Classical Physics
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Lecture No. # 29

The question raised was just as we have phase transition from a Para magnet to a Ferro magnet. Do we have a phase transition, corresponding phase transition electric field, other substances with permanent electric dipole moments in the pulp, just as there are permanent magnets with permanent magnetic moments, we have got in the bulk. The answer is yes. These are called Ferro electrics and this is called the Ferro electric transition, pretty much the same as Ferro magnetic transitions, but the mechanisms could be slightly different. Generally Ferro electric transitions are all also accompanied structural phase transitions namely there would be crystalline phase transitions, which would lead to shapes of unit cells or whatever which have permanent dipole electric moments. But they fall in pretty much in same kind of class.

As we will see even the magnetic transitions; there are huge numbers of that many numbers magnetic transitions. We are trying to look at the simplest of these. So, that brings us to where we were trying to make an analogy between fluids and magnets. In the process we had to write down the equation of motion or the equation of state for a Ferro magnetic substance, which connects the magnetic field. The magnetization and the temperature just as for the fluid the equation of state connects the pressure the volume and the temperature.

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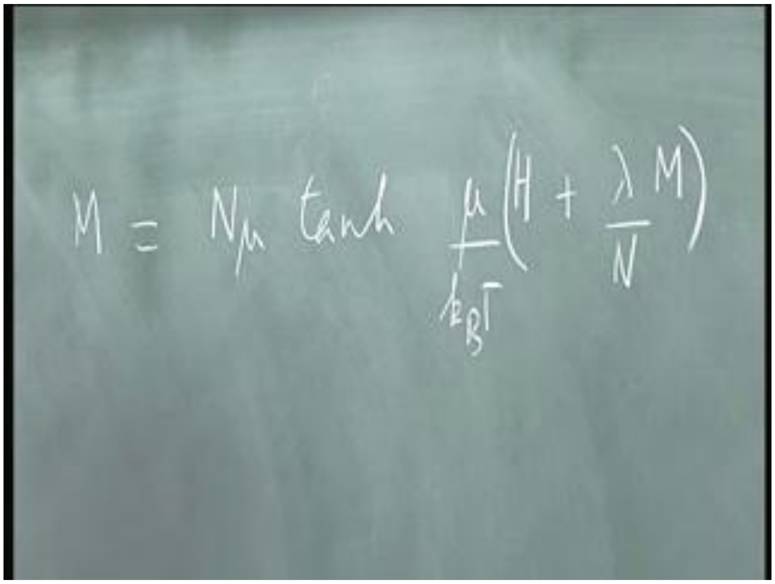

So, this is where we were at and we reached a state that the magnetization M in a simple model an up down model for instance was of the form N mu tan hyperbolic. If you recall it was mu times the external field over k Boltzmann T. This was a typical equation of state between M H and T for a Para magnet, but in the case of Ferro magnet I pointed out that. You have to modify this by writing this as H plus lambda over N times M itself. Namely, a electric field a magnetic field and effective field that is proportional to the magnetization itself. It is some kind of approximate treatment of the internal field inside Ferro magnet.

Now the question is this kind of equation of state is my claim is shows that there is a Ferro magnetic, Para magnetic to Ferro magnetic phase transition. If we examine what this equation of state does and the behavior near the critical region would be very similar to that of fluid. Near it is critical region, near the vapor liquid transition this is our target to show this. So, let us examine this equation here and ask what kind of magnetization, you have it is an implicit equation. So, you cannot solve this explicitly in analytical form, because on the left hand side you have binomial and right hand side, you have a transcendental function. So, you can do this numerically so, various values of H, but our primary interest is to find out if there is a magnetization which is brought to zero, when the field is zero.

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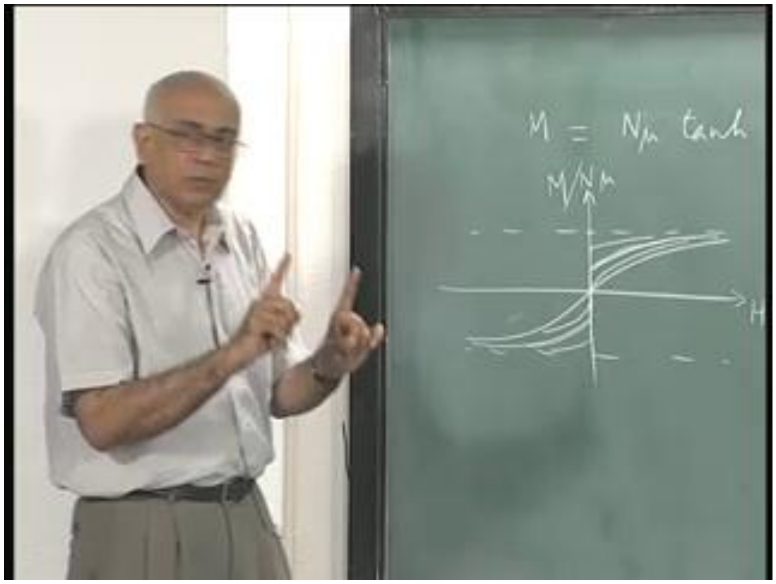

So, recall H versus M graph for the Para magnet M divided by N mu. For a Para magnet these are saturated in this fashion and for a Ferro magnet. We know that the behavior is something like this and then there is something hysteresis loop it comes back and goes off. Now, we should ask whether this hysteresis is taken into account in this model or not and the answer is not in general, because we have to ask where?

Hysteresis comes from and the way it arises is when you have complicated substance like real magnet. The bulk there are little magnetic domains in this sample and hysteresis occurs due to the existence of these domains. These domains take some time they click into place click into right orientation the magnetization inside. These domains into the right orientation once the field becomes sufficiently strong. If you have a single domain specimen, there is no hysteresis everything behaves exactly in the same way and either there is an up magnetization or a down magnetization. So, we will talk about this discuss separately time, if time permits let me ignore hysteresis and talk just about the single domain specimen.

In which case below the critical temperature, the magnetization does this and then the rest of this loop is lost this portion is present. And then it jumps again this avoids hysteresis remember that the original. If I simply continued with the way that the Para magnet would behave this is what it will do if I draw this again.

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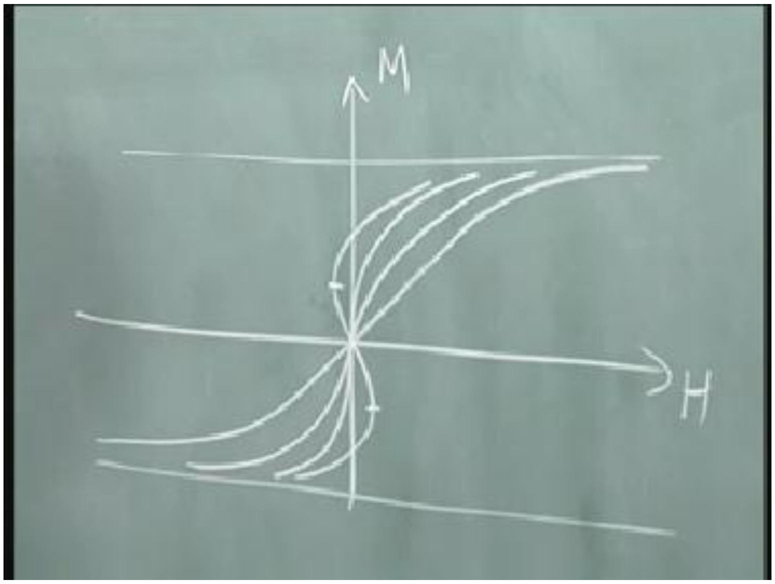

You have a situation where the slope goes like this. This is H versus M i increase, i decrease the temperature then the slope becomes steeper and steeper. Presumable at some finite temperature called the Curie temperature instead of t equal to zero this slope becomes like this. And what would you expected below that if this is a continuous process I would expect the curve after that to start doing this so, it will be tilt over.

Now, would create a problem because this region of the curve from here to here is unstable. It has the same problem that the P V graph had when you went up, when you came down to minimum. You went up to maximum between the minimum and maximum, if you increase the pressure the volume increase and that was unphysical in exactly the same way this region is unphysical.

Because you are increasing the fields in the positive direction and the magnetization in the negative direction and that is unphysical once again. So, this portion is removed that leaves you with this, which is precisely? What hysteresis is? So, you have this kind of behavior and then it jump and off there, but now I am saying just as we had the Maxwell tie construction got rid of hysteresis in the fluid. Here, if you look at a single domain specimen, then of course there is no hysteresis and all that happens is that this curve comes along this and jumps discontinuously and goes here and on the return path it does retraces the path so, this is what happens in a real Ferro magnet?

There is however a critical temperature at which this linear graph becomes a higher order graph here. There is an inflection point and after that this things separates out in this fashion. I will talk about the free energy and how this mechanism operates? But as you know the temperature further, it keeps doing this; till it hits the saturation magnetization at very low temperatures as you approach absolute zero. It is clear the slightest positive field, because thermal agitation is so unable at very low temperatures destroy the order causes the magnetization ordered in the direction of the field, and the other direction the smallest negative field will cause complete order. So, this is what the family of isotherms look like the analog of what happened in the liquid gas state?

