Numerical Methods and Programming
P. B. Sunil Kumar
Department of Physics
Indian Institute of Technology, Madras
Lecture - 17
Solution to Liner equations: LU decomposition number

We have discussing a solution to set of linear equations that is what, we what doing the last class we continue with that today. So we have system of linear equation of this for that is a_{11} and x_1 plus a_{12} x_2, a_{1n} , x_n equal to b_1. So, n such equation for n unknown x_1 x_2 x_3 up to x_n, probably discussing was method for solving such has some linear equations by the different methods that is a what a looking at.

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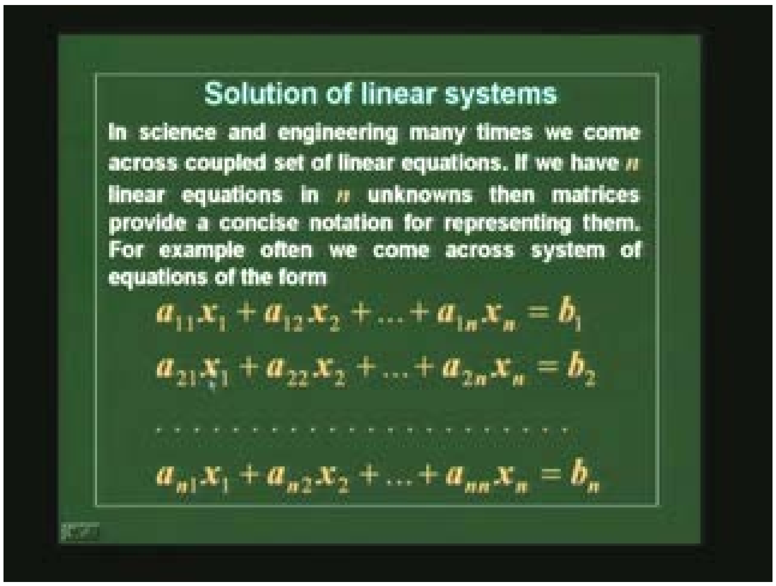

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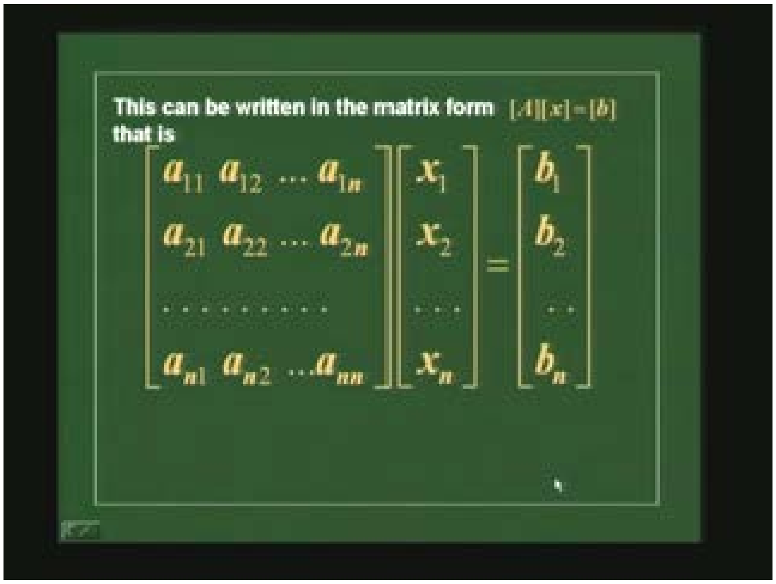

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

So we saw one method is that we could right of the form the matrix right, and then we could inverse of the matrix and multiplied on this side to get the solution. So, inside of the doing the directly finding the inverse of the matrix, we were looking at what the method call Gaussian elimination. So just is it mind you that, we were looking at this Gauss elimination scheme, you looking at the Gauss elimination and then which we had attained the matrix as a_{11}, a_{12} etcetera a_{1n} and a_{21} a_{22} etcetera a_{2n} We right matrix of this form and then our idea was the since the set of linear equation can now be return has in this form as show you.

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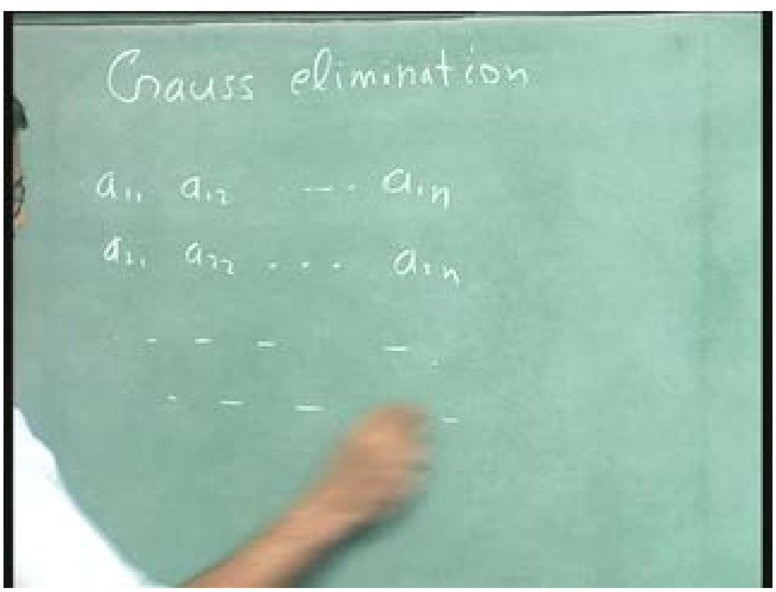

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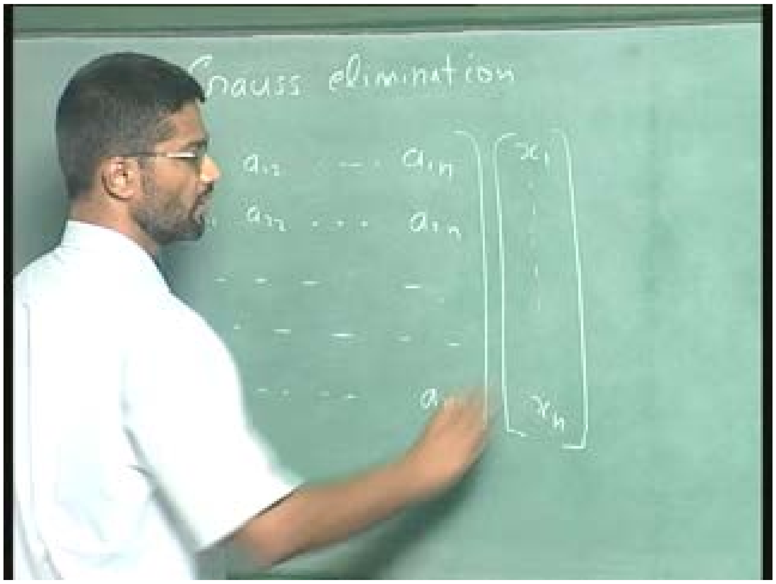

Okay, since we can right the set up linear equations in this forms and if we could reduce the, we also saw that at if I multiply any of the this rows, okay by a constant either side of the equation, this equation that is this side, this side by multiplied by constant or if I take a linear combination of this, replace one of this rows by linear combination of two rows and similar thing here also and then I will not change the solution of the equation. Hence we used that method to eliminate, the all the columns below the first row and similarly, all the column are below the second row here all elements below the second row, the second column and all the elements below the third row in the third column etcetera to get a matrix which is non zero only above the diagonal the and diagonal and above. So that was idea of the Gauss Elimination Scheme.

