Numerical Methods and Programming
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Lecture - 24
Systems of Non-Linear Equations

Today we will look at the system of solutions, for a system of non-linear equations in a little more detail but we will looking at equations of this form that is f_i of x_1, these are unknowns, so we will look at system of equations of this form that is f_1, f_2, x_1, x_2 etcetera up to f_n. So we will look at this in little bit more detail, how to solve a system of equations is non-linear equation of this form using Newton Raphson technique which we have already used for single equation, single variable case. So we will just look at how to do that in a case of system of equations.

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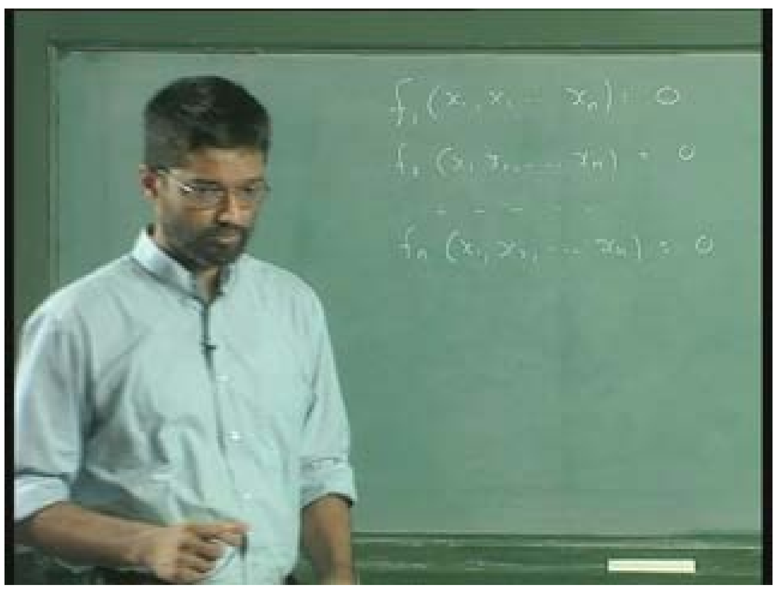

So the basic idea is the following that you have equations of this form n such equations for n unknowns and then we would tailor expand this function around set of guess values and then we have that guess values. So x_n guess values we would tailor expand this function around that guess value and then we have the tailor expansion now given by this that is f_i of x_1 plus h_1, f_i x_2 plus h_2 etcetera all the way to x_n plus h_n.

So h_1, h_2, h_3, h_n are the increments are in the variables x_1, x_2, x_3, x_n that is now given by f function evaluated at the guess value plus h_j into del f_i by del x_j equal to 0, that is this is the sum here, sum over j, so h_1 del f_i by del x_1 plus h_2 del f_i by del x_2 etcetera. So that is will be n such equations for each write of the f_i you will have such equations this f_i here too, okay that is end such equations.

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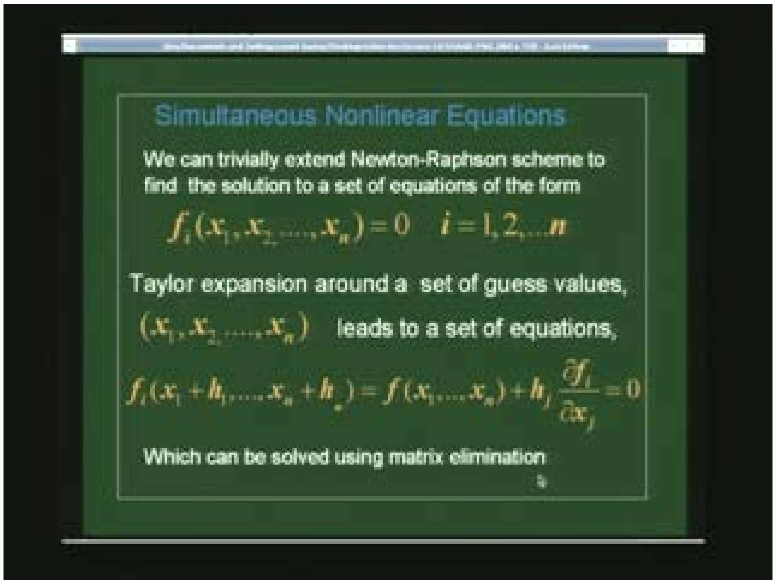

So in short, so we have the equation of this form that f_i of x_1, x_2, x_n equal to 0 for i going from 1 to n and then we have the guess values x $_i$ for again i going from 1to n and then we tailor expand this function that is f_i at x_1 plus h_1, x_2 plus h $_2$ to x_n plus h $_n$. We write that as f_i at x_1, x_2, x_1 plus h_1 in to del f_1 by del f_i by del h_1 plus h_2 into del f_2 by del f_i by del x_2. So all the way up to h_n into del f_i by del x_n right, this is for all i going from 1 to n. So that is what we have, so this is the, so now we have all the i from 1 to n this is the expansion we want to solve, so we obviously put the left hand side equal to 0 this term equal to 0 is what we are going to put and we are going to solve for this that is what we did for the one variable case, one variable case we have one equation and one term here.

So now we have a series of such equations, so but then we have series of such equations we can solve them by the metrics method we know that is the series of equation now we written as h one del f_1 by del x_1 plus h_2 del f_1 by del x_2 plus h n del f_1 by del x_n equal to minus f_1 of x_1 x_2 up to x $_n$ and the next would be h_1 del f_2 by del x_1 plus h_2 plus h_n del f_2 by del x_n equal to minus f_2 of x_1, x_2, x_n right. So we have the equations the n equations such like such that last one ending with minus f n of x_1 x_n that is the set of equation which we have to solve, so we have this set of equations to solve, okay so and we know all this coefficients are evaluated at x_1, x_2, $_3$, x_n that is our guess value right.

So then we have this derivative evaluated at that and then we have to solve for h_1, h_2, h_n etcetera. We have we know the right hand side of the equation because that is again the function evaluated at x_1, x_2 etcetera. So we will see the specific example where we use this okay the function may be two variable, so will be two equations and then we will solve that the equations and get the answers that is may be x_1 and x_2, okay that implementation we will see in a program here.

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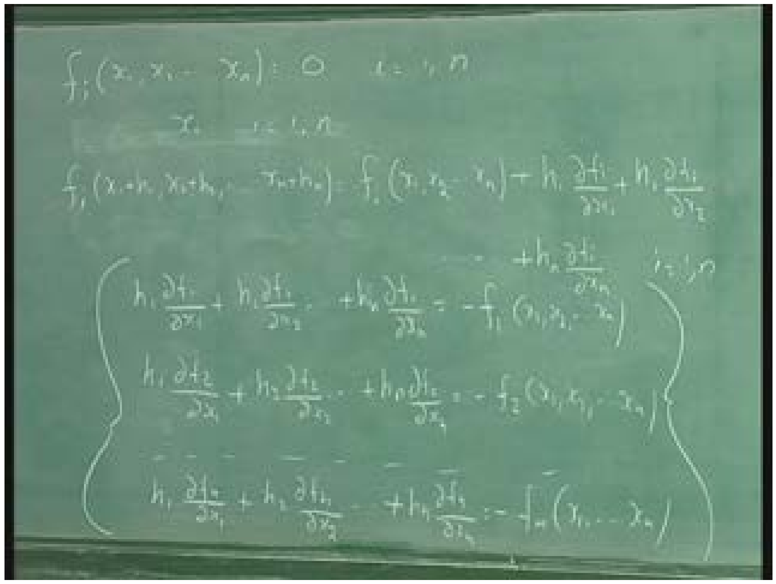

So the functions which will be solving out of this form, so we have 2 functions the first one let me called as f_1. So f_1 is the minus x square plus x square xy, x square plus xy minus 10, that is the one function and the other function f_2 which we will solve is x_1 minus y plus 3 xy square minus 57, the two functions of this form. So f_1 equal to x square by plus x y by minus 10 and the other one which we will look at is minus function value at x plus 3square y square minus 57 that is the function value, okay that is the function which we are going to solve. So the derivatives would be we have listed this derivative that is del f_1 by del x_1 del f_1 by del x_2, so $x_1$here is x and y_1 x_2 is y which we written as x plus y so we have kind out by del f by del x, okay so that f_1 by del x that is basically 2 x plus x and then we want to find del f_1 by del y which is x and then you have del f_2 by del x which is 3 y square and then we have del f_2 by del y which is 1 plus 6 x.

So given this, given this 4 quantities that is the derivatives and this function value derivative and this function value. So we can go ahead to solve this by Newton Raphson technique that is what the program going to look at okay so this program is uses the elimination technique, elimination, gauss elimination with your thing to solve this equation. Okay that is what we see here, so I have the main program in which now I have the solutions bit are to be given as an array, my guess value is also given as array. So x_1 x_2, so the mapping is x_1 is now x and x_2 is y on this thing which I have written here.

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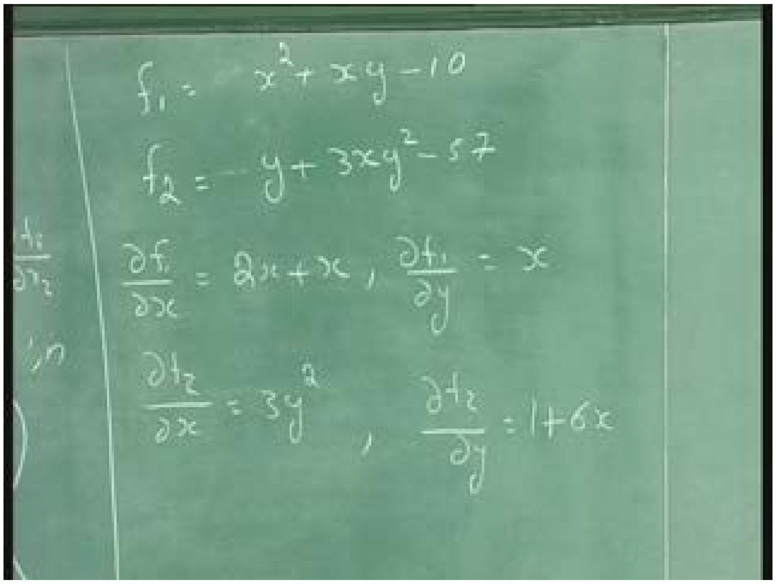

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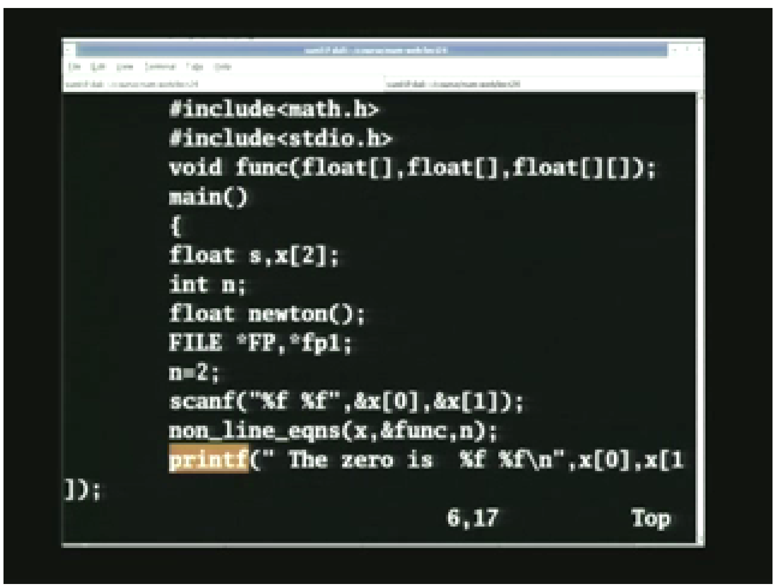

Okay, since we have more familiar with x and y instead of x_1 and x_2, okay that is the mapping so the array elements now solution now will contain now an array of x_1 and x_2 so my guess values. So x_1 and x_2, so this program requires the main program, so it calls its going to call a function called non-linear equation the solutions, so where we will pass the number of equation which we have number of unknowns that is the order of this, the method this array one dimensional array x that is the this case 20 and one that is 2. So n equal to 2 what we are going to pass and then we will pass a pointer to the function func here is called okay which is which written this function something which we provide which returns the function value and it is derivative the derivatives at all wherever while we will ask for in the x.

