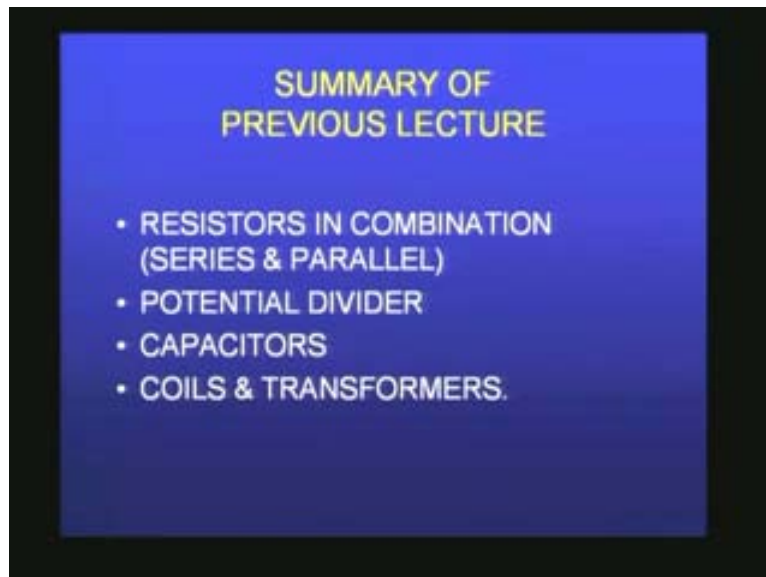


**BASIC ELECTRONICS
PROF. T.S. NATARAJAN
DEPT OF PHYSICS
IIT MADRAS**

**LECTURE-4
SOME USEFUL LAWS IN
BASIC ELECTRONICS**

Hello everybody! In a series of lecture on basic electronics, learning by doing, we now come to the fourth of the series. Before we go on to the lecture we will look at what we saw in the previous lecture.

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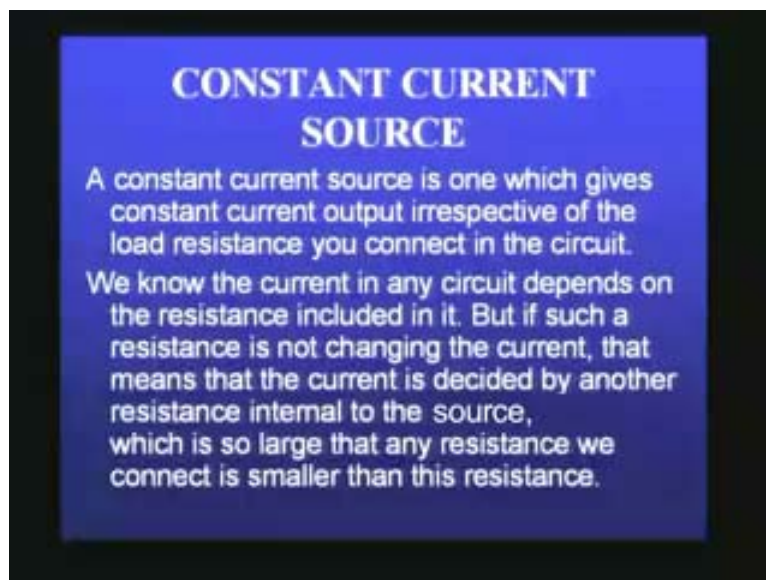


In the previous lecture we studied about resistors in combination. For example series and parallel combination of resistors, potential dividers, how voltages can be divided and a smaller voltage can be obtained from a larger power supply by using a set of resistors in the potential divider form and then something about capacitors, different types of capacitors, electrolytic capacitors, non-electrolytic capacitors and we also briefly saw about coils and transformers, inductances and transformers which are very, very essential in several electronic circuits.

In this lecture we will look at some of the useful laws and theorems. We have already seen one important law which is basically ohms law. We also did an experiment to see how ohms law is obeyed when I have a voltage source and a resistor. The voltage across the resistor is proportional to the resistor and the current is proportional to the voltage when the resistance is constant. All those things we have seen already. Before we go further into the laws and theorems I thought we will just briefly look at the constant

current source. We should first know about current sources. We have already seen about voltage sources in the previous lecture. Now we will study about current source. Just as voltage source is something which gives a voltage output, obviously you can now understand current source is something which gives a current output. The constant current source is something in which the current is a constant. Just as you have in a constant voltage source, irrespective of the value of resistance I connect across the output terminals, the voltage remains constant in the same way in a constant current source irrespective of the value of resistance I connect in the circuit the current will always be a constant. The constant current source is basically different from the constant voltage source.

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If I have a simple circuit with a voltage battery or a voltage source and a resistance, you already know that every voltage source is also associated with an internal resistance. Therefore when I look at a voltage source I should always remember there are two things in this. One is an ideal voltage source which has got zero internal resistance; added to that is the internal resistance which is corresponding to the voltage source that we are looking at. Therefore any given voltage source can be equivalently represented by two items. One is an ideal voltage source which has got zero internal resistance as you all know and a series resistance which is actually the internal resistance of the voltage source that we have.

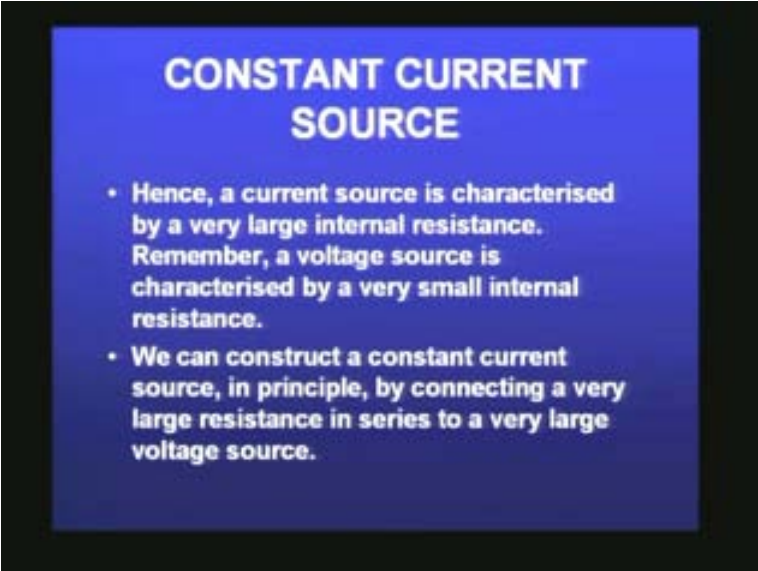
In the similar manner if I connect any load or any resistance, load means resistance, then the moment I connect the load the value of the current in the circuit will be different. Because current by ohms law is given by voltage divided by total resistance. The total resistance in this case is the internal resistance of the voltage source plus R_L the load resistance that I have connected. So you can imagine immediately that the current in a circuit of this type will always depend upon the resistance that I connect as R_L . When will this current be a constant? When will this current remain constant in spite of the

resistance that I connect as R_L ? Only when I have very large resistance as the internal resistance of the voltage source you would find the external resistance that I connect cannot alter because it has been decided already by a very large value of internal resistance and therefore the external resistance which is much smaller than the internal resistance cannot alter considerably the current in the circuit.

You can immediately recognize that a constant current source should naturally be characterized by a very large internal resistance. So how do I define a constant current source? A constant current source therefore can be looked at as something which has got a very large value of voltage source in series with a very large value of resistors. Together they can form a current source. A current value is decided by these two. For example voltage divided by the internal resistance will be the current that you will get. Because the internal resistance is a very large value, whatever the value of resistance that I connect which are smaller considerably with reference to the internal resistance you would find they will have no control on the current and therefore the current output will remain a constant for several ranges of resistances I connect at the output.

So if I extend this concept and then look at an ideal current source, the moment I say an ideal current source you can immediately imagine that an ideal current source should have infinite internal resistance. Because I say ideal I can go to the maximum value I can think of. The maximum value is infinite resistance. So, an ideal current source is characterized by infinite, internal resistance. Because it is ideal it cannot be practiced; it cannot be realized in practice. Similarly in an ideal voltage source, perhaps you may recall, the internal resistance will be zero. In no practical voltage source, the internal resistance can be zero and therefore they are all not ideal. The other point I wanted to stress is that we can build a constant current source by employing a very large voltage source and a very large resistance as internal resistance.

