Numerical Methods and Programming
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Lecture - 20
Eigen Values and Eigen Vectors: Bairstows Method

Today we will review, what we learn about Eigen values, the calculation of Eigen values in the last lecturer and then go ahead and look at, so then otherwise, are computing this Eigen values and Eigen vectors.

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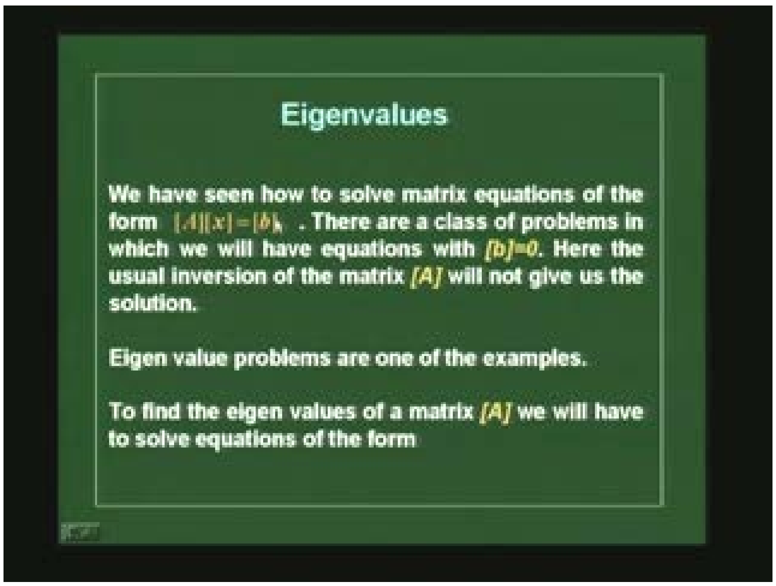

First, we saw that Eigen values problem is as basically, finding the solution of an equation a minus lambda i equal to 0 for matrix of form a. A system matrix equations a_x equal to b in a matrix equations a_x equal to b, the right hand side of equations 0 and then how do we solve that is why we are started this Eigen values problems and the importance of solving equations right hand side of the matrix equations 0, that is what we started the discussion on the eigen value problem and we went to you said would be interested in solving equations of this form that is a minus lambda i and x equal to 0 that is eigen values equations, and we said that this has this kind of equations have a non-trivial solution are provided the determinant of the a minus lambda i_0 that leads has to the what is called the characteristic polynomial that is determinant of a minus lambda i equal to 0 of the matrix of the form a is the characteristic equations for the eigen values.

So this characteristic polynomial, we learn the way of constructing this, that is using the fadeev laverrier method. We show that how to construct given a matrix a minus lambda i, its characteristic polynomial which is lambda n minus p n minus 1 lambda n minus 1 etcetera. So we show that this can be constructed by finding the trace of the series of the matrices given by regression relations starting that fact that p_n minus 1is just the trace of the matrix a and then, we can construct matrices of a into the b_n which is b_n was the first start with a itself, so it is a into b_n minus p_n minus 1i. We can construct matrices form and take the traces of that and that trace which goes has the coefficients of the this polynomial of this characteristic polynomial.

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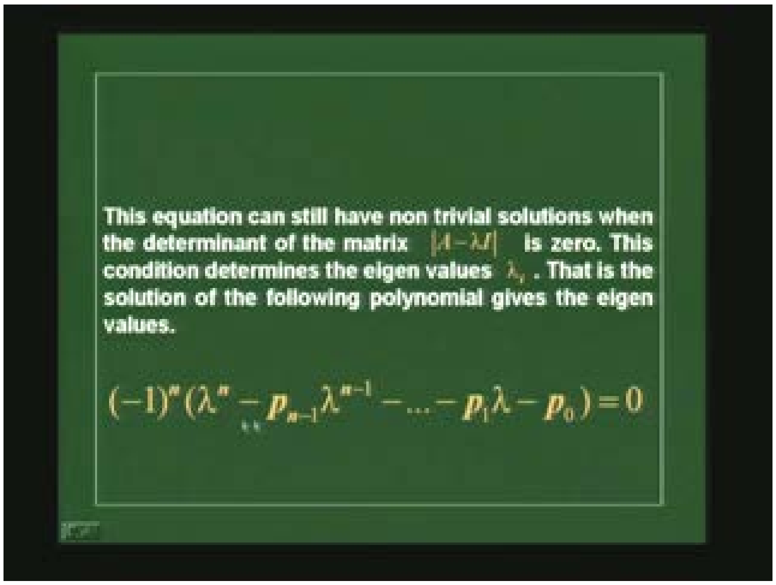

So we saw that and then, we wondering how we solve the equations, now we have the polynomial but we are roots of this polynomial, if the polynomial in lambda. So the roots of the polynomial all our eigen values so the eigen value, so the finding the eigen value the problem of the finding the eigen value was then that way splitted into 2, one is to find the characteristic polynomial which we said for large matrix is finding the characteristic polynomial itself a non trial problems, and we say that use the method use this which we method the fadeev laverrier method for that. And then once, we have the characteristic polynomial, we saw the program also in the last lecturer, of constructing the polynomial coefficients and we have the characteristic polynomial then the equations is how to find the roots of that characteristic polynomial and so now, that leads us to a general problem of finding the roots of internal polynomial and we said that one way to do that is use what is called the Bairstows method.

So in the context of the eigen value problem is the solution of the characteristic polynomial, are even in the more general sensex solution of n th order polynomial is on the roots of the n th order polynomial which is want to be find out of 0 of the n th order polynomial are equations in a roots of an equation of this form that something of the left hand side equal to a_0 plus a_1 x plus a_2 x square of a_n x_n. So the Bairstows method of applies a technique of this form that is we say that, we would find roots of the this polynomial in pairs of consequent variant that is, we allow for the roots to have complex, this is n th order polynomial could have complex roots then, we get them in pairs such in the roots an complex consequent and that general enough, if the coefficients of this polynomial is real then that is general enough that is we do not lose anything by making this particular, taking this particular approach.

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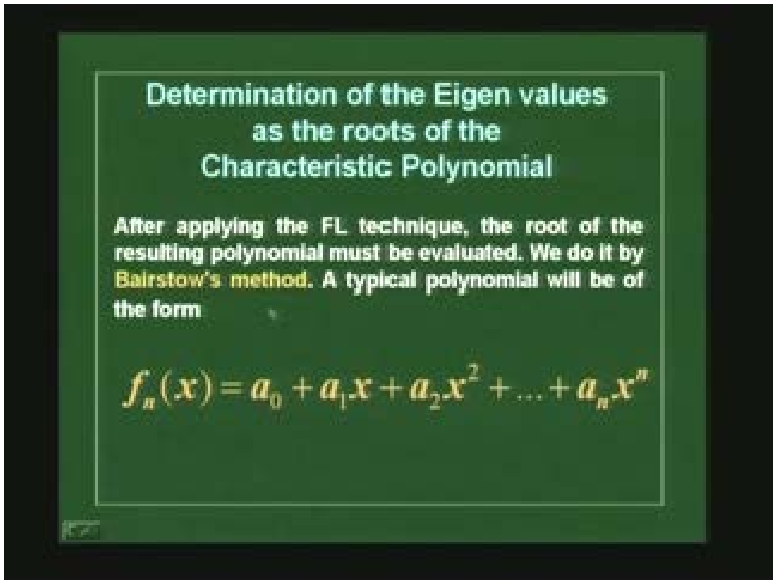

So, if you want find roots of this polynomial, n th order polynomial in Pairs, that is we want to allow for complex roots and then, the complex roots come and consequent pairs and then, best method is to and that is what Bairstows said is to divides polynomial by quadratic polynomial are this factorize this polynomial has the product of a quadratic polynomial multiply by n minus 2 order polynomial right. So that is what be discuss in the last class.