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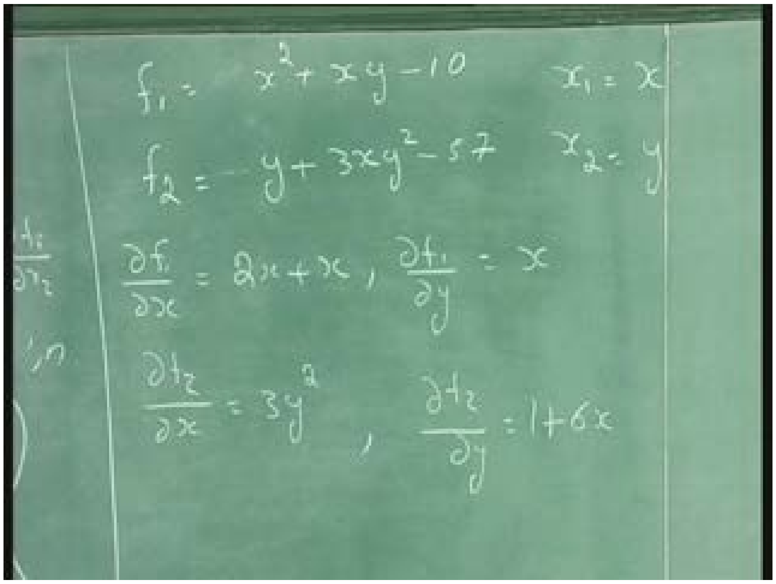

So then, so this a main program basically just reads of the guess values from the screen and then calls this function to which it passes those guess values it also pass the pointer to that function and the order the number of equations or the dimension of array x that also been passed and it gets the solutions back in x this print it out x_0 and x_1.

So the solutions comes back where we give x as guess value solutions come back as a_x here and sense repeat this again, since we have the passing the function here because we declared outside the main okay and we can see the arguments here this function takes in are 3,2 one dimensional array is and 2 dimensional array. So the 2 dimensional array is the first derivative of all the functions with respect to all the variables that is the 2 dimensional array and this is the solution of the of the x value x_1 x_2 that x array and this is were function values return that is, that is the program is the main program.

So then we just called that function that is that is the function non-linear equation function and function of course takes the x as an array taken here and this is our function pointer here now called function as I said three arguments 2, 1 dimensional array and 2 dimensional array and m is the integer which is the order. So I used m to dimension my metrics here inside this program okay this program require general you can solve it easily extend this to m_x extend this to m to a dimensional object I am solving this whatever we have solving here this is the specific example of m equal to two but this is this program is general you can take you can pass in here I am should be able to here.

Okay, so now this is the array element some working array is which we have declared here this is the 2 dimensional array and one Dimensional array 2, 1, 3 dimensional array this is something which we require to solve the gauss elimination, solve the non-linear equation using gauss elimination okay. So we start with some error defined as the 1, this set equal to 1 okay and now remember m is the order of all metrics here and then we have we just print out all the metrics just to show how the metrics are evolving, okay we called the function here and pass the x value now here one more point, we have pass the function here we called the function here. We passed the guess value the first round, the first round before the element before the iteration start, we call for the function and that function, that is the guess values and this is where the function value this derivative will return and this d actually return negative of the function values, okay so my d in the program is written as d of i is written as minus f_i.

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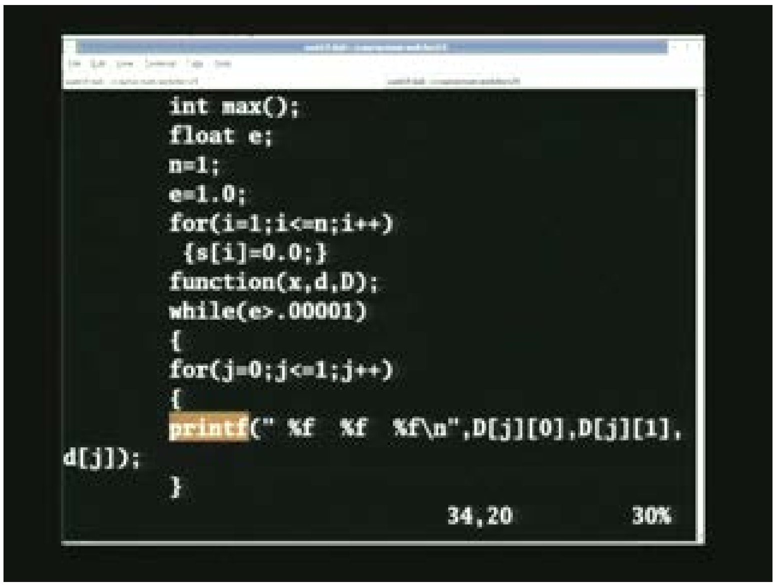

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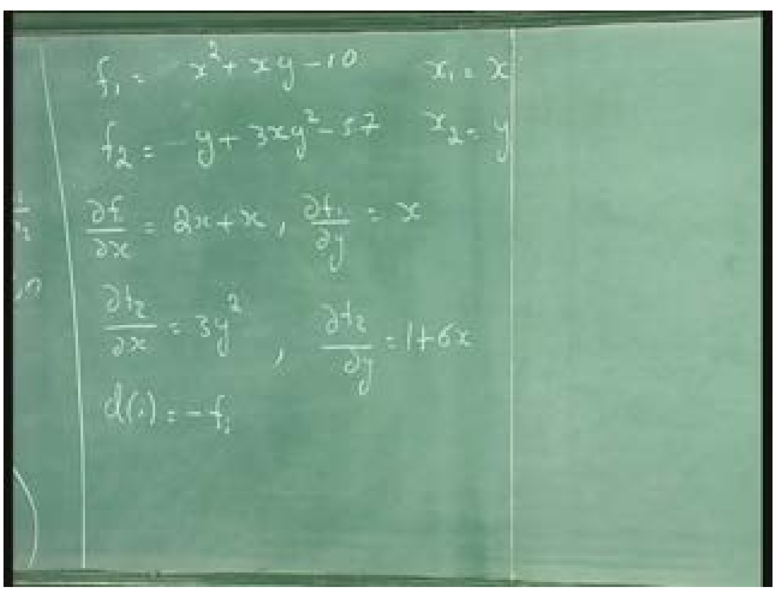

Okay because that is the right hand side of the metrics equation that is the right hand side of the equation minus the function value this is gauss elimination right hand side of the equation so that is this will be of minus of that D will get the second derivatives, that is the simply the program and then we will start our iteration loop here. Okay starting with we will eliminate, use the gauss elimination process to get the solutions the gauss elimination process we will know, we have done it in earlier classes, okay this is with pivoting okay that is the part program.

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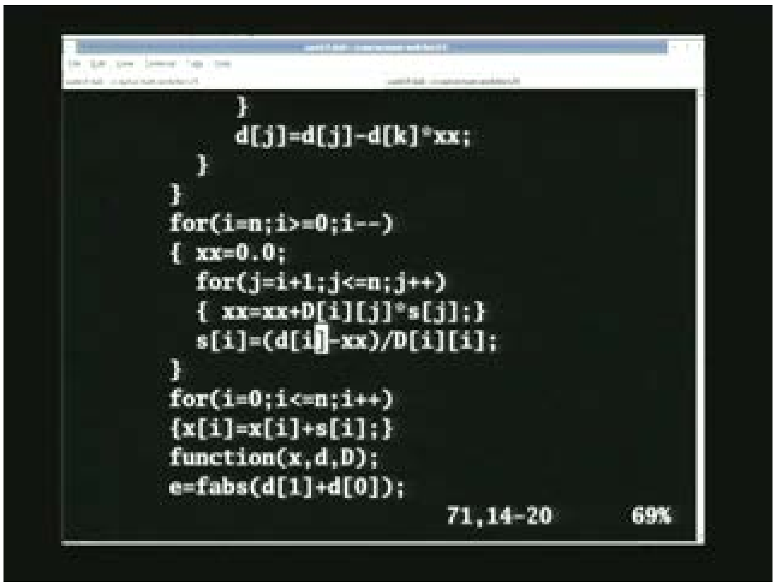

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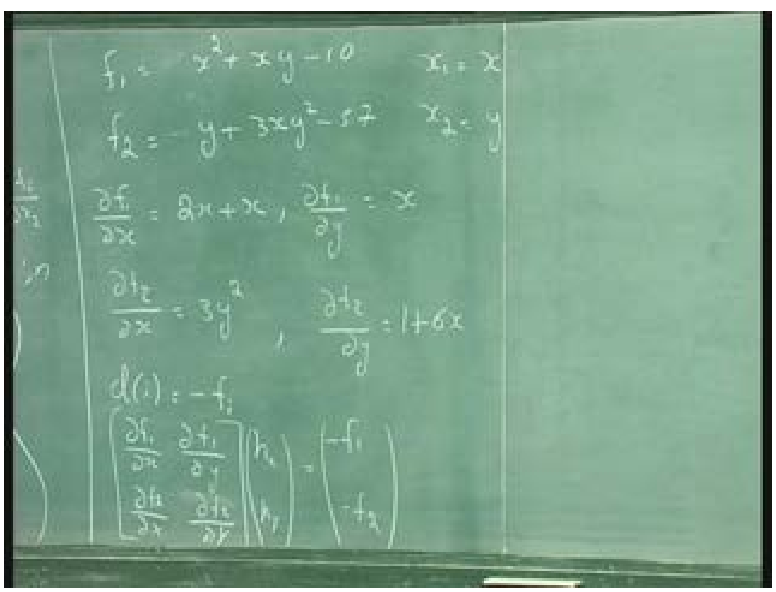

So once we come here we have solution here and then we call the solution basically s is the solution of this equation okay so we write now metrics equation of this form, so that is del f_1 by del x and del f_1 by del y del f_1 by del f_2 by del x and del f_2 by, okay that is our metrics equation. So that into we will write h when we write h x and h y instead of h 1 and h_2 we will call it as h_x and h_y that is equal to minus f $_1$and minus f_2 right that is equation we are going to solve once we get the solution of this equation would be h_x and h y okay which I called as s here. So this whole program here is the gauss elimination with pivoting that is what it is doing.

Okay from here onwards gauss elimination with pivot okay and so once we come here we have the solution s_i we have the solution. Okay once we have the solution we have to update our x values okay now that is done here, so since x_i goes to x_i plus h_i okay h is now is s here okay, so now we have the guess the new value okay and then we will evaluate the function and then we will call the function derivatives here again with the new x value and declare the error here is d of 1 plus d of 0 that should be 0 the function I am just using the function here value itself here. We need not use this as the error as we now we should actually used x of i earlier of the x of i minus x of i new divided by x of i new the error value. So I am just using simply for the convenience here as the function value but this function value has the error of course we were looking for the function the points at the function goes to 0.

So I can use the function value as error but as we seen earlier this problem when function goes to 0 very slowly that is the derivatives near the 0 is very small this does not very accurate but this is just easy way of doing it that we could also define as x old minus x new divided by x new some lower all x in this particular cases okay then i just print out the values here error which we get in this printf function okay that Is the program okay now this is to for pivoting and here is the function so the function value evaluated here this is our func function, so it gets the x value and it gets the returns the function value negative of the function value and it derivatives into 2 dimensional array.

So this program has to be supplied depending upon what function this set of equations you want to solve and it properly diminished, okay here is the only two variables we just put everything as two dimensional arrays if there are n variables and it could be n arrays. Okay now we have the function value here is minus plus of x square plus xy minus 10 and other f_2 is x plus 3 xy square power function xy square minus 57.

So then the derivatives here I have written the derivatives here which i have written on the board so that 2 x plus x_2 x plus y first derivative of derivative of the derivative function y of 0 with x of 0 and the derivative of the function y of 0 of with x of 1 and derivative of function y of 1 with respect to x_0 and the derivative of the function y_1 with respect to x_1 that is where the derivatives are.

Okay now we run this program and see what we get. Okay we run this, so now it is waiting for two guess values. So we give 2 values as 1 and 2, okay and the program converges pretty fast and then goes to solution as 2 and 3 that is the solution of this equation, we see that function value goes to 0 at this point printing out function value here lets look at what we are printing out. So we are printing out the x_0 and x_1 values. So that and the function values okay now I put in the minus sign here we will actually printout the function value this is the error were the functions goes to 0 the 0 also will go to 0 because it just sum of the 2. So the function value goes to 0 at this point.