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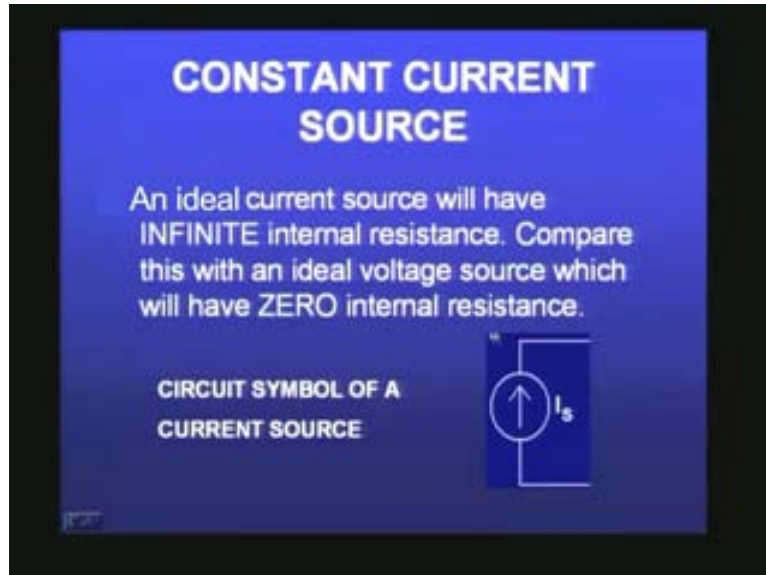


CONSTANT CURRENT SOURCE

- Hence, a current source is characterised by a very large internal resistance. Remember, a voltage source is characterised by a very small internal resistance.
- We can construct a constant current source, in principle, by connecting a very large resistance in series to a very large voltage source.

These two combinations along with an external resistance will form a reasonably good constant current source. With this background I will show you on the screen that an ideal current source will have infinite internal resistance and we have to compare this with an ideal voltage source which will have zero internal resistance.

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The circuit symbol of a current source is shown here which is just a circle with an arrow inside. The arrow direction shows whether it is a source or a sink. In the I_s here, I means current, S means source. So I_s is a current source and the arrow indicates that it is a current source. It can also be a current sink. When I invert the arrow that you see on the screen then the current will be in the other direction. It will be flowing towards the ground. Therefore current will have to come from the outside load into the ground and therefore this becomes a current sink. So except for the direction, the property of the current source and the current sink are identical. You do not have any confusion about it. You would find that such important concepts will be coming again and again in various circuits and therefore you should remember a normal voltage source will always have an ideal voltage source with a series resistance and normal current source will have ideal current source and an internal resistance which is of very high value. These two concepts we should remember for all our future applications.

There are also other concepts which perhaps you already know. I assume all of you have got a background in basic electricity and therefore you would have seen Kirchoff's laws; Kirchoff's voltage law and Kirchoff's current law. But for completeness let me briefly review the concepts of KCL or Kirchoff's current law and KVL or Kirchoff's voltage law quickly. What is Kirchoff's current law? At any instant the sum of all the current flowing into any circuit node is equal to the sum of the current flowing out of that node.

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KIRCHOFF'S LAWS

Kirchoff's Current Law

At any instant the sum of all the currents flowing into any circuit node is equal to the sum of all the currents flowing out of that node:

$$\sum I_{in} = \sum I_{out}$$

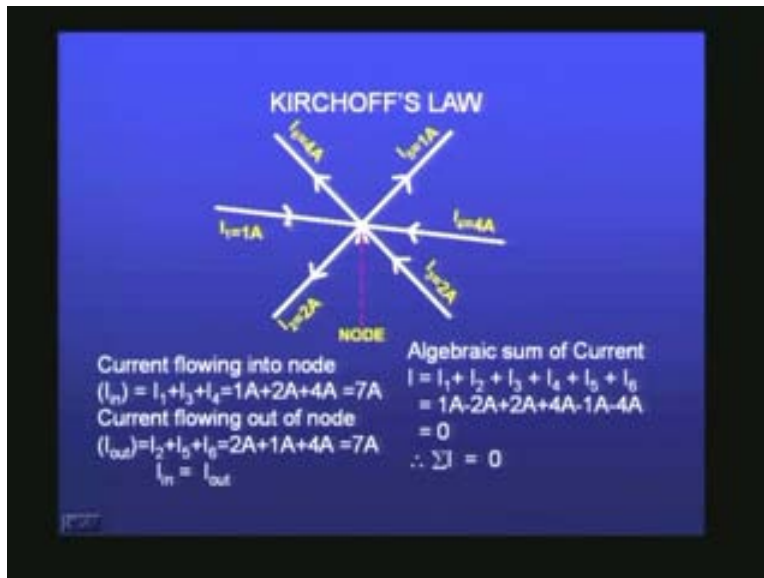
In other words at any instant the algebraic sum of all the currents at any circuit node is zero:

$$\sum I = 0$$

What is a node? A node is a point where you have several wires or several current conducting wires come together. Therefore it is a junction point of several wires. So if I have such a thing then what Kirchoff's says is that whatever current that is flowing in towards that point will have to flow out. There cannot be any storage or staying of current in such nodes. This is the essence of KCL or current law. I have shown it on the screen with an equation. Sigma that is sum of I_{in} all the current flowing into the node is equal to sigma of I_{out} the sum of all the current going out. This can also be explained in a slightly different way. In other words at any instant the algebraic sum of all the currents at any circuit node is zero. That means I add all the currents algebraically. What do I mean by algebraic? If the current is flowing into the node it is positive, if I assume, then if it goes away from node it is negative and therefore the total current, algebraic sum of all the current at this node will be equal to zero. Therefore the equation sigma of I equal to zero represents this concept. You can use any of these. You can either say current flowing in is equal to current flowing out or you can say the algebraic sum of all the current flowing into a node is equal to zero where automatically you take the vector direction of the current whether the current is flowing in or flowing out.

I have shown in the next screen a small example where you can see I have shown a node which is the center point. You can see that currents are coming from different directions and they are also flowing out from the node in different directions. I_1 is for example one ampere which is flowing into the node. I_2 is two amperes flowing out of the node and I_3 is two amperes flowing into the node, I_4 is again flowing into the node, four amperes. I_5 is minus one ampere flowing out. I_6 is four amperes flowing out. If I want to apply Kirchoff's law then I should sum all the current flowing into the node.

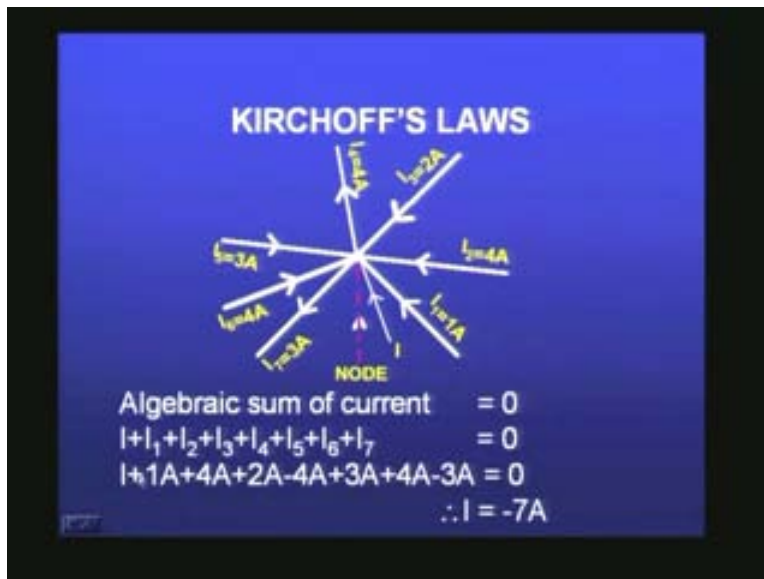
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That means you can see here current flowing into the node I_{in} is equal to I_1 ; I_1 is flowing in which is one ampere; I_3 which is also flowing in which is two amperes and I_4 it is again flowing in which is four amperes and if I add them $1A + 2A + 4A$ that gives seven amperes. So the sum of the current flowing into the node is $7A$. If I look at current flowing out of the node I_{out} in the next line, you can see it is I_2 which is actually two amperes; I_5 which is actually one ampere and I_6 which is four amperes and when we combine them all together, again you find it is equal to $7A$. Therefore I_{in} , the current flowing into a node is equal to I_{out} , the current flowing out of the node. In this case both are $7A$. The other way to do to is to look at the algebraic sum of all the currents. That is what I have done on the right side. The algebraic sum is I is equal to $I_1 + I_2 + I_3 + I_4$ everything I sum it together; but make sure that if it is flowing in I keep it as positive. For example I_1 is taken as one ampere plus and I_2 is minus two amperes because it is flowing out and I_3 is plus two amperes; I_4 is plus four amperes it is flowing in again; I_5 is minus one ampere because it is flowing out of the node; I_6 is minus four amperes again it is flowing out the node and therefore if I now find the total, you can find the total is zero. Therefore the algebraic sum of all the currents flowing into a node is equal to zero. In both ways you find that Kirchoff's current law is obeyed.

I have given an example. What is the advantage of knowing the Kirchoff's law? You can see in this figure I have got one node in which I do not know the current. I want to find out what this current is. I have all the other currents. I know all the other currents. The simple way of looking at it is the algebraic sum of current flowing into the node should be equal to zero. So I , which is this current I do not know plus I_1 plus I_2 plus etc, all the things I put together should be equal to zero. I , I do not know; plus I_1 is one ampere as you can see in the figure and I_2 is four amperes; so plus. I_3 is again flowing into the node, two amperes; I_4 is flowing out of the node therefore minus four amperes and the next I_5 is three amperes; I_6 is plus four amperes; I_7 is minus three amperes because it is flowing out and if I look at the total and evaluate I can always get the value of I which I do not know.

