Numerical Methods and Programming
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Lecture - 26
Higher Order Derivatives From Difference Formula

But remember we are talking about finding the derivative of a function numerically that is given a set of data points, how do we find the derivative on one of this defined data points or on some other points in between and we looked at the method in which we would interpolate we will construct a polynomial which will interpolates all these points and then find the derivative of that polynomial.

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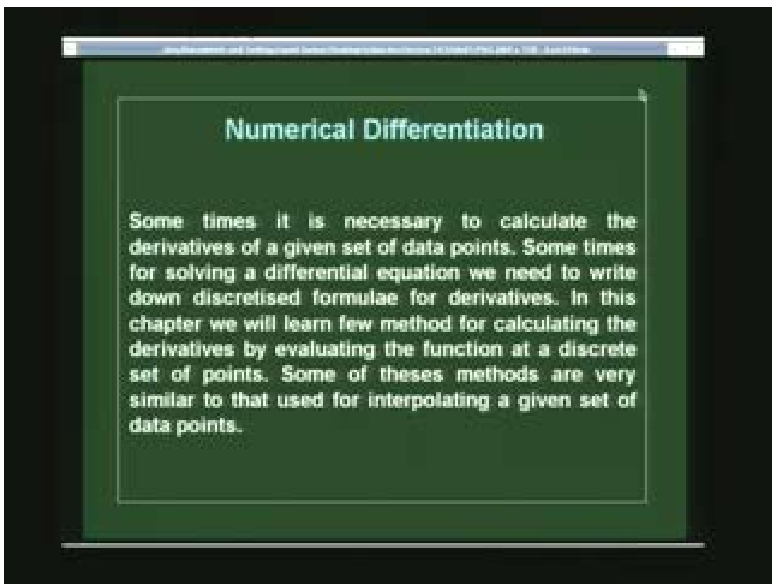

So we will first look at an implementation of this algorithm and then we will look at other methods of finding the derivatives like the finite difference key etcetera, so this is something which we have looked at in the in the last few classes that is we have a set of points given by x_0 x_1 etcetera and we have the function values of those points and then we find out the polynomial which interpolates that set of data points or the or the set of we have a polynomial which goes through all those data points and that polynomial of order n is defined in this fashion by in this fashion using the Newton's key and I guess you remember, that these points f x_0 x_1, f x_0 x_1 x_2 etc.., are the coefficients of the first order second order term etcetera or can be obtained by constructing a difference table.

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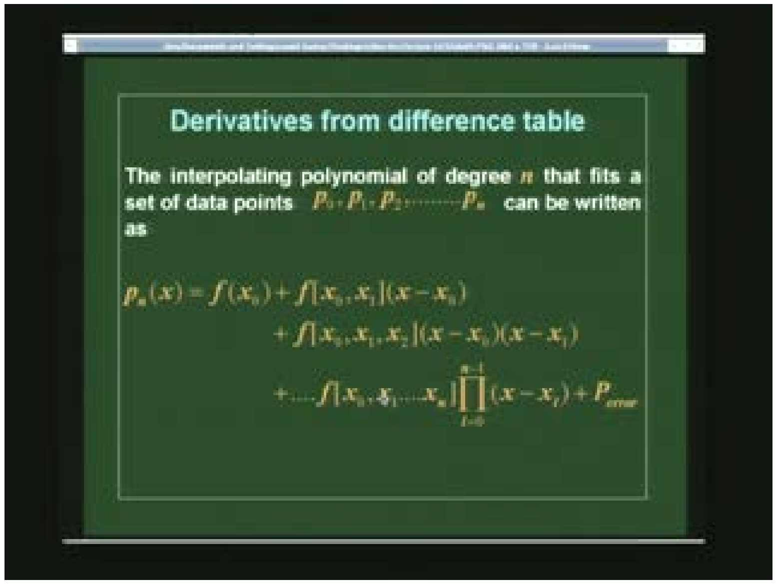

So that is what we would be looking at I will just write down this difference table once again and the we will look at the implementation of this this algorithm to find the derivative once we have the difference table and we have the coefficients right and then we can construct the derivative simply by this method. So that is we write, now the derivative as again in terms of another polynomial and this look at that this this terms become slightly different for example, the first order terms there are two octave and similarly, there will be the nth order term there will be n plus 1 octave.

So that is n plus 1 sums in the in the nth order terms n minus 1 term will have n such sums. So that is the polynomial and we will just look at the a program which calculates this for a given function before we look at the program, let us look at this coefficients of the polynomial once again that is we will construct the difference table and then look at how we get this coefficients. So that is listed here. So we have the that is I have listed here five points here for example, so 6 points that is 0 1, 2, 3, 4, 5 and these are the tabulated x values, ok need not be equidistant it could be any 5 values, okay 6 values x_0, x_1, x_2, x_3 and x_4 and these are the functional values.

Okay, so now these terms in the square brackets are the coefficients of the polynomial rite so we have the function value here and then the next term would be the difference between this function value divided by x_1 minus x_0 and the difference between this function values is divided x_2 minus x_1. So this is the first order divided difference what we call the divided difference the first order divided difference and then you have the second order divided difference and then you could write the third order divided difference here as x_0, x_1, x_2, x_3 minus f of x_0, x_1, x_2 divided by x_3 minus x_0 etcetera.

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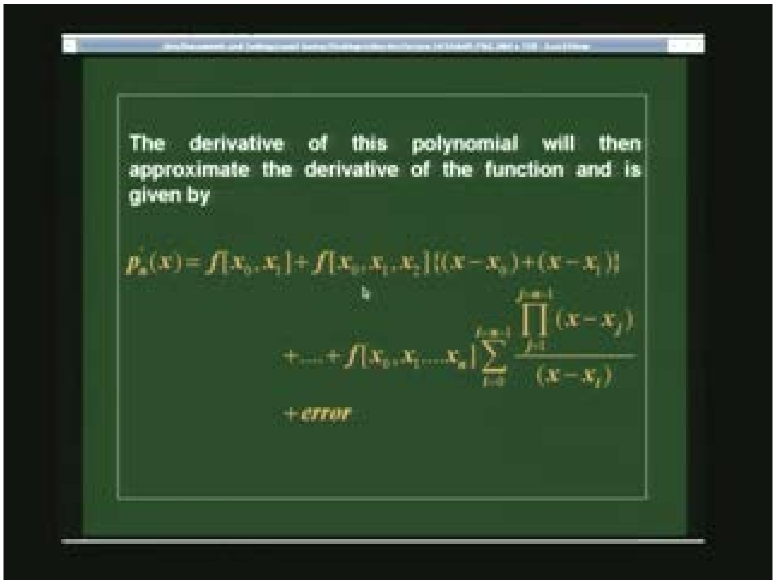

So you could write down them all these points. So we could construct a divided difference table like this and then read off these points, ok we done this earlier in the case of polynomial interpolation this is exactly the same thing.

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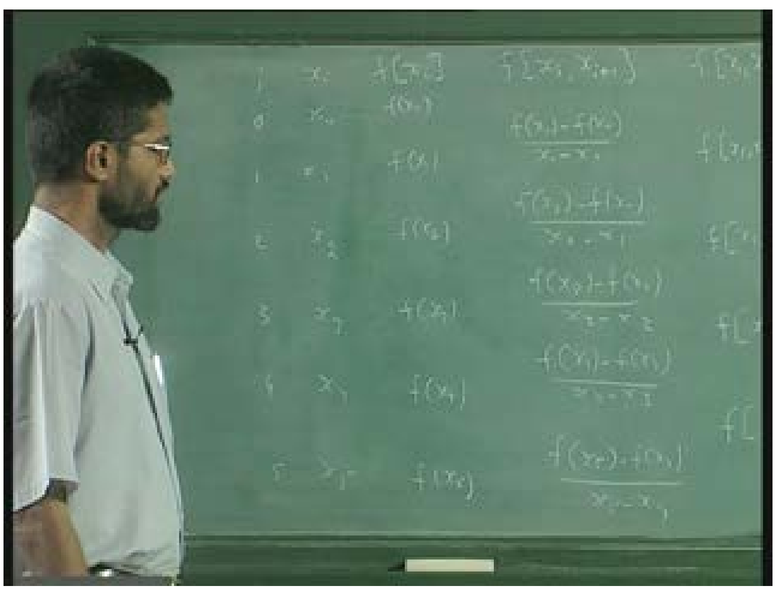

Okay this is something which we have looked at in the in the last few classes and also notice the way we construct a polynomial here is we just write down this as first order term, second order term and the similar depth up to the nth order term, for an nth order polynomial we do not, we need not write it like this we know that we can write it in a nested form that is starting from the nth order term we could write it as in a nested form we have seen that earlier.

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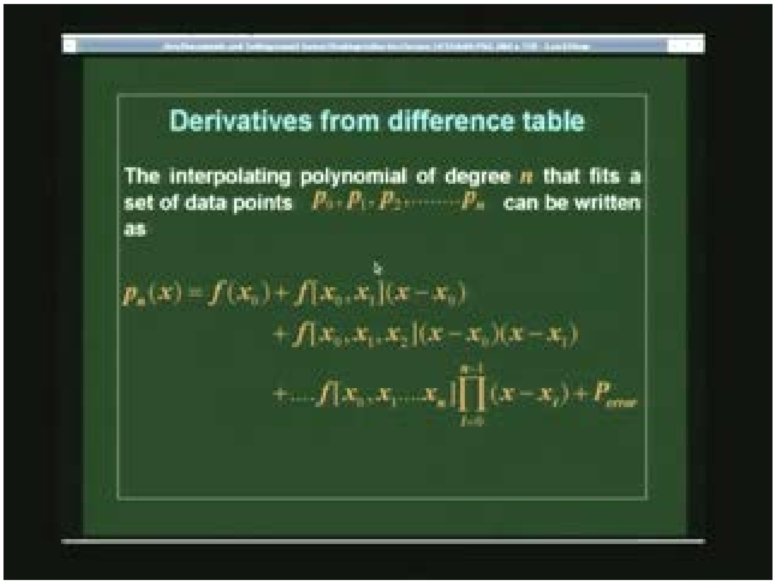

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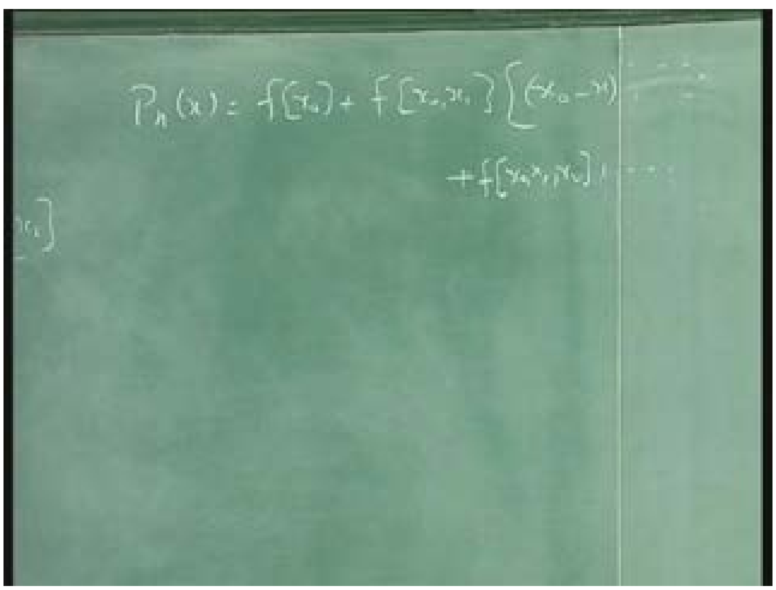

Okay so we could write the polynomial P_n of x if you remember as f of x_0 plus f x_0 x_1 into right. So x_0 minus x plus f of x_0 x_1 x_2 etcetera right. So we could this in a nested form like this, we have seen this earlier, so that is what we would see in the in the program first and then however but when you go into calculating the derivatives it is convenient to write it down in this fashion. Okay so it is convenient to calculate the derivative by writing it into a separate function separate terms for each first order second order and higher order terms.

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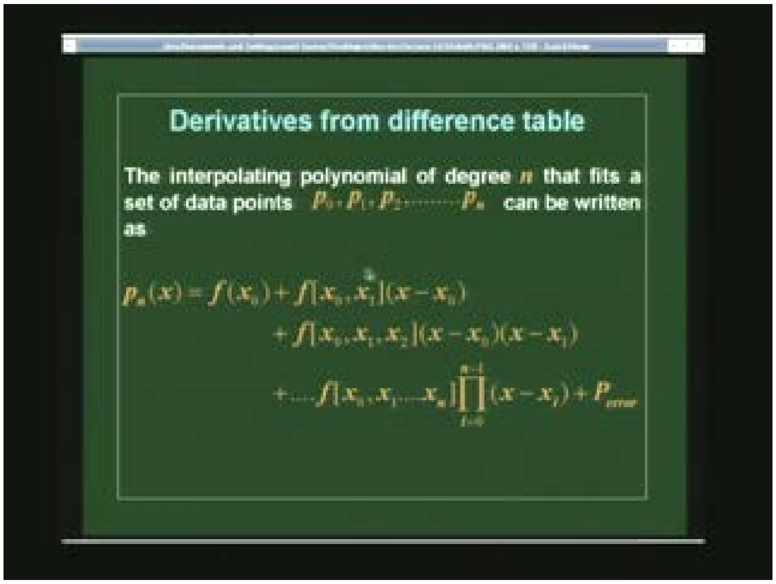

So here is a program which would implement this particular algorithm for the Newton's form of the polynomial, so what we do is, we just take a function x of i as a f of i which is exponential x, okay so that is the function which we are going to do.

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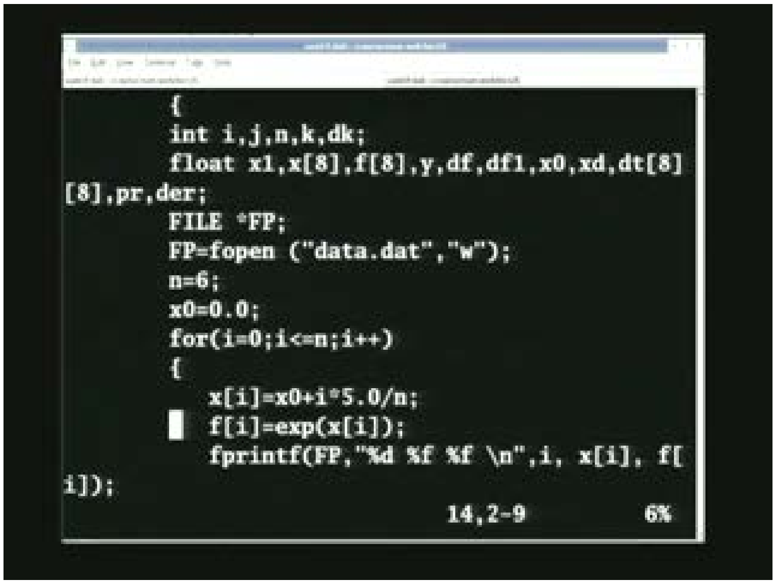

So what we do is, we take this function f of x which is e to the power of x and then we will tabulate this function in a set of, set of discreet points and then we will construct a polynomials which goes through this function, this set of tabulated points and calculate the derivative that way we can actually compare its the derivative of this function which is the function itself.

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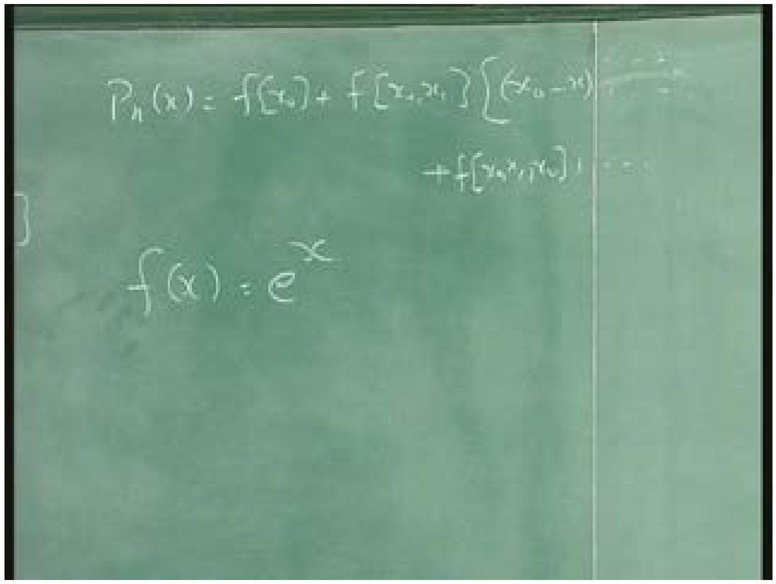

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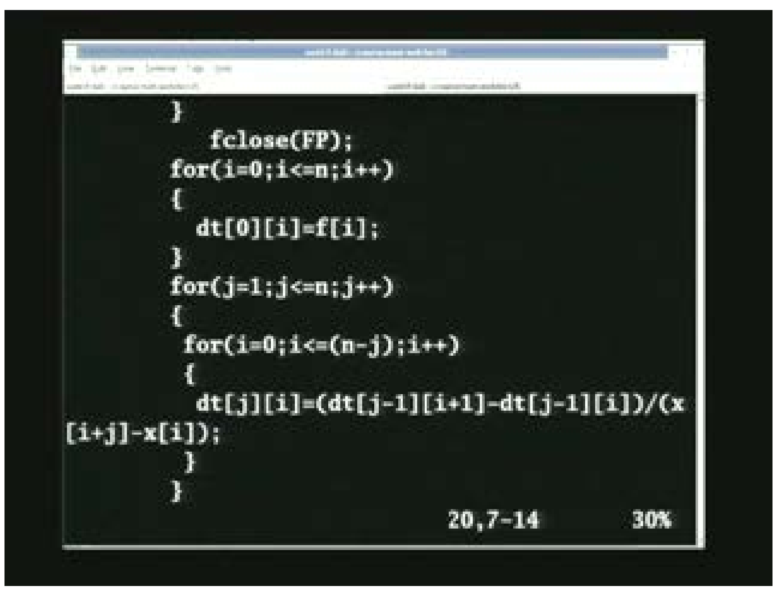

Okay that is an easier comparison for demonstration purpose. So we just calculate the first part of this program is actually just calculating that or tabulating that function, so the function is exponential x, so I just take a set of equidistant points need not be equidistant in this particular case we just take for convenience a set of equidistant x values. So starting from 0 to n, so there are n plus 1 such points, so where n_i fixed as 6 or 7, so 7 tabulated points and then we have the function value of those points which is given by exponential x is print them on to a file for that, we can compare it with a interpolating polynomial and then here I compute now the divided difference okay so I compute the divided difference in to two dimensional array.

