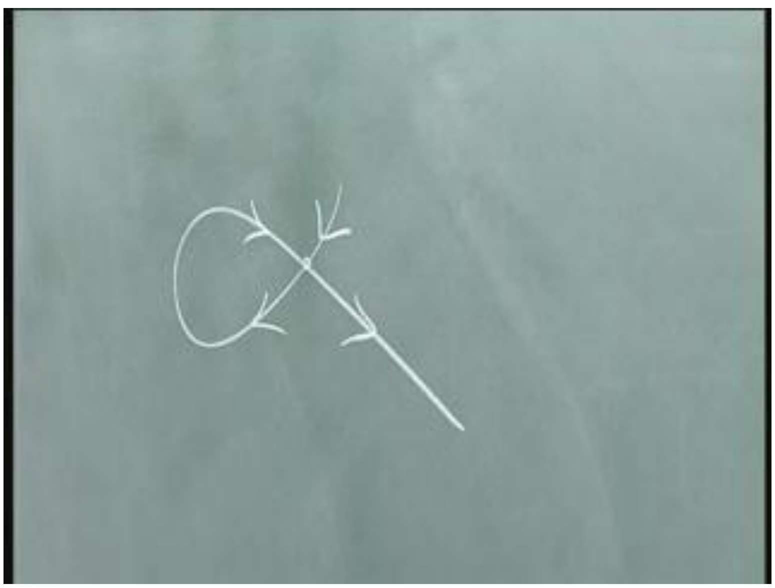
Classical Physics
Prof. V. Balakrishnan
Department of Physics
Indian Institute of Technology, Madras

Lecture No. # 18

Let us look at the quiz paper, and then we start with the question one, this is true or false questions. I will read out the question, and then do with the answer. In case of doubt please raise the question, and then you clarify this. The first parts set the phase trajectories of a non autonomous dynamical system can intersect themselves or each other. We know for sure that for an autonomous system they cannot do so for the non autonomous system there is no such constraints at all, because the rules of the game change with time, therefore if things intersect themselves after sometimes, it is a different rule and different trajectory.

(Refer Slide Time: 01:47)



So, possibility of this kind exist, so phase trajectory can come along go through here, and although at this point, you might ask is in the feature determine uniquely in terms of the present. The fact is you reaching this point at two different times, and the rules of evolution are different in the two cases. In one case, it would have been this; in the other case, it would have been that. So, it is quite clear that it they can intersect themselves, if it is non autonomous and that is the true statement.

The Euler-Lagrangian equation of a system do not change, if an arbitrary function f of q 1 to q n and t of the coordinates and time, it is added to the Lagrangian. This statement is not true, if the total time derivative of such a function is added, then of course this statement is true. But an arbitrary function cannot be written as a total time derivative of another function.

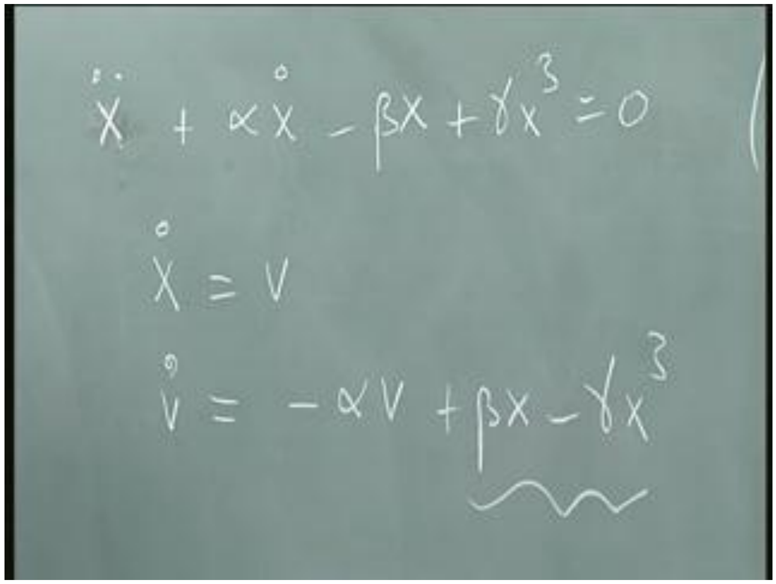
And therefore, this is not true completely, there are functions which do not have primitives, which cannot be integrated this not definitively, square root of sin x; there is no function of which square root of sin x is the derivative. So, in general you cannot write an arbitrary function as total time derivative of another function, and therefore this statement is not true, so please read all the classes in the question, all the statements completely. Leaver's theorem says that a volume element in the phase space of a Hamiltonian system does not change under time evolution.

And the statement asked is, this is true only for time independent Hamiltonians, and it is not true for time dependent Hamiltonians; and the statement is false, because Hamiltonian flow we saw is volume preserving regardless of whether there is a explicit time dependence in the Hamiltonian or not. It is another matter that the Hamiltonian is not a constant of the motion, if it is explicitly time dependent, that is certainly true because, d h over d t is equal to delta h over delta t and that is not 0, so that is certainly not so.

But, the fact that the measure is preserve, in phase space the volume element is preserve that is certainly true, even if you have time dependent, so the statement is false; north space theorem applies to both continuous symmetry transformations, for example rotations and discrete symmetry transformations for example, reflections. And it is a false statement, you cannot define in another current remember, the way we derive the conserve quantity from (( )) theorem, which was to ask what happens, if you make an infinite decimal transformation.

And if a transformation is discrete, there is no infinite decimal version of it passive, if you have a reflection in a mirror, either you have a reflection or you do not, I mean if you reflect twice you get back to wherever is like parity, so there is no infinite decimal transformation; and therefore, the theorem does not apply as it stands, it is a false statement.

(Refer Slide Time: 04:51)

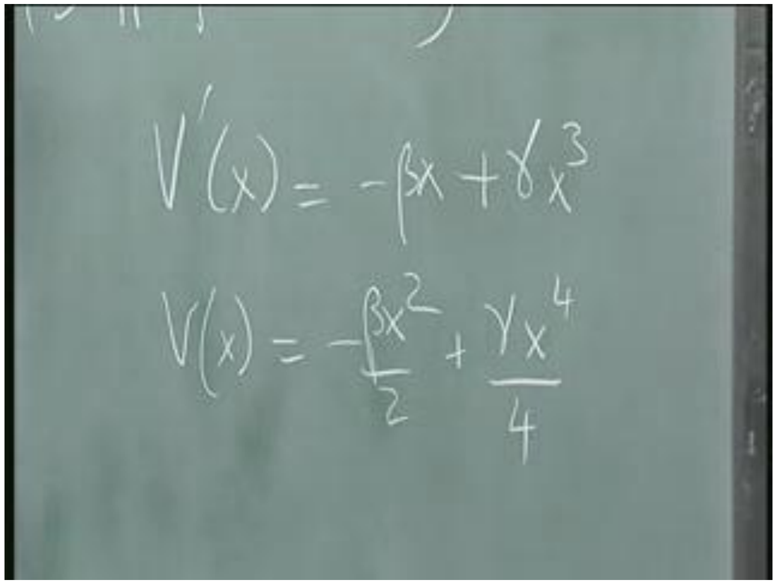


The next one was a damped non linear oscillator, has a equation of motion x double dot x double dot plus alpha x dot minus beta x plus gamma x cube equal to 0.

This is a very famous model of a non linear oscillator, it is called the Duffing oscillator, write the term is a and it is a little bit of history attach to it, it is prototypical model of a system which has a nonlinearity as well as dissipation; the dissipation comes, because you have a friction term here, with the positive coefficient. And if you did not have this nonlinearity this would look an inverted oscillator, because you see you could write x dot equal to v and then, v dot that list on is equal to minus alpha v, so that is a friction term as you can see straight away, plus beta x minus gamma x cube, this portion quite evidently comes from a potential; the derivative of some potential with respect to x.