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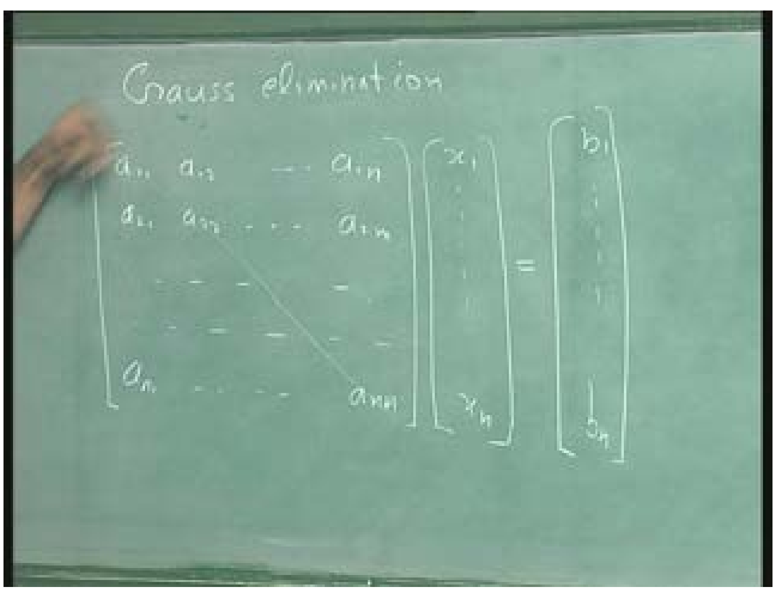

So for that we use the, we replace this one, a_{21} for example, by a $_{21}$minus a_{11}, a_2 multiply, if we divide this whole row a_{11} and multiply by a_{21} and subtract from this, that is what a doing to get this to all elements this row a_2 i for example, there is all elements or this row get replace by a_{2i} will go to a_{2i} minus a_{1i} divide by a_{11} into a_2 into a_{21}. So that is what do right to it, okay so if do this all the elements in this row get replace by, this row minus this row multiply this factor a_{21} by a_{11} and then, we can see that when i is 1 that a_{21} will go to 0 in the case because, a_{21} by a_{11} by a_1 that will go 1, a_1 my a minus a_{21} that will go to 0.

So, such this I could all the by all the rows the rows and similarly, I will do a_{3i} agas a $_{3i}$ minus a_{1i} divide by a_{11} into a_{31} etcetera for the all the rows and then I could introduce into 0, at the end of this operation I would have obtained a matrix of this form which is picture 0's all below the diagonal side, has all below can 0 below the diagonal elements. So we have all 0 below the diagonal elements and above non zero above the diagonal elements all the elements will change many time this for example, this row would change one time and this row all change n time etcetera, n minus 1 time this row change for each elimination round all the rows would change.

Okay and then, we will get matrix which is non zero only above the diagonal elements and hence, I can once have obtain the matrix like that I can find a solution of this equations because I can do back substitution for example, the equation now the first last equation would be a_{nn} x_n equal to b this is all would have change in the process I will do the when I do the operation I also change this side ok I would get now a_{nn} I like an equation a_{nn}, a new a_{nn} that is changes n minus 1 time into x_n will be now equal b_n which row will change n minus 1 times 2.

So from that I can solve this significance I can get x_n, x_n once, I go to x_n here, I can go back the next step, the next equation have the 2 values x_n and x_n minus 1 and x_n also already know x_n. So I can compute x_n minus 1 etcetera, so such that has called back substitution.

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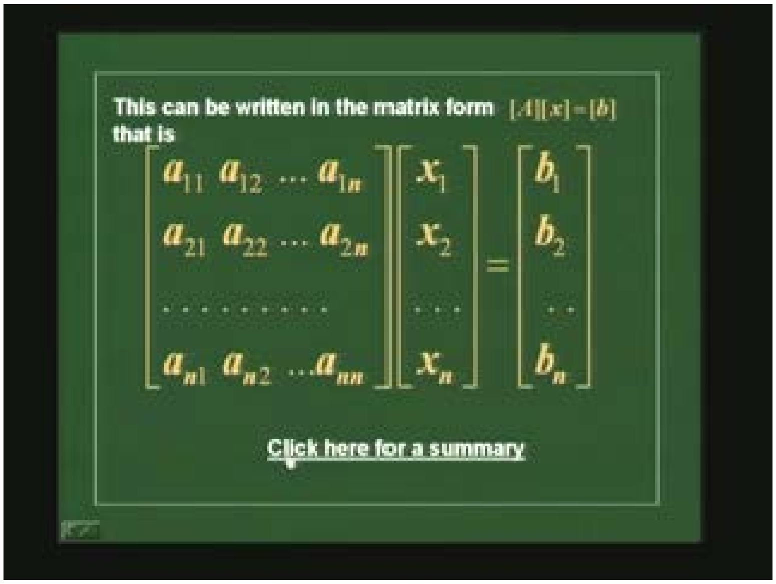

So, to summarize what we do in the Gauss elimination is to solve a matrix of this form, okay if you want to solve a set of equation of this form, okay we would right this equations in the form of the matrix, so the matrix would be given by this, so we will righted in this form and actually we do you programming actually righted as 4 by 3 matrix for the fourth column will be fill by the right hand side of this equations. Okay, this is convenient because the another do operation on this rows we also do the operation this rows as to convenient to righted in this fashion. And then, now we would be subtract from this rows, okay this row from the subtract this row divided by 3, all elements of this row divided by 3 and multiplied by .1.

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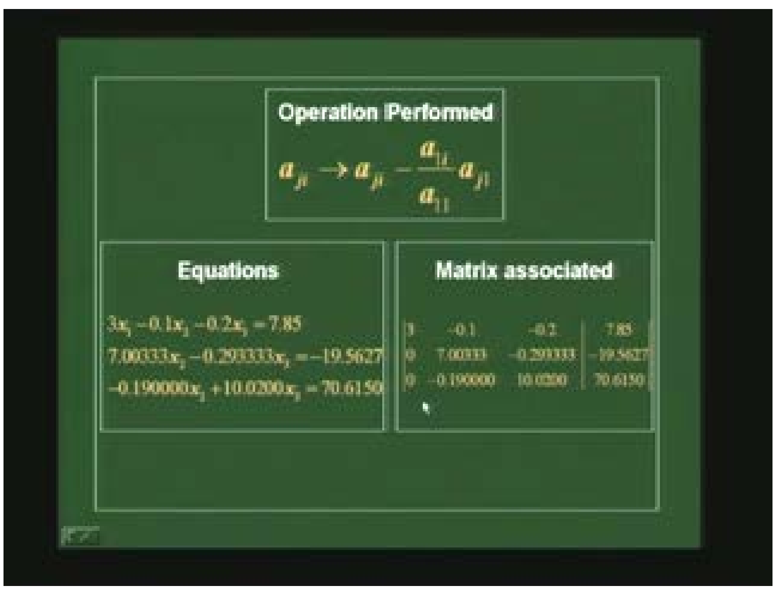

