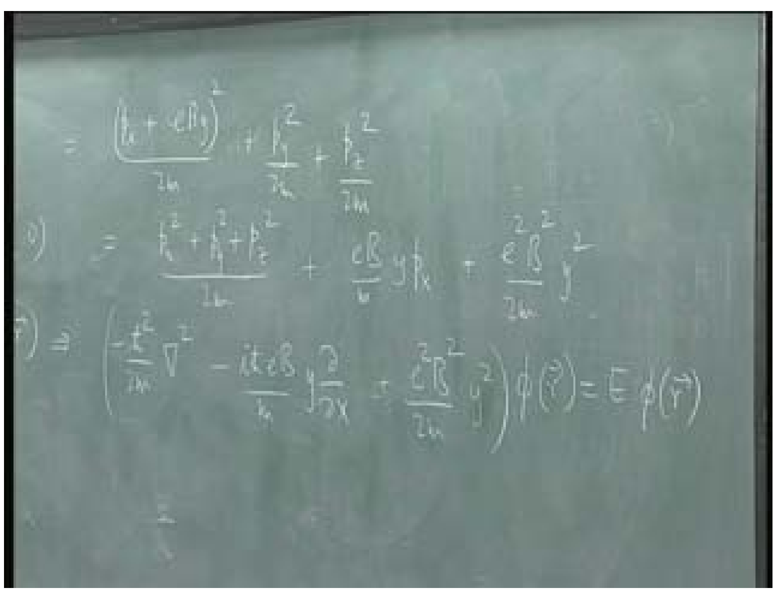

So we can write down H as (p_x + eB_y) whole squared over 2 m + p_y squared over 2 m + p_z squared over 2 m. notice that this (Refer Slide Time: 40:01) portion of the Hamiltonian which has a y and this (Refer Slide Time: 40:06) part here commute with each other because p_x and y commute with each other. There is no problem with that. But the B_y and the p_y of the Hamiltonian don't commute with each other because y and py don't commute with each other. So that's why it's not a trivial problem. Otherwise it would be totally trivial and that's why you get these non-trivial results there. And what does the Schrodinger equation look like now? its = so let's rewrite H as px squared + py squared + pz squared over 2 m + eB over m y p_x, I can do that because y with p_x commutator is 0 and therefore y p_x is the same as p_xy and i cancel the factors, + e squared B squared over 2 m y squared. Therefore H phi(r) = E phi(r).

That's the time independent Schrodinger equation in the position basis and the E has already been found. The E is characterized by a quantum number n and a wave number k z. but whatever it is, they are some real numbers. this equation gives you - h cross squared over 2 m del squared, that comes from this (Refer Slide Time: 41:48) portion of the Hamiltonian and then what is this (Refer Slide Time: 41:57) term do when it acts on phi? Its y p_x acting on phi but then its in the position basis. y just multiplies the wave function but what does p_x y do? It is - i h cross delta over delta x. so there is a term which is - ih cross eB over m y delta over delta x. and of course you could have written as delta over delta xy. It doesn't differentiate. That acts on phi and this (Refer Slide Time: 42:30) term here doesn't do anything. It is just e squared B squared over 2 m y squared on phi (r) = E phi(r). That's the eigen value equation. Now of course it's clear from this equation that the only complication is arising because of this y here (Refer Slide time: 42:57) and the y here. Otherwise it looks like free motion. So let's take the wave function to have a trial form and what would my guess for the trial function be?

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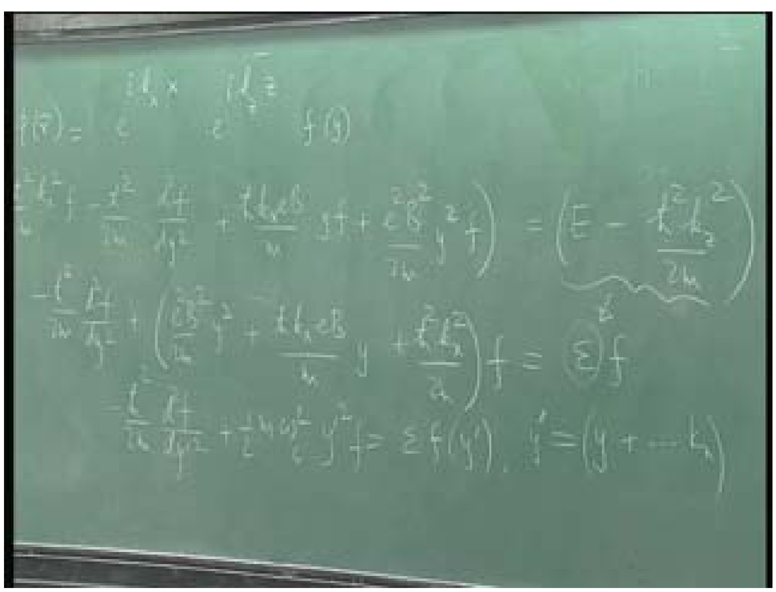