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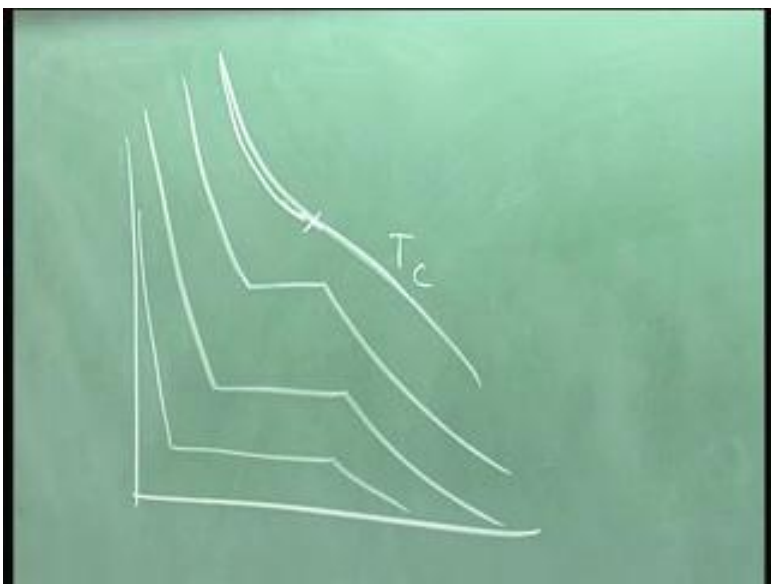

It says all those isotherms in the liquid gas case, it is there is a critical point and then there was this and then there was this and so on. All these graphs are collapsed on to a single line and they are to this line that is the only difference. Because of the certain technical reason, which I will explain there is a symmetry. In this problem which is missing the other problem and this is the reason why all these graphs are collapsed. So, this is the family of isotherms the analog of the family of isotherms our job is to show that this critical isotherm, which has a inflection point here.

Where the slope is zero and the curve, which is also zero exactly the same thing would happen on the critical isotherm, which does this. So, you can see if you tilt this by 90 degrees, it has exactly the same sort of this is our task to show that at the critical point. You do not have linear behavior here, but you have a higher order behavior, because it must be an odd function and after that it splits on and becomes discontinuous. So, this is the picture we would like to explain.

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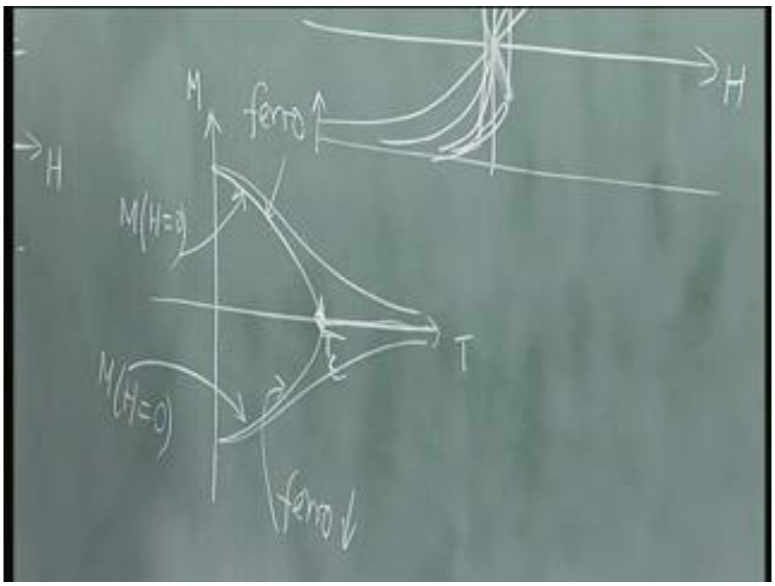

Now, what would happen? If I plotted the same graph, but with T here and M here, then if a small positive value of the field, I plot all these lines, these intercepts here. I get a graph here as T decreases M versus T and of course it will reach saturation.

So, if I start here and plot all these graphs from the smallest value to the largest from very high temperature to very low temperature, the graph would start looking like this, if I detect this from this side for a magnet in which the magnetization is downwards that graph would look like this, which is symmetric on the other hand, if I plotted only these intercepts here, that is the magnetization in the absence of an external field. There are two possible branches either I have an external field, which is switched off from the positive side or from the negative side. Because of this discontinuity I have two branches of curve and those would starts at zero T and then it goes up in this fashion.

So, it is clear? That graph would come along like this at this value T c, it would start taking off and hits the saturation on this side and hits saturation on this side. This completely symmetrically I have drawn it accurately this graph here is the M in the absence of the field zero and these two is M in the absence. But this thing would correspond to a Ferro magnet and let me call it Ferro up to show that it is positive direction and this is Ferro down. So, we have two phases earlier we had a liquid and gas, but now you have got a Ferro up and a Ferro down. Here is the critical temperature also called the Curie temperature of magnets and above that it is a Para magnet. So, the critical point is here and the analog of this is the V versus P diagram here.

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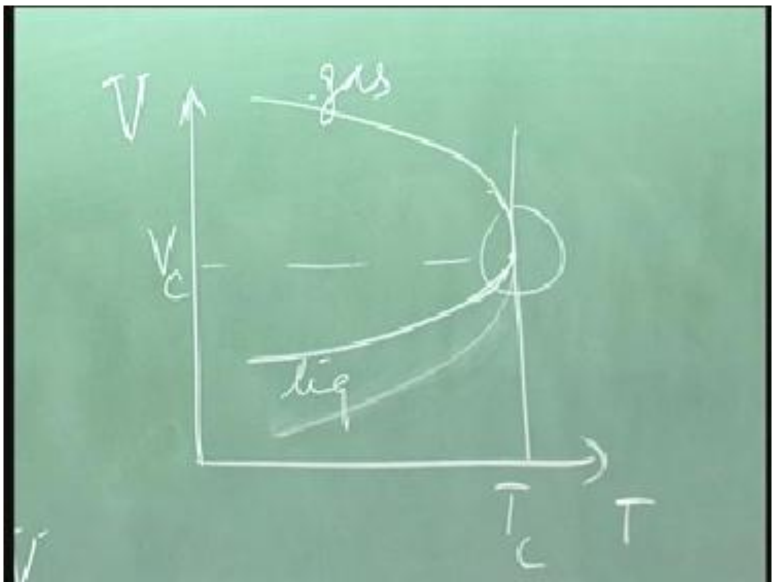

So, this is P versus V and this graph very loosely of this kind this is equal to V c and this is equal to T c. So, you can see that this graph is more or less getting reproduced there except that here, there is tremendous asymmetry. This curve is not a parabola or anything like that; because we know in reality this here the small volume thing is a liquid. So, that does not change all that much and this gas that changes a lot volume of the gas changes a lot temperature than the volume of the liquid. And it is an unsymmetrical curve, but our focus is at this point here, where it turns around and that is parabolic.

I am going to establish actually parabolic goes like the square and exactly the same thing happens here. So, this line of phase now the third graph, we have to draw of course is the P versus T diagram.

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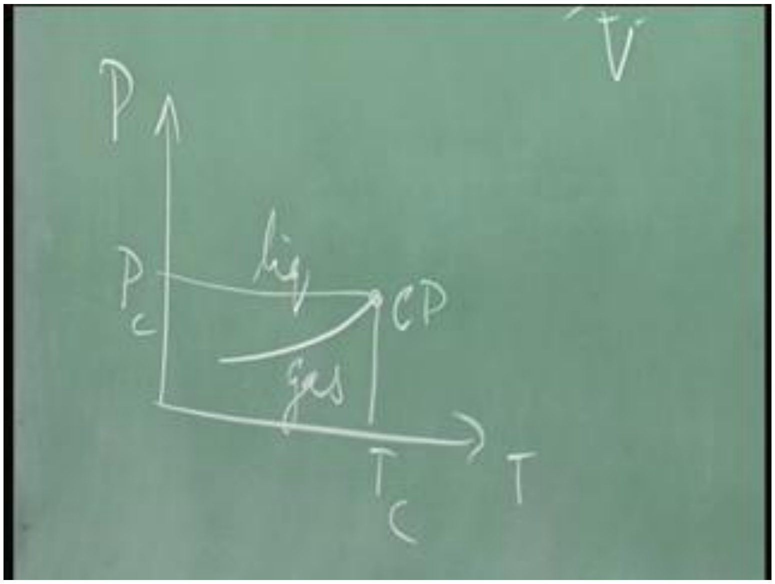

That we already drew here is T, here is P, this is liquid, this is gas and this was the critical point, this was T c this was P c critical pressure. The question is what is the analog that this comes H versus T? What is the analog?