When you subtract from here, I get 0 here and then all elements of this row divided by .3 multiply I am divided by 3 and multiply by .3 and subtracted from this I would get 0 here, that is what is would do in the next step. So that is, basically we do this operation a_{ji} goes to a_{ji} minus a_{1i} divided by a_{11} which is the matrix, this element into a_{j1}. So if you do this, all the element of this matrix, this matrix this rows guess multiply by a_{j1} by a_{11} and then you subtract from this rows and this row, so then you would make you would eliminate x_1 from this 2 equations, the last equation, we the eliminate x_1 and corresponding matrix would be then, given by this. And then we will do this operation, this particular elements that would do elements, this elements that would eliminate x_2 from this equations by multiplying this, by subtracting from this equations, this equations divided by 7.033 multiply by minus .1900.

So, if I subtract from this and I get the 0 that is what operation I would next. So I would that operation the second column, okay and then I would get equation, this will the x_2 is now gone from this, okay and corresponding matrix would be this one. So now, I can do the back substitution because now, I know can the solve this equations x_3 equal to 70.0843 divided by 10.0120, so that is the back substitution which we are going to do and that will give us x_3.

So once you have an x_3 I can go back into equations and solve for x_2, okay so that is what, so we can continue this process in general, we can write in this form and get all the solutions as 1 by 1 x_2 and then next x_1 etcetera. So that is what would be solve, so in the process that what we see that, when we eliminate column, the all the elements in that particular column, we need to multiply this quantity this row by quantity which as a_{21} by a_{11} a_{31} by a_{11} etcetera, here all the denominator a_{11}. So if a_{11} is very small or 0.

Okay 0 will not work, this method will not work if will a_{11} any of 1 diagonal elements 0 then this method will not work or if it very small, then will have a problem that we because here in this would be amplified right because, we would be dividing by small number. So error would get amplified, so that is the problem in this method which have, which have discuss in the last section.

So one way to eliminate such thing is to reorganize have my matrix, such that I will reorganize my matrix, okay such I make sure that the diagonal elements are always largest, large element. That is what calls pivoting scheme. So first what will do is I will find out this all from column, this column I look at the first column, ok so I look at the first column and then will which see the large, the largest element to that column. So just see which one is largest element in that column and okay and then I will take that particular role that is a in this case a_{11} was smaller than a_{21} and a_{21} was the largest element in the first column and then, I would write this matrix, this row first, at this equation a_{11} a_{21} a $_{21}$ into x_1 x_2 x_3 x_n equal to b_2 that equation goes first. That means I will interchange this rows 1 and 2.

So that is the idea, so first I take this column and I find out which is the largest element in that column and then I would interchange between that and the first row, so the first element have the largest in that column. And then, I would go ahead by the elimination scheme and would eliminate the rest of the elements in this column an then I look at the next column, okay below row2, row 2 and below and I will find out which is the largest element in that column, and then I would again interchange rows in that column and then I would make this element the largest in that column.

So in this way I would always the largest element in a particular column as diagonal element of that particular column in a row number. So, that is call pivoting scheme, okay such what we would do the next thing, so we will do this Gauss elimination with pivoting okay, so to repeat there is in there is any the first when I have matrix here before I eliminate this even before I do this operation to eliminate now, the elements in that column. I will first find out which is the largest element in this column x, okay then I would take this turns out to be the largest element. So that is this a_n, a_{n1} was the largest element I had, when I interchanges this 2 rows, I would write the equation down, this equation up, now that we have interchange now b_1 and b_n.

I would do interchange of that, and then I will go head I will scheme an eliminate all of this, all of the column and then I look at the next row and next column and again find out which is the largest elements and I would interchange, okay so that way proud of make sure that this very large that is the largest number possible in that column. Okay, so that way I can reduce the possibility of error, okay that is called Gauss eliminating with pivoting, okay so that that is means summarized here, so you go from row 1 to row n and choose i which is the larger than case such has such that a_{ik} is the maximum in that, that is means finally you have to find out what is the maximum value, maximum number which is which is modules you are not interested in the sign here. So just we could find out modules and then final maximum value of that quantity, and then I have interchange rows I and k.

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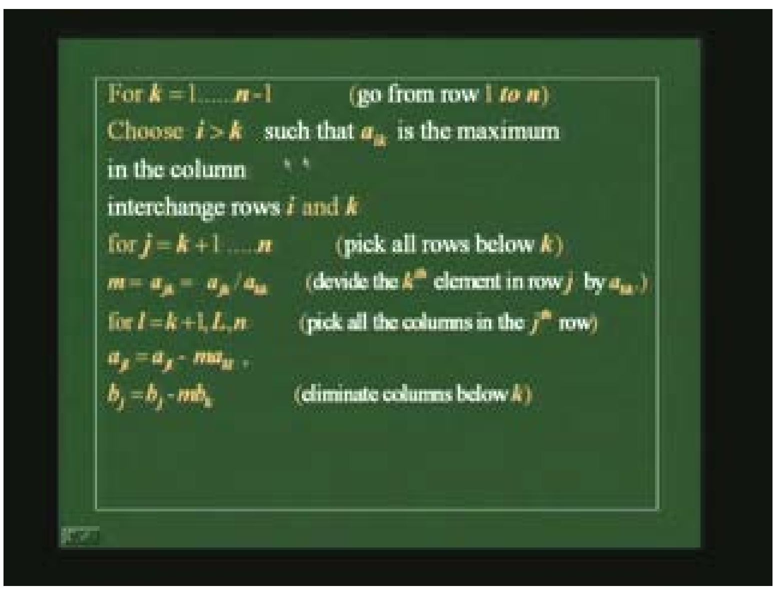

So for any operation, so for any column we will do that, that is I go to the I go to from 1 to k here is right side keep doing this. So first I do that then I will go below this then I will go then the next element I go below that etcetera and I interchange that and then I would do the divide by this a_{jk} by a_{kk} element and than subtract and I do that and than subtract elements and eliminate the column 0. So that is the normal procedure, so this gauss elimination with pivoting only thinks with do is that we could find out, always find out the largest element in a particular column and then interchange the rows and have to keep track of that row interchange, if also not interchanging beyond.

So this is defiantly a much more stable much more, much better way of solving the set of linear equation, then the simple Gauss elimination. So, now before have we go and see the implementation of this in, this in a algorithm in a program we could look at the some other ways of doing matrix some others way to solving the set of linear equations and another method is calling l u decomposition is also useful for finding inverse of the matrix, so we will go throw this before we accept look of the program.

So in this particular case in the l u decomposition, such an equation as a_x is equal to b, where as we have matrix a and x, we have I column vector x and that is equal to column vector b, okay now this is return as, this matrix a would right has l matrix and matrix u at time x equal to b. Okay, now this is our column vector x_1 x_2 and x_n and b is the b_1 b_2. And this is the lower diagonal matrix and this is the upper diagonal matrix, so one we will write is one lower then matrix a as the product the lower diagonal matrix and upper diagonal matrix.