That should be phi(r) = some non trivial function of y because this y sitting in this coefficient and y sitting here (Refer Slide Time: 43:22) but everywhere else its just derivatives. and you know once i have a derivative, the eigen function is just e to the power i times, the constant times the coordinate. So let me take this to be e to the power i k_x x e to the power i k_z z where k_x and k_y are constants to be determined and that times some function of y and that's non trivial. We don't know what it is. It's some function of y and that would be a good trial guess for the wave function. If it doesn't work then of course we go back and ask is there a better solution. If i put that in, the first term is - h cross squared over 2 m and then if i differentiate with respect to x, this portion i get a - ik whole squared. So it gives me h cross squared kx squared over 2 m. and then in the y term, it is nontrivial. It's - h cross squared over 2 m and then d 2 f over dy 2. Multiplying all the exponential factors and so on, this remains. - ih cross eB over m y times delta over delta x. that pulls an ikx. So it's + h cross k_x. So the - i cancels with the + i there and gives me h cross eB over m and k_x. So this is fine and there is a y sitting here (Refer Slide Time: 45:30), I can't do anything about it + e squared B squared over 2 m y squared. There is always an f throughout.

The whole thing is = E times f. the exponential factors are cancelled out. Now there is ought to be a k_z squared too. So it's + h cross squared over 2 m k_z squared. So let's move that to the right hand side. so this is = E - k_z h cross squared k_z squared over 2 m on f. we know at the back of our minds that E is actually h cross squared k_z squared over 2 m + h cross omega c times n + ½. So that's why it's not unreasonable that the right hand side has that combination. It doesn't depend on k_z anymore. So the z direction motion is quite trivial. The x and y is another story because they are sort of mixed up. You got these 3 terms (Refer Slide Time: 47:06).

What would you suggest? so this is - h cross squared over 2 m d 2 f over dy 2 + e squared B squared over 2 m y squared + h cross kx eB over m times y + h cross squared kx squared over 2 m f = epsilon f. this term here (Refer Slide Time: 48:00) is epsilon. We are heading towards the harmonic oscillator as we can see which is what we expect. We know the energy values are almost like harmonic oscillator here. Then what should i do to this (Refer Slide Time: 48:19)?

Let's pull out an e squared B squared over 2 m and see what happens. It's better to pull out an omega c squared because it's going to look like a harmonic a harmonic oscillator. So this is what i am aiming for. so multiply by is - h cross square over 2 m d 2 f over dy prime 2 + 1/2 m omega c squared times some y prime squared f = epsilon f (y prime) by completing squares. This (Refer Slide time: 49:33) term here has got a y squared term, a y and a constant term. So this can complete the squares very easily. Omega c is the cyclotron frequency which is eB over m. and what's y prime? It is (y + ... k_x). y prime is shifted by a certain amount which depends on this k_x.

This is the equation of a simple harmonic oscillator in the coordinate y prime. Therefore, the potential is parabolic and this is y prime and y prime squared and therefore the energy eigen values are h cross omega c times n + ½. Those are the square integrable wave functions. So we can write the solution down.

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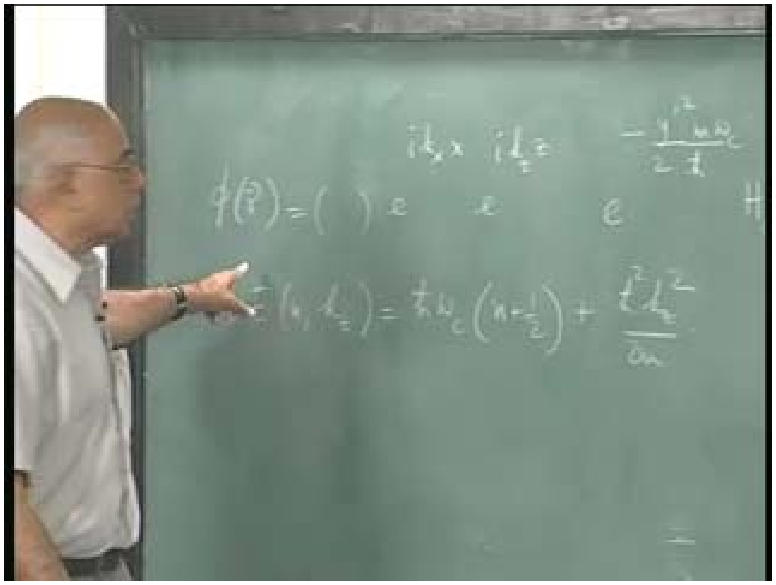