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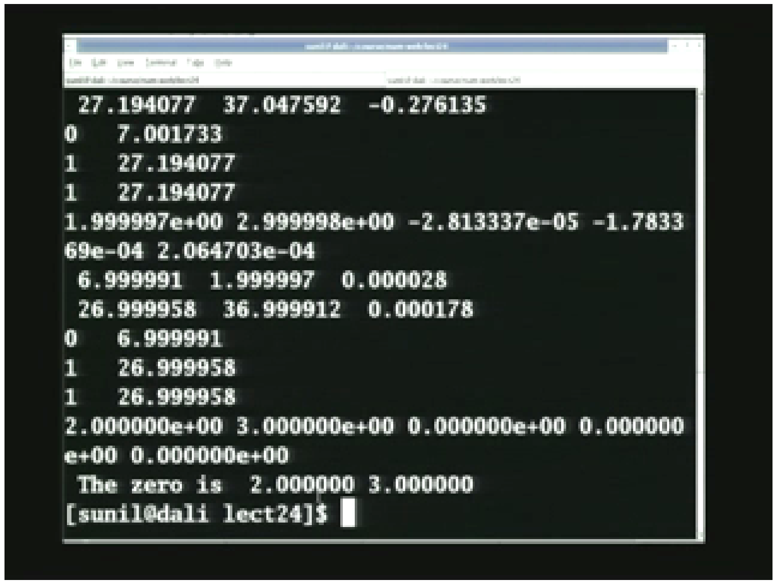

So now after this we will go to the next topic of the of this course, so far we have looked at in this particular section, we have looked at solution of systems of non linear equations we have use them first started with the system of variable one variable equations that is we looked at the equations of the this form.

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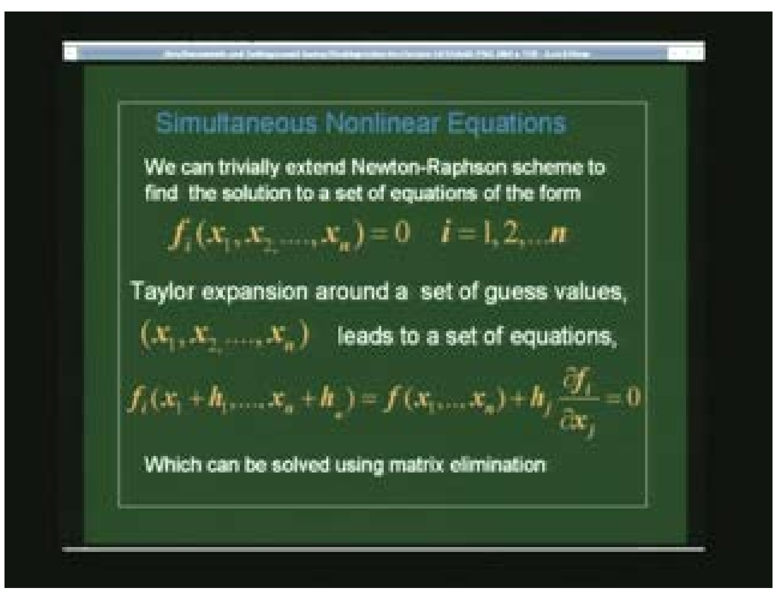

So we will looked at the equations of the form f of x equal to 01 variable equations okay and then we found many methods one method which we looked at was bracketing methods that is, so we set you know mean or the midpoint method we called it we call midpoint method midpoint method is something which we looked at to find the solution of this. So this method had two guess values on either side of the 0 okay and then we looked at after this we looked at false position method false position method that also require that we have guess values either side of the 0 that is we have guess values x_1 and x_0 and x_1 and such that f of x_0 f of x is greater than 0 and f of x_1 is less than 0.

So for the both methods we needed that okay that is methods which needed two guess values on either side of the 0 and then we looked at methods which does not need that for example, we looked at fixed point iteration, fixed point, this iteration scheme we wrote the function f of x minus as g of x_i minus x equal to 0 and then we use the iteration scheme as x_i plus 1 equal to g of x_i that is fixed point iteration and then we looked at what is the most popular method that is Newton Raphson.

Okay, so we Newton Raphson scheme in which we need derivative right, so in this in this particular case we need the derivative of the function we wrote iteration scheme as x_i plus 1 equal to x_i minus f of x_i divided by f prime of x_i and then we replace this derivative by its different scheme by using different scheme that is we called secant method. Okay in the secant method we simply replace the derivative by f of x_i by f prime of x_i we replace this saying by f of x_i f of x_i plus 1 divided by x_i minus x_i minus 1.

So again then this require two guess values for the secant method but we do not need on to be on the either side of it 0 these are the methods we are looked at in the used to solve equations of this form and then we generalize that into a system of equation of this form and using the Newton Raphson method. So this can be this particular a method which we also generalize into a system of equations that is the summary of solutions to the non-linear equations which we have looked at.

So the next topic which will be looking at numerical differentiation, so as we saw in this particular method itself some cases we need to find derivative of functions. So in the case where the function is what we want to find is set of data points, okay so we want to find we do not have a functional form, okay we only have a set of data of points and then we still want to find for example, where the where it actually cross the 0 etcetera, what is the x value at which it cross the 0 and then we need approximate scheme for finding derivatives that is one of the reason why we went to this method in this particular in this case of for systems of non-linear equations the solution to non-linear equations.

So the many cases like this in engineering and science were we need to find derivatives of function, derivatives of a function derivative case of a line which passes through a set of points. So approximate scheme to find derivatives, so this function value could be written by another program okay in that case we do not know what the derivative of the function is but then we should find that using some kind approximate numerical scheme. So that is what we have to discuss now and after we discuss the differentiation methods we will go in to the integration schemes.

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Okay we summarizes the when we need to find to calculate the derivative if you just give data points that is where most useful this method which we are going to look at most useful when in this kind of particular case where we have only a set of data points given and we want to find the derivative many of the optimization scheme which we would be interested in looking at would need derivative of the function evaluated and then we need to find that some method evaluate that and some times even for solving differentiation equations we need to write down this credit formula for derivatives.

We may want to convert for example, if your partial differentiation equation we will look at the cases later, the partial differentiation we will again have to convert them into a set of discrete equations and then again we need to write down the derivatives in terms of some set of discrete in a in a discrete using discrete formula. So there are many phases were actually this important, so here we will look at some methods calculating this thing the first method which we looked at that using interpolation, okay because we already learned how to interpolate a curve between a given a set of data points. So we will look at that now.

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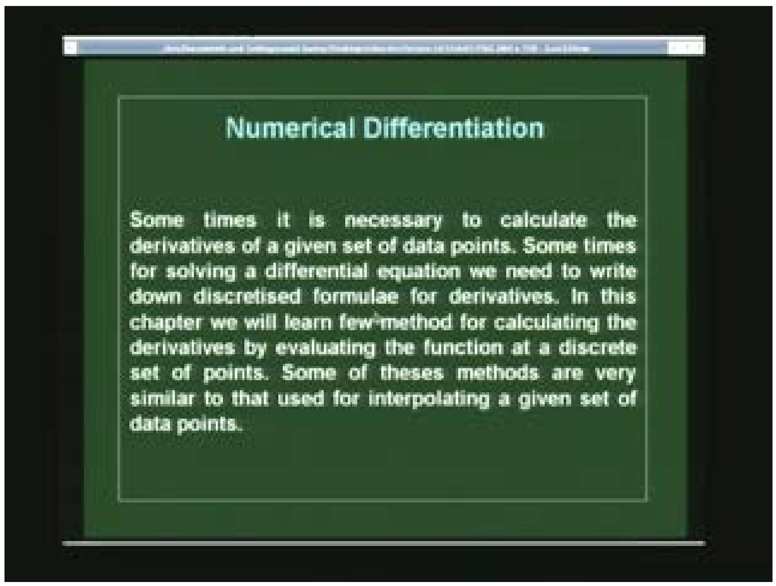

So let us say we have is set of points, so we have some function f of x here verses x and we had a set of data of points. So the set of data of points we were asked to find the derivative of this function at any point here, I may intercept the derivative at this point. So that is this x value, let we call them as some x_i. I want to find the derivative of this line which passes through this point okay at this x_i. So what I have is only a set of data of points, I am interested in finding the derivative. So the question is how, so one method is that we know how to get a curve, a curve is going through this point by the interpolation scheme right. We know how to get a curve using polynomial interpolation; we can actually get an equation for a curve passes through all points.

So once we have the equation for that kind of polynomial. So let me called them as n th order polynomial p_n of x which passes through all the points that is this function polynomial takes p_n of x_i, f of x_i. So this polynomial passes through all this points and then I can simply find the derivative of this polynomial so my f prime the derivative of this line at any point between the derivatives would be just derivative of this polynomial evaluated at that point. So that is one advantage of getting a polynomial like that we know how to write down this polynomial for example using newton's form, we know this can written as f of x_0 plus remember there is a function called x_0, x_1 minus x_0 plus f as x minus x_0 into up to x minus x_n minus 1. So this something which we have seen before okay so this we have set of data of points I can write down a polynomial which goes through all this points this in this fashion in this Newton form.

So these are the coefficients which we can evaluate, okay all terms in square bracket okay the coefficients of this polynomial okay and that is related to this thing can give as f square bracket x_0 and f square bracket x_0 x_1 coefficients which are relate to the function value at x_0 and $x_1$and we can evaluate this using different stable we have seen this earlier. So we will just write down that expression again just you remind you of what we have learn in the polynomial interpolation.

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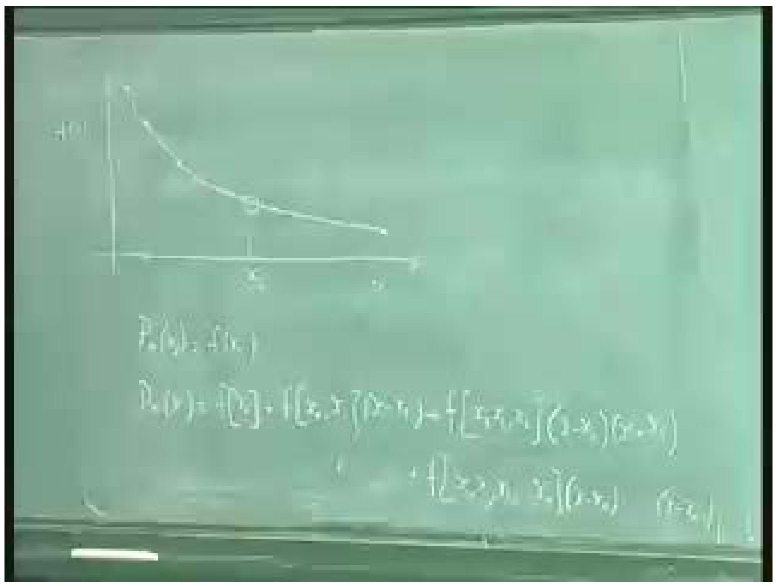

So for example f of x_0 is same as f of x_0 and f at x_0 x_1 that is the second the coefficient of first order term here is f of x_1 minus f of x $_0$ divided by x_1 minus x_0 and the coefficients of this third order this term was that is f at x_0, x_1, x_2 you remember is f of x_0, x_1, x_2 minus f x $_0$ x $_1$ divided by x_2 minus x_1 etcetera. So we constructed a different kind of stable and then as a kind of recursion thing we had a regression relation from which using which we could compute this 3 third order terms from second order term and fourth order terms from third order term etcetera.

So here at x_0 x_1 is this is x_1x_2 will be similarly, f x_2 minus f x_1 divided by x_2 minus x_1 x. So we know how to construct this polynomial in short we really know how to construct this polynomial and once we know the polynomial and then I can find out the derivative of this polynomial and then i can evaluate that polynomial at any point so the derivative of this polynomial simply would be p_n of x at any point x the derivative of this polynomial as we see this another polynomial with x_0 x_1 plus. So up to, we will have f of x_0 x_n and then we will have here is sum of all the terms. So notice the second term the derivative will have two terms here right one is x minus x $_1$ and another is x minus x $_0$, we have two terms here x, x_0, x minus x_0, x minus x_1 okay then this is the similarly last term have sum of i going from 0 to n minus 1and j going from 1 to n minus 1,0 to n minus 1and then we will have x minus this is phi, x minus x_j divided by x minus x_i okay that is the term which we will have. So I will summarize this again here, so you look at this. So we have the polynomial which goes through all the points given by this right.

