Basic Electronics
Learning by doing
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Lecture – 26
Mathematical operations

Hello everybody! In our series of lectures on basic electronics learning by doing let us move on to the next lecture. Before we do that let us quickly recapitulate what we learnt in some of the previous lectures.

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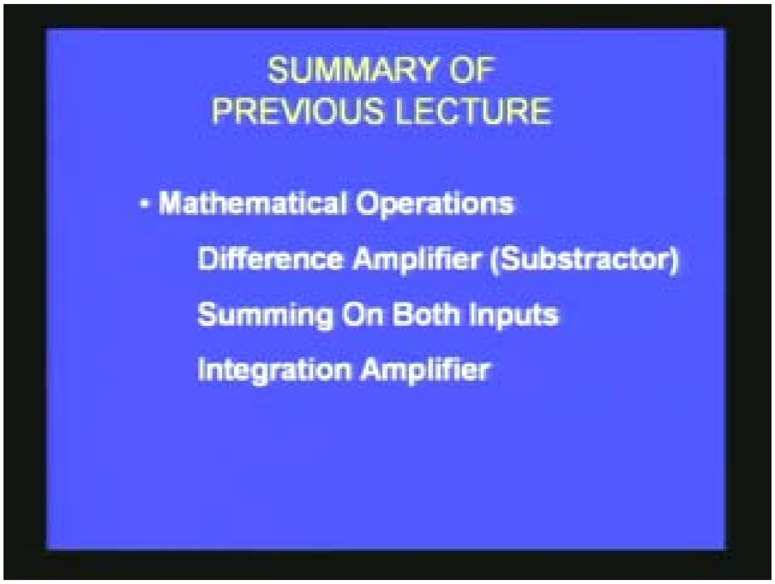

You might recall we were discussing about the various mathematical operations that can be performed using an operational amplifier. There are several applications like multiplication by a constant, summing amplifier, difference amplifier, integration amplifier, etc. In the last lecture you might recall we discussed about the difference amplifier and also how we can sum on both inputs the inverting and the non-inverting inputs and we also tried some simple problems as an illustration and then we discussed in detail about an integrating amplifier using operational amplifier. Now in this lecture we will try to look at the next mathematical operation namely differential amplifier. I want to show you couple of slides which we have seen already in some of the previous lectures about the various mathematical operations that can be performed using an operational amplifier.

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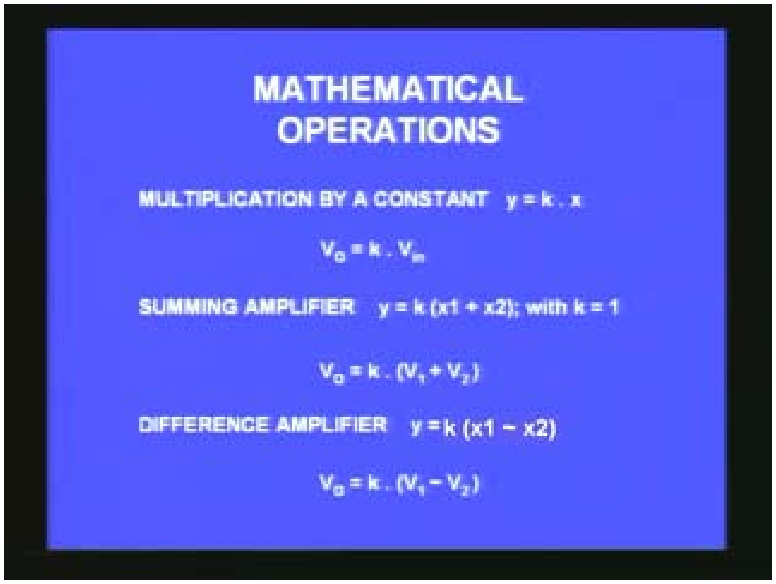

For example on the screen you can see multiplication by a constant where the output is some constant multiplication of the input. Then a summing amplifier where the output is the sum of the two inputs with a constant factor if you want and then difference amplifier where the output voltage is proportional to the difference in the two input voltages. Then we also saw integration being performed where the output voltage is a time integral of the input voltage V_{in} dt. V output is equal to k times integral V_{in} dt.

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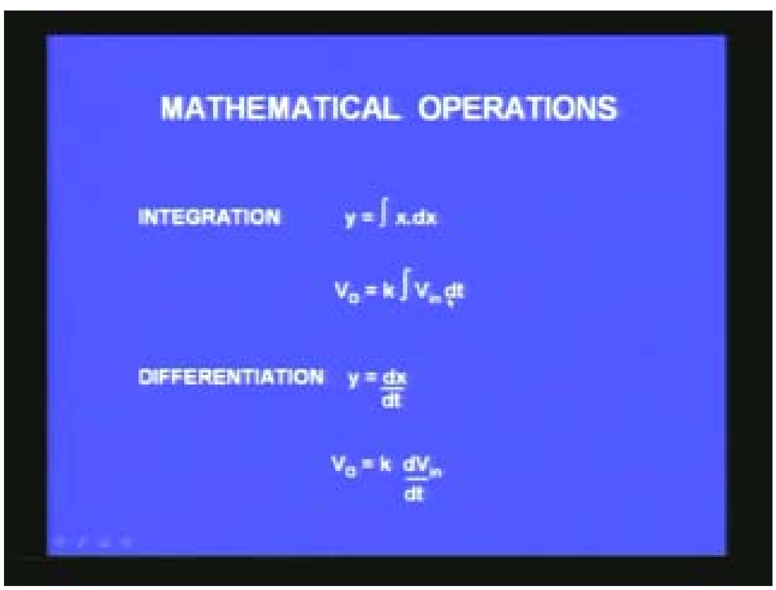

We saw some examples of these. We also saw a demonstration. Today we will discuss on the differentiation circuit where output voltage is going to be some constant multiplied by the time derivative of the input voltage dV_{in} by dt. V output is equal to k times dV_{in} by dt. How do we do that?

