**Classical Physics**
**Prof. V. Balakrishnan**
**Department of Physics**
**Indian Institute of Technology, Madras**

**Lecture No. # 38**

We were looking at Lawrence transformations in particular I said I would discuss the transformation properties of the electromagnetic fields themselves. Let us first write the Lawrence transformations in a comfortable or convenient passion.

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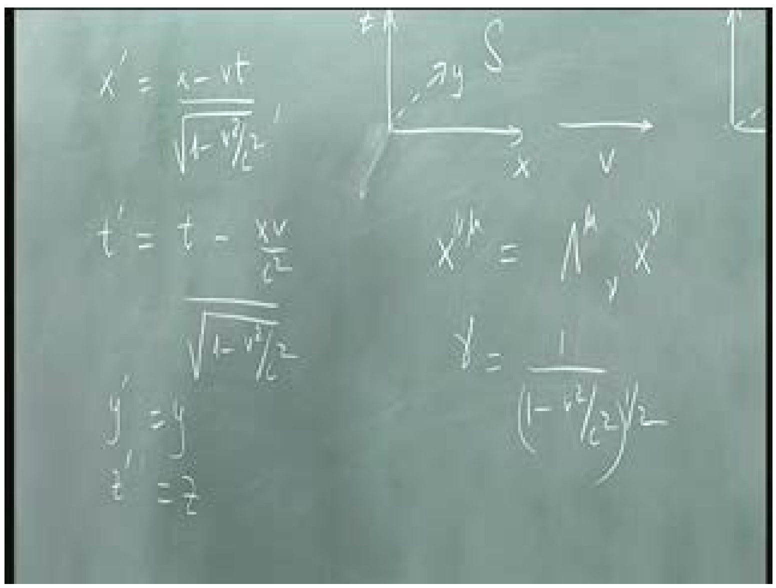


For instance if you look at a transformation, where x prime is x minus v t over square root 1 minus v square over c square, t prime is t minus x v over c square over the square root, z prime equal to z. We have in mind something where frame I kept labeling in to this direction, actually you should have done it properly, here is x, here is y going into the board, and here is z and this frame is moving along x p v, so that we have x prime, y prime, z prime. Add this x coming out of the board and moving in the y direction so the earlier figure, this is the right way to do this.

This is S, and this is S prime and these are the Lawrence transformation equations, then recall that we wrote this down in compact notation, as x prime mu equal to lambda mu nu x nu, and this matrix lambda of coefficients was what k v your told you, what the Lawrence transformation was like. And if you recall in this particular instance, this lambda matrix was of the form, gamma minus gamma v over c minus gamma v over c and gamma, and then there was 0 is everywhere else this passion; where gamma was equal to 1 over 1 minus v square over c square to the power of half.

Now, of course you know, this implies the relativistic addition of velocities, law of velocities is that velocity is do not just add up linearly, but you know that if you have a frame, if you have the frame S prime moving with x p v 1 c, and then a frame as double prime moving its p v 2 with respect to x prime. Then you know, the law of addition of velocities, for velocity is in the same direction, is v is v 1 plus v 2 over 1 plus v 1 v 2 over c square, this is the law of addition of the velocities, and that is well known provided that the two velocities on the same direction of course.

Now, of course we been talking in terms of v over c, so this is really v over c is v 1 over c v 2 over c and v 1 v 2 over c square, this suggest very strongly that the correct variable to look at is not v or even v over c, but rather something where that becomes a linear law of additional velocities. And what is this v do, after all this v can go all the way from 0 minus c to plus c, because it is a single component. Now, therefore, v over c goes from minus 1 to plus 1, and what variable, what mapping from x to some function of x takes you in a monotonic way, in a one to one way, between minus 1 and plus 1.

Tan hyperbolic tan hyperbolic x runs from minus 1 to plus 1, and for x sufficiently small it is actually linear, it is essentially x, so it look like the right variable to describe, to use is really not v, but tan hyperbolic v over c. And as v goes from minus c to plus c, this goes from minus 1 to plus 1 is a bounded quantity, so let us put u equal to this in earlier, then of course its a trivial mandite to verify, that gamma is 1 minus tan hyperbolic square and of course, it use is trigonometric identity says cos hyperbolic square minus sin hyperbolic square is 1, then immediately this implies that gamma is equal to cos hyperbolic u.

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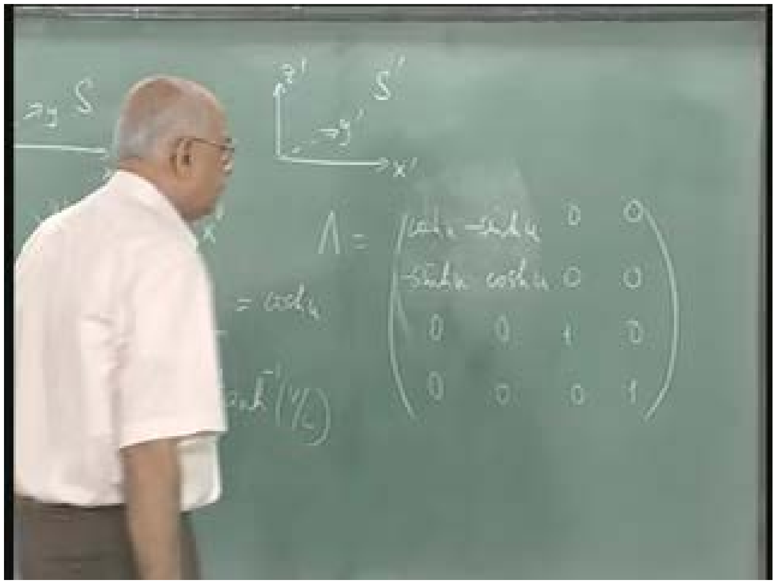

And of course, then v over c itself is tan, so we have sorry we should write v over c v over c equal to tan hyperbolic u or u equal to tan hyperbolic inverse v over c tan u, that is the transformation; its immediately tells you that gamma v over c is sin hyperbolic, so that gives us a very simple way of writing the Lawrence transformation equation. It simply says replace this by cos hyperbolic u sin hyperbolic u the minus sign, minus sign hyperbolic u and cos, there almost starts looking like a rotation.

Except in the case of the rotation matrix you had something like cos alpha minus sin alpha minus sin alpha cos alpha, because orthogonal this matrix is pseudo orthogonal. Because, remember that the that the orthogonality condition it has to obey is lambda g lambda transpose equal to, where g was the matrix concern, so this is a very convenient representation.

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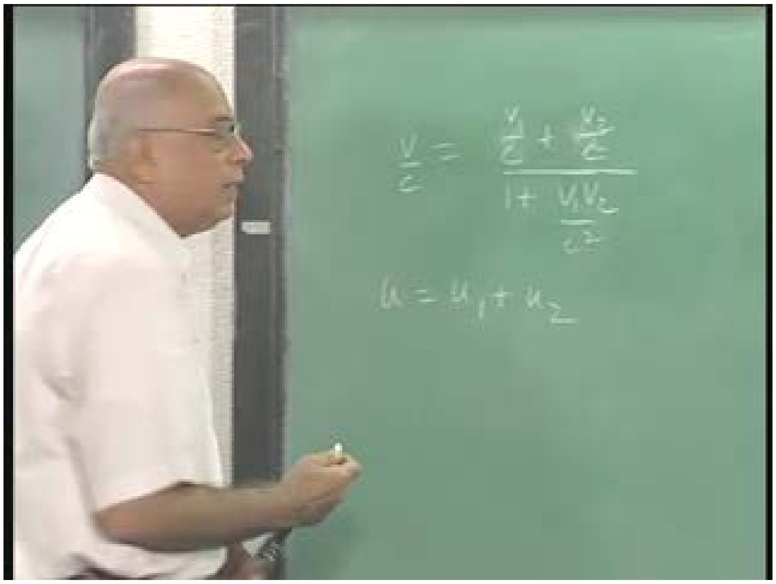


And this immediately implies by the way that u equal to u 1 plus u 2, if I define tan hyperbolic v 1 over c as u 1 and similarly, for v 2 this immediately implies u is the 1 plus u 2, so Newton was almost write Newton Galileo etcetera, velocity is do not add up; tan hyperbolic v over c is they add up, if they are in the same direction.