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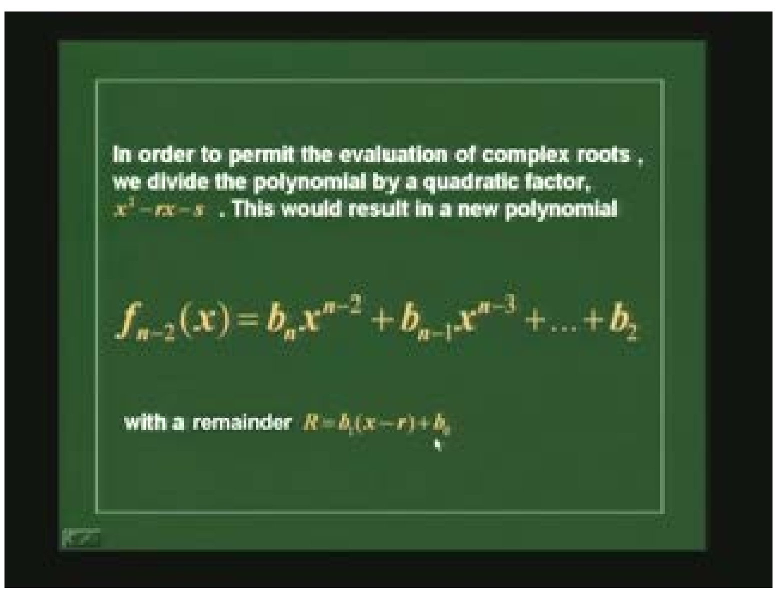

So we said that f_n the n th order polynomial can now be return has x square minus r x minus x into a polynomial of order n minus 2 which is now given by b_n x_n minus 2 plus b_n minus 1 x_n minus 3 up to b $_2$. So then, we say, if we have arbitery value of r and s then we will have a remainder to this, that is b_1x minus r plus b_0. Now the advantage of this method is the we can determinant given r and s, we can determinant this coefficient b_1 b_n minus 1, b_2, b_1, b_0 etcetera by relation of this form. So that give that great advantage that, so we have the polynomial whose nth order polynomial, whose coefficients was give by a ns and then we divided that by x square minus rx plus s and we got the coefficients of polynomial of order n minus 2 or and order of n minus 2 whose coefficients of b and 2 b_2, so that is given here. And then, we have remainder which has the b_0 and b_1 which is given by b_0 $b_1$into x minus r plus b_0.

So that is what, idea we have and we can even determine the b's. So the question is how do we determined r and s, so then we said that, we could do a kind of iterative approach that is we could say that, we start with r and s and we will compute, what the remainder is which will be given by b_1 x minus r plus b_0 and then, we could just then r and s such that the remainder is 0, so that is the idea.

So for that what we do is, we assume this and we start with a guess value of r and s and we could look at using the relations which as just now so the using this relation we could compute and 0 b_1 that is $b_2$would be a $_1$plus r $b_2$plus s b_3 and b_0 is a_0 plus r b_1 plus x $b_2$and we could value of $b_0$and $b_1$with the 0 or not if they are not 0 then we could say that okay I will expand this b_0 and $b_1$and around the value r and s which I just to assume.

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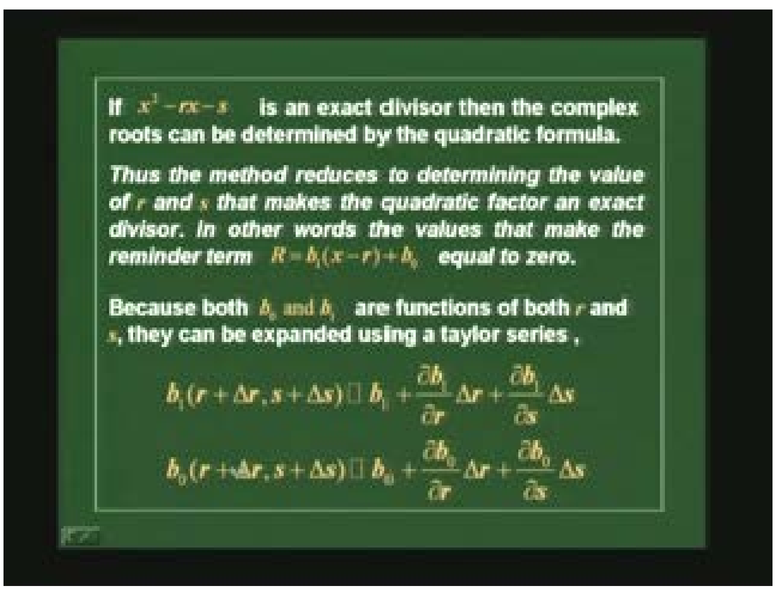

So when I write $b_1$and r plus delta s plus delta s has $b_1$plus del $b_1$by del r into delta rplus del $b_1$del s into delta s and similarly, b_0 r plus delta r s plus delta as has b_0 plus del b_0 del r and delta r plus del b_0 by del s into del s. I can write it like this and then I would say that okay, so the new value of b_1r plus delta r s plus delta is at r plus delta r plus delta is at b_0 r plus delta r under plus delta s should be 0.

So I will find the value of delta r and delta s such that, this is 0 so this left hand side equate to 0, I could find the delta r and delta s, s is delta is satisfied so that is what approach is, so that would leadership and equations of this form then del b and del r and delta r plus del $b_1$delta r s del s into delta s equal to minus b_1 and del b_{10} by del r into delta r plus del b_0 by del s into delta s equal to minus b_0 and this is a simple linear equation and we can solve is by elimination immediately. So now the wide at become do that provided we know what this derivate r del b_1 del r del b_1 by del s del b_0 by del r del b_0 del s etcetera, so provided we know that this derivatives are, we can determine delta n r delta s.

So we assume r and s and we compute b_0 and b_1 and if there not z_0 what we do has to expand the value b_1 and $b_0$around r and s and find out the delta r and delta s such that the new values is b_0 and b_1 and 0. So that is what approach here when he actually expand that of course we take keep term only of to the first order in delta r and hence the new value of delta r and delta s we get, may not make the $b_1$and b_0 as 0 right. So when we expanded this here, we are expanded only up to first order in delta r and delta s. So this value may not go to 0, even if you solve this equation this left right hand side equation with putting b_1 at r plus delta r and s plus delta s, is 0 that is, if you solve this equation resulting delta r and delta s may not make the b_0 is 0, because we terminate it at first order but then we could do this iteratively and then obtain value is delta r and delta s such that b_1 and b_0 and 0.

So now the question was how to determine this derivative. So for that Bairstows shows that the way we constructed this $b_1 b_n$ minus all the b_n b_i is from the a's, we could use the similar method to construct the derivative of the del b_1 by del r del be 1 by del s etcetera from the b_1 that is, inside that ok we could right regression relation of this form that is c_n is equal to b_n c_n minus 1 equal to b_n minus 1 plus r c n etcetera, if form and then we could take the derivate of del b_1 by del r and del $b_1$by del s from plus from this equations that was iterate right.

So we would get this equations has that is del $b_1$by del r and del $b_1$by del s has you know has c_2 and so del b and del b_1 for example del b_1 by del r, so b one here would be b_1 plus r c_2 right. So del b_1 by del r equal to c right c_2, so we could substitute that here ok c_2, c_3, c_1, c_2 etcetera that is del b_1 by del r, del b_1 by del s, del b_0 by del r, del b_0 by del s from this relations so we have the a is and then have the b is that is a's are the coefficients of the n th order polynomial and b is the coefficients of the n minus 2 order polynomial and then we construct this c is from the b and there is just care as the coefficients that is in that is the derivative of del b_1 by del r del b_1 by del s etcetera.