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To understand that you can recall some of the earlier circuits that we already saw. One of them is just a simple network with one capacitor and a resistor in series connected to a signal source V_{in}.

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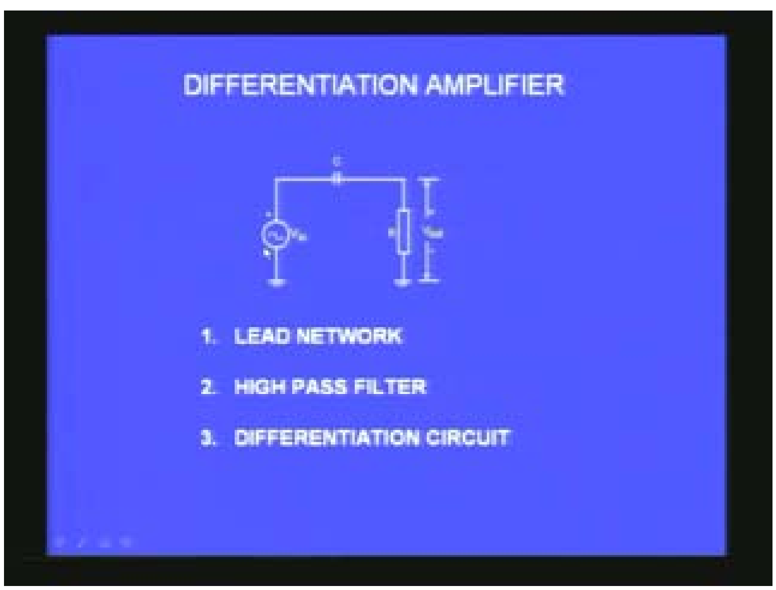

This we call by different names earlier. For example this was called as a lead network when we discussed about the frequency response of rc coupled amplifiers. We discussed in detail about the behavior of the simple network which involves one capacitor and one resistor in series and you take the output across the resistor. When you do that it can also be considered as an high pass filter because when the frequencies are very large the reactance offered by the capacitor which is inversely proportional to the frequency will be very, very small and almost all the voltage will go through the capacitor which will become almost become a short and what you get will be a full output. But when the frequency is very low, very close to dc then the reactance offered by the capacitance will be very large and very small amount of voltage only will be able to pass through and it passes high frequency component and it does not pass low frequency component close to the dc. That is why this is called a high pass filter. High frequencies will be passed through this filter. Incidentally this can also be called a differentiation circuit. That is reasonably simple to understand. I will try to explain to you how you can do that when we actually go to the circuit.

A very simple differentiated circuit I have shown here using an op amp where again the same network which I discussed a moment ago is being used. You have a capacitor and a resistor.

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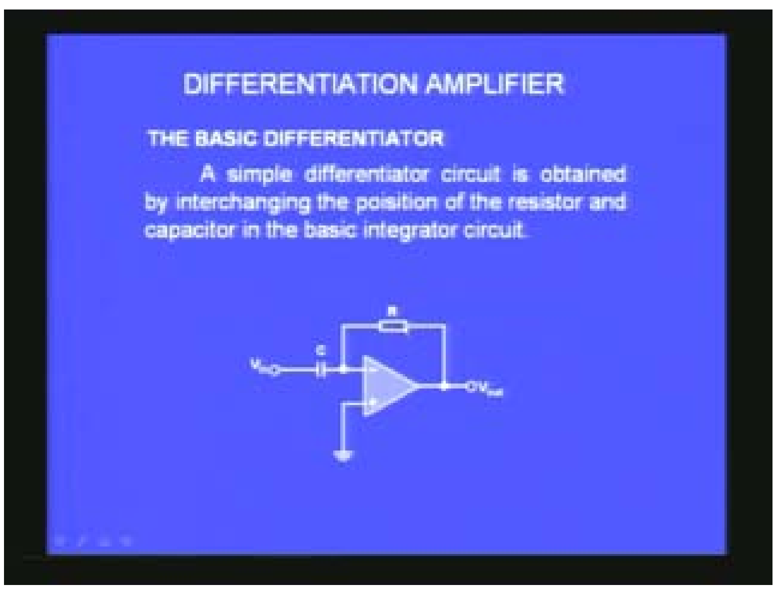

You know in this circuit this input is actually connected to ground, the non-inverting input and this becomes a virtual earth. The minus side this point becomes a virtual earth and whatever voltage I apply is actually between this and this point and that means it is only across a capacitor and similarly here if I measure the output voltage I always measure the output voltage between this terminal and the ground and now because this point is equivalent being a virtual earth to ground what I actually measure is the voltage between the output terminal and this point which is nothing but the voltage drop across the resistor which is what I am going to measure as an output voltage and the input voltage is applied across the capacitor. What is the consequence of that? I assume an input signal, a sine wave y is equal to A sin omega t where A is the amplitude and omega is the angular frequency. A sin omega t is a standard sinusoidal voltage that we have. If I differentiate this dy by dt will become omega A cos omega t.

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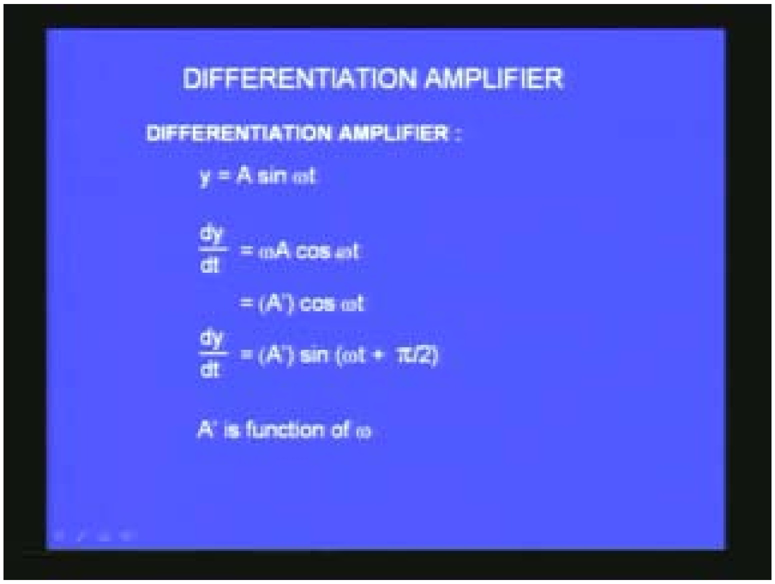