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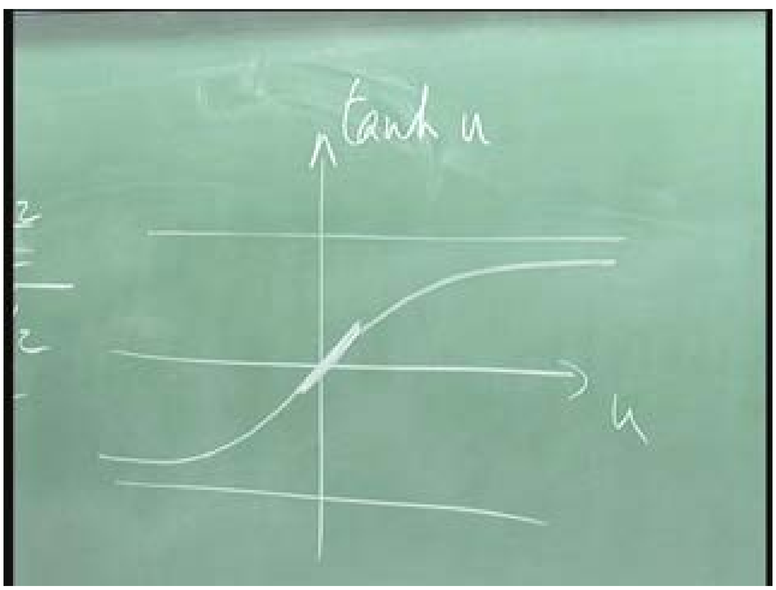

And of course, for sufficiently small values of v over c, so if I plot here u equal to plot here, tan hyperbolic trying to see how to write this, tan hyperbolic u versus u and of course, this quantity goes discussion here, and here its linear approximately linear of there; that is the non relativistic region v much much smaller than c, this quantity is essentially linear. Otherwise, you see that this saturates on both sides, and it adds up, this quantity here u equal to tan hyperbolic v over c is the rapidity, in the relativistic law of addition of velocity, a velocities in the same direction say that the rapidity is add up, not the velocities.

But, it these quantities tan hyperbolic inverse v over c is what adds up, and that is the discovery that is the special relativity, we could have started with this law and then, deduce backwards the Lawrence transformation equation two. So, very convenient way of writing this lambda of course, a given lambda the given general lambda would be this, it will be in some arbitrary direction, then of course if you make a boost from this frame to another frame, in an arbitrary direction.

The way to write that transformation matrix would be to first take this quantity, take this frame rotated to bring it to this, bring one of the axis to the direction of the velocity transformation, move it there and then rotate back, so that these axis are parallel. So, the general transformation would be much more complicated as you can see, but we will written down its special Lawrence transformations in which the velocity is actually of the second frame, the axis of parallel oriented parallel to each other, in the velocity is along one of the coordinate axis, otherwise its much more complicated to write down.

I should also mention here right away that, if you wrote if you went to frame S prime from S by moving along the x axis, and even from S prime to S double prime by moving along the y, the y axis of the z axis, you write the result is not boost from this frame, to the frame S double prime it is a boost and rotation in general. So, this means that the boost themselves do not form a group, because the composition of two boost in different directions is not an equivalent boost, it is not equivalent to a single boost, but it is a boost along with the rotation, the rotation form a sub group of the Lawrence group

Because, they are only they do not affect these the time component at all, but only this special components in they are a sub group, SO (3) is a sub group of SO (3, 1), but the boost do not form a sub group that is why the Lawrence group is fairly non trivial group to deal with. Now, that we know this, let us ask what happens to the electromagnetic field under a Lawrence transformation, then we come back after we deduce that, we will come back and write down what happens to a charge particle, in an electromagnetic field, what is the actual equation of motion in the relativistic case, will do that next.

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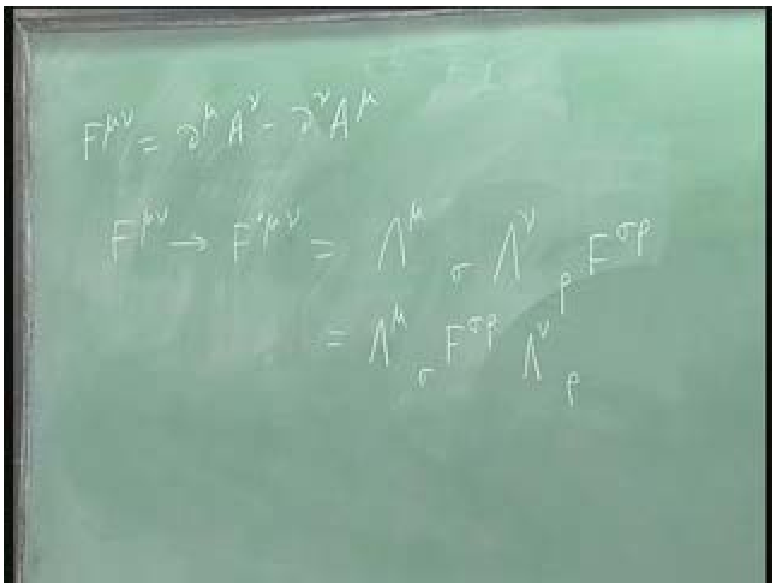


But, first let us look at what happens to the field tensor, recall that F mu nu was del mu A nu minus del nu A mu, now under a Lawrence transformation given by a matrix lambda, you get F mu nu goes to F prime mu nu and that is equal to the index nu is like a vector index. So, this is lambda mu sigma and show is the index nu that is lambda nu rho F sigma rho that is the transformation equation. Because each of these as like a vector index and there is a lambda matrix corresponding to it, and you have to contract over sigma and rho. But you can write this in a slightly more convenient form, we could write this as equal to lambda mu sigma F sigma rho you can bring the F inside here.

And this is contracted this thing here is the mu sigma the element of this Lawrence transformation matrix, multiplied by F sigma rho, and then on the other side you have a lambda nu rho, but that unfortunately is not equal to rho nu, if it were you could just contracts, so this is lambda nu rho in this passion. Had the rho been here and the nu been there, then you could have contracted and just written it as product of matrices, but you have the same problem as before which we tackle, so what yes

Yes yes

I am saying the rule yes, the rule of addition of velocity, this is not correct, the resultant of two velocity is v 1 plus v 2 is not equal to v, its not equal to v that is exactly the point.

No, no no please let me careful; no I am not saying the law of addition of vectors is wrong in a given frame of reference, not saying that ordinary vector algebra will apply of course, I am not saying that when you add two four vectors they add component by component in the usual passion. And of course, if the time components are not relevant, but only the special components they would add up also, the quantity is that are the transform in a given prescribed way under Lawrence transformations of four vectors, not three dimensional vectors.

So, you see when you go from one frame to another frame, the x component of this original vector the one component, does not add up there is not the same as a one component there, when you go, even when you rotated changes its makes up with the y and z components. But when you go to another Lawrence frame, another inertial frame, it makes up with the zero components as well, so it is the four vectors that are now, the quantity is which add up. Now, what I am saying is that when you add velocities, these are three velocities, this is d x over d t d y over d t etcetera, these components do not add up like they would under ordinary rotations, they do not behave like vectors.

The three components of a four vector do not form a vector in any sense, under Lawrence transformations; they would continue to be a vector and rotations of course, in the conventional sense. So, the law of additional velocity is not valid, not the original one that you wrote anyway, because time gets mixed up that is why its not valid, and now the further point was that there is something like the law of additional velocity, were the rapidity is add up as long as these are in the same direction. But the moment you have different directions then of course, this is no longer truth much more complicated, and back that is the point. And the further complication was that boosts in different directions, when you composed to boost the answer is not an equivalent boost, that related to the fact that the velocity is do not add up.

