Numerical Methods and Programming

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Lecture - 21

Solving Non-Linear Equations

Today, we will look at some of the methods to find solutions of non-linear equations that will be the topic of discussion today. So solutions of non-linear equations, non-linear equations, okay that is what we would want to discuss today. So we have saw in the last lecturer that we were we were find to solutions of polynomials of the form, we wrote some polynomial of the form a_n, we had a_0 plus a_1 x plus a_2 x square of this form. So then we have to find the roots what we will call the roots of these polynomial equations, we have saw in the case of characteristic polynomial in order to determine the Eigen values of the metrics.

So this is one example where we have to find the solution of non linear equations for the roots of this polynomial is nothing but, saying that is equal to 0 and finding the solutions of these equations. So we will discuss some of this methods, we saw one method of doing that, that is dividing this by a quadratic polynomial and finding the, I mean factorizing this polynomial in to a polynomial of this order, we will write p_n of x as some p prime n minus 2 of x in to some other polynomial which you would call in this case x of y_2 of x.

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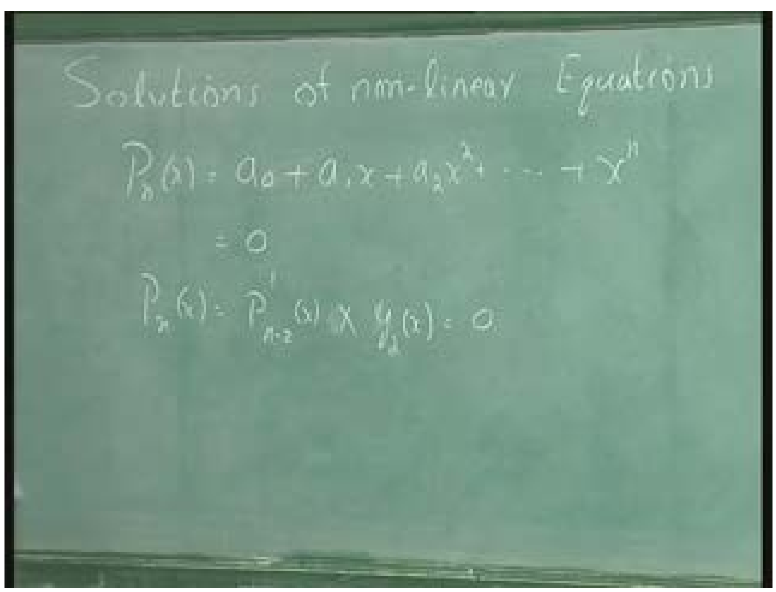

So we had a quadratic equation, quadratic polynomial multiplying this n minus 2th order polynomial and then we could find the roots of this is equal to 0 has definitely has one root has y_2 of x equal to 0, y_2 be a quadratic polynomial and we know how to find close form solutions of quadratic polynomial. So in general finding the solutions of non-linear equations will not be so trivial and it look like many times that we have complicated non-linear equations and which were not of this simple quadratic form.

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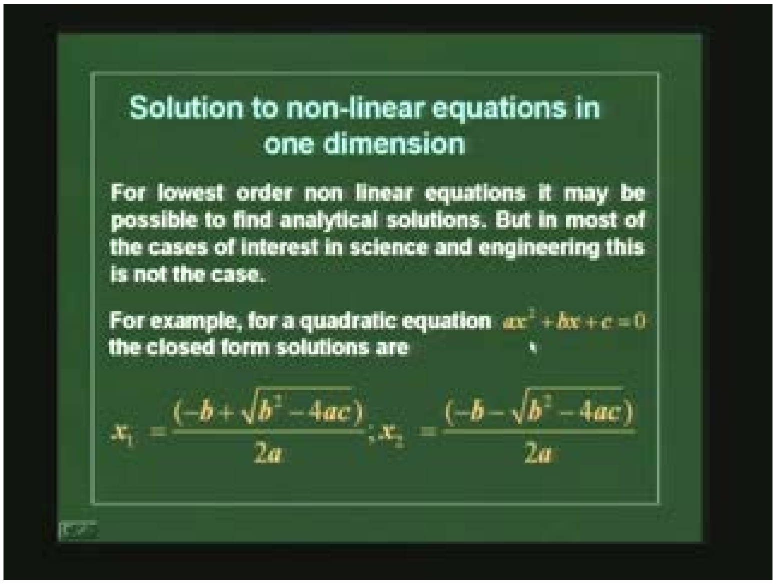

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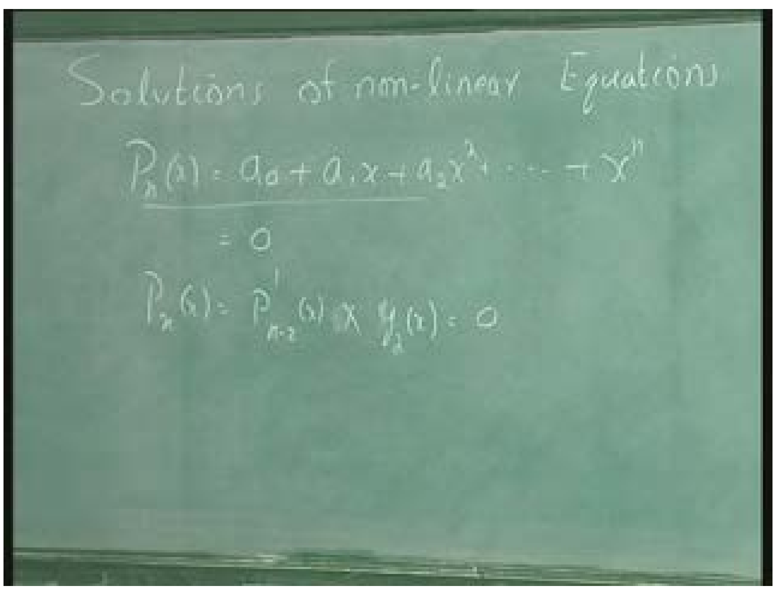

So one example is of course is to get the non-linear equations into a factorize the non-linear equation, non-linear functions which you have into a quadratic a functions multiplied by another functions and then we can find the one of the roots as the roots of this quadratic equations, that is what we did in this case of polynomial equations, but not all the non-linear equations in polynomial form so we may not be able to do this factorization quite easily.

In this case, it was easy to do this factorization and try this as quadratic function and then finding the solution of that. But this may be not always easy and it will not always easy to find in the general case of non-linear equations, it may not be easy to find a closed form solution of this form at all. So here in the case of quadratic equation, we know that we have closed form solution of this form but if this more complicated functions it is not even obvious that we have closed form solutions.

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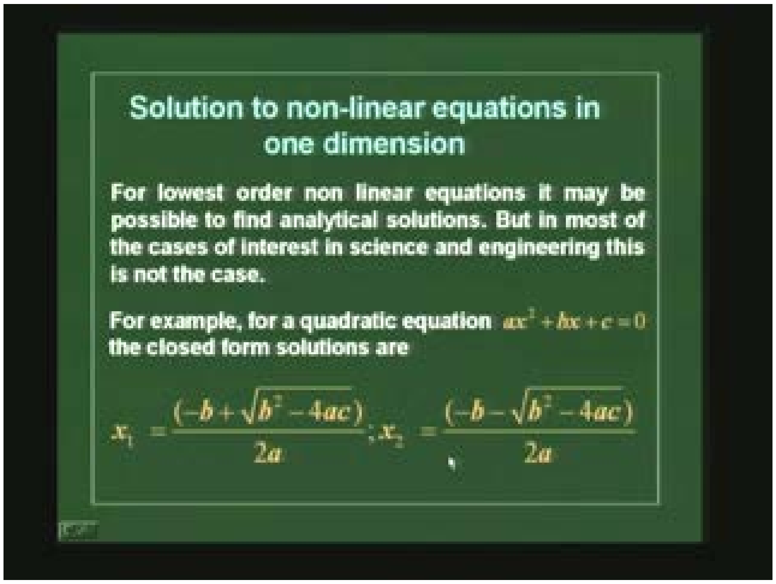

So the questions is what we did do we obviously we go in to some numerical technique of finding the solutions of that. So in the case were closed form solutions are difficult, we could um are impossible to find okay we would solved non linear equations of this form f of x equal to 0 in using some numerical technique. So one of this ways, first thing you should do when you get an equation of this form that is any non-linear function of f of x and we have to find the roots of this equation that is, if you just given an equation of the form f of x equal to 0 and if you asked to find the roots of this equation. What we do is to find is this plot this function of f of x, if this is the simple function to plot, okay then we would just plot, then we would have especially, if it is a case like this and you would plot f of x versus x.

So we would plot this function okay, so that and then let us say that function that is our y equal to 0, that is y equal to 0 that is f of x_0 line and then if you have the function behaves something like this okay now all this crossing point of this solutions of this equations, or in another sense the solution of this equation of this same as it 0 of this function write so we have just to find where this functions cross the axis, the x axis of the f of x equal to 0 line that will be the simplest method, okay if you or at least you will get an idea may be may not be very accurate by visual thing but at least the simplest method and it is as first might be a good method to find at least approximate value of the zeros of this function f of x. So this are the points and this let we call them to an x_1, x_2, x_3, x_4, so we have x_1, x_2 or x_3, x_4 okay, now these are the four points which this functions goes to 0 this is a solution of equations.

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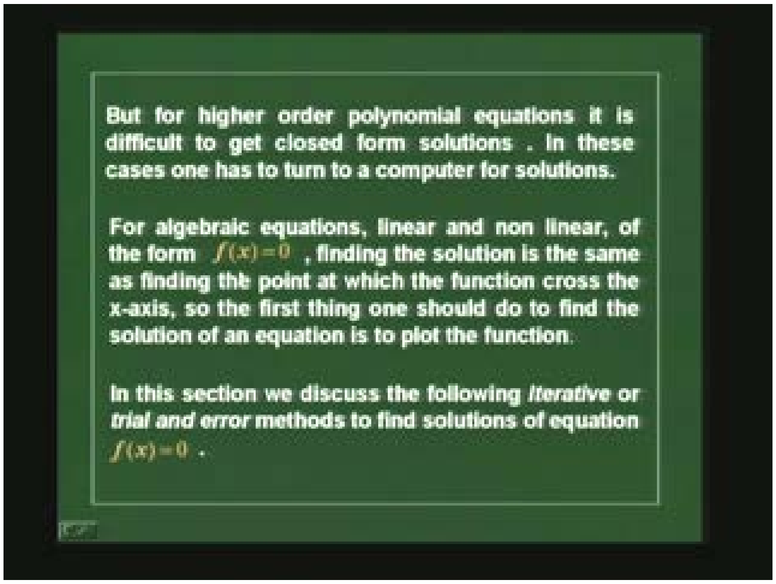

Okay, at least we found 4 such roots that is a simple graphical method to do that. So another important property that whenever, this function near the 0 right, near 1 of the 0 near x_1, x_2, x_3 and x_4, this function would change sign right, that is the 0. So obviously this negative here and positive on this side, so when the function goes pass the solution one of the roots write and the root change the sign okay, we can choose that we changes the sign that in another method that is, we could use in the iterative schemes. We would have use this property write the function changes sign when it goes to the root and it is a very special case where it just touches the and then goes away we will not discuss such cases okay.

So here is cases in which this function crosses the axis that f equal to 0 line, so changes the sign, so we can use that property in a iterative schemes that something we were using so in general way is to first do this graph plot this function and then look at the solutions by inspection by that is called the graphical method or do make a guess for the solutions even by plotting we could get by guess for the solutions because the accurate value may be difficult to read of a graph but then we know that the function changes sign at this point okay, so we could iterate around that point and get the value to the desired accuracy.

So in general we have four different methods of doing this okay, so of all first one which i discuss is the is the graphical method and all others that is the methods of successive bisection which we will discuss in detail Newton Raphson's iterative method or secants method, all this methods are basically iterative methods make a guess solution and you iterative around that all this 3 methods. So I would classified this as 2 basic methods one of them is graphical method and other is iterative method. We will see, so the graphical method is what I just describe here on the board that is just plotting the function that will crosses the 0 axis.