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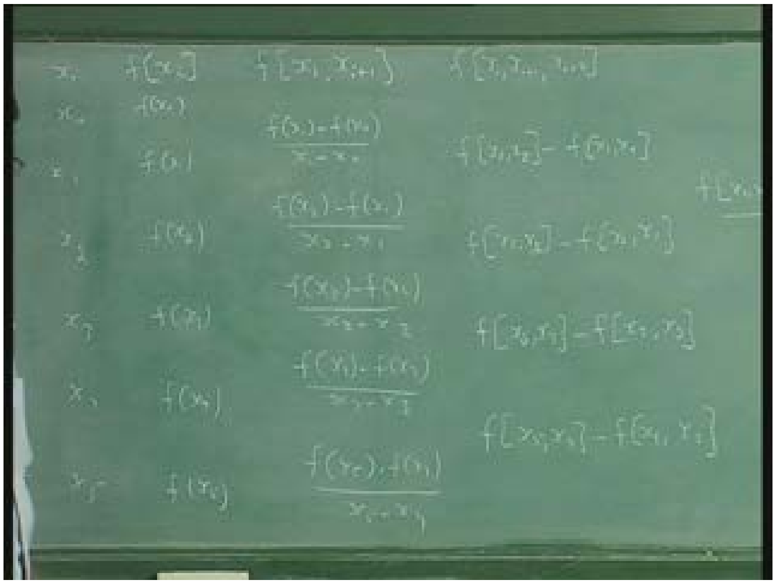

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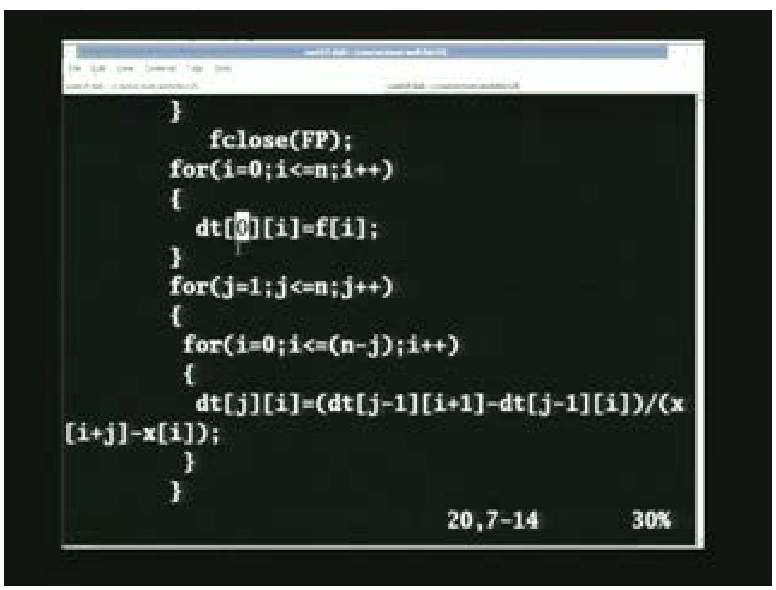

So this the first, so the first term i is the index of this rule, so I have 2 computing this in to a this divided difference in to a two dimensional array. One index for this row this columns and one index for this rows, so the first index here is for the columns, okay that this of first order divided difference which is starting from 0 and when this one that is of second order divided difference etcetera.

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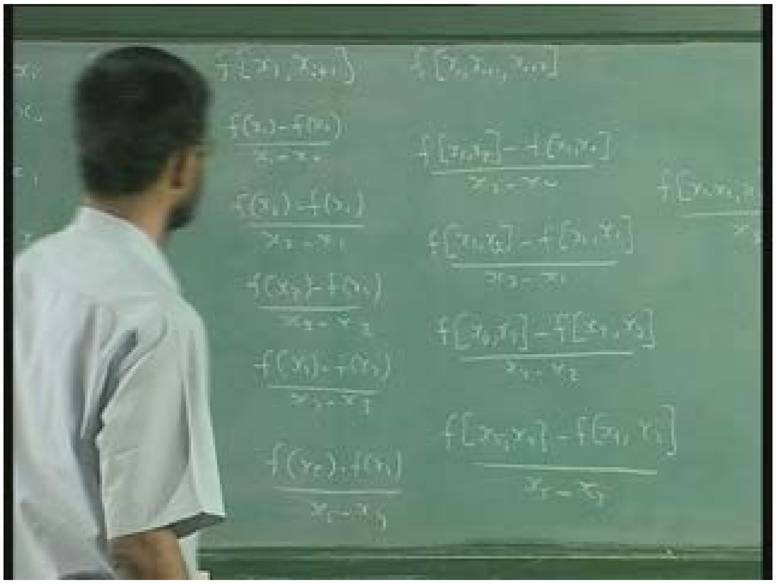

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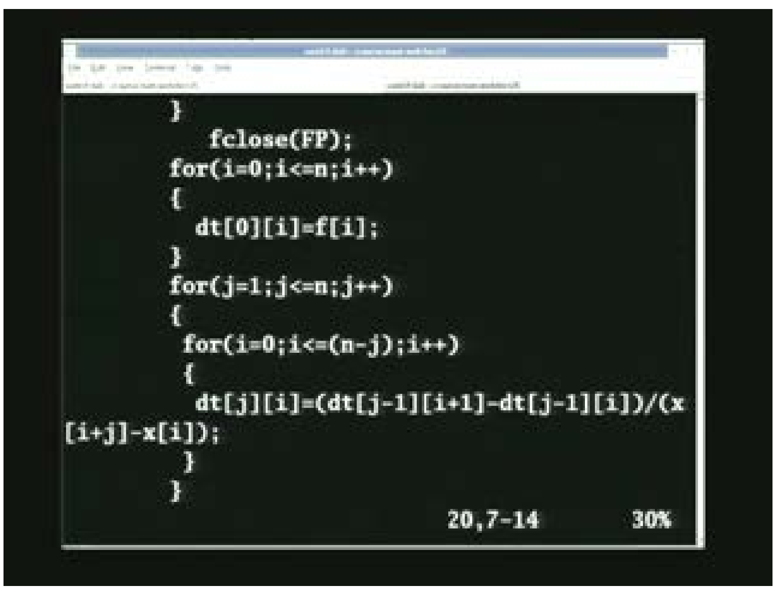

So here this is 0 th order divided difference would be just the function itself and the first order would be the difference between the function divided by x_1 minus x_0 which is one interval and the next would be here I have to write x_2 minus x_0 and here similarly, x_3 minus x_1 and this x_4 minus x_2 and this x_5 minus x_3 and the sec, the second order divided difference. So this is the index for the order of the divided difference and this the index i which tells you the how we runs through all the columns. So that is the first, so the divided difference is actually tabulated in to this two dimensional array dt.

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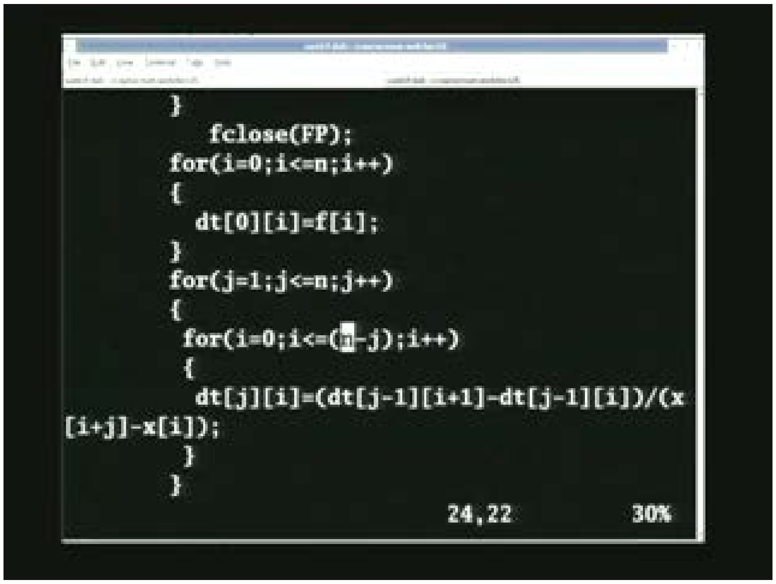

Okay that is the first I constructed the 0 th order one and then I go into a loop which is j going from 1 to n, j goes from 1 to n so 0 is already done this the order of the divided difference we have n points tabulated so we have up to n order in the divided difference. Okay n plus 1 points tabulated, we n points in the divided n order in the divided difference. Okay n plus 1 because it goes from 0 to n and so I have n plus 1 order divided difference starting from 0 to n and then we so there this the construction of the divided difference so I take so each for each order, so the number of number of elements in that divided difference will be n minus j, so each order j.

So if you take the 0 th order I have the n divided difference points that is 1, 2, 3,4, 5, 6 here and I take j equal to 1 that is the first order then I have only 5 of them, that is 1, 2 3, 4, 5 and when I take the third order second order I will have only 4 of them, ok the 6 tabulated points so there is only 4 of them etcetera so that is we go from i starts from 0 to n minus j.

So that is this loop and i compute the divided difference for each order, so once i have as a difference between the previous order divided difference divided by x_i plus j minus x of i just notice that x_i plus j minus x of i. So the denominator keeps increasing in distance as we go along So here I have ignored the fact that is equidistant and we know that if the points are equidistant we can simplify this procedure I have just writing a general program which is actually works for any tabulated any set of tabulated points need not necessarily equidistant.

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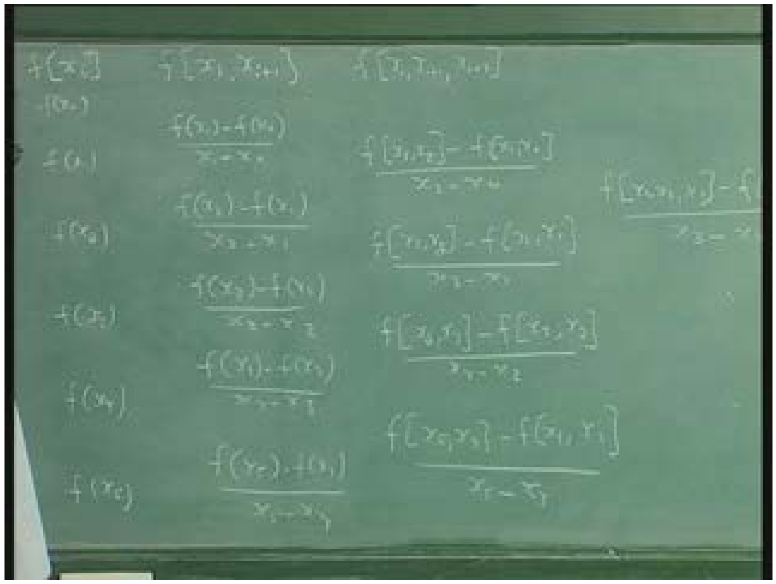

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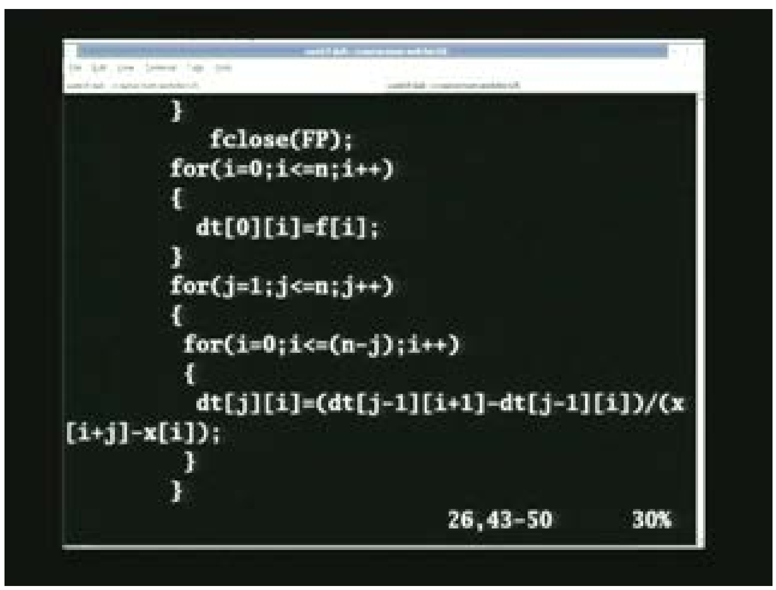

So once you have the once you have that divided difference, now we can compute the derivative polynomial and the derivative the reason why I did this is a two dimensional array instead of just one dimensional array which we done in the polynomial interpolation. Remember, in the polynomial interpolation we actually computed this divided differences into a one dimensional array the reason being that we actually need to store only one set of divided differences values. Okay so if, we have always enter the divided differences table at the top, so if we have always entering the divided differences table at the top that is we can we can evaluate a function value at any point starting from x_0 to x_5 in between any points using this polynomial which we wrote as p_n of x by entering the divided differences always at the top or we could also enter somewhere in between.

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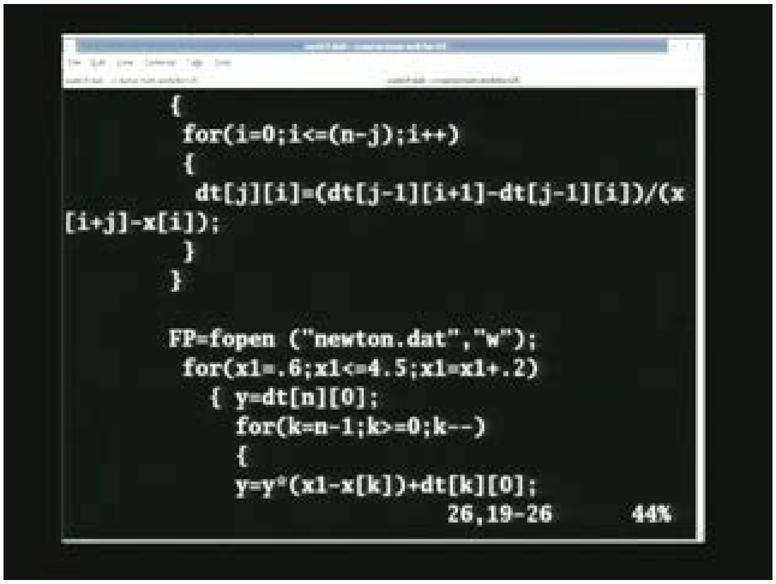

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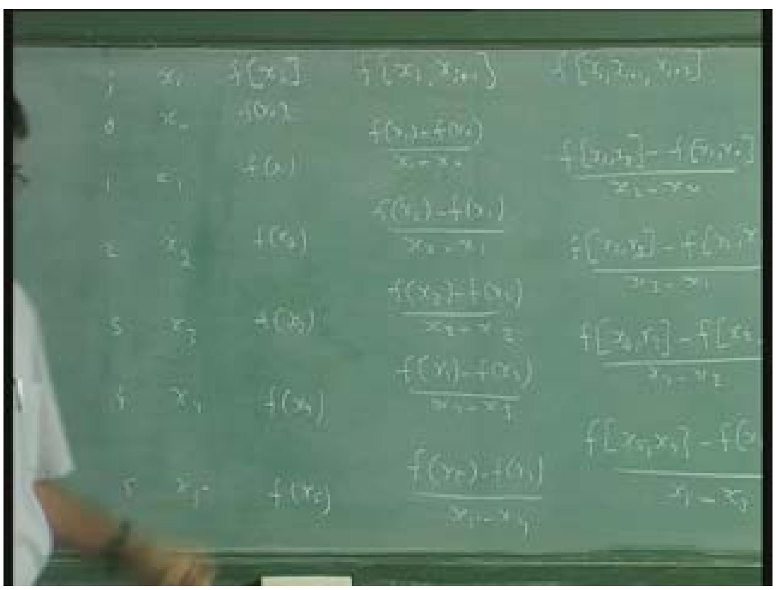

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So if we are entering it always in the top we will be using only these set of only these set of values, ok we will use only these set of only these set of divided differences values. So we need to store only that. So I need only a one dimensional array to store that. So that is what we done in the polynomial interpolation case. But here, we want to have a little more freedom that if I want to compute as we saw in the last class that if I want to compute the derivative of this function, ok anywhere in between 2 tabulated values. So we should wanted to have freedom that I want to enter the difference table of this points for example, if I wanted to compute the derivative at any x in between x_3 and x_4, I should have a freedom to enter the difference table at x_3 or x_2 or x_1 or x_0 anywhere.