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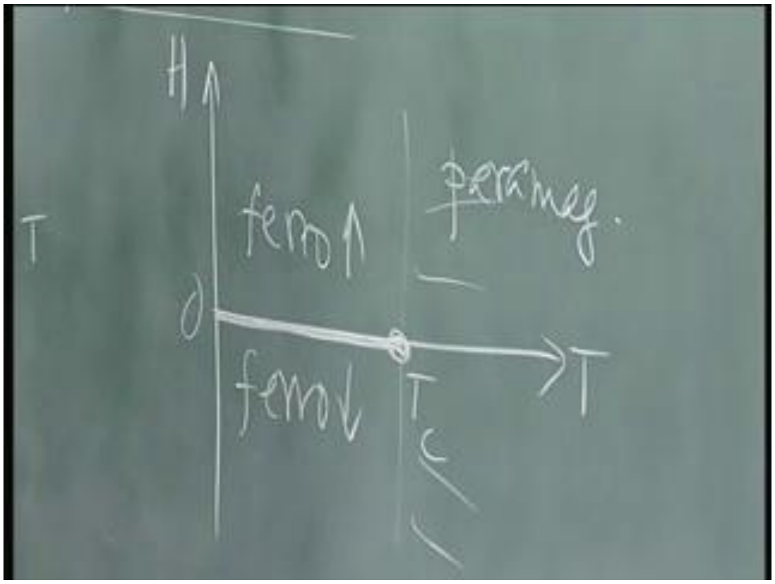

So, that is what we have to figure out since H can be positive or negative, we need to have force both up and down. So, this is H and this is T and here is T c and what is the critical value of the field at, which this transition takes place. Obviously zero, because it has to go up to a Ferro down and it has to switch the field from positive values to negative values and therefore this is also H c H c is zero itself critical value is zero itself. Now what is the analog of this graph? This is the graph of first order transitions discontinuous phase transitions, where the volume changes discontinuously as you go from the gas to the liquid or liquid to gas.

Across this line to have a change in volume and abrupt change in volume and that is why it is a discontinuous transition, but when you hit this point the difference in volume goes to zero. This is the difference in volume and that is going to zero at this point and then you have a continuous phase transition at the critical point same thing happens here. When you are here, when you are here if you switch the field here at this value of the field and that is the magnetization. If I take a small negative field this is the value of the magnetization so, there is a jump from this point to this point and it is discontinuous and it continues goes on being discontinuous till you hit the critical point. When the change in the magnetization goes to zero at this critical point.

So, this is also a continuous phase transition, because it comes along this and continuously moves out of this point it is a non-zero value. So, again a line of first order transitions ending in a critical point, but what is that line it is clear by symmetry up and down are distinguished in this problem. Completely whatever configurations you have in the magnet, when the field is up some regions would be some magnet moments would be up and the rest would be down. On the average there is an up magnetization, if you reverse the field there is exact symmetry. So, whatever is up now is down and whatever is down is now up for every configuration for which we have some moments up and less down.

There is an equal and opposite configuration of the same energy, when it is reversed. Therefore by symmetry it is clear that the line of transitions is this itself cannot be anything else. That is the line across, which there is a phase transition. This phase is Ferro up, this phase is Ferro down and on this side is Para magnet. The analog here is a gas the liquid and the homogeneous fluid and in the P versus T diagram also this side. You have the liquid gas and on the right of it you have it is a homogeneous fluid. So, this is the analogy, but now we have to answer a serious question how come the slope here is positive? But the slope here is flat of this curve of first order transitions well by symmetry.

We have argued that this has to be flat it is better to see in a slightly more systematic way or rigorous way. What is the slope of this graph here? What is the slope of this curve given by this is the boiling curve of the system? It tells you the way in which the boiling point changes increases as you increase the pressure and what is the formula? That is gives you boiling point as a function of pressure.

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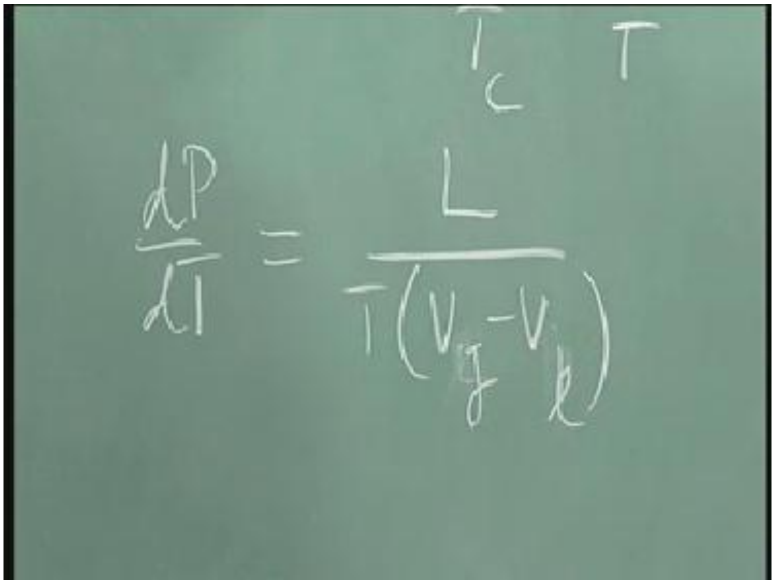

And what does that equation say? It says d P over d T equal to well dredge your memories pardon me latent heat somewhere, right latent heat. So, what is the formula? The formula that is typically given is that the latent heat divided by the temperature multiplied by there is something like this; ring the bell yes or no?

Yes.

No.

How did you get through grade eleven l is the latent heat of boiling? Or whatever the vaporization latent heat of vaporization. How much is it for water? How many units? I do not know. What units are used these days 540 calories per whatever ok.

So, that is the latent heat. This boiling temperature and this is the difference in volume between what and what how do I know? It is this is gas and that is liquid. How do I know this is positive? We are arguing in circles. If you know this slope is positive then the denominator has to be positive provided l is positive. So, how do I know? Which one should I use here? Because I know there are situations where this slope could actually become negative, if we looked at the solid liquid transition. Then of course the slope could be like this or in the case of water. It is like this and that is because the volume the specific volume of water at zero celsius is actually greater than that of ice floats on water that does not always happen.

When a substance freezes it generally contracts, but in the case of water few others like this much for instance as soon as it freezes. It expands, because it gets into a more open structure. So, how do I know? Which phase is which? And you cannot tell me this is g. I could have made l that g then it would have become negative. The answer of course is what you mean by L? You mean the heat supplied or the heat removed going from one place to another the other way. So, the correct way to write this equation is without ambiguity to say what is L over T? This is d Q the amount of heat you supplied divided by temperature.

So, what is that? It is the change in entropy so, really this is delta S over delta V and you see you recognize the Maxwell relation here. This is the Maxwell relation, it is one of the Maxwell relation delta S over delta V at constant T at the boiling point is equal to delta P over delta T at constant volume. Now, there is no ambiguity, when you say delta S it could be phase two minus phase one over volume two minus volume one or it could be volume one minus entropy one minus entropy two. It does not matter; you have to tell me what the initial state is? What is the final state? And then this delta S is well defined no ambiguity anymore.

Now, with this in our case we could write this as equal to S liquid let us write this as S gas minus S liquid over V gas minus V liquid. Now, you can tell this is going to be positive or negative and so on. I could have written it the other way inverting the sine both in numerator and denominator. It will not change anything at all. Is this bigger than that?

Yes.

Indeed, because the gas is highly disordered and therefore, it is entropy is much bigger than that of the liquid. What about the volume that too is always positive? The volume is always this difference is always positive. We have two large numbers and this difference is enormous huge difference in the denominator. When water gets into steam at room temperature by how much do you think the specific volume changes at boiling point one atmosphere. By how much do you think by what fact do you think volume changes? 16, 100 or something like that. You change the entropy so, the slope here is very small because you have a large denominator both numerator and denominators are positive.

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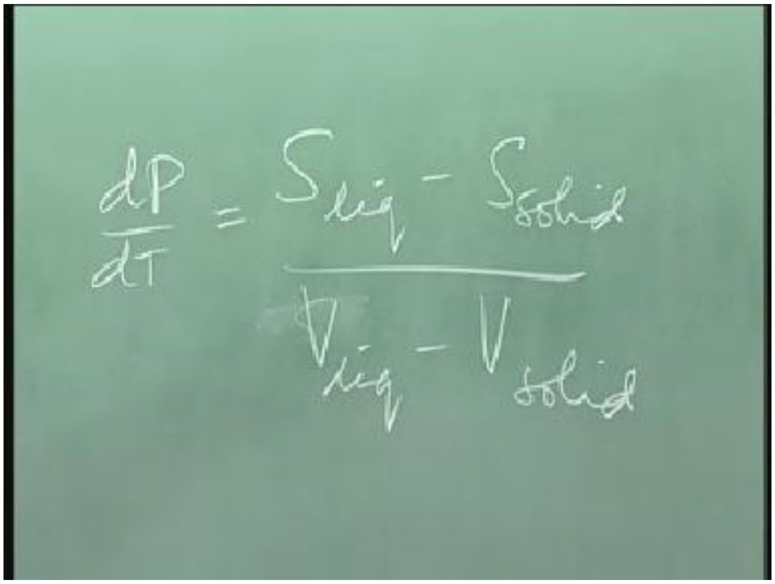

If you did the same thing for this graph, if you do the same thing for the liquid solid transition, then d P over d T equal to S liquid minus S solid crystal over V liquid minus V solid. Now what you think is happening is this positive or negative? It is always positive, because the liquid has higher entropy than the solid, which is an ordered state. But you see the denominator would depend on the interactions in the structure in most cases V liquid is greater than V solid, but there are exceptions V liquid is smaller than V solid like water. Water is trying to now crystallize in an icosahedral order near zero Celsius, it is very close packed, but icosahedral cannot tie three dimensional spaces.