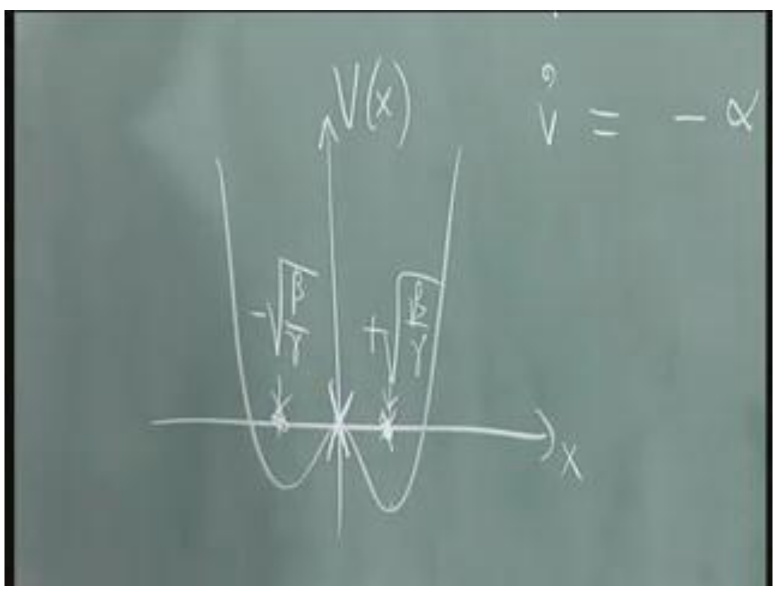
But you see if it were a simple harmonic oscillator, you would not have this nonlinear term, it have a linear term and it would appear with the minus sign, because the restoring force to take you towards the center of oscillation, and this would correspond to a plus half k x square potential. But in our beta x square potential here, you got a plus sign for the beta.

(Refer Slide Time: 06:22)



So what is the potential look like, we know that V prime of x equal to minus beta x plus gamma x cube, because that is what the forces V minus V prime of x, so therefore V prime of x is minus this; this is what the force was, the f of x is minus V prime of x, and that is given by this.

(Refer Slide Time: 06:55)



So, what is V of x apart from a constant V of x is equal to minus beta c square by 2 plus gamma x 4 over 4, and what is that potential look like. If I plot as a function of x, I plot V of x near the origins since alpha, beta, gamma are all positive constants given that it is part of the information, this is an inverted bowl, and that bowl takes to up this, a gamma x floated to x up, so it is clear that this potential looks like this, this function.

So, it is a potential with an unstable critical point at the origin V equal to 0, x equal to 0 that is a critical point and that of course, an unstable critical point, because you have an maximum of a potential. But you have two minimum, one at this point and one at this point, which I given by taking this quantity and equating it to 0, where the force vanishes; so you have roots at x equal to plus or minus square root of data over gamma and those are these two points.

And they correspond to minimum of the potential and therefore, these could correspond to stable oscillations, if you did not have dissipation, if you did not have this term then of course, they are stable oscillations. The moment you have this term, it is clear that things are going to fall into it depending on whether it is under damp or over damp, things are going to fall into it as asymptotically.

So, they will be two stable as asymptotical, stable critical points at plus square root of beta by gamma minus square root beta by gamma these two, they would be stable and this guy is unstable. So, this oscillator this is a very famous example of nonlinearity I mention, the more general problem is when you take this oscillator not 0 on the right hand side, but you apply a sign of seidel force to it, so you have a forced damped nonlinear oscillator.

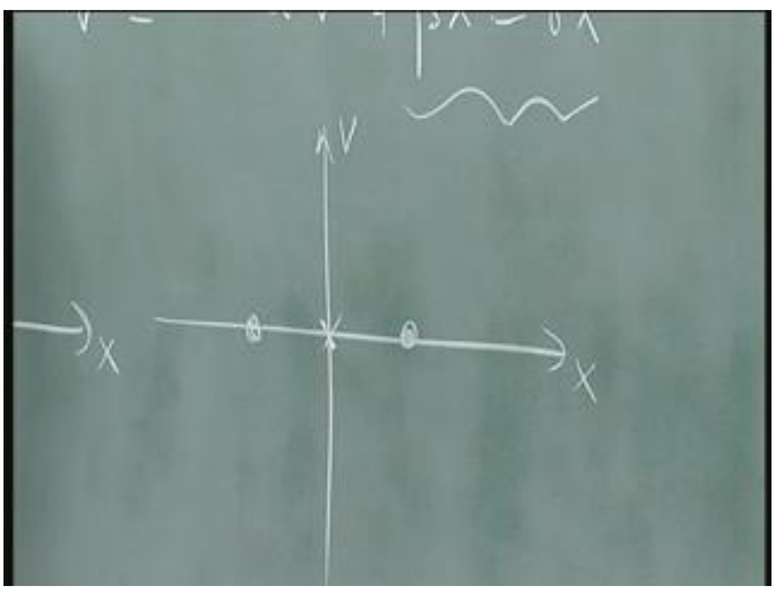
And the forcing would be perhaps go like A cos omega t, where A is the amplitude of the external force and omega is the frequency of the external force, this is a non autonomous system. Because you putting in energy into the system or removing it you, are actually putting a time dependent force on the system, so this is non autonomous, this dynamical system is non autonomous. Once you put a t here, and it is an exceedingly complicated system to analyze unbelievably so, in fact the full analysis is not completed probably we never will be.

The reason is you have a parameter space in which you have one parameter alpha, two beta, three gamma the amplitude of um forcing four, and the frequency of forcing five, now a five dimensional phase space of parameters are very rich one; all kinds of things can happen, you could get a one of them, you could say that a time scale, I fix the time scale such that, alpha is equal to 1 beta is equal to 1. You still have four of them left, and that is a very complicated, this system is a two dimensional phase space, 1 degree of freedom but, once you put forcing unit it becomes non autonomous.

And then, you have to look at it in extended phase space x v and t and this system could display kayak's even a simple thing like this can display kayak's. And in fact is does for certain parameter value it or equal to large values, it displays chaotic behavior; it displays all kinds of other behavior. One interesting question is suppose, I do not have this 0, is it possible to have a periodic trajectory, even though you have dissipation, even though you have this thing here, is it possible to have some initial condition for which the system becomes periodic; that is it damps out, but then it has enough energy to cross here goes over to that side, then its energy is given to the system while, you can see and then does it become periodic.

Is there an isolated periodic trajectory in this, and the answer in this case is no, it is a rigorous theorem you can show this I will do that later, to show that this system cannot have an isolated periodic orbit, and indeed everything would fall into this point or that point.

(Refer Slide Time: 11:15)



So, all points in the phase space here, plot x versus little v phase space, there is an unstable critical point here, there is an asymptotically stable critical point here, as an asymptotically stable critical point here.

Then the interesting question you can ask is, if I start somewhere here, every point is some phase trajectory. If this is my initial condition would it fall into this or would it fall into that, and that is not easy to answer; it turns out once again that depending on the initial conditions you would fall into this or that. And all points which fall into the left attractor or called points belonging into the base in of attraction of this critical point. And are the basin of attraction of that critical point, and these basins of attractions are very integrate in structure. In general arbitrary non linearities they would have very little factor structure. So, the whole of phase space could be incredibly mixed up in a crazy fashion, some parts would fall into the basin of attraction of this, the small neighboring initial condition could make you fall into this and so on.

Think about physically, I start of it starts oscillating in this and then a friction occurs and now, it is possible that the system comes here; and then as does not have enough energy to cross that, and then it is eventually fall in to this. But, if it have in infinite decimally greater energy it would fall into that, does not have enough energy come back here, due to the damping falls into that. So, you can see immediately the small changes in initial conditions could make you about this attracter and that attractor; it is for these reasons that this model is very popular books written on this single model, and the Duffing oscillator.

The next question was the Laplace linear lance and vector the constant of the motion in the case of the, yeah sorry.