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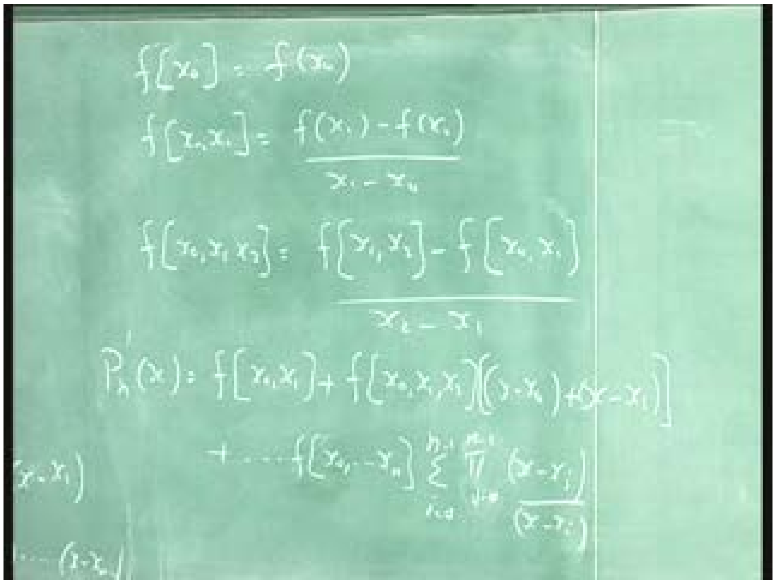

So we have the nth order polynomial given as f of x_0 because the first coefficient is just the function value evaluated at x_0 and then these square brackets comes as I told you related to the function value as f of x_0 minus f of x_1 divide by x_0 minus x_1 etcetera, and the last term would be just a product of all the terms up to n minus 1 that is the nth order polynomial which we have here.

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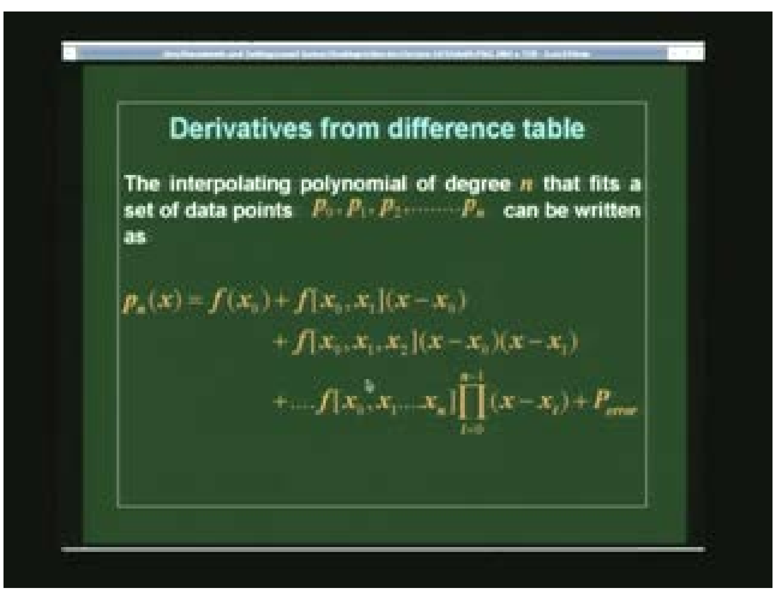

Okay and then we can find the derivative of this polynomial the derivative of polynomial is another polynomial which starts with this coefficients f of x_0 x_1 and then it goes all the way, to all the way to last term which is now x minus x_j product divided by x minus x_i. So the second term will have two term similarly, the last term will now have n steps, okay the second term here is two terms the second order term of the polynomial is the first order term in the derivative which will have two terms and similarly, n minus 1 the last term will have n minus 1 such terms.

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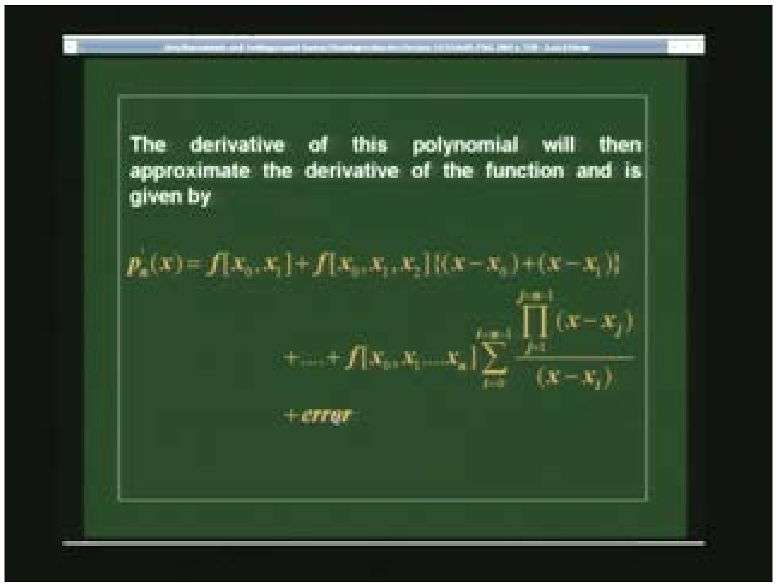

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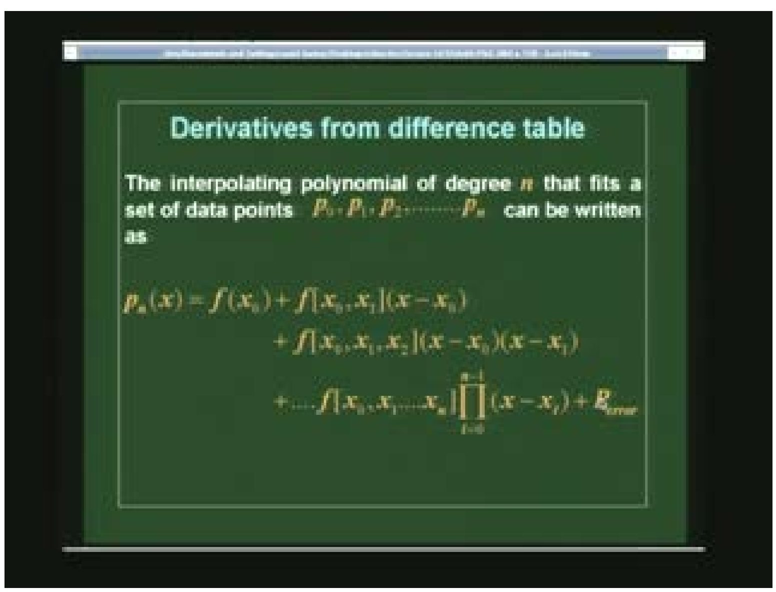

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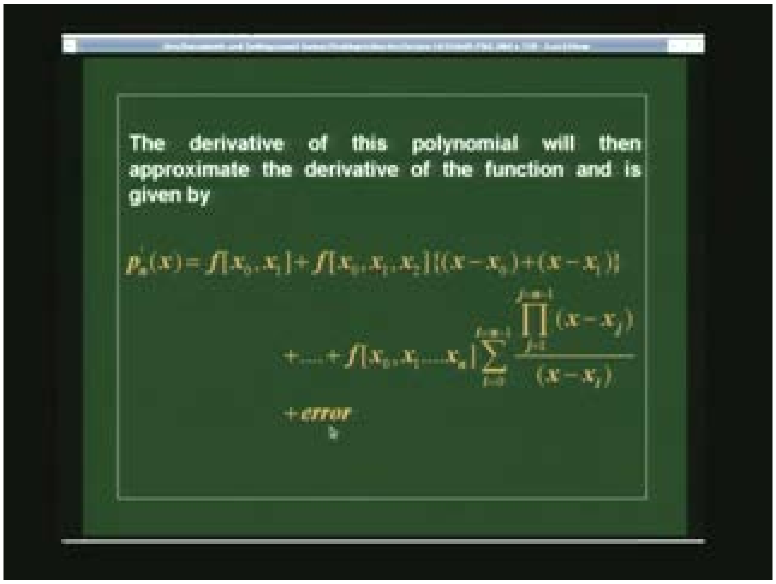

Okay each of them will be a product of n minus 1, each of them will be a product of order of n minus 1 that is the polynomial which we have. Okay now we can evaluate this polynomial at this point and then there is some error of course infinite series which we are using there is some error term here. Similarly, there is an error term here, this is the polynomial error and then the term and the error here but we know what the error is kind of estimate what the error is because we have seen earlier that in the case of polynomial the errors here okay is like the next term in the polynomial.

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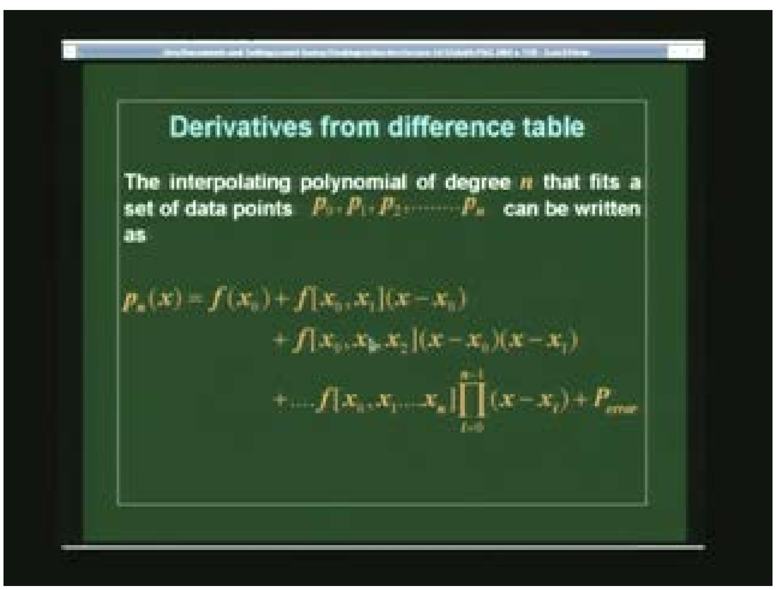

You have seen that okay, so if you look back if you remember earlier we have written this earlier that in the case of polynomial interpolation that the error in the polynomial nth order polynomial is like the nth order plus one polynomial so it is like the next term here, this is the error the order of the next term in this, in this expansion

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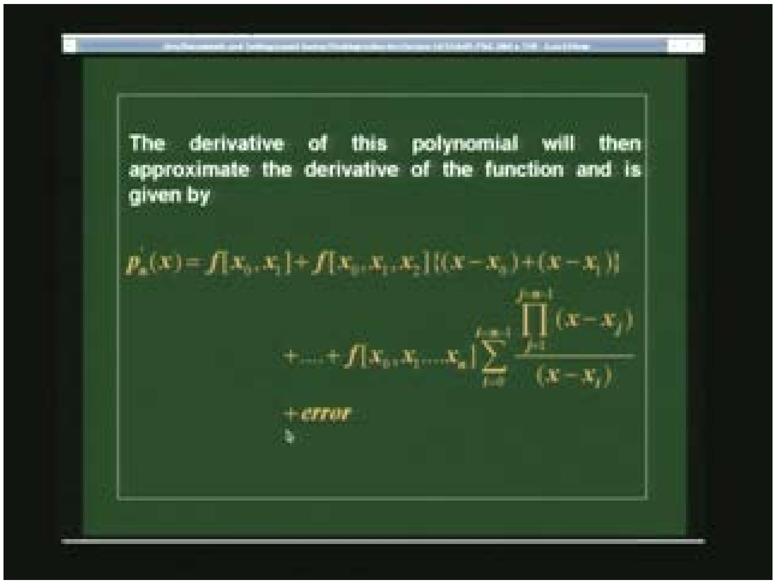