So, ends up because of the nature of it are bonds freezing into an open structure and that open structure is less dense than that of water, which had incipient icosahedral order and therefore this solid becomes bigger than the liquid. And then you end up with the negative slope a very high slope, because this difference is always small liquids are practically incompressible. This difference is very small and therefore the slope is very high, but the sign of the slope is not done it depends on what happens to difference in volumes? So, that is the reason what is going on there? What would be the analog here in this case?

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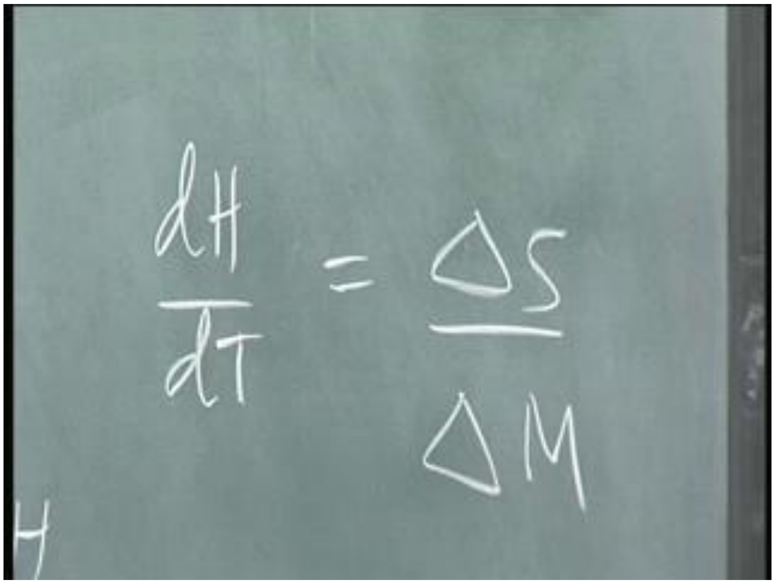

Well I have to write d H over d T over delta M. And now the point is when you have a Ferro magnet in this state up there? Here is your Ferro magnet.

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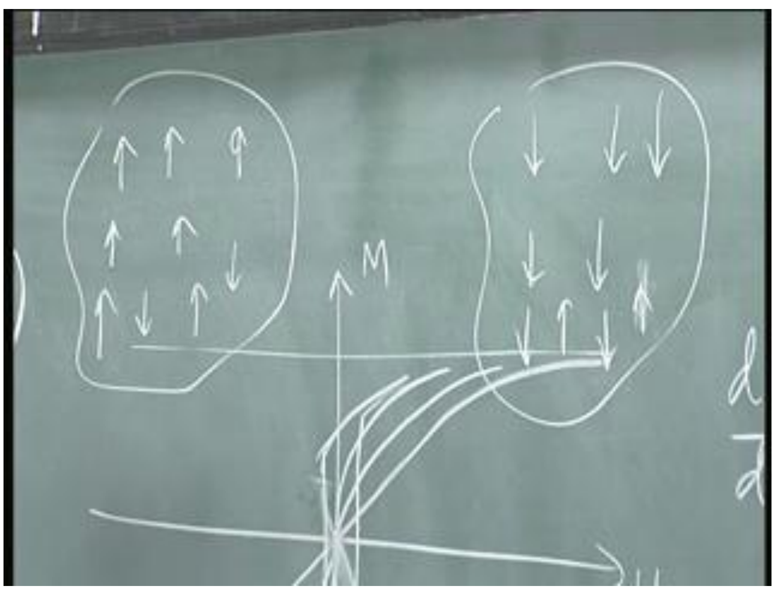

Most of the stuff is like these magnetic moments. Here are like that here and there are guys like this, but on the average it is half. Now the down this phase would have exactly the opposite whatever is opposite down and whatever was down was up. In this fashion there are this is a exact symmetry between these two configurations and therefore for every such configuration possible you have one such configuration possible there. Therefore, the entropies are exactly the same on the other hand the change in the magnetization is different is non zero, it is from here to here. So, we have a situation where the denominator is non zero and the numerator is zero and therefore, this is equal to zero.

So, that is the reason why the Ferro up to Ferro down coexistence curve has a flat slope it cannot have any other slope, because if it had a slope like this. It could end at this point and besides, if it had slope like this. There is no reason why it should have slope like that, because they are completely symmetric up and down and the only graph that is completely symmetric is in fact the flat. Now that is a good thing too, because you see if you start here at this pressure and I want to make it a gas. What should I do? There are two ways of doing it, one of them is I lower the pressure keeping the pressure constant. Otherwise, I heat the system keeping the pressure constant I keep it in an open beaker and I heat it.

So, I follow this path then of course I cross over into the gas phase, if the slope was not zero here. In principle you could go from a Ferro up to a Ferro down by starting here and simply heating it. Now, who ever heard of heating the magnet and making the change of the magnetization change direction you have to cross this line? And therefore, it is obvious that this slope is indeed zero and this is the curve. Now, I could ask with this analogy so, close how come there is? So, much estimator here and the answer of course is these phases are very different. But there is a very interesting law called the law of rectilinear diameters, which says there is a change of variable possible here.

So, instead of V I have change of variables the linear change such that you have a parabolic shape. So, there is indeed some line here about which there is symmetry of this curve and that is an experimental fact. There are theories to explain why there should be so, and so on, but they become model dependent. Right now, we are trying to see what can be said about model dependence? But this is what is going on? So, we have analogy between the two situations the three graphs here and the three graphs here and the task is going to be to see. What is the behavior near the critical region, but before that let me show you from this graph itself. We started doing this that you do get behavior of this kind or the zero field magnetization with…

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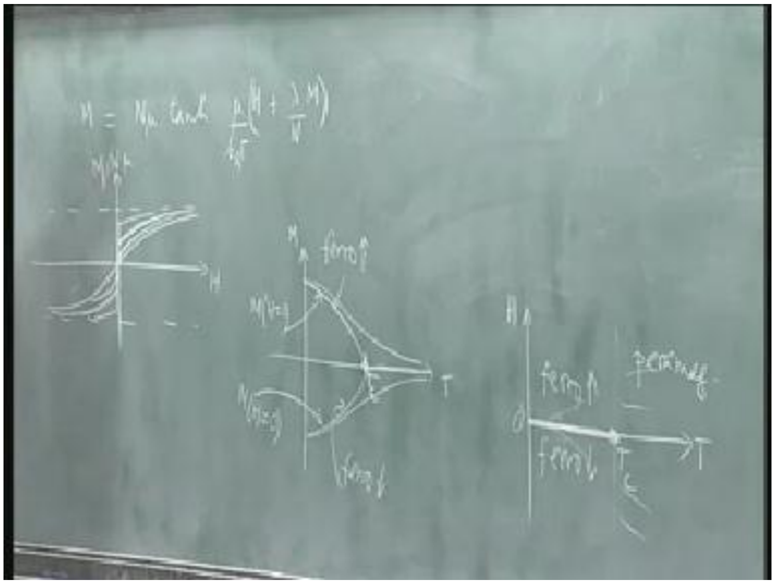

Because if I let M 0 denote by M 0, this guy here is the so called remnant or spontaneous. This is like what we call it? Spontaneous magnetization in which the magnetization, which you get in the absence of the field in the Ferro magnetic case, it is non-zero. We would like to ask can we derive an expression for what this intercept is going to be like? And the answer is very simple.

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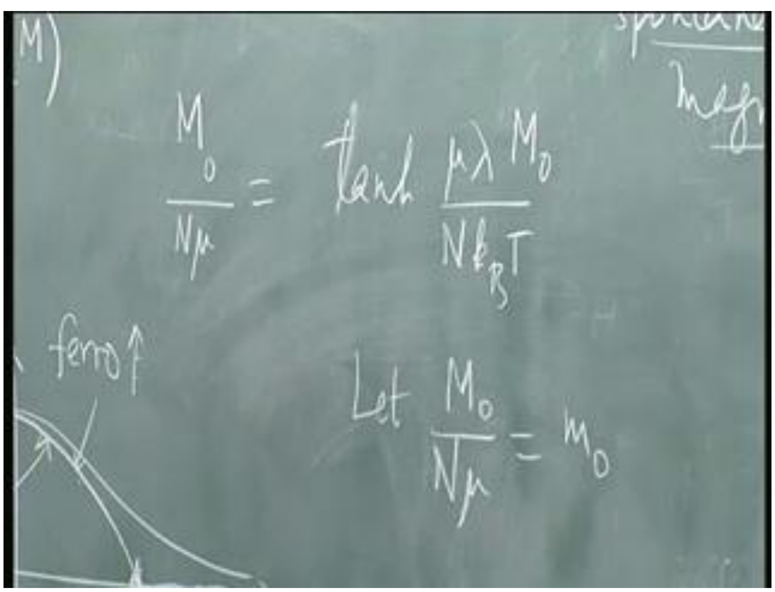

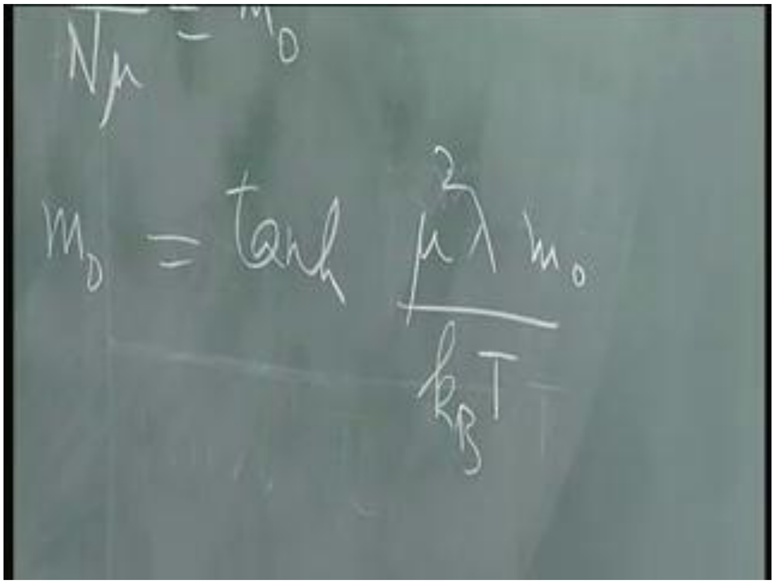

Because if I do that. If, I put M equal to, if I put H equal to 0 and M naught over N mu equal to tan hyperbolic lambda M naught mu N k B T. Let us give it a symbol; let M naught over N mu equal to m naught magnetization per atom, per moment in units of this natural unit. Then what does this become this equation becomes m naught equal to tan hyperbolic mu square lambda m naught. That is the transcendental equation, we have to solve; and that is easily solved.