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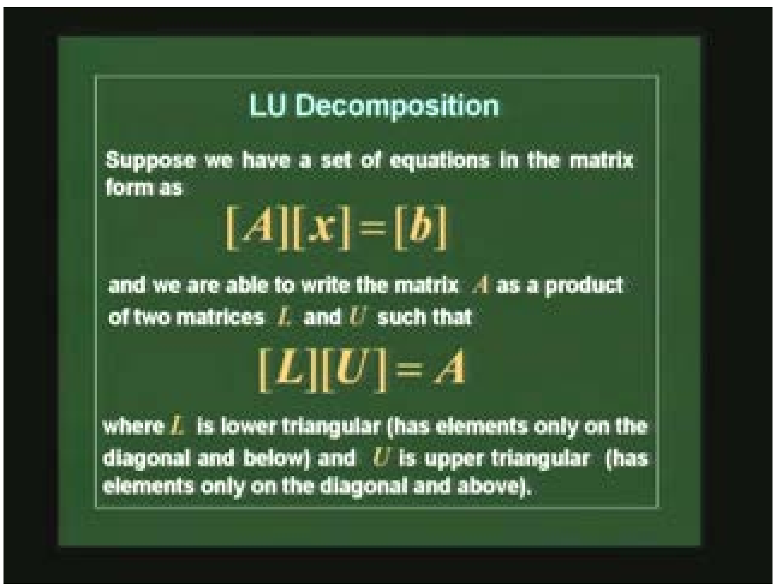

Okay that is call l u decomposition that is what you return here a x equal to b is now b in a will be now return has product of matrixes this l is the lower triangular matrix, lower triangular elements, that means elements has non-zero only and the diagonal and below, so lower triangular its call l it has elements non-zero, only long the diagonal and below the diagonal and u which is upper triangular matrix which has only the elements non-zero along the diagonal and above okay, that is we can right this matrices as this form okay is the example l and u, we call the elements $Alfa_{11}$ $Alfa_{21}$ and $Alfa_{22}$ etcetera and this is $beta_{11}$ $beta_{12}$ and $beta_{22}$ etcetera.

So side here, so we know the matrix a will be return has to be product of this 2 matrixes. So our equation now will look like this, so we have LUX equal to b right, okay and then if you so we have do first UX and okay and then multiply by l okay so when have we have if solution, let say for equation of this form that UX equal to y, and then we can take that solution y and then since by LY equal to b. So we know b okay and let say we know l that we could write the given matrix a has an l and u which product and l and u and then I know this right hand side of this my matrix equation that is b and then I could first solve LY equal to b and get the y and use that here and then solve for UX equal y to get the x that is I know you. So this way the 2 step process I would solve.

So wonder, what so advantage of this, so you see this LY equal to b, so l and u are triangular matrices that as I said, since l is the triangular matrices that is l is this one okay its lower triangular matrix. So I could right this equations now LY equal to b in this following form. So, LY equal to b will be now return has LY equal to b. So l is now Alfa $_{11}$, 0, 0 up to Alfa $_{1n}$ and then Alfa$_{21}$, Alfa$_{22}$ up to this 0 and Alfa$_{31}$, Alfa$_{32}$, Alfa$_{33}$ up to 0, so all the way up to so last would be Alfa$_{n1}$ Alfa$_{n2}$ up to Alfa$_{nn}$. So we have equation like this, okay lower triangular matrix we had y_1, y_2 up to y_n that now that is equal to b_1 up to b_n, that matrix is equation here.

So now, we can easily solve this because, we can do for what this substitution that we know first equation would be, Alfa$_{11}$, y_1 equal to b_1 okay, so which again solve for y_1 because I know Alfa$_{11}$ and next equation would be Alfa$_{21}$, b_1 plus Alfa$_{22}$ b_2 equal to Alfa$_{21}$ to y_1, Alfa$_{22}$ y_2 equal to b_2, so since I know y_1 I can from this y_2 so etcetera. So I can forward substitution, very easily I can solve for this y_1 y_2 y_3 y_n etcetera. So once you have all the y's, I goes back to this next equations, that is LY equal to b, if so sorry, I have y go back to this equation that is UX equal to y, now u is upper diagonal, upper triangular matrix. So you can see this before, right in this gauss eliminate exactly what we had, we are the upper triangular matrix and then the upper triangular matrix we can do the back substitution, so for this equations b_x equal to y which we just solve in the earlier.

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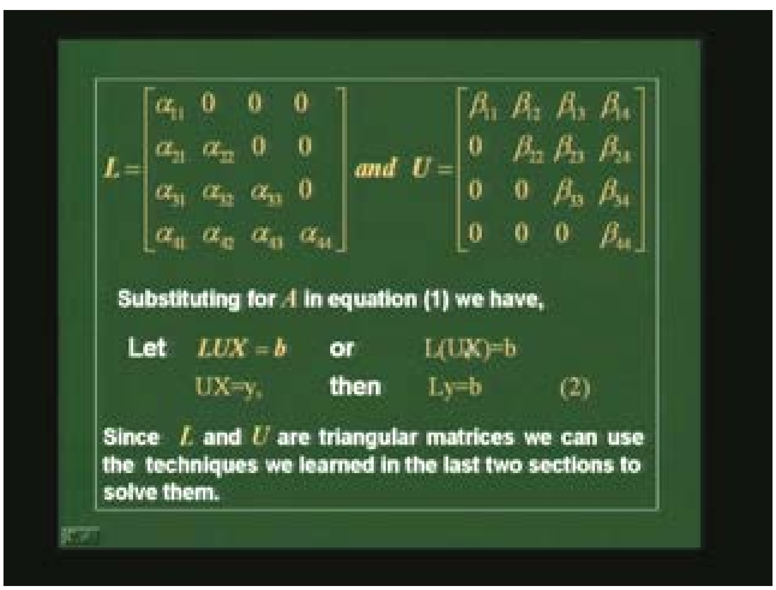

So what, so advantage is that, we do not have worry about the keeping track about b is here. So we have the matrix a and b which is broken b into 2 factors that is always L and U and LU equal to a. So which is it solving a equal to ax equal b equation okay we switch allowing two ways, okay we do not have to worry about the changes in b, only thing is find out what is L and U, and then we to do back substitutions. So once a found in L and U, we do one back substitution, one forward substitution to find out the y's and then we do one forward substation to find out the y and one backward substitution to find out the x value, that is what will do.

So then, now it is left to finding the values as, finding the matrix l and u that given a matrix a and so now that term set to be difficult. So we given l and u as a, so right we right l as this matrix as solve for $Alfa_{11}$ $Alfa_{21}$ $Alfa_{31}$ etcetera and the u as this upper diagonal matrix $beta_{11}$ and $beta_{12}$ x. So and then u that is equal to a. So now, we can so equate this both side which is expand the matrix right, that is matrix equation and then right n square equations of this form is Alfa ,so we have 2 sets of such equations that will be n square equations that is $Alfa_{il}$ $beta_{lj}$ equal to ai_j for i less than or equal to j now these are important, so you write them down here again what we have is two equations, that is 1 is $Alfa_{il}$, $beta_{lj}$ sum over l going from 1 to i, that is what we have so l equal to 1, l equal to i, okay so that will be given by a_{ij}. Okay, this is for i less than or equal to j.