So then this, the error here should be derivative of that that term that way we can actually estimate the error. So I am writing down the error this is the nth order term okay is then is a product of all quantities x minus x_i, i going from 0 to n and now there is a pre factor here pre factor we know is the nth plus order derivative n plus 1 factorial that was the we have seen earlier

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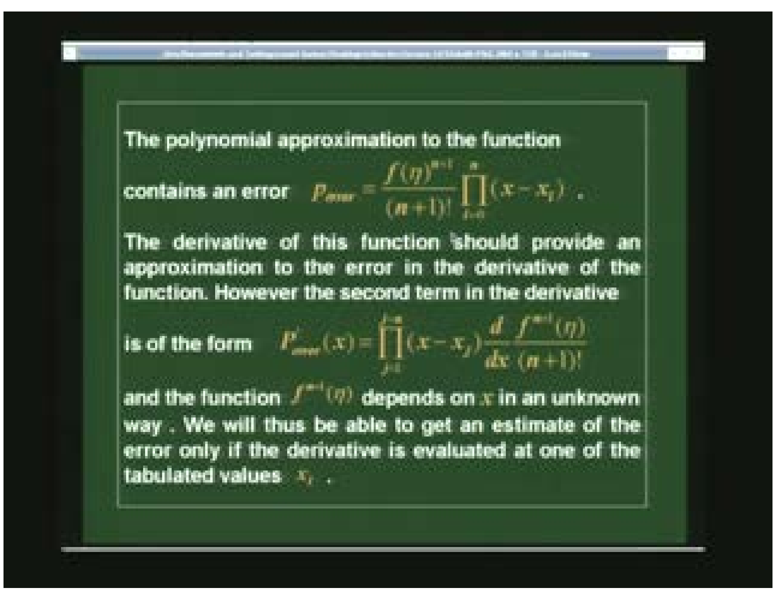

So we have seen that the error in the polynomial is of the order of the next term in the polynomial that is we have nth order polynomial is like the n plus 1 th order term polynomial. So the n plus 1 th order term is simply this, x minus x_i multiplied by i going from 0 to n, the product of x minus x_i, i going from 0to n and the pre factor we have known is nth plus order derivative of the function the true function divided by n plus 1 factorial evaluated at some point between x_0 and x_n that is the error in the polynomial so we can take the derivative of this that is del p by del x that should be the error in the derivative.

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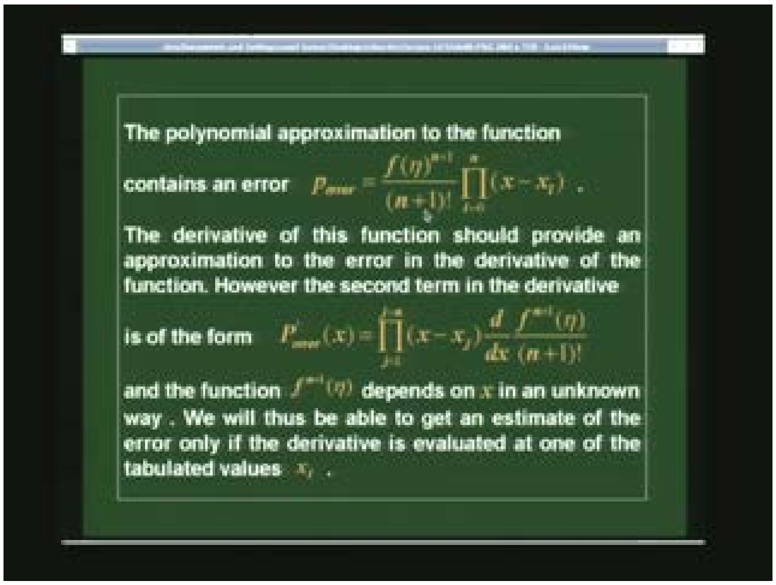

Okay, so the idea here is to that if you have set of points given we want to find the derivative at anywhere in these points not only on the points it tabulated but even in between this, then we would think I can actually give a interpolation, I can write a polynomial which interpolates all this points and then i can use the polynomial to find the derivative we would think to find the derivative to this point why I should take all this point, okay I could simply I could take two points here may be 3 points or 2 points draw a line then find the derivative there.

So there is obviously some error in that and that error is of the order of the polynomial which we use okay that is the polynomial, if we use the polynomial nth order to fit to interpolate this set of data points. We have the error the error in the polynomial is like the nth plus one order term okay and hence the error in the derivative will be of order something nth order will be the derivative the error in the derivative.

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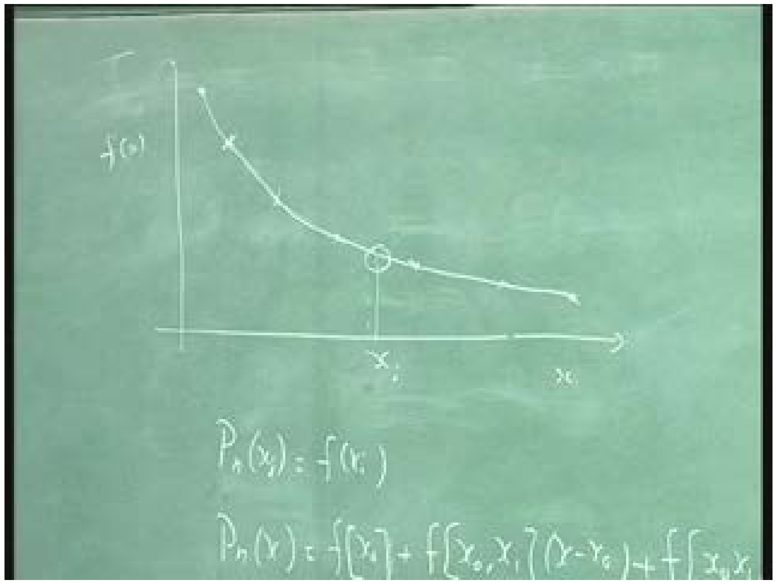

A quantitative value is difficult because of the following reason, that is the error we can write like this but we do not know the function actually right because we do not know the real function which goes through this point we know only approximate function which is our polynomial. Okay so which we will not be able to evaluate this n th plus 1 th order derivative obviously. Since we do not know the functional form of this the derivative of this with respect to x which actually have two terms that is one is the derivative of this part multiplied by this and the another one is the derivative of this part multiplied by that right because this actually a function of x Etta is some value um of x between x_0 and x_1 okay so we need to know what that is.

We need to evaluate of this derivative that particular point the derivative of the function of that particular point but we do not know that function right not able to get complete quantitative value of the error but we can get a good estimate of the error okay, since we do not know the functional form of this f Etta do cant a quantitative error but we can get a good qualitative idea about the error in the in the derivative

So you see that if you evaluate the derivative at that tabulated point this is x_i of the tabulated points and then the this quantity this only one term in the error because the other term goes 0 we have only one term in the error that just a derivative of this part with respect to x of course. So any value which is tabulated, we can actually use this the derivative of this with respect to x as the error again since we do not know the functional form here is again get a qualitative error but in un tabulated points we cant even get that because simply we have no idea what this quantitative would be with that caviar we will look at particular example of using this idea of this polynomial interpolation to get the derivatives and get some feeling which gives some feeling for the error.

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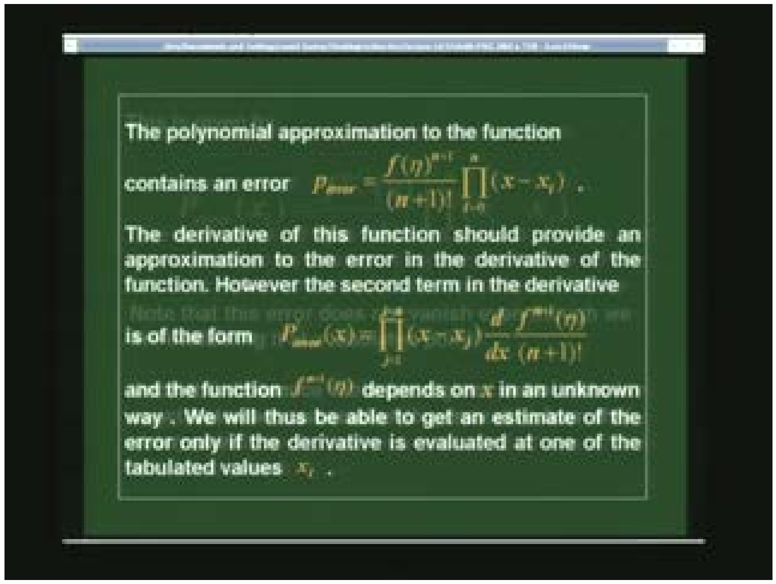

Okay to summarize the error of the tabulated points the tabulated points error in this method of calculating numerical derivative by interpolation would be of this order that is n plus one derivative evaluated at some point Etta between x_0 and $x_1$divided by n plus 1 factorial into a product x_i minus x_j. So now this is evaluated at x_i a product of x_i minus x_j going from 1 to n excepted j equal to i. So that is the error. So this is useful because this may way of calculating the polynomial um using calculating the derivative using a polynomial is useful because it is easy to program, once we have the difference table for the polynomial then we can read of for the derivative also very easily from that.

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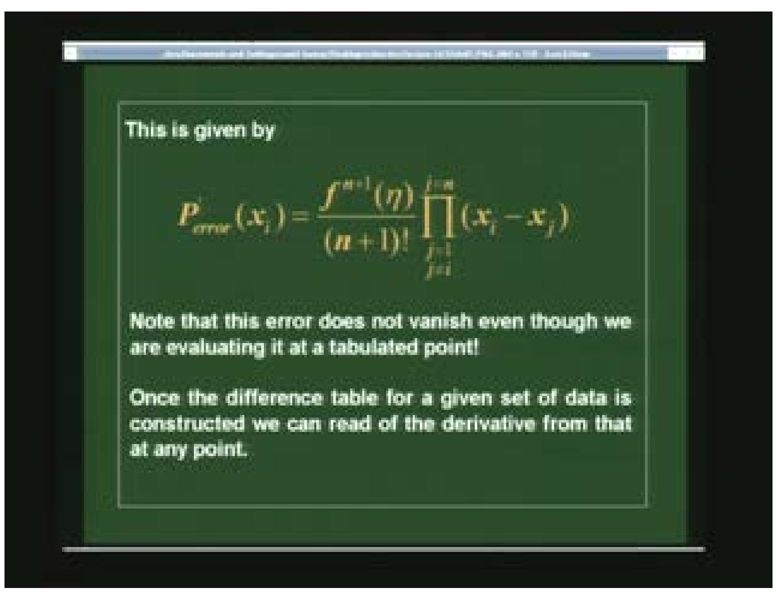

Okay, we will see that for a particular derivative of the function f of x equal to x square minus 2 x plus 2. So we will see this example okay, so now remember what difference table so the difference table i am showing here in this 1 is same as what is given as here write for this we can continue all the way to x n as simply the function value at x_n that is the first order first column in the table, okay that is I have index i going from 0,1 to 4 there are 5 terms given here. So, I have and then I have the x values tabulated at that point 0, 1, 3, 4 and 6.

Okay normally this is this given to us and we construct the polynomial just to demonstrate the error in this, in this whole method we have actually using this functional form of the f here okay to get this number this also something which encounter in real life. Okay if you know the functional derivative is functional table we do not know this table but this is to demonstrate that, we construct this table from a known function and look at the error in our derivatives.

So I use this function here f of x equal to x square minus 2 x plus 2 actually tabulated this functional value function values functional values at x equal to 0, 2 x equal to 11, 3 is 5, 4 is 10, 6 is 26. So we have tabulated at this at this 5 different points that is the first order and the next one would be compute the f x_i, x_i plus 1 okay that is terms of this type f x_0 x_1 and the next would be f $x_1 x_2$ etcetera. So that will be, that is equal to f of x_2 minus f of x_1 divided by x_2 minus x_1 and we go on like that for all the pairs. So we take each of the pairs compute the next one and then we take each of this compute the next one etcetera.