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I is equal to -7A. So you do not have to worry about the direction of the current in this case because automatically the sign will show the direction of the current. Because we have taken all current flowing into the node as positive I get minus seven. Therefore in this I, the direction of arrow I have shown here which I did not know at that time is actually wrong. It should be in the other direction because now after solving the problem with Kirchoff's law I find the current is flowing out and the magnitude of the current is 7A.

Let us move onto the next which is Kirchoff's voltage. Just as we have the Kirchoff's current law, a Kirchoff's voltage law is also very useful in solving simple network problems and then trying to know unknown voltages. Kirchoff's voltage law says at any instant the sum of all the voltage sources in any closed circuit is equal to sum of all the voltage drops across different resistors and inductances in the circuit. This is another law of Kirchoff's which talks about in this case a closed circuit. A closed circuit can have several voltage sources or batteries and several resistors all connected in series or parallel in different combinations. Another way of saying that is if I sum all the voltage sources together that should be equal to all the voltage drops across different resistors that I have used. The other way to do that is I can always look at in the same way as in the KCL. The algebraic sum of all the voltages around a closed circuit across different components should be equal to zero.

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KIRCHOFF'S LAWS

Kirchoff's Voltage Law

At any instant the sum of all the voltage sources in any closed circuit is equal to the sum of all the voltage drops in that circuit:

$$\sum E = \sum I_z$$

In other words, at any instant the algebraic sum of all the voltages around any closed circuit is zero:

$$\sum E - \sum I_z = 0$$

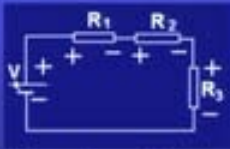
In this case, in the equation you can see sigma of E where E represents the voltage sources all the E_1, E_2, E_3 , several of them minus sigma of I_z which is I into, actually I should have put a R here, so the voltage drop across different currents is I into R which will give me the voltage drop across different resistors; they must sum up together to zero. That is what KVL shows.

Let me take an example. You see on the screen there is voltage source E with the polarity shown and three resistors $R_1, R_2,$ and R_3 all connected in series to this voltage source.

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KIRCHOFF'S LAWS

- In the circuit shown, The voltages across the resistors using Ohms law are IR_1, IR_2, IR_3 .



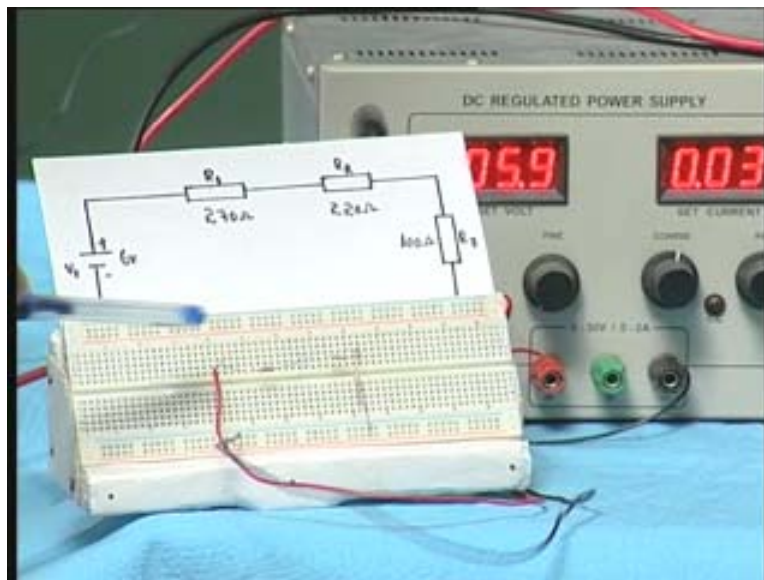
- The + and - for V is different from those for the resistors. The + is shown at a higher voltage.
- Hence $V = IR_1 + IR_2 + IR_3$ or $V - IR_1 - IR_2 - IR_3 = 0$

What Kirchoff's law says is that this voltage V should be equal to if there is a current I flowing through that which we know will be $IR_1 + IR_2 + IR_3$. So the voltage drops across the three resistors are IR_1 across R_1, IR_2 across R_2, IR_3 across R_3 . If I add them all

together because you find they have the same sense of direction. For example current entering is plus, current leaving is minus; again this is plus and minus, plus and minus. Therefore you can see all of them can be added together. But if you come here at the voltage source you find I am coming from minus to plus with reference to voltage. Therefore if I want to add them algebraically if I take V as positive all the other things will have to be minus so in the second line you see $V - IR_1 - IR_2 - IR_3$ is equal to zero is one form of KVL or V the total voltage source in the circuit loop is equal to $I_1R_1 + I_2R_2 + I_3R_3$ which is written here. So either you can equate the total sum of the voltage sources to the voltage drops across different resistors in the circuit or you can say the algebraic sum is equal to zero across a closed loop. So this is Kirchoff's law. We will quickly go and see the Kirchoff's law on the table and then will come back to look at another example of Kirchoff's law where it can be applied.

Here I have a breadboard which you are now familiar with. Okay. You can see I have one, two, three resistors connected in series and this red wire is going to the power supply here which is showing 6V. So I am connecting 6V here. The ground is common. This point and this point are ground.

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This is connected to plus of the power supply. There are 270 ohms here; 220 ohms and 100 ohms all connected in series. So it is a very simple circuit. We have one voltage source and three resistors. That is what I have shown here. What Kirchoff's law says is that this voltage, 6V should be dropped across all these three resistors totally. Therefore the current $IR_1 + IR_2 + IR_3$ should be equal to 6V. That is what it says. Now what we will do? We have a multimeter which I have switched on and I have connected to voltage and it is now measuring DC volts. So I have two leads of the multimeter. Now what I do is I will measure the voltage across each of these. For example let me connect this to ground, one of the terminal and other terminal I connect here to the positive end. You can read it. It is reading 6.04 voltage; I suppose you can see that.

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What I have applied is 6V. I have measured that. This is the voltage source. Now what I am going to do is I am going to measure the voltage across each of the resistors. So let me connect to one end of the resistor the negative terminal of the multimeter, to the other end I connect the positive terminal. So now I have connected the voltmeter across the 270 ohms here and voltage is around 2.77. I hope you can see that. The voltage here is 2.77 voltage.

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Now I will remove the voltmeter from across 270. I will connect it across 220. So I connect the positive to one end; I connect the negative to the other end. Now let us see what is the voltage? The previous one was 2.77, now it is 2.24.

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So if you add them together they come to approximately about 5V and now the last resistor is R_3 , 100 ohms. I will connect the voltmeter across this 100 ohms here and at the bottom and you can see the voltage is about 1V.

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So totally $2.77+2.24+1$ together gives me 6V approximately. So you can see whatever voltage I connected here is divided across each of these resistors. As Kirchoff's law says the voltage source should be equal to $I_1R_1+I_2R_2+I_3R_3$ and therefore Kirchoff's law is verified.

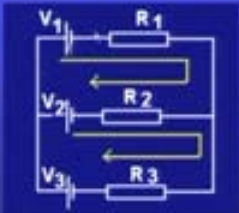
Now let us take a second example. In this case you can see that it is a slightly more complicated circuit. It has got three voltage sources V_1 , V_2 and V_3 as you can see on the screen and it has got three resistors R_1 , R_2 , R_3 .

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KIRCHOFF'S LAWS

Consider the second circuit:

There are two loops. The equations using Kirchoff's Voltage Law (KVL) are:



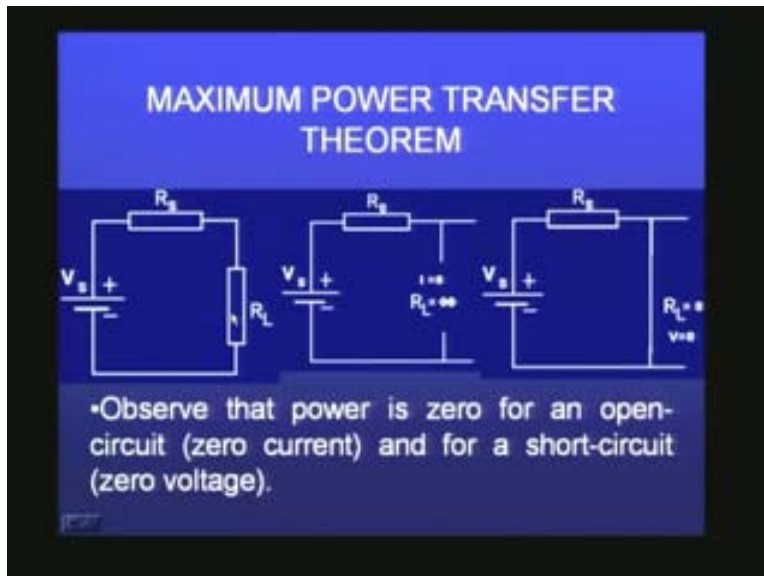
$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 - V_2 = 0 \quad \text{and}$$

$$V_2 - (I_2 - I_1) R_2 - I_2 R_3 - V_3 = 0$$

So they are connected in this way and now you can see you have two loops. There is one loop in this direction as you can see on the screen and there is another loop here which involves V_2 , R_2 , R_3 and V_3 . Right. So if I now try to apply Kirchoff's law for this then you can see V_1 minus $I_1 R_1$ minus $(I_1 - I_2) R_2$ minus $V_2 = 0$ because I assume the current here is I_1 ; here it is I_2 ; here it is I_3 ; So $I_2 - I_1$ is current flowing through R_2 minus V_2 . That should be equal to zero. The algebraic sum of all the voltages V_1 , voltage across R_1 , the voltage across R_2 and the voltage V_2 all together should be equal to zero. Similarly for the next loop V_2 minus $(I_2 - I_1) R_2$ minus $I_2 R_3$ minus $V_3 = 0$, the voltage source, this together the algebraic sum should be equal to zero. So by using this you can find out an unknown value of resistor or unknown value of the voltage across any of the resistor. So Kirchoff's law is very useful in solving several problems.