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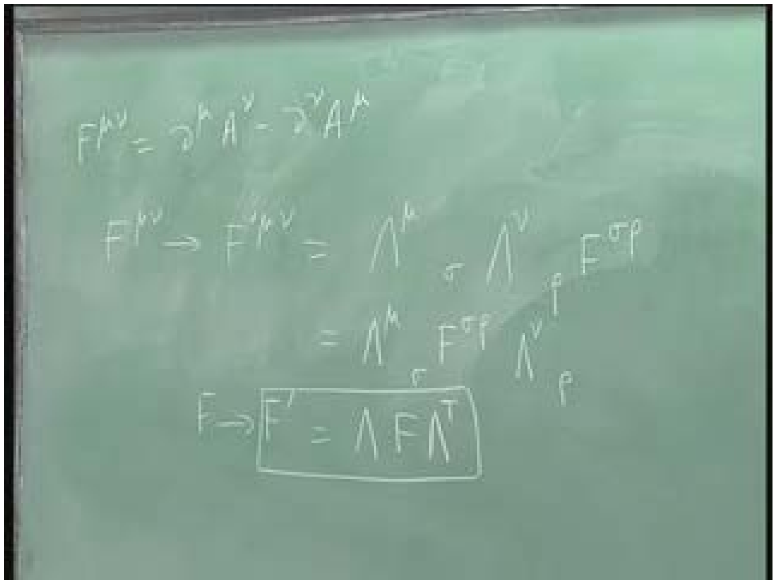


So, this is the rule now for the transformation and of course, its easy to see what we should do, this F as a matrix goes to F prime, if I write these components as a matrix goes to F prime which is equal to there is a lambda here, there is a F here, and there is a lambda transpose here. Because you need to bring the rho here, take the new out, so this is the transformation law for a field tense, and of course once you tell me what lambda is as a matrix, for instant this guy here, and you tell me what the original F is, I put this n and I compute what this F prime is. And therefore, I know all the electric and magnetic fields components, in the new frame of reference.

I am going to leave that is an exercise to you, to do it in this case namely in the case when you have the frame S prime which is moving with respect to x along the x direction, the axis oriented parallel to the original one with a speed v. So, you need this lambda and of course, it happens to be equal to its own transpose, then you put that in here you compute this and you find what F prime is, and remember the 0, 1 0, 2 0, 3 components of F, where minus A x over c minus c y over c and so on.

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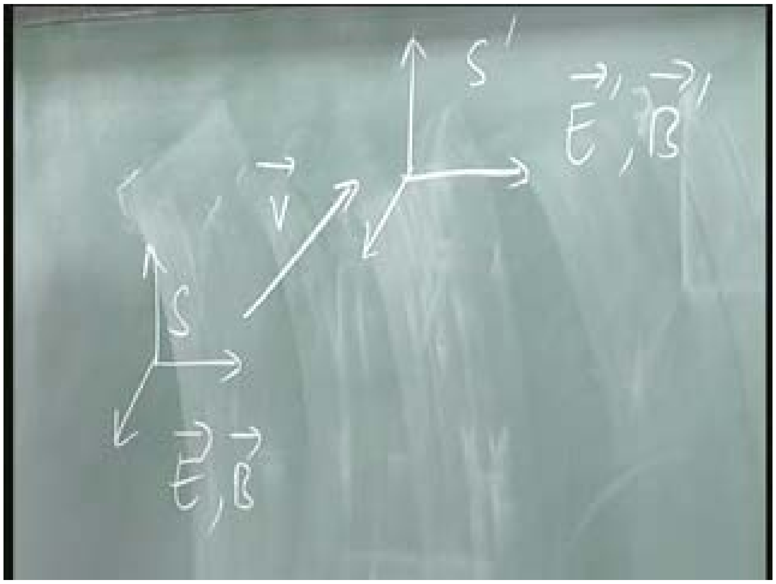

And then the phase space components, the 1, 2 1, 3 and 2, 3 components, so the magnetic field components, when you do this algebra here is the result that you get, so now suppose I have a frame S this passion, and it is boosted to a frame S prime without a rotation just boost it in some arbitrary direction v, along some arbitrary direction with the magnitude v. So, this frame S prime is not moving along the x axis of the original thing, but in some general direction, then the electric and magnetic fields in this frame, if I denote those by E and B, and those in this frame by E prime and B prime, these guys are related to these things by linear transformations which looks like this.

The component of E along this direction of the boost, let me call it E parallel, so E parallel prime is the component of the electric field in the new frame, along the direction of the boost. Let us happens to be equal to E parallel itself, so the electric field does not change, the component along the direction of the boost does not change. Similarly, B parallel prime remains equal to B parallel, no change in the electric and magnetic fields along the directions of the boost. But the transverse components E perpendicular prime, and that is the vector, because if you have the component here, the transverse one can be anywhere in this plane and it is a two dimensional vector, that quantity is gamma times E perpendicular.

So, it is multiplied by this gamma factor 1 over square root 1 minus E square C square plus V cross B over C, 19.49, I need not write B perpendicular here, because V cross B parallel is 0 anyway, so might I just write it as B. So, you see something like the Lawrence course coming out here, in this case its not an accident, this is exactly how its transforms and that incident is a reason for the Lawrence course, this is precisely this term that axis.

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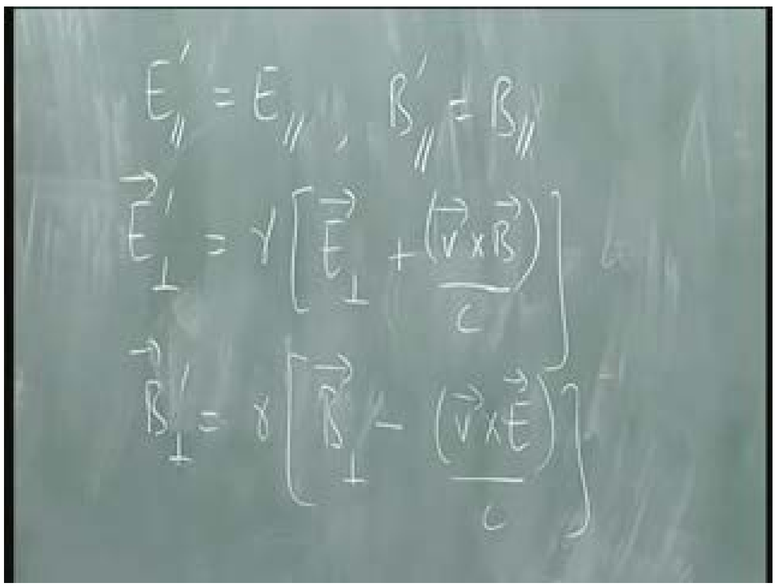

Similarly, B perpendicular prime is gamma times B perpendicular minus V cross E this time over C, so this is what the new magnetic field looks like, its gamma times the original magnetic field the transverse component minus V cross E, and it would as well B E perpendicular here over C, so this is what comes out from this relation here. Of course, if you rotate the coordinate system also, then the components of E get mixed up with each other in the components of B get mixed up with each other, but there is no um of course mixing up between E and B. But what a boost does is of course convert electric fields, magnetic fields and part and vice versa, and these are the exact transformations equations, again I leave this an exercise, work this out let us go on to what happens to a particle, in a charge in an electromagnetic field.