Sin will become cos on differentiation and this omega A is new amplitude of this cosine function and I call it by a new variable ….. A prime. A prime cos omega t is now the differential of the input voltage. dy by dt is A prime cos omega t which can also be written as A prime sin omega t plus pie by two where a phase factor is introduced because the sin has become cos it has to have a 90 degree phase I rewrite this equation as dy by dt is equal to A prime sign omega t plus pie by two. This if you compare with the y is equal to A sin omega t the input signal they are of the same type. This is also a sin wave this is also the sin wave expect for a phase factor and the A has now become A prime. The amplitude has become A prime. But it has got some very specific consequence and that is this A prime is nothing but omega multiplied by A where omega is angular frequency, two pie f. Omega is a angular frequency and it is equivalent to two pie f where f is the frequency of the input. The amplitude is going to be a function of frequency. This has got some very important consequences. What is that?

The value of A prime is no more going to be a constant. If I send larger frequencies, high frequencies 100 kilo hertz, 200 kilo hertz sine wave the amplitude for that wave will be very large. If I send smaller frequencies 100 hertz, etc the amplitude A prime can be very, very small. If I have a specific signal may be a square wave or even a sine wave always there will be some noise voltages which will be coming into the game at the input. Along with the signal there will be some noise added to that due to the extraneous reasons and these noises can have different frequencies. We have no control over them and this spectrum of the noise can involve very high frequencies as well as low frequencies.

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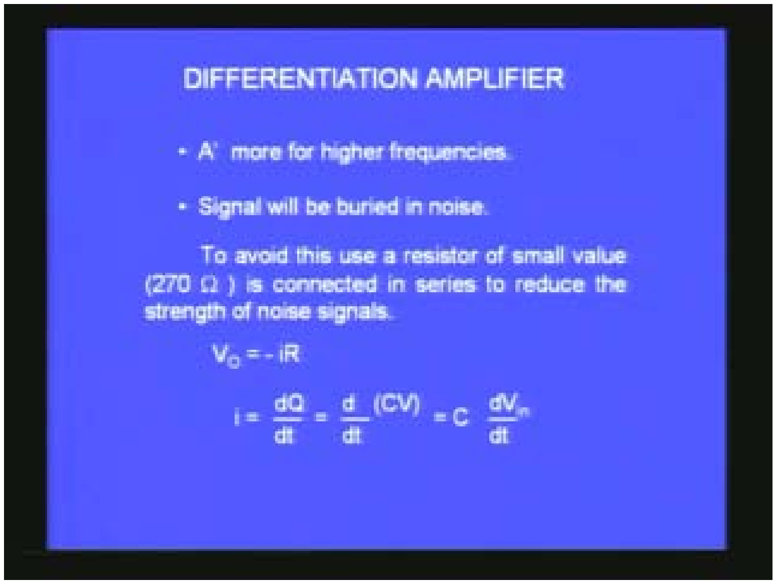

If the frequency is higher that is preferentially amplified by my differentiation for the circuit and in the output more of the noise signal will be amplified than the signal and ultimately it may happen that the entire signal will be completely buried in the noise and when you use a differentiation circuit you have to be very, very careful with reference to noise signal. How you are going to avoid this? I will explain to you in a moment. Before that I wanted to tell you the output voltage is going to be -i times R which is very simple to understand because you can see that whatever is the current that is going to flow in as soon as you apply the V_{in} this current almost all of them will flow through R because very small current only flows through the op amp. This we have seen number of times earlier and the input bias current of the amplifier is going to be very, very small. I will explain in more detail about what the input bias current is all about later on. But you can for the moment imagine that the current actually passing through the amplifier is going to be very, very small because of the large input impedance associated with the amplifier and almost all the current which is flowing due to the ac signal applied through the capacitor will only travel through R the feedback resistor and V output is nothing but i into R where i is the current flowing through the R and this voltage is going to be negative because the current is flowing toward the output from the ground and this voltage is going to be negative and that is why I say the output voltage V output is equal to minus i in to R where i is the current flowing through the system.

We also know from fundamentals i is the rate of flow of charges. i is equal to dQ by dt the rate of charge flowing and that is equal to d by dt of CV because the Q is associated with the capacitor at the input and the relationship between Q and the capacitance is Q is equal to CV. Differential of CV divided by dt in this C is a constant V which is the input voltage is going to vary and I can take this C out. This equation i becomes C into d V_{in} by dt. This is the current. Now we combine it with the output voltage. The V output will now become i into R that is - RC or CR into dV_{in} by dt.

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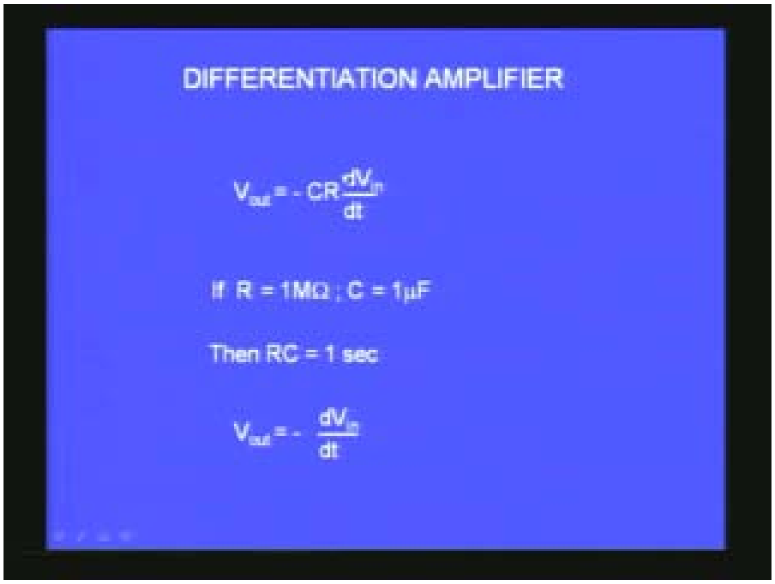