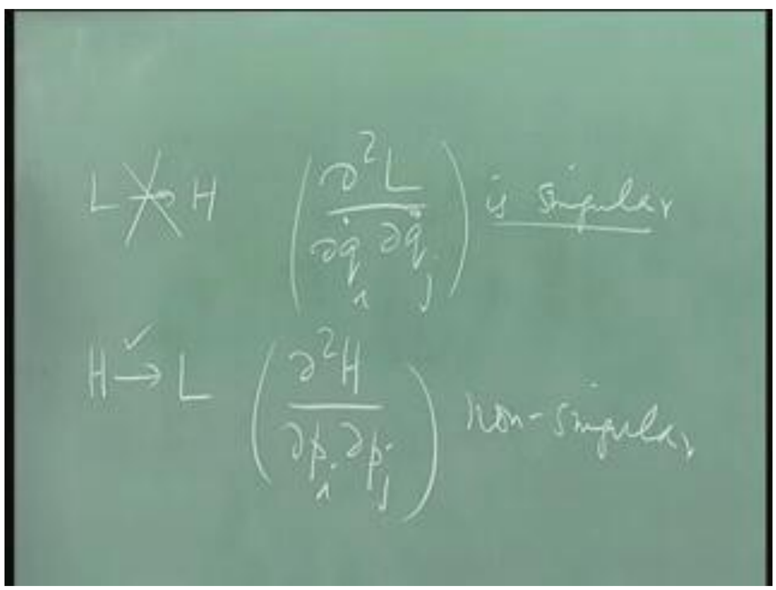
(( ))

The linear matrix yeah sure, I mean if you do that in the original case, there is no centum, there is no Eigen value with real part equal to 0, so the analysis will give you reasonably character also; we will give you characters also sufficiently close to one other critical function. So, this is a linearizable problem yes definitely, and if you did that you discover this point is unstable, and these points are stable.

The Laplace linear lanes vectors the constant of the motion in the case of a potential minus k over R, in other words in attractive 1 over R potential. But, not in the case of a repulsive 1 over R potential, this is not true, because its lengths vector as nothing to do with whether the case positive or negative, the constant of the motion and both cases. It is just that when case negative the orbits are ellipses, there are bound bound orbits on the other hand, the bounded orbits. If k is positive then you have a repulsive potential, and then the orbits are all hyperbolas, but all the orbits are guarantee to be cornice sections in this potential anyway.

But this is nothing to do with whether the Laplace linear lence vector exist or not, that is a symmetry of the problem and there is nothing to with whether k is positive or not. And we derive the relation d A over d t equal to 0 and never made use of the fact, the k was the positive or negative.

(Refer Slide Time: 14:37)



The Lagrangian transformation, from the Lagrangian to the Hamiltonian can be made only the Hessian matrix due to L del, so the statement is the transformation from L to H can only be made if d two L over delta q i dot delta q j dot is singular, this matrix is singular. That is not true you can make this transformation, only if the matrix is non singular that is the whole point, but does it singular there and if you add your eyes open you would have seen that it say singular, therefore the statement is false. But the next statement where the Hamiltonian was into the Lagrangian, so the question is can you go from the Hamiltonian to the Lagrangian the condition required is that this matrix is non singular, so if this is a singular this is not possible, if this is non singular this is possible.

So, in all cases the second statement is true, this is only to find out you got 8 AM, you are a weak or not.

(( ))

In general it will be asymptotically stable, so the statement the ask was the system has to asymptotically stable critical points, and that is true, they are asymptotically stable, they are not stable.

(( ))

Yes

No, no

(( ))

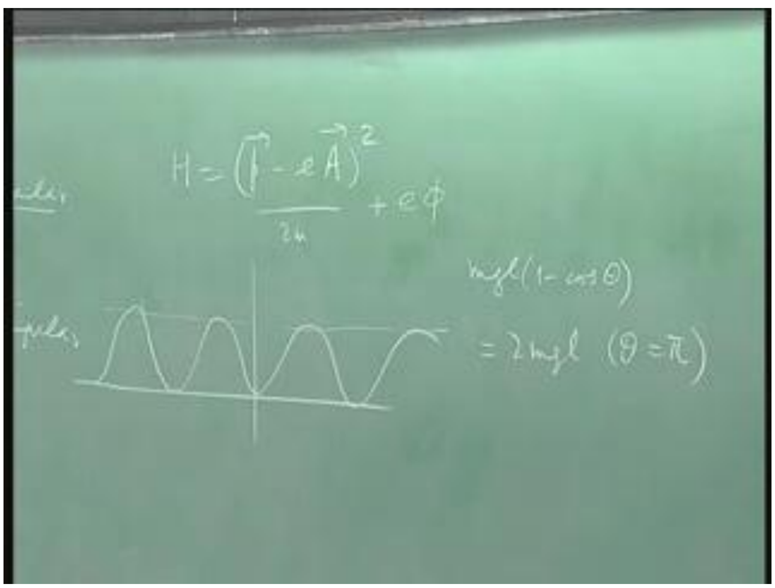
That is not true, no if the real part of the Eigen value is negative, then its asymptote is stability, if the real part is 0, then it is a center the real part is positive it is unstable, that is it; whether, it is a stable asymptotically stable node or asymptotically spiral point, stable spiral point that depends on the details of the parameters. But, that is not; both of them are true when the real parts of the Eigen values are negative that is all we made.

So, whether it is a spiral point or whether it is going to be a node would depend on how heavy the damping is, if I start of something here, if the damping is very high it could just trickle down here and stop. On the other hand, if the damping is not sufficiently high, then it could go here could oscillate here and come back here with the smaller amplitude, and then go back and keep doing this about this point and stop here, that would be a spiral poin. And that would depend on the size of the damping relative to the other coefficients, but in all cases asymptotic stability for those two points.

And the next one, I was surprised at the answers, because the ball bounce is up and down in the horizontal on a on a horizontal flow under the influence of gravity, the statement is, if the collisions with the floor are assume to be perfectly elastic, and A resistance is neglected the time period of the motion is independent of its amplitude, that is not true, we know that if I take this object and drop it. The time it takes to hit the ground is going to depend on the height of course, and if it is a ball it is going to bounce up and down, so this is…

The people who take the true and I was wondering why the effect of JE has on off, the Hamiltonian of a charge particle moving in a time depended electromagnetic field is case invariant.

(Refer Slide Time: 18:15)



We know the Hamiltonian, we know it is H is equal to p minus e A whole square over 2 m plus e phi, it got explicit dependence on the scalar and vector potentials. So, obviously cannot invariant, it got explicit dependence here, so it is not case invariant, not true; the question is how do this change if you make case transformation, and that is what you ask in the fill in the blanks at the end.

What I found interesting was a people are said, yes it case invariant here, and then have calculate a non zero changes, whenever I see that kind of contradiction I am remained of this guy who was asked, how many chapattis you could eat on empty stomach and he said 19, so the guy told in the jokes said immediate, because after the first one, you eat in the first one your stomach is not empty anymore. So, you go and call his wife why and ask how many chapattis can eat on empty stomach, and he said 4 and he says oh god, if you said 19 are told you a good joke, it is not a mutually contradictory things. In the fill in the blanks, the first question was Lagrangian, is a simple pendulum Lagrangian and you are ask for the energy of the separate case, that is twice energy, I think most people have it right.

Because the 0 that we got the potential goes like this etcetera, and the separate tricks corresponds to this energy, so you basically have mgl times 1 minus cos theta and this is twice mgl of theta equal to pi. So, that is the maximum of the potential and the whatever is the energy there, that is the answer twice mgl.

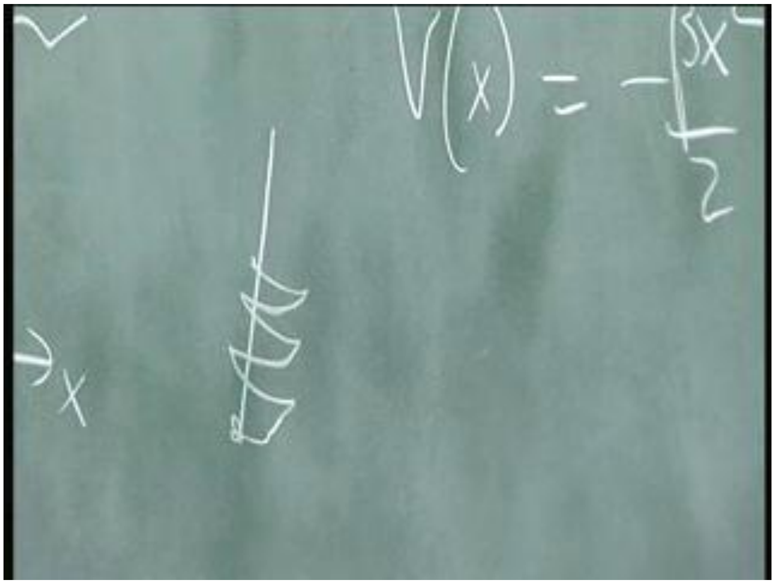
Now, the next question.