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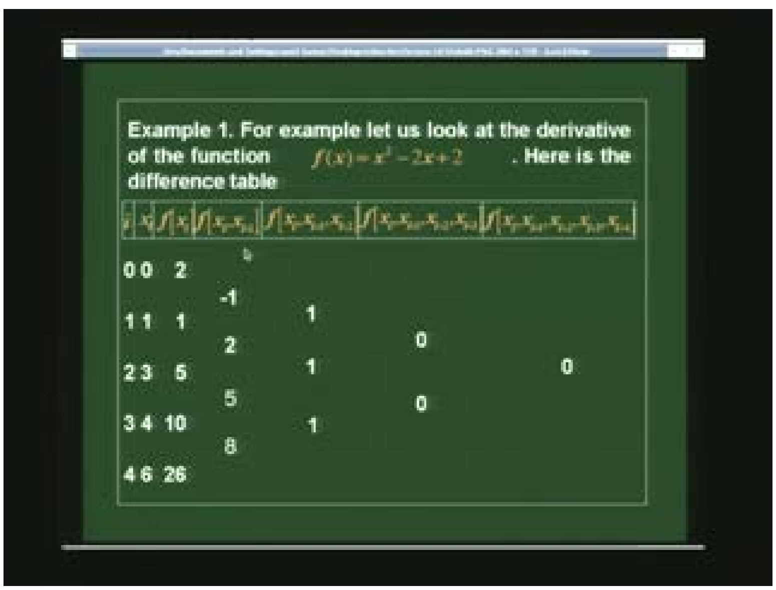

Okay, so we can see that this is the function values this is the difference between this, this two points difference between the two points difference between these two points divided by difference between these two values and this is the difference between these two values divided by difference between these two and this difference between 26 and 10 divided by the difference between 6 and 4and then the next level that is the third order term in the polynomial this is given by the difference between these two that 2 n minus 1 is here and the x value difference between these two write and the next one is difference between 5 and 2 divided by difference between 4 and 1 x equal to 0 this is third order term and you can do the forth order term in the polynomial and then the fourth order term and the polynomial etcetera, fourth order mean function with 4 variables required coefficient which can take 4 points in the consideration and the fifth point etcetera.

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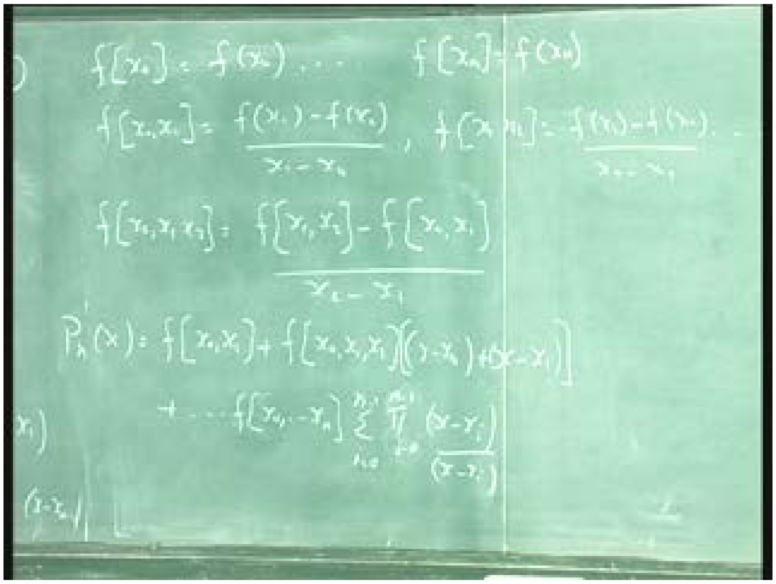

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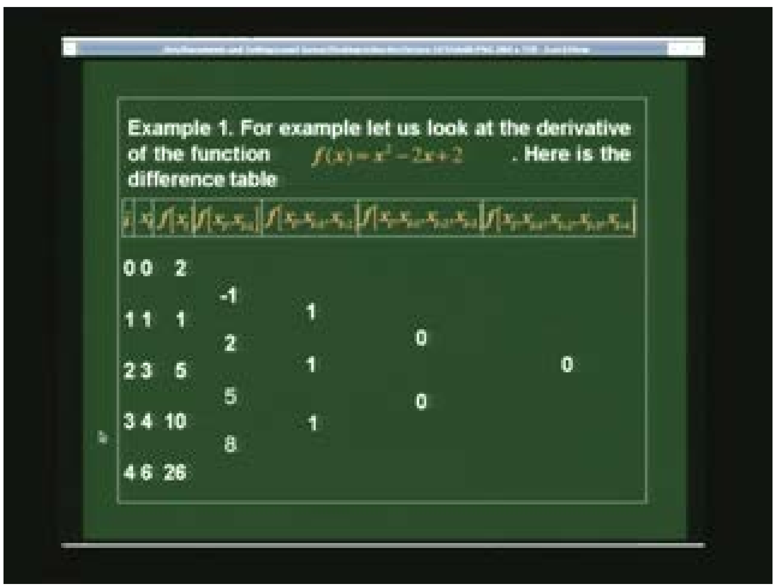

This thing are 0 because some polynomial this is whole function of the polynomial of order two, we are obviously getting higher order terms 0 now let us say we want to evaluate the we have this table we have difference table constructed.

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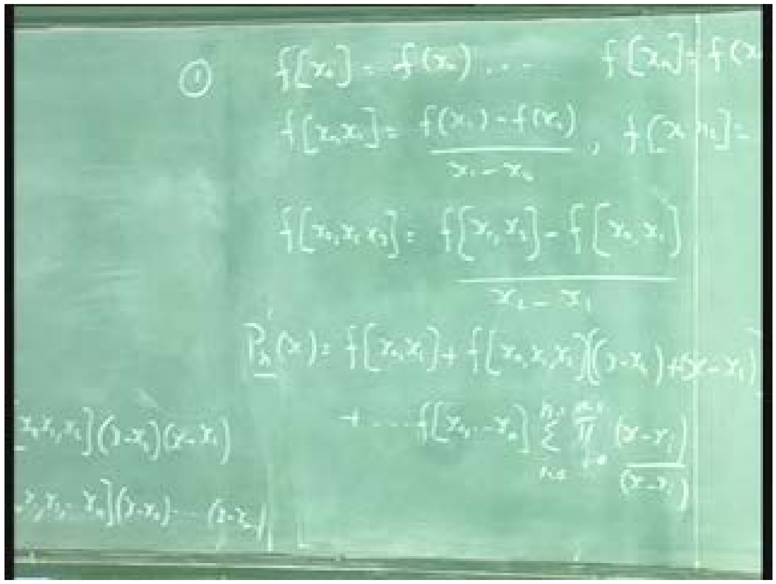

So it could be any set of data points given to you from some experiments, some simulations, some other functions I have given you this function value at this tabulated value of x and then we construct this difference table and from the difference we can write down the polynomial. So since we have the all the quantities write down this polynomial then from that polynomial, we can write down the derivative of the polynomial so let us evaluate the derivative at any point lets say between 3 and 4, okay now if you want to evaluate the derivative between 3and 4, I could take second order polynomial, third order polynomial or first order polynomial.

Okay so we will see that we will take polynomial which goes through for example 0, 1, 3 and 4 or we could take a polynomial which goes through 1, 3, 4 and 6 right. So we can construct two such polynomials that is two polynomial of this same order one goes through 0, 1, 3 and 4 okay that gives the third order polynomial and there is another polynomial which goes through one three four and 62 such polynomials which we construct to evaluate the derivative between 3 and 4, since both this polynomial passes through 3 and 4 we should be able to evaluated derivative between 3 and 4 using that.

So that is what we see. So x equal to 3.25 for example we will take and we try to calculate this polynomial. So the derivative of such polynomial third order polynomial would be is given here. Let us say we construct the third order polynomial which goes through from this table, okay if I want to construct a polynomial which goes through 1, 3, 4 and 6. So we have 4 points, so the polynomial will be given by 1 plus 2 into x minus 1 plus 1 into x minus 3 plus 0 into x minus 3 into x minus 4 etcetera, So let me write it down here.

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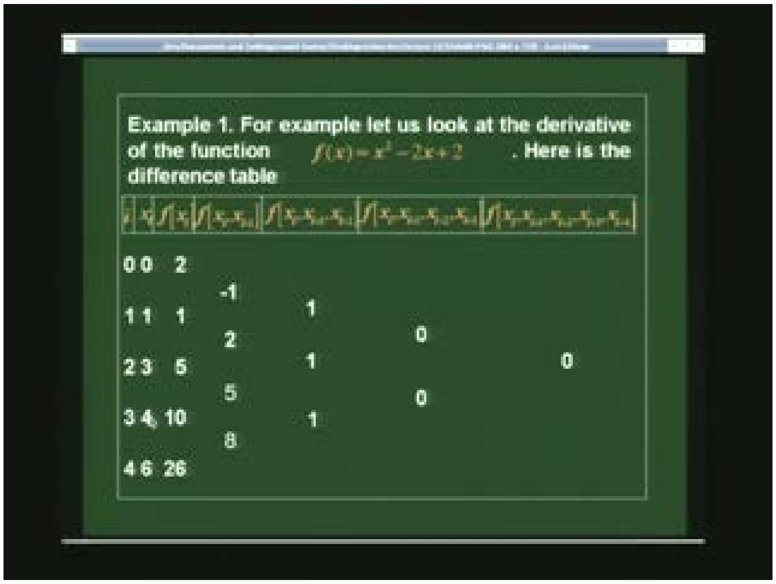

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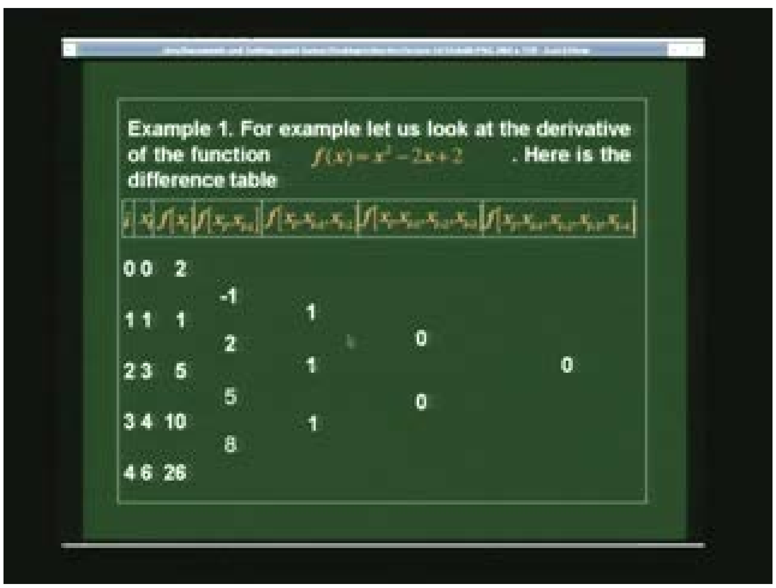

So we were looking at the polynomial which goes through one function x 1, x values x 1, 3, 4 and 6, and 1, 3, 4 and 6 that is the function. We were going to prove x_i is given by that 1, 3, 4 and 6. So polynomial of third order polynomial p n of x we will write as f of x_0 is 1 now f of x_0 in this particular case f of x_1 because we are going to start from 1. So x_0 is 1, so 11 plus 2 into x minus x_0 is 1, 2 is the next term okay if f of x_0 x_1 is 2, f of x_0 x_1 is 2 that is the next term and then, so 2 into x minus x_1 and then we have the next term that is f of x_0, x_1, x_2 that is f of 1, 3, 4 that is 1 again.