So once we have that, we can solve this equations and obtain the delta r and delta s to our satisfaction that is, satisfaction in the sense is we put the tolerance for the value and then get the b_1 and so get the and we get the del r delta s such that b_1 and b_0 r is 0. So that is the method, so we just see an implementation of this method in the code here.

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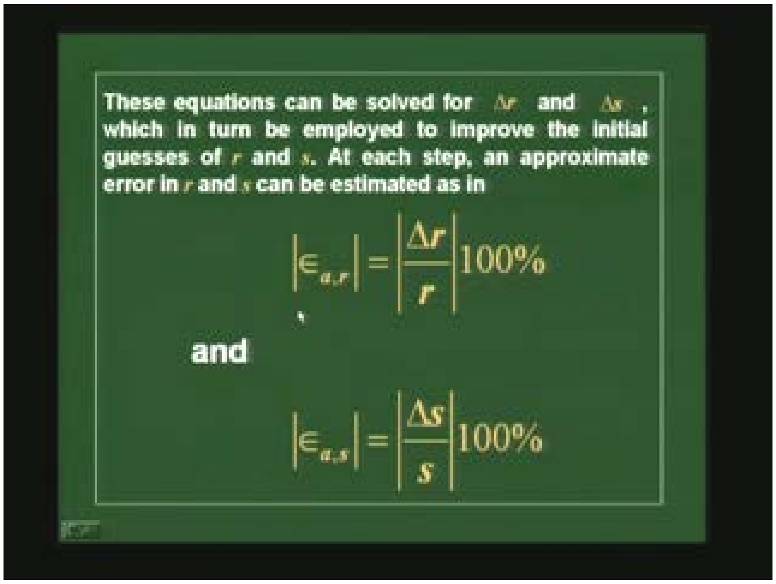

So we have the set of coefficients, the set the polynomial which as the coefficients of that of this form, okay that is coefficients of this polynomial we have so that is shall I just. So we will see just these are the coefficient of the x nth term, this is coefficients of the x the power of the 0 term and this the coefficients of the last one of this coefficients of the n th power of term so its actually coefficients of x the power of 5, okay that is a $_5$ and then this coefficients a_4, a_3, a_2, a_1, a_0. So thus we are solving fifth order polynomial here. So we start solving with a fifth order polynomial. Okay the coefficients are a_{51}, a $_4$ minus 3.5, a 32.75, a 22.125 and a $_1$ minus 3.875 and a 01.85 or shortly be x the power minus 3.5 times x to the power of 4, 2.75 times x to the power 3, 2.125 by the x to the power of 2 minus 3.875 x and 1.25 plus 1.25 equal to 0 that is what you trying to solve. So that is the polynomial which we have.

Now we are trying to factorize this polynomial and get the roots. So that is what we have, so that is, fist this program ok started and defined all the find all the appropriate variables here, all floating point and then I open this file which has the show coefficients dot data coefficient dot data and read out all the coefficients using this fscanf function I read out all the coefficients in to the array a. So once I have all this coefficients, so then I know the order of my polynomial. So this n gives me the order of my polynomial and n equal to 4 is been 0 to 4 that is means to 4 that is 5.

Okay so then what I to do I have to know trace that rights to I have to know do this this calculation here, ok so we have construct I have the a n matrices so I have the an arrays given by just now you show that I have the polynomial a_n, okay so that is all the a_0 to a_5 I have a_0 a_1 a_2 a_3, then a $_5$and then from that now I need to construct the b_n is so that is first step so that is done here. Okay so as long as order of the matrix, order of the polynomial is greater than 2, I have to construct I have to continue the trace in tell order of polynomial is 2 right so first I take the fifth order polynomial and divide the fifth order polynomial by x square by minus r x minus s that is what be looking at, we took a fifth order of polynomial and now I am going divide that by x square by minus rx minus x and then I have to construct now b_5 to b_2, b_5,b_4, b_3, b_2. I have to construct that is what done here. So I put m equal to n here and this is same as n here write now the grater than to I constructed I use this I use relay that it b_n and a_n, b $_n$ equal to an b_n minus o1 equal to a_n minus 1 plus r b_n from b_i, i goes from n minus 2 to 0 is given by a_i plus r_i plus s b_i 2 so that is here.

ok that is what the whole thing is doing here right, that is first think b equal $_m$ and b_m minus n equal to b_m minus 1 plus r b_n and then I go and construct all the polynomials. So I did that, so j equal to minus 22 greater than 0 and I come out here so if this all the b value. So I have all the b values here ok now once we have the b values and then now construct the c value right, so because the next step will find out the del b_1 by del r and del s etcetera. So that I do that by so I know the del b_1 del r to del b_1 del s to construct that, I need to see it relation and then I can said del b_1 by del r c_2 and del b_1 by del s is c_3 etcetera. So from this relation.

So that is what I am going to do. So that is c is has to be constructed and here is c is constructed so once have c is here okay exactly same as the now all the b is here about place by a and all the a here replace by d that is what the c constructions, and then have the matrix this 2 by 2 matrices which is c_2, c_3, c_1, c_2.So that matrix and right hand side minus b_1 minus b_0 remember that and that is what is constructed here, so the 00 element is c_2 the 0,1 element c_3 and 10 element c_3 and c_1 and 11 element c and then right hand side is given by the d column that here that is minus b_1 and minus b_0.

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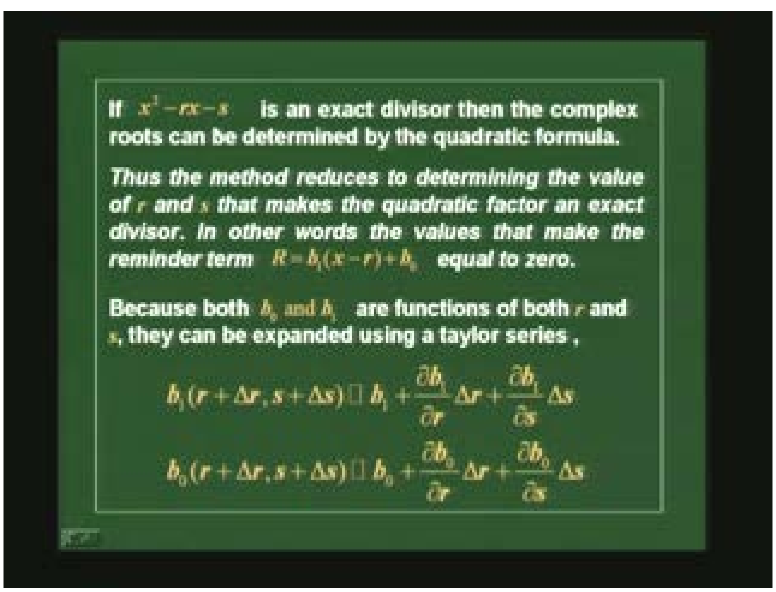

So I have all the matrices and I just saw I inwards this, I solve this particular equations that is c_2 c_3 matrix c_2, c_3, c_1, c_2 into delta rs delta equal to minus b_1 minus b_0 that why inverting the matrix c_2, c_3, c_1, c_2 and multiplying into b_1 and b_0 that is what done here is c in this, in the case of non-linear function fitting in only a function to set of linear points is exactly the same. So we have this relation here and now, we use that particular r and s, delta r and delta s value which obtain by actually multiplying the inverse of the matrix this d matrix, so the little the d matrix that is the d_1 b_0 and I can get delta r and delta s and say r equal to r plus delta r and s is equal to s plus delta r.