(( ))

By definition from me, then there is no question of any kind other kind of motion at all right, it would sort bending it would sort clipping and so on, so from the beginning for me a simple pendulum is a friction less device, in which you have like massless rod and heavy bob, and it is a rigid rod. So, that both rotations and vibrations are possible otherwise, it is complicated even if it is bob with the string, the string should be inextensible otherwise you run it a problem right, so then it is a very complicated problem.

Since, we raise it let me point out, that one of the very complicated problems, the reason we introduce the action was also, because of this kind of problem, if you took a simple pendulum of the kind east describing namely heavy bob attach to a massless string. And let us say the string cannot be it is not extensible, it is a fixed line; now you start oscillating in and as you oscillated suppose, you slowly sort in this string. So, imagine for example you made a whole in the table, and you put in the pendulum through it and it is oscillating there, and then you gradually draw the string out, so that the length of the string becomes less and less.

(Refer Slide Time: 21:30)

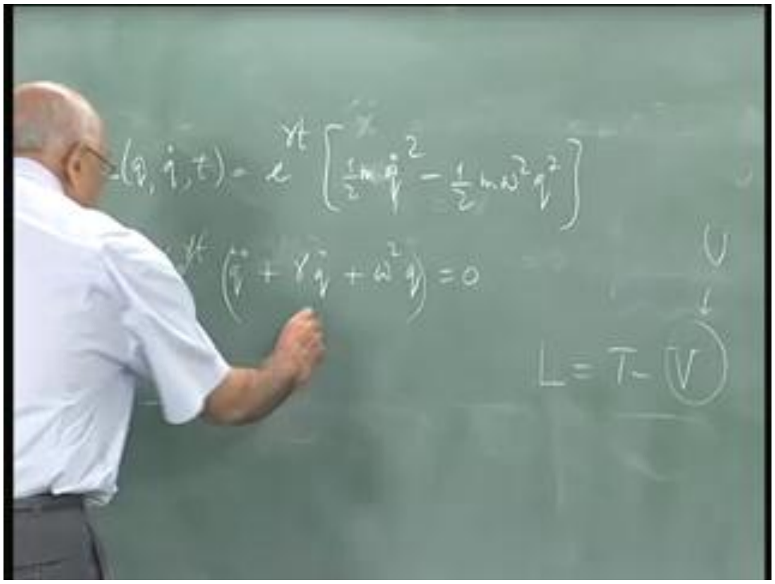


Then of course the motion of the pendulum is going to look like, this it is going to start here, and then it is going to go here, it is going to keeps doing this, it is going to go backward. Now, then the question is what happens to the frequency, because length is changing and you know the time period is 2 pi square root of L over g and L is very slowly being decreased, therefore you know that the time period is going to decrease slowly, but the question is can I solve this problem, and this is one of the very, very important problems in mechanics.

Yes, you can using the fact that the action of the system is actually in adiabatic invariant, another words the action does not change essentially, so this is one of the reasons for introducing the action in classical mechanics, that they can be uses adiabatic invariants when you put up the system sufficiently slowly, it turns out that the quantities at remain constant of the action variables. And that is one of the reasons, in this the many, many papers on this including a famous paper by Chandrasekhar himself, on this pendulum in decrease and by little word on other famous people, but the pendulum we have in mind is much simple device.

The next question give you Lagrangian, and not surprisingly most of you discovered that this equation of motion was in fact, that of a damp simple harmonic oscillator, so you see you got dissipation in this problem, and I can still describe it by Lagrangian. So, the lesson is suppose to be that dissipated system could under suitable conditions, still be describing by Lagrangian and takes us it is fairly non trivial thing.

(Refer Slide Time: 23:15)



Because, we have L is a function of q q dot and t and its e to the gamma t times the harmonic oscillator [m q dot square minus 1 half m omega square q square, this portion alone is a Lagrangian for this simple harmonic oscillator, the undammed ordinary simple harmonic oscillator. But, I have an explicit time dependent factor here, and if you compute the equation of motion, you discover that q double dot plus gamma q dot plus omega square q equal to 0.

Actually it gets this whole thing gets multiplied by m e to the gamma t, and it is not clear to me why you do not score of these things, because neither m nor e to the gamma t is 0, so you really get this. And a fact that you leave it at that point with m e to the gamma t and not bring the equation all the terms to the left hand side and so on; to me the symptoms of laciness which perhaps is reflection of the semester that we people are all in.

But, you would not do that in the JE, for the simple reason that you are worried about some shortsighted or myopic examiner giving you 0, where you should actually got 1, in which case your entire word line feature history would have got changed, always have found this very puzzling how it is that human being kept adopted to things. And of course, by the time you come into your 4 th year there are so many exams you gone through, that exams do not hold any terror for you, I had an experience where the students comes to an exam without a pen and he borrows my pen, the reason is you has a tooth brush there instead of this pen by mistake.

Of course, when you took the joined and examiner no doubt at all, that is pair and suit of either side of the exam hall, and what should binocular to see whether you written the roll number down correctly, not to mention the fact that you are 4 water bottles, and 6 pair pens, but after 4 years things become, so this is equation of motion of a damp harmonic oscillate. So, the important lesson damped dissipated systems sometimes can be return in terms of Lagrangian.

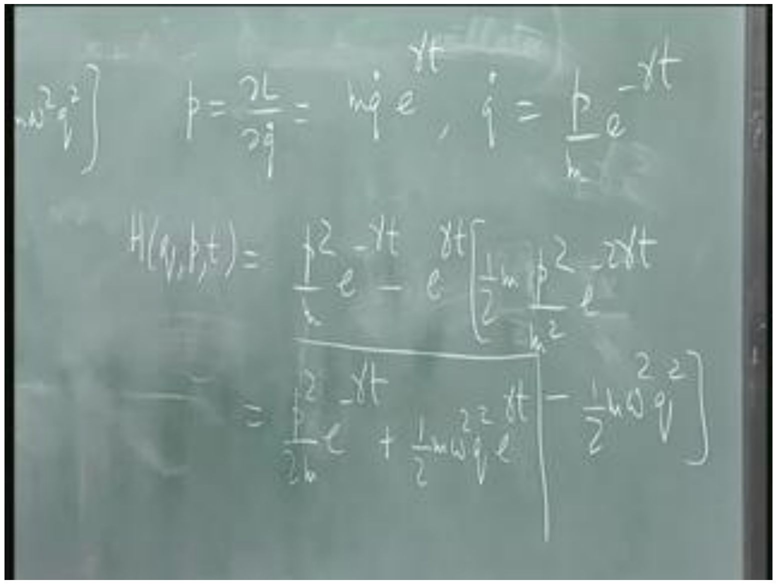
(( ))

It is a general prescription for by which you can write down not a potential energy you know, that for conservative systems of kind we have in mind, the Lagrangian can be written as T minus V very often, in the leap in Newton’s equations. But this V could be replace by something called U, which is called the rally function which for certain kinds of dissipation would play the role of a potential, and therefore you can write the Lagrangian equations down once again. So, the answer is in general no, but in special cases yes, you can do, what is interesting is we know the phase trajectories in this system, they are going to go and fall into the origin, and that is very clear.

(Refer Slide Time: 26:11)

So, is this a measure preserving flow in q and q dot is this measure preserving, because wherever I start all these fellows are going to fall in into this point here, so it is clear the entire phase space is going to swing to a point, no matter where you start it is going to swing your point. So, its volume preserved in phase space no, in this phase space no, but now you can ask can I go to Hamiltonian, can I take the system and go to Hamiltonian, so let close your eyes and do that, let us see what happens to the Hamiltonian here. We know at the back of our minds, that this is a dissipative system and therefore, any Hamiltonian would be explicitly time dependent, if it is is exist.