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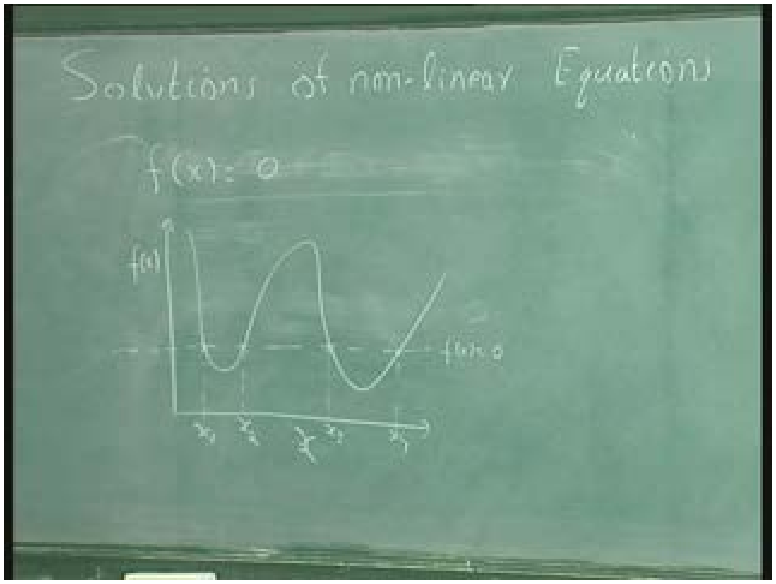

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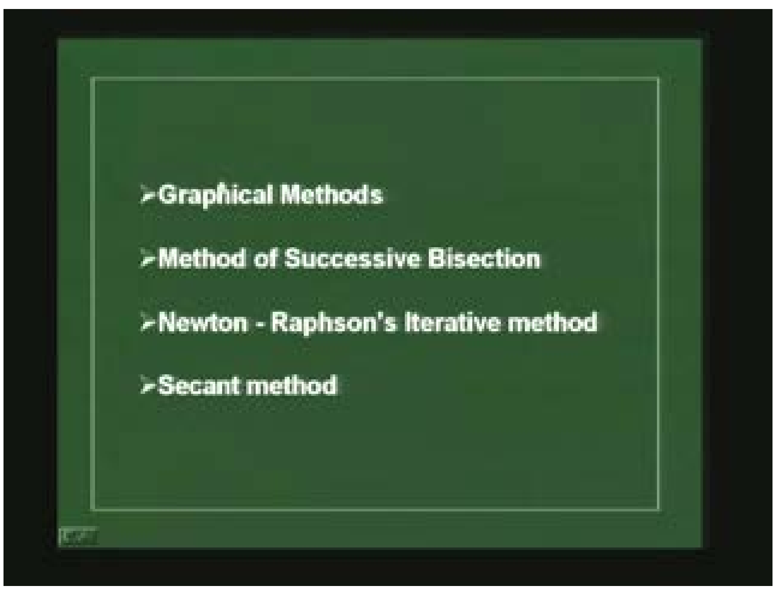

And now, we will look at the method of successive bisection, so that is what we will do just use property that the function changes a sign at this file. We will make a guess value we guess enough to make one guess we will make two guess, okay so we will do that i will re plot the function again. So let us say that, x equal to 0 line and my function is something like that this, okay so then i will make i will make 2 guess okay. So 1 x_1 and 1 x_2 okay we choose x_1 to be such that positive that is call this as x_1 and this as x_2 my x axis. So that is the idea, we have to pivot 2 points x_0 and x_1. So x_0 and x_1 we choose x_0 and x_1 okay, so such that f of x_0 and f of x_1 are always opposite sides okay.

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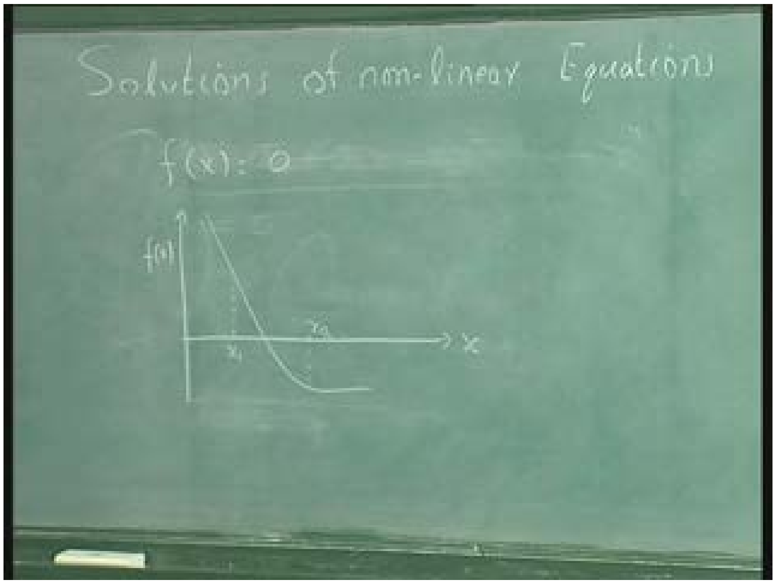

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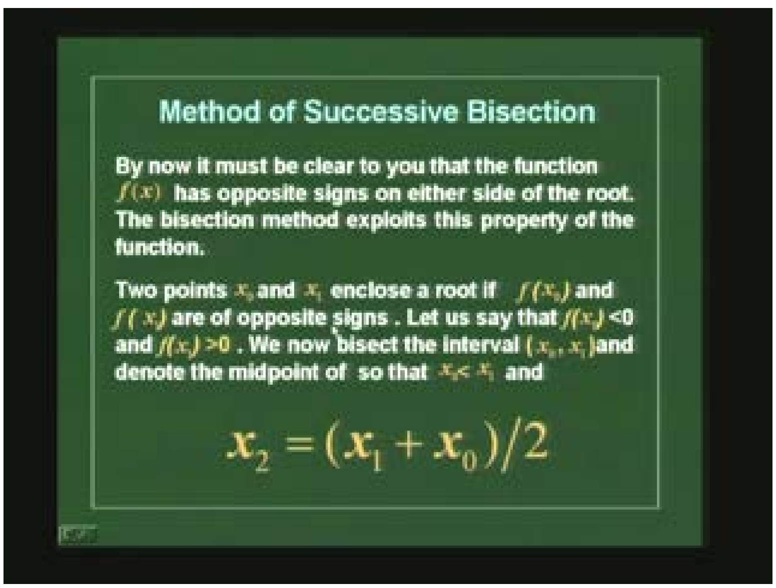

We choose f of x_0 to have to negative and f of x_1 to be positive is choice. So the basic idea in the method of successive bisection is choose to make a first make a guess, okay such that of two point such that that is on either sides of the root, okay now that can be dangerous in this we will discuss for example, the initial guess we reasonably good that if the function has a multiple roots and if you choose in the functions as multiple roots, now let us say it goes like this and if you choose one to be here and one to be here, that if you choose your initial guess to be some were here to be here again one here and then again the functions changes sign of course this is positive here this is negative here but then we will not iterative in to correct root.

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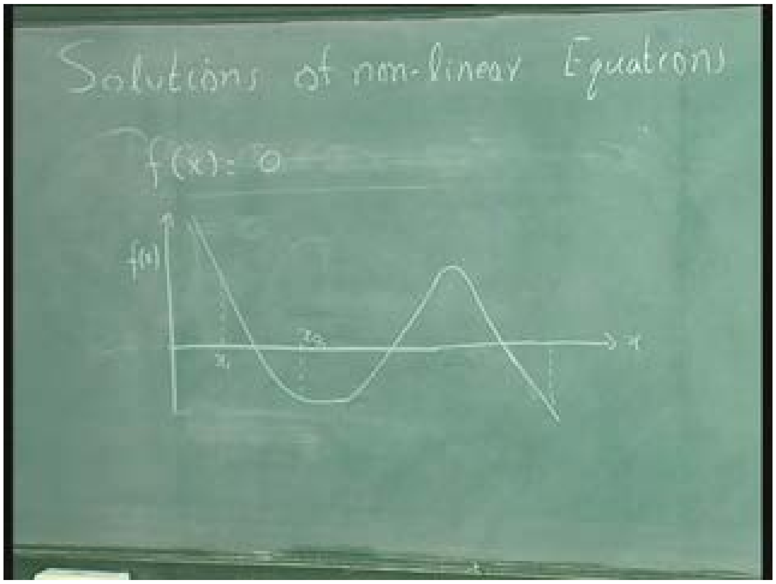

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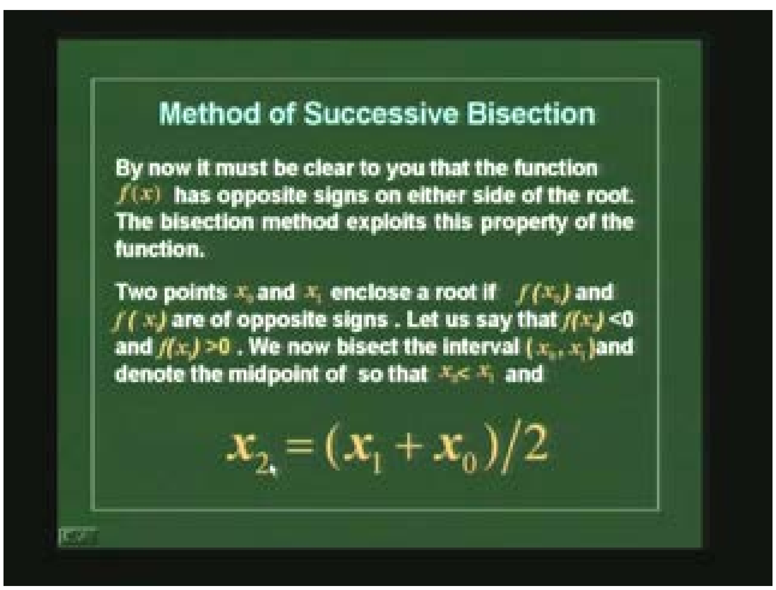

So it has to be reasonably close the solutions and we have to have an idea there are no multiple roots, we do not have multiple roots within this interval here okay. So that is one of the problems with this successive bisection, we have to make initial guesses which are reasonably close enough, close in the sense that we make sure that one root between this

x_0 and x_1 and then what we do is to find the midpoint of this, we take x_1 plus x_0 by 2 as a new x_2 value.

So we, so now we find which have x_1 here x_0 here so which is x_2 is equal to x_0 plus x_1 by 2, okay that is some were will come now here and then we will find the function value some were here that point okay, so we have the function value at that point that we will draw till fast we can demonstrate it better okay the x, x_2 be better some were here.

So the x_2 now f of x_2 from this graphic f of x_2 is less than 0 okay. So the two possibilities we find x_2 we have x_0 and x_1 we have found x_2 is a mid point of x_0 and x_1 and then we find f of x_2 possibilities of course it could be an three possibilities, three possibilities actually, if you are lucky, so normally either 0 greater than 0 or less than 0 the two possible, if it is less than 0 then what we will do is we replace, we will through away the x_2 and replace through away x_0 and replace x_0 by x_2. Okay now in this case, we will say now $_0$ is now x_2 okay that is this is x_0 is been shifted to this point and then we again find the mean of this two, okay now next time we find then mean that we will some here okay on this side of this thing.

Okay so, then so that is the case the next iteration we will have f of x_2 greater than 0, so then we will replace x_1 by x_1 equal to x_2. So the next plot we will see this is x_0 the mean some were here. Okay then I will get a positive value of function will be there value in that case i will through away that value x 1 here the x_1 here and x_0 here. So I am closing on to the on to the 0, okay that is a very simple idea of successive bisection.

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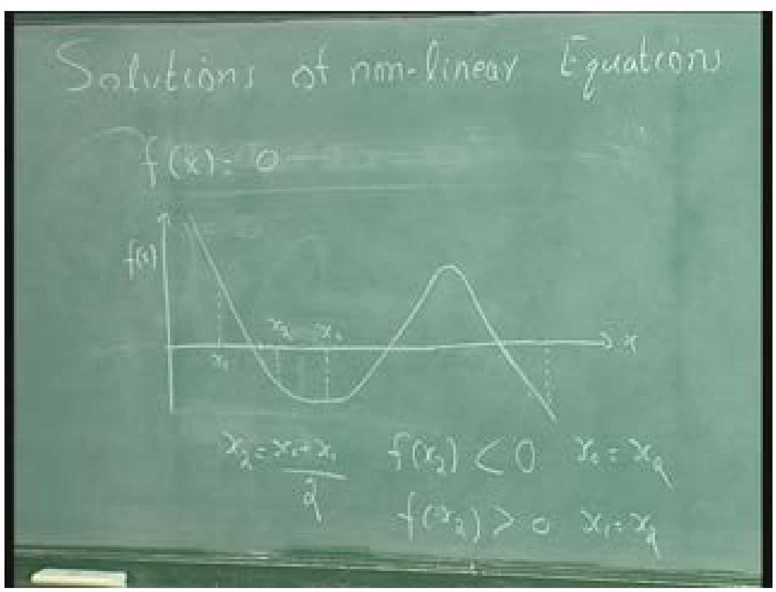

We will summarize that here, okay so we have if f of x_2 is greater than 0 then the root we know that x between x_0 and x_2, so we will replace x_1 by x_2 and if it is less than 0 then we will replace x_0 by x_2 and now we will search interval between 0, 1 and 2.