In this representation phi(r) is of the form some normalization constant to normalize the harmonic oscillator solutions and then there is an e to the ik_x x e to ik_z z times f (y) but this f(y) = e the power - y prime squared over 2, measured in units of the appropriate length. In this case, the length scale is set by m omega c over h cross H_n of y prime squared root of h cross where these are the Hermite polynomials. That completes our solution. This is what the wave function looks like. Corresponding to this, you have the energy level h cross omega c (n + ½) + h cross squared k_z squared over 2 m.

What's the meaning of k_x? It's appearing here and it's appearing here (Refer Slide Time: 53:00). Think of it physically. The magnetic field is in the z direction and the charged particle classically would move in the xy plane with the helical motion on the z direction. the projection of motions would be circles. Where is the center of the circle? The energy doesn't depend on where the center is. It can be anywhere in the plane. So there is an infinite degeneracy and that appears in quantum mechanics through this number k_x which doesn't appear in the energy at all. So every level is infinitely degenerate. In the sense that the levels for different values of k_x, we have different wave functions. The Hermite polynomials would be centered at different points.

y prime depends on k_x but the energy would be exactly the same which would correspond classically to a particle doing a helical motion anywhere. So the levels are infinitely degenerate. It's not symmetric because i choose the gauge to be - B_y, 0,0. You could covert it to a gauge where its - 1/2 B_y, 1/2 B_x, 0 and then the symmetry between the x and y would be restored. In fact, it doesn't matter where you can rotate in the xy plane. If the field is in the z direction, the whole system is invariant under rotations in the XY plane. So which axis you call x and which you call y or what combination you choose completely irrelevant. But there is a one parameter degeneracy which has shown up here (Refer Slide Time: 55:00).

Physically, of course you don't have this because you put this whole system in a box. Once you put it in the box the levels in the xy direction would get quantized completely. then this would make sense under the conditions that the characteristic length scale in the problem, given by the cyclotron motion that gives you a length scale which is this (Refer Slide Time: 55:36), equal to lambda say. This is the characteristic length scale. That's the pitch of that's the radius of the cyclotron orbit. if that is small compared to the length of the box, then you can see that the box really is irrelevant.

This would be a good approximation here. And the degeneracy would actually be proportional to the ratio of the size of the box to this lambda here. It turns out to be proportional to the square of this. That would measure the degeneracy and when the box goes to infinity, the degeneracy becomes infinity, but if you put it in a box and the box is very small, then this whole thing goes out of the window. You have to put the potential due to the box in. that's a different story. As long as L is much greater than this thing here (Refer Slide Time: 56:34) and where L is a size the apparatus; the system that you have. It's a good approximation. Now what we have to do next is to ask, suppose I chose a different value, suppose i made a gauge transformation on it and choose a different A, what's the guarantee that things are not going to change. Well, you can see that the earlier portion where I looked at the commutation relations between pi x and pi y, that depended only on B not on A. therefore this was completely general. It didn't depend on the gauge at all. The energy levels don't because they depend only on the magnetic field cyclotron frequency. It doesn't depend on the vector potential or anything like that.

The wave functions appear to have some dependence on this specific choice of gauge. That's why y came out as a special variable here. It could have been anything else. This is a physical argument but we have to make sure that indeed, this is the case. So what we should do next is to ask what does a gauge transformation do to the wave function? In particular it should not change the modulus square of phi. It shouldn't change probability densities but that's hidden here because you may argue that the y prime was dependent on y and y came because i chose that particular gauge. What if I had chosen a different gauge but i get similar answers? This we have to answer.

So what I will do next time, which is tomorrow, is to show you that the wave function is actually gauge invariant. It also undergoes a phase transformation. Simultaneously the field and the wave function will undergo a joint transformation and then the system will be gauge invariant. And in cases where you have singularities, where you have non simply connected regions, then the phase of the wave function is actually measurable. And this was pointed out very late in the history of quantum mechanics and goes by the name of the Bohm-Aharonov Effect. So we will do that next. We will talk about it. Let me stop here today.