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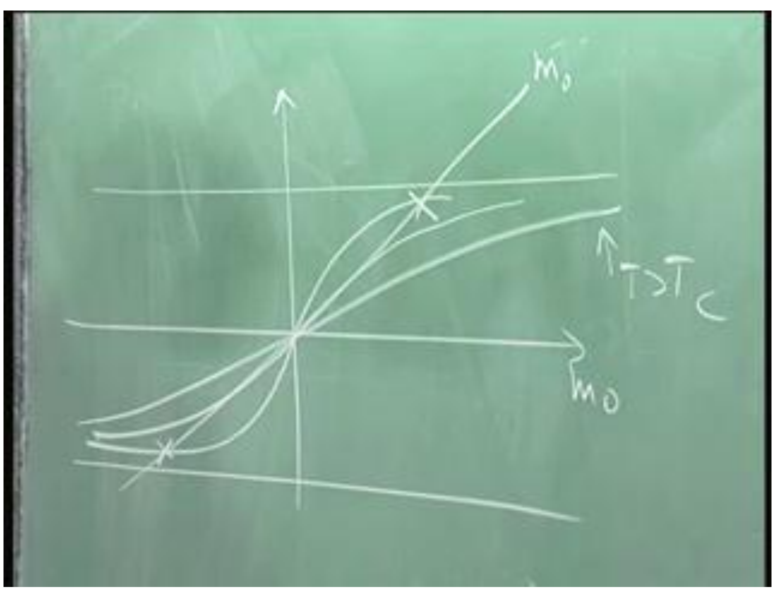

We plot this to do numerically we plot m naught here towards, whatever is on the right hand side and this is the line m naught and you have to plot the right hand side. We know that this saturates at m naught equal to infinity plus minus infinity saturates at plus minus one, but the slope at the origin is mu square lambda over k B T. So, there is a case this is the saturation so, there are situations where the curve goes like this. And this is what T greater than T c is? And for T less than T c the curve would go like this. Therefore, you would have a loop here and a loop there. They would correspond to the two intercepts, this intercept and this intercept. When would that happen, if you lower the temperature sufficiently there is one curve, where the slope is exactly 45 degrees here and goes off and after that it splits off the root splits and that happens, when the slope is one.

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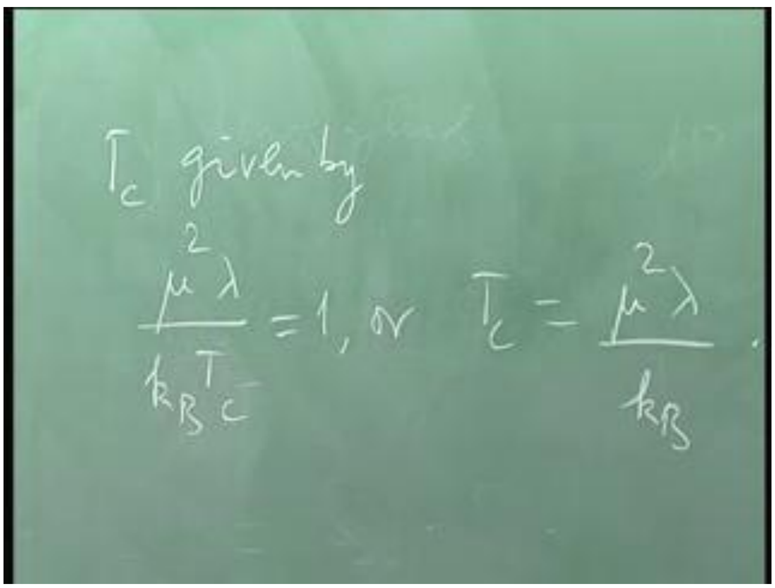

So, T c given by the condition mu square lambda over k Boltzmann T equal to one or T c equal to mu square lambda by k B its non-zero. So, at that point for temperatures below that the slope gets higher unity at the origin and you have two non trivial roots root in the middle continues. So, it is like saying this root still continues this is m naught and this is m naught and this roots still continue and our job is to show. That is exactly unstable that root is not a thermodynamically stable state, because that would be the Para magnetic branch and now you have gone into the Ferro magnetic phase. So, the Para magnet becomes unstable so, how do we do that? We cannot do that from here.

You really need to do that by asking what is the (())? And that is where the lambda theory of phase transition comes in we have mentioned that next, but this is the mechanism should be and incidentally you can play with this. And do the same graphical construction even with edge small positive or small negative and you would end up with these curves, because purely numerically. It is easy to deduce these types, but what is interesting is that the critical temperature? In this model depends on this constant lambda and this lambda was the multiple of m naught in the model that I pose for the effective field. So, while here it is completely empirical you just chose the internal field to look like this. Any microscopic model of magnetism should be able to predict what this lambda is? Or what it replaced?

So, the job would be to find this quantity, but notice that. I also remove this N in order to make things in terms intensive variables, but that is a small technical detail. The fact is there exists a non-zero critical point a non-zero critical temperature below, which you have Ferro magnetic. So, that is the first present we get from this the next thing. We have to do is to find out what is the behavior of this graph? Because this looks like there is a cusp here and I would like to know what does this cusp do? What sort of behavior do we have at this point? Similarly, what kind of behavior do you have for the susceptibility? It is this thing is suggesting very strongly that the susceptibility, which is a slope at the origin, is actually going through infinity.

Because there is a stage at the critical temperature, when the susceptibility becomes infinity the slope is 90 degrees, but after that on the other side you can still continue to define the slope.

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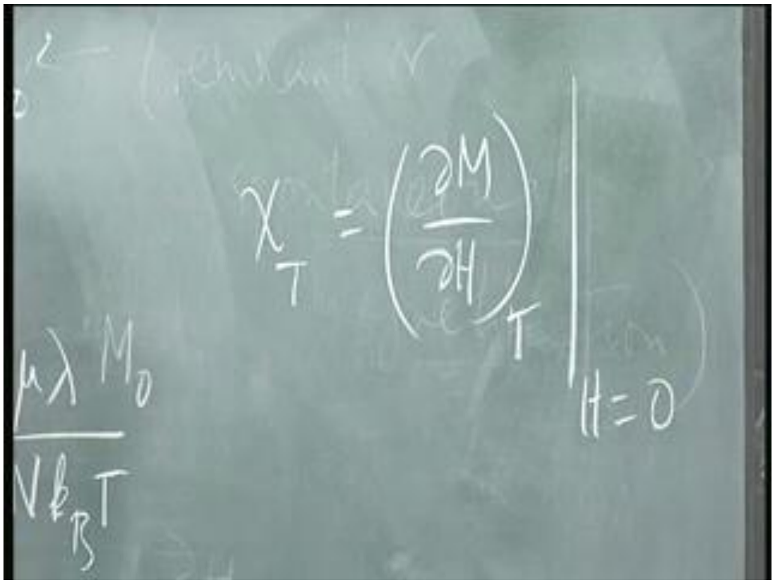

You can still continue to find the susceptibility as chi T equal to delta M over delta H at H equal to 0 at constant temperature evaluated at H equal to 0. You can continue to define this even if the function is discontinuous M itself is discontinuous, but it is slope could be defined very well. Because you see the slope here and the slope here by symmetry are exactly the same. So, there is no problem in defining the right slope or the left slope and you get exactly the same for the both. But meanwhile the susceptibility go up to infinity.