Let us move onto another very important theorem in electronics which is maximum power transfer theorem. What do we mean by that? I will take the small example. I have a battery here which is connected to two resistors R_S and R_L . What is this R_S ? This R_S and V_S as you can see is nothing but an actual voltage source. Any voltage source can be represented by an ideal voltage source V_S which has got no internal resistance and R_S as the series internal resistance across that. So together they form any actual voltage source. R_L is the resistance I want to connect this voltage source to. I have two resistors in the circuit; R_S which is internal to the source and R_L which is connected externally by me.

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When will the maximum power be delivered to R_L ? There are two resistors. Therefore the voltages will be shared. You know that series resistors share voltages. So these two resistors will share the voltage and power is nothing but voltage into current. So R_S into I , R_L into I will give me the voltage or I square R_L and I square R_S will give me the power across these two.

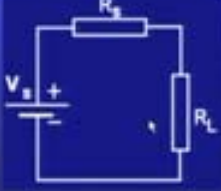
If I short this, that means when I have zero resistance this is shown in the last figure here. R_L is zero means I short this; remove this R_L connect it by a short. That means it has got zero resistance. So the power is I square R . R is zero and therefore the power becomes zero and voltage also is zero as you can see and therefore you can have a zero power when I have short and when I open that means when I just release R_L and keep the two terminals open without connecting anything. If I have that you can see because I have an open circuit this corresponds to infinite resistance. R_L now is infinity and I is also zero because I have infinite resistance the current will be zero. So here you have a case of current being zero and therefore what is the power? I square R . I is zero and therefore power is zero. So in both these cases when I have a short or when I have an open the power delivered is zero. So these are the two extreme cases. Somewhere in between if I keep increasing the value of resistance R_L from zero to some value or up to infinity how will the power be changing? That is what we are looking at. When will the power be maximum with reference to the voltage source? When can the voltage source deliver maximum power to the load resistor? It cannot be at zero value or it cannot be at infinite value; that we have already seen. So when can that provide maximum load? The theorem says that maximum power will be delivered only when R_S is equal to R_L . R_S is the internal source resistance; R_L is load resistance. So when the value of the source resistance and the load resistance match, they are equal then there is maximum power transfer. You should always remember this is different from the condition when I will have maximum voltage across R_L .

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MAXIMUM POWER TRANSFER THEOREM

Voltage Source

•When a load resistance R_L is connected to a voltage source V_s with series resistance R_s , maximum power transfer to the load occurs when R_L is equal to R_s .




You can see maximum voltage. The voltage is divided by the two resistors because series resistors divide voltages and the voltage division will be proportional to the resistance and therefore the larger the value of resistance, the larger will be voltage across R_L and therefore when I want maximum voltage, you find R_L should be maximum with reference to R_s but when I want maximum power, not voltage then you find R_L should be equal to R_s . That is what is maximum power transfer theorem says.

We can list out all the various relationship between the load resistances, voltage value across the load, current across the load, power across the load by set of equations that I have shown on the right here.

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MAXIMUM POWER TRANSFER THEOREM

•Under maximum power transfer conditions, the load resistance R_L , load voltage V_L , load current I_L and load power P_L are:



$$R_L = R_s$$

$$V_L = V_s / 2$$

$$I_L = V_L / R_L = V_s / 2R_s$$

$$P_L = V_L^2 / R_L = V_s^2 / 4R_s$$

The condition for maximum power transfer is R_L should be equal to R_S , the internal resistance; V_L the voltage will be $V_S/2$ because V_S is the total voltage; R_L and R_S are equal and therefore the total voltage should be divided equally among the two resistors. Therefore the voltage across the load will be half of the voltage source. Similarly the current across the load I_L will be V_L/R_L . We all know that and if you substitute the value of V_L by $V_S/2R$ and power is voltage by current, you will find V_L square by R_L , that is the power V square by R or it is V_S square by $4R$. So this is the maximum power. V_S square by $4R_S$ is the maximum power that will be delivered to the load R_L .

How do I get this? How do I say R_L is equal to R_S ? Is there a way to prove? I have just briefly taken a simple way of proving it. P_L the power output is voltage into current. That is power; definition of power. What is voltage in this case? V_S into R_L by R_S plus R_L . Because we have already seen R_L and R_S provide a potential divider, the voltage across R_L is total voltage multiplied by R_L divided by $R_L + R_S$. This we have already seen that is the voltage. Similarly current is voltage divided by total resistance. V_S is the voltage; total resistance is $R_S + R_L$.

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MAXIMUM POWER TRANSFER THEOREM

- $P_L = \frac{V_S R_L}{(R_S + R_L)} \cdot \frac{V_S}{(R_S + R_L)} = \frac{V_S^2 R_L}{(R_S + R_L)^2}$
- For P_L to be Maximum, $\frac{\partial P_L}{\partial R_L} = 0$; and $\frac{\partial^2 P_L}{\partial R_L^2} < 0$

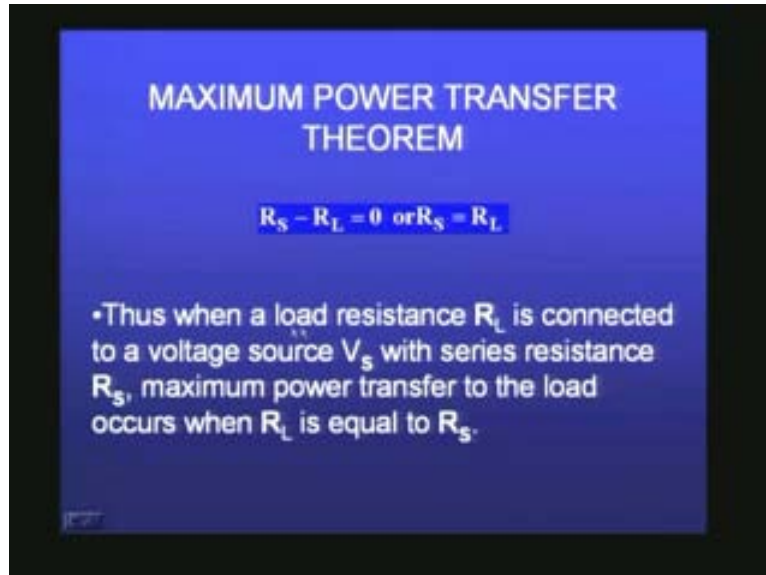
we can find $\frac{\partial P_L}{\partial R_L} = V_S^2 \left[\frac{(R_S + R_L)^2 - R_L \cdot 2(R_S + R_L)}{(R_S + R_L)^4} \right] = 0$

$$\left[\frac{(R_S + R_L) - 2R_L}{(R_S + R_L)^3} \right] = 0$$

So this is the total power. When I simplify this, it is V_S square R_L divided by $R_S + R_L$ whole square. So this is the value of P_L . When I want maximum P_L with reference to R_L , I can differentiate. All of you know calculus; dP_L by dR_L should be equal to zero if I want maximum value for this and $d^2 P_L$ by dR_L^2 should be negative if I want maximum. If this dP_L by dR_L gives me zero, gives me an extreme value either maximum or minimum and $d^2 P_L$ by dR_L^2 tells me whether it is a maximum or minimum. So we can find dP_L by dR_L for these functions from our knowledge of calculus and I have done that here. V_S square R_S plus $R_S R_L$ whole square minus R_L into two into R_S plus R_L divided by R_S plus R_L whole square by the V by U formula and ultimately when I simplify R_S plus R_L minus two R_L divided by R_S plus R_L whole cube should be equal to zero from which I can say R_S minus R_L to be equal to zero

or in other words R_S should be equal to R_L . That is what is the maximum power transfer theorem already mentioned.

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Thus when the load resistance R_L is connected to a voltage source V_S with a series resistance R_S , the source has got series resistance R_S , then the maximum power transfer to the load occurs when R_L is equal to R_S . This is very, very important concept in electronics. Later on you would find when I build an amplifier and connect it to a loud speaker, usually loud speakers are coils with magnets and a diaphragm; you all know that. It is a simple copper coil and the resistance of the copper coil will be very, very small. An amplifier is some thing which will magnify an input voltage at the output to a large value and during that process when I look from the outside of the amplifier, the amplifier can be imagined to be a voltage source with the resistance, the internal resistance that we already talked about and that resistance can in general be a very large value and therefore when I connect an amplifier directly to a loud speaker you would find there will not be any power transfer at all. That means sound output from the load to the loud speaker will be very minimal unless I make the internal resistance or the output resistance of the amplifier is equal to the very low resistance of the loud speaker I will not get any sound at all. So one has to be very careful while connecting loud speaker which is very low resistance load to an amplifier which might have a large output resistance and there the concept of maximum power transfer comes directly and therefore it is a very important theorem.