This shows very clearly that the output voltage is the time derivative of the input voltage multiplied by a constant which comes into the game because of the value of the capacitance and the resistance that I have used in the circuit. If I choose R is equal to 1 meg ohm for example and C is equal to 1 micro farad the product RC will cancel out. It becomes 1 and RC becomes 1 second. RC is the time constant. We normally measure it in seconds. RC will be 1 and the V output will become - dV_{in} by dt This becomes a simple differentiating circuit.

Now I will explain to you about the circuit. This is the basic circuit which involves one capacitance and one resistor. But that is slightly modified here for a very specific range of frequencies in the range of 1 kilo hertz and I have used a capacitor whose value is 0.01 microfarad and a feedback resistor which is 10K and I have also used a resistor here which is a very, very small one 270 ohms. I have used 270 ohms in addition in series with the capacitor. Then there is a 10K resistor in the feedback and I have also used a 10K resistor for the other signal for connecting to ground. This 10K I have added for balancing the input bias current differences between the two input about which I will discuss in more detail later on.

This is my differentiating circuit. Why I have added this 270 ohms at the input is because in a differentiating circuit the output gain is going to be a function of frequency. Higher frequency components from other noise sources can be preferentially amplified and the output will become buried in noise. To avoid that I introduce a 270 ohms so that whatever noise that is generated at the input side will be dropped across the 270 ohms. The 270 ohms will make sure that not much voltage passes beyond this point and the noise current due to the noise will be very, very small and they will have very little effect at the output. Because the value is too small, 270 ohms it will not greatly affect the input signal which will be in the order of volts. If I apply then I will get that.

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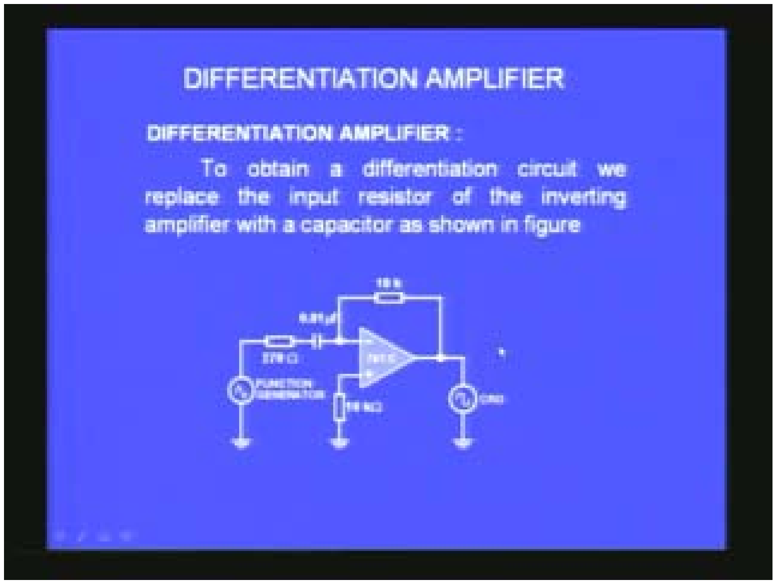

How do I check whether it is performing a differentiating operation? If I give a sine wave I have no way of knowing it because a sine wave after differentiating becomes a cosine wave with a phase difference of pie by two. I would not be able to distinguish between the input and the output with any characteristic features and I would rather go in for the well known fact that we have already learnt in the integrating circuit that when I apply a square wave to an integrating circuit I get a triangular wave. We all know differentiation is the inverse of integration. What will happen if I apply a triangular wave at the input? I must get a square wave. For an integrator if I apply the square wave I get the triangular wave. For a differentiator if I give the triangular wave I should get a square wave as simple as that. We would try by applying a triangular wave at the input and we will try to see whether we get a square wave.

I have shown you here a picture. In the first one the input is a triangular wave and the output is indeed a square wave. Now differentiation of a ramp; a ramp is constant multiplied by time. This is the time axis. This is the voltage axis. If the voltage is continuously increasing with reference to time that means it should be …. time. There is a finite slope associated with straight line. It is a constant multiplied by time and differentiating it should become a constant. That is why it is a constant here and when it comes down again a ramp then it has to be again a constant with reference to the negative side. Therefore it is a constant here. The square wave is formed from the triangular wave. We will also see in the actual demonstration. If I give a square wave as the input what will happen?

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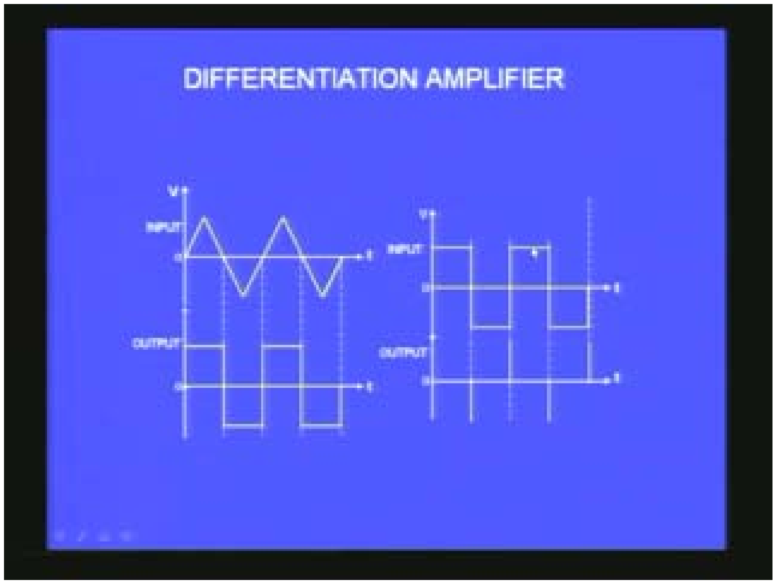