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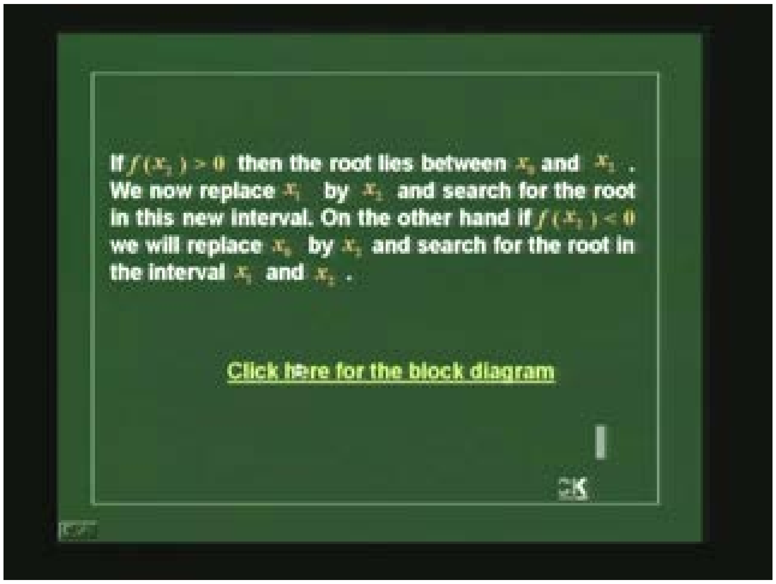

So that is a, that is a basic idea, so then we will have lets take a function of this form, okay complicated function f of x equal to “667.38” by x square into 1 minus exponentials 1 minus x exponential of minus “0.146843” x minus “1.5”. So it is not a easy solution of this some non-linear equation, so we will take two trail points which enclose the root x_0 and x_1 write and we will make the f of x and f of x_1 are opposite sides.

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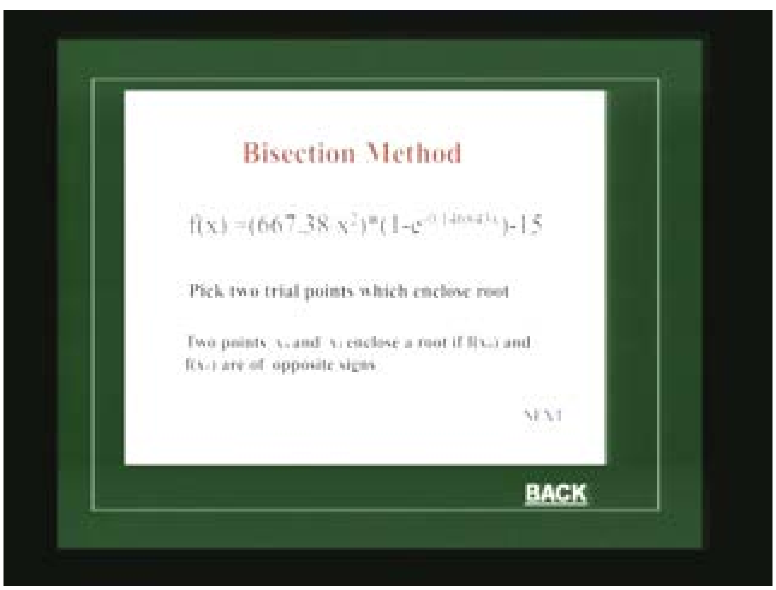

So I am just summarizing that, what we discuss here and so then we will have root like this, that function behaves something like this, that we have roots x_0 and x_1 here, okay x_0 is positive, x_1 is negative “2.5” and “5.5” and the root some were here.

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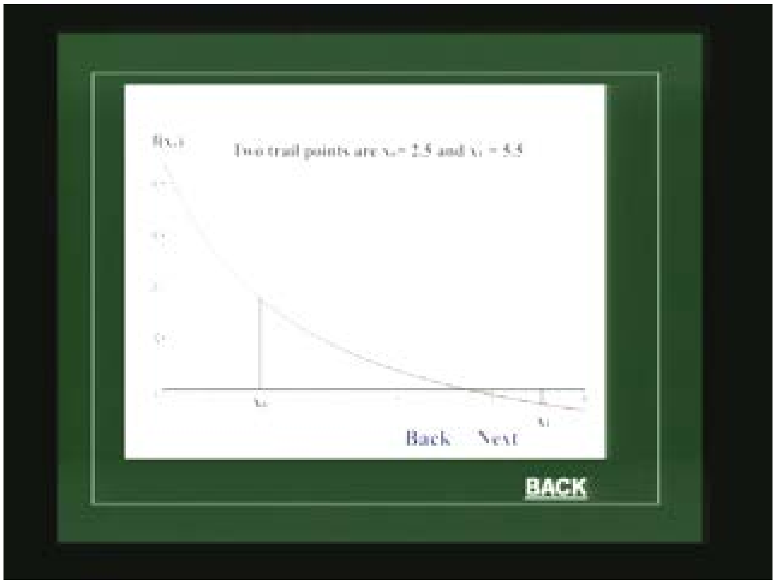

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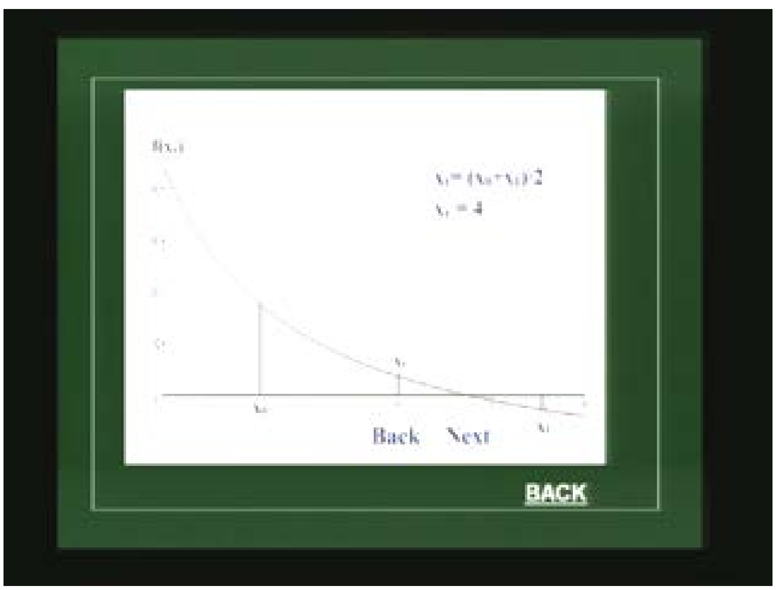

From the graph you can see but we have not plotted the graph, you do not know where it is so we have taken two values x_0 and x_1 okay such that x_1 is positive and another is negative and then we found the mean of that, that is equal to 4 which we shifted the x_1, x_0, x_0, x_1, some were here 4 and the function value there is positive right that we can see. So we can replace the x_0 by that so our x_0 has shifted here, we can see in successive bisection okay we will continue we will doing this and then you can see that the this two comes closer x_0 and x_1 closer and thus the way finally will arrive at the solutions. We repeat this bisection procedure okay and we always enclose the roots, we will always make sure we always enclose the roots in each interval. Okay and then as soon as we reach the accuracy which we need the precession we need stop, so in this the function the root “4.72” okay that is what we will see. So we have to see, we have to look at accuracy when we reach the beside accuracy we stop it, so the question is what is desire accuracy okay that also we have to worry about.

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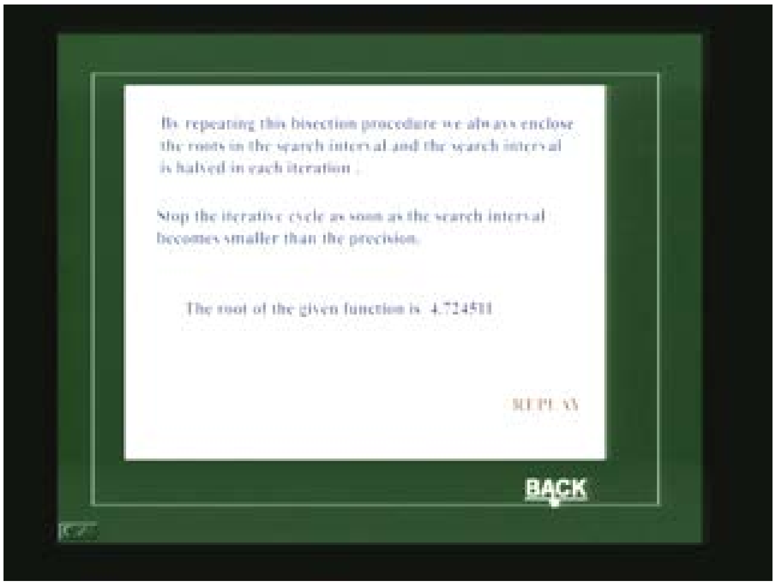

For example, in this particular case we will see that we will bisection procedure with each iteration, with each iteration, we will half the interval write because that is the scheme here each iteration here the interval is halfed. We will start from this then we will go to that and then we will go that we will so we will keep the halfing the interval, so in a about in a about ten iterations the interval would have shanked about 1000.

Okay so now we have to be careful that we have to stop it some were when it is the machine precision otherwise it will get into infinite loop. So we should not ask for accuracy which have beyond the machine precision. So that is we something we should be careful about. So now the next question is how do we will come with the an idea of what is the accuracy which we requires what is the accuracy which we can get, that is what we summarize here, that so we have to terminate this iterative scheme algorithm okay when we predetermine criteria for the allow error right, so that is what we have to do.

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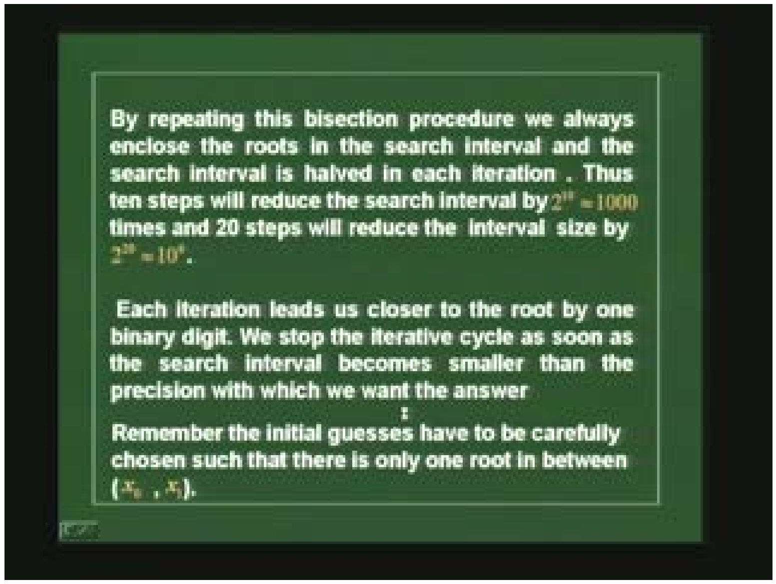

So we cannot ask the function ask the iteration to be continued to be till f of x goes to 0 exactly because is the floating point operation. So we can go exactly to 0 the function cannot go to 0 and also because when you do this scheme x_0 plus x_1 by 2 x_2 we will have a some kind of round of error is coming okay beyond some iterations, we will not able to get any more accuracy.