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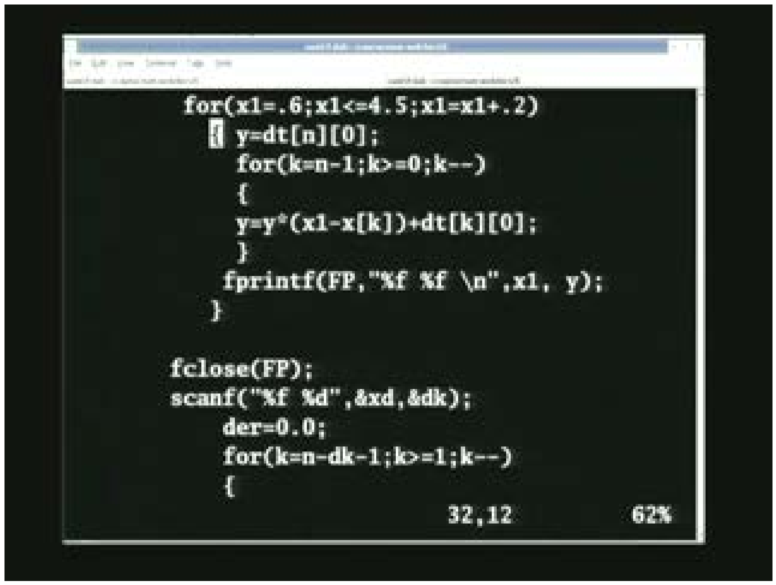

So that is the reason why I store this divided differences here into a two dimensional array to just get the freedom for that. Okay then here is first we just calculate is to see everything is correct, so we will first calculate the polynomial itself okay. So now the polynomial is constructed here. So there are three loops as, we can see that polynomial construction now involves this finding a product of x minus x_i.

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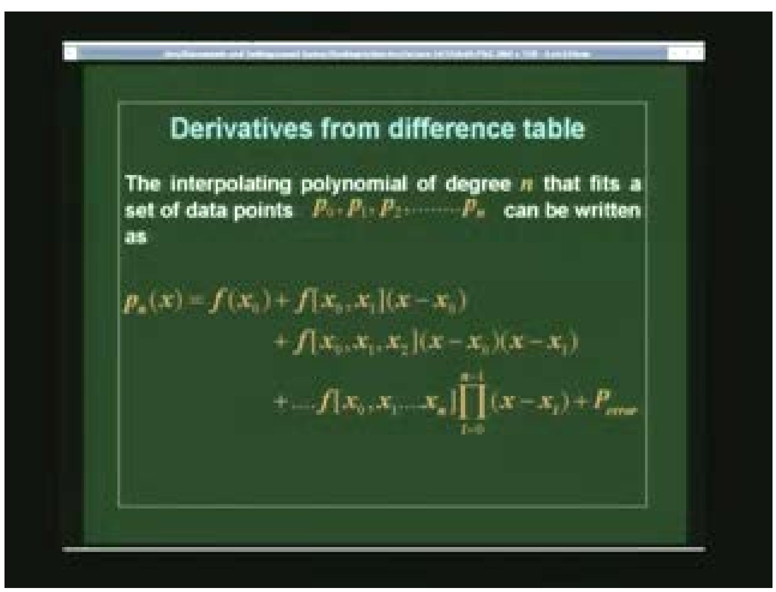

$$P_n(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots f[x_0, x_1 \dots x_n]\prod_{i=0}^{n-1}(x - x_i) + P_{error}$$

So for the nth order term we will have to find the product of n x minus x_i. So that is x minus x_0 into x minus x_1 into x minus x_3 up to x minus x_n minus 1 right. So we have to find that product first then multiply that by the coefficient okay and then added to the next product and then multiply it by the next coefficient etcetera that is what we have that is what we have been doing here.

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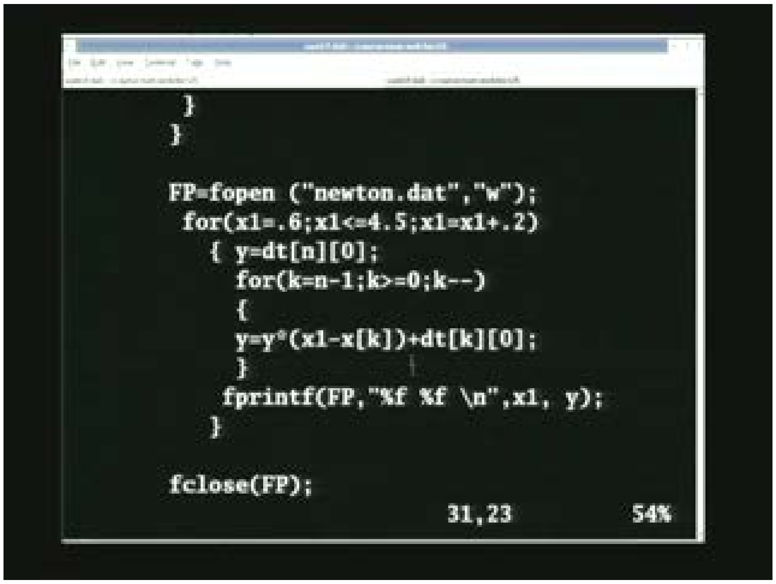

So if we look at this first I will find the, I will start with now computing this this value this polynomial value starting from point 6 to 4.5. So we have we have the function tabulated from 0 to 5. Here also a function tabulated, so we are starting from x 0 equal to 0 to an i equal to 5 so x 0 to 5. So we will evaluate the polynomial we will construct the polynomial and evaluate it at the other set of values that is starting from .6 to 4.5.

So now if I take an x value and if I want to evaluate this polynomial now I am not writing this in a nested form I am writing term by term. So this this is not term by term, this in the nested form. So I start from the last term that is d last highest order, highest order divided difference that is dt of k, 0. So remember here I am entering the difference table from the top, so I am entering it in the top, so all the row is the 0th row for every column this is the 0 th row that is every column the top element is by a coefficient. So I start from the highest order ok the highest order is k.

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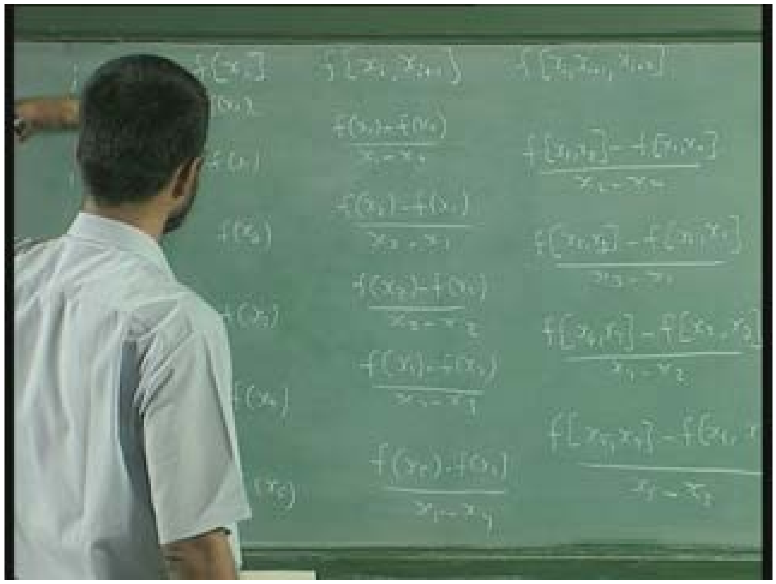

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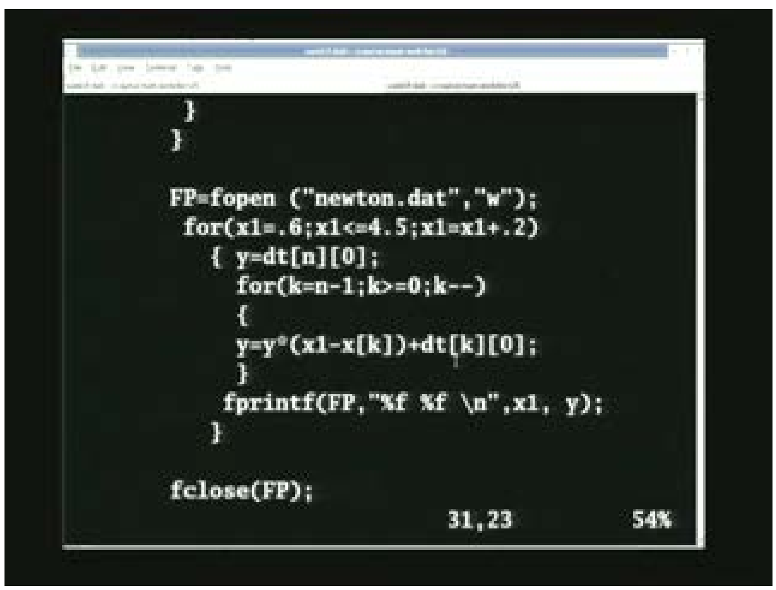

This is the highest order I start from k that is so the last term k equal to n minus 1 that is my highest order. So I start from them actually the highest order is n highest order is n ok I start from there. Okay I start y as the highest order coefficient and then I find the product of that with that with x_1 minus x of k, okay x_1 is the point at which I want to evaluate the polynomial and then I had this term dt next coefficient. So let me write this thing it becomes clear. So the way I am constructing the polynomial is from backwards.

So I start with my highest order divided difference that is x_0 x_1 up to x_n, that is my highest order divided difference and then I multiply, I multiplying this with x minus x_n.

And then I add to this another term which is f of x_0 x_1 up to x_n minus 1, okay then I put a bracket there and I multiply this by x minus x_n minus 1 and then I had another term which is f of x_0 x_1 up to x_n minus 2 the next coefficient into this and then I put a bracket there and then i multiply this by x minus x_n minus 2 etcetera. So I continue with this way till I reach f of x_0. So that is the way I am constructing, so I am going it polynomial like this I am constructing this in this in this fashion starting from the last term when I am constructing it backwards.

So this the nested form of the polynomial for each term you add you have to put in 1 more bracket to show that this term is actually the nth order term in the polynomial because it keeps get multiplying by x minus x_n x minus x_n minus 1, x minus x_n minus 2 till it reaches x minus x_0. So that is what it is said, we will this is n minus 1this is n minus 2 and this n minus 3 till it gets x minus x_0 in the end. So this this will be then by the time it reaches x minus x_0 this term would be multiplied by x minus x_n minus 1 x minus x_n minus 2, x minus x_n minus 3 up to x minus x_0. So that is the implementation here.

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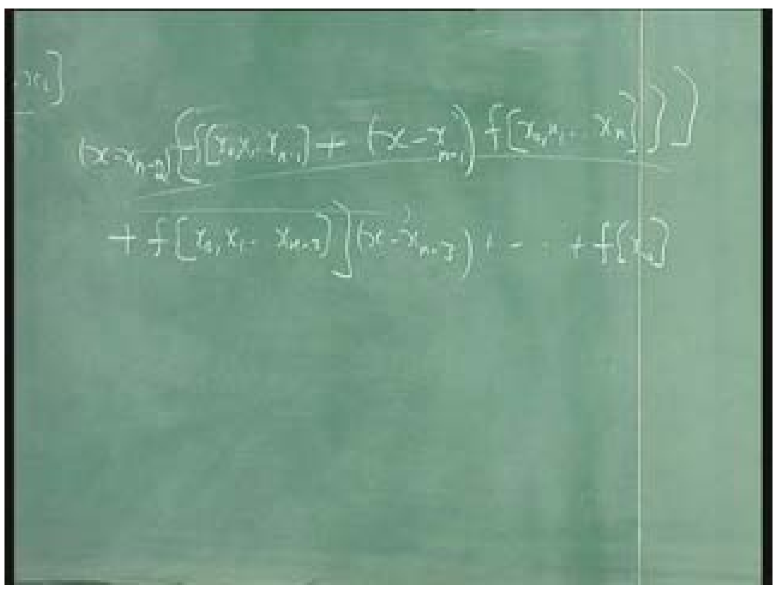

So you start with the highest order divided difference and then you multiply that this y is now just this highest order term multiplied by x minus x_1 minus k, k is n minus 1 to start with add the next order term that is n minus 1, 0 this always 0 because we are on the top of the divided difference table now that becomes y, to that y now you multiply the next term that is n minus 2. So $x_1$minus x_n minus 2 and add the next divided difference that is n minus 2 etcetera 0 that is the nested form of the divided difference table. So that will give you the polynomial and I write down the function the value at which I am evaluating it and the polynomial value to a, to a function to a data file called Newton.dat.

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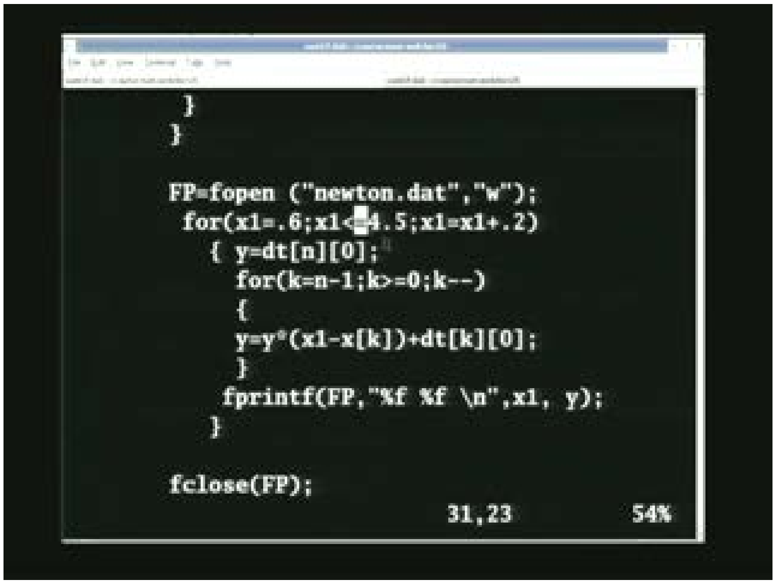

So that we just see that we just run this, okay so before we run this I just want to show you one more thing. So then after we evaluate this polynomial now, we have the polynomial and the set of data points and then I want to evaluate the derivative at any point.

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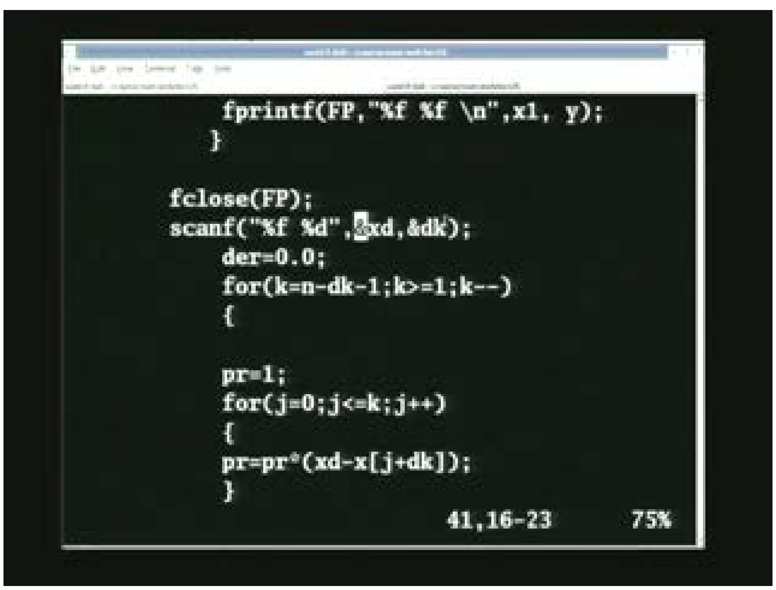

So we have to actually enter the program and wait for some wait for you to enter which value at which value you want to evaluate the derivative. Okay so that is x_d is the value which I wanted to evaluate the derivative and dk is the number is the point at which I want to enter the difference table as I said that if I want to evaluate the derivative at any point. Let us say between x_2 and x_3, I could enter the difference table at x_2 or x_1 or x_0, okay so it weight for this index to be given and at what value you want to evaluate the derivative and at what point you want to enter the difference table, so this our two things you have to see.

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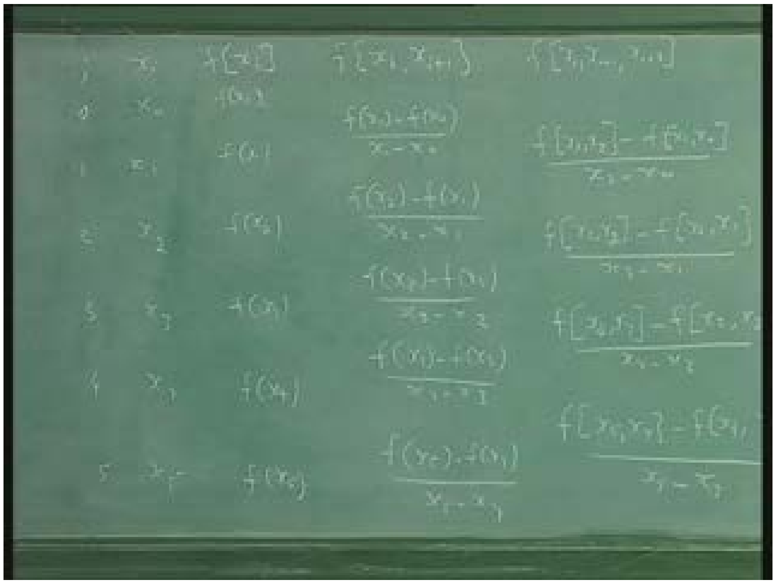

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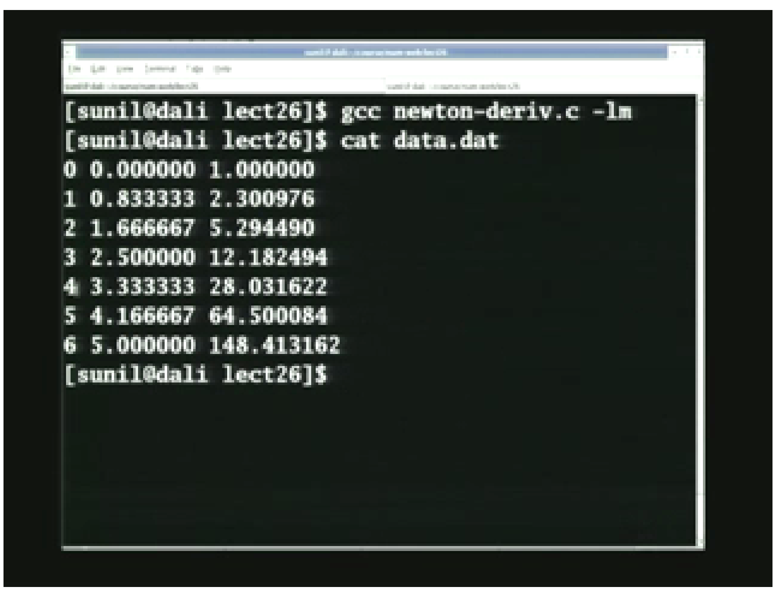

Okay so we will just look at the, okay so that is the tabulated value okay so 0, 1, 2, 3, 4, 5, 6 so the 7 points. So these are the tabulated x values or tabulated function values of exponential x. So what I was trying to tell you is that if I want to find the derivative let us say 2.7, okay of this function if I want to find the derivative of 2.7 then I should be then I can enter the difference table at n equal to 3 or n equal to 2 or 1 or n equal to 0. I cannot enter n equal to four because then it will exclude 2.5, 2.5 to 3.3 terms all the all the values from 2.5 to 2.3 but I can enter it anywhere earlier.