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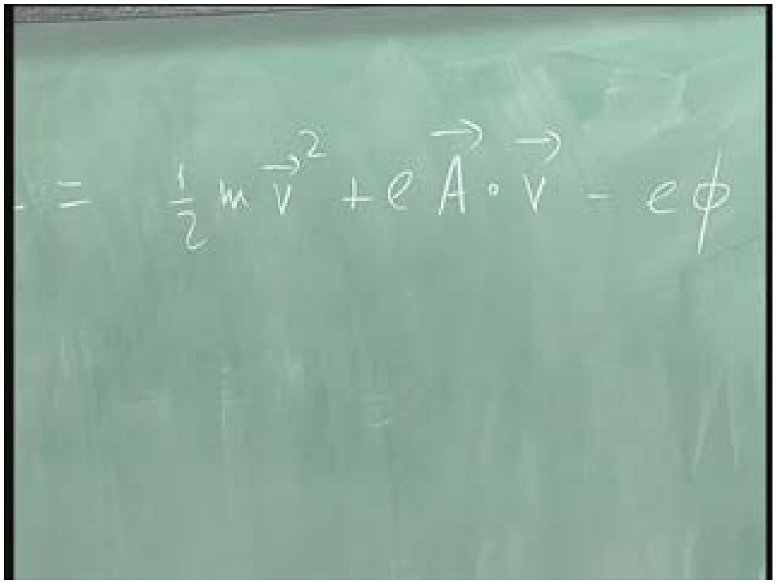

Now, recall that are original result was the following, they started off by saying the Lagrangian was equal to 1 half m v square plus e A dot v minus e phi, that was the Lagrangian which we wrote down for a charge particle in electromagnetic field, and it was Taylor in such a way that we got the Lawrence force equation, we got m d v over d t is a charge times electric field plus V cross B. Now, we should now ask what happens in the relativistic case, first we should ask what is the Lagrangian for a free particle in the absence of an electric electric or magnetic field, for a free particle I know the momentum is going to be conserved, but I know the momentum is not a linear function of the velocity; its m v divided by 1 minus v square by c square, I need to produce that.

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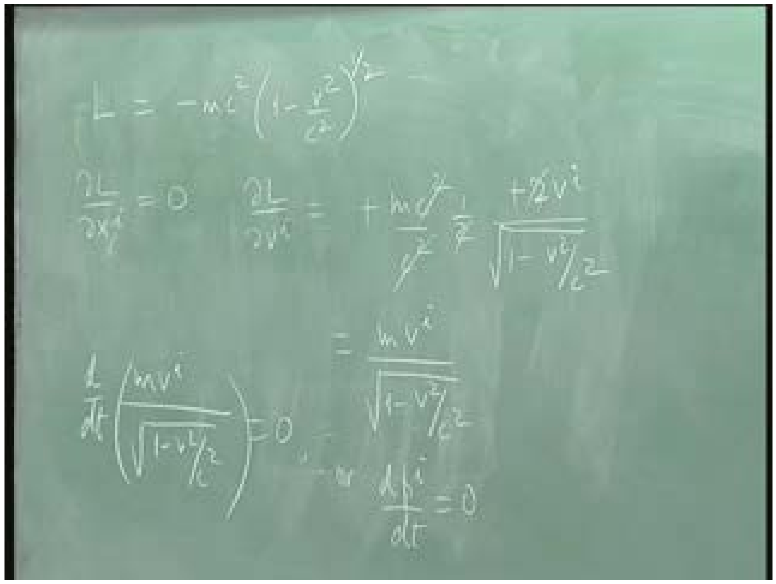

So, let us write the Lagrangian down for a free particle, and this is the Lagrangian that produces m c square 1 minus v square over c square to the power of half, consider this Lagrangian then I explain where it came from, what is delta L over delta x i this is equal to 0, because its no dependents of free particle, there is no dependent on the coordinates. But delta L over delta V i this quantity is minus m c square and then there is a half 1 over square root 1 minus v square over c square, and then minus 2 vi over c square, so this cancels, this c square cancels and that is equal to m v i over square root. If I differentiate with respect to v i any of the velocity components, this is what you get.

And the equation of motion the Euler Lagrangian equation of motion says, d over d t m v i over square root 1 minus v square over c square, this quantity is equal to 0 that is it. And you recognize what is inside there, it is just the component of the momentum, the i th component of the momentum; so the Lagrangian does produce the right equation of motion. So, this is the free Lagrangian, now where it comes from you can actually deduce this slightly more fundamental considerations, but I am not going to do that I am more interested in writing down, what happens when you have a charge particle there.

Now, the fact is that you want this quantity L d t this action to be invariant, and if you write this in terms of proper time this is the same, as gamma L d to have by proper time I mean the time, that the particle c is in its rest frame, and I want this to be invariant this guy is invariant. Therefore, gamma L must be invariant and that tells you it must be equal to a constant, and if you choose the constant, then v goes to 0 the constant is minus m c square that is the give is correct normalization, then L becomes m c square times 1 minus v square over c square to the half.

I am not going to go further into this, we are not going to discuss this now, the reason is that I would like to see what happens to this, when you put in a charge here, and put in an electric and magnetic field, and then see if you get the right answer, right experimental answer for this. Lagrangian are some level are always written down based on experience, trying to get the correct equation of motion in the simplest cases, and after that you generalize it. So, we convince that this Lagrangian is, as its stands and now you put in a charge, put in a charge, put in a charge on this particle and switch on the electric and magnetic fields.

Then nothing much happens, this remains e A dot v minus e phi, I will see a little later where this comes from, what does this look like it actually is a simplest variant you can write down, which involves linearly the current, as well as the electromagnetic potential. It is essentially j mu A mu, now what happens well delta L by delta v i gives you an extra term here, so this is equal to m v i over square root 1 minus v square over c square plus e, I should be a little careful here, let me write this in velocity for all components, so let us write this plus e A exactly as before.

But, this is the momentum, canonical momentum by definition, so this is equal to P that is always the definition of the momentum, the conjugate momentum is delta L over delta r dot here, we call that earlier we just had this, now we have this extra quantity here, what is the Hamiltonian, so let us settle that also. The Hamiltonian of this particle is P dot v minus L, when you must eliminate L in favor over eliminate v in favor of P, you must write everything in terms of p and r. But that is not so easy to do this time, because you can see that earlier when you had m v you could just write v equal to P minus A over m, but now you have this factor also sitting here.

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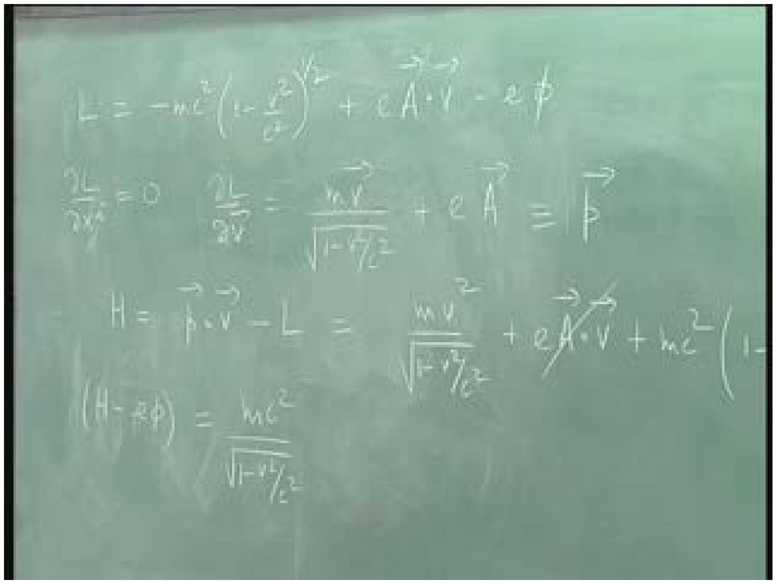

So, it is not that trivial, we need to work on this a little bit more, and see what happens, but let us see what happens we put this in this is equal to e dot v, so that is m v square over square root 1 minus for the moment, let me retain the v depends here k dot, so that is v plus e A dot v minus L and that is plus m c square 1 minus v square by c square to the power half minus e A dot v plus e phi. Then of course, this term cancels out which is the whole reason for doing this, therefore H minus e phi is this guy here, is equal to m over square root 1 minus v square over c square times v square plus c square minus v square.

I have added these two terms, this is m c square and I took this factor out here, so it become 1 minus v square over c square a multiplied by c square, and then the v cancels; so its m c square over this guy. I made a mistake, I made a mistake, its look its, and this implies that H minus e phi whole square is equal to P minus e A whole square, then multiplied by c square times this plus m square c 4 by use this quantity, I have m v square and then I add this square of that plus m square c square the v square cancels out.

So, whole idea is to eliminate this quantity, and do that by squaring then you get a 1 over 1 minus v square c square, and if you have a v square and top you want to cancel that and you do that by adding this v square, so this term you add and I fix the dimensions by putting the c square. So, this is the relation which replaces the free particle relation, the free particle relation you recall was E square is c square p square plus m square c 4, and all that happen when you um go to put in the charge particle is that, H goes to H minus e phi P goes to p minus e A, but phi and a are the components of four vector h and p are the components of a four vector.