Maximum power transfer can also look at in terms of current sources. I took only the voltages sources; but you also look at in terms of current sources. I have shown a circuit here. I have a current source and current source has got an internal resistance which is called G_S . It is represented here in conductance. G_S is source conductance. Source conductance is one by source resistance. You know that conductance is one by resistance. It is nothing but $1/R_S$ and the load resistance is also represented by a load conductance G_L

which is nothing but $1/R_L$ and therefore for maximum power transfer R_L should be equal to R_S , we have already seen, which in this case corresponds to G_L is equal to G_S . $1/R_L$ should also be equal to $1/R_L$ or G_L is equal to G_S . Similarly I_L is equal to $I_S/2$.

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MAXIMUM POWER TRANSFER THEOREM

•Under maximum power transfer conditions, the load conductance G_L , load current I_L , load voltage V_L and load power P_L are:


$$\begin{aligned}G_L &= G_S \\I_L &= I_S / 2 \\V_L &= I_L / G_L \\&= I_S / 2G_S \\P_L &= I_L^2 / G_L \\&= I_S^2 / 4G_S\end{aligned}$$

Here the current is dividing because G_L and G_S are in parallel; you know that parallel resistors divide current; series resistors divide voltage. Parallel resistors divide current here and because they are equal, the two resistors divide current equally and therefore I_L becomes $I_S/2$, the source current divided by two and the power P_L is I_L square by G_L or in terms of G_S , I_S square by $4G_S$. This is the law corresponding to constant current sources. Current sources when you employ and look at the maximum power transfer here. There is also another case where instead of DC you can think in terms of AC circuits. When you have AC circuit you will not only have simple resistors you will also have reactances or impedances like for example capacitive reactance or inductive reactance. So in general you can call them as load impedance Z_L .

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MAXIMUM POWER TRANSFER THEOREM

Complex Impedances

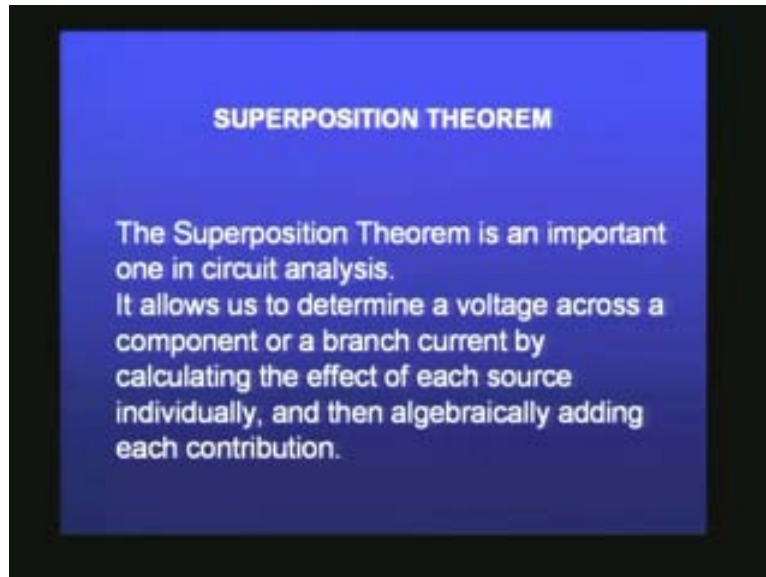
When a load impedance Z_L is connected to an alternating voltage source V_S with series impedance Z_S , maximum power transfer to the load occurs when Z_L is equal to Z_S^* (the complex conjugate of Z_S) such that R_L and R_S are equal and X_L and X_S are equal in magnitude but of opposite sign.

When a load impedance Z_L is connected to an alternating voltage source, AC source, V_S with series impedance Z_S in that case we also call the internal resistance as internal impedance. Right. That is Z_S . So maximum power transfer to the load occurs when Z_L is equal to Z_S^* . What is the star? The star is a complex conjugate of Z_L . Normally you will represent Z_L as $R+J$ times X_L or X_C or whatever depending upon whether inducting load or capacity load. Therefore in general it can be a complex. That means the phase can be different even though the magnitude can be same. So you would find Z_L and Z_S should obey the relationship that Z_L should be equal to the complex conjugate of Z_S which is the internal impedance. This is what the maximum power transfer says with reference to impedances in alternating current sources. Actually if you look at is R_L+X_L or X_S then the real parts R_L and R_S of the two complex quantities Z_L and Z_S and R_L should be equal to R_S and X_L and X_S in which X_L is the reactance of the load, X_S is the reactance of the source. These two should be equal but opposite in direction because the complex conjugate will be opposite in sign. That is what the maximum power transfer says with reference to impedances.

Let us move on to another theorem which is again very important in solving network problems and that is superposition theorem. Superposition is a very important concept in electronics as well as in several other fields of physics. For example superposition of waves you have heard. What do you mean by superposition of waves? If in a medium there are several waves the resultant amplitude due to all the waves present at any given point in space is nothing but the sum of the individual amplitude of each of the waves. We have also seen similar situation with reference to mixture of gases. For example Dalton's theory of partial pressure that tells you that if I want to measure the total pressure due to a mixture of gases then all that I have to do is I should fill the volume with each of the gas independently and find out what is the pressure at that point due to each of the gas independently and ultimately I will add all these values together to obtain the total pressure due to the presence of all the gases simultaneously in the volume. So superposition theorem is very useful in several fields, even in quantum mechanics in physics superposition is a very important concept.

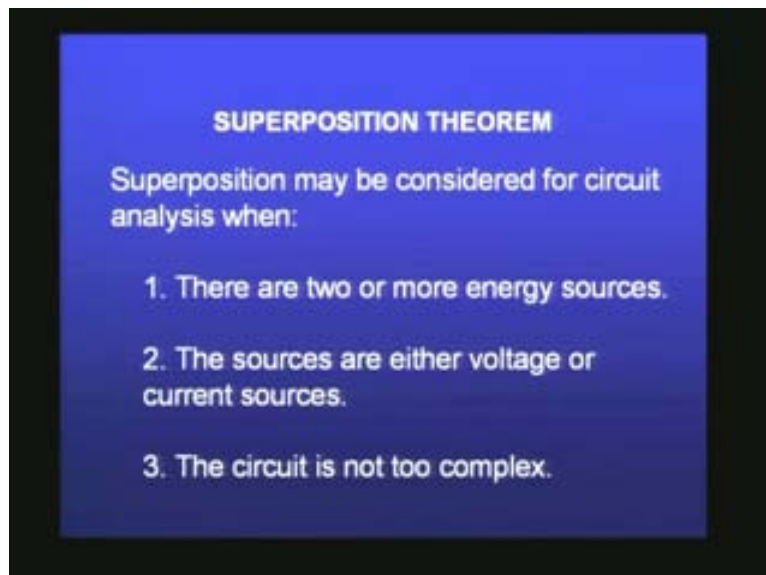
So let us look at the superposition theorem with reference to electricity and electronics and superposition shows in what way it helps. It helps in determining the various components of voltages and currents in different segments of a circuit.

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So it is very important and very useful. Now how do I do that? In any given circuit you can have more than one voltage source. You can have two or three voltage sources, you have several resistors.

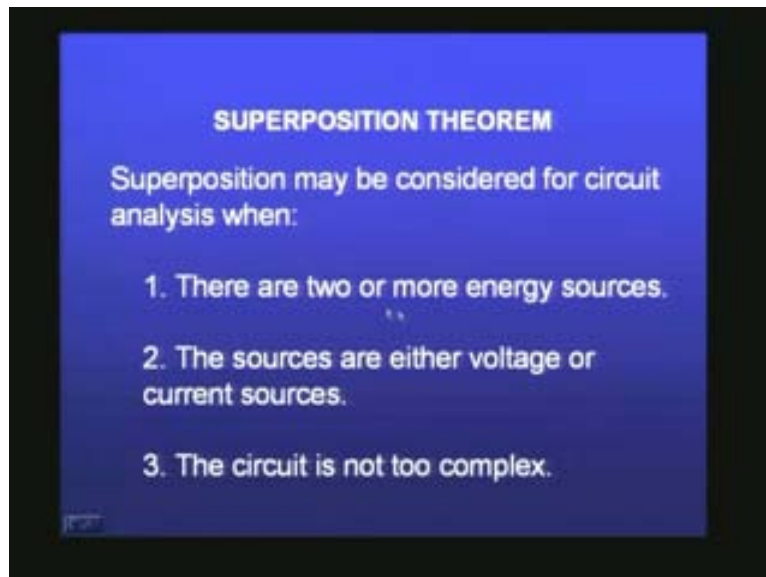
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Let us confine ourselves to simple cases of resistors and voltage sources. How do I find out a current through any one resistor due to the presence of several voltage sources?