So that is what the first equation is, which is the all both equation it is come from this equations that is, so this row multiply this column equal to element a_{a1},a_{ij} th element. So we have the first equation, this valid for all i less than or equal to j. So remember, just notice at that the limits of the sum is x is l going from 1 to i, okay l going to 1 to i $Alfa_{il}$, $beta_{lj}$ equal to a_{ij} is the just will right down in the matrix we can see that is to be Eli bole has to get.

Next equation is $Alfa_{il}$ $beta_{lj}$ equal to a_{ij} but now limits are l equal to 1 to l equal to j, okay now the next equation, so the next equation is again the $Alfa_{il}$ $beta_{lj}$ equal to a_{ij}. Okay now, that is for i greater than j and now the sum l going from l to l equal j, so l going 1 to i here will going to j here this for i greater than j, this is for i less than into j here. Okay now these are the 2 equations which can solve okay. So what we have is n square such equations and we have n square plus n unknowns ok because, the diagonal elements are non zero for both Alfa and beta elements ok such non zero elements both are Alfa and beta matrix. We have n square plus n unknowns may be n square equations, so we have to so the exam choice, so that we choose that has the Alfa matrix has it diagonal elements 3 equal to 1 that is what we will do. So we will say that alfa matrix this that is the that is the lower triangular matrix has all its diagonal elements equal to one its once we set there, so that is we already set for $Alfa_{ii}$ equal to one so then we have to only unknown ,so then we can solve like this it is what we do will first we will see that x plus ij we will solve for the which choosen i okay and all for j values grater than i the use this equations and all for j value less than i is the equations and then we have change the i value and then go again to, for example, I do first i equal to 1 into i equal to 1 and then j can going from 1 to n just we will have at j going to n.

So we will use this equations so we will say I equal to 1 and then 1 we have j equal to 1 is it so j equal to 1 and that case I so I is 1, so l goes to from 11 so just to be only one element in this i $Alfa_{11}$ beta l is 1, j is the 1 equal to a $_{11}$ so that is that $beta_{11}$ is equal a in 11 and then we can change it j equal to 2, j equal to 2 and same i equal to 1, we can j equal to 2. So now again, we will have only one elements here because i is only1, so it has a_{i11} and beta l is 1 j is to 2 $beta_{12}$ is equal to a_{12}. So that is $beta_{12}$ is equal to a_{12} by following this we can easily seen that all the general equations is the $beta_{1n}$ is it a_{1n} that is $Alfa_{11}$ is 1.

So we can solve that very easily and then get the ok now one we choose i equal 2 then we will go to i equal to 2 and then we will see that for I, so for j which is one, we cannot use this equations because i is 2 and j is the 1 this equation is satisfied because the reduce the this equations okay. So for j equal to 1, we will use this so we will have only one element l going from 12, l going to 1 is j is 1, so we have Alfa i is 2 Alfa_{21} beta_{12} beta_{11} j is 1 beta_{11} equal to a_{21} okay, that is we will we have okay and from this, we know what beta_{11} just a_{11}, so will write Alfa_{21} as a_{21} by beta_{11} which is a_{11}. So note is, that is Alfa_{2} such has got Alfa_{21} and beta_{11} and beta_{12} all the way to beta_{1n}.

So then next would be j equal to, right j is equal two we have to use. So when you use j equal to 2 again you will go back here then we right will equations now i is 2 and j is 2, so l goes from 1 to 2 the two element two factors in the sum so will have Alfa_{21} beta_{1j} is 2 and so beta_{12} plus will you one we will right. So we have now i equal to 2 and j equal to 2, so when you do that we are going to use to this. We have two parts that is Alfa_{21} beta_{22}, j equal to 2, so beta_{22} beta_{12} is plus then and then Alfa_{22} beta_{22} equal to a_{22} that is what to be here. So we already know what is Alfa_{21} is, we have found out one here and the know what is beta_{12} that be find out from here okay and this can be every think get the beta_{22} from this and then we will write beta_{22} has a_{22} minus Alfa_{21} is a_{21} minus a_{2} a_{11} that is a_{21} minus divide by a_{11} into beta_{12} was a_{12}, so that is okay.

So Alfa_{22} is one so that is so we can continue in this fashion. So we can do all for j equal to 3 etcetera. So you will find that in general I can solve for all of this, I can get beta_{2i} okay for all elements the from all j values is higher than 2 okay all values is higher than 2, I can use this equations and then, I would get and equation of this form that is I will get this b, a_{2j} minus a_{21} divided by a_{11} into a_{2j}. I would get so would do so write the b_{2} next element to beta general element all the element beta $_{2j}$ as a_{2j} that is what I do I get here because i is 2 and j is any j value so a_{2j} minus I will have in general the solution as a_{21}, a_{21} divided by a_{11} into a_{1j}, a_{1j} that is what would do so that a general solution which he can easily check. So, why I want to do this here because of the following reasons.

So I do this elimination by this two-step process okay that is 2 equation. Okay now, I am getting Alfa and beta elements by full picking up and than doing for i all I greater than j this equations i less than j this equation and use this using this we can actually solve by going back and forth by solve I can solve for all alfa and all then all beta provide Alfa_{ii} is equal to 1 now, such as now than this now will notice that the elements which we are getting okay we will find the first row of the beta is that simply just same as the matrices itself okay, the first row the beta is the same as the matrix itself a_{1}, beta 1 a_{1} and so we are writing matrix in the form, if you remember that we had the matrix as Alfa_{11},0, 0, 0, 0 and Alfa_{21} Alfa_{22}, 0, 0 etcetera.

Okay, so we had that matrix and then we had beta $_{11}$ beta_{12} up to beta_{1n} and then beta $_{22}$ beta $_{2n}$ and this side elements 0, so the last what beta_{nn} that is, that was our a matrix. So okay, that is what a so now if and a_{21}we know diagonal elements one here, that is what choose into okay right and we find Alfa_{21} is a_{21} by a_{11} okay, so it is similarly a_{21} by a_{11} and, in general then beta the next element beta here okay, so beta_{22} is a_{22} which has the a element here minus this a element this row okay divided by a multiply by a_{21} by a_{11} okay, that is what here getting here a new element here. The beta_{22} is a_{22} minus a_{12} multiply by a_{21} by a_{11}.

So now, that is exactly what we did when we did by eliminate, elimination okay the gauss elimination. So it term of so what we actually getting this this beta matrix which we get here, okay is it exactly the same as the matrix which you get on gauss elimination ,okay the simple gauss elimination what we get beta as the matrix the finally upper triangular matrix, that matrix is the same as this beta matrix and the factors which we use to eliminate , to do the gauss elimination okay that is, we are taking the first row and we take the second row was subtract from the second row we was subtracting the first row multiplying by factor and that factor is a_{21} by a_{11} to eliminate the columns below the matrix a, below the first row all the elements in the row, since all the elements in first column below the first row we multiply the first row by a factor which is a_{21} by a_{11} and similarly, the next one by a_{21} a_{11}. So the whole row was multiply by a_{21} a_{11} and that is the exactly what goes in as Alfa$_{21}$ here.