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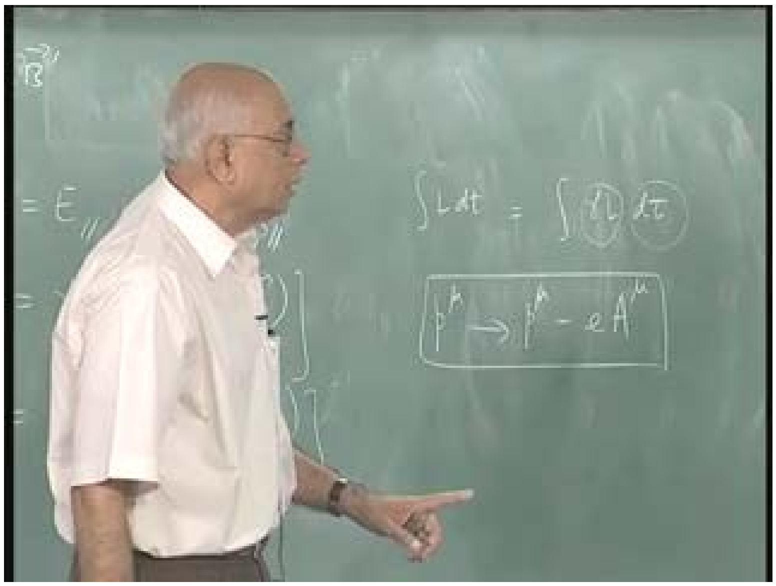

So, its evident that all that has happen is that p mu goes to p mu minus e A mu, so switching on the electromagnetic field just changes the definition of the four momentum, canonical four momentum, its subtract of e A mu that is all it has, and you have exactly the same relation as before. What about the equation of motion notice, now that the canonical momentum this guy here is fairly messy, messy function of this as well as that.

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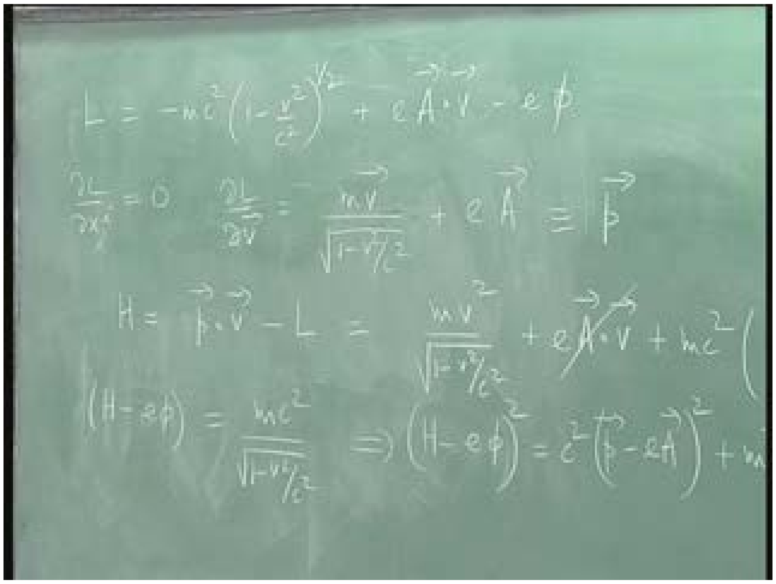


But what about the equation of motion, when we almost there, because that down we know the equation of motion says the Hamiltonian we have.

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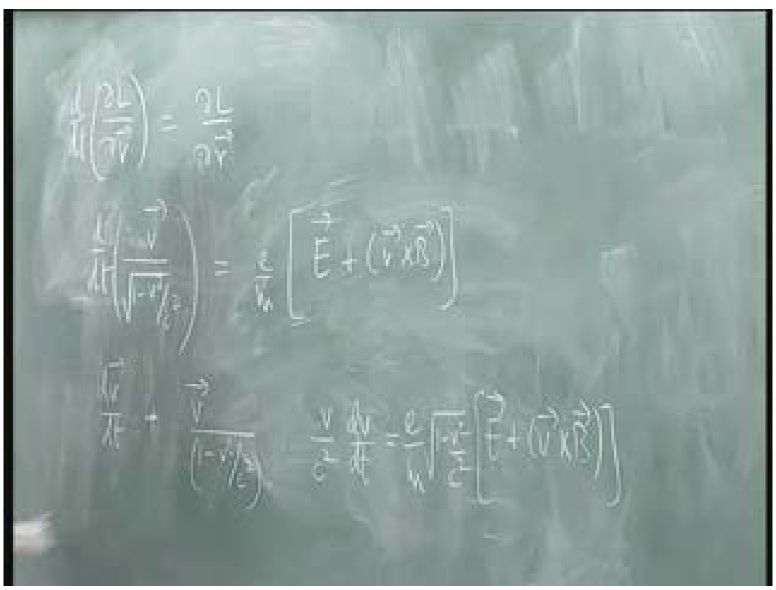


Now, the equation of motion says, d over d t delta L over delta v is equal to delta L over delta r component by component, and the L is sitting here all that happen was in the changing in the momentum, because of this term this extra factor here, the definition of the momentum the velocity dependence got a little more complicated here, everything else remained exactly the same as before. delta L over delta r, the r dependency is going to come from the r dependence of the vector potential, and this scalar potential exactly as before, and all the manipulations we did earlier, could still go through there be no change at all, the only difference would be this quantity here this derivative.

So, the next few steps would be identical to what was before, and the result is exactly what we got before which is d over d t but now, m v over square root one minus v square over c square this quantity here, this is equal to e times E plus V cross B. All this business about the total derivative with respect to time of A and so on, all that is exactly as before, all the manipulations are exactly as before, and this is the equation of function. This is the momentum, mechanical part of the momentum not the canonical momentum, it is the if you like the kinetic part of the momentum, but its got this extra term here, this is the exact equation of motion.

Same Lawrence force the same everything as before, but its not m d v over d t, but we have to deal with this and we would like to get an equation for the acceleration itself, that is the whole point to see what kind of equation you get, that becomes little trick here, but let us do it. So, this says first let us move this m out, then it says d v over d t 1 over the square root, then I differentiate this, so its equal to minus 1 half this a v sticking out here, and I differentiate this quantity its 1 minus v square over c square to the power 3 half there, and then I have to differentiate the v here, so this is equal to minus 2 v over c square, sure I am going to make mistakes here d v over d t multiplied.

So, let us get read of the matrix plus sign, get read of the 2 here and its that is it, this is equal to e over m E plus V cross B, and let us multiply through by this guy this square root, so this becomes just this factor here, and then this is equal to e over n square root 1 minus v square over c square. Let us move this term to the right hand side, so that’s equal to minus v d v over d t and then it is a v vector there v over c square and whole thing is divided by 1 minus v square over c square.

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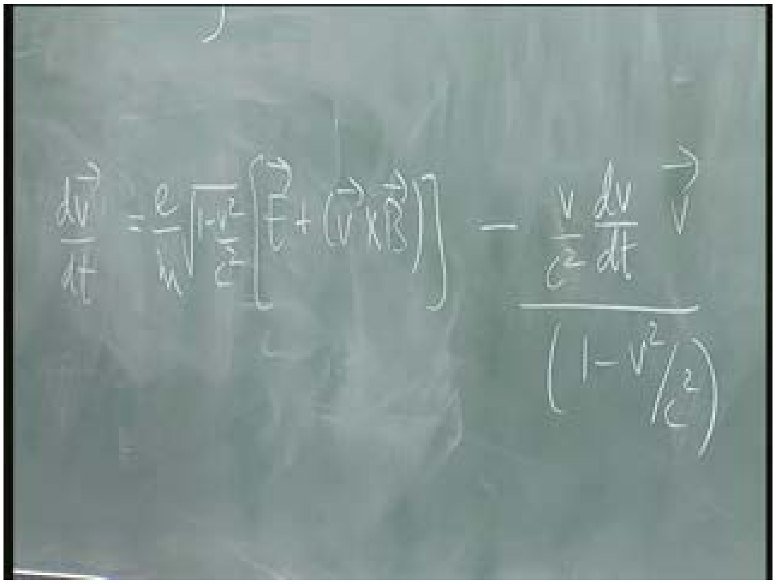

That is d v over d t, that is not very helpful because, you still have this term sitting here the d v over d t, so its not very helpful, but notice the following, notice that if I write, I need to do this in a slightly better way, I need to do this more efficiently.