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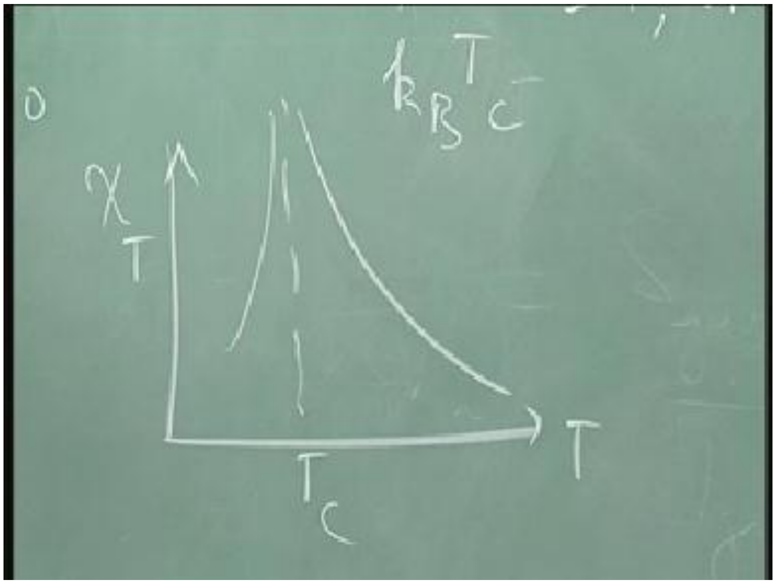
f 

So if I plotted the susceptibility as a function of temperature T and I plot the chi T. At the critical point is of this fashion and the claim is below, that it will fall down in this fashion. We would also like to derive the Curie Weiss law from these relations so; let us see how to do that? So, we look at this equation of state and see what we can prove? There are several things we can do the first of which is to actually find out what is the solution to this equation?

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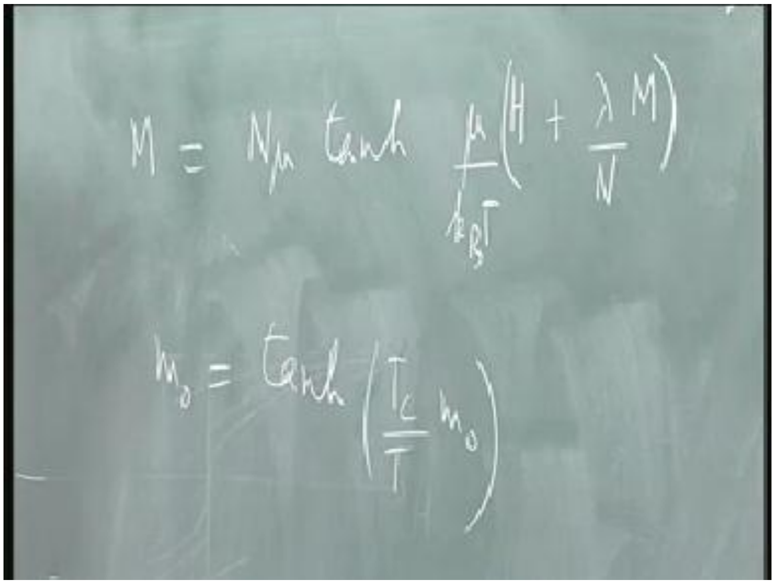

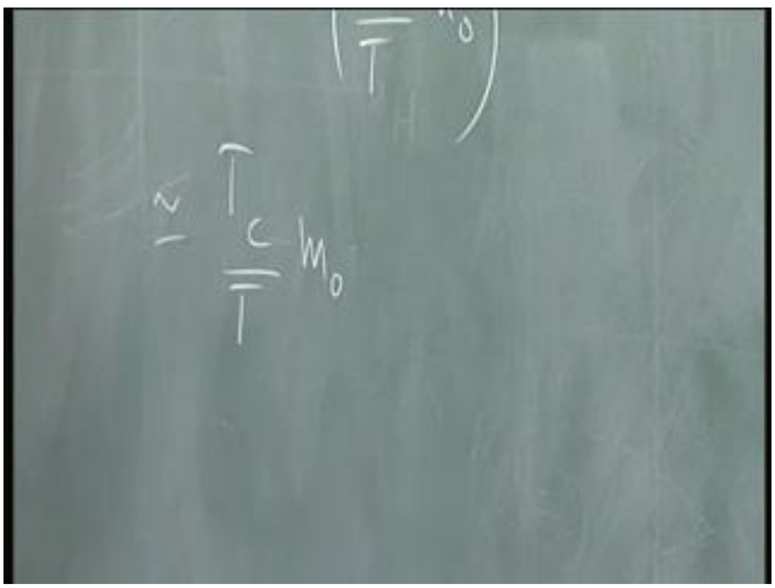

So, let me write this in a slightly better way m naught equal to tan hyperbolic mu square over lambda k T m naught, but mu square lambda over k is T c. So, we could write this as T c over T, because T c has now become a parameter instead of constants mu square lambda k Boltzmann and so on. We just replace it with T c so, it will tell me the behavior. So, it is obvious that solution to this at T equal to T c this cancels the only solution to m naught is tan hyperbolic m naught is zero, but below T c. You have other solutions the question is can I extract those solutions?

What should I do? I am going to keep T slightly less than T c so; the slope is slightly greater than one. So, what should I do pardon me expand this, because I know I am in the region, where the magnetization is essentially 0 T plus T c. In this region here m naught near T and T c in that region, I can expand this. So, that T is approximately T c little less than that so, T c over T is little bigger than one and m naught is approximately zero. I should expand this function what should I do? M naught here, what will be the next thing? Well a correction the correction this will be a higher power of m naught.

Now, what would that correction is pardon me cubic plus or minus, (( )). So, sure it is minus is right, make sure it is got to be minus. Well the plus would keep going coming down. It is going to get saturated so, there has to be a minus sign.

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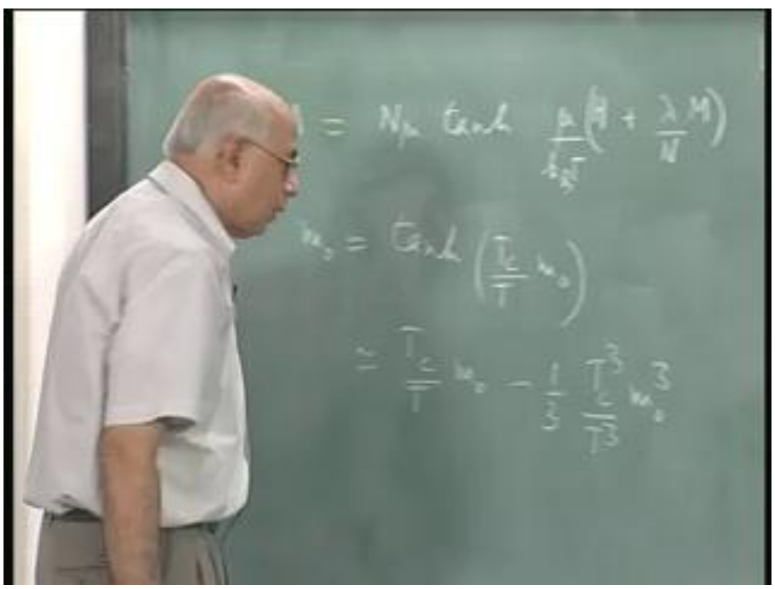

So, the next correction otherwise it increases so, there is a minus. What should it be proportional to… that is a constant one. Third I would guess? Well cot hyperbolic remembers was one over x plus one over three. So, tan hyperbolic and then what cube? This whole thing cube T c cube over T c m naught and I want the solutions of this equation. I want these two roots this guy always is a root, I want the root here and the root here. So, what should I do? Can I cancel m naught? Yes, because I want to root other than zero.

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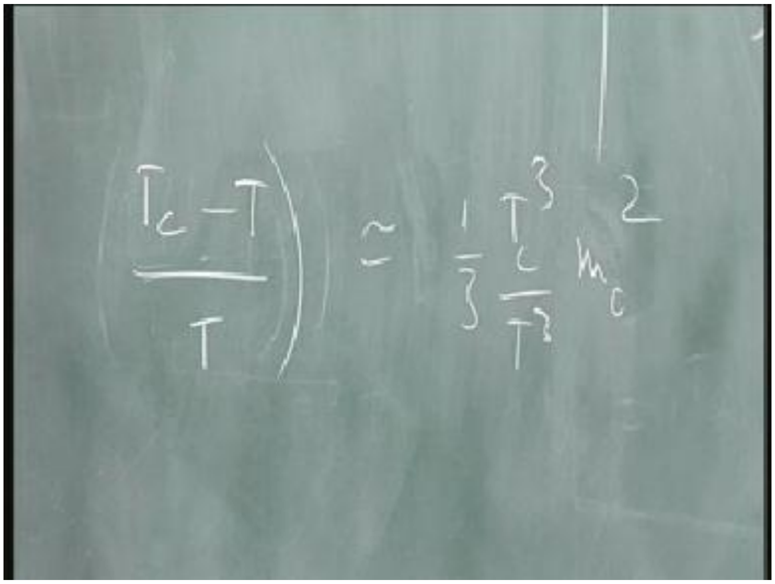

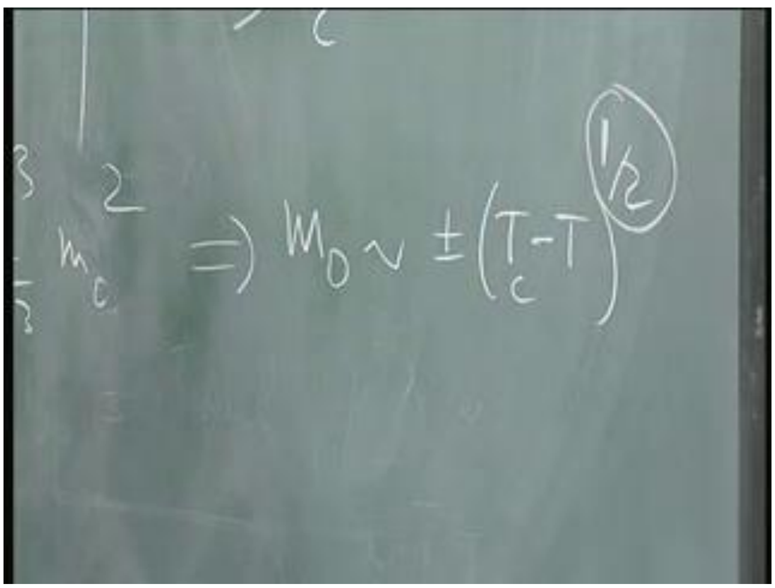

So, this says m naught times one minus T c over T is approximately minus one third T c cube m naught cube and I cancel the m naught so, this is square. Remember this is a negative quantity, because T is little less than T c. So, let us get rid of that by writing this by getting rid of this plus sign and writing it as T c over T minus one, which is T c minus T on top divided by T.