Then the superposition theorem tells you just as in the other case assume that all the other voltage sources are not connected in the circuit. Assume that only one of the voltage sources is connected. Now calculate the current through the resistors due to only one of the voltage sources. Then remove that voltage source, introduce the second voltage source. Find out what is the current through the same load due to the second voltage source. Then remove the second voltage source, introduce the third voltage source, find out what is the current? So finally you cover all the voltage sources in the circuit one by one like this and obtain the current due to each one of the voltage source and sum them all together at the end. Algebraically you sum. Some may provide current in one direction, some may provide current in other directions depending upon magnitude of the voltage and sign of the voltage source how it is connected in the circuit. So in general these total current if I add that is the current I will get when all the voltage sources are present simultaneously in the circuit. This is what superposition theorem says. It is very simple but we can understand. If you really look at it in solving problems it is very, very useful once it is put as a theorem it becomes very convenient for us to apply the theorem and solve very complicated situations in electrical and electronics circuit. So I have listed many of the important points that we should remember in the superposition theorem.

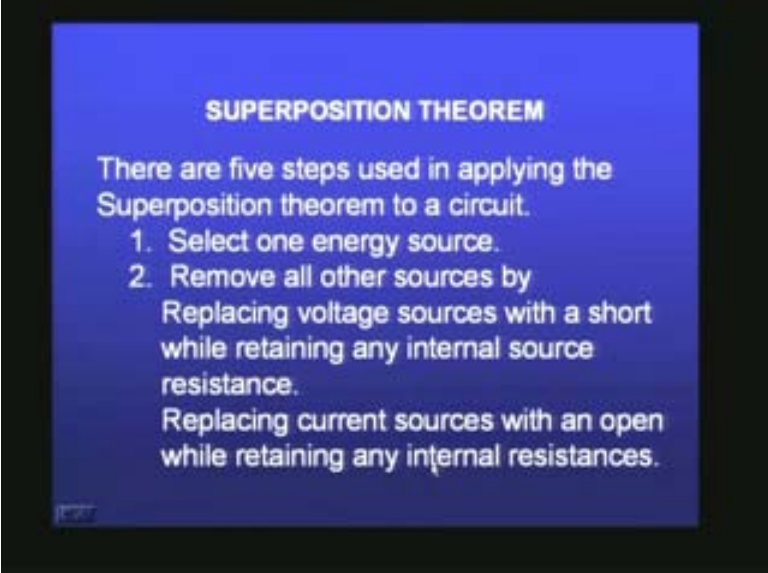
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There are two or more energy sources you can have. The sources are either voltage or current. You can also have current source. The circuit is not too complex; if it is too complex it becomes very difficult; may be in a simple case we can easily do that. There are five steps used in applying superposition theorem. You should remember that. That is select one energy source only at a given time. Second remove all the other voltage sources or current sources. How do you remove? You must always remember an actual voltage source is also having a series resistance and actual current source also has a series resistance. Therefore when you remove the voltage source, if it is a voltage source you should replace the voltage source by a short. If you remove the current source you should replace it by an open circuit. You should keep the circuit open at the place where the

current source comes and so replace voltage sources with a short while retaining any internal source resistance of the voltage sources. Replace current sources with an open while retaining any internal resistance that you may have.

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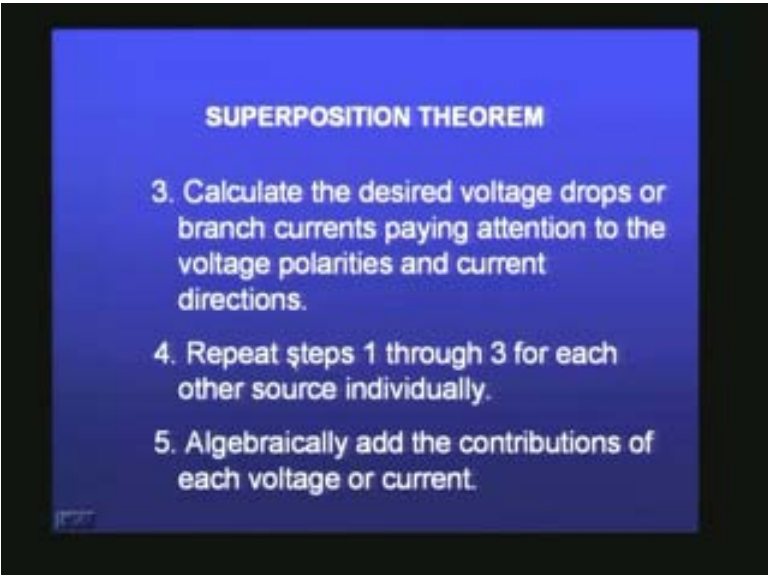
SUPERPOSITION THEOREM

There are five steps used in applying the Superposition theorem to a circuit.

1. Select one energy source.
2. Remove all other sources by
Replacing voltage sources with a short while retaining any internal source resistance.
Replacing current sources with an open while retaining any internal resistances.

Then calculate the desired voltage drops or branch currents paying attention to the voltage polarities; how the voltage polarity is connected and then apply the Kirchoff's law and simplify the circuit if it is a parallel series combination and obtain the voltage. Do that first for one voltage source.

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SUPERPOSITION THEOREM

3. Calculate the desired voltage drops or branch currents paying attention to the voltage polarities and current directions.
4. Repeat steps 1 through 3 for each other source individually.
5. Algebraically add the contributions of each voltage or current.

Repeat the same thing for the second voltage source, third voltage source as many as you have and then finally add all the current or voltages due to all these voltage sources in any given branch and that will give you the result.

I have now taken an example. On the screen you can see two voltage sources V_1 and V_2 and again for simplicity I have taken three resistors R_1 , R_2 and R_3 . Now what do we do? The principle is very simple. Superposition theorem says select only one voltage source. I have selected the voltage V_1 and replaced the second voltage source by a short. Therefore I have just connected a straight line here where the voltage source V_2 was there and this is now a first circuit.

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SUPERPOSITION THEOREM

For the circuit above

1. Select one energy source. V_1
2. Remove all other sources V_2 :

by replacing voltage sources with a short while retaining any internal source resistance.

• Replacing current sources with an open while retaining any internal resistances.



I want to find out what is the voltage developed across R_2 . That is what we want to do. There is no current source here; so there is no problem. I will find out, due to this V_1 , the voltage across R_2 . I will keep it safe and then I will remove. Next time I will remove the voltage source V_1 , introduce V_2 and put a short in place of V_1 and find out the voltage across R_2 and I will add these two voltages to obtain the total voltage.

I quickly take an example I have V_1 is equal to 6V, V_2 is equal to 12V here and the resistors R_1 , R_2 , R_3 . R_1 is 100 ohms, R_2 270, R_3 is 220 and we want to find out what is the voltage across R_2 .

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SUPERPOSITION THEOREM

Let us consider an example with the following data: $V_1 = 6V$; $V_2 = 12V$;
 $R_1 = 100\Omega$; $R_2 = 270\Omega$;
 $R_3 = 220\Omega$.

Find voltage across R_2 .

We shall use Superposition Theorem and solve it.

We shall use superposition theorem as an example. Now what we do? First we remove V_2 , connect a short in that place. V_1 is 6V. I have 100 ohms here; 270, 220. If you look at the circuit you find this is nothing but R_3 actually is coming in parallel to R_2 . Therefore the total resistance is nothing but the parallel value of R_2 and R_3 . So let us find out what is R_{23} , which is parallel value of R_2 , R_3 and that is shown here. R_2 into R_3 divided by $R_2 + R_3$. We know the formula and that is around 120 ohms when you do the calculations.

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SUPERPOSITION THEOREM

$$R_{23} = \frac{(R_2)(R_3)}{R_2 + R_3}$$

$$R_{23} = \frac{(270)(220)}{270+220}$$

$$R_{23} = 121 \text{ Ohms}$$

So these two R_2 and R_3 can be replaced by a single resistance, 120 ohms.

Once I know there is only 120 ohms here, 121 ohms here as R_{23} , I have only two resistors with one power supply and I want the voltage across this which is nothing but simple potential divider arrangement.

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SUPERPOSITION THEOREM

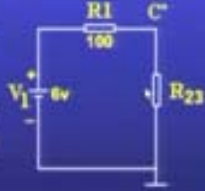
- we now have the circuit on the right We will now solve for the VC' by voltage divider theorem

$$VC' = V_1 \cdot \frac{(R_{23})}{R_{23} + R_1}$$

$$VC' = 6V \cdot \frac{121.23}{121.23 + 100}$$

$$VC' = 6V \cdot (0.548)$$

$VC' = 3.3V$



So what is VC_1 in this case? It is nothing but V_1 divided by the resistors combinations R_{23} divided by $R_{23}+R_1$. That is the potential divider arrangement and when I do that and find out the value it is coming to be around 3.3V. So 3.3 V is the voltage when I use only one voltage source. Now I go to the other side and I keep V_2 and short V_1 here. I have again a similar situation R_1 and R_2 are coming in parallel because there is nothing else here and what is the parallel value of R_1 and R_2 ?


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SUPERPOSITION THEOREM

$$R_{12} = \frac{(R_1) (R_2)}{R_1 + R_2}$$

$$R_{12} = \frac{(100) (270)}{100+270}$$

$R_{12} = 73 \text{ Ohms}$



R_{12} , I call that, is nothing but 73 ohms by a similar calculation. So now I have a simplified circuit for V_2 which is 220 ohms, R_3 as a series resistor and R_{12} is 73 ohms. Now I find the potential divider of these and what is the voltage across R_{12} , I calculate that here and you find it is coming out to be 3V.