What is the differentiation of a constant? The differentiation of a constant is zero and you get a zero volts here in the output. But what is the differentiation of the vertical line? The differentiation in the VT curve is nothing but the slope of the line and the slope of this line is actually minus infinity going down to infinity and I get a straight line going up to minus infinity but due to actual practical limitations you don't get this line going up to the infinity. But it is having a finite length. You have again a constant voltage here on the negative side and that again should become zero and here it is a positive going pulse towards infinity that is why I am getting here and you can see the square wave when I apply at the input of a differentiating circuit I should get series of sharp pulses at the transition point and I should get zero volts whenever the voltage is constant +V or -V etc. These two will be the two different wave forms that I wanted to explain to you how the differentiating circuit is reacting.

The gain which is a function of frequency, the omega is equal to V_{out} by V_{in} and the V_{out} is across the raw R feedback resistor and V_{in} across Z_c. It is - j omega RC because Z_C is -1 by j omega C. It becomes -j omega C (not in slide) RC.

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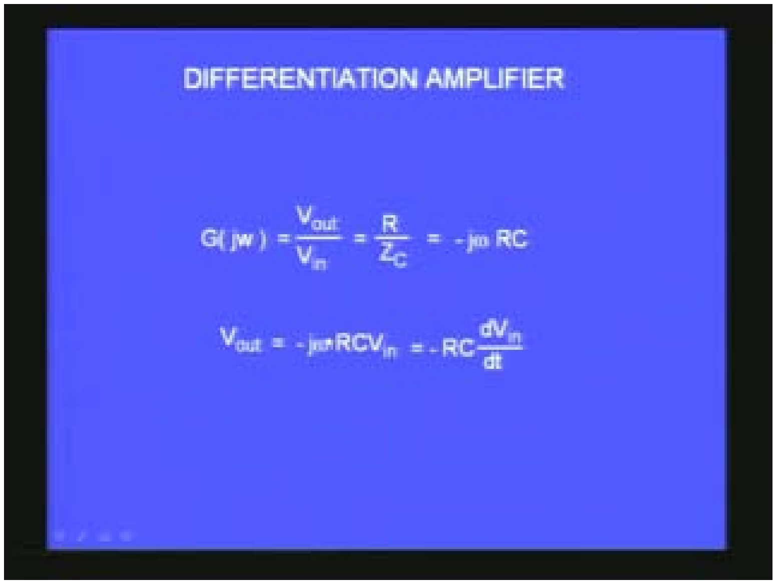

The V output is - j omega C RC into V_{in} from this equation and that is equal to -RC dV_{in} by dt because - j omega V_{in} is nothing but the differentiation of a input signal dV_{in} by dt. The dV by dt is nothing but I by C; current divided by C.

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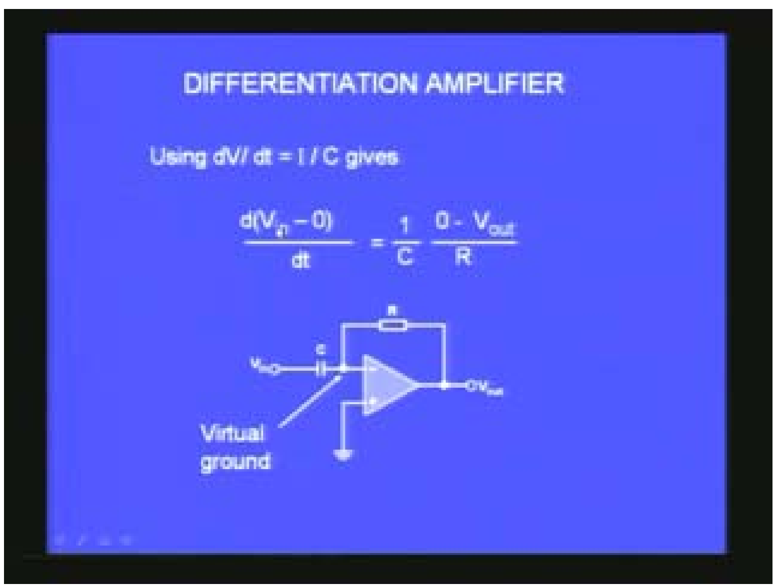

Here at the input side the differential voltage is V_{in}-0. This is zero here. This is V_{in} by dt is equal to 1 by C zero minus V output by R at the output and that is also the same as what we have already seen. If you look at the frequency response curve it will be something like this. It is because it is a superposition of two curves. One is due to the op amp about which I have not yet discussed. But I will discuss at some detail later on.

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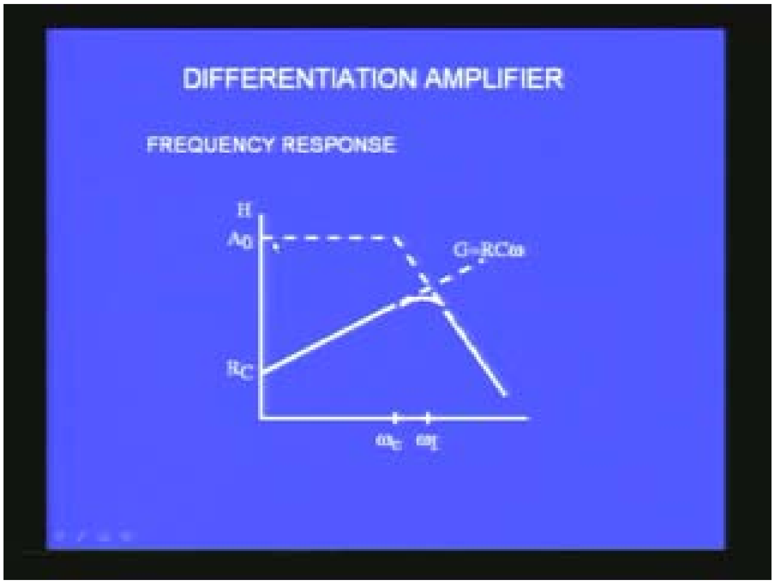

You have a constant gain after some frequency and then it roles of with a constant slope at the end due to the internal capacitance and this is the bandwidth graph corresponding to all the board plot, corresponding to the op amp and this is the response of the RC circuit that I have and both of them will give me a resultant which is actually shown by the solid curve here. Basically we also saw that it is going to be a high pass filter and that is what also you have seen from the frequency response.