So just is short cut once appear just striated away delta r as in expression and then, what is the r and s values I am getting what is b_0 and b_1 and that step is print that out here. Okay this print statement prints b_0 b_1 r and s, okay such what print statement print out, so actually printout the order of the polynomial the r value ok that I can see the print value the print out the order of the matrix, order of the polynomial in that step r value, the order to begin with five then r value the s value, the b_0, the b_1. So once we got the b_0 and b_1 we go back here and check what is the b_0 b_1 values are, we want to be 0.

So I can here it value of point 0 0 1 so b_0 and b_1 r not 0 that will go continue with new value, will continue the iteration for right, so it will continue the iteration till, so the new c value this will actually can change new values can be even out side loop it does in change because "a" is not change in this loop, okay for this b is change. So the new change will come, so we continue iterations ok this change because r and s has change okay so b will change because r and s will change new r and s, the new r and s value will calculate b and c and will continue iteration till obtain the new delta and r and delta s and will continue till b_0 and b_1 is 0. Okay that is the first step.

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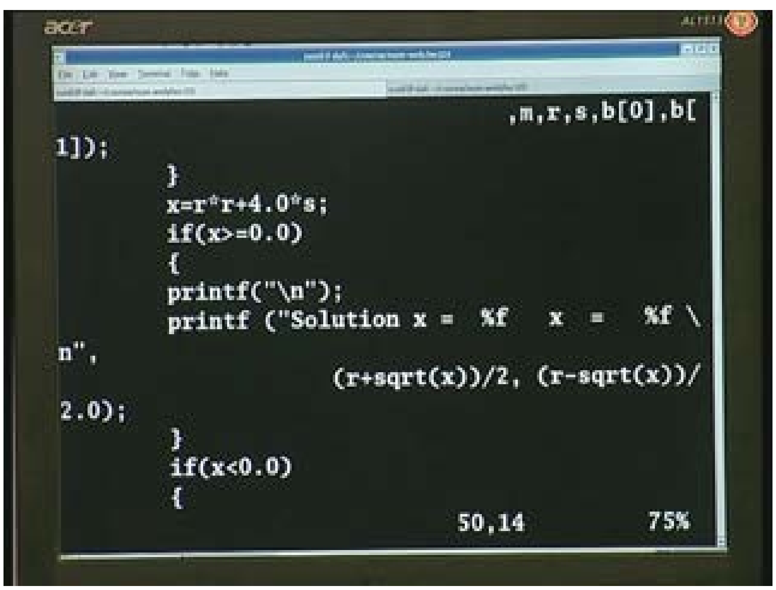

So once we done that, we have a found a solution okay the found in r and s okay, then what have to do, we have to know find the solutions once we have found has been one factor right, so we have the found the factor of the form x square minus r_x minus s, if you found r and s value which factorize our n th order polynomial to n minus truth order polynomial multiplied by x square minus r_x minus s. So once we have that we know that solution of this equation straight away as r plus r minus root of r, r square minus 4 has divided by 2. We do the root of the equation, so that is want to be use just find our r square, so this minus square r square plus 4 is, we could find out the solution of if 1 r_1 s that means is found and factor of the polynomial that is x square minus rx minus s equal to 0 and that the solution is that x equal to r right, plus or minus root of r square plus 4s the minus 4. So plus 4 has divided by 2, so remember we have the found r and s values so we have finding the what is quantity r square plus 4s, so r square has plus 4s can be negative okay, because we allow for this complex root next roots.

So when you x square minus r x minus s equal to 0 the roots of that quadratic equations can even be a complex number. We allow that the complex consequence, ok you say r plus or minus root of r x square plus 4 of by 2. So r_x 4 x square can be negative that will give has complex consequent. So we have to see that x_n is in this c program, we can not handle complex number that way, so we have to first find out whether r square plus 4s plus negative r, if at has positive we could straight away root of that and right that solutions here is, r plus square root of x square x is the remember is, r square is 4 is okay. So r plus square root of x by 2, r minus square root of x by 2 has to my solutions if this is negative then we are take the root with other side that we should find the square root of minus 6 added infinite of I, so that is solution now will be r plus i square root of minus 6 and r minus i square root of minus x.

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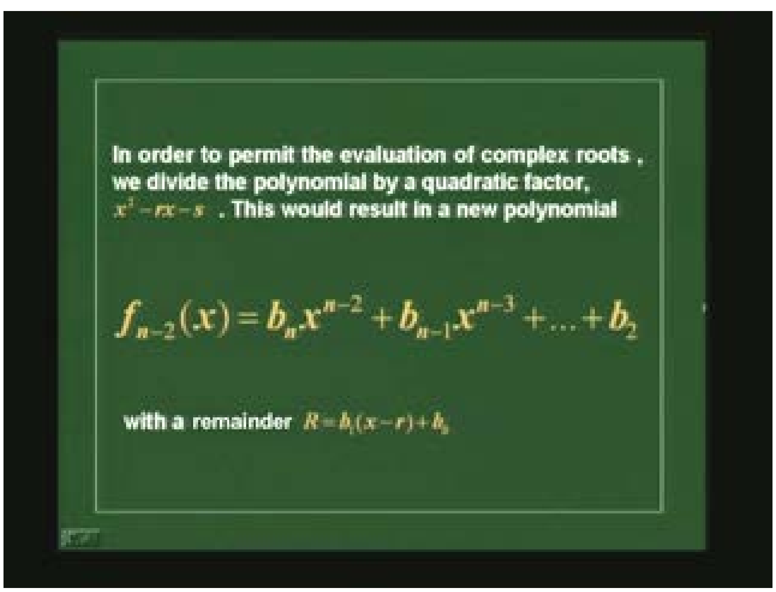

So we will have r, so once r square plus for x is negative however solutions will never be r plus i square root of minus 6 divided by 2 and r minus I square root of minus x divide by 2, so that is what return here. So now the real part of r by 2 the imaginary square root of minus x by 2, okay I read them separately now same see does not allow complex numbers. Okay, so r by 2 and square root of minus x by 2 and r by 2 square root of plus x by 2 minus x by 22 variable and n minus so that is what I am going to print out. Okay then once we have the finish that okay, we will then come to we will have would be left with this is quotient polynomial may be factorize that okay we divided the n th order polynomial by x square by rx minus s and now what is left with and n minus 2th order polynomial right now, this case we have the fifth order polynomial. So we have the third order polynomial that is quotient polynomial.

So I print out here, the coefficients of that quotient polynomial, okay and that will be printed out here and that it will be a_j is what it will as a_0 of the quotient polynomial right would be the b_2 correct that is way, we have return our polynomial remember. So we have return have a polynomial has this way, that is the quotient polynomial is given by this one that is the x the power of 0 order term coefficients is a_2 so that what is call a_0 right the original polynomial, so we the program I want to replace all the a value is the new a value after for the question polynomial, so that is a_0 become b_2 and similarly, we go to all the way a_3 now b_5. So that is what here, so n is now as 5 in our case. So 5 minus 2, so such has highest power now x cube and the coefficients of the b_5.

So once we have, b is the correct, b is the constructed we can get the quotient polynomial right solve this, then we will go back the quotient polynomial and then I simply go back here. Okay and then continue that process I read know I know reduce my order by the m minus 2 and then we will continue that and then will get it new polynomial a cubic polynomial and again I factorize the cubic polynomial has a quadratic into linear set the find the linear, so we find the linear solutions is the quadratic polynomial and then we can read of the solution from the linear program that is what we do. Okay and we can see that here, this implementation, this I will see, show you, so seen the coefficients data I know will see. So okay.