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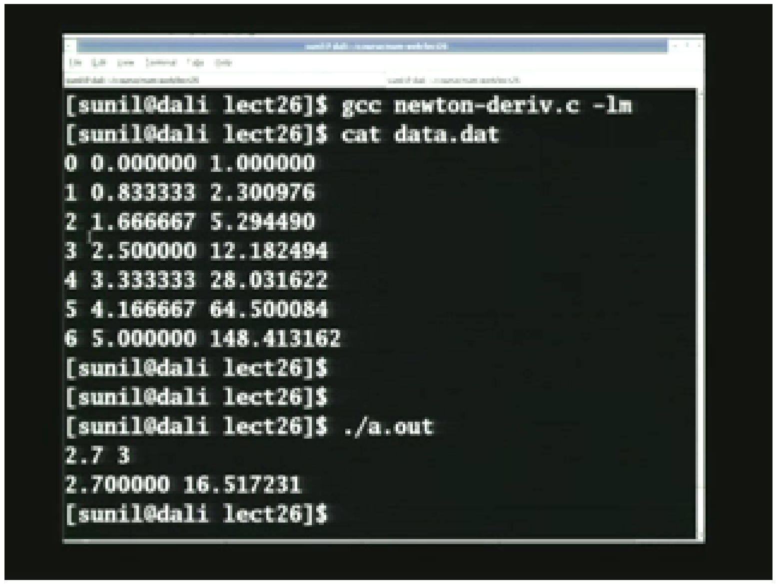

So that is what we would be say doing, okay so if I want to, if I run this program okay it waits for the x values let us enter it as 2.7 and then let us say we will enter the divided difference table at 3 that is .3. So it says that the derivative of the function at 2.7 is 16 which is off by a large amount. Okay so, it is clearly off 2.6 is now exponential 2.7 is not 16 we know that, okay it is clearly of now what we have done really is to find the derivative at 2.7 we have entered the difference table at 3. So we are constructing a polynomial which is basically linear.

So we are constructing a linear polynomial in this case not linear polynomial. We are constructing a polynomial which is of order 1, 2, 3, 4 of order 3 and then we are finding the derivative of this point. So it seems to be off by a large margin, so but this will keep improving as we as we enter at earlier and earlier points as we can see that if I enter the same value evaluate the derivative of the same value but I enter the difference table say at 1 okay or at 2. Okay so we get a slightly better value that is 15, now if I enter it 2.7 and if I enter it 1 it slightly even better value 14.9 but you can say it is slightly fast converging into the correct value. Okay how are we computing this derivative, so it is here that you enter the value at which you want to compute this and the point at which the difference table you want to enter.

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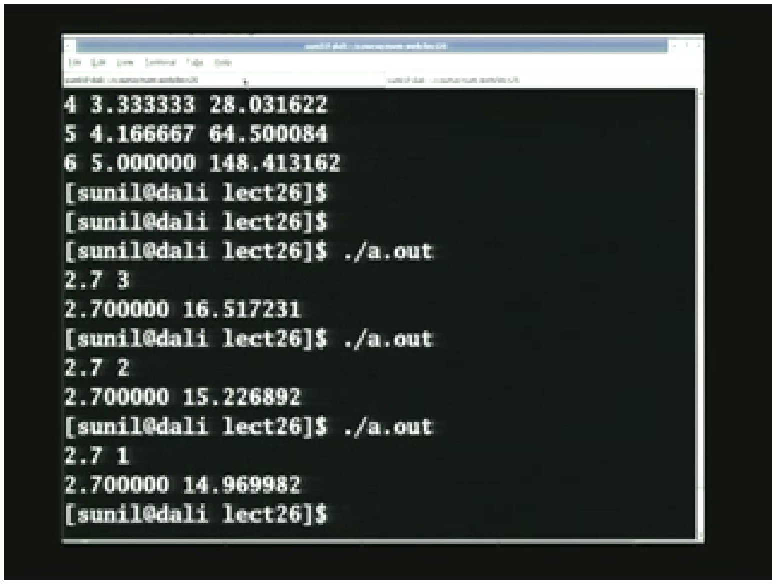

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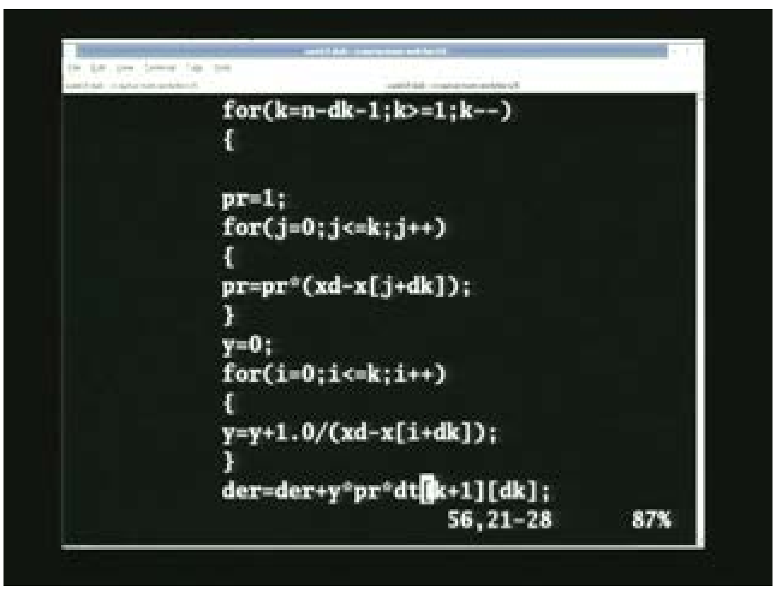

Okay and then this function now has to compute now 3terms okay, so I will show you that okay, so it has to compute this product right for every term okay, so therefore the last term that is the nth order term. So we have nth, n plus 1 nth order term okay we will have some n minus 1 such sums right. So the second order terms the coefficients is second order derivative divided difference we have 2 first order terms similarly the n plus 1 th order divided difference you will have n minus 1 sums of n minus 1 order terms that is we will have x minus x_0 x minus into x minus x_1 up to x minus x_n minus 1. Now divide that divide by x minus x_0 x minus x_1 etcetera.

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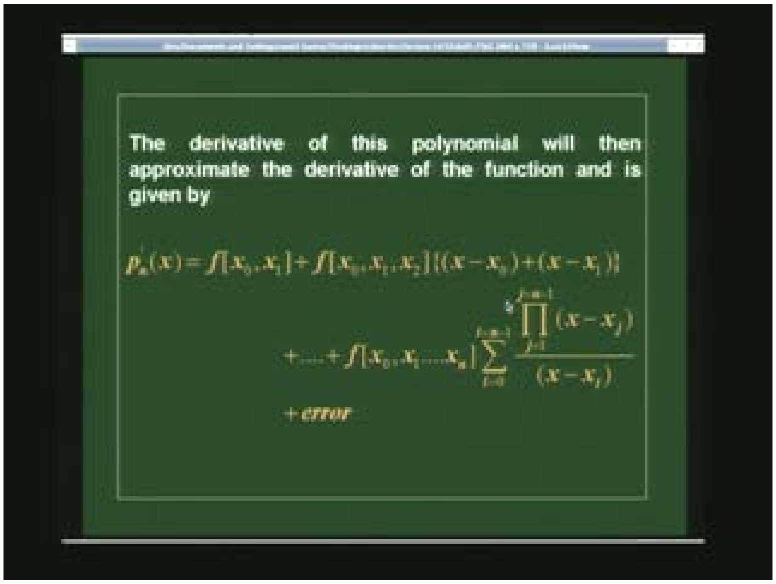

So you will have a term a table like this so basically we have to compute this term okay we have seen this earlier we have writing down the derivative of the polynomial in the Newton's form here. So the first order term now becomes a constant and the second order term becomes now the first order terms is the derivative but in that there are two such terms similarly, the n minus 1, n th order term of the polynomial n will, now become an n minus1th order term because we are finding the derivative but then there will be n minus 1 terms in that okay, that is the n minus 1 terms inside that function itself okay that is what we want to evaluate.

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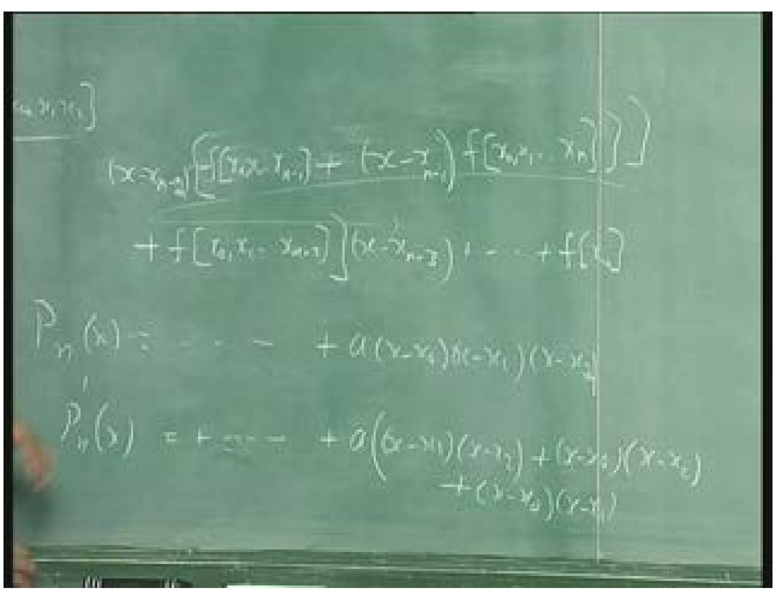

Okay so remember that if you have if you have a polynomial, a term in the polynomial has let us say, a term in the polynomial. So many terms and then we have one term which is some coefficient times x minus x_0 into x minus x_1 into x minus x_2 etcetera. So if I take the derivative if that the derivative of this polynomial, we will now have this term as a into, so we will have many term again and then we have a into, so we will have first derivative of this x minus x_1 into x minus x_2 then we have plus x minus x_0 into x minus x_2 and then we have 1 more term which is x minus x_2 x_0 into x minus x_1.

So that is the third order term in the polynomial will now, third order term in the polynomial will now become the second order term in the polynomial in the derivative but there are three such sums. So that is we have to program it in this way instead of writing for each term like this we have general formula which writes this term particular term as again this particular term we will now write slightly differently, we are saying that we can write it as a product of 3 terms divided by x minus x_0 x minus $x_1$etcetera.
Okay I will write this this particular term as, so we will write a into so we will now write as x minus x_0 into x minus $x_1$into x minus x_2, okay we will write it like that .and then we will write it as sum of we write this again.

So the derivative so we have the derivative of the function so that had, so this is the first order term, so we will have some term first order term and then we had a term which is a x minus x_0 into x minus x $_1$and x minus x_2 plus a $_x$ plus x minus x_2 and then we had x minus x_1 into so we had this 3 terms right and then we may we may have higher order terms and then we are concentrating on the third order term this I can write as a into x minus x_1 into x_0 into x minus x_1 into x minus x_2 into 1 by x minus x_0 plus 1 by x minus x_1 plus 1 by x minus x_2 .Okay that is what I can write it as.

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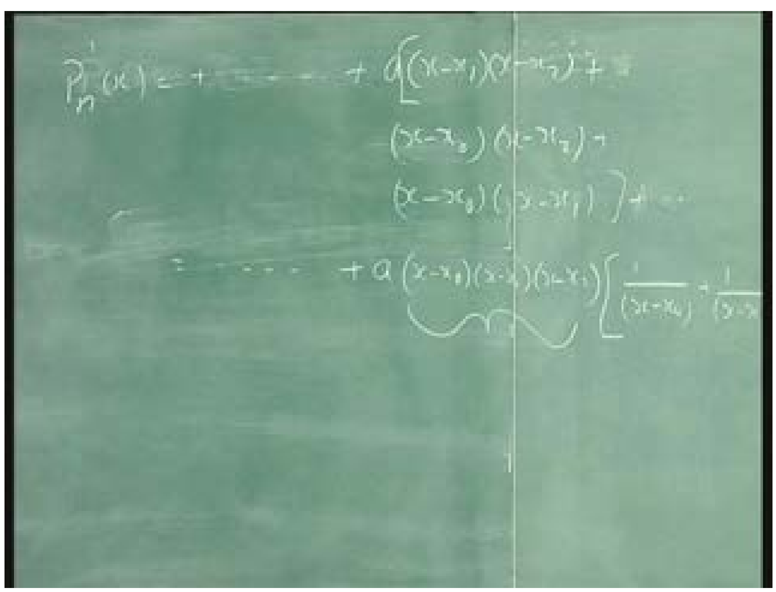

So that is the x minus x_0 terms x minus x_1 terms this is x_0 this x_1, okay so we have these 3 terms written as in this fashion okay so I have a product here and I have a sum. So that is what I am trying to do here okay, so I will have a product and a sum so this is product first which can be come out of a sum okay and then a sum of 1 over x minus x_i multiplied by the corresponding divided difference coefficient, okay that is what that is what this term is doing here so I have a sum of x minus x_i's here, so again the d_k is the points at which I enter the polynomial.

Okay that is also important okay, so I need to know which point I enter in the divided difference so it is not always be on the top okay that is the sum which Ido1 minus i do a sum of going from 0 to k of all the points which is that is the order the term right. That is the order the term in the polynomial the derivative polynomial and I have a sum here for that term that is 1 plus x_d the point at which I enter as x minus x_i or x_i plus dk because dk is the point which I enter the polynomial and then there is product here.

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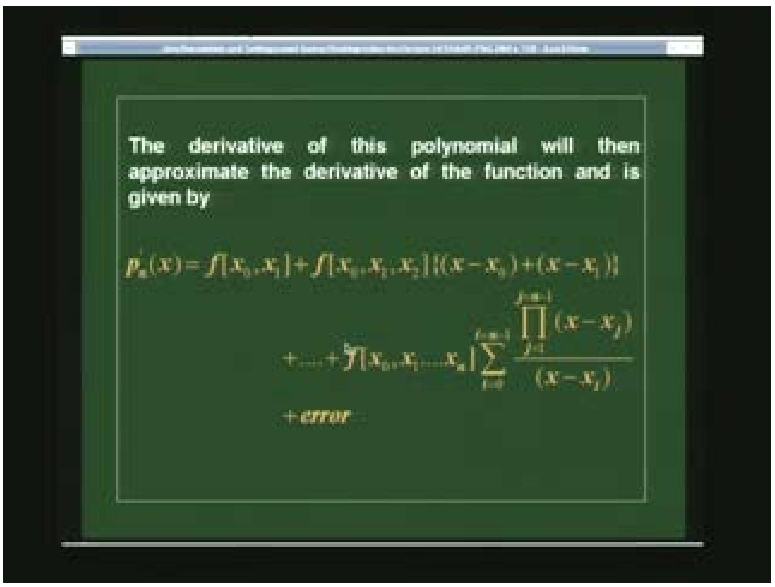

So first I find the product x minus x_i and then I take the sum which is 1 by x minus x_i summed over all the i's and then I take the product of that sum and the, this sum and the and this product p_r and y, I take the product here and multiply that by the coefficient the corresponding divided difference coefficient okay. So now it comes to be k plus 1, so we have checked what point we have to enter the divided difference, okay so when you run the and the sum how many terms will be there in this polynomial it depends on where you enter, so if you are entering it at the top that is dk is 0.

Okay there will be n plus n terms in the polynomial because the nth order polynomial will have n terms in the polynomial and the derivative will have n minus 1 terms and here that is when you enter in the top if you enter anywhere in between, so if I say in the case 2.7 we saw that we could enter it in 4, 3, 2, 1, 0 etcetera.

So if you enter it later point that is somewhere here and then you will have only that many terms in the polynomial right. For example if I enter this point then I will have a term the polynomial I will have a term containing this and then 0, 1, 2, 3, 4, 5 etcetera there will be 5 terms.