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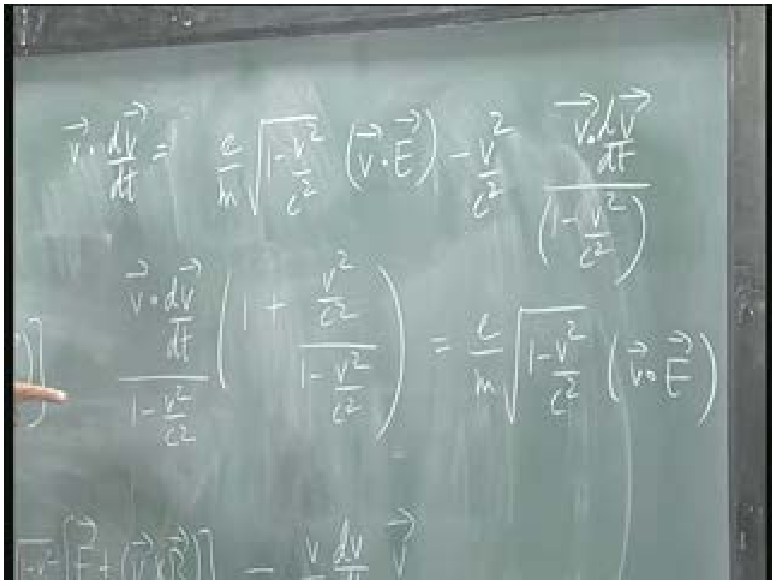

If I take this equation and dotted with V on both sides, then I get v dot d v over d t equal to e over m square root 1 minus v square over c square v dot E, and this term is 0, and I get a v square here, its seen to an extra v sitting somewhere, minus v square over c square, we go boldly where no manner gone before, so v square, v d v over d t over 1 minus v square by c square. I got it with v and I write that at v square I retain this v d v over d t there is a 1 over c square. But I know that this can be written as v dot d v over d t, and that is the crucial point, for any vector v d v over d t is the same as v dot d v over d t, because each of them is equal to 1 half d over d t of v square, so that is the basic trick.

Once you do that in variant good shape, because its says v dot d v over d t equal to v d v over d t, no, no this multiplied by 1 plus v square over c square over 1 minus v square over c square, I bring this to this side is that correct now, this is equal to e over m square root 1 minus v square over c square v dot F. And of course, this thing here reduces now to 1 minus v square over c square, and the v square over c square cancels and all, and this is v d v over d t, so you put that back here, so I am going to substitute for v d v over d t over this fellow here, I am going to substitute this.

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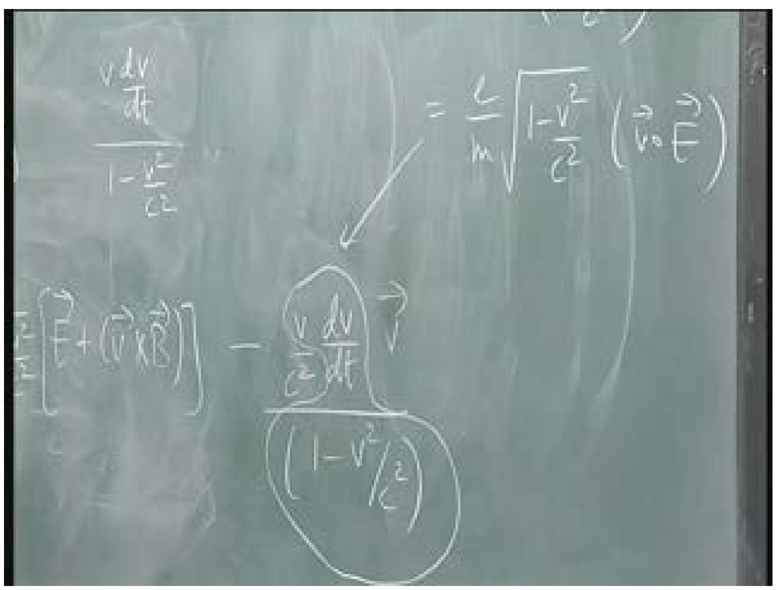

I am going to substitute this equation, so the only factor that remains, this is the common thing and the only thing that remains is this whole portion goes away, there is a E over m square root which comes out, and there is a v over c square v dot.

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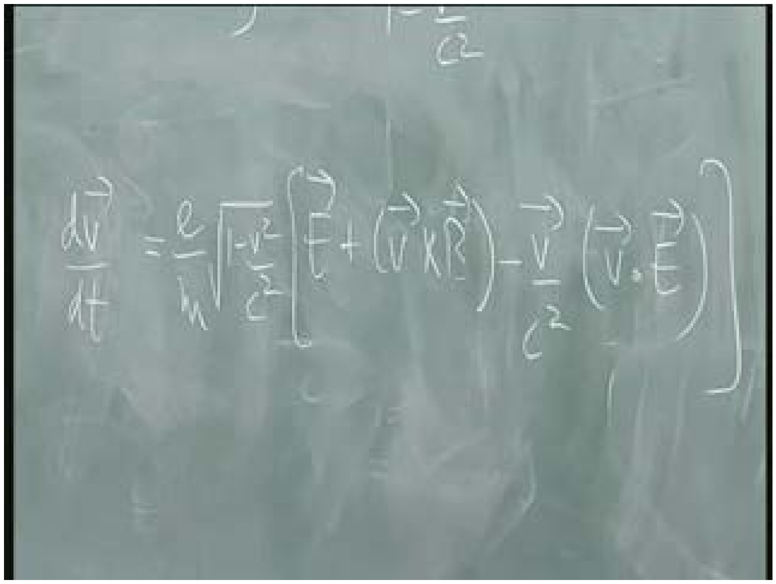

So, this is minus v over c square, that is the exactly equation, so you see it is a basic identity was this and you have to use this, use the fact and then I get this, so this is the exact equation of motion which replaces the original equation in the non relativity case. Now, you see that when v over c is negligible, then this goes away this term goes away and you back to the original , this is the equation for the acceleration of the charge part. And over the rest follows you can go on from here, and find out what it does, I do not want to I do not have time to introduce things like the regarded potential and so on, so forth, that it sees.

But physically you can see very simply that the electric and magnetic fields are closely linked to each other, they are related to each other extremely intimately, they really are the same field, and when you switch on when you couple it with matter then all you do is this replacement p mu to p mu minus A mu. That is equivalent to saying, that the interaction the interaction Lagrangian between matter, and the electromagnetic field is minimal, its of the form j mu A mu multiplied by the charge, this is all that been set, this coupling is sufficient to produce, whatever you need.

Now, this is the beginning of relative, the relativistic dynamitic of charge particles in a, in a magnetic fields, and the extra term here notice is not a trivial term, because this function of v is not trivial, its not a linear function nor is, this leads to a large number of , so this is all I want it say about relativistic electro dynamic.