Now I have to quickly show you a demonstration of the circuit, a simulation of the circuit. Here on this screen I have a bread board. I have the dual supply I have the function generator which generates square wave in this case and I have an oscilloscope on this side. Now I make auto set up. That means op amp goes in and the various components will go and occupy the appropriate places we defined and all the wiring is being done with reference to the power supply, the input voltage or the function generator and the oscilloscope, etc. For reference I have also shown you the actual circuit on the bottom screen here. You have 270 ohms in series with the capacitor which is a 0.01 micro farad and you have a 10K resistor which is connected to ground at the pin number 3 which is the non-inverting input and you also have a 10K resistor which is at the feedback between the 2 and 6 terminals of the op amp. It is the exactly the same as what you have here and that has been wired.

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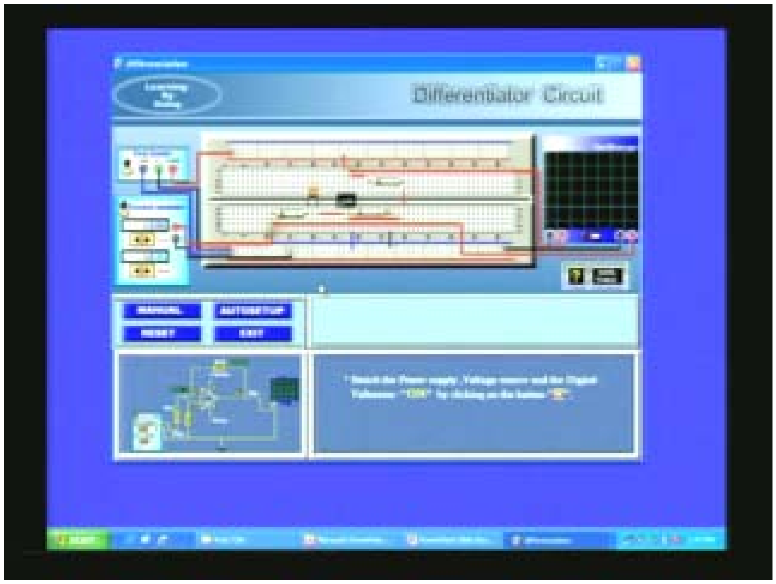

Now I will switch on the power supply. I switch on the function generator and I also switch on the oscilloscope. If I now apply some 500 hertz and about 1 volt the triangular wave given as the input is actually a triangular wave function generator produces a square wave and the amplitude is multiplied by two because of the value that I have used here in the specific circuit. A triangular wave given to the differentiating circuit should result in the square wave at the output. That is what I wanted to show you.

Now we will also try and do an actual demo of the circuit. I will show you the demo now. Here I have the differentiation amplifier, differentiating circuit. It is exactly the same as what I discussed already. You have the 741 here and you have the 0.1 micro farad and 270 ohms and the 10K in the feedback and there is another 10K at the non-inverting input connected to the ground.

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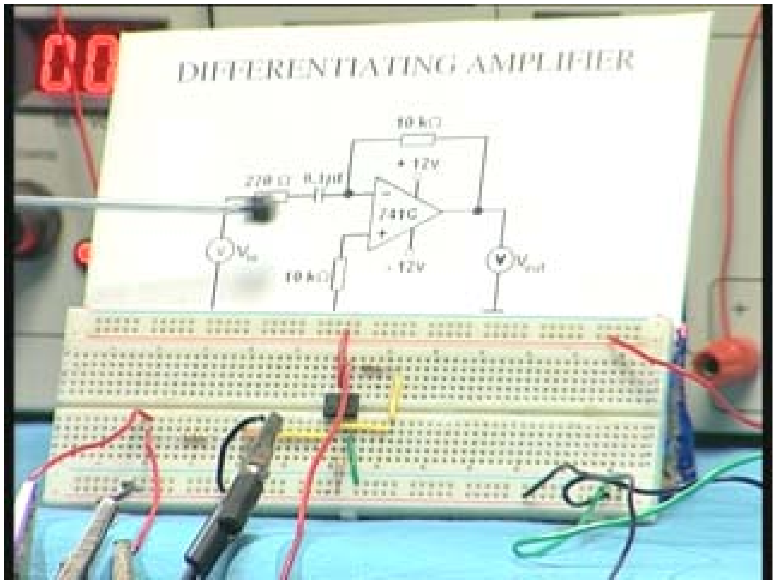

You have an input which is actually obtained from this function generator which can be varied in frequency. You have a sine square triangular wave etc all those outputs are there. These knobs are for applying different frequencies. I have right now applied 1 kilo hertz. I have pressed this button. It corresponds to 1 kilo hertz and this measures approximately the amplitude which is about 200 millivolts.

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This is the input to this circuit which is also wired here you have the same op amp and the wiring is exactly similar to what I showed in the simulation also. This is the dual supply and the plus minus voltages are applied along the rail sockets and both the input and output is monitored in the oscilloscope and I have got a triangular wave at the top which is actually the input and what you see in the bottom is a square wave which is the differentiated output.