So when I run this program the first one step, I will tell you they are we give to initialize guess of the r and s okay that is scheme and here okay once we have to red out all that a coefficients will give the initial guess for r and s. Okay, so we have the initialize guess for r and s and that initialize will go head and solve for the correct quadratic polynomial that is the correct r and s values and then we have n minus truth order of polynomial left and we have to factorize that for that we will use the new obtain r and s has a initialize guess and continue.

So we will give only one guess we could have tenth order polynomial but we are to one guess value of r and s and then will continue use that for the next lower order that is what you doing here. So we had one guess for r and s has to be given r and s and such the saw the program wait for that it for the same would ok and then its next round after the finish is the finding the r and s value the prints out the solutions and then actually will go back here right, so when it is goes back in this m equal to m minus 2 and then it will goes back okay.

So when I does that, when it will goes back from here, okay it now does not ask you for new r and s value, okay the r and s value is outside this loop when it takes, whatever the value r and s values obtain for the previous factorization has the guess for the next one.

So that is what we say. So you start which some case value let say minus 1 and minus 1 i given minus1and minus1. So then it will some other expanding out the order of polynomial which is 5, I given minus x, minus1 as the starting value for r and s by the computed new r and new s which is minus 644 and s is 4.138 for the b_0 for offering being 0 right, so b_0 is 11 and b_1 is minus 10. Okay, so obviously my guess was predict bad, okay it goes back and then compute for the new r and s as the gauss again delta r and delta has computer new r and s so that transferred minus .5 and 0.4 and the b_0 was reduce to 2.1 and b_1 has minus 1.80 it is again a is of 0 its continue iteration, okay and then finally comes to b_{10} and b_{00}. So it conversional fast and then it is gone to r has minus .5 and ss plus .5 so that is the r and s value for this.

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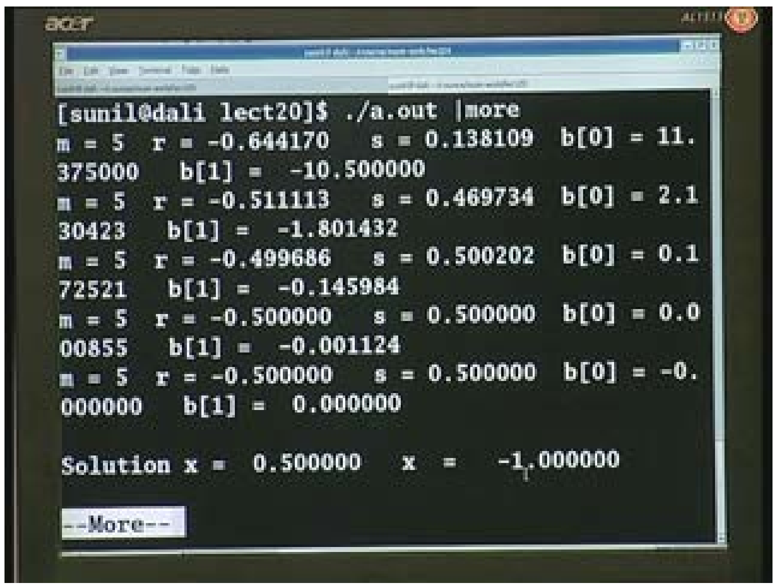

So we have one factor that is x square plus half x minus half as one factor right, okay that is r is minus half s plus half and corresponding solution are x equal to half and x equal to minus 1 right x square plus half x minus half equal to 0 of to x square minus x plus equal to 0 so x equal half so x equal to minus 1 is our solutions.

So then we have now left with an equal to m with three polynomial right that is what our left with so left the coefficients of that x cubic1 remember x cube term is the coefficients of the x the power of 5 times earlier, so x cube term as one has the we have the polynomial which is left polynomial as x cube minus 4 x square plus 5.25 x minus 2.5, that is polynomial which has left just left.

So now I start with r equal to minus .5 is equal to plus .5 has initial guesses in a compute the new r and s values again we can see that the b_0 and b_1 has to bad to begin with and then it converges, it converges to b_0 equal to 0 the b_0 equal to 0000021 and b $_1$equal to 00005 which is much less than the tolerance I have given. I given it all point 001as the tolerance. So that you have a new set of r and s values r equal to 2 and s equal to minus 1.24. So now, the new polynomial, new factors that is x square minus 2 x plus 1 point two 5 equal to 0 that is solutions will it fine now the term of here r square 4 s r square is 44 is 5 minus 5, so r square plus 4 is negative.

So however roots are now complex numbers, so one root is 1 plus half i are other root is 1 minus half i, obviously because they are complex consequent and we have a quotient polynomial which is now linear which is x minus 2 for 1 per minus 1 per 999984 just 2. So x minus 2 equal to 0 or x equal to 2 is a next solution, so if you found all the roots for the polynomial this polynomial, if found one root as 2 and one as1 plus half i and otherwise 1 minus half i and if found earlier root has half and minus 1. So, half minus 1 plus half i, 1 minus half i of the different polynomials, different values of the different roots which we obtain from us.

So we have all the Eigen values, so that is one method of the computing i eigen values other ways of doing this may be depending upon the problem, we will have to choose what is the fastest method, so we will look at the sum over methods, so we just summarize method. So we just saw the computed r and s for each for the polynomial and we look at the quotient polynomial till us static or cubic or linear I say ok just we linear which as the philosophy solution x equal to minus x by r that is what be done okay and what our the quadratic polynomial which we get as a factor. We just solve that and obtain the solutions.

Okay then we said that would also briefly mansion in the last class that at we could is not the only way to compute eigen values and eigen vectors you could use vector power method. So in the power method, we use the fact that the Eigen vectors for what is calls I completely basis that means that given any vectors. So here n by n matrix even any vector column, vector n column vector you can write that has a linear combination of the eigen vector so any arbitery vector. “V”, I given V as the arbitery vector here.

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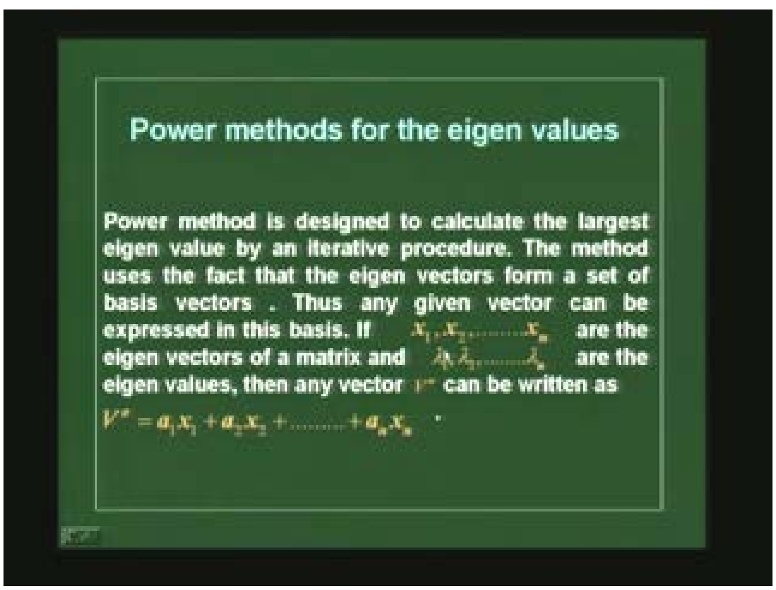

So any arbiter vector can be return as a linear combination of the eigen vectors, let say x_1, x_2, x_3 up to x_n where the eigen vector each other them or column vectors say x_1, x_2, x_3 up x_n of the column vector which has the eigen vector of the n by n matrix a that is it. So we now organize here, what it means Eigen vectors and Eigen values. So if this equation for eigen vectors of this the matrix a and then any arbiter vector then I can any write has linear combination of this coefficients of this vectors x_1 and x_2 etcetera remember each of them our column vectors this is fact, I can use this fact to construct the eigen values and eigen vectors.