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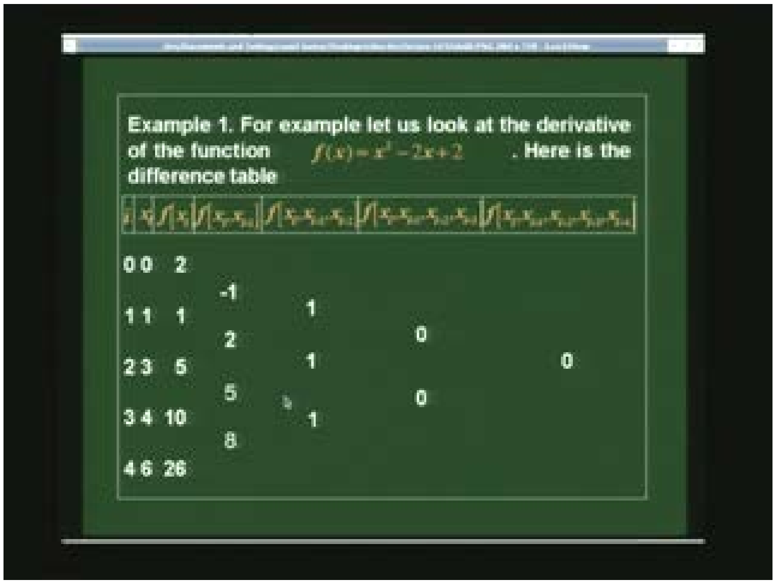

So the next term that is 1 into x minus 1, x minus x_0 into x minus x_1 that is 3 okay and the next term in this thing would be plus the next term next term coefficient is 0 okay that does not we will write it 0 into x minus 1 into x minus 3 into x minus 4. So that is the third order which we will have the derivative of this order polynomial at any point would be is 2 from this plus now there are terms are here. So 1 into, so we have term x minus 1 plus x minus 3 and the last term is okay that gives the polynomial, so that 2 x minus 2we got the answer of the derivative at any point.

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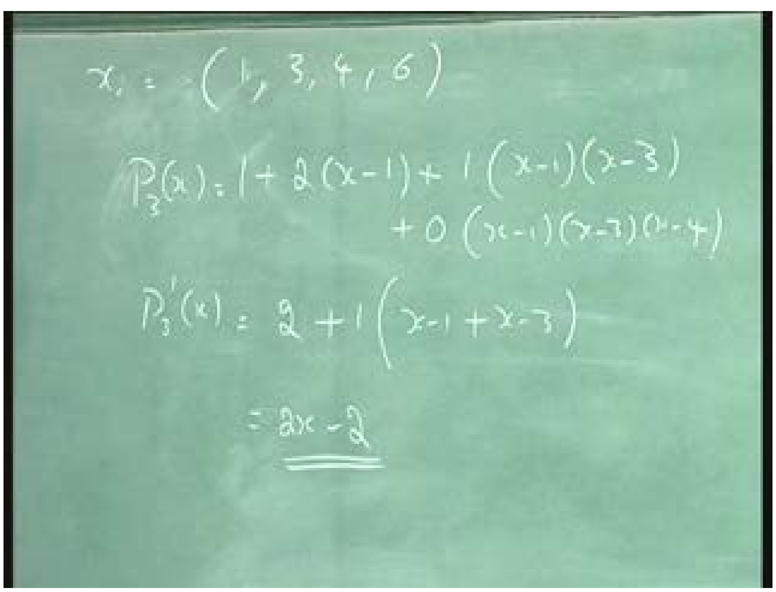

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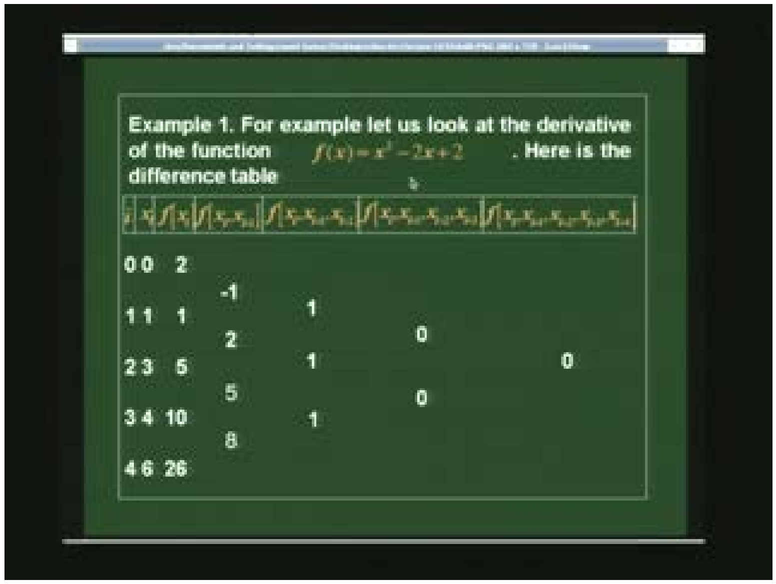

So you can see the from the function here which is x square minus 2 x plus 2 the derivative 2 x minus 2. So it looks like this the third order polynomial which we constructed and we took at the derivative the derivatives exact. So that we will always true in this particular case is exact what we just did is was this 1, 2, 3 and 4 equations x value which we took was x_1, x_2, x_3, x_4 and we constructed the polynomial and we found the derivative 2 x minus 2, we can do exactly the same 4, 0, 1, 2 and 3 and we would find that we get exactly the same derivative this case 2 x minus 2.

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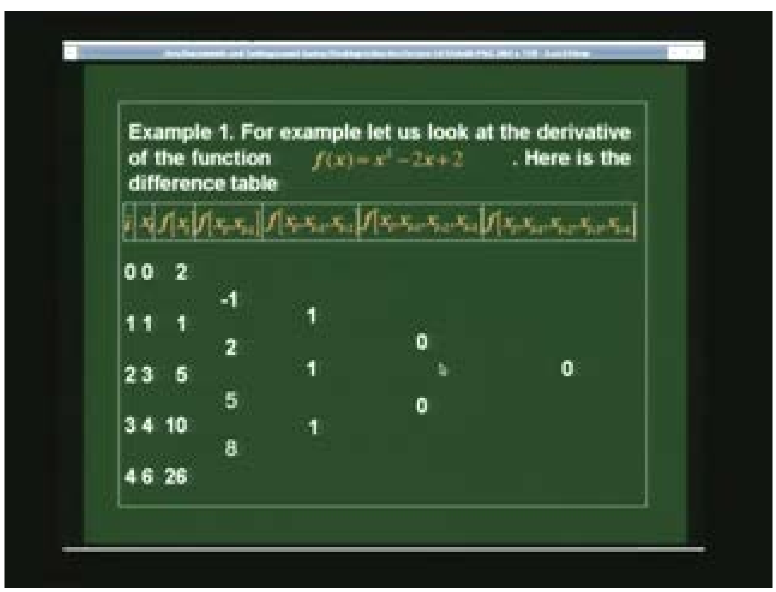

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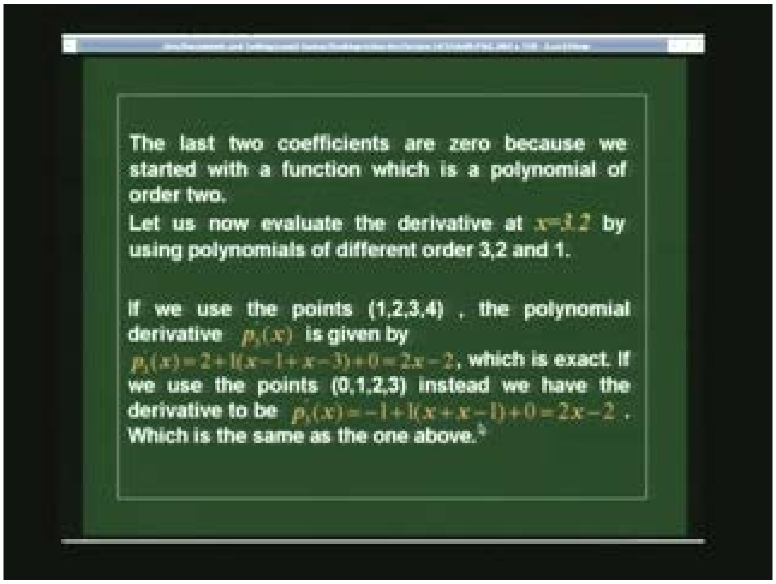

Okay so what we have to do is take 0,1 and 2, 0, 1, 2, 3 and that is x value 0, 1, 3 and 4 and we can read of the polynomial from this using this set of values now that is 2minus 11 and 0 and we can write down the polynomial and derivative which is again given by 2 x minus 2. Okay so which is the same as above, so we get 2third order polynomial which goes through this point x equal to "3.2" which we want to evaluate the function and we find that before evaluating it the derivative is exact because we knew we were looking at function of this form the values tabulated from the real function which is x square minus 2 x plus 2.

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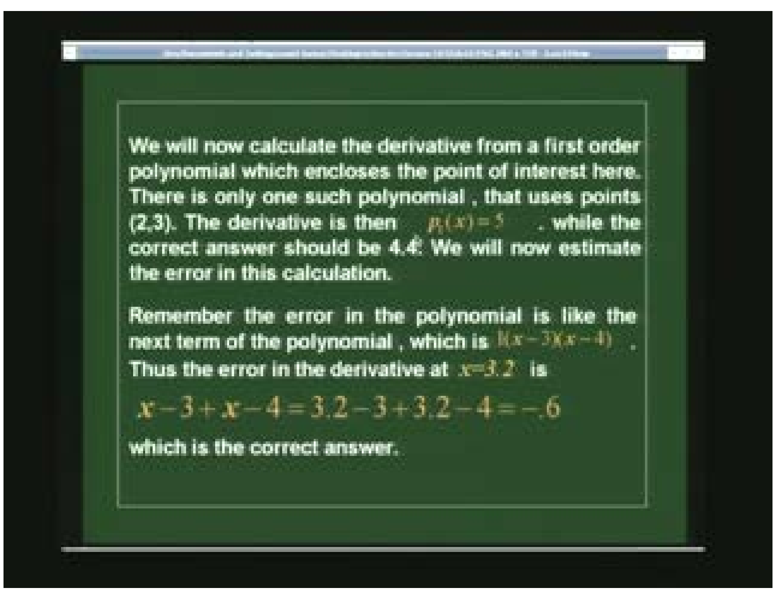

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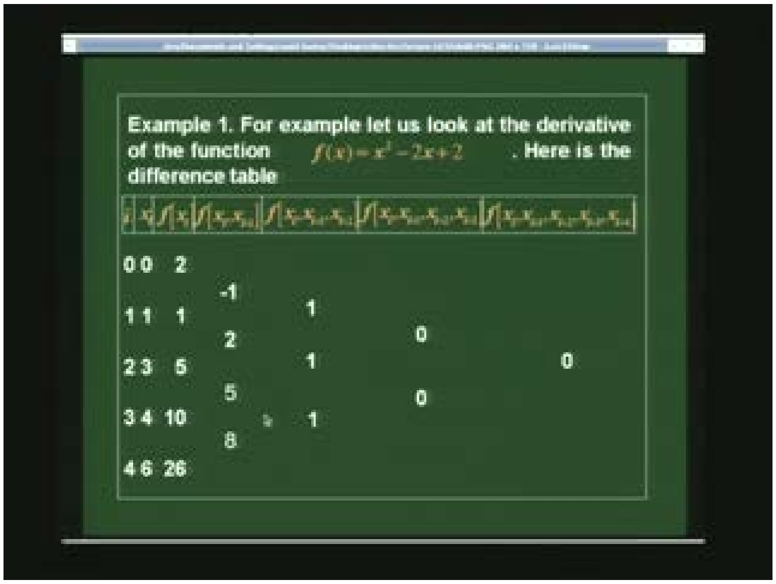

Okay so then, now we can go just look at the second order polynomial that is the derivative being first order we can look at that and you compute what the what the error in that would be. So here in this case, we just look at a first order polynomial and if you look at the first order polynomial which goes through 2 and 3 for example, okay let us see, we want to evaluate something here, okay some derivative between 3 and 4, so we just construct a linear line which connects 3 and 4 3, 5 4, 10 as the function value okay and then we will evaluate the derivative between that is the polynomial is given by 5 plus 5 into x minus 3.