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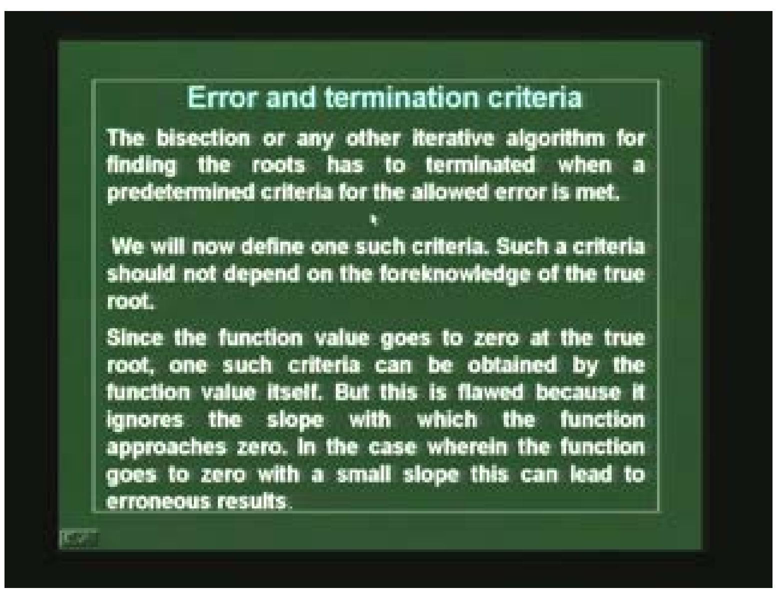

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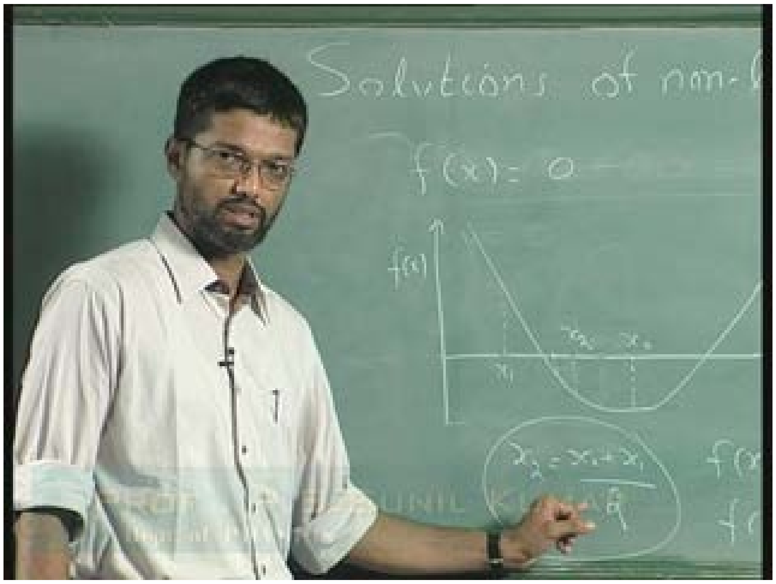

So we have to define a proper criteria. So we cannot of course define a criteria that the depend on the knowledge of the true root right. So we do not know the true root so we cannot define a criteria which based on the actual knowledge of the root because we could actually have multiple roots solutions and we have to do automated program so we cannot have define the criteria based on the knowledge this is algorithmic details.

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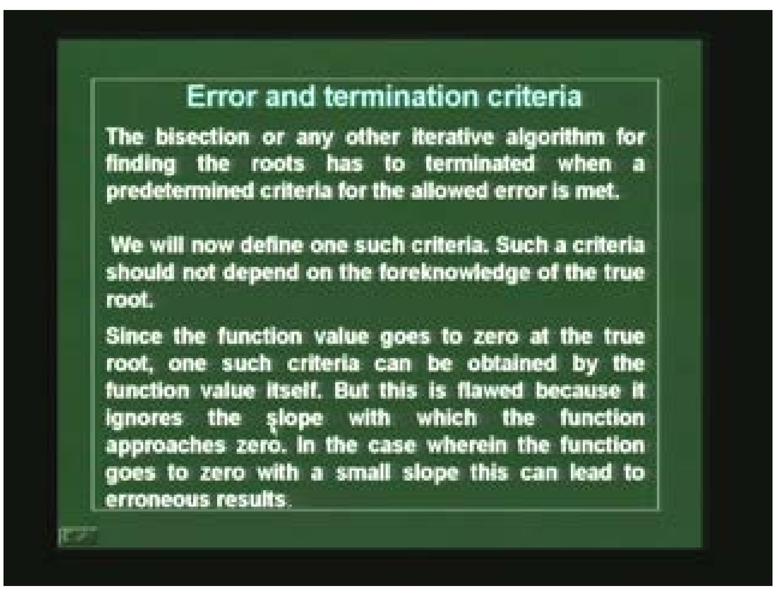

So since the functions goes to 0 the true root the one such criteria is that function value itself, okay now here is the problem okay because the function value we could say that this kind of program, this kind of function it might work well okay what is saying that we know that the function goes to 0 at that point we could say that okay I could have when the function vales reaches the 0 the function f of x reaches f of x closes to some epsilon, okay that is epsilon, the small number which is in the machine precision and then I can stop this iteration that is looks reasonable way of doing it that but again the problem is here because the way the function approach the 0.

Let us see that, is in kind of function okay but let us say the is function something of this form. We will say that that is again my 0 axis, x axis that is my f of x now, we will say the function is something which goes to 0 rather slowly it is like okay something like this the some function which goes to, which goes like that okay, in that case now we know the root is here this slope could be very small, in that case if you put in f of x equal to some small value is my root that could be very far away value it could be here or even from away here because this value is so small, we could even have the whole range of x values for which we get the same answer that f of x equal to epsilon.

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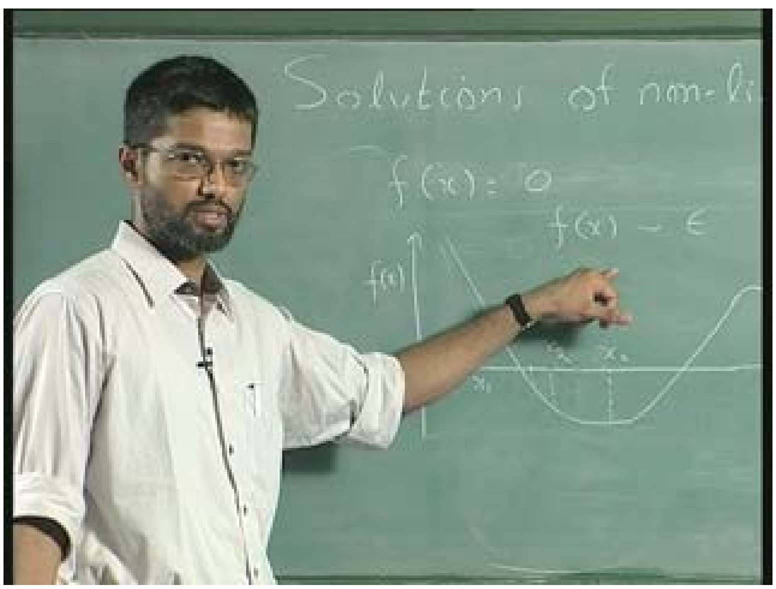

So when the case in this particular case, in this case we are looking at 0 of the function, so the function going to 0 write is not a good criteria because if it goes with small slope okay this can lead to an error message. So that is one case where it would fail. so the best approach would be that look at what is in the each step, what we get in each step that is, you find the new x_2 value we find a new x value write as of now root and we compare with the old value and if the ratio of this two that is a better approach.

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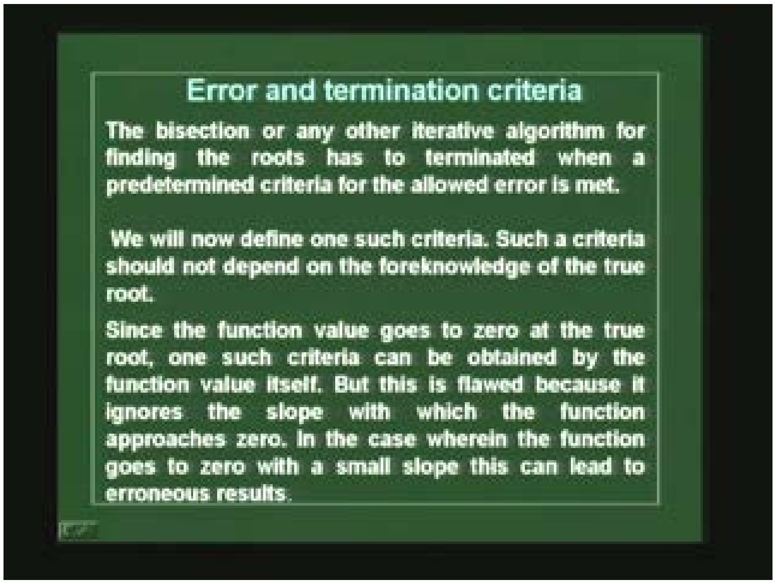

Okay in this case, what we called the x_r new is the what we obtain from bisection x_0 plus x_1 by 2, so we did this method here okay.

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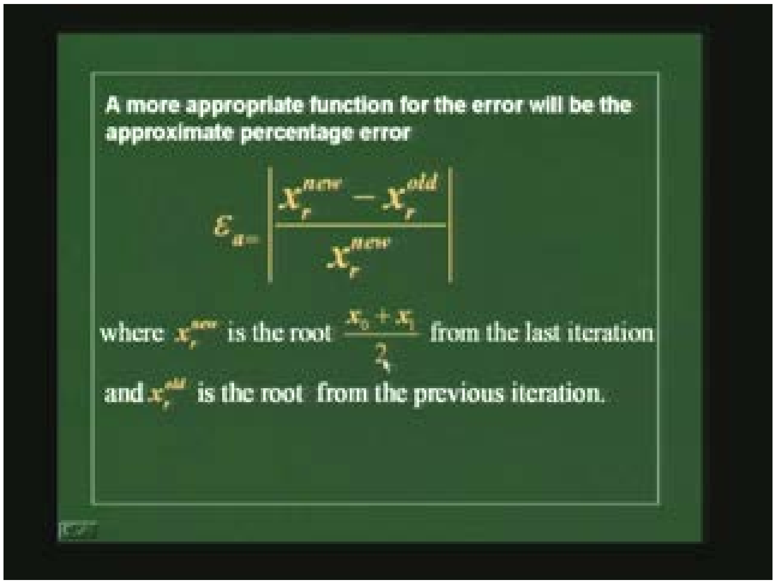

So here we did that x_0 plus x_1 by 2 and we brought it here so that is a_1 value then we took this a x_0 and then we did another iteration x_0 and x_1 by 2 and we got the value here okay so the x_2 in the first step this is the x_2 in the second step this is called x_2 and this is called x_2 new okay then I can x_2 new minus x_2 old divided by x_2 new mod of that. So that will be the one criteria of epsilon okay.

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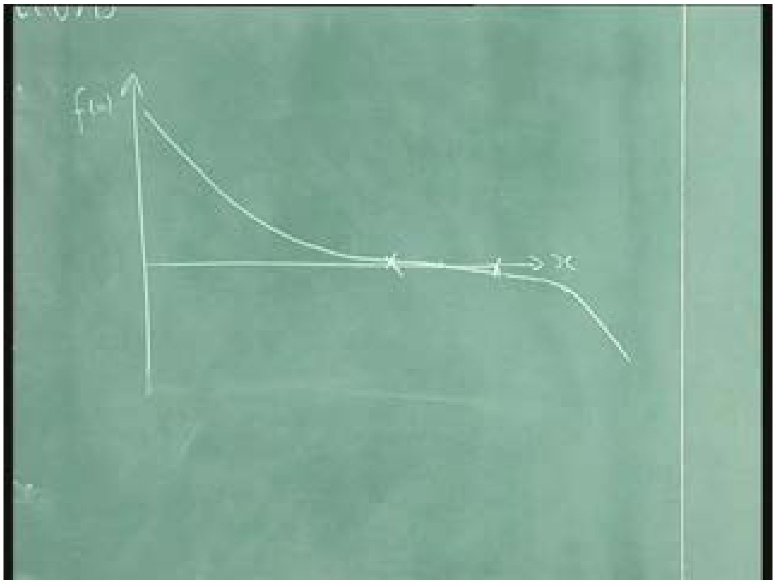

So instead of looking at the function value itself, we will look at each iteration how much this is jumping that will be okay in this kind of cases okay so if this is not jumping too much write the roots are jumping too much but will be close to the actual value. So that is the idea, so that is a better method because this does not depend on the slope of the function.

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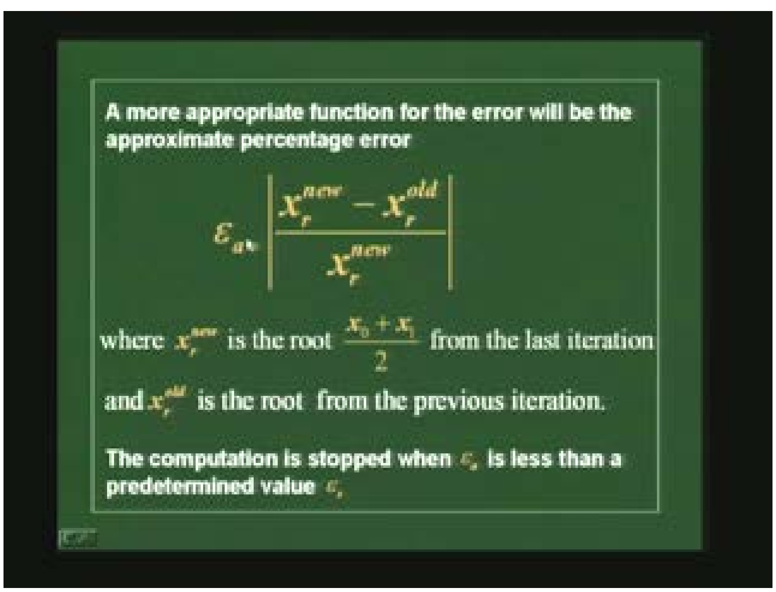

So the slope it does not depend on the slope of the function, so even with the cases the slope is two small this would work okay. So remember here, what I call x_r new, what is obtained from the bisection that is x_0 plus x_1 by 2 and then we will called x_r old what we obtain from the bisection the previous iteration step okay, so then we should can epsilon a which is now normally again the machine precession the pre determinant value it normally should be higher than the machine precision. So the round of error does not take to an infinite loop okay. Pay attention should be paid to that fact.