So let us say lambda 1, lambda 2 up to lambda n are my eigen values and have ordered in such a way that lambda 1 is my largest eigen values and then lambda 2 is next large etcetera. I assume that no 2 Eigen values have the same are the first the largest Eigen values are not to be same are not the Eigen value the same, I assume that this element or this work in the particular case the power methods. We have a assumption that to make is that no eigen values are same or physics term we say this note integer address ok and then we would write any arbiter vector v_0 which is taken arbiter vector v_0 and multiply by the matrix a, so just taken a arbiter vector v_0 which I know can be return as just we solve a x_1 plus a x_2 plus a x_3 write which has so that we can write in a arbiter vector has this form and then multiplied both side this by matrix a, so I get a v_0 call it I call at b_1.

Okay so that v_1 is now a_1 into a x_1 plus a_2 which remember each this is a column vector and this is now n by n matrix write so each of the column vector and this is n and by n matrix, so I can multiply this and I get the vectors b_1 then I know a x $_1$ by the eigen value equations that a x_1 that is lambda x_1 lambda 1 x_1 and a x_2 the 2 x_2 write.

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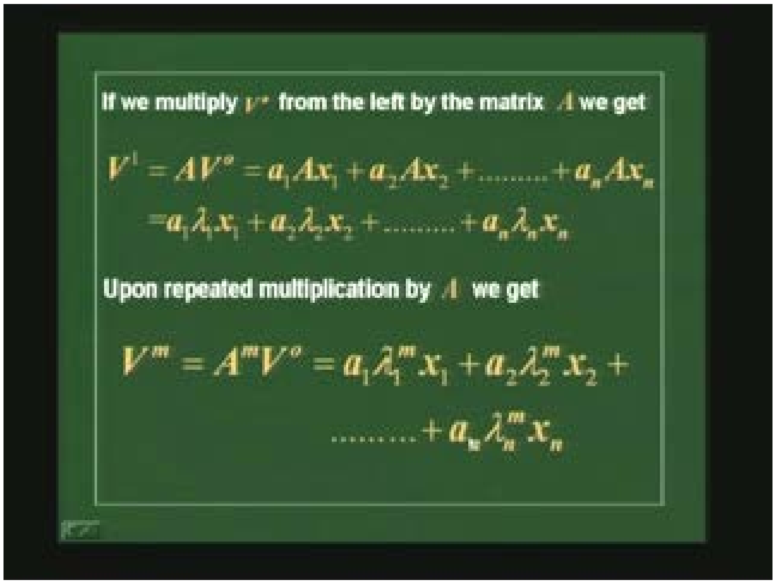

So I can write the equations ok as this equations has a_1 lambda 1 x_1 plus a_1 lambda 2 x_2 a n lambda n x n and then I take this again multiply by a and take the v_1 and multiply it by a and right hand side again by a. So get the 2,so V_2 will be a1 lambda 1 x_1 plus a x_1 a2 lambda 2 ax_2 plus etcetera and then ax_1 plus lambda x_1 so we to will be a 1 lambda 1 square x_1 a_2 lambda 2 square x_2 etcetera or if an repeated multiplication of this matrix how does the right side by a I will get v_2 the power n has a 2 the power of n v note plus a_1 lambda 1 power to the m this to be m. So if I multiply this by m times if I multiplied by m times and then I get it is a m is v m a_2 the power m v_0 has a 1 lambda 1 is power m x_1 plus a_2 lambda 2 is power to them m x_2 plus x 3.

So multiply by m times and then if lambda 1 m if lambda 1 is largest Eigen values this right, I could take if I take lambda 1 m out of this matrix m out of equations I will have a_1 x_1 plus a_2 lambda 2 by lambda 1 to power x_2 plus a $_3$ lambda 3 by lambda 3 by power m etcetera up to a_n lambda n by1power to m x_n but we said lambda1 is largest eigen value and that case lambda 2 by lambda 1 is number is smaller numbers and lambda 2 by lambda 1 to power m is almost 0 right any number which is less than 1 is the power which is the large integer is 0 right.

So for example, if so if the ration between lambda 1 and lambda 2 lambda 2 by lambda 1 is point 1 by the time we have multiplied by matrix 44 times which point 1 to the 4 of 4 is already the very small number so what is in depends in short what we find if the ret again arbiter matrix b and keep multiplying by this matrix a and then what we end of earth since this term will dominate everything everything else is go to negligible we get that the result of multiplying and arbiter vector v node the column vector v node by the matrix a m times m being the large enough an integer is that you get and right hand side a_1 times lambda 1 to the power of power m x_1 and everything else is 0 from this can be read of what is the eigen values is what the eigen vector.

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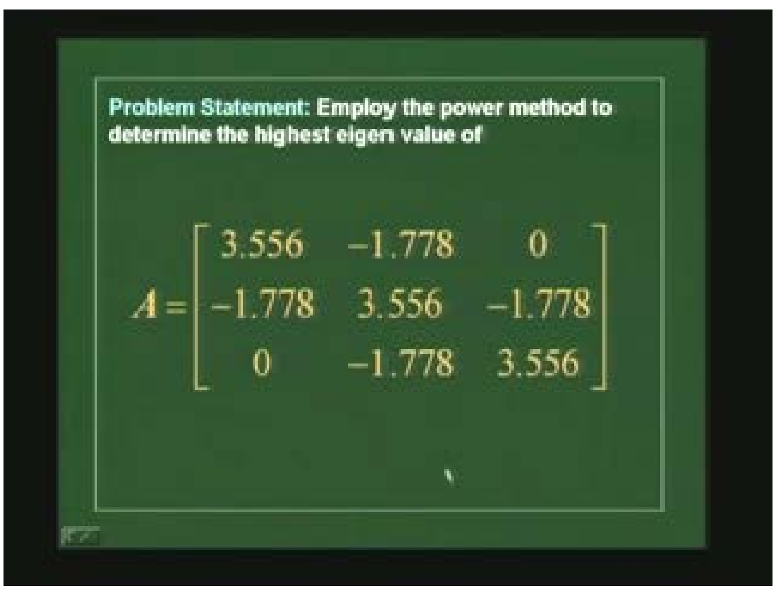

So if you taken the arbiter vector multiplied by a matrix a m times and we normalize what we got the right hand side that what you get is the last eigen vector is the corresponding to largest eigen value and then factors is multiplying set is the largest multiply eigen value and we will see the will see implementation of this. Okay so this is the in short something, say that if I multiplied by n times I just simply get on right hand side of the Eigen vector corresponding to the largest Eigen value. Okay so here is a small example of that.

Okay, so we will see this that is I have matrix a I want to find the value a and eigen vectors so we will first determined in the highest eigen value are the eigen vector corresponding to the largest eigen value and then we will see how to generalize to get all the eigen values so first we determined in the largest eigen value.