(Refer Slide Time: 26:58)

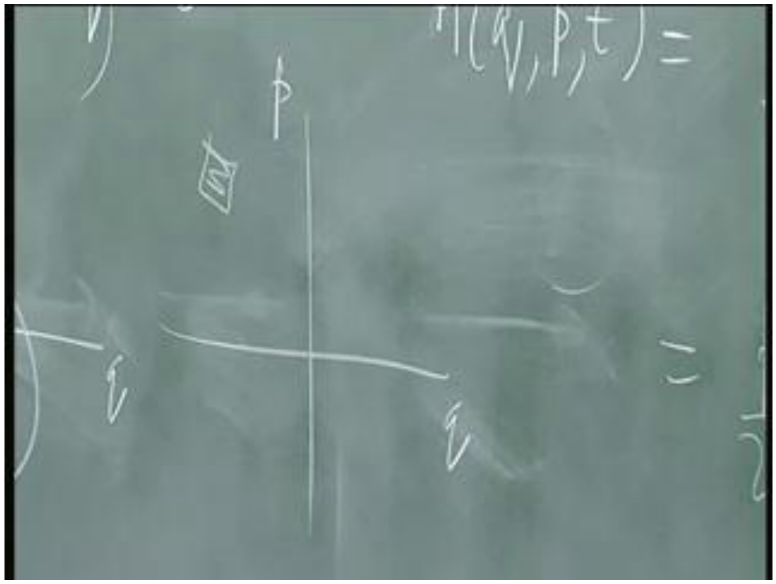


So, let us see whether we can do that we have to define P is delta L over delta q dot and that terms out to be m q dot e to the gamma, so please notice p is explicitly or time dependence, in this formally. So, solve for q dot equal to p over m e to the minus gamma t, so what happens to the Hamiltonian H is the function of q, p and t and that is equal to p q dot. So, that p square over m e to the minus gamma t minus the Lagrangian, which is e to the gamma t times 1 half m q dot square, but q dot is this here, so p square over m square e to the minus 2 gamma t minus 1 half m omega square q square.

So, this works out to p square over q m e to the minus gamma t, it is gives you minus gamma t plus 1 half m omega square p square e to the plus gamma t, it got explicitly dependence, so in the p q phase space is measure preserve, it is a time dependent Hamiltonian or area element preserve in the p q phase space. They are, look at our first part C of the first question, is a (( )) theorem says the volume element in the phase space of a Hamiltonian system does not change magnitude, and thermal evolution, this is also true for time dependent Hamiltonians, so it is true here to.

So, you see this begins to tell you even the simple example that, the change from q from q dot, q and q dot the q and p could be quite non tribute; one of the great advantages of changing to the Hamiltonian frame work is that the flow becomes measure preserving. Even if the time dependence in the Hamiltonian, in a sort of crude sense you can you can see, in a crude way you can see what is happening.

(Refer Slide Time: 29:22)



Because you see volume elements in the q p phase space, little volume element would not change as time goes long. It is a non autonomous Hamiltonian, so you must really look at the extended phase space with t also sticking out.

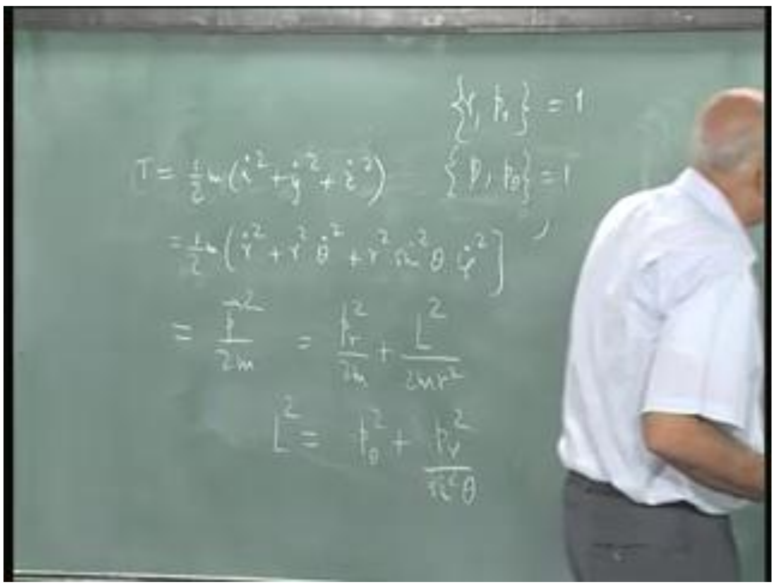
But, in a very crude sense, we know that for a damp harmonic oscillator, if I solve these equations q goes like e to the minus gamma t over 2, it is called a damping factor and p q dot also goes like e to the minus gamma t over 2. So, in this phase space look at what happens to p, this goes like e to the minus gamma t over 2, this goes like e to the plus gamma t, so p goes like e to the plus gamma t over 2, q this goes like e to the minus gamma t over 2 and therefore, d p d q does not change.

So, this is an interesting problem, where you see that the fact that it is time dependent explicitly, does not change the generals statements about the preserving of the measure, and the fact that you have a very complicated kind of shrinking of volume in the q q dot phase space, can be overcome by changing to a new variable, this one of the great advantages of the Hamiltonian framework. And this is the simple instance explicitly see this happening.

(( ))

Yes, so Hamiltonian system, now the question is can you call this conservative, see the fact is those things are useful when you did not have, when you had only autonomous systems, the moment you have non autonomous system then this distinction becomes completely meaningless. When the third question was the orbital angle of momentum, I was little surprise by the answers here, but let me write this down, the number of people can close to the answer, but ended up writing the kinetic energy rather than the angle or momentum.

(Refer Slide Time: 31:35)



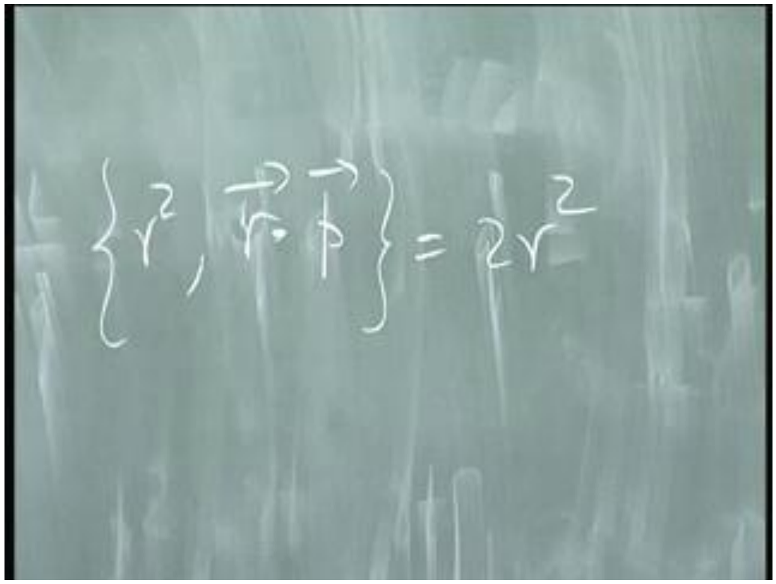
So, you see the kinetic energy of a free particle is 1 half m in partition coordinates its x dot square plus y dot square plus z dot square, if you wrote this an angular variables, spherical pole is coordinates, then of course it is equal to 1 half m r dot square plus r square theta dot square plus r square sin square theta phi dot square this is certainly true. But if you wrote this in the Hamiltonian framework, then this is equal to p squared over 2 m, where p is the momentum the momentum vector dotted with itself.

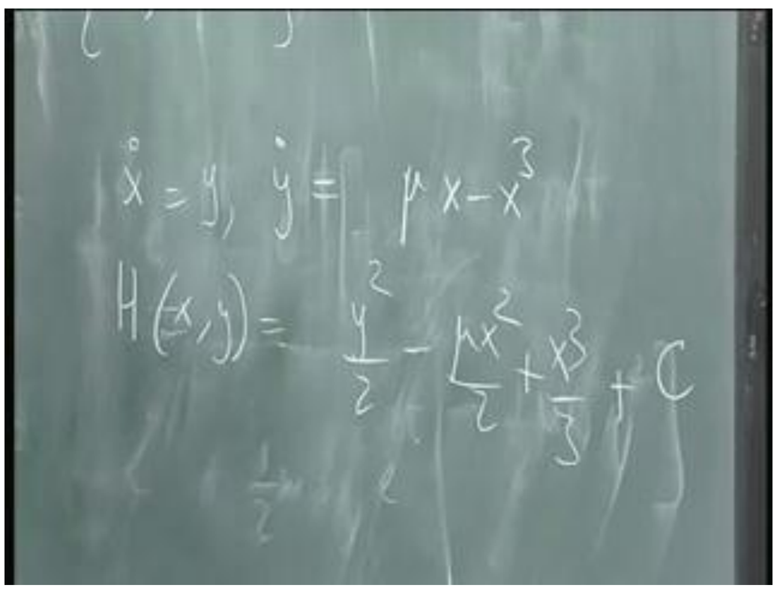
And of course, you could write it as p x square plus p y square plus p z square over to m, but if you wrote this sense spherical polo coordinates, then this is equal to the radial momentum squared over 2 m plus the angular momentum squared divided by twice the moment of inertia and this is 2 m r square. Now, what is L squared, L square if you work this out all you have to do (( )) cross p and square it, then this turns out to be equal to p theta square plus p phi square over sin squared theta and that is it. There is no 1 over 2 m r square that k m only when you wrote it as part of the kinetic energy, otherwise it is dimensionally wrong, so whenever you write these answers check the dimensions, and that way you would not have answers whether the first time one physical set of physical dimensions, and the second term is another set of physical dimensions, this is not possible.