Okay now here is an example of such a routine, you can do it with the calculator okay we have the function of the form 1 minus x square plus log 1 plus x equal to 0 okay. So I make some guesses, okay some guess is tabulated here, okay the guess values here x_0 is ".5" and x_1 is 2 so x_0, x equal to ".5" this is a positive number and for x equal to 2 this is a negative number is easy to see that okay. So then my initial guess x_0 and x_1 are actually, what we called brackets the roots, brackets the root it is on either side of the root and then i can take mean of this that x_0 and x_1 again "1.25 " then I look at the function value that "1.25 " values that happen to be positive.

So you remember now, this is positive x_0 is on the positive side x_1 is on the negative side here okay. So f of x_2 is positive, so what I would do is, I would just replace this 1 by x_2 okay, now I am bracketing my solution between "1.25" and 2 negative the step you can see so, in this case you to find epsilon a i have used x_0 and x_2 this is what I am going to replace. So I check, I take x_0 and x_2 this is what I am going to replace in the first step next step is obvious what I am going to replace but here its is x_0 and x_2 difference between these 2 by "1.25" as by as my epsilon, so then I replace that by "1.25" okay and then i took the mean of that, so I got "1.625". So x_0 and x_1 and that function value that point is negative.

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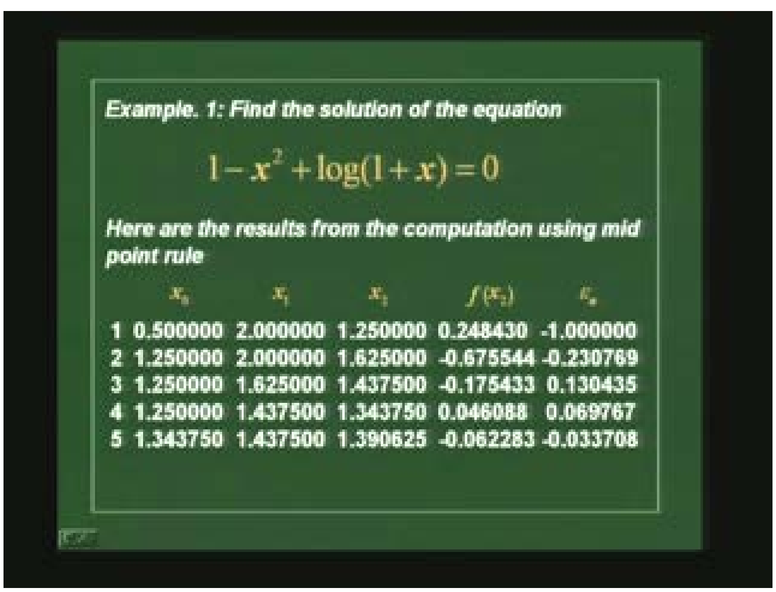

So obviously, I am going to replace as x_1 by this is new value by a new bracketing a new "1.25and1.625". So we continue with this process then we got the function value to be negative it is not necessary that we need to jump of it on either side right. We have in this case the function values is negative again, we will replace x_1 by the new value by the new value is "1.4375". So now we have the new bracketing "1.25and 1.435 and you can see that error absolute error that is the magnitude of this is decreasing pretty fast.

So in a about, in the case 15 iterations, okay we have gone into very small error, okay and we find that a new our root is about "1.36389" okay we have a accuracy up to, that. Okay, so that is what we have obtained in this thing, we can see that the error in this function gone to in this particular case of the function is already going to 0 and you can see that here that the function is already going to 0 very slowly here. Okay the but the epsilon a you can see how fast is decreasing in this case, okay so definitely this is a better method of determining the accuracy of our root by looking at this epsilon a, x_2 which is x_2 new minus x_2 whole by x_3 that is an example of one such method.

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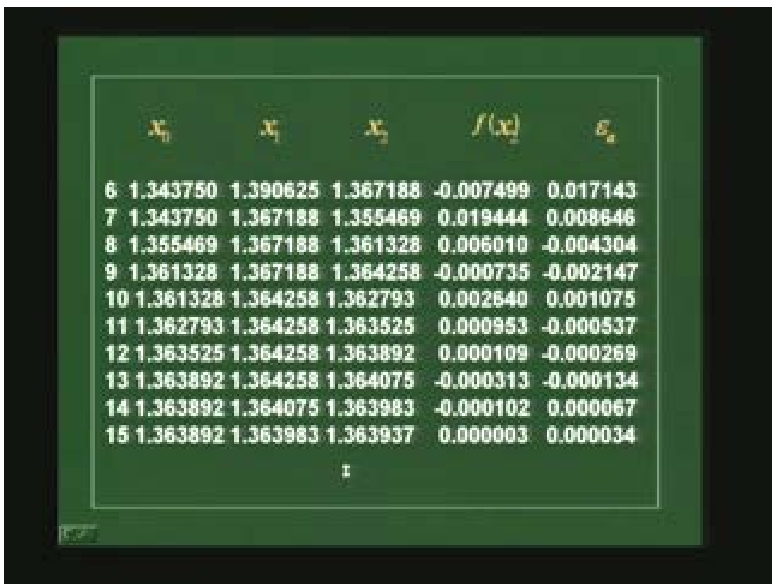

	x_0	x_1	x_2	$f(x_2)$	ε_a
6	1.343750	1.390625	1.367188	-0.007499	0.017143
7	1.343750	1.367188	1.355469	0.019444	0.008646
8	1.355469	1.367188	1.361328	0.006010	-0.004304
9	1.361328	1.367188	1.364258	-0.000735	-0.002147
10	1.361328	1.364258	1.362793	0.002640	0.001075
11	1.362793	1.364258	1.363525	0.000953	-0.000537
12	1.363525	1.364258	1.363892	0.000109	-0.000269
13	1.363892	1.364258	1.364075	-0.000313	-0.000134
14	1.363892	1.364075	1.363983	-0.000102	0.000067
15	1.363892	1.363983	1.363937	0.000003	0.000034

So now we could use similar method and another one which is now that was the a simplest way to do bracketing and then do the mean and take the mean retain every step and we could slightly improve on that by using, what is called a method of false position this method again uses same bracketing technique that is, we need two guesses x_0 and x_1 with f of x_0 less than 0 and f of x_1 greater than 0 either way does not matter the notation we are going to use choose f of x less than 0 and f of x_1 greater than 0.

The difference is again that this value of x_0 and x_1 we will go to new values of new value and replace one of x_0 or x_1 depending upon the new value what we get is x_2 f of x_2 what we get is positive or negative the function is positive or negative is same as that as bisection method not very different so what is different is that the new value is not the mean of x_0 and x_1 we will try to be little more clever in this particular case.

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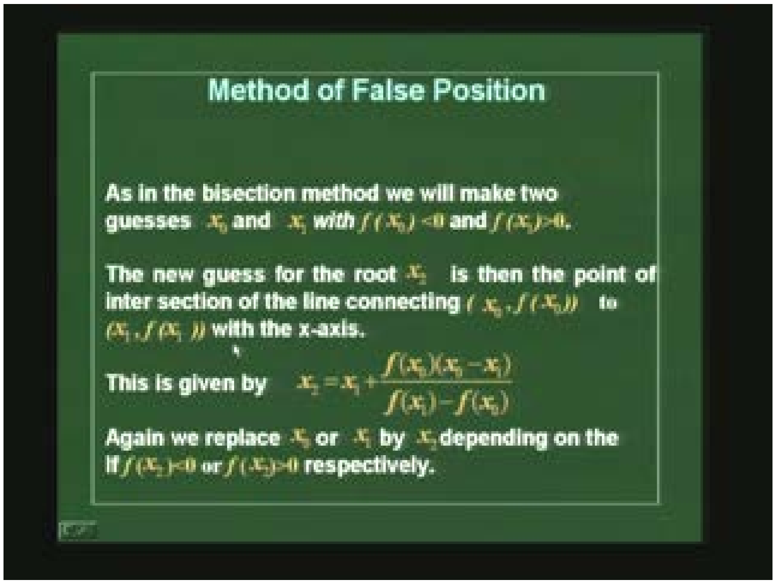

So now the new guess here is to obtain here by looking at where line connecting the 2 x_0 f of x_0 z_1 f x_1 is the meet x axis, that is a here I will show that show that here again. So you would had in this case f of x_0 let us we take this, we had f of x_0 and x_1 that is our initial cases okay, so we choosen an f of x_0 to be less than 0, so the f of x less than 0 and f of x_1 greater than 0 so now in the bisection method what we did was to simply choose x_1 plus x_2 by 2 that is what bisection method in this case.

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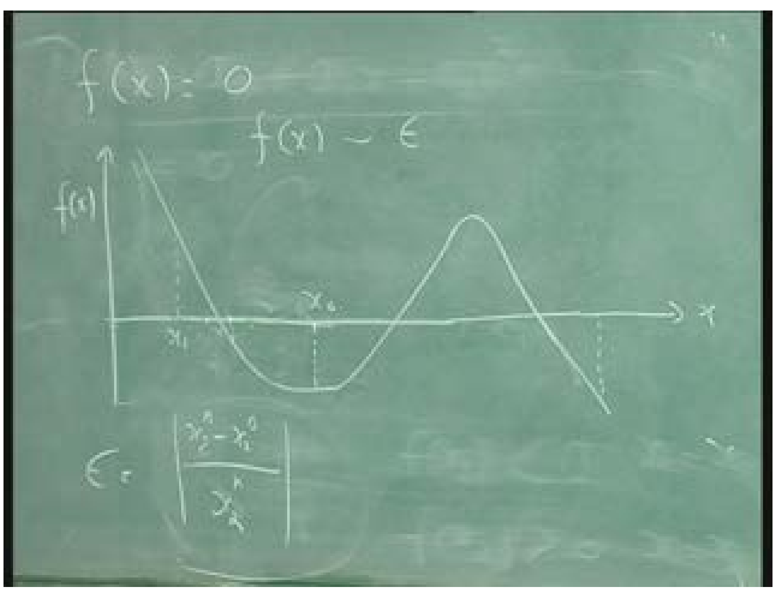

Okay in this method of false position what we do is, we take a straight line connecting this point to this point. Okay we will draw a straight a line, okay like this from this and then we say this is where that line meet the axis the x axis okay that is a new solution that will be our x $_2$ we write the equation of this straight line okay and find the intersection of that straight line with the x axis that is what it is so the new position. So we know how to write the equation a straight line which connects 2 points x_0 f of x_0 and x_1 f of x_1 and definitely meet the x axis somewhere, okay so then that if this crossing the x axis because changes a sign. So it will get that intersecting some were of that x axis, so then we will replace we will get the new x_2 value which is intersection point then the formula, we can easily derive from that we can write the equation of straight line x_0 which connects x_0 f of x_0 and x_1 f of x_1 and we take 0 of that function, that straight line where it crosses the 0, okay that is gives this a new value x_2.

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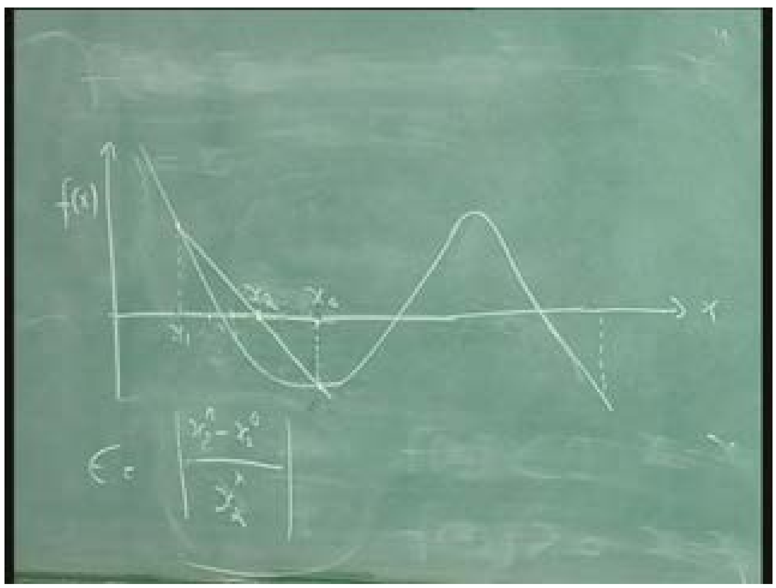

So then again, we will replace x_0 or x_1 by x_2 depending upon whether it is positive or negative in that cases x_2 being f of x_2 being negative, so in this particular case we will replace by x_0 by x_2 we have now of this. Now we have a point here and in the next straight line would be we will have something like we have to draw straight line between this point and this point that again cuts the axis here.