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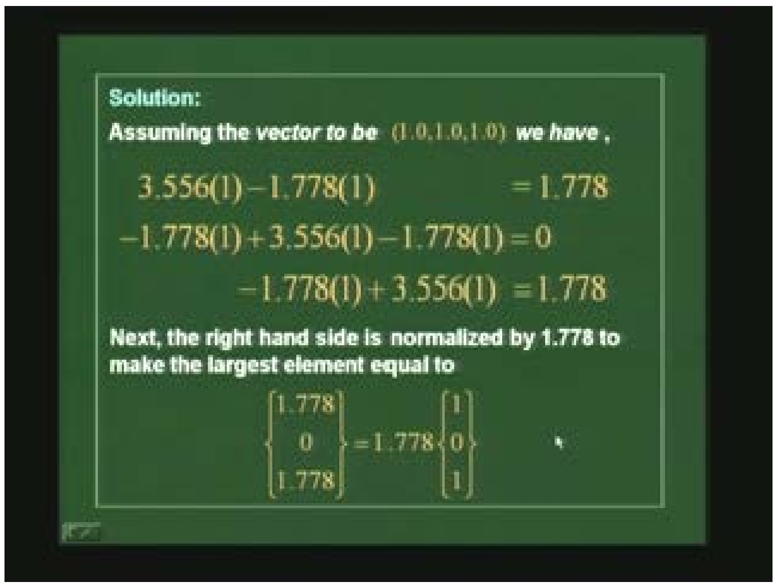

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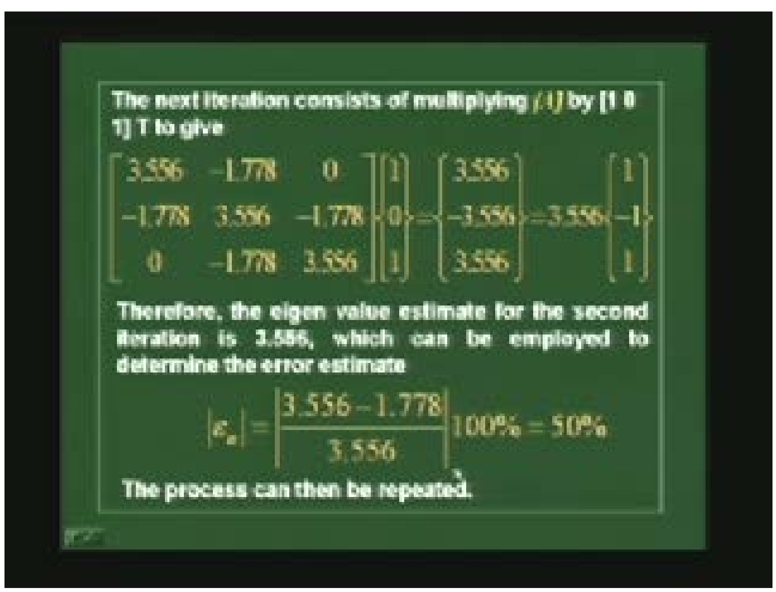

So we have the matrix we can matrix and then we want to find out now, so we start with sum arbitery vector okay, so 3 by 3 matrix the arbiter vector is 111 is simplicity so took 11 is simplicity and I said I multiplied that vector that matrix by 11, okay I got something and right hand side as 1. 718 778, 0, 1. 778.

So now I make the largest element on this column as 1 and so now I have got, what I got was, I took the matrix a multiply by 111 and I got some number multiply by 101. Okay that is my first iteration, now what I do, I multiply again this one, I multiply by vector my matrix I take this vector 101, I would reply by “a” again.

That is I want to do. So that is the next step, I multiply the matrix by 111I got 101,ok and then I multiply 101by the matrix again, the next step the iteration number 2, I multiply it by 101and I get something new that is 3.556 minus 3.556,3.556 that is I make the largest elements on this column vector as 1. So I take the 3.556 how to column in this one column now I multiply matrix by 1 minus 11 write so I can each step I can compute percentage error right. So the previous number I got here eigen value was 1.778 and now I got 3.558, so the error is about the 50 percent, I can continued the iteration till this error the percentage till the error that is the deviation of the previous value the next iteration value is very small.

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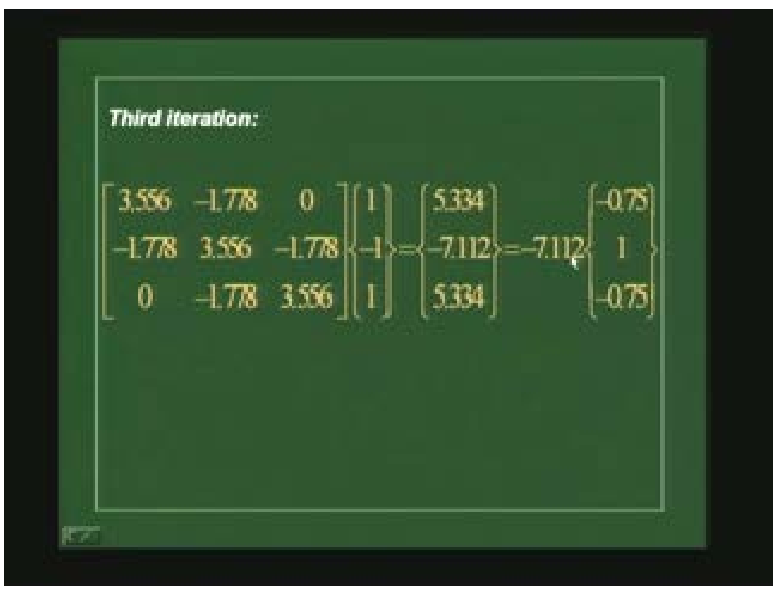

So I repeat the procedure, ok so now I take 1 minus 11 and multiply by this matrix a again and I got now minus 7.17112 and minus 0.751,0.75. So I guess you would follow what I am doing. I just taking the arbitery vector started with arbitery vector multiply by the matrix a and I got vector here column vector I made the largest element in the column vector has 1 and then took that column multiply by the matrix again, okay so now the third iteration of the process and we had the previous iteration, we had got minus 1, minus 1, 1 as the vector multiply by the matrix again I got minus 0.751 then 0.75 minus 7.112. So, okay so error again so the previous error that is the value obtain this step minus the value of 10 the previous step divided by the value of this step is the percentage error, that is depends of high third iteration the second iteration is only it was only 50 percent here, it is one 50 percent because our change side. So that as long as this is large, this is not we are not obtained our goals, but this is not decrease monotonically that is what we just saw.

Then the fourth iteration we took that minus 0.751 minus 0.75 we got them previous step and then multiplied by the matrix a and we got minus 6.223, 0.714minus 0.714, now we can see that we are reaching some were ok we reaching to value closing enough is started with minus 0. 751 minus 0.75, we ended up with minus0.714, 0.714 and 1 if this truly eigen vector you know what we get if truly eigen vector this product should be give exactly this same here with number here right, so that number here would be there eigen value ax equal to lambda x that is our idea in this iteration scheme and reason this work is because we are because the largest eigen value dominates that is what we just solve right okay.

Okay so the fifth iteration the 714 had gone 708, so we are almost converging. So we converging the closing to.7 ok that is the switch converging and to that Eigen value converges on to something like .6 minus 6.07. So this method this way we could constrict the eigen vector and eigen value corresponding to the largest value this to eigen value so this is the power method this is once is the power method we take arbitrary vector and keep multiplying that by the matrix a.

So now we each time you normalize a vector to make the largest element in the vector has 1 and then take that vector and multiply by this vector again once again you drop this and when in then reach the final hands the when we actually we have the correct eigen vector and the matrix multiplied by that eigen vector should give us the eigen value times the eigen vector, is the vector itself that is the idea.