Now, what are the physical dimensions of these guys, what is the physical dimensions of p theta, angular momentum and p phi angular momentum, theta does not dimensions at all, each of these guys as dimensions of angular momentum. What is the physical dimension of p r, it is clearly not the physical dimensions of angular momentum, it is a radial momentum, so it as a dimension of energy multiplied by mass square root of…So, this is got different physical dimensionality, it is obvious because, r and p r the portion on bracket is 1, in the portion bracket of theta p theta is equal to 1 and so on. So, it is immediately clear, that the physical dimensions of p r would be different from those of p theta, the radial moment you would have a different physical dimension, because it is a length sitting there and r.

Then the question about portion brackets that was straight forward, and most people have the answer, just a piece of algebra.

(Refer Slide Time: 34:42)

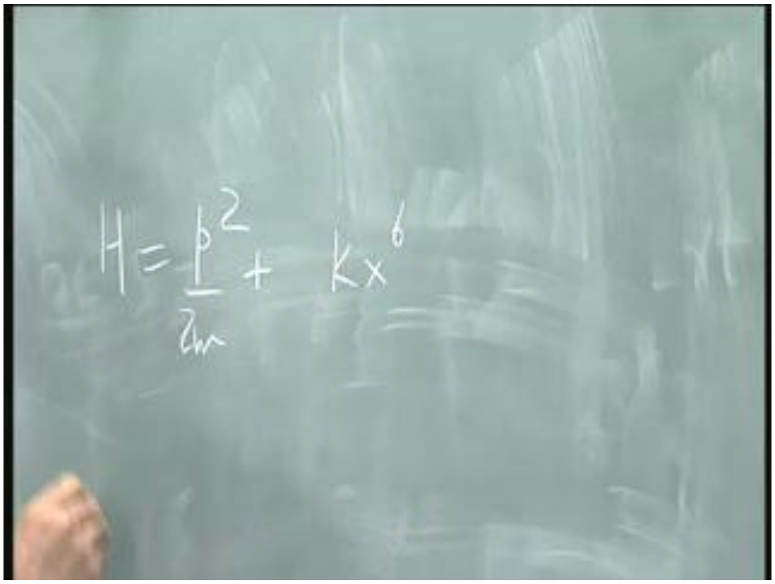




So, we know that r square r dot p is equal to 2 it is twice r square. The next one was the Hamiltonian, you given x start is y, y dot is mu x minus x square all you have to do is to integrate this, and everyone has got this answer right.

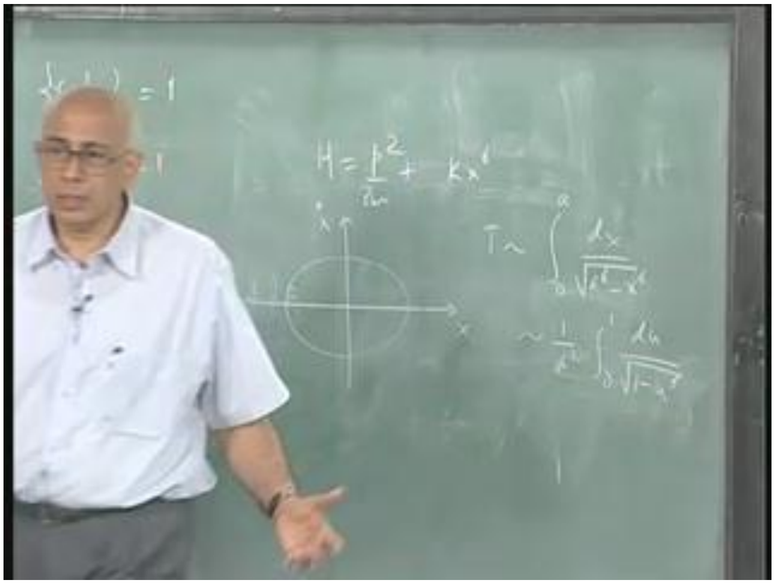
So, x dot equal to y, y dot equal to mu x minus x cube, there are people who write mu like u, please do not do that, because u look like a velocity. So, H of x, y is in fact equal to y squared over 2 minus mu x squared over 2 plus x cube over 3 plus in general some arbitrary constant which is irrelevant, then the nonlinear oscillator sestic oscillator, I think that goes like potential goes like k x to the power 6, the time period is proportional to the amplitude to the power minus 2.

(Refer Slide Time: 35:56)



So, you have a Hamiltonian, which is p squared over 2 m plus k x to the power 6.

(Refer Slide Time: 36.04)

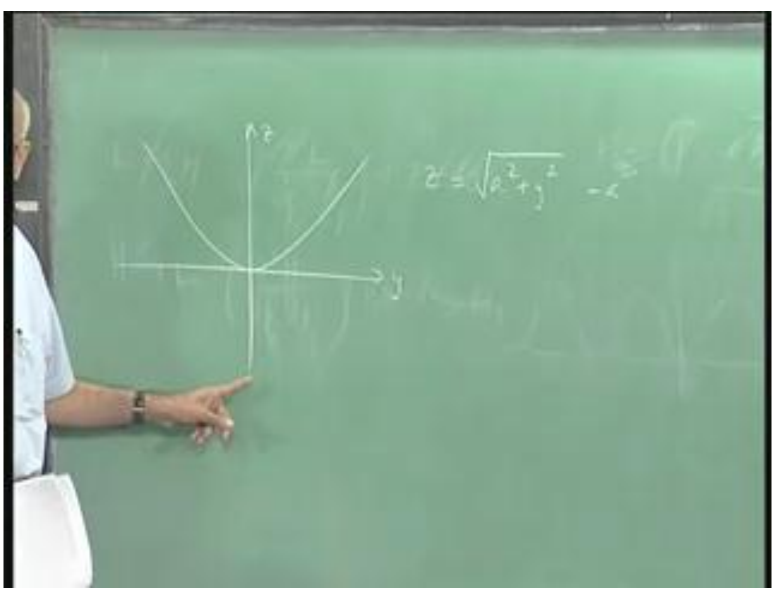


And you are ask for the time period, where the phase trajectory is in the x x dot space for example would be some kind of ovals of this kind not ellipse is, where x dot square plus x power 6 is equal to positive constant, and you are ask to find what the time period is, and the t goes like integral 0 to a 4 times this the x over x dot and x dot would look like e minus the potential. But, the e is proportional to a to the 6, so a to the 6 minus x to the 6, so apart from some constants this is what the time periods.

And all you have to do is to put x equal to a u, make a change of variables, you get an a out up there, and you get an a cube here this becomes 1 and therefore, this whole thing goes like 1 over a square integral 0 to 1 d u square root of one minus u to the 6. Like explained on an earlier occasion, this integral exists, it is just the square root singularity at u equal to 1, some finite number and then it is 1 over a square, so it is a to the minus 2, since I said t is proportional to a to the r, r is minus 2 not plus 2 some people are lost a mark, because they wrote plus 2 and I am sure that the new what is going on, it is just that did not read it carefully.