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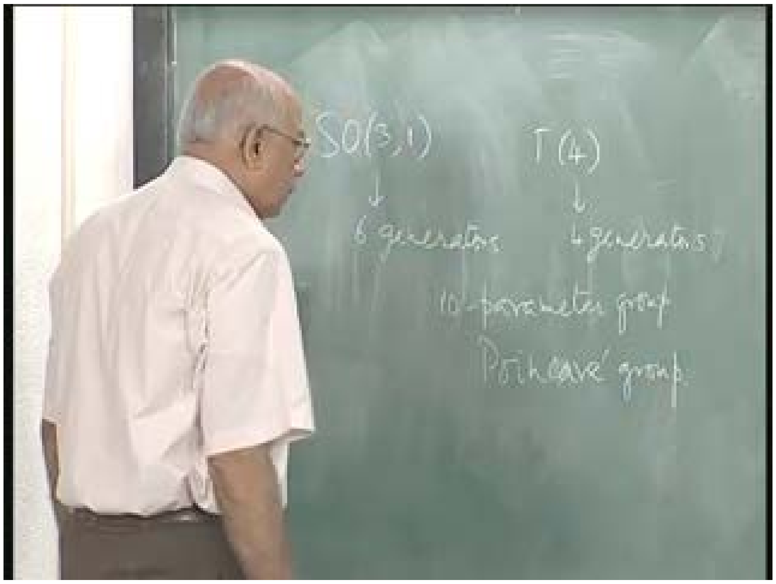


Let me say of few words about, the Lawrence group itself there is a final statement, I mention that this special Lawrence group the group of transformations talking about, is SO (3, 1) which is a set of boosts, the set of rotations, they are all homogeneous transformations. Now, the question you could ask is what about translations, what about moving the origin of space time, both in space as well as time, now of course you can do that, in the laws of physics are invariant under this group, not just under the Lawrence group itself.

And that group is the translation group in four dimensions, T 4 its the group of translations where I change x to x plus A, y to y plus B and so on, and T to T plus some T naught, that is an abelian group of transformations, because you can do these things and any order you commute. This is not an abelian group rotations, around compute among themselves boost on compute among themselves and so on, much more complicated group. There are six generators here, and there are four generators here, so together they form a ten parameter group, this ten parameter group is called the Poincare group or the in homogenous Lawrence group.

And this of the principle of relativity essentially says that the loss of physics are invariant, form invariant under transformations belonging to the Poincare group. In quantum mechanics these as serious consequences, its says something about what elementary particles can do and cannot do and so on, but that the classical level we could still ask, how are these, how is this group generated and what is the algebra of generators, what is the lie algebra of generators, this is not all together trivial and the reason is as follows. Suppose, I take a point x a space time point and I make a Lawrence transformation on it, belonging to a SO (3, 1), it could be a rotation, it could be a boosted it could be combination of these guys, this goes to some lambda x.

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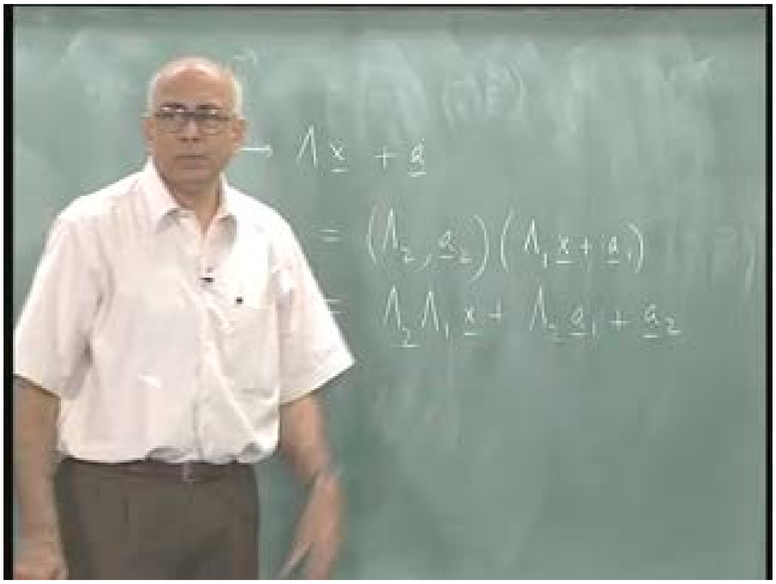

If in addition I also translate this space time origins, then this is lambda x plus a, some four vector a which stands for the shift in this origin of coordinates as well as in time, I do two of these successively I do one first, and then I do another. So, I start with an x and I apply lambda 1, a 1 to it and successively after that I apply lambda 2, a 2, in the question is what do I get at the end, this is equivalent to apply and this is a 2 on lambda 1 x plus a 1; that is equal to this lambda 2 is applied to that I added.

So, this is equal to lambda 2 lambda 1 x plus lambda 2 a 1 plus a 2, it is not equal to lambda 2 to lambda 1 on x plus a 1 plus a 2, because the first translation has a second Lawrence transformation acting on it, for instance it could be a rotation acting on it. That tells you that the translation part, the homogeneous part in the inhomogeneous part or not decouple from each other, this is not a single group property, this is a group composition law when the group composition law is not trivial, because this lambda acts on that guy in that translation here.

Therefore the group its not just the direct product of SO (3, 1) the T 4, but its SO (3, 1) with T 4 and this is called the semi direct product, however this is a sub group of this full group here, the inhomogeneous Lawrence group and its an abelian sub group. At the moment you have an abelian sub group lots of good things happen, you can induce the representation of the full group, using the properties of the using the fact that there is an abelian of group here, this is part of part of group theory, the theory called theory of induce transformations, I do not do that here.

But I want to just motivate for you finally, what happens when you go to quantum mechanics looks at this, for that let us look at the generators of this group this inhomogeneous group, this part has four generators which are called the translation generators, it is the four momentum.

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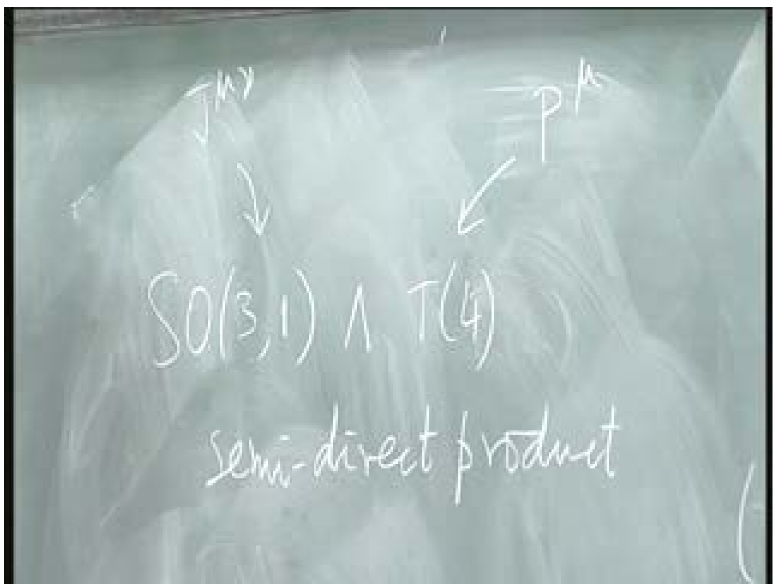


This part is six generators and the generators look like this J mu nu, whereas earlier you had J 1, J 2, J 3 for angular momentum, now you have six of them, this is anti symmetry and you have six of these generators of which the phase space components would be just the angular momentum part. But the others would also involve boost, they also include the boost generators, now together this obey a complicated algebra, since these guys form an abelian of group the competitor algebra for these simple that of course 0.

And then these fellows here, good form good form in a le algebra here, which would be typical le algebra which you have for something like a rotation group excepts its pseudo rotation group, SO (3, 1) I am not going to write that down, will have a long combination of J's linear combination of the J's. Then the interesting thing is what happens when the J's is the competitor of the J's is with p and those are not 0, because of this property, I would not write this algebra down here.

But, now you can ask how do I label representations of this group, and whenever you have a lie group of this kind, you have an algebra and there are operators in this group, there are operators in this algebra there are certain functions of the generators of this algebra with which all the generators commute.