So it comes very close very fast okay this is very much faster, little more complicated in the case formula as of as simple as x_0 plus x_1 by 2 but we have to find solution of this equation its very simple and then you can find a new root here next time and you can see that comes to very close that is your actual root. So that is okay so we remember again that we assume the function as opposite sign either side of root is important, okay if you to remember okay.

So what is the method of false position, we assume that we the function supports its sign which is always true right and then we will make 2 guess x_0 and x_1 such that they are negative and positive and then we will find the line connecting f of x_0 f of x_0 and x_1 f of x_1 and we will find the point at which that line crosses the x axis and let the point be

x_2 okay and then we have the expression for x_2 and then we will replace x_0 or x_1 by x_2 depending upon whether f of x_2 less than 0 or f of x_2 greater than 0.

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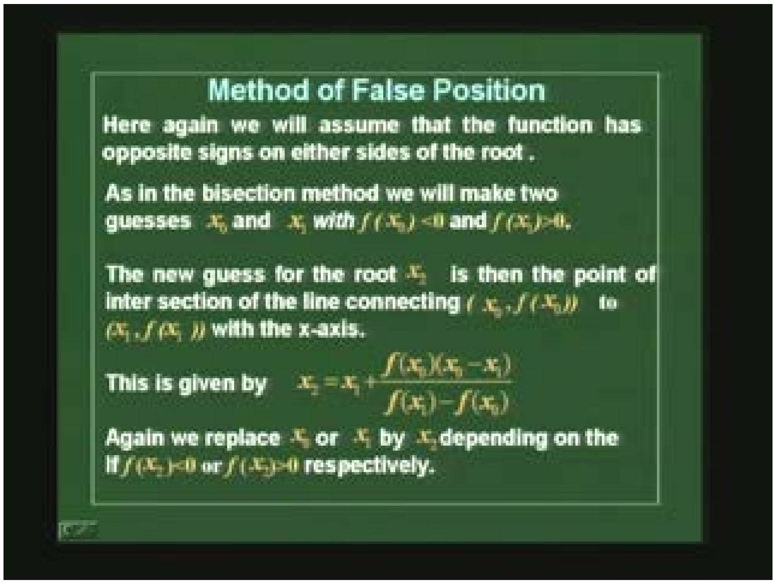

Okay so again we will see in the summary in a in a block diagram here. Okay so here we will see again that false position method, so again we have take the same function as the too, okay and then we again taken the two functions that encloses the root.

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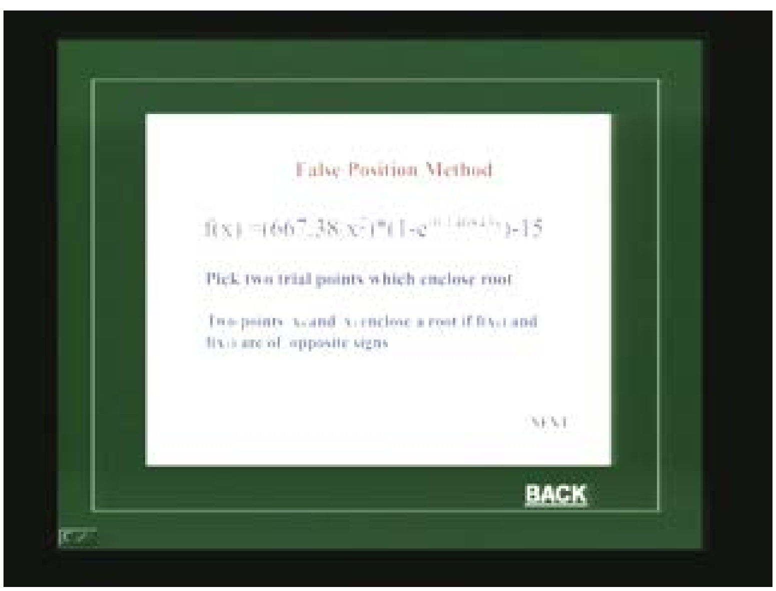

Okay again the same as in the bisection in the same function same as in the bisection case and then now, we will draw a straight line connecting this two functions this two function points okay that is what we have done that our new plot here write that, we will shift x_1 to that new point okay, so we will have x_1 sifting to this new value here.So the x_1 as shifted that point and then we again have a new line now connecting this x_0 and that function value that is shifted to here that is the next step. Okay we will keep new doing this iteration again and again. So we will continue this iteration till we desire the accuracy. So that is what we have shown here.

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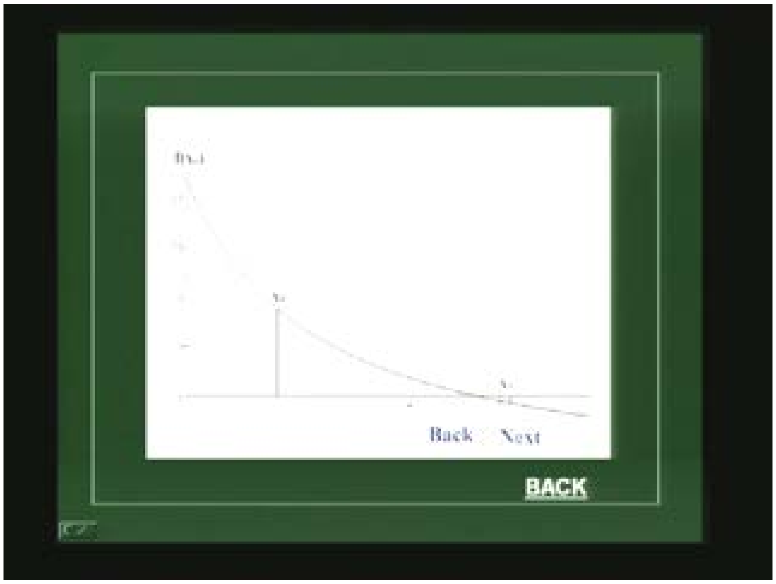

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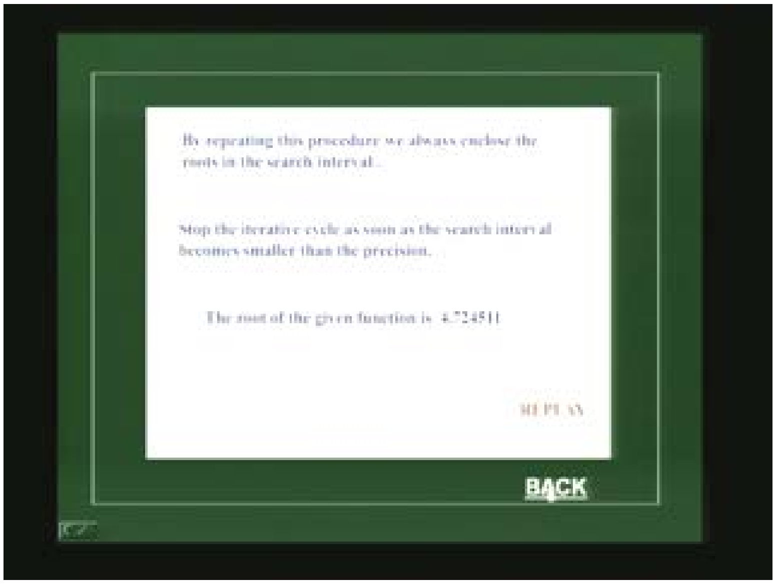

So thus in this sense it is very similar to what we saw in the case false position the bisection method not very different and we will again see an example of use of this function using log of 1 minus x square plus log of 1 plus x as we saw in the earlier case.
Okay and we will see that this converges much faster than the one which we saw earlier that is the bisection method. I will also see that the error is decreasing monotonically okay so but there it is a warning here there are problems with bisection method.

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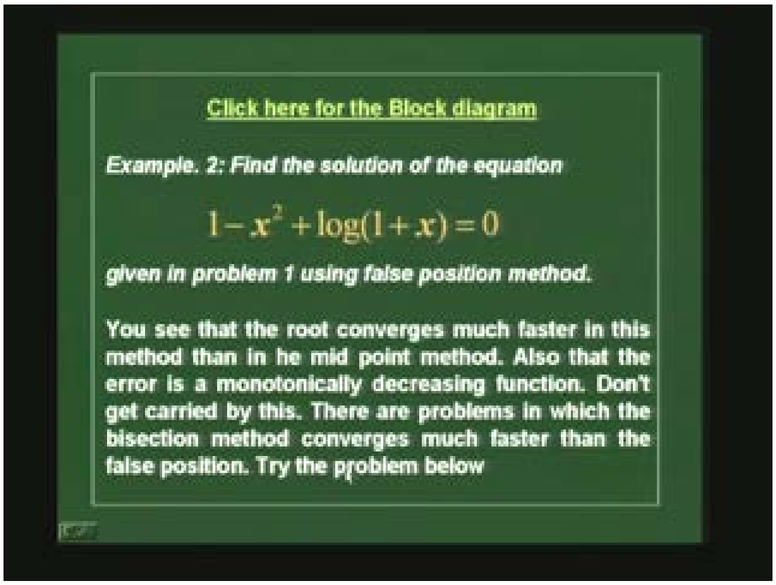

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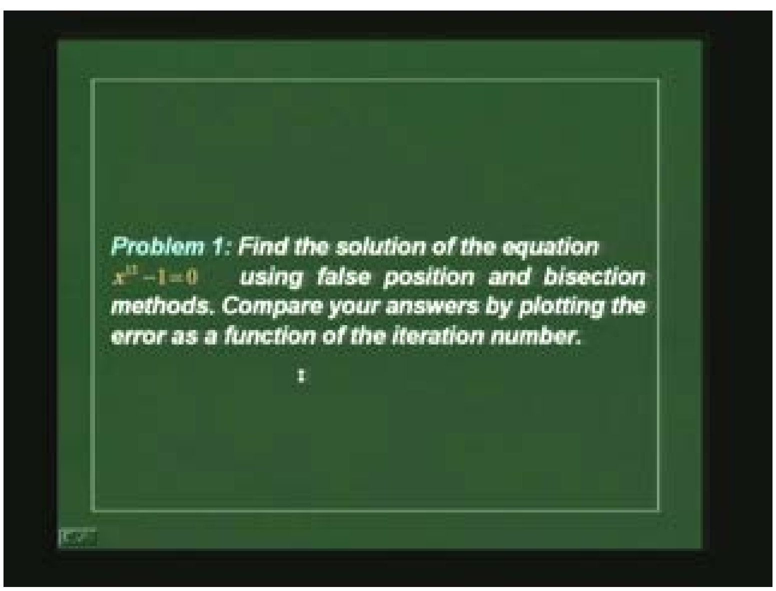

Okay, so that is you can see that yourself if you try to solve an equation of this form, so you encourage to you look at this form the function of very simple function x to the power 12 minus 1 equal to 0, okay and you compare you do this with the bisection method and also with the false position method and compare your answers by plotting the error as a functional iteration number and see how does it converges.

The 2 methods you can try that all this things that were by taking as x_2 point straight line connecting this two initial guess crossing the x axis or by just taking the mean of this x_0 and x_1. So that is what we have seen, so far so in this both methods need a good guess about where the 0s are that also another point. So we compare the 2 methods the 1 thing is for bisection method, this false position method which converts much faster than in some cases and but the both methods require some drawbacks or common drawback that is 1 is that both requires the 0s, all the 0s to be listed.

So we need to know how many 0s are there and we need to know where the 0s are okay before we proceed to find the answer 36.45. So one thing is to how to list all the 0s, so the one would be of course to plot the then look at it okay, so then the simplest scheme to list all the 0s will be to start from one end of the interval whenever you look at all the points within which the function changes sign all my time changes the sign again here is the problem that we have to go in a step size which is smaller than the distance between any 2 roots.

So how do we list all the 0s that is the question write in that question? We will say that okay start from 1 end and keep going towards the along the axis and every time it changes the sign i know that i have passed one root. Okay I have 10 here and I passed here get another 0 changes the sign x, but again this a problem that steps side we use to go right.