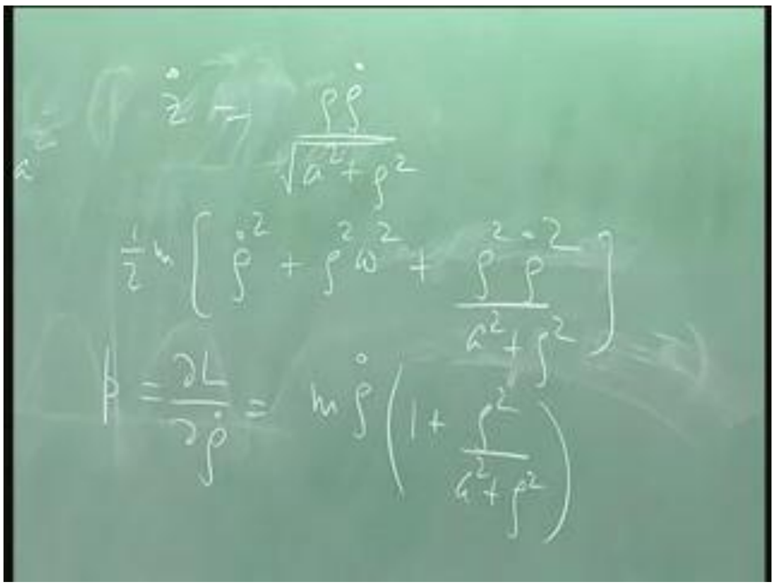
Then this is the bead moving on a hyperbola, we work out the case of the problem case of the, of the circle, so there are some pupil who wrote down the answer for the circle, that remain did mean of this you know, how many chapattis can you eat (( )), forever it shall be circle.

(Refer Slide Time: 37:59)



So, this hyperbola look like this here, and this is z this is y and you are give z equal to square root of a square plus y square minus a square minus a, so that is the upper branch of this hyperbola, there is another one here intersecting at two way, and then of course you have to write down what the kinetic energy is, and you wrote the kinetic energy down its z dot equal to this becomes the rho square.

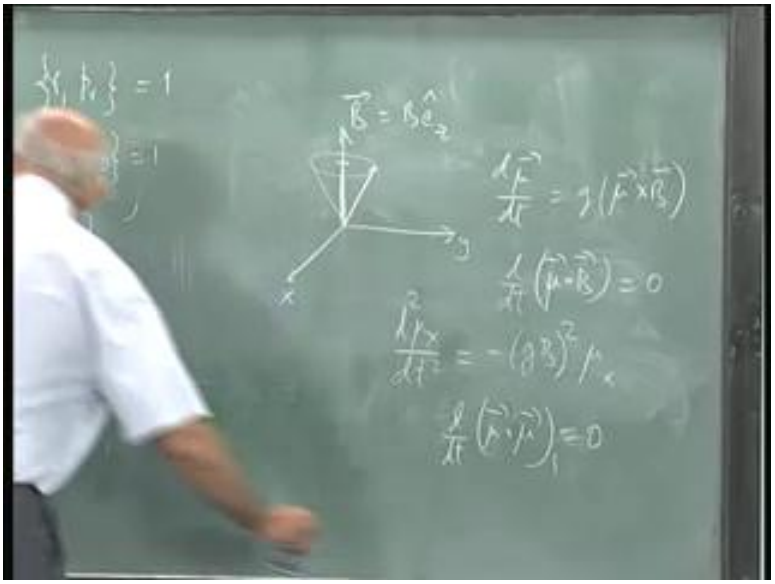
(Refer Slide Time: 38:35)



Once you start rotating, it become rho rho dot divided by a square plus rho square square root of this, that is what z dot is. And then you have to compute 1 half m rho dot square plus rho square omega square plus z dot square which is rho square rho dot square over a square plus rho square, and you have to remember to remove this square root because you squaring this. And then p equal to delta L over delta rho dot, so it becomes equal to m rho dot times 1 plus rho square over a square plus rho square.

In fact, that you have in written it as a square plus 2 rho square over a square plus rho square has been condoned, but this is the answer; there is no square root or anything like that, that is the momentum conjugate to radial variable rho, the actual distance rho. The next one was in over problem the precision of a particle of a magnetic moment in a constant magnetic field, and I assume that this problem is well known to you.

(Refer Slide Time: 40:07)



So, you have a magnetic field in this direction, when you have the x y plane, and your tool d mu over d t equal to g times g mu cross B, and you see you can resolve mu into the z component along the field and two transverse components, take dot product on both sides with respect to B. So, take B dot whatever it is, then you can see d over d t of mu dot B equal to 0, B is a constant field does not change you take dot product take it in, and the this is 0.

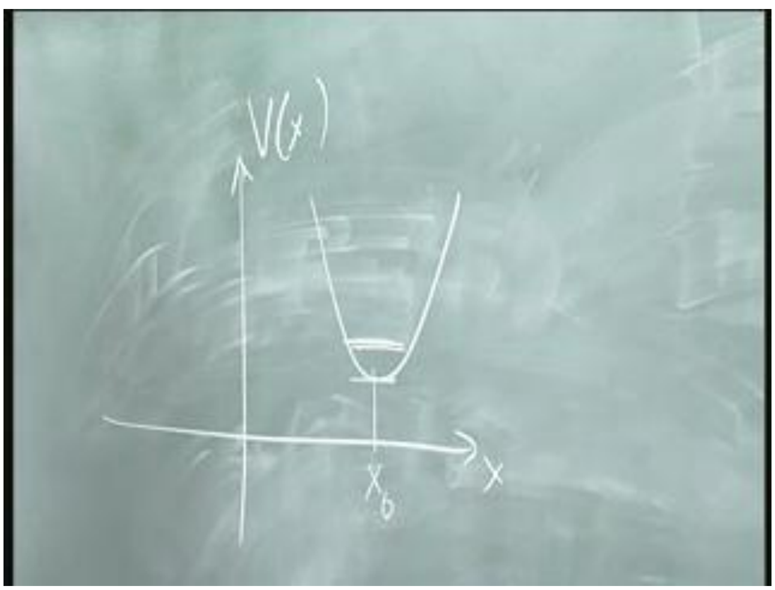
So, it is says the component of mu along the direction of the magnetic field does not change, the other two components should precise around, and as you know uniform circle a motion in the x y plane is equal to two simple harmonic motions are right angles, so you know the answer mu x is a simple harmonic oscillator equation.

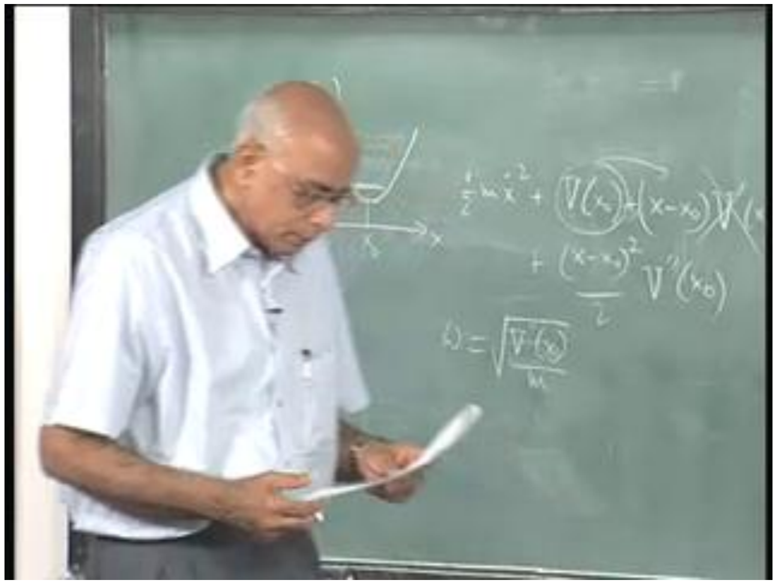
So, it is clear that you going to have d mu x d 2 mu x over d t 2 equal to minus g B whole square mu x; and similarly, if a mu y to a base exactly, the same equation except the initial conditions are (( )), now I would not accept your writing d mu x over d t or d 2 mu x over d t with the d mu y over d t on the right hand side, because any of you tell me what is the equation for d mu y over d t? So, that is not complete in answer, you have to say what is the equation from mu x, so you can only involve mu x I should be able to solve for this thing.

So, writing in terms of another unknown does not give me the equation for it, that is not acceptable, incidentally you could also ask what is mu dot this equation on both sides, that two is 0, so it is clear that d over d t mu dot mu equal to 0, in other words the magnitude of mew is preserve. So, the component of mew along this field is preserved, and the magnitude is preserved therefore, the only thing it can do is to move on the clip of a cone, it can precise in the type of a cone.

Because you could ask cannot you do this, and come back, why this have to make the circle and that is what this equation true, it does not do that it makes full circle goes around, so the tip of this traces is a circle or the x and y components, to simple harmonic oscillations. So, the famous problem of precision in the magnetic field, the next one surprise me, because you just set the simple potential as a simple minimum at some point x naught, and ask for the frequency at this point, and that is harmonic oscillations, so the frequency is extremely straight forward to write down.