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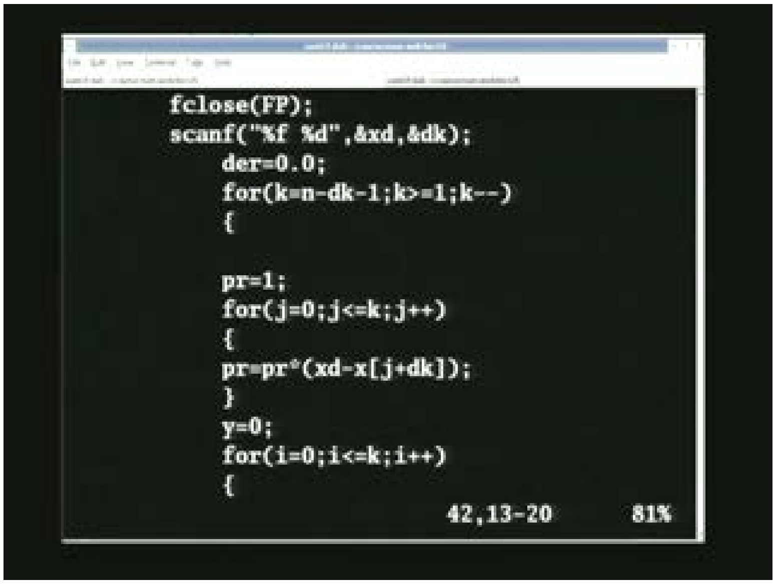

But if I enter here for example then i will only have 1, 2, 3 and 4terms okay, so I will have less number of terms as you come at only three terms 1, 2 and 3 terms so and as if you enter here you will have only 1 and 2 term and then if you enter this point then you have only 1 term in the polynomial.

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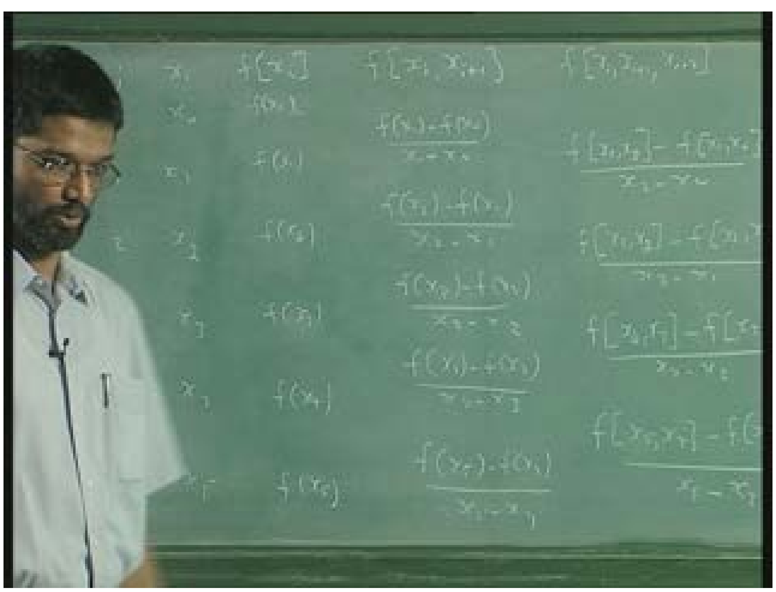

So as you enter the later in the in the difference table you have less number of terms in the polynomial, okay so that is what is reflected here.So this is the basic total number of points in the polynomial which goes from n minus dk minus 1 so if dk is 0 it goes from n minus 1 to 1. So actually it goes to 0 is added separately so n minus 1 to 1 in this in this term it goes but if you enter later ok then we have less number of terms in the sum and this in the polynomial.

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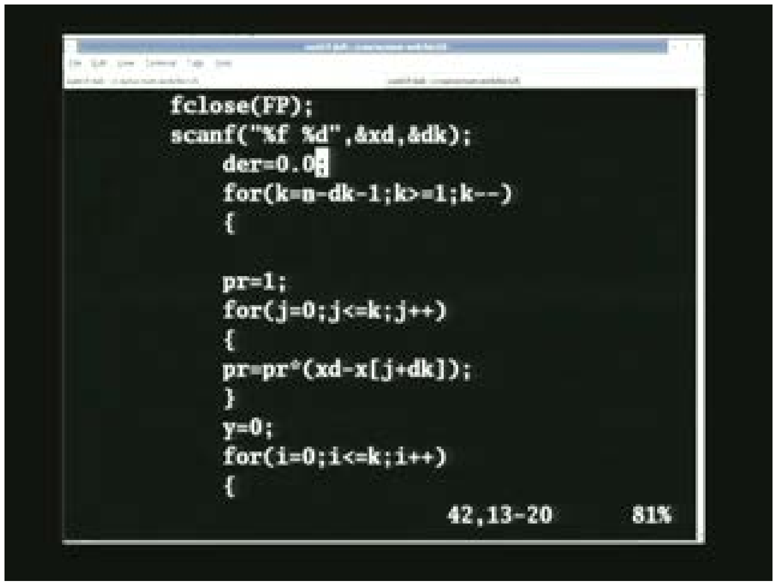

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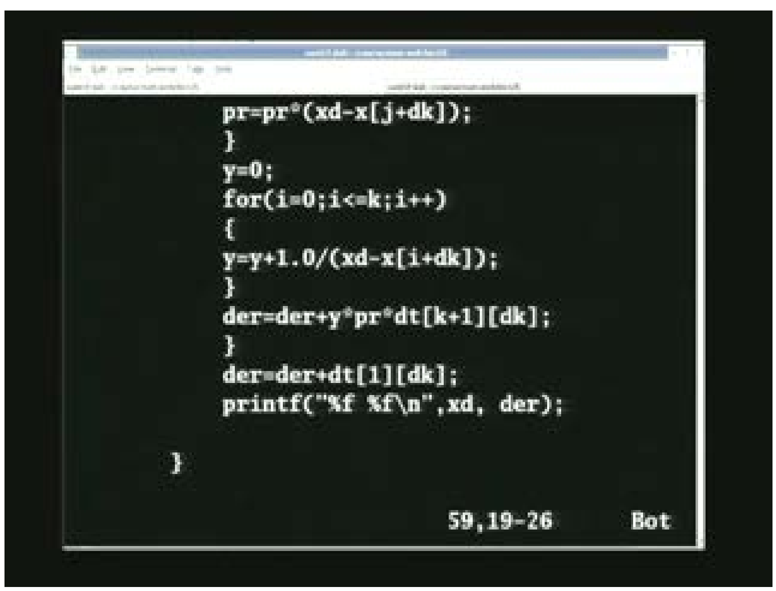

So I find the product of each term for these terms and is and I multiplied by the corresponding divided difference coefficient and I get the derivative and the last term is added separately here as 1 plus dk, 1 dk the last coefficient then I print out the derivative and that is what we just now saw so basically it is the same thing. So if I do this polynomial derivative am just constructing this in this fashion, okay so if it is the third order polynomial and this will be by a third fourth order polynomial if we have the third order polynomial this will be my last term there will be a second order term and there will be a first order term there will be a 0 th order term in the polynomial that is what we just now saw.

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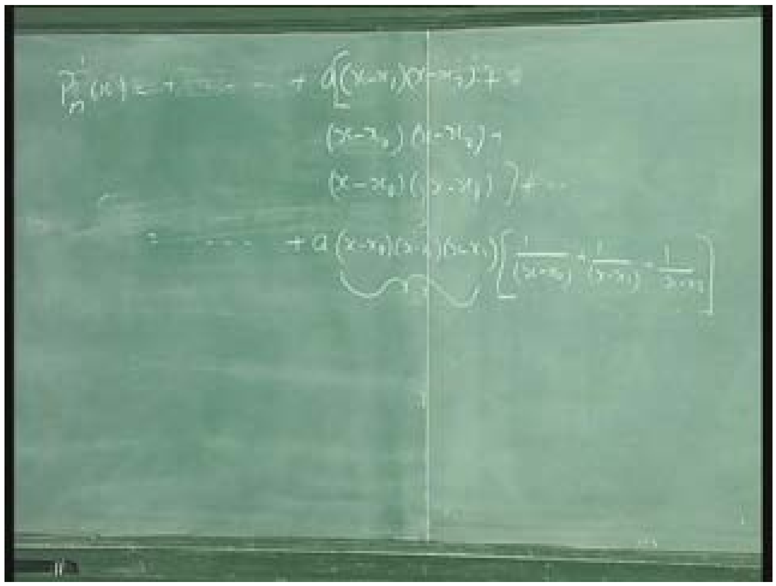

So you can plot so you can plot this polynomial as we seen earlier, so what I have done is now I have this this squared points as my tabulated values now I, I constructed a polynomial which goes through that and then I have evaluating this derivative at any point a while then I could evaluate the derivative for example, we took at two points seven some where here okay by constructing by using the polynomial which goes starts from here and then going all the way end I could even stopped it there I use it all the way up to the end or I could use the polynomial which starts from here and goes all the way up to that okay I could use the full polynomial or I could only use a part of the polynomial construction okay that is what I have get just see.

Okay so now we were we can use the same program which we just now saw we could use the same program to actually construct this this derivative using the method of we have using instead of using the divided difference we could use the deltas that is we do not use this this denominator instead of using the divided difference we just take the difference okay that is, we do not we do not use the denominator okay, that is that we can do in the case where the points are equidistant okay that is x_1 minus x_0 is equal to x_2 minus x_1 equal to x_3 minus x_2 etcetera. So that is then the same program can be used but we could write the divided difference slightly differently and that is what we have seen in the in the earlier class.

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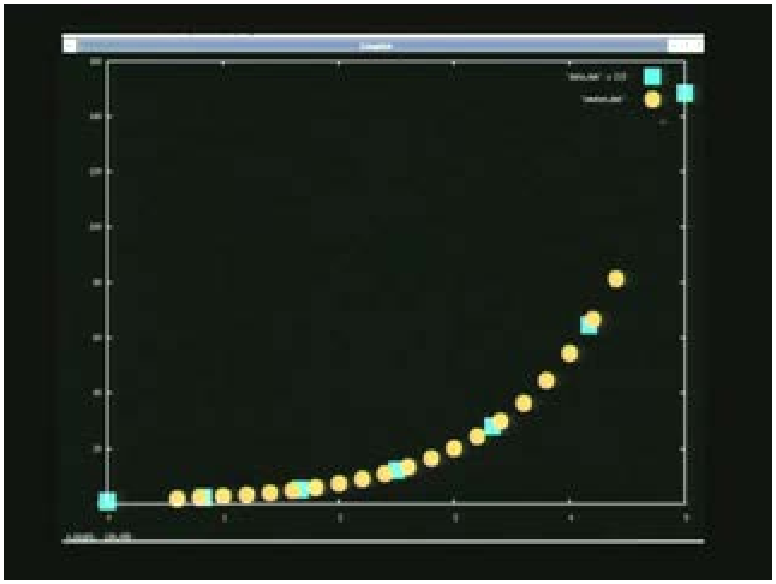

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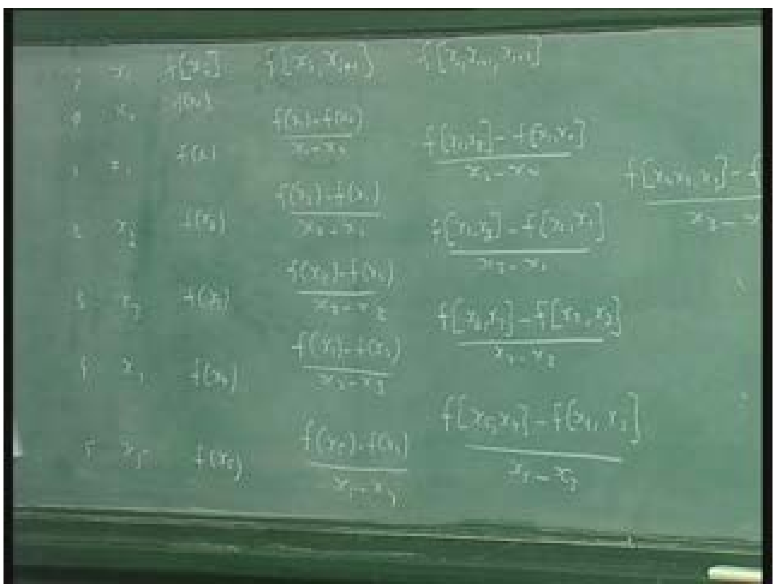

So we will see an implementation of that in a in a later class. Okay right now we can go into the difference formula which we have started discussing yesterday and then we look at how to construct now instead of a polynomial how to write down a difference formula for the for the derivative. That is the question was that if I want to evaluate the derivative of the polynomial.

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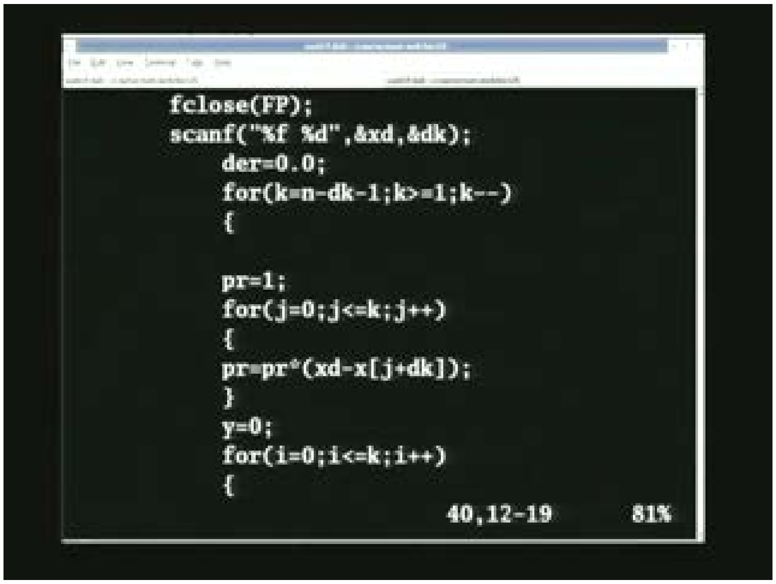

Okay if I want to evaluate the derivative of the function at any point in between the tabulated values then I need a polynomial, I need some smooth function. So as I said that you could have the derivative by approximating the set of discrete points by a smooth function and finding the derivative of that. So that is the method which we use which I want to evaluate the derivative at a point in between a set of tabulated values but if I want to actually evaluate the derivative at the tabulated value itself then I could I could construct that by writing down a discrete formula for the derivative and that is what we called the difference formula.

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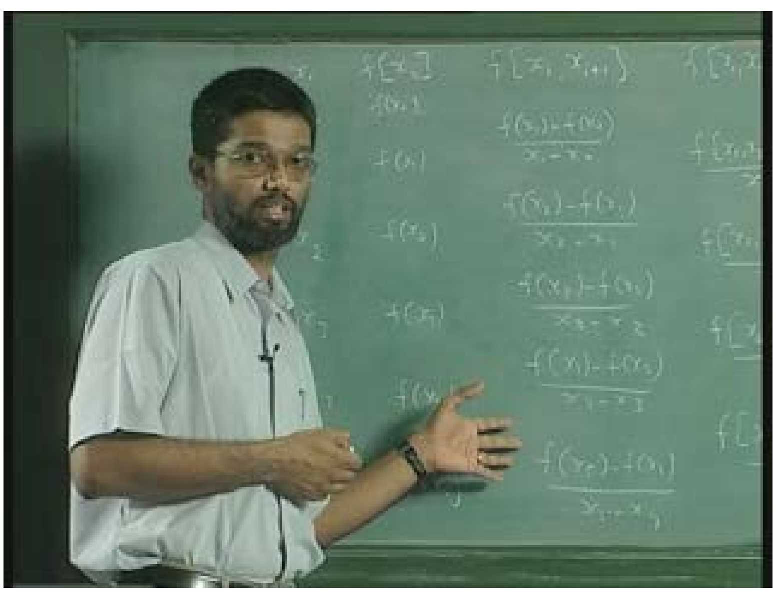

So writing the derivative as a difference formula, so derivative of a function as a difference formula and that is what we saw one thing which we saw yesterday. Okay we will continue with that and write down and see how to construct different formula difference formulas using a general scheme again based on the polynomial interpolation idea ok that is what we look at now again

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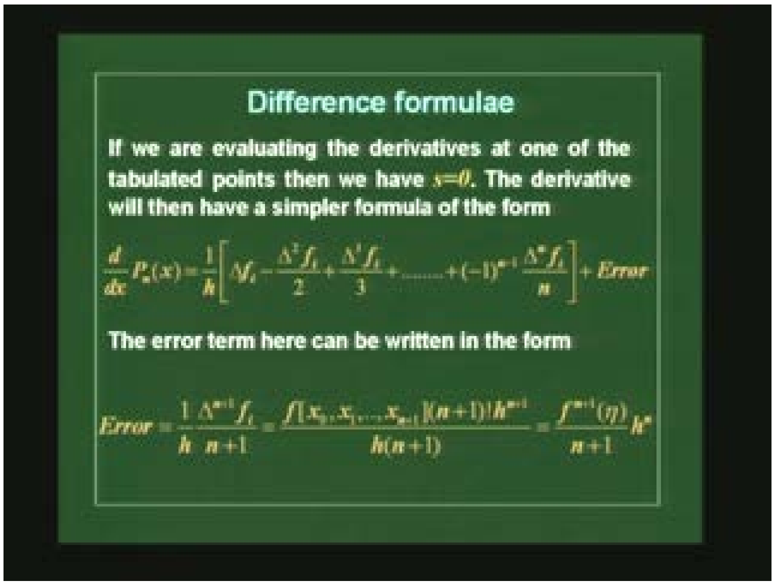

So as I said just now that if I want to write down the derivative of the function at one of the tabulated points and then we could enter the polynomial at a value, at the same value then construct the polynomial that is, if I want to evaluate the derivative of the function okay this this function and, let us say x_3 evaluate the function the derivative of the function at $x_3$then I could enter the polynomial here. Okay at this point and then construct and then look at what the derivative is, so we would do this in the case where the points are equidistant so when the points are equidistant we have seen that we do not need to write this difference formula in this fashion, okay we do not did this in the class last class that We can write them as just the differences can be now written a without just the differences not the divided differences and then we can construct this f from the what you called the deltas.