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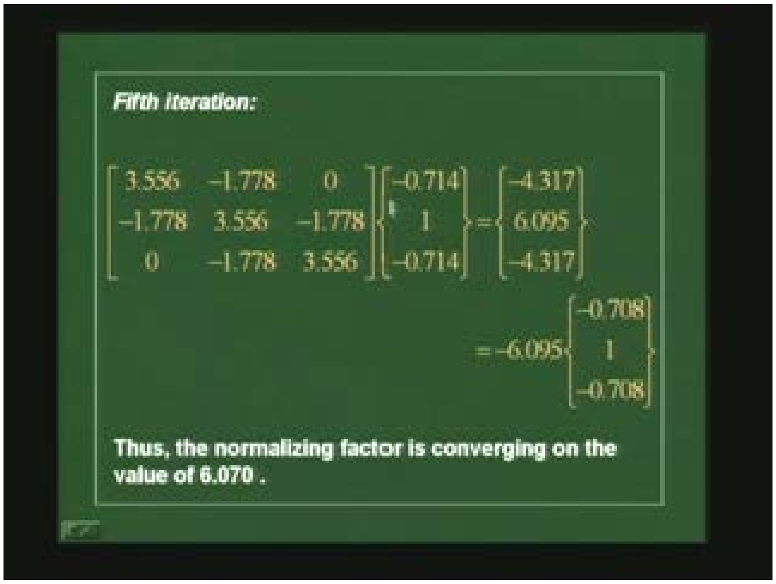

So now, we could ask the question with the generalize this. Yes we can generalize this, ok we can generalize this to obtain even the, even other Eigen values. Okay so that is the for example, we can very easily, see how to get the smallest Eigen value. So now I said that lambda 1, lambda 2, lambda n was the eigen values of the matrix a, ok and now if we can lambda 1, lambda 2, lambda n are the eigen values of the matrix a that will be known that lambda 1 inverse, lambda 2 inverse, lambda n inverse are the eigen values of matrix a inverse. So let us say, if lambda 1 or lambda 1 is the happens to be smallest eigen value of the matrix a then lambda 1 inverse would highest eigen values of the matrix a inverse and so we could just simply find the largest eigen value of a inverse that will give us smallest eigen value of the inverse of that give up the smallest eigen value of the matrix a and we know the eigen vector is the change write the eigen vectors the matrix a are also eigen vector of matrix a inverse.

So immediately we can see that by using the power method, we can determined wide the compute a inverse which also how to learn, how to do so provide can compute the matrix a inverse I can compute the eigen values the largest eigen value and the vector corresponding the largest eigen value and the smallest eigen value and then eigen vector the corresponding small is eigen value and again come computing the matrix a inverse you could use to fadeev laverrier method which has to be learnt in the last class.

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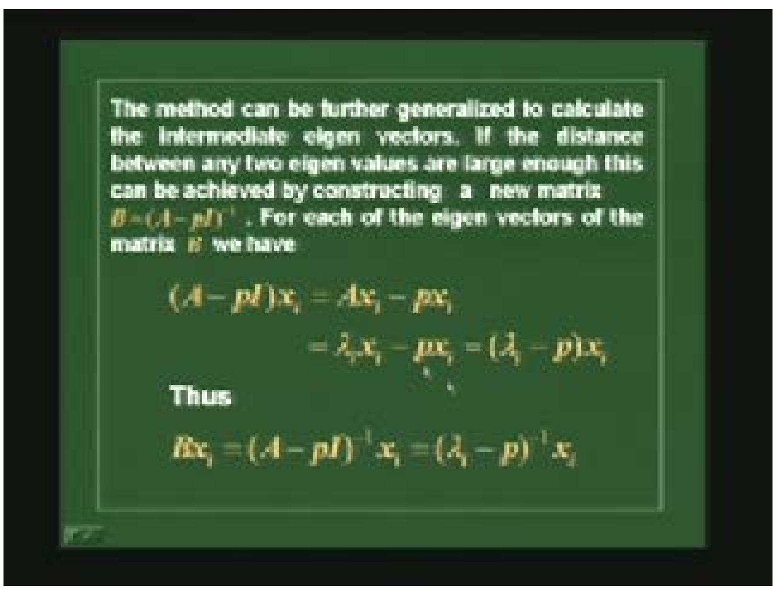

So, now this method, so we have this idea are we can generalize we can get all eigen values because now what be could do is required construct this methods and this matrix a minus pi ps and then new matrix b a minus phi inverse and construct eigen value the largest eigen value of this matrix b. Now let see, what the is me so let me right down the equations and a minus phi, so a is our original matrix and b subtract sum number times identity matrix from that and we write down that matrix equation here so etcetera its column vector so we have a x_i minus p x_i you know right. So a x_i minus lambda x_i and p x_i p x_i was and then ps, now just diagonal matrix.

So this will simply lambda minus p x_i so the p i diagonal matrix so this would simply give this p x_i, p is number so lambda i minus p x_i, so we know if I construct eigen values of b now that it will be lambda i that is p inverse will give be eigen value of that right. So if I can how and idea above what the eigen value of is are then I can take the number p which is the close to eigen values and construct this give matrix b which a minus p inverse and then the largest eigen value of this matrix would be something close something were largest eigen value this b matrix would be lambda i minus p inverse where lambda i is the eigen value of the a matrix. So only the disadvantage is that we need to have an idea about where the Eigen values are, but again the some ways of guessing what the Eigen values are. Okay so we could use for example just use a theorem, that if you just take the diagonal elements.

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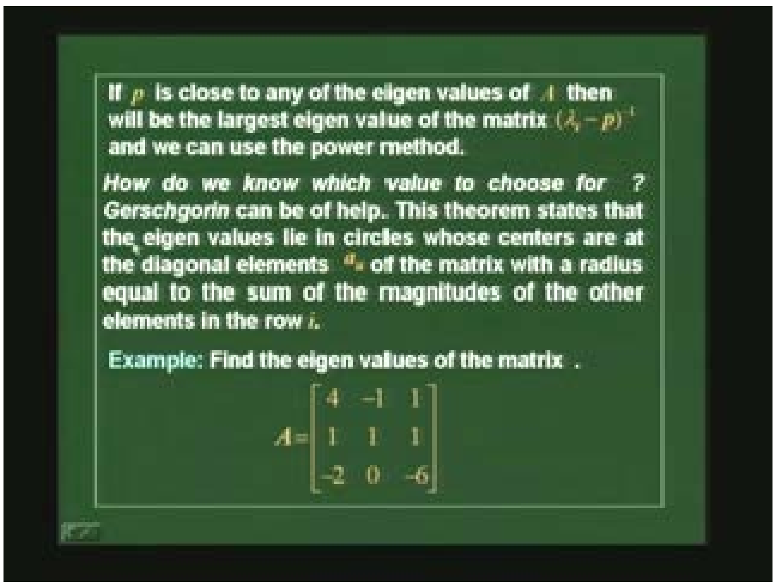

Okay, so we could just theorem Eigen value is the lines circle whose centers that the diagonal elements. So if for example here it will be Eigen value is this matrix would be around 4, around 1, around minus 6. Okay and there magnitudes will be around in this value now how far how far from the 4 a table b, okay how far from 4 is eigen value is b will be and that the distance will be given by the sum other the magnitudes of the other elements in the row I, that is this will 4 plus or minus 2 and this will be 1 plus or minus 2 and this will third Eigen value will be minus 6 plus or minus 2.

So I can use that, so I can use p so I can choose p as 41minus 6 and construct the new matrix as a minus 4 i inverse and look at the largest Eigen value of that. So that will give me the Eigen values and Eigen vector which is closed to 4. So in this way I can construct all Eigen values and all the Eigen vectors. So that is advantage of power method we will both the eigen values and eigen vectors this by simply by constructing finding the product of and matrix with in arbitrary vector. So that is the summary of our discussion on eigen values and eigen vectors and next we will go into again solutions of linear equations then we will continue from the discussion which we had on the polynomial, the roots the polynomial. We will go and look at the solution of others a non-linear equation that is what would be discussion in the next class.