And similarly, we can show what goes on Alfa$_{22}$ as Alfa$_{22}$ is what goes for Alfa$_{31}$ here, okay the next element Alfa $_{31}$ here is the factor which here to multiply the first row in the gauss elimination to get ride of the elements in this column, in this row here and the first column. So in summary what I am writing say is that, this operation which do here okay, do get this beta and Alfa matrix okay finally produces this operation kindly produces a matrix beta which is the same as the upper triangular matrix which as got from the gauss elimination scheme, upper triangular matrix is the gauss elimination scheme now going to beta matrix here that is u matrix here and the factor which is use to eliminate those, a column elements below the first and second row and all the elements in the nth column below the nth row, okay was that factor which use to multiply that was is the element Alfa $_{ij}$ that is what would be fine okay that is what would be fine from the simple scheme okay. So that means we are just using the gauss elimination itself we could get both Alfa and the beta element are the l and u matrix, we can actually get from the gauss elimination okay and then that we would see the actually small calculation this one.

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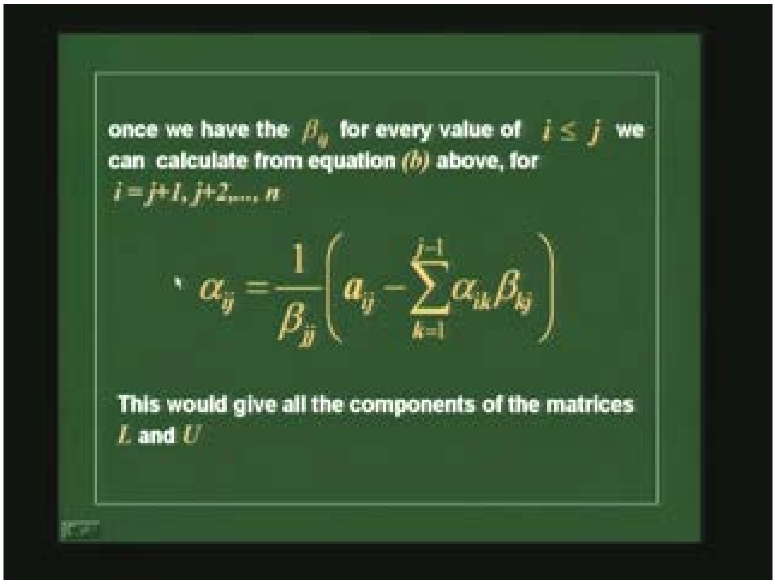

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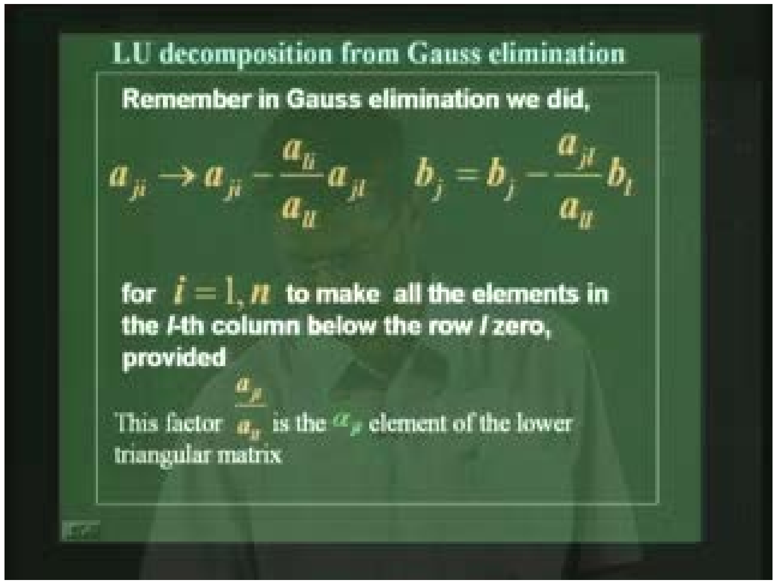

Okay I will show you that so only difference is that gauss eliminate remember, we actually whenever we transform the matrix a, we also transformed this b okay. So we did that okay in the gauss elimination scheme okay but, in this case here, we don not have do that what we do is which is simply store the factor in the Alfa matrix okay that is what we doing okay we simply store the factors as a_{li} by a_{ll} in our Alfa matrix, Alfa matrix this is the ij th element of the Alfa matrix.

We just store this in the Alfa matrix and then we will do a 2 step elimination provide two-step process, one forward substitution, one backward substitution to get , so this factor this are the factor which is as the store and the Alfa $_{jl}$ element in the lower triangular matrix okay. So we will see in the this in a small we can use a small example to see this, okay we try to solve in equation of this form, so less have in the equation form this form see yesterday I see in our example that is 1.1 and 6, 2, 1and 1, 4, 2. So we want to solve this equations of this form x_1, x_2, x_3 equal to 7.65,okay let us see, we have any question to solve equation of this form. So what we do, we as I say eliminate these two element and then this element and we would get upper triangular matrix okay.

So that what we would be do it right. So will do that by first is multiply by this row by 2 all this element by 2 and subtract from this, okay that will go to 0. So that multiply by this 2, so 2 into this row minus this okay 2 into this row, subtract from this row, this row go to 0, so that will get it as 3, now the new matrix would be 3, 2, 1.1, 0, so when I multiply by 2, I will get it minus 1 here and I will get minus 1.2 here. And then, I would multiply this row by 1 by 3 and subtract from this to get 0 here, so it is 1 by 3 is my factor okay, so factor here was two remember to get this and the factor here was is 1 by 3 is so when multiply by 1 by 3 and then when I get 010 by 3, 4.9 by 3 as the factor ok that is the factor has get now, I got 1 by 3 factor here, and I have to factor 2 and then I can now, further reduce this right by multiplying this row by 10 by 3 and adding to this okay then will goes to 0 this will get added to multiply by 10 by 3 okay and added to this.

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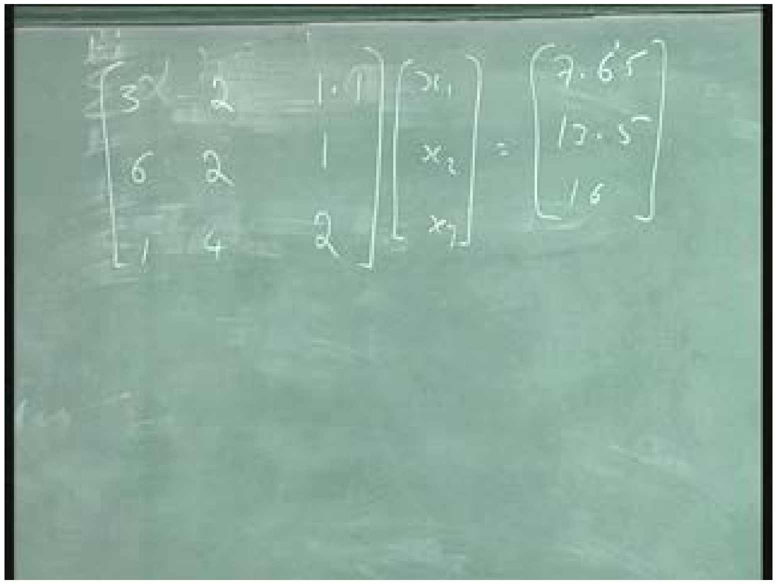

So this will change will become this multiply by 10 by 3 and added to this and this will go to 0, so that factor is a minus 10 by 3. So that is with the factor minus 10 by 3 going here, I would get is 3, 2, 1.1, 0, minus 1 and minus 1.2, 0, 0 and then this term out to be just minus 1.1 by 3.