So, what lesson does it tell you finally? This is harmless this guy is fluctuating this is around T c so; this is going to essentially set it equal to T c the point the place. That is going to be zero is this factor also is essentially unity apart from higher order corrections. So, what lesson does it tell you? What does it tell you about m naught? This implies that m naught is proportional to plus or minus T c minus T to the power half square root.

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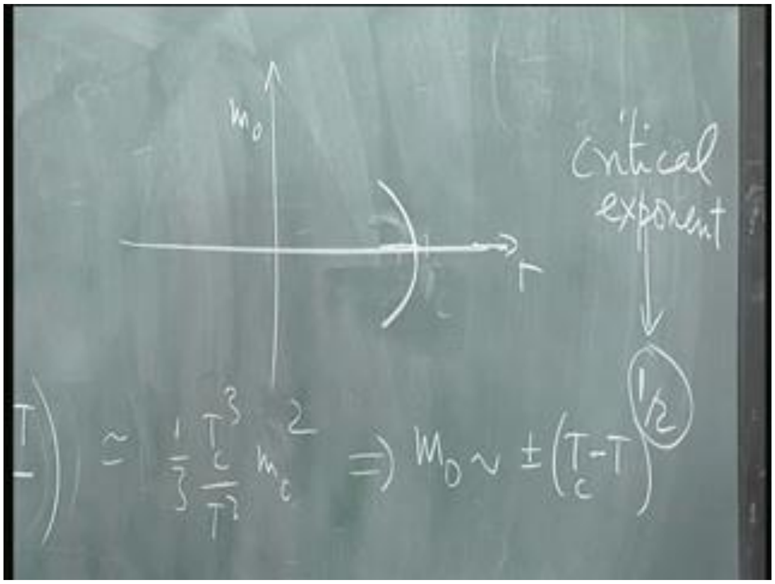

Therefore, what is the slope of this graph? What is the slope of this graph at this point? It is 90 degrees; because it is a square root. You have to differentiate one over square root; you really have this, which is the nature of this.

This half is called the critical exponent in this case it is called the magnetization exponent, because that is what the spontaneous magnetization does exactly the same thing is true for the liquid gas. I am not going to prove this; I will give it to you as an exercise to start with the Vanderwaal's.

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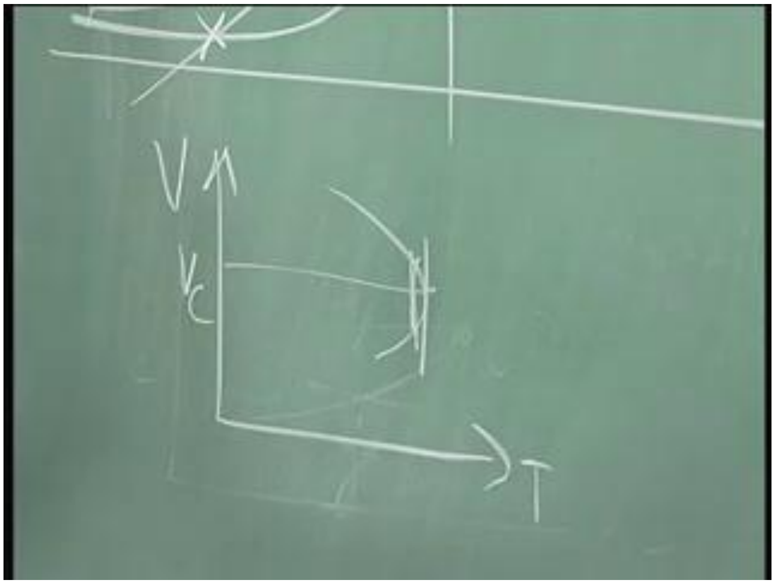

Then to show in the volume versus temperature graph in the vicinity of the critical volume this difference here, vanishes like the square root of T c minus T same half. So, while you know that the gas becomes liquid due to some mechanism. Totally different from what a magnet does the half is exactly the same in both cases?

The half is arises here, because both these models are examples of what are called mean? Field theory model and in other words the Vanderwaal's case, we assume that the pressure was not P, but had a correction minus a over V square. Because of the attraction between the molecules, but that attraction, we said was every molecule. Since we have a molecule on the average, it has a course and attractive contribution proportional to the square of the number density. That is how we got one over V square? In the same way here, we assume that every magnetic moment is seen by every other magnetic moment and the internal field at any point is proportional to the magnetizations.

So, we have exactly the same philosophy and that is what led to the same critical exponent. Now the question is does the experiment show this or not and now you have to ask how accurately? Can you get to the vicinity of T c (()) temperature and the answer is yes. In most cases depending on the experimental set up you would see essentially something like that, but fortunate thing is you do careful experiments near T c. Then you do not see your half at all, you see there is something very different is one third or 0.3 or something like that. This is quite different, but you have to take very close to the critical point. How close could you get there are précised criteria for this? It turns out that temperature is one of the hardest things to control temperature to milli degree accuracy. It is a very big achievement criteria.

Going below that higher energy is even hard unlike other quantities like frequency. Frequency is the most accurately measurable physical quantity. You can measure frequency to one part, which has incredibly there are clocks out. There are physical clocks astronomical objects, whose pulsars, whose rate of change of time period can be computed to one part to 10 to the 19. They maintain those are very accurate, but temperature notoriously, very difficult to control, but depending on the system and say more about this.

You can determine what the critical exponent is and it turns out that this exponent is not quite half, but at the moment. Let me retain this now as a mean field critical exponent the first such exponent we are going to talk about more. The next question is how does the critical isotherm? We know that the critical isotherm in the fluid case had it had a inflection point.

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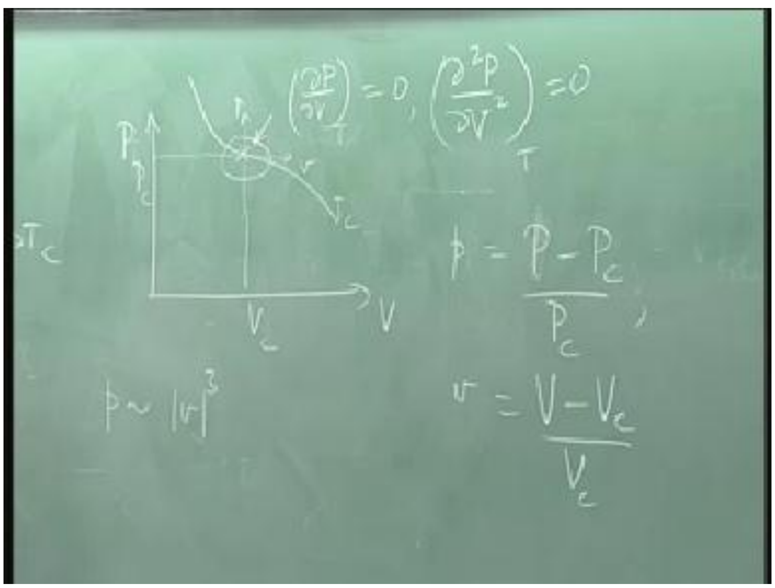

So, we know the following is true here is V here is P this isotherm had a inflection point, at this point delta P over delta V at constant T was 0. This is at T c it is an inflection point so; you had something which came along like this. Now I am not going to do this again, but I leave you to show that if in the vicinity of this point which by the way is V critical and P critical. If I define little P equal to be equal to P minus P c and I make dimensionless little v equal to V minus V c by V c and shift origins to this point. When this is like saying this is v and this is p out her and I ask what does this look like at that point? Then it is easy to see it is not linear.

It is coming down it has a flat here and then goes down so, it would look like p is proportional to v cube before, after linear. The next one would be a cube and you can show this from the Vanderwaal's isotherm, by actually computing P c V c and T c and then going to these reduced variables doing the expansion of state. So, what you can show? That p goes like this, here this three here is also a critical exponent and in this case a mean field critical exponent. It is a cube three, in general it is something else and the standard symbol for that critical exponent is delta. In a theory of phase transitions there have been a whole lot of critical exponents have been identified and fixed name has given to these exponents. That one is called delta, this one here is called beta and it is not the inverse temperature.

They just ran out of symbols so, they used this critical exponent and the mean field is a half delta is (()). In reality delta could be something different, it could be 5, 8 it could be larger and so, on. So, you have again correction to mean field exponent the question is what does this critical isotherm of ours do in this case? That is we have to look at this so, we go back here and let us try to rationalize this everything.