So we call this as now as delta and this is as del squared etc.., and we had an I here to tell you where in the polynomial which we are entering , okay this is del i_3 .So that is the, so all these things now becomes just del just the difference not divided difference. So we use these notations to distinguish between the difference and the divided difference. So now this just the difference of the functions and the value of the functions. So that will be this will be actually deltas and this will be this in the first order and this will be difference in the del 2 that will become del 3.

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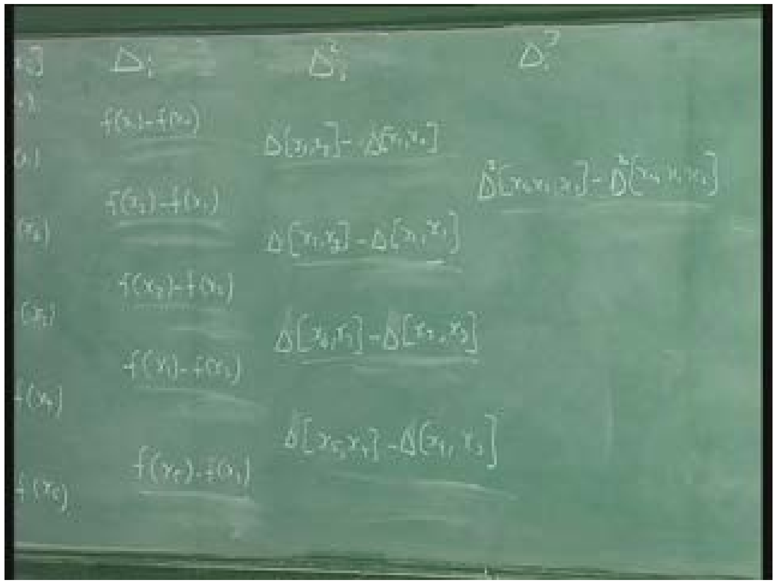

Okay and using this we could actually construct the polynomial and the polynomial we know that it would look like s into s minus 1. So we I do not have the polynomial here.

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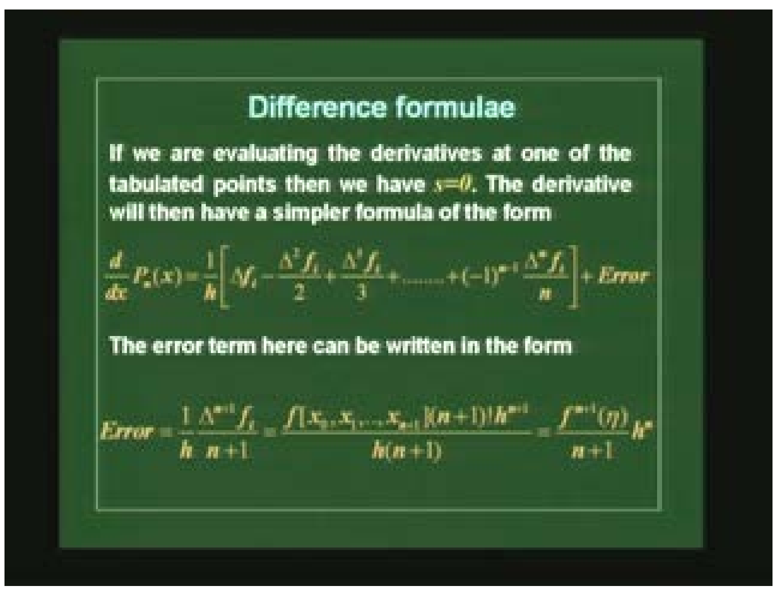

So we will have the polynomial now at the end of x at any point actually as we saw as f of x_0 plus and s into delta plus s into s minus 1 by 2 into del 2 etcetera. So we could write it the polynomial in this fashion .we have seen that earlier. So then we can construct from this polynomial the derivative of that with respect to x and we have seen the x now instead of using this s here.

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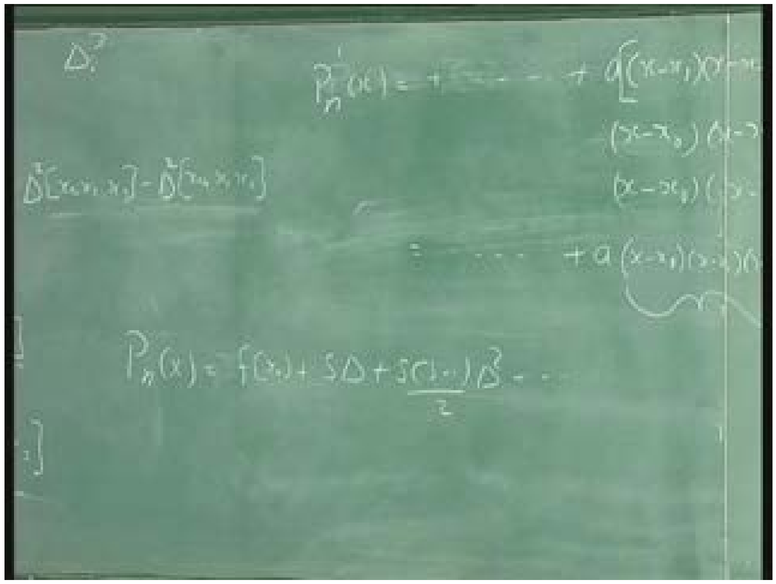

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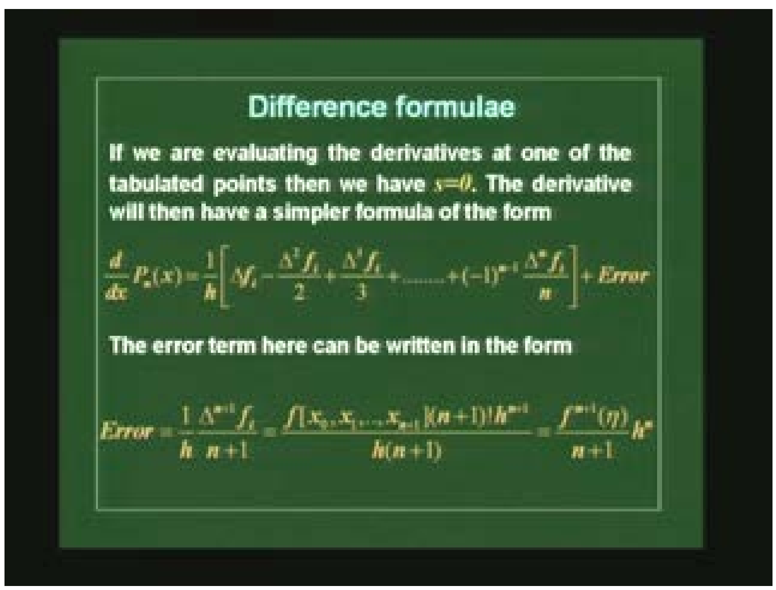

So again to remind you that s was x minus x_i divided by h. Now h equal to x_i plus 1 minus x_i difference between any 2 points which is the same. Because it is a equidistant set of points. x_i plus 1 minus x_i. So that is what we had.

Okay and now we have so we could write this thing owing things in this s way table and we know that the derivative of this with respect to x is, we wrote dp n of x by dx we wrote it as ds by dx. So we wrote it d by ds of p_n of x because as we can see this is written in terms of s into ds by dx and we know ds by dx is simply h_1 over h, so this becomes 1 over h into dp n by ds that is the formula I have written. So if you do that if you substitute that polynomial into that formula then we will get it in this fashion.

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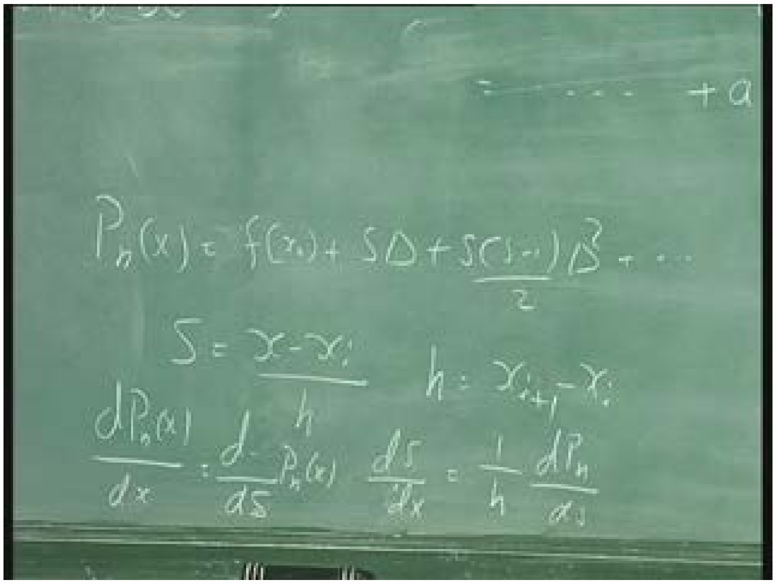

Right that is we get the first term would be this one and the second term would be the minus sign so that will be del squared by 2 etcetera if s equal to 0. So that is if I find the derivative by entering if a derivative at any point at any point by entering at that point okay.

So if I evaluate this derivative at some i value ok and I evaluate this s at that point starting from that point that is I start the difference table from that point. Okay I enter the polynomial at, not the polynomial the derivative the derivative by entering at the polynomial at the difference table at i itself, so I evaluate this at x_i okay by entering the polynomial at x_i, the difference table at x_i then it will become x_i minus x_i by h then it becomes 0.

okay and then my derivative of the first term would be delta the derivative of this with respect to s right, this first term would be delta and the second term will have only a minus one term. So minus 1 term minus delta square by 2 minus delta 2 by 2 etc.., that is what we have alternating plus and minus sign

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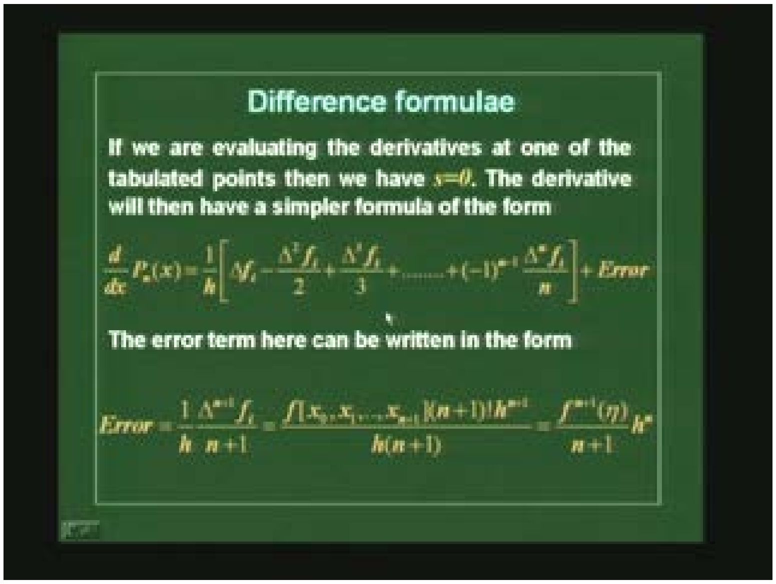

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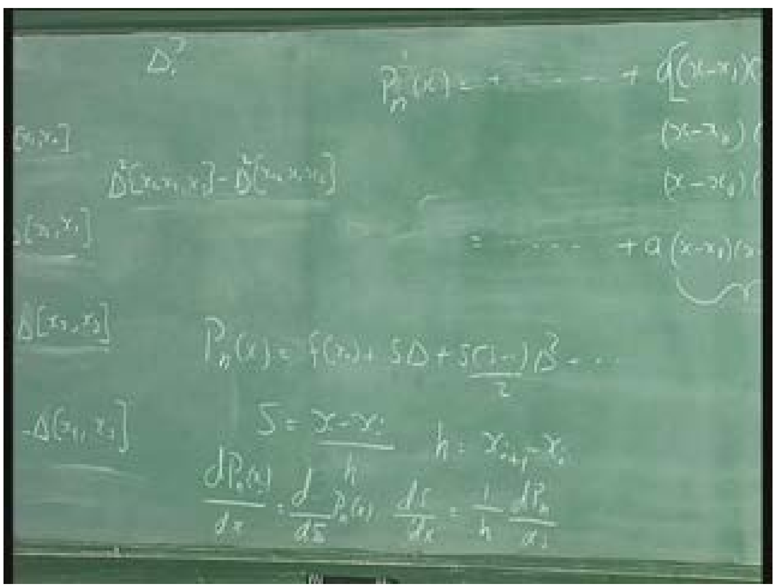

So this is what is summarized here. So that is the difference table which we had and then we went on to construct the higher order derivatives. So we said that okay now I can write it in 2 different forms that if I enter s equal to 0 and if look at only these 2 terms okay and then I have this as in this fashion that is del f_i and del 2 by 2, okay that is only 2 terms, okay and then I can get this by a difference formula from this now substituting for delta f_i and delta 2 f_i, I can get a difference formula of this form remember delta f_i is f_i plus 1 minus f_i and the delta 2 is the difference of the first order differences from the difference table.

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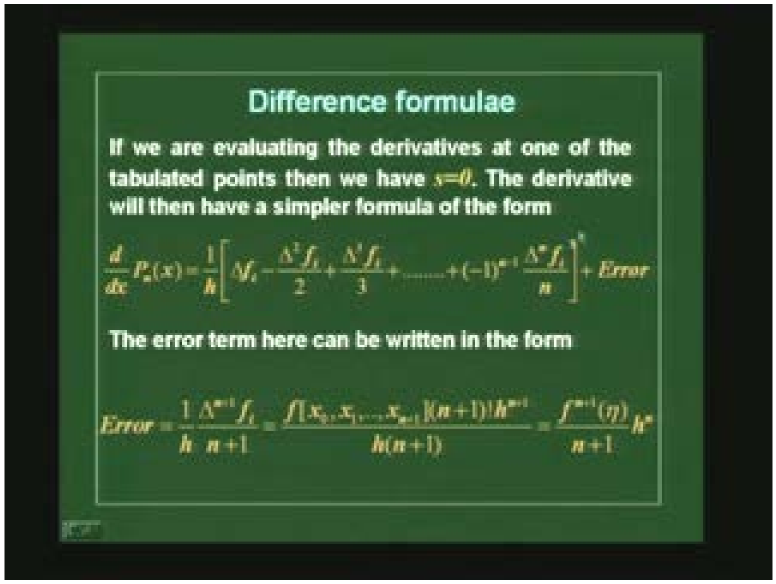

So then I can construct a function like that ok. So then if I can do better then, I can get a order of h squared accuracy in this difference formula using evaluating the function at three different points i plus 1, i and i plus 2. So we turns out that if I enter the polynomial at i minus 1 instead of i itself that is if I want to find the derivative at 3, I enter the difference table at i minus at 2 okay and then my s will become x_i, x will become here this is x_i minus x okay.

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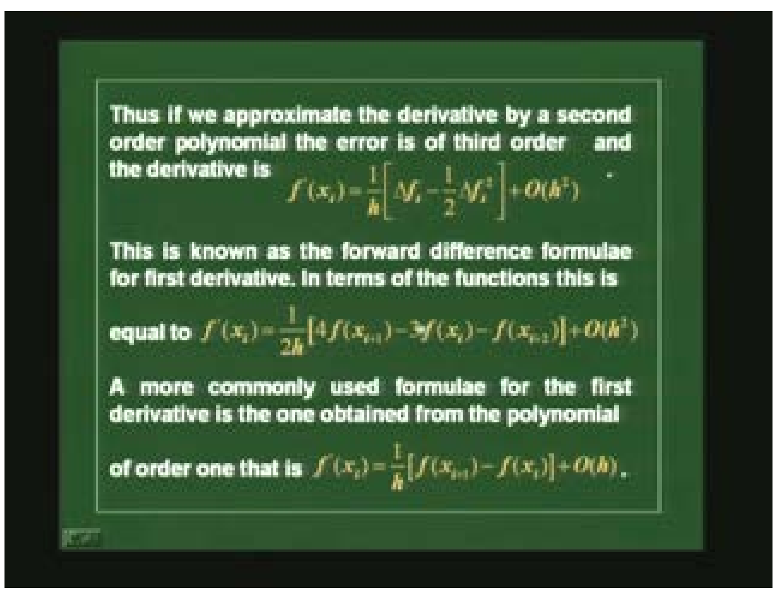

So if I that is correct if I enter the polynomial at x_i minus 1 okay and then this x_i, so if I then it will become s become x_i minus x_i plus 1 minus x_i by h, okay which is 1. Okay so I could instead of entering at 3, I could enter at one value earlier okay that is this will become i minus 1, I do not enter the polynomial at the difference table at i, I enter it i minus 1 and evaluate the derivative at i. then s is x_i minus x_i divided by h and that will become 1.