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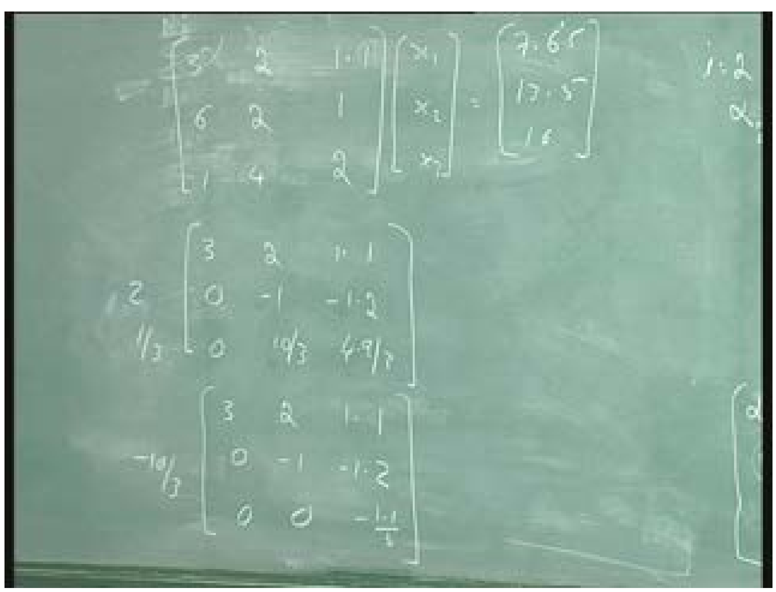

Okay I got now, so upper triangular matrix so what to my client is there this that this is my beta matrix in the beta matrix exactly, this beta matrix is exactly the same as this okay, what is my

Alfa matrix now, Alfa matrix is also diagonal elements is 1 okay, and $Alfa_{21}$ this 1 would be just 2 okay and $Alfa_2$ $Alfa_{31}$ will be this 1 by 3 by 3 and Alfa $_{32}$ would be minus 10 by 3. So that is the clear, so my beta matrix is, so is Alfa matrix now is 1, 0, 0 and then I have 2, 1, 0 and then have 1 by 3 minus 10 by 3, 1. Okay that is my Alfa matrix now, ok this 3 factors, I put in there okay I am beta matrix, now there is now 3, 2, 1.1 and 0, minus 1, minus 1.2 and 0, 0 minus 1.1 by 3 okay that is my beta okay now see my very simply that that the multiply which is 2 will give this matrix a back okay.

So I actually then, have decomposition of this matrix into upper triangular and lower triangular matrix by do this okay then can easily see that, so now you could actually solve this now, we are not done in any think b equation at all is it is as it is, so my question now is that this into x_1 x_2 x_3 is equal to 7.65, 13. 5 and 16 okay that is my equation. So what are going do is to is to do again two-step process, okay I am going to say that my I have LY equal to b, so this are the set of equation which we have this LUX equal b and then we wrote UX_l and UX equal to b that is must we have now 2 equations set UX equal to y okay and then we have LY equal to b, that is what to you thins so first what will do we do LY equal b and solve for y and then we substitute that here and get the UX. Okay that is what going do x, so we will use this equations LY equal b, so we know l okay, so we have 1,0,0 2,1,0 1,2 by 3 minus 10 by 3,1 into sum y_1 y_2 y_3 that is equal to 7. 65, 13.5, 16.0 okay, we can solve this is, this equations okay.

When you solve the equation we would find that y_1 y_2 $y_{3.}$ We will get y_1 as, so you can do this y for you substitution as a set, so is y_1 will out to be 7. 65 it is straight away, ok and then we can find y_2 well be equal to minus 1.8 and we get y_3 as 10.45 that is forward substitutions, so we will get y_1 strait away and then you have 2 y_1 plus y_2 as 13.5. I know y_1 so I can put you y_2 that and then I have 1 by 3 y_1 minus 10 by $3y_2$ plus y_3 as 16. So I can get the y_3 from that so this will lead to that is a division is to, so once we have to y_1 y_2 y_3. Okay and now I can use the equation UX equal to y okay now going to say this UX is y that is what put here right, so now I can UX equal to y, so I will do that y you better x being 3, 2, 1.1 and 0, minus 1, 1.2 and 0, 0 minus 1.1 by 3.

Okay so and then I have that x_1 x_2 x_3 which as the solution we want to fine and right hand side would be now 7.65 minus 1.8 and 10.45 okay that is what to do now we are to do back substitution here. So we are to do first x_3. So minus 1.1 by 3, x $_3$ equal to 10. 45. So we can solve that and then we will get that x_3 as minus 28.5 by just going to this first 1.1, x_3 by $3x_3$ equal to 10.45 and then I have equation that minus x_2 plus 1. $2x_3$ is equal to minus 1.8 right can I know x_3 here I can substitute that here and then would give me x_2 has x_2 is equal to 1, okay that is what we will get and then I can write into last equation that is $3x_1$ plus $2x_2$ plus 1 point $1x_3$ is equal to 7. 85.

So from that I can get x 1 and terms out have to be so x_2 terms have to be 18 and x_1 terms out to be 1, okay can see do this so calculation and then would then you would get this solution, so what are the done so far, so done these a matrix has l terms u terms x and then LUX equal to b. So we took UX as some y, okay will solve from LY equal to b and we got y from here and then we put that back into this equation and then UX. So now with we solve this and got y and then okay and then set this UX is y so that UX equal to y is here and solve, as solve for x that is general method and that is can be programmed.

Okay I can be return has the simple all together to solve this, so what is the advantage is so we don not have to do have to do anything at the b values okay we don not to transformed form the b values and also notice that then, we do not need any extra space to store l and L and U because you had one matrix a right now that matrix a, is Split into 2, now matrix two do different matrices l and u and we an chosen the diagonal elements of the l to be 1 that is has to the know that is no need store back we are not going to be use that we are going to know this one. So in this equation here later already assume that this one.

So we do not need to store the diagonal elements, so that the diagonal elements and above is a matrix can be use to store b this u, okay so diagonal elements and above of the original a matrix can be use to store the u matrix this upper triangular matrices okay. So that is a, so then have a right is the a equation okay then a matrix okay, so this part of my u matrix can be simply stored in the a matrix itself okay and the elements below the diagonal elements, diagonal of the a matrix can be used to store the, l matrix.