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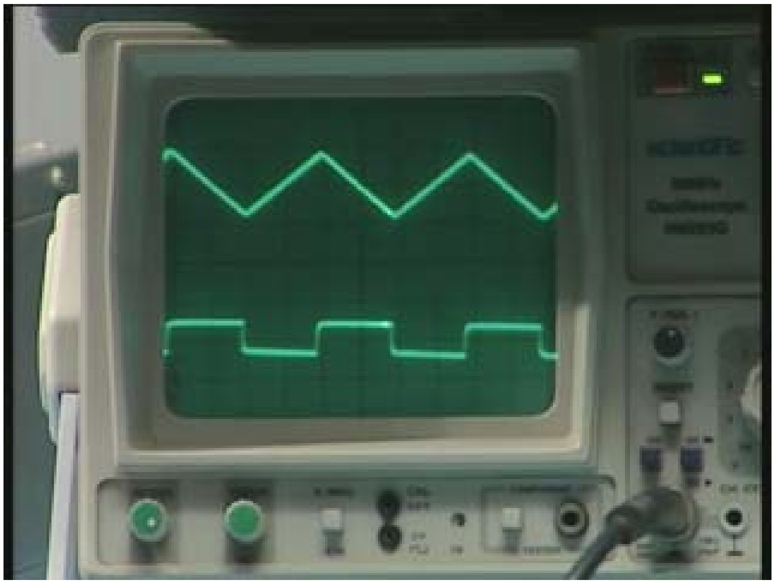

Now if I change the frequency you can see the output is also changing correspondingly.

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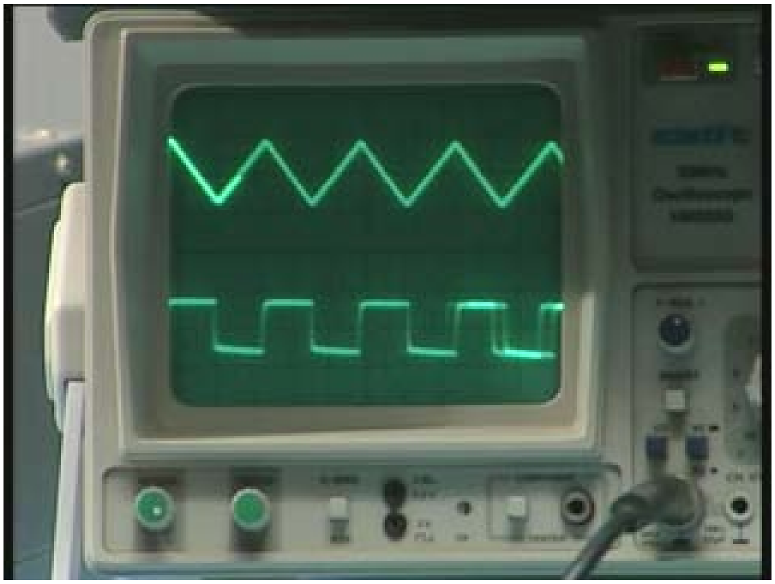

There is of phase factor which you are not able to exactly see but the triangular wave becomes the square wave when I apply through (or) from? a function generator. I also mentioned to you what would happen if I apply a square wave at the input of the amplifier. I now change the input to a square wave and the two sharp pulses corresponding to the transitions high to low and low to high corresponds to the differentiation of the square wave.

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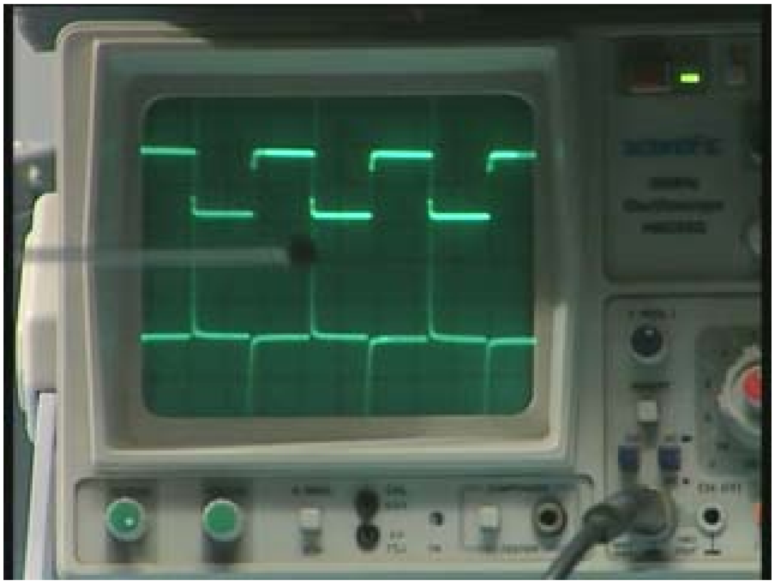

So this is the output wave form with sharp spikes corresponding to this transition from high to low and the square wave will give me very sharp pulses. If I put the diode correspondingly I can get rid of either the bottom pluses or the top pulses and I can use it for triggering other circuits. As I change the frequency the output also changes. If I reduce the amplitude the output also shows slight variations. If I give a sine wave both of them are sine wave because one is a sine wave the other is a actually a cosine wave. There is only a difference of about 90 degree phase which cannot be very precisely seen here but you can certainly see that there is a phase difference. If you look at the corresponding peak positions they are not in the same vertical line. There is a relative shift and that corresponds to the cosine wave. The bottom one is the cosine wave the top one is the sine wave.