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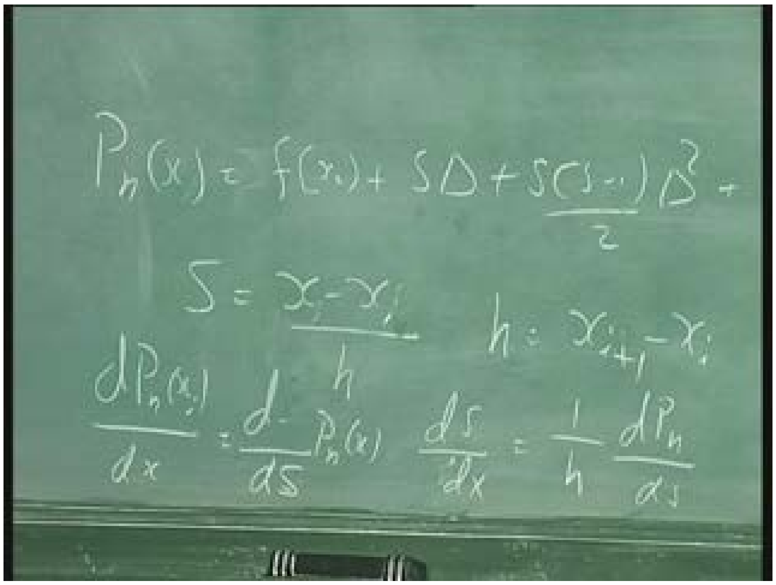

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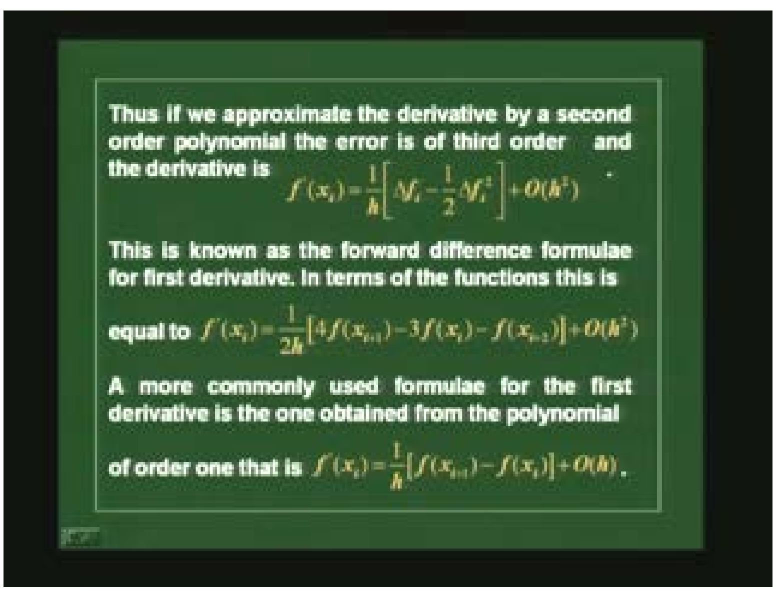

So s becomes 1 and the formula turns becomes even simpler for the same order of accuracy, you could get by entering it x_i minus 1 a difference formula of this form which is f_i plus 1 f_i minus 1 by 2 as a order of h squared that is called the earlier one was called this is this is what we called as forward difference formula and it is of second order accuracy forward difference formula of second order accuracy and here is a again formula of the derivative in terms in terms of the function values now at i plus 1 and i minus 1 which is now again the order h squared accuracy.

And this is called a central difference formula. So that is we have seen in this case. So now we are now converting the functions the derivatives a set of discrete values remember if we have to write down these formulas as we see we need to have the values of the function tabulated at equal intervals otherwise, this is inaccurate or this cannot be constructed. So that one has to remember just having a set of tabulated points and taking the difference is not enough, we need to have the function values tabulated at equal intervals to write the difference formula for that derivative and then we could construct higher order derivatives using the same scheme. Here is the general scheme for constructing the higher order difference.

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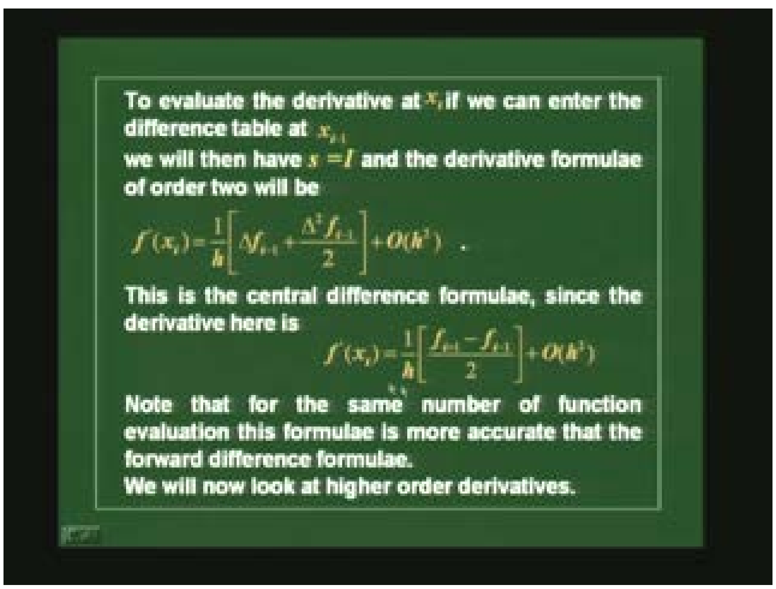

As we said that so we will write down the general formula, so to write down the divided difference, the difference formulas for higher order derivatives so what first we saw is the first order derivatives. So first order derivatives we saw that we can have a central difference formula or a forward difference formula either of them using a second order polynomial or first order polynomial. Okay we can have a forward difference formula or a central difference formula and then now we will look at higher order derivatives. Okay higher order derivatives can be obtained in the following form that each is write down a general operator for finding the derivate. So by constructing these variables this e and delta.

So delta we know f of x_i plus 1 minus f of x_i from the difference table we just saw that it is just f of x_i plus 1 minus f of x_i and then we have now, so what is called as stepping operator e ok which takes the function f of x and then gives a value of f of x plus h. Assume that this a stepping operator e, so our idea would be to relate e to delta. So delta is f of x_i plus 1 minus f of x_i remember delta f is, so delta acting on f is f of x_i plus 1minus f of x_i and e acting on f is taking f to f plus f into x plus h, f of x plus h or e f of x f of x_i is for equally for equally spaced data, e into f of x_i is equal to f of x_i plus 1 okay.

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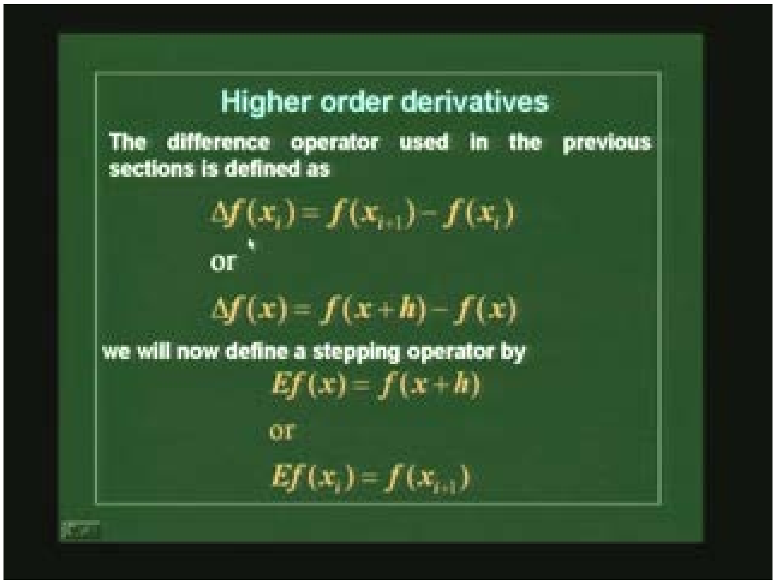

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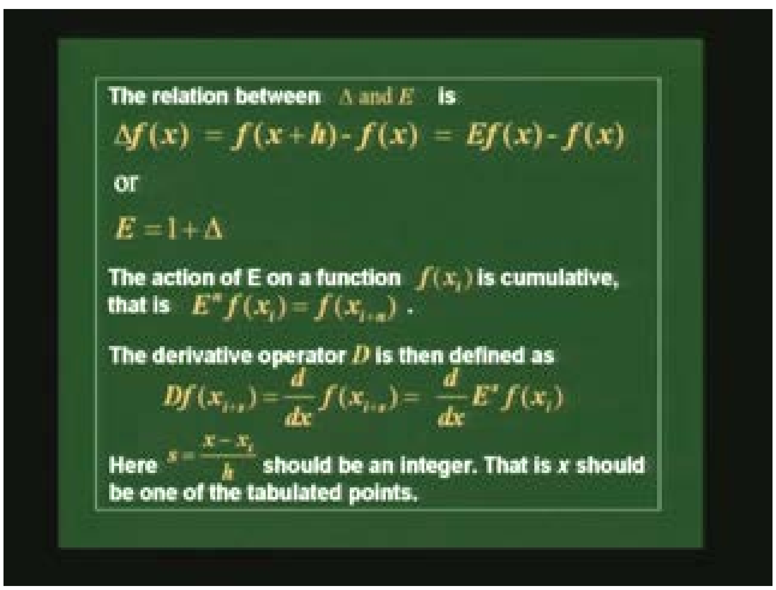

So now we can use that, so this two conditions that we just wrote down. So what we said was we had one function called delta f of x_i delta f of x_i which is f of x_i plus 1 minus f of x_i and then we said that we have something called e f of x_i. So e acting on f of x_i will give us f of x_i plus 1. Okay that is our equation remember and then I can now write delta f of x since f of x_i plus 1 is f of x plus h, delta f of x is basically f of x plus h minus f of x right. So that is what it is this is nothing but now I can substitute this here okay and then I can write delta f of x_i as f of x_i plus 1 is e into e times f of x_i minus f of x_i okay or now we have a relation that delta is equal to e minus one or e equal to 1 plus delta okay that is the relation which we get.

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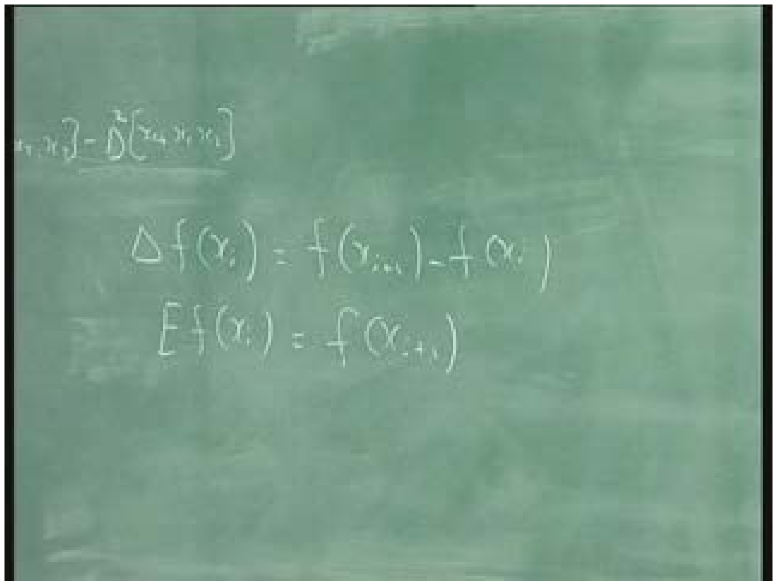

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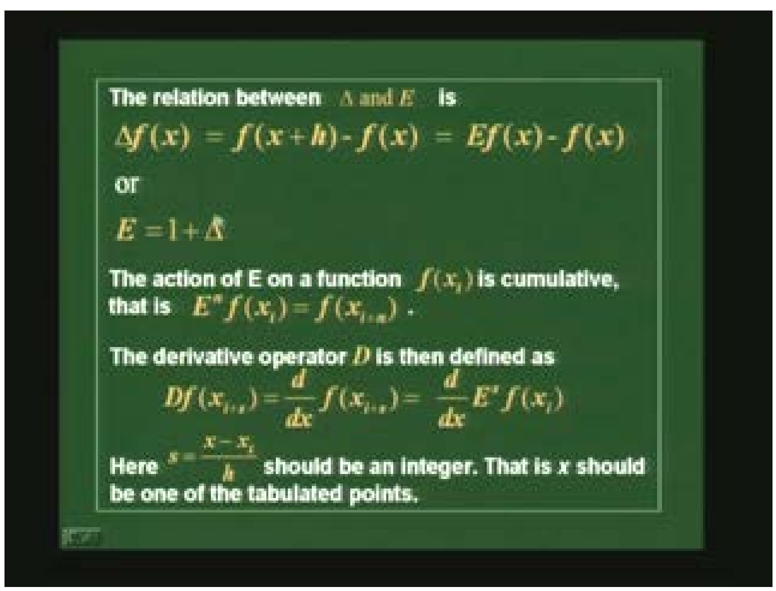

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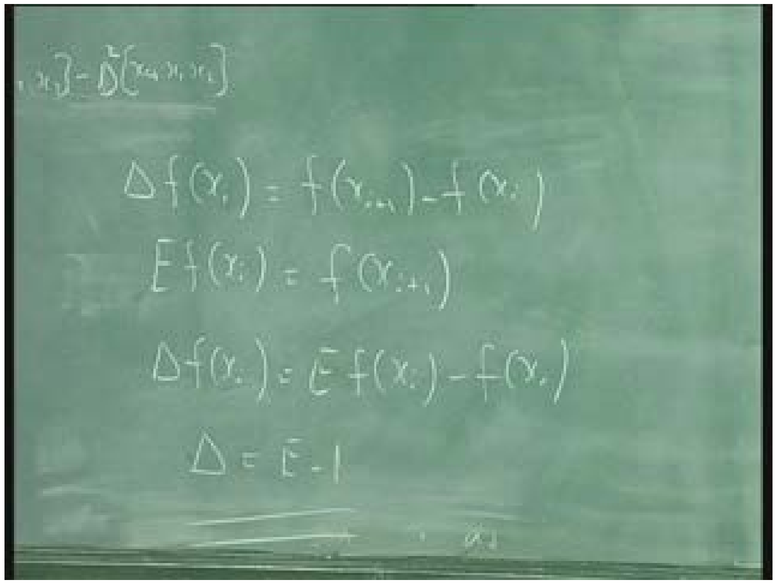

So we have a stepping operator e which is related to delta by this formula e equal to 1plus delta or delta is equal to e minus one ok that is what we are going to use that. So we have this stepping operator which is 1 plus delta. So we can use this cumulatively that if we use this stepping operator n times then f of x_i will go to f of x_i n, f of x_i plus n okay that is the that is the cumulative use of that okay and then we will use we will now we will define a derivative operator and that is what we are going to use to find all the higher order derivatives. So let us look at the first order derivative first ok derivative operator acting on any function f of x_i any function f at x_i plus s is simply the derivative d by dx of that function at i plus s

Okay now I know that f_i plus s is d by dx of f of f at x_i plus s is e to the power e to the power s that is e acting on s into f of x_i remember s, is remember s is an integer now in this case which is x minus x_i by h .Ok so again we have to enter this all the thing is valid even we enter the we want to evaluate the derivative at one of the tabulated points that is, this s has to be an integer if s has to be an integer then x has to be one of the tabulated values right. So that is the point. S is x minus x_i by h, h is the difference between two tabulated, nearby tabulated points, ok this has to be an integer that is x has to be one of the tabulated points. Okay then I can write it in this fashion, d derivative operator acting on f at i plus s is equal to derivative d by dx of the stepping operator acting s times on f of x_i.

Okay so now we have that, ok then we can now write it down in this is fashion I am repeating that again. Okay, so now I know d by dx can be replaced by ds into ds by dx and we now just know that ds by dx is now 1over h. Okay we just saw all these, so this one over h into d by ds of E^s f of x_i. Okay so this d by dx is been replaced by d by ds, okay since s is x minus x_i by h, okay d by dx I can write as d by ds into ds by dx and s is x minus x_i by h, so ds by dx is1 over h we just saw that okay

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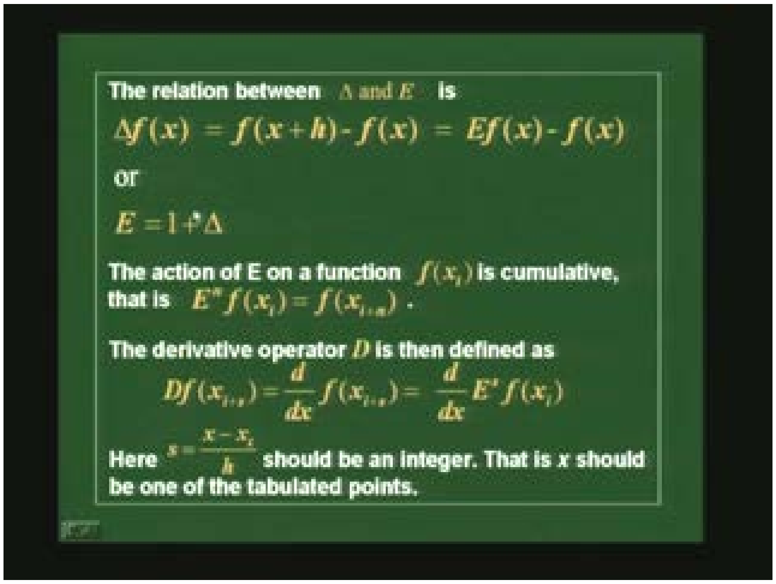

So now I can substitute all these into the formula to get a divided difference formula, okay, so that is the difference formula for the derivative so let us see how we will do that okay we use this following property that is the derivative of log of e to the power of s f of x_i. I can write this as d by ds of, so I can write the log of e to the power of s f of x_i as log of e to the power of s plus log f of x_i but f of x_i is just a number. Okay, so it is same as log e to the power of s right.