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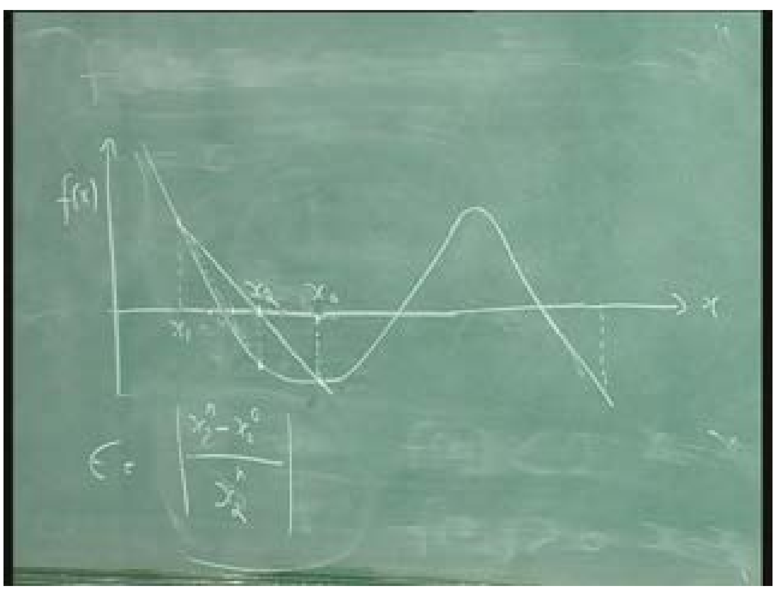

We will see in a implementation in a program, when we look at the program that the step sides which you would see as we go along x axis has to be smaller than interval between

these roots, if you go larger interval that we might miss okay, we might jump from here to here and we have the larger step sides and we say we are passed the root, we will miss both the root okay that is possible we have to be small enough to an increment okay, the increment use to be very small then the question is how small what is the value of that.
So that is the problem with this okay, this method is we need to have an idea about the other side. Okay, it is an advantage is to have to use methods which does not this kind of bracketing method that is something we should be looking at that now okay so again we one more problem is here.

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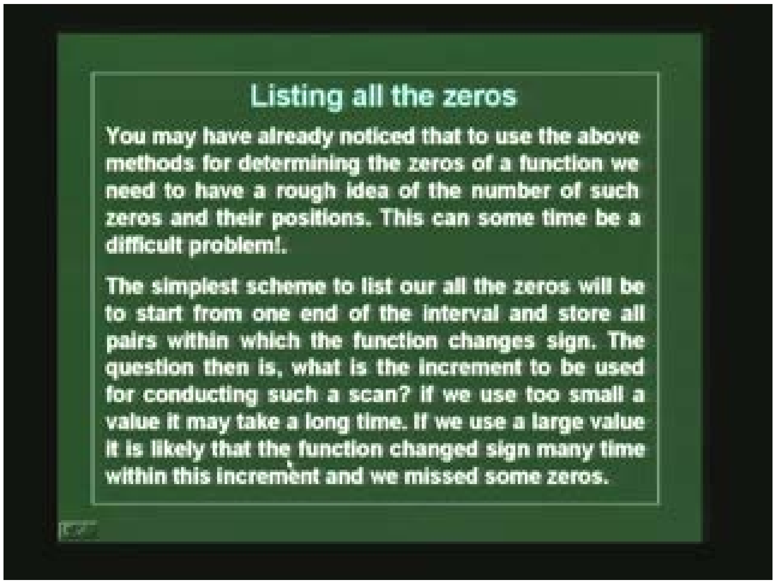

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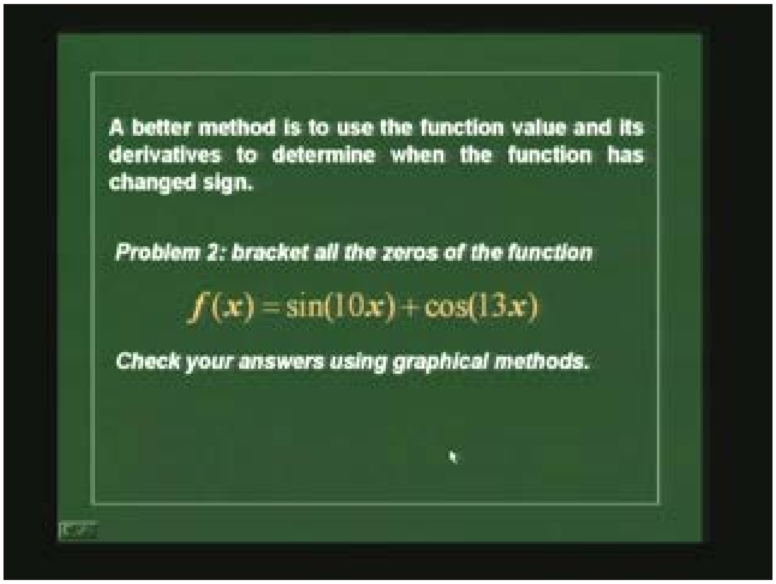

So we will try to bracket all the 0s of this function, of this solution the bracket all the 0s of solution by some methods by using different steps sizes, sin 10 x plus cos 13 x. So you could just bracket all the 0s of this function, so do not try to solve for it but just bracket it, plot the function and you can see the you have obtain all the 0s which is just to demonstrate that you would have problem with the step size determination. I have to get just feel for this step size determination problem.

So we see that the common problem with both bisection and method of false position is the determination of 0, that is the biggest hurdle otherwise, it is extremely simple method both are extremely simple method to implement. We just have to make one guess and then we can continue with doing that finding 0 and once you found 1, 0 once you found, we started with 1 x_1 and x_2, we found 1 x_1 $x_2$1 function then we could take little above that.

Okay and then again the bracket this solutions. So once we have found 1, 0, let us say in this case, we find solution one, so the one solution from the axis that is our solution axis okay then the next solution to find the next solution, we could start with the initial guess with one value slightly higher than this and then the another value where the function has changes the sign to again we look at the function changes the sign then we start with the other value there and then again you can bracket that it will converge to that that roots once you found one root, it will easy to get another root but again when you search for the zeros, then we will search for the points function changes the sign.

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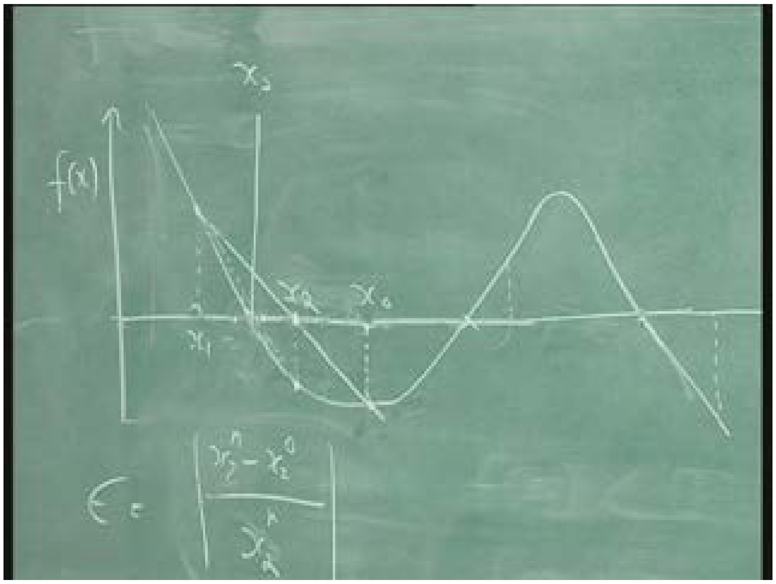

Okay, you have to be careful that is a most important point here in this both in this methods and the two important points is to take care when you write a program on this is 1 is that is to make sure that you have enclosed all the roots and the second thing is that the accuracy you demand that x old minus x new divided by x new modulus of that should not be more than the machine precision, such that we will not get into infinite loop because of the round of errors that something you should be careful. Okay now we will look at some methods which does not use the pre knowledge of the 0s of the function, okay they are called direct methods. So this is iterative, we saw then the methods we need some idea about solutions, so now we will look at some methods which will not use that. Okay, what is the previous thing, what we saw that is an iterative scheme and we saw 2 guess solutions on either side of the 0 of the function.

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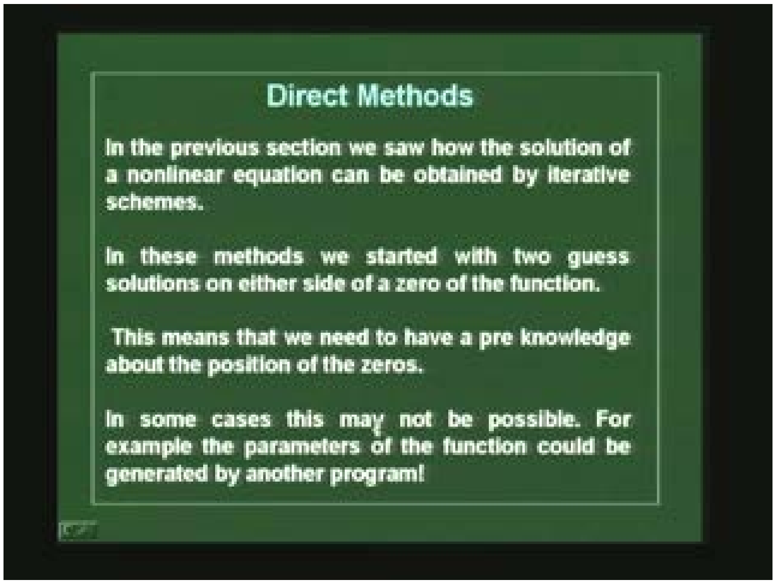

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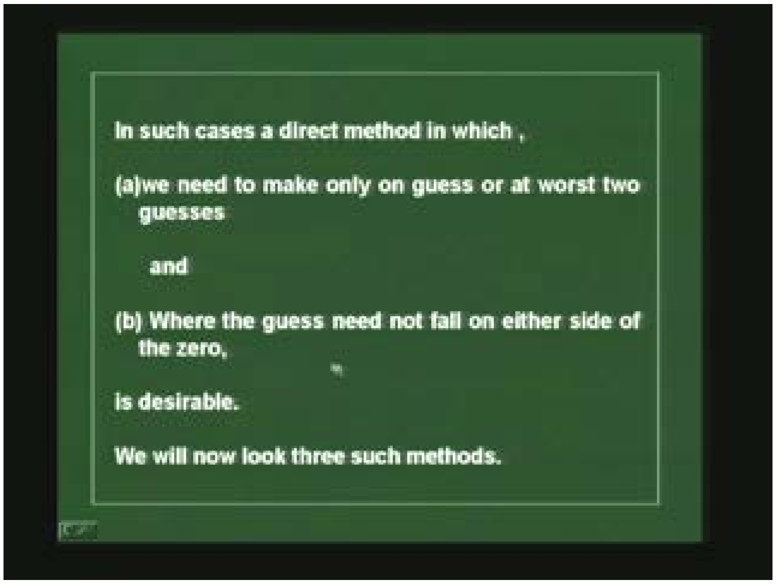

So we need pre knowledge of the 0s, okay that is a basic example. So now, we will look at cases we need we need two guess on either sides 0 is not needed because we might have a the function value itself given by some of the program which will not be able to scan an whole thing and find out where the 0s are.

Okay in that case, we need a slightly different method, so basically what we are saying we need to make the best or worst we need to make only one guess it will be very nice method in which we will not make any guess at all, we have to find the solutions of non-linear functions okay in which you do not have make any guess that will be the best or it at worst one method which we will make one guess. So we do not have one idea about were the 0s are okay that is one guess, it can be on either side of the 0 okay we do not have one side 0.So it can be any were on either side of the 0.

Okay, so we will look at some methods uses the form. So the one is fixed point iteration so in the fixed point iteration method, we will convert the function f of x that is the next method which we will look at. So it is called fixed point iteration method, so here what we need is again we need one guess.

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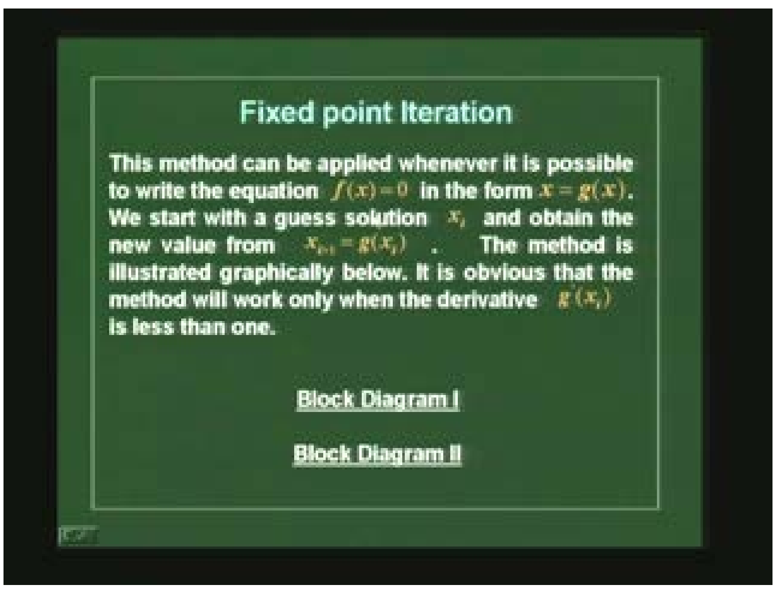

So we have the function of the form f of x equal to 0. So that is what we have okay so as we before some function f of x equal to 0 now what we want to do is we will make one guess that is x equal to x_1 the solution we will say x_1 is our guess the first guess is x equal to x_1 and then from that we want to go to 0 that is the value at equal to 0. So in the case of fixed point iteration, we need to write the function f of x as g of x minus x. So that is the must you should be able to write the function f of x as g of x minus x, and then we can see that f of x were goes to 0 is equal to x same as x is equal to g of x. So this equal to 0 implies right that implies x equal to g of x that is the basic idea behind the fixed point iteration. So, we start with one guess and we have this g of x equal to x it is like a map.