So we do not need any other storage apart from the original a matrix itself okay, see if we are dealing with large matrices this definitely very helpful because storing two dimensional array is always a big problem on a computer okay. So here is a this particular l u decomposition method will actually help us to solve the problem okay and then, we can also that forward and backward substitution is extremely easy to program okay, that is what to you see in a program after these. So here is a program which implements is the l u decomposition of the matrix and the find solution use that to find the solution of linear equation, okay here is the program.

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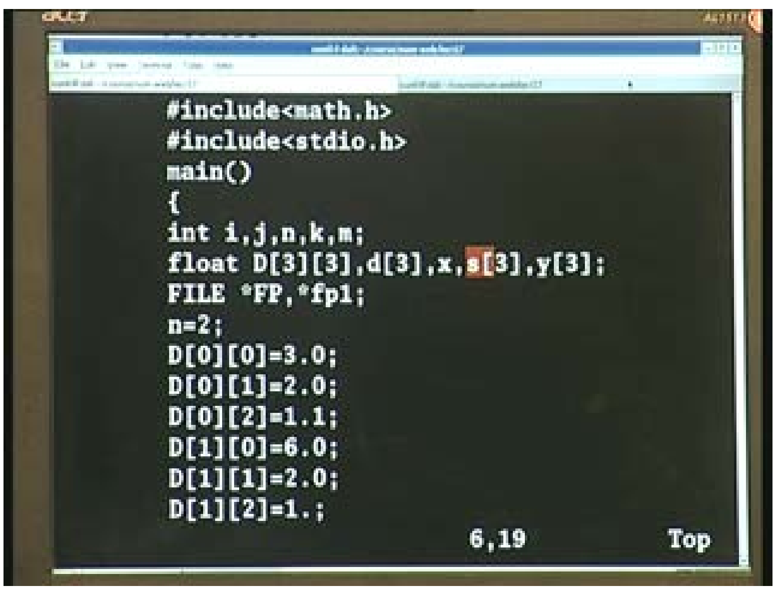

So we have the matrix d which would be the, which is the matrix of the decompose little d column vector is the solution, is the right hand side of the linear equation and we will get the solution in this column vector and now intermediate value remember, we have this l, we had LY sequel d that is that solution of that. So we are write the matrix d has now l and u, so l and u into x equal to d is our equations in right from that l into l into u and d l and u into x equal to l and u is equal to s here, not x equal to d is the linear equation. So now that u into s is what would be recognize is y and then solve LY equal to d first, LY equal to d solve first and then when will use that y and then you say use u into s is equal to y as the next equation and solve for y, so that is the to actually the solution s has the intermediate value which would get from the equation l into s is equal, l into y equal d that is solution we will get from again in y.

So here is matrix d okay, so which is the initial matrix is given now, you want to the solution are right hand side equation is are here, the d 012 and then do you have the, we will print there out here and which see the matrix first see matrix equations. So I will just show, so such that full matrix okay so we have the matrix element 32 as 1.116211 and 142 and right hand side of the equation is 7.65, 13.5 and 16. So this a linear equation of the form 3 x_1 plus 2 x_2 plus 1.1 x_3 equal to 7.65 etcetera, so that is a full matrix and then we have score here this would now l and help us put the find split the matrix into Land U which remember, we are using the method here, using here the elimination process we use the elimination, the gauss elimination method to get the l and u matrices that is what we are going to do. So first we check k equal to 0 the first row, and then we will go from j equal to 1 to n, that all the rows below that and then first column and the eliminate the first column for the first column of all the all the rows which has do first right.

So okay, when you do that all the columns get it effected when I have an change but an the factor which we need to multiply the first row to eliminate all the rows in the first column is this 1 that is d is j is 1, d_1 into k, k is 0. So d_{101} by d_{00}, that is first element that is the factor which you use right, that the factor which use this x here and that is factor from all the elements in that row okay for the all the rows and then same for the all the rows we will do that this way, we will in this row factor change will factor will change so one row one factor right, so one row whole row is multiply by 1row by 1 factor okay which is x that given a row j which is every thing below k the factor is djk divided by d_k, okay that is same for the all the elements in one row. So now that factor in the lu decomposition we discuss that factor goes here as the element in that column right, so as set for every row in the column right for every row, we have the same factor right for every here, for the same factor so when you take the k equal to 0 that is the first column, ok then we eliminate every elements below that first row using different factors, for different rows right that is what the factor here is the djk by dkk.

So I take a row j, ok now I have factor, ok and that factor is now stored in the djk element because d jk element it is going to be 0 right and then another take another j, ok now new djk new factor djk by d_{kk} that is goes into the second I take the first row and get d_{1k} by dkk that is d_1 is 0 by d_{00}, that is goes has the d_{11} element d_{10} element which is now actually eliminated. So I store the factor that is the eliminated of my l matrix right, so l matrix is compose of this factor which is used to eliminate columns okay, that is when store that okay then I take next column for the next row that is j, so j I change write then I have new factor okay there is d_0 by d_{00} and then d_{20} is stored in that factor x is stored in d_{20}.

So the end of the process I will have the one round for k equal to 0 if I did is the first column, first column is now first column below the first row, first row is also 0 because that is what I will show here. Okay it is all the first column below the should be all count to be 0 but what I do here is I store that element which I have to multiply, okay in that column okay, in just now that is what I show me here okay so now I look at this I had to multiply this rows by 2 and subtract from the second row first row by 2 and subtracted from the second row, first row by 2 and subtract from second row, to make the second row is 0. So that factor row is actually stored in that element this any way to 0. I stored that here itself. So this is part of the l matrix this part of u matrix and, this part of the l matrix. Similarly, I have to multiply the first row by 1 by 3 and subtract from here and sum third row to make the third row, first column 0 and that is 1 by 3 stored there.

Okay, so stored here, so this again part of my l matrix is. So similarly, I will do now for second column okay, now this all the second column also do the same thing and then after I finish all that okay I have the second column done now I have the second column to make this second column the last element 0, okay I have to multiply this one by again I have divided by this .33 and multiply by 2, so that factor is being store here. So this is now by this time I have completely decomposition matrix so now this the first row and I have the Land U matrix both put in this okay, so the u matrix you know s given by this whole element 1 2 3 right and the l matrix is given by the this 1 2 and 3 elements, okay everything else is u matrix okay.

The diagonal elements of the l matrix is we know is 1, so we do not need to store that, these 3 are the or the elements of the u matrix. Okay, then we have seen that I would change than right hand side because I do not need to change to right hand side okay, and then I do I do the 2 forward substitution to get the y matrix, that is this one a given here the y matrix you obtain by the forward substitution, so first element in does in change he can see for this can change here and other to change and then I do the backward substitution to get the solution that is a_x.

So that this a solution of the UX is equal to y and this is solution of LY equal to d, okay I got y here and then I substitute UX equal to y and I got a solution here and this the come and just compare of the its comparing then will substitute of this solution and back into my equation to get the right hand side correct yes, I get almost correct the round of the errors you can see here, okay that is what we will see. So I the, I am not using any pivoting technique in this, okay so I will show the pivoting techniques implemented in the program in the in the next lecture.