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SUPERPOSITION THEOREM

- we now have the circuit on the right We will now solve for the V_C'' by voltage divider theorem

$$V_C'' = V_2 \cdot \frac{(R_{12})''}{(R_{12})'' + R_3}$$

$$V_C'' = 12V \cdot \frac{72.98}{72.98 + 220}$$

$$V_C'' = 12V \cdot (0.25) = 3V$$


So the two voltages are one is 3.3V or 3.288V, close to 3.3, the other is 3V. So all that I have to do is I have to add these two here. By Superposition theorem that comes out to be 6.288 or approximately 6.3 V. So you can see the values calculated.

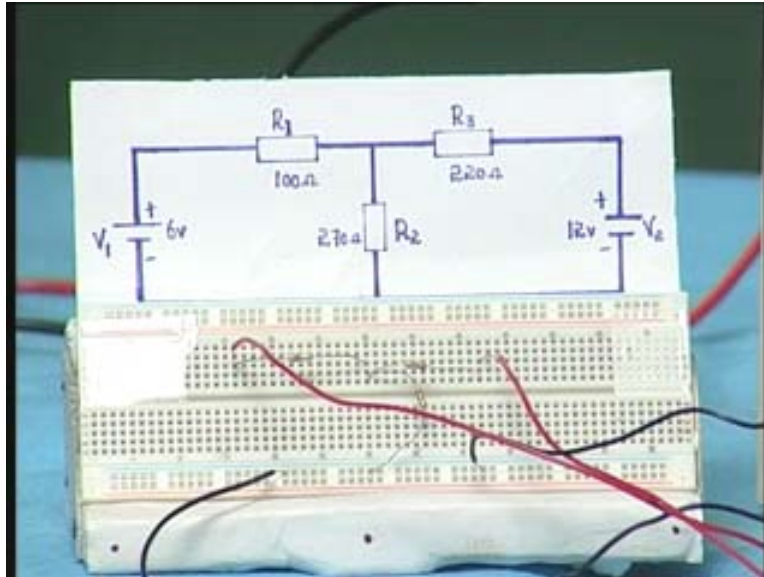
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EXPERIMENT

- Values Calculated: $V_C = 6.3V$
- With V_1 alone: $R_{23} = 121, Ohms$
 $V_C = 3.3V$
- With V_2 Alone: $R_{12} = 73 Ohms$
 $V_C = 3.0 V$

VC is 6.3V by superposition theorem. If I take V_1 alone I have 121 ohms and 3.3V. When I take V_2 alone it is 73 ohms and 3V. Therefore now let us quickly go and do the experiments and find out whether we get these values. I have actually wired a similar circuit on the table. Right. So here you see I have 6V. You can see that is drawn from the power supply and I have another voltage here that is drawn from the last power supply which is capable of going from 12V to 15V and the 12V is set there; 6V set there and I have three resistors 100 ohms R_1 , 270 ohms R_2 and 220 ohms R_3 .

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Let me switch on. Now I must measure the voltages. Here it is showing 12V. I should keep it to 6V. Let me measure the voltage using the multimeter. So I have connected between zero and plus and now let me set this to 6V that is about 5.9 V and let me measure the voltage on this side using the multimeter. I will connect it voltage range and measure DC volts. Now you can see 12V, 12.08.

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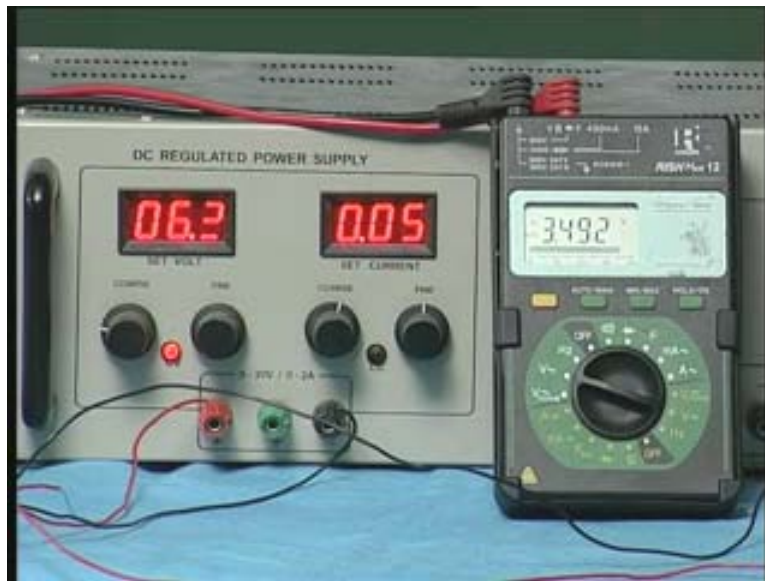
So this voltage source is 12V on this side; this voltage source that is here is 6V. Now I want to measure what is the voltage across R_2 . What we will do is directly we will measure the voltage. So I will connect the voltmeter across R_2 and the other end of the voltmeter I connect to the ground. So what is the voltage I get? I get 6.28. You remember we calculated 6.288. Now it is 6.24 or something like that here. So the value is reasonably close. Why is it different? It is not 6.28 because these resistors have a tolerance band. By now you know all resistors have a tolerance. That means what is 100 ohms need not exactly be 100 ohms. What is 270 or 220 need not exactly be 220. We calculated 6.288 with all the value exactly there. But now it can be different because these are all preferred values and therefore you find it is around 6.23. We can actually try and check. I now remove the 6V. I connect it with the short and I measure the voltage. Now the voltage is 3.02.

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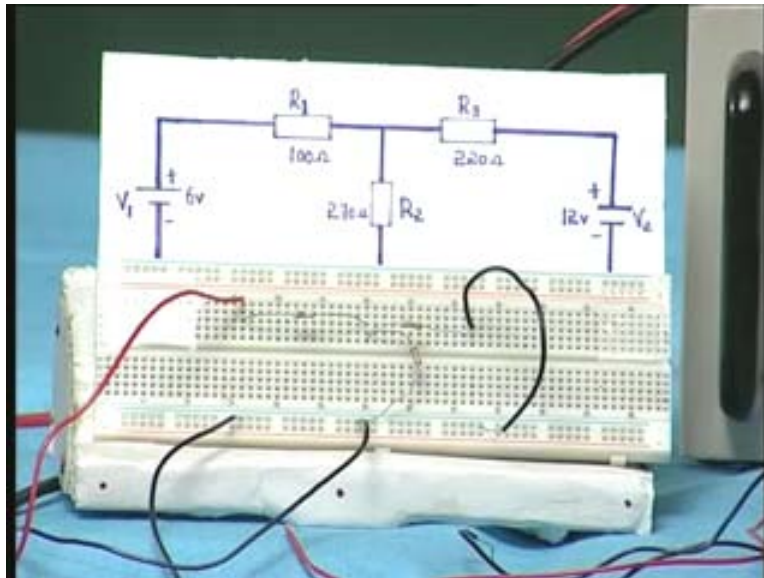
That we have already seen. When I have a 6V shorted and only 12V is there it is 3V. Now what I will do is I will connect 6V again and remove the 12V and replace it with a short. I will measure the voltage here is around 2.4 or something. Right. It has come down is it? Yes; Six volts. Now it is about 3.5. So 3.5 and 3 that gives six point some thing.

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So that is the voltage we got when both the voltage sources were connected together. So superposition says that when you have only one voltage source find out what is the voltage across R_2 and remove this and connect the other voltage source; here you connect it by a short and measure the voltage. Then you add them together you will get the total voltage which you will get when both the voltage sources are available. This has been very easily shown. We can actually also measure resistances if you want. For example after connecting you remember the total resistance I can now put it in resistance mode and measure the resistance. I will switch off the power supply. Now I want to measure the resistance after shorting on this side. I want to measure the resistance across this point. This is total resistance. This is nothing but 100 ohms plus a parallel value of 220 and 270. I short here. This is 100 ohms plus 220, 270, etc gives me around 219 or 220 ohms. That is the parallel. Similarly I can measure the effective resistance from this side also and then I can compare with the calculations that I have done already.

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So just I have briefly shown you how superposition theorem can be verified by having two power supplies and two resistors and we have seen that it exactly follows the idea that we already look at. Thank you. We will go back.

You can see on the screen the total voltage was around 6.3V calculated. We also got around 6.2 and 121 ohms. I just now measured 120 ohms; 119 ohms for this and we also measured this is 3.2 and this is 3 and totally that gave 6.2.