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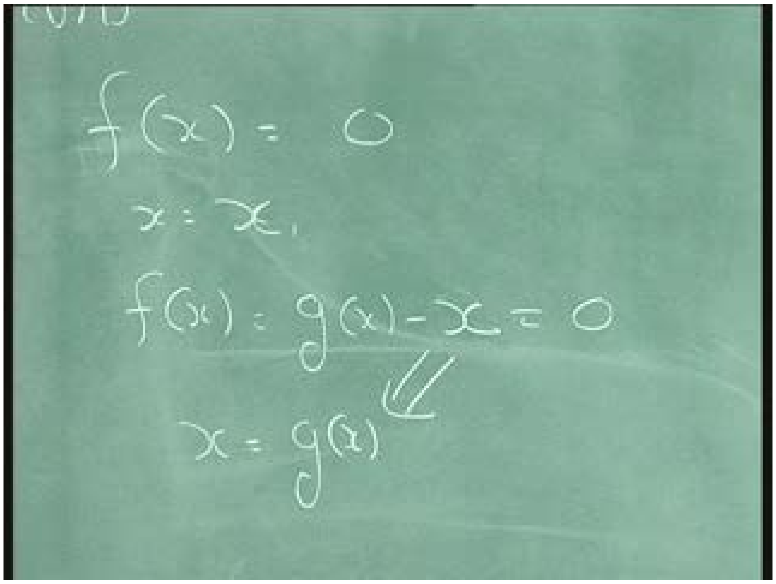

Okay, then we will say that next iteration point is obtained by we start with one guess is x_i the next point is x_i plus 1is given by g of x and you can see that this is actually a solution your guess x_i this is a solution, then this will be the identity right. So you get new x_i plus 1 you will get same x_i that is the idea that would be so the error will be g of x_i now I will show this is method graphically again here so when we will come back to here and discuss where this method can fail the method can actually work better we will look at that now.

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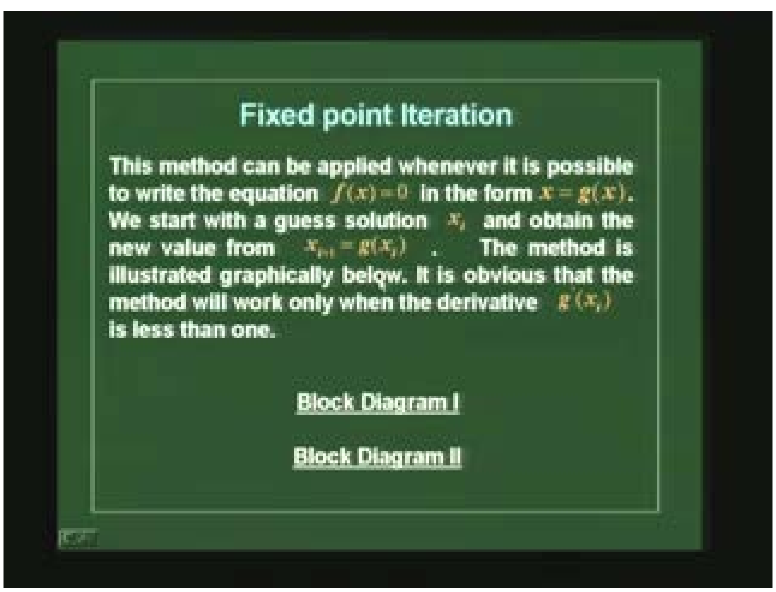

Okay here is the fixed point iteration, so we will write f of x equal to 0 as x equal to g of x okay, and then we will start with some guess. So we have converted f of x equal to g of x we start from f of x_i plus g of x_i. So then we will continue with this, so that is the basically graphical representation, so we have function like this and basically what we are trying to find this y equal to x line is intersection point that is our idea. Okay we will start with this x equal to 0 and then we have to converge into root okay

So we started with some function this is graph here shows x and g of x write we started with some guess x equal to 0 and then we can generated the function g of x some guess and we have a new x value and the new x value x, g of x because new x value, okay that gives new g of x here, okay and then gives the new x value so this is a line y equal to x. So by writing x equal to g of x basically what we trying to find is the is the graphically means this g of x function likes that then y equal to function likes that we want something that try to something satisfies both intersection point of that.

We have y equal to x, y equal to g of x that satisfies the our x value should satisfies both so that means that, we need where y equal to x line intersection line x g of x. We want to find the iterative graphically it is very easy um but iteratively have to find out that is what we have to shown here. So we started with x_0 value write that is the g of x corresponding to this given by this, write that is our next x value right.

So that x value is corresponding to g of x, so from that x value we found g of x that is here. Okay that is our next value is x_2, okay and then we have found g of x from there then we will finally converge on to this point slowly but converge in to this point here. Okay so again here we will see that it will work only when in the slope of the g of x line is less than 1 otherwise it will diverges.

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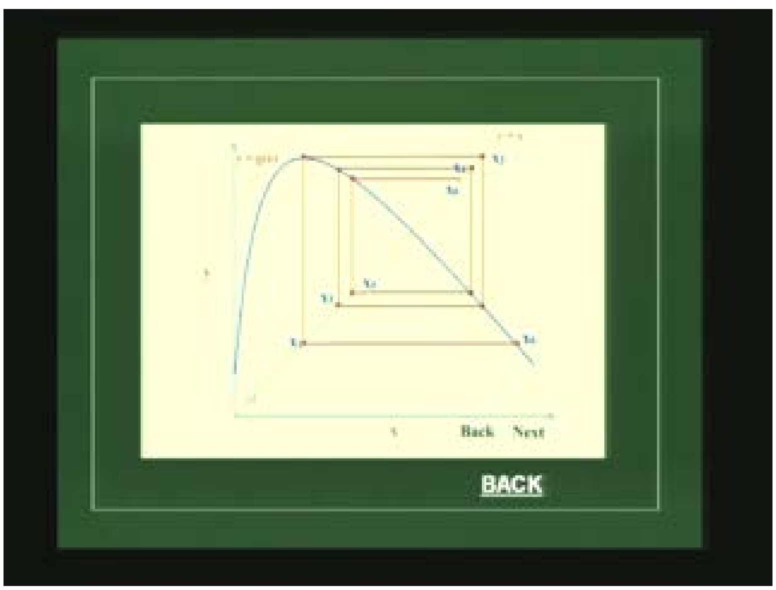

So we will make progress here rather slow and one more important point remember that it will not converge when the slope of g of x line is greater than 1 that is can be shown here okay now here is the function you have to find the 0 so here you need initial guess was here we could find here it will converge here. Okay now this is a the function g of x function as slope less than one it is actually negative slope here and you started from here and it will converge very fast here but we can say it started from we stared from where else or the initial guess was only one initial one guess, we have to make for 1 x_0 but we made it some where here okay and hoping that converge we will on to this point but we will not because, the slope of this function is here this is larger than 1, so what we basically find is it does not intersect is this goes away.

So that is the problem with this method. So we should be able to write the f of x equal to 0 as x equal to g of x option and we also need the initial guesses at points where g prime of x that is a the derivative of g is less than 1, okay the g prime of x should be less than 1, okay the 2 values have to be satisfied so that is something which you can easily see so by just taking the x g of x_i plus 1 minus g of x_i okay divided by x_i plus 1 x_i that is will be the slope of this and comparing with our x_i plus 1 x_i minus we are writing this equation here as x_i plus 1 equal to g of x_i and so we can easily see that g of x_i plus 1 minus g of x_i divided by x_i plus 1 minus x_i okay you write something like this so g of x_i plus 1 is in our case x_i plus 2.

So g of x_i is x_i plus 1 that is what we get okay and we have x_i plus 1 minus x_i and we want, we want this to be this is a this is the distance between the two guesses, two roots the distance between the two guesses or iterative results in the iteration in the i plus 1 step this the distance between the roots at the i th step and we want to be less than 1 okay this is the g prime

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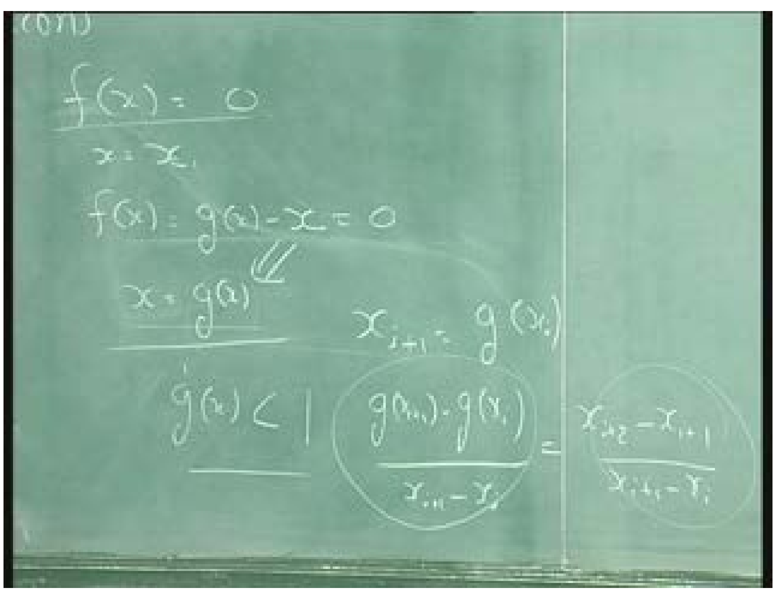

So we know the g prime has to be less than 1 to be work otherwise it will be diverge, so we need the g prime that is the derivative g prime to be less than 1 for this iterative scheme to works thus it will works be the drawback for this particular scheme. So even though only one guess as has to be has to be made this still have a drawback here that is that we need g prime of x to be less than 1 probably 1, that is important a problem with this method, again you could find the solution of the as a problem you could look at function of the form x minus cos x equal to 0, this is the easy problem where we saw obviously can be written as x minus g of x now g of x can be easily identified as cos x here and you just you know find the roots, root of this start with x equal to “0.5” and you can see how fast it would converge.

So we will find the g of x value and then that is this is also written f of x value here, this has to go 0 here that f of x is now x minus cos x write, so I look at the g of x, now g of x is the new x value and again find the g of x here and replace x by that and then you know both by iterations scheme and finally, you have to reach x equal to g of x. So you will find in the particular case, so we have gone to something like that 15 iterations and it is okay the error the error is still not very low but the function has gone to close to 0 that is again the problem of the slope okay.

Okay, so the again one more problem which would be to find the root or equation x minus e to the power of the x plus 2 equal to 0. So you could do this problems with all the 3 methods now which you so far looked at, that is the bisection method, the method of false position and the this fixed point iteration. Okay so we have 3 methods by method of bisection, the method of false position and fixed point iteration. so you could do this problem that is x minus e to the power x plus 2 equal to 0 with all the 3 methods, you could in the case of fixed point iteration we need only one guess so we could do it from x equal to “.5” and x equal to “1.5” as the two different guesses, that is the summary of the method which so far we looked at. Okay, so now we could in the next class probably we would look at some other iterative scheme again another iterative scheme but again which uses only one particular one only one guess.

So the guess problems in which we need methods or the methods in which we need two guess are only fixed point iteration, the method of false position and bisection that is the two methods in which we need two guess and we need two guess such that bracket with 0s and here many 0s many roots and we need many such cases which will bracket of all the roots in the case of fixed point of iteration we need only one guess that is, the root could be anywhere but it could be such that the function derivative of g of x that is now method we write f of x minus g of x and we need the derivative of g of x that around that point to be less than 1. Okay now, we will look at the another method as called Newton Raphson again an iterative scheme okay which is similar to the method of the fixed point of iteration. So that is what we will discussing in the next class, we will also look at the some of the implementations the of the ideas of the methods which we discuss in the next class.