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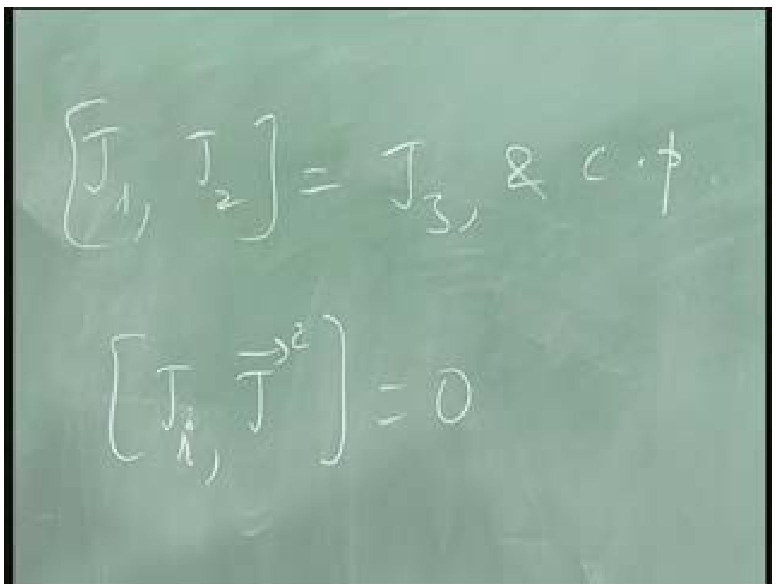

And those are called the casino operators, with give you an example from the angular momentum algebra, remember that there you had J 1, J 2 equal to J 3 and cyclic permutations, you had J I, J equal to epsilon  something of this kind, but you also know that every component here, J i with J square is 0. That is why you could label representations of this rotation group by the Eigen values of J i square, and that is why when you have the hydrogen atom for example, you discover that the total angular momentum is a constant of the motion.

Then the quantum numbers that you need does not principle quantum number, and there is angular momentum quantum number, and since there is there is any component of the angular momentum is also a constant of the motion, you can simultaneously Eigen states of that component the total angular momentum, and the total energy and the Hamiltonian. So, that is the reason you need a three quantum number n l and m, in exactly the same way you have to find the casino operators of the invariant, of the inhomogeneous Lawrence group. And you would label representations of the inhomogeneous Lawrence group, in terms of the Eigen values of those casino operators, this is the more complicated algebra that one, but it turns out that there are two casino operators here.

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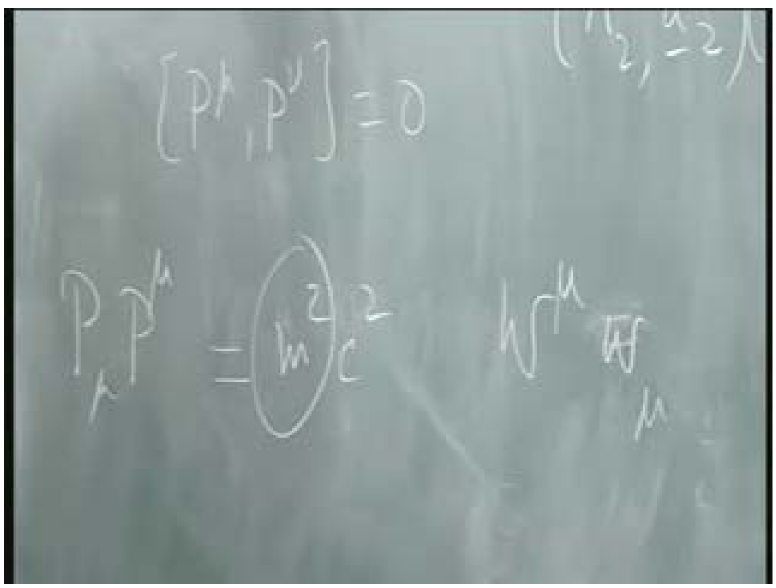


And these casino operators one of them is easy to write it just P mu P nu, and if you now apply this two elementary particles whose way functions are suppose to form what a called reducible representations of this group, then this quantity here is proportional to some m square c square of something like that, this value of m is a label. So, this m the rest messes particle immerges as one of the labels of the casino operators, it is an intensive property of that particle, of the representation.

The other one, the other casino is more complicated to write down, you have to introduce a combination of J's and P's of these J's and P's and there is a vector of quantity called W its called the  operator, and this guy here W mu W nu, which involves the J is which involves the Pp’s and so on. This quantity is also I mean invariant and value in the rest frame of this particle of this object, turns out to be proportional to m square times it involves S, this is infact, it comes S, S plus 1 or something like that.

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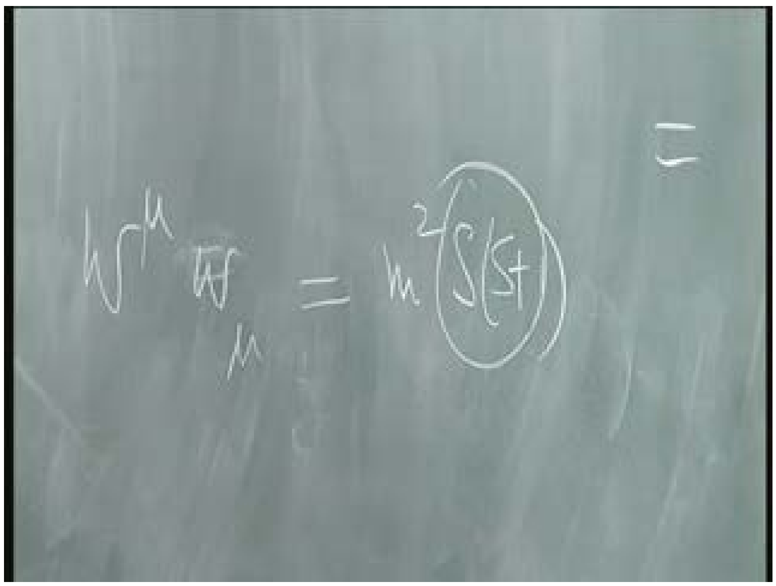

So, it involves another another Eigen value, and this is what is called the spin of the particle, so this is where the mass and spin come from, and the although there set this in a very wage way here, we cannot justify it unless I do the quantum physics, quantum mechanics of it. The point I want to emphasize is the fact, that the loss of physics or invariant under inhomogeneous Lawrence transformations, when you go over to quantum mechanics, and look at elementary particles from instance it has performed implications there, so properties like the mass and the spin of a particle immerge from this requirement.

The really immerged as interstice properties, which come out from this requirement, other properties like charge, vary on number, other quantum numbers, they do not come out from these space time requirements from this space time symmetries or anything like that they come out extra properties all those. But these two properties emerge in a basic way, due to Lawrence invariants, in this thing I want to emphasize and end with, so it has very deep and very perform consequences as we seen throughout, electromagnetism itself is rightable, understandable in a very, very simple passion, once we use the fact that its completely relativistic invariant theory.

That was the original motivation for special relativity, and happens to be a good accident in some sense because, like travels with the fundamental velocity, we have this entity which travels with the fundamental velocity. Therefore, its equations are motions are intenstically relativistic, there is no non relativistic approximation to the Maxwell equations, they are fully relativistic at distance; there is no non relativistic regime for a particle that moves with the speed of light.

Whereas, from the material particles you can go all the way from 0 speed right up to c on the other hand, for something moving that something which is got 0 response, it moves at speed Cc and its fully relativistic. You could also ask what is the equivalent of the  equation for Fortran's, the answer is the Maxwell equations themselves quantizes, taken in the quantum sense that is in fact the set of um evaluation equations, Fortran field. That is part of quantum electrodynamics, you would not get into that here, but I hope that throughout, able to give you a some flavor of what classical physics is like, where it is taken as and so; large chunks of classical physics have been left out.

Large part of classical dynamics would not handle at all, very important portion we neglected completely is general relativity, what happens when you have gravitational fields, we have talk about that at all, that is another very major part of classical physics. The numerable other part which you have in touched , but I started off by talking about mechanics in the general sense, dynamical system, we going through Lagrangian, Hamiltonians and so on.

Move slowly into what happens, when you have many interacting degrees of freedom with some statistical mechanics, thermodynamics and finally, moved into considerations of symmetry little bit about groups. And after that you moved into special relativity for little bit, and then its most prime beautiful example of special relativity, and operation namely electromagnetism itself. So, that is been a kind of broad overview of classical physics, partial but I hope still convert you some of the ideas involved in this field, and left you with the anticipation of what we have, I had of as in quantum mechanics, let us stop here.