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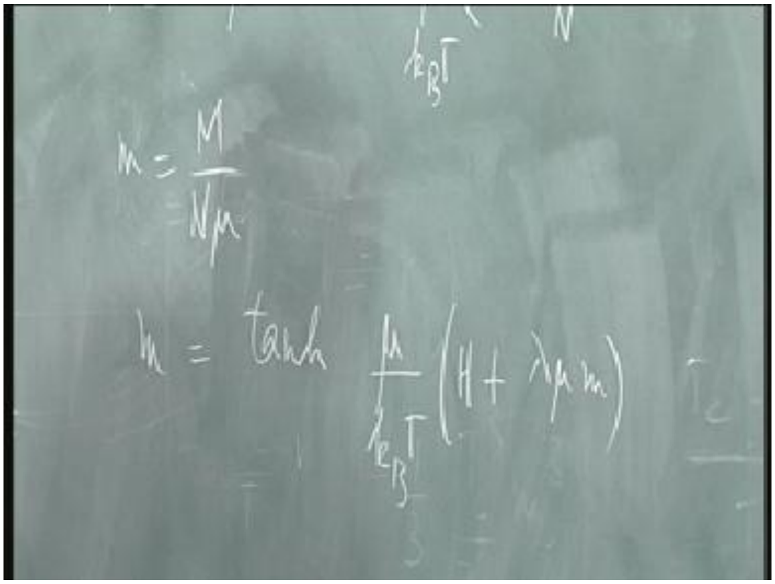

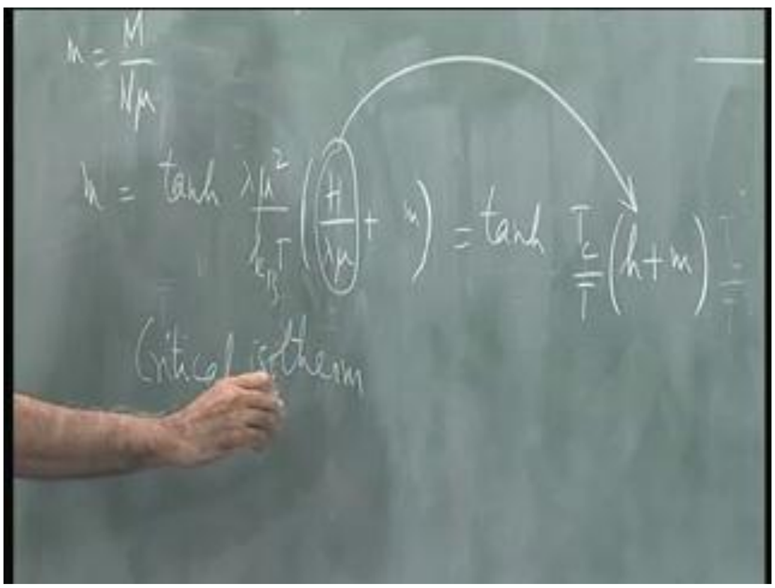

So, I put m equal to M over N mu, this becomes m equal to tan hyperbolic mu over k Boltzmann T H plus lambda mu m correct. Let us take things out so, that we have the lambda and the mu square so, I take this out here and H divided by lambda and this is equal to tan hyperbolic.

Now, lambda mu square over k was I believe our T c so, this is once again T c over T. Let me call this little h plus m so, this H over n mu I call little h in some units, just the magnetic field.

Let me read this fashion and the critical isotherm is given by m equal to tan hyperbolic h plus m, because T equal to T c on the critical isotherm and what am I trying to do?

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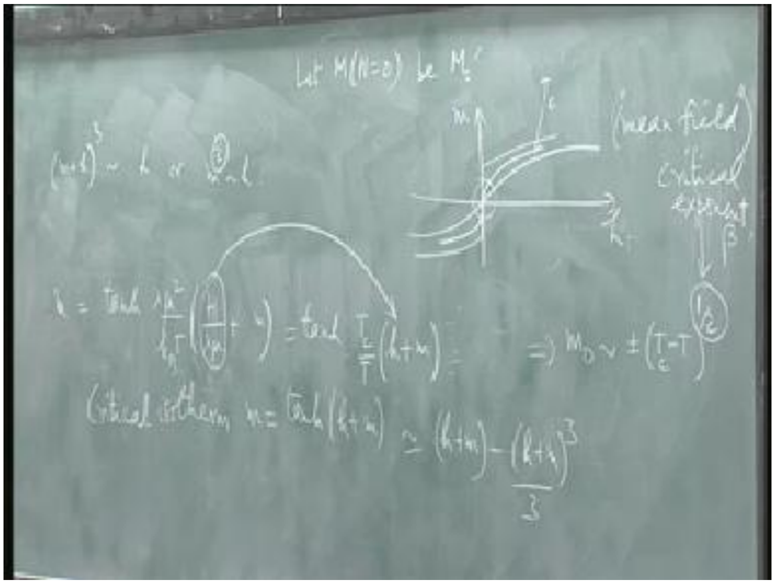

I am trying to find in the h versus m graph, I know the Para magnet looks like that and I know that Ferro magnet looks like this. But the critical isotherm looks like this and I am trying to find this cubic curve. So, this is at T c and I am trying to find trying to show you, that it also cubic curve from this graph from this equation.

Now, both m and h are near zero so, you are right here. In this region the critical value of the field that, we already saw is 0 H c 0 and m critical is also 0. So, therefore this is equal to h plus m minus h plus m whole cube divided by three and that is m. So, what is the conclusion? This m cancels on both sides and the equation you have to solve is precisely the earlier equation. It says therefore, that m plus h cube goes like h and both m and h are small. When you solve this equation you discover that this is even smaller so, keeping first order term. Here, it immediately is clear; that you can neglect this m cube is proportional to this. Of course you can refine this so, on but our point is not that our point is to show that at the critical point, you have a cubic curve.

So, it is exactly the same the same three appears, here as appears here. Even though these are very different systems all together, this is not an accident. Work this out and do this consistently solve this equation consistently, mere h is equal to zero m equal to zero solve it consistently and then you discover that h is like a cube. So, the mean field exponent has come out, the same for two different systems all together. We have no reason to expect this at all, by the way what does the susceptibility do? That is not hard to find, because all you have to do is to find the slope of this graph. This point goes back to this equation here, for T slightly greater than T c, T is slightly less than T c finds the slope for m versus h.

If I solve in this equation and you will discover that k i T is proportional to one over modulus T minus T c with a power one in exactly the same way. That you will find here on this graph from this side. What is the analog of the isothermal susceptibility, the isothermal compressibility and you discover k T equal to minus one over V delta P over delta V T. And you know that this fellow goes to zero and the reverse of this kind is exactly the same, also has the same one over T minus T c. In general, however this is not one; this is the Curie Weiss law. We have derived it now, but this is not one, but it is a critical exponent and it is generally denoted by gamma not the ratio of specific heats.

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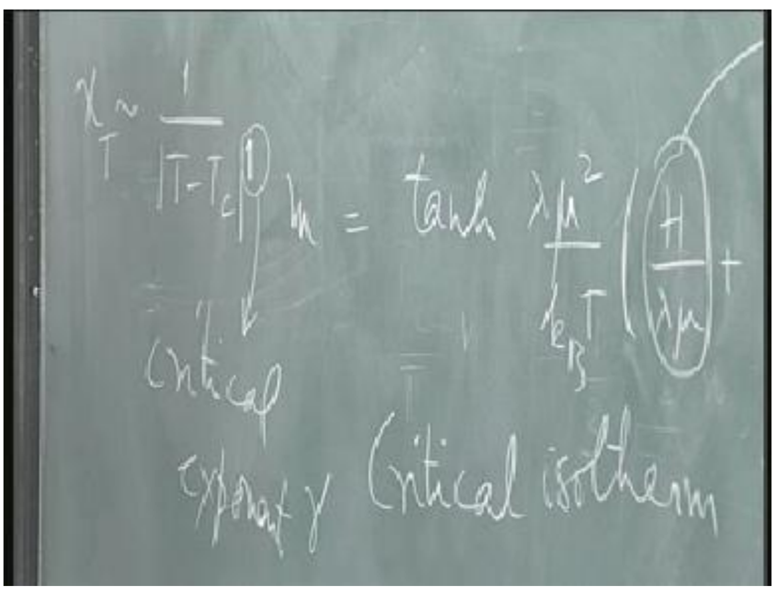

This gamma experimentally for three-dimensional magnets could again depend on the kind of magnet. You have order of one and one third about 1.3 not 1. So, there is a deviation from the mean field critical exponent, which we found here gain for deep reasons, but mean field will give you in a very simple way. Our next task is to find out, where are all these things coming from, is there a simple phenomenon logical theory. We can give to see when the phase transition would occur? What would be the general way of doing this? You know including all these systems together and this is what Landau and I will talk about it next time. Landau's theory of phase transitions and he did this way back in 1937 and he actually showed that ideas following from broken symmetry, would lead to of these predictions of phase transitions or at least a analysis of these phase transitions.

Today the starting point of critical phenomenon is Landau's theory; this is the phenomenon logical theory. And from that we get all stuff put on elaborated upon (()). Let us take off from here to starting point so, it is good idea to have this empirical theory simply, because it applies in so, many different things.