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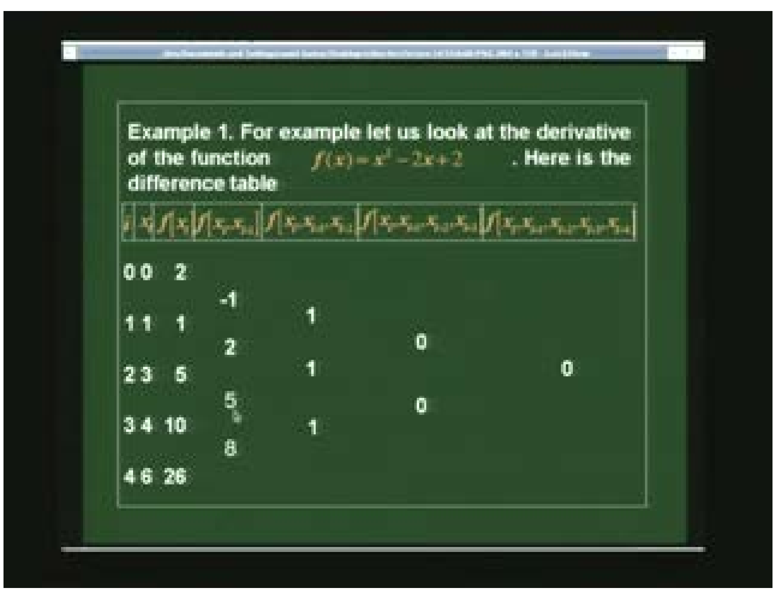

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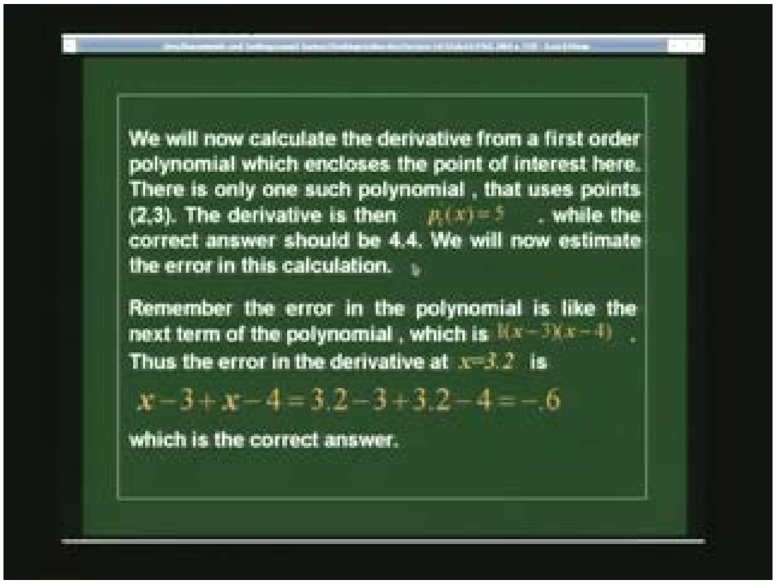

So the derivative would be linear function would be 5 plus 5 into x minus 3, okay and then you just look at the derivative of that okay that will be 5 that is what we will find here okay so the derivative is simply 5. So the polynomial, remember the linear polynomial is goes through these two points okay would be just 5 plus 5 into x minus 3.So the derivative of that would be just 5so we have a constant derivative is 5 but we know that the derivative 2 x minus 2 with x equal to "3.2" which is "4.4" so we have half "5.6" by going to a linear interpolation between two points which brackets the which you want to evaluate the derivatives is we get an error " 4.47" "4.6" we just wrote down the polynomial of order 1 okay and we said that 5 plus 5 into x minus 3.

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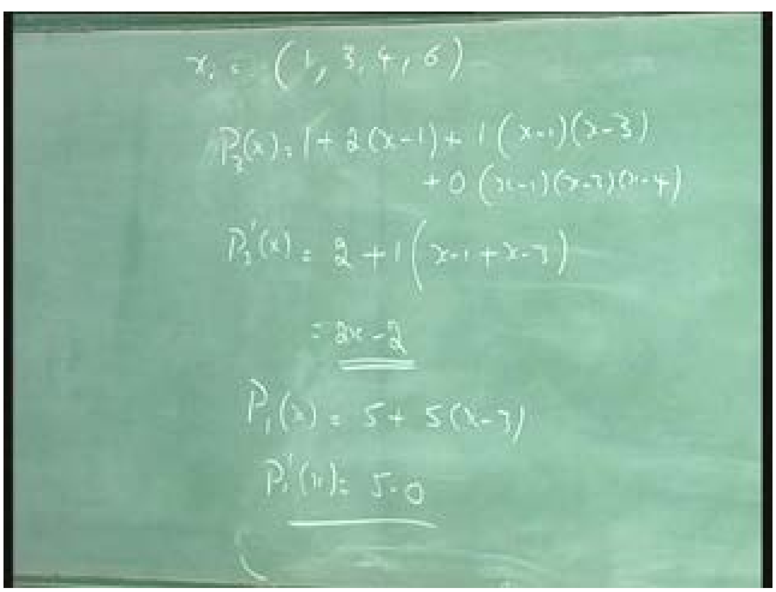

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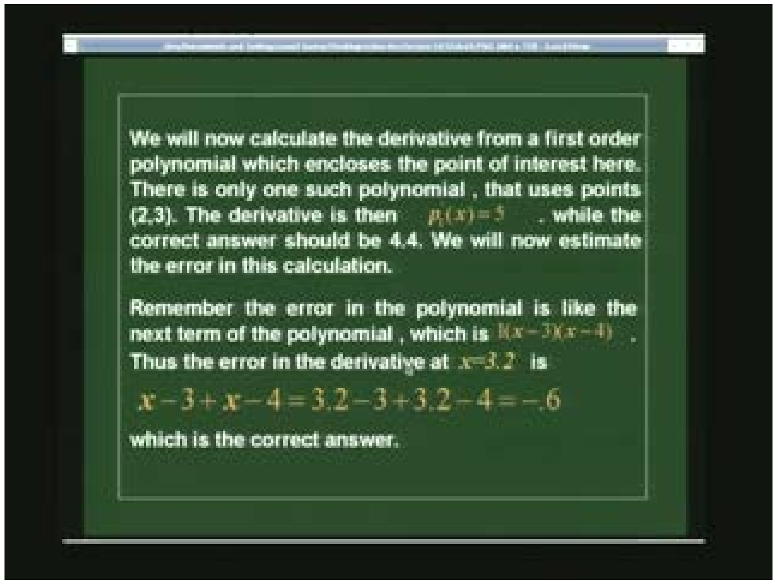

So we had used x i as 3and 4, so we got 5 plus 5 into x minus 3 the derivative was equal to 5 but at 3 point 2 the derivative should be 4 .4, okay so the error is the point 6 and then now we use the idea, the error should be next term in the polynomial what is the next term in the polynomial. So what is the error in the polynomial, so the error in this polynomial would be some coefficient here which I do not know some coefficient or if I just write the second order polynomial. So then this term should be 1 into x minus3 into x minus 4 that is the term which I written because after 5 the next coefficient is 1.

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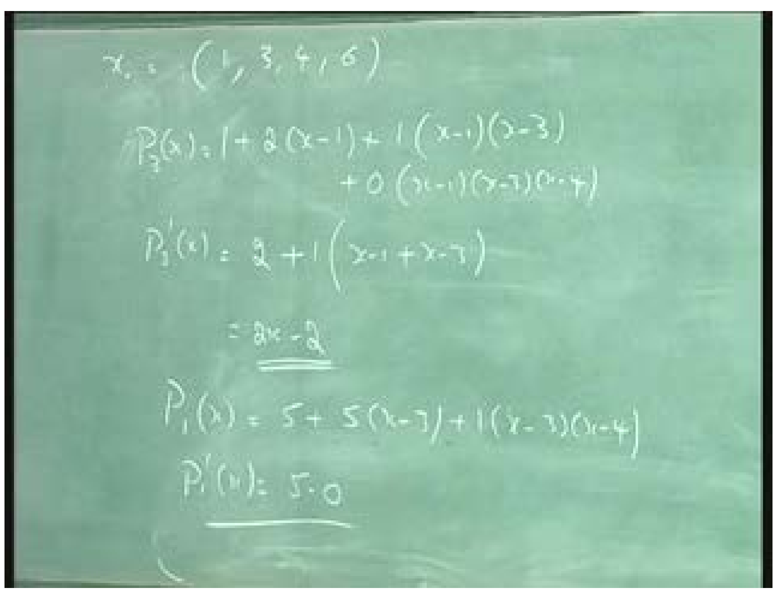

So the next polynomial term is actually is x minus 3x minus 4, so this is my error term. Okay if I write a first order polynomial this is my first order polynomial the error in the polynomial is like that, okay the next term in the polynomial so that is the error in the polynomial. So then if I what is my error in the derivative that will be of this order and that would be 1 into x minus 3 that is 3.2 that is minus 3 plus 3.2 minus 4 that would be the error in the polynomial evaluated at x equal to .32 or in general it should be the order of that type that is equal to, if I substitute x equal to 3.2, I will find that the point is 6 that is what I summarize here okay.

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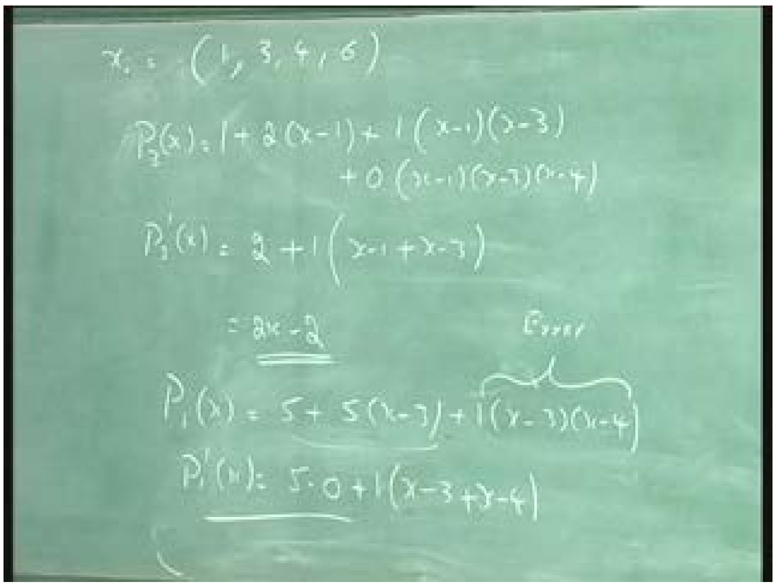

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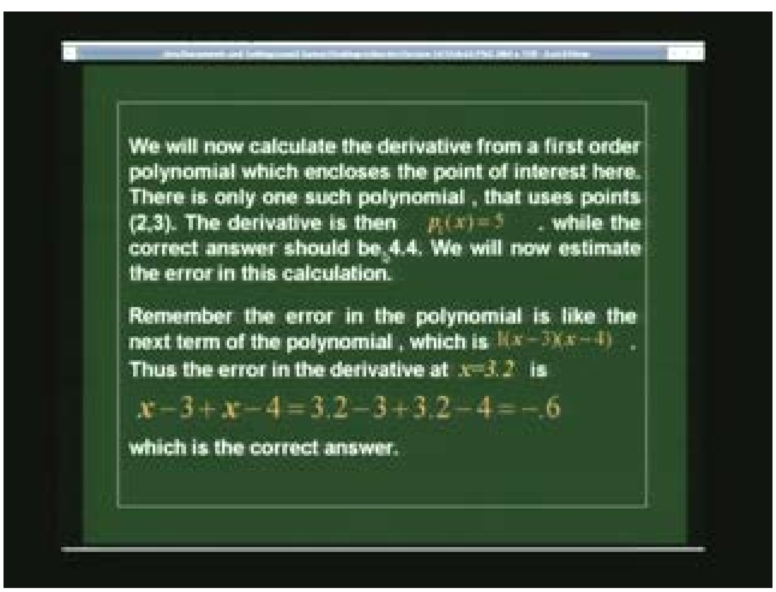

So if I use a first order polynomial and then I used the fact the next term in the polynomial. Okay so then I write the next term in the polynomial and then I can that I can compute the error from this the derivative of this the derivative and if I evaluated at x equal to “3.2” plus minus .6, so our value which we found was 5 and the error was point 6 so we know actual values “4.4” so you see that it gives the good idea here in this particular example it is exact but it may not be exact always but it will give us a good estimate of what the error is by just finding the derivative of the next term in the polynomial.

So that is the we will look at this is one method of finding the derivative of set of data points derivative in between given set of data points by actually interpolating by this points by polynomial and then taking the derivative of the polynomial and we know that if you have the difference table, then it is easy to construct then another polynomial which can be written in terms of a difference table values. So that is one method, we will look at another methods in the finding the derivative and discrete derivative in the coming classes.