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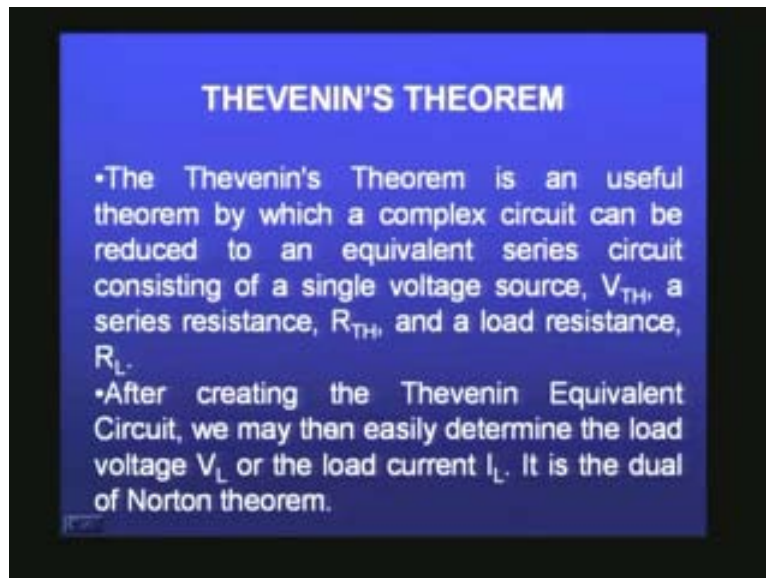
EXPERIMENT

- Values Calculated: $V_C = 6.3V$
- With V_1 alone: $R_{23} = 121 \text{ Ohms}$
 $V_C = 3.3V$
- With V_2 Alone: $R_{12} = 73 \text{ Ohms}$
 $V_C = 3.0 V$

I have not measured this resistance; but in principle we can measure this resistance also in a similar fashion.

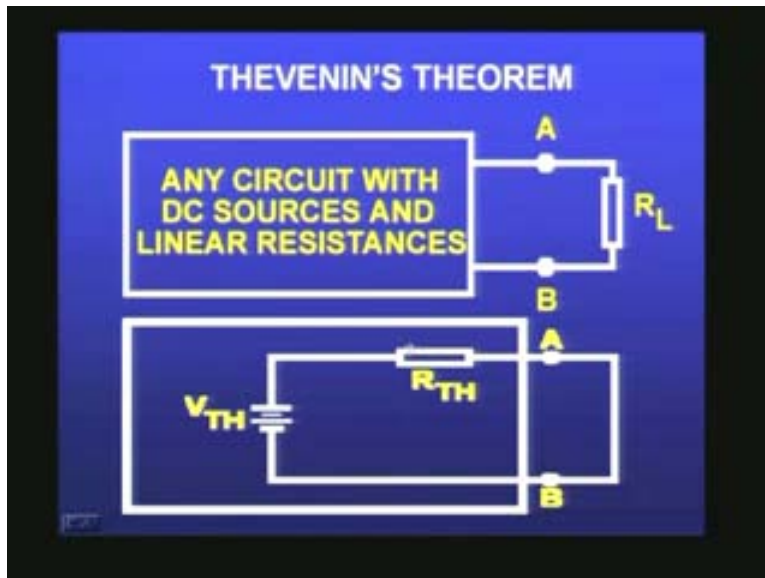
Now let me move on to a more important theorem which is called Thevenin's theorem. Thevenin's theorem is again a useful theorem in network applications and what it says is if I have very complicated circuit that can be simplified by replacing the complicated circuit with just two components. The entire circuit which contains several voltage sources, several resistors can be replaced simply by two components. One is Thevenin's voltage source and the other is Thevenin's resistance. So any complicated circuit can be simplified into two components one voltage source, one series resistance or Thevenin's voltage source, Thevenin's resistance as it is called. There is also another similar theorem which is called Norton's theorem which is the dual of Thevenin's theorem.

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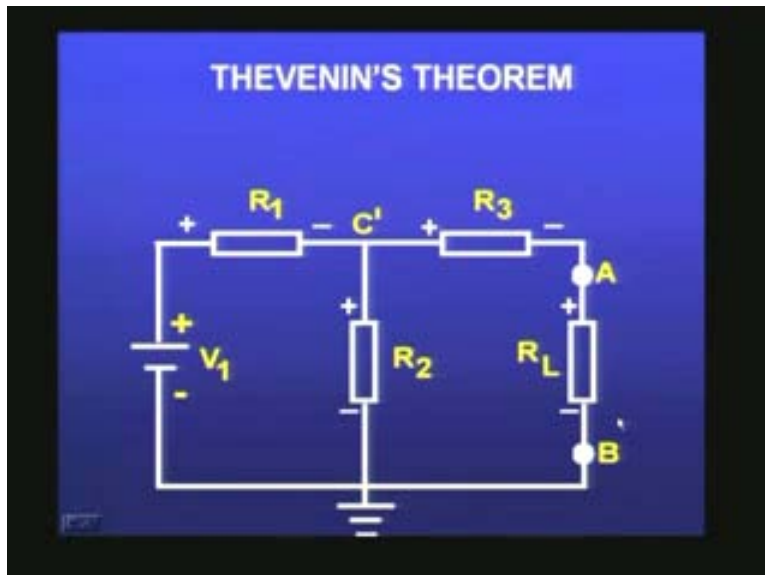
Thevenin's theorem talks about voltage sources; Norton theorem talks about current sources. Let me just briefly explain to you and perhaps we will try some experiments of Thevenin's and Norton in the next lecture. Right now I will just give you principle of the Thevenin's circuit. Here you have on the screen any circuit with several DC sources and linear resistances connected to a load. What Thevenin says is you can now replace entire circuit here by just an equivalent circuit which contains only one voltage source which is effective Thevenin's voltage source and one resistance in series which is effective Thevenin's resistance.

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The whole complicated circuit can be easily simplified; I can put my R_L and the whole circuit becomes very simple.

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So that is what Thevenin's theorem says. Let me quickly go to Norton's theorem. Norton's theorem is the dual of Thevenin's theorem. It again helps us to simplify very complicated network with one current source which is called the Norton's current source and one conductance which is called the Norton's equivalent conductance.

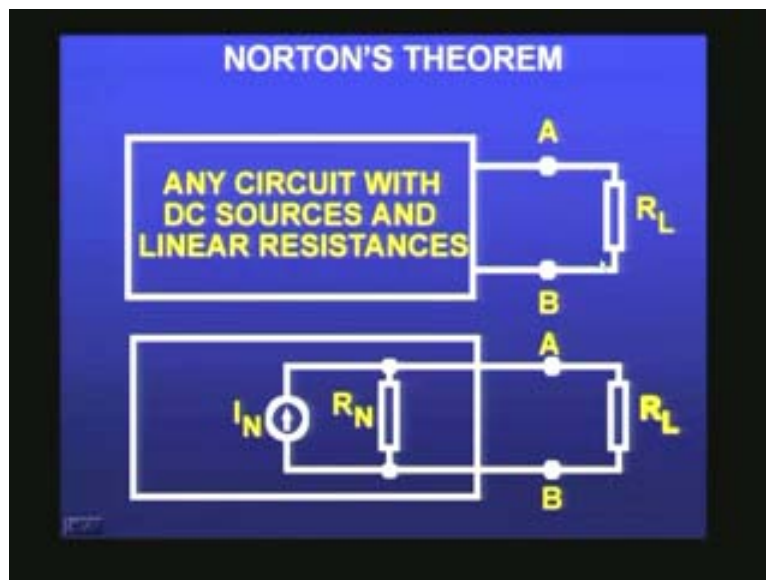
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NORTON'S THEOREM

Norton's Theorem is the dual of Thevenin theorem. It simplifies a complex network into a current source called the Norton Short Circuit Current (I_N), a parallel Norton Equivalent Conductance (G_N) or Norton Equivalent Resistance (R_N), and a parallel load resistance. After creating the Norton Equivalent Circuit, we may then easily determine the load current I_L .

For example any circuit with large number of sources and currents can be replaced by one current source I_N which is Norton's current source and one R_N or G_N which is Norton's conductance or Norton's resistance connected to the R_L , the load resistance.

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So this is Norton's theorem and the other one is Thevenin's theorem. They help us to solve very complicated network into very simple scheme and then try to obtain the voltages and currents, the equivalent circuits of any arms; if you want to find out what is the voltage or what is the resistance you can use Thevenin's theorem and Norton's theorem; solve it, simplify it and then try to obtain the actual values of voltage and current. What we have done in this lecture is that we have looked at the basics of the current source and then we looked at the basic ideas of Kirchoff's law, review of Kirchoff's law, Kirchoff's voltage law and Kirchoff's current law and then we also

looked at maximum power transfer law. The maximum power is transferred to the load when the internal resistance is equal to the load resistance. We also saw different conditions corresponding to the impedance and then we looked at superposition theorem which helps us to solve very complicated circuit and then obtain the current or voltage in any branch by trying one by one the different voltage sources and current sources; find out the effective voltage and then sum them all together. That gives me the total voltage or current in the branch due to the presence of all the voltage sources that corresponds to superposition theorem which helps us in our several circuits. Then we briefly looked at the statement of Thevenin's theorem and Norton's theorem which helps us to simplify very complicated circuit into very simple voltage source and series resistance or a current source and a conductance, Norton's conductance. So we will see some applications of Thevenin's theorem and we will also see an actual experiment where Thevenin's theorem will be obtained; that means Thevenin's equivalent circuit will be obtained for a complicated network and then we will also see the applications of Norton's theorem and Thevenin's theorem next time. Thank you very much.