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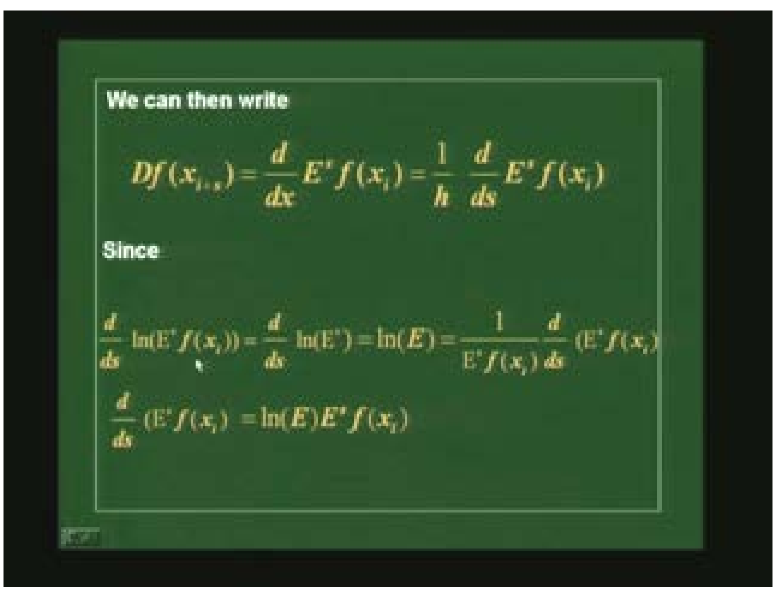

So what I am trying to say is, that I write d by ds of log e to the power of s into f of x_i okay which is same as d by ds of log e to the power of s plus log of f of x_i right, but this is a constant this is same as d by ds of log e to the power of s which is nothing but d by ds of s log e, okay that is equal to log e.

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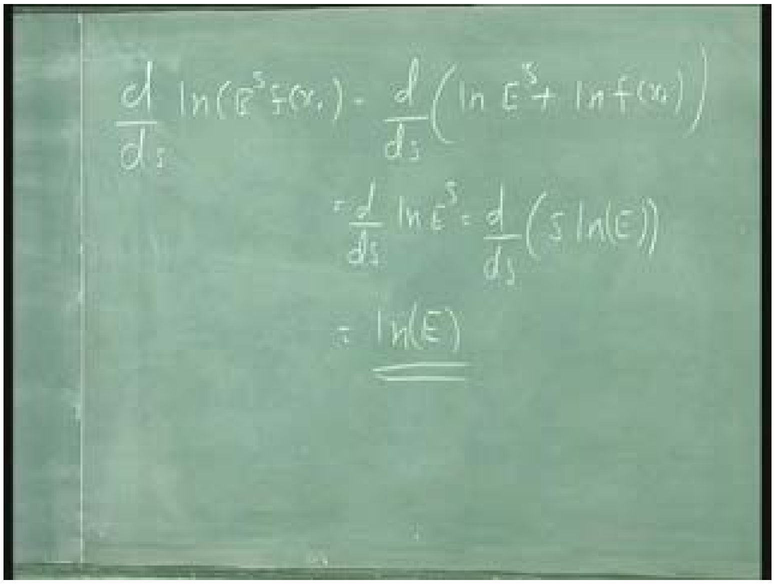

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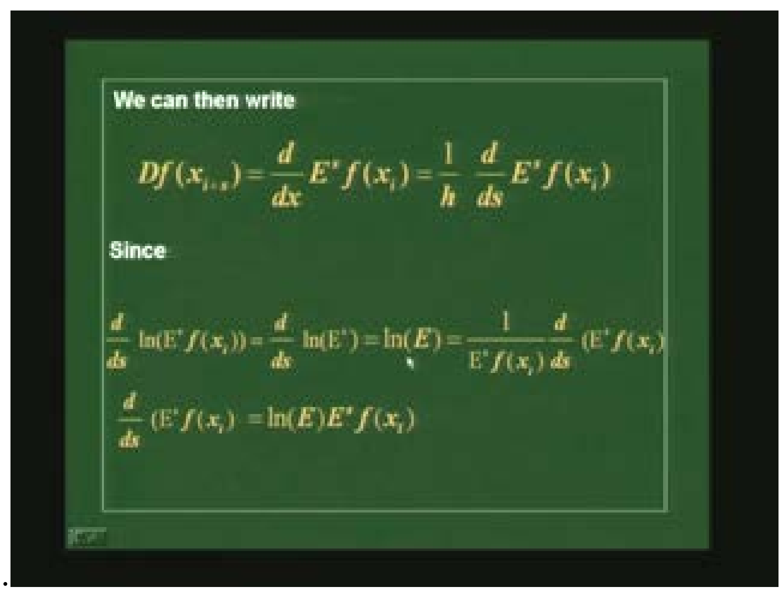

Okay that is what I get, okay so it is just log of e, okay d by ds of e to the power of s f of x_i is just log e so then but we if I just construct if I just only look at this part this is log of derivative of a log that is 1 by the argument multiplied by the derivative of that argument that is this 1 over e to the power of s f of x_i d by ds of that argument but d by ds of e to the power of f of x_i okay, has thus that is this part log e into e to e power of s f of x_i. So I am using this function. so I basically use the d by ds of log e to the power of s f of x_i is log e and I also write d by ds of log e to the power of s f of x_i as 1 over e to the power of s f of x_i d by ds of e to the power of f of x_i and then from this I am saying that d by ds of e to the power of s f of x_i is e to the power of s f of x_i into log e okay now I am not going to use this, then for s equal to 0 okay, now again we will do that, so we just said that d of f x_i right.

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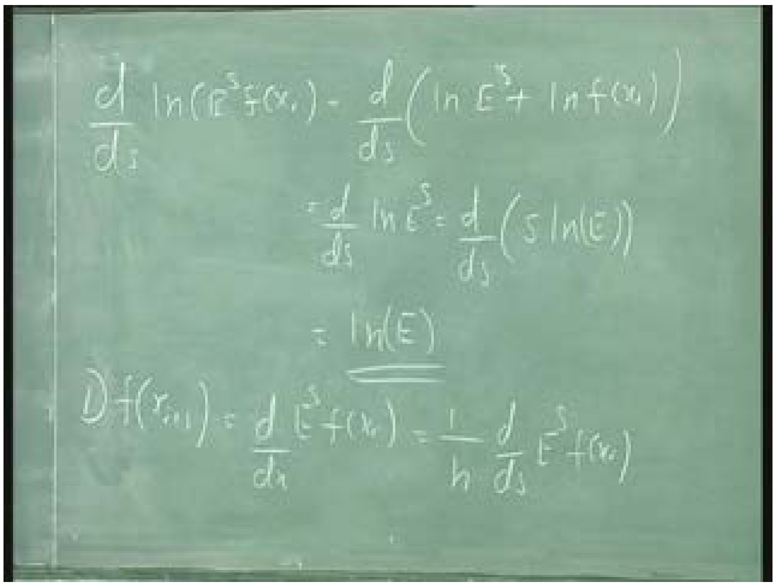

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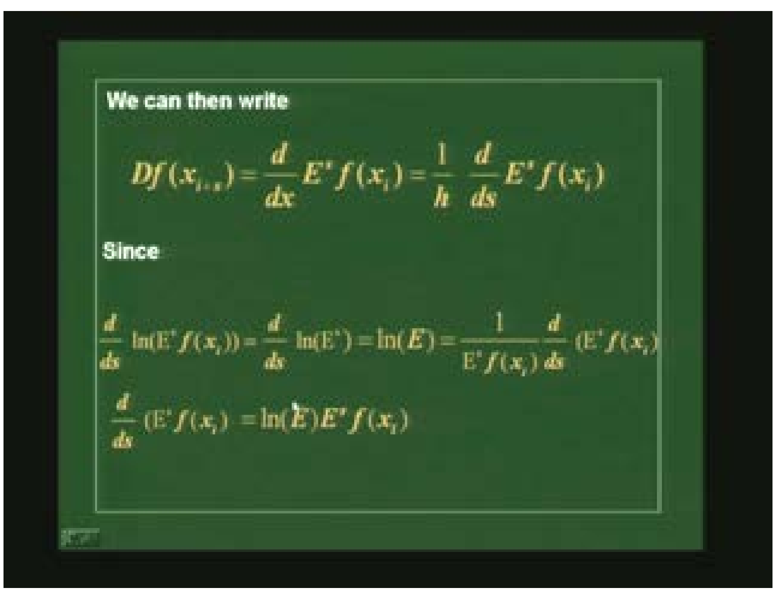

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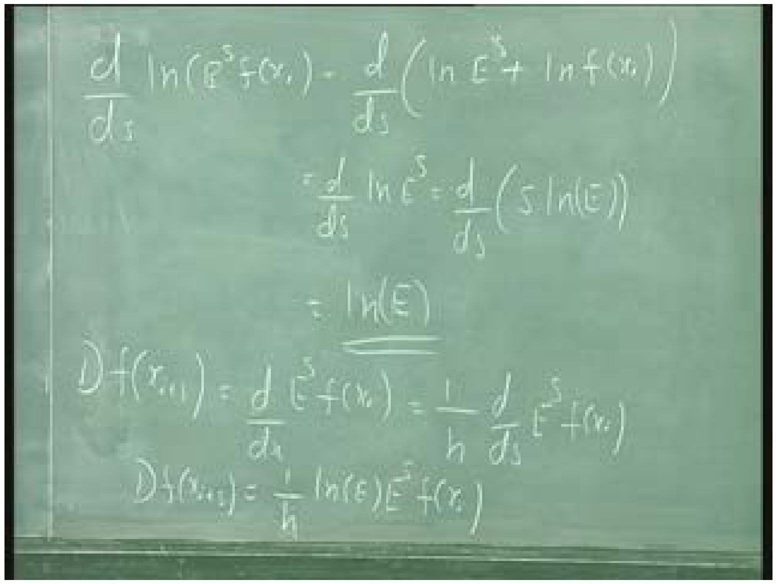

So d of f xi plus s, we said is 1 over h d by ds of e to the power of s f of x_i okay but we are saying d by ds of e to the power of s f of x_i is log e, e to the power of s f of x_i okay. So let us write this, okay so we saying that d by ds, d by dx of so what you want to write were d of f of x_i. So we are going to write d of f of x_i plus s. We will write that as d acting 1 to the power of s that is d by dx of e to the power of s f of x_i. Okay and that is we wrote that as 1 over h times d by ds of e to the power of s f of x_i, now we are seeing d by ds of e to the power of f of x_i is what we are going to replace it by this function here, that is log e, e to the power of s f of x_i.

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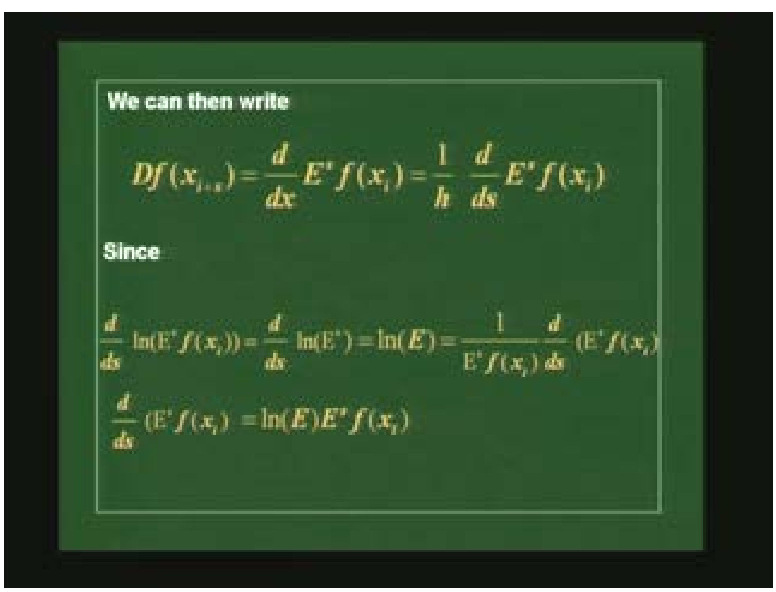

Okay so we are going to replace this by this as 1 over h as log of e, e to the power of s f of x_i okay that is d of f of xi, so d of f of x_i plus is 1 over h. So d of f of x_i plus s is 1 over h log of e, e to the power of s f of x_i. So for s equal to 0, the derivative of the function at x_i is equal to 1 over h log e times e into f of x_i. So log e into f of x_i, so d f of x_i is 1 over h log e into f of x_i or d from this I can write, so from this expression for s equal to 0, I can write d f of x_i as 1 over h, log e into f of x_i. Okay or the d as operator, we can write as 1 over h times log of e, that is the d operator is and we know e is 1 plus delta. So I can write it as 1 over h times log of 1 plus delta, so d acting on any function is 1 over h time log of log of 1 plus delta acting on that function at any value x_i.

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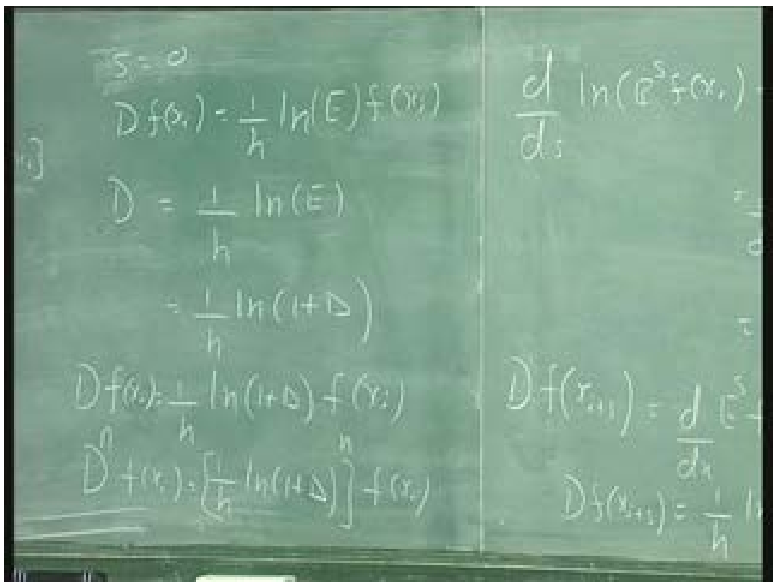

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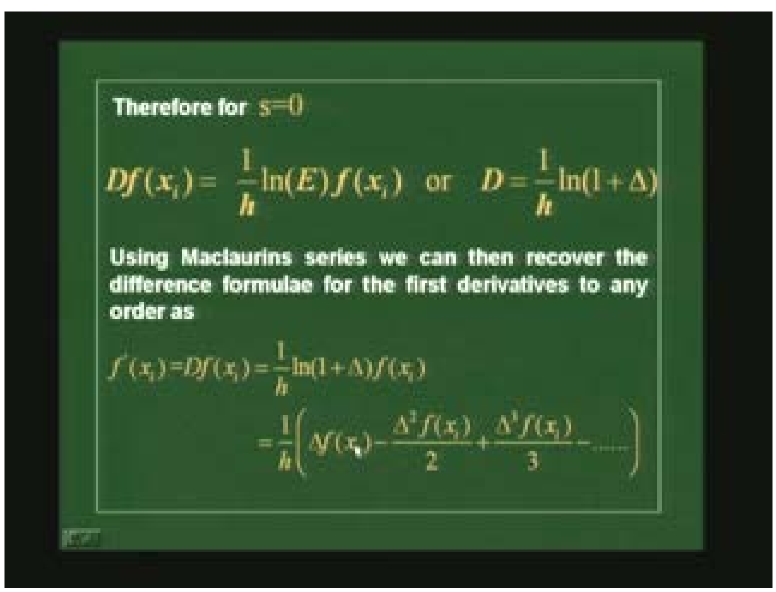

Okay that is the point which we want to write. Okay now, if I want a higher order derivative I simply have to write it as d to the power n f of x i and act this n time, that is over h time is log of 1 plus delta acting on to power n acting on f of x_i. Okay that is the formula ok and then I can expand log of 1 plus delta, Okay in a series, okay and I can write that I can write expand log of 1 plus delta for example, d of f x as is this as far we have just concluded okay but I can expand this log of 1 plus delta and keep it on to any accuracy in delta

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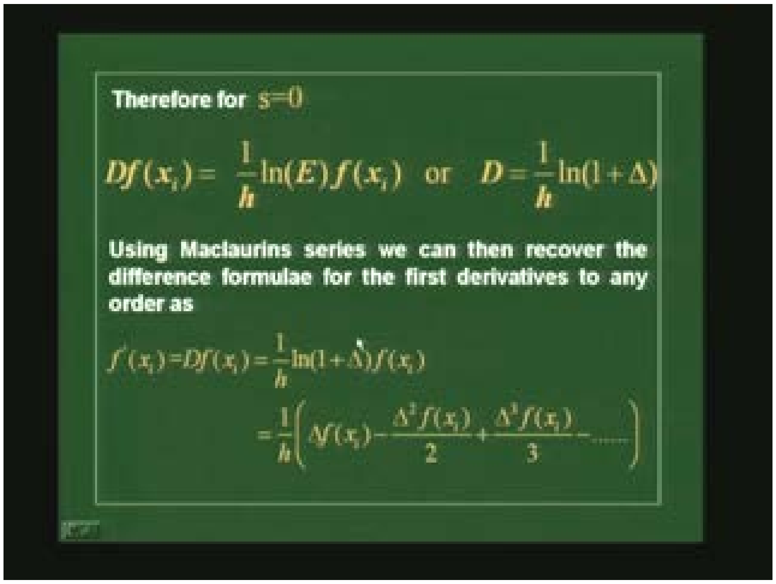

Okay I can use this difference table and I can expand this log of 1 plus delta okay and I keep any delta of any order I want, Okay that is the idea which is used to construct higher order derivatives using the difference table and then we can write down the difference formula for any order in the derivative consistently starting from this operator which is 1plus delta, d is plus 1 over h, 1 plus delta

Okay we stop here and then look at, in the next class, we look at the implementation of this in a program and then we start discussing integration and the integration of a set of discrete points.