(Refer Slide Time: 43:16)





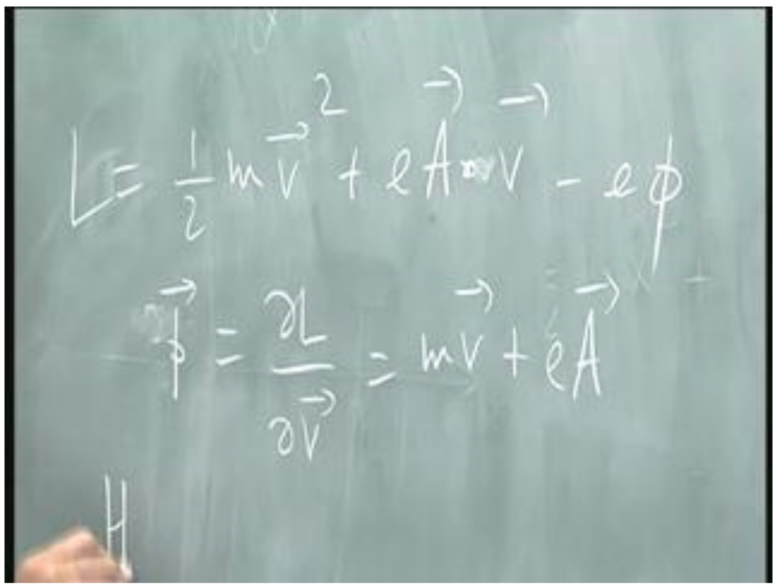
So, as a function of x V of x has some minimum somewhere at some point, x naught a simple minimum, and you are ask what is the frequency you as small oscillation about that point here.

So, all you have to do is write the energy down as 1 half m x dot square or velocity square plus the potential energy and do Tyler expansion at this point, so the first term is V of x naught is some constant plus x minus x naught V prime of x naught, but that 0 the whole point this being a minimum is that V prime of x naught is 0. So, the lots are people who got answered with V prime of x naught or V prime of x of course, the frequency is a constant, you cannot depend on where you are in this problem, we asking for small oscillation about the point x naught; so whatever derivative occurs must be add the point x naught right plus.

The next term, so this thing is a written in this form, plus x minus x naught whole square over 2 V double prime at x naught, so this is of the harmonic oscillate a form except you shifted the center of the oscillation to x naught instead of the 0. But, that was in change the frequency, and the frequency omega equal to the square root the shrink constant divided by the mass the shrink constant, the role is played by V double prime x naught. So, it is just V double prime x naught and you must in forget them us, so that is the answer.

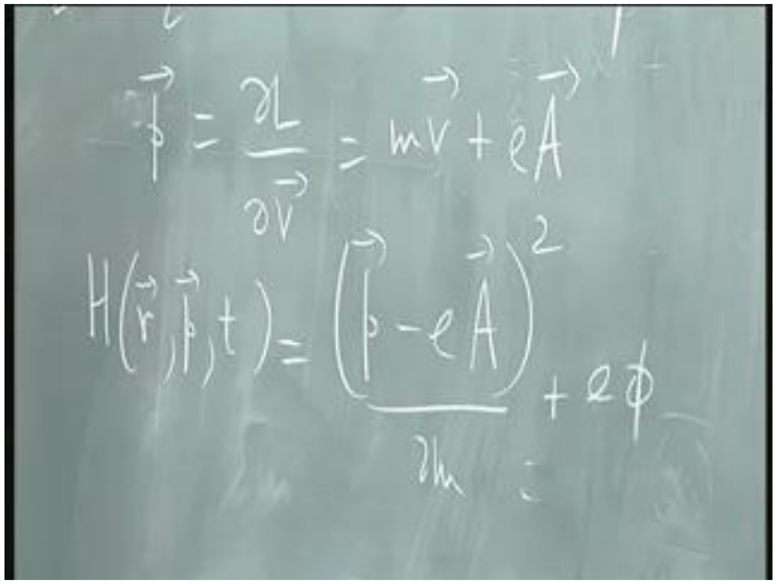
And last of all um the Lagrangian of a charge particle in a electromagnetic field is given to you, and the you make a gauss transformation and you ask what is the Hamiltonian do, and we know that Hamiltonian will change. Because, in general remember gauss transformation on the electromagnetic fields is like a canonical transformation on the Hamiltonian. And we know that the Hamiltonian under a time dependent economical transformation, it is not the old Hamiltonian return in the new variables. But, there is also in extra partial derivative term, which is a partial derivative of the so called generating function of the canonical transformation; and that is what we will emerge here.

(Refer Slide Time: 45:47)



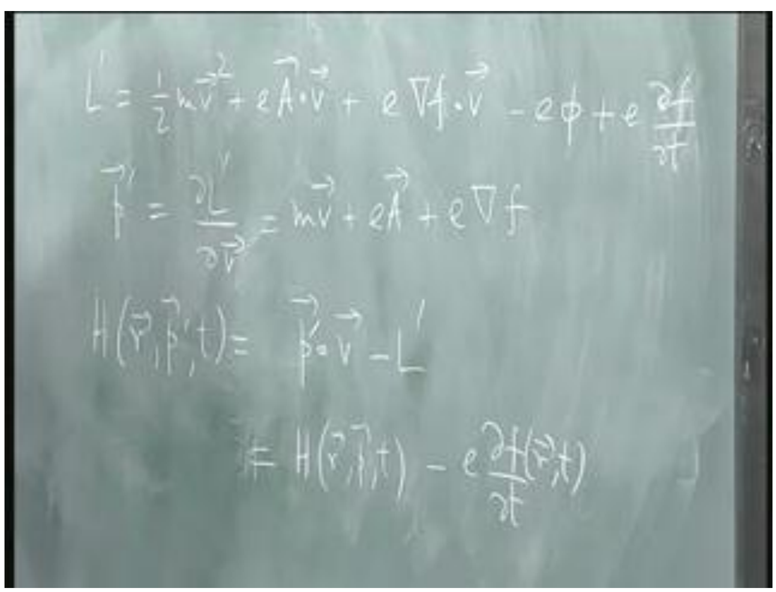
So, you start by saying L is equal to 1 half m V square plus e A dot V minus e phi and these a functions of r and t therefore, the momentum p is delta L over delta V equal to m V plus e A we know that.

(Refer Slide Time: 46:11)



And then the Hamiltonian which is a function of r p and t turns out to be p minus e A whole square over 2 m plus e phi, and now I come along and say, I do not like this I make a gauss transformation.

(Refer Slide Time: 46:35)



So, I write L prime equal to 1 half m v square plus e A dot v plus e grad f dot v minus e phi plus e delta prime delta f over lambda t, so I change a to a prime which is a plus grad f and phi to phi prime, which is phi minus delta f over delta t, and this is what the L primes. Therefore, the p prime in this case the delta L prime over delta V, and that is equal to m v plus e A plus e grad f, you got an extra piece you have to put that in. And then the new Hamiltonian of r p prime and t is equal to p prime dot V very most eliminate for V and write in terms of these other quantities, minus L prime p prime dot V minus L prime. So, this term just flows around it gets cancel down both sides, once you do this and then this thing reduces to, and then you discover this becomes equal to H of r p and t, which is the original thing here, minus very important to remember this is a partial derivative otherwise, not true.

If you made a gauss transformation that does not explicitly depend on time, then it is equivalent to canonical transformation, which does not depend on time and therefore, the Hamiltonian will be just, the new Hamiltonian would be just the old Hamiltonian function in the new variables; it is substitute in terms of this new variables. Otherwise, it is an extra partial derivative term in that system.

So, this is this is not the total derivative, the Lagrangian changes by a total derivative, L prime the first from L by d f over d t d over d t of e f, but the Hamiltonian differs minus e delta f over delta t it is very different, it is a very different quantities; so that was some people did get it, but not too many. Let me quickly go over problem set three in case you have it.

We are out of time, that means so we should stop here, and let us do this I would like to spend some more time, so perhaps tomorrow morning, we will go over problem set three, and we could do the following if it is okay with you, otherwise we go on to something else. And that is either we will look at all the earlier problems see it is tomorrow, or I start a new topic it depends on you.