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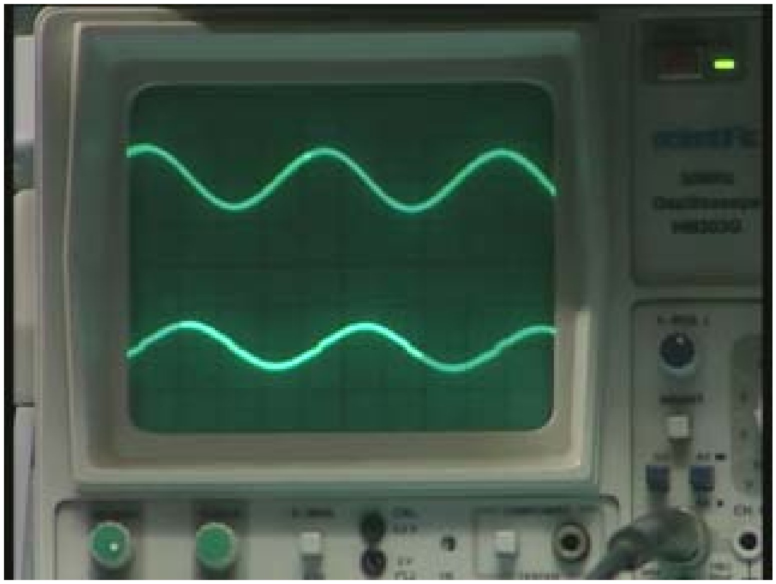

I will again come back to the square wave and you can see the spikes. This circuit performs what is known as differentiation of the input signal with reference to time. We have seen the demonstration as well as the simulation of the differentiating circuit. We can also have several variations of the differentiating circuit. For example I can have multiple signals. In this circuit I have used a summing amplifier but I have used two capacitors on the two input side and both the output will be the differential of the two input signals.

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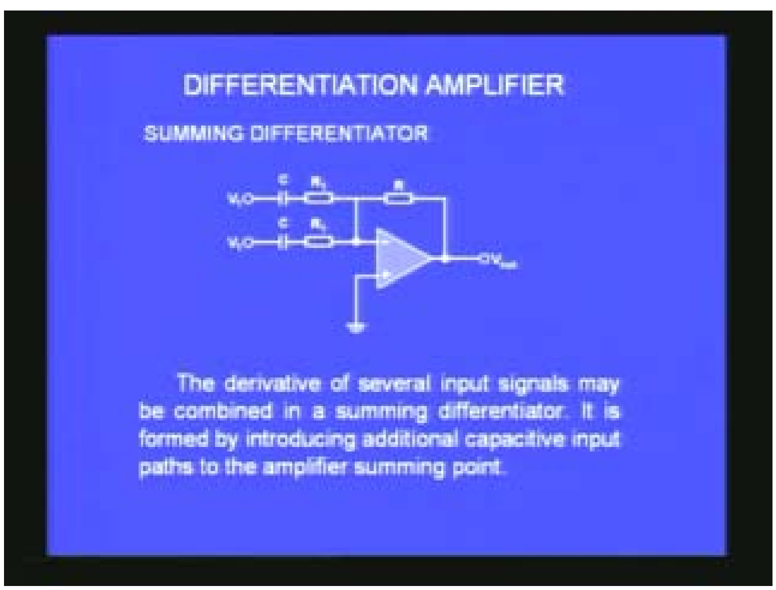

It is a sum of two terms. Each of the term will correspond to the differentiation of the V_1 signal and the V_2 signal and that is what I will get. You will have the summing as well as differentiation perform on the signal. Similarly I can also have another differentiating circuit where I have used both the input terminals. The non-inverting terminal and the inverting terminal both of them are used here.

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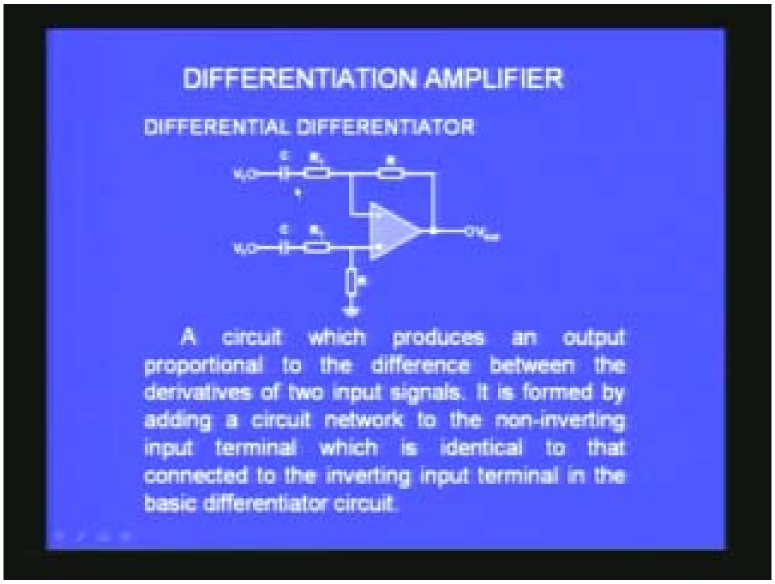

This is another variation of the differentiating circuit which also can be used depending upon the requirement. When you want to apply to different applications you can try to use these types of circuits. With these we have completed most of the mathematical operations that I wanted to discuss. I will not stop here because I should justify why these mathematical operations are important? How they are going to be useful in different applications?

I already mentioned to you that these operational amplifiers are very, very useful in analog computation when you want to perform analog computation; that is on analog signals when you want to apply some mathematical equations. I also mentioned to you that most of the physical phenomena can be represented by way of a differential equation. If you are able to have a differential equation and solve the differential equation you would have reasonably solved and understood the basic phenomena you wanted to understand corresponding to the boundary conditions which we have to apply corresponding to the differential equation. It becomes convenient for us to provide an electrical analog of the actual physical phenomena by constructing certain operational amplifier circuits with which I can perform the solution of the differential amplifier. I can obtain the solution of the differential amplifier instead of mathematically or instead of actually working with the real system I can take an analogous system in which the variables are simply voltages, currents and I can perform the differentiation or the differential equation can be implemented electrically and solve the differential equation. Thereby I would have solved the actual real time real problem that I used to handle.

These mathematical operations that we discussed are useful in such applications. As an illustration I want to give you two different types of application. One is solving simultaneous equations and second one is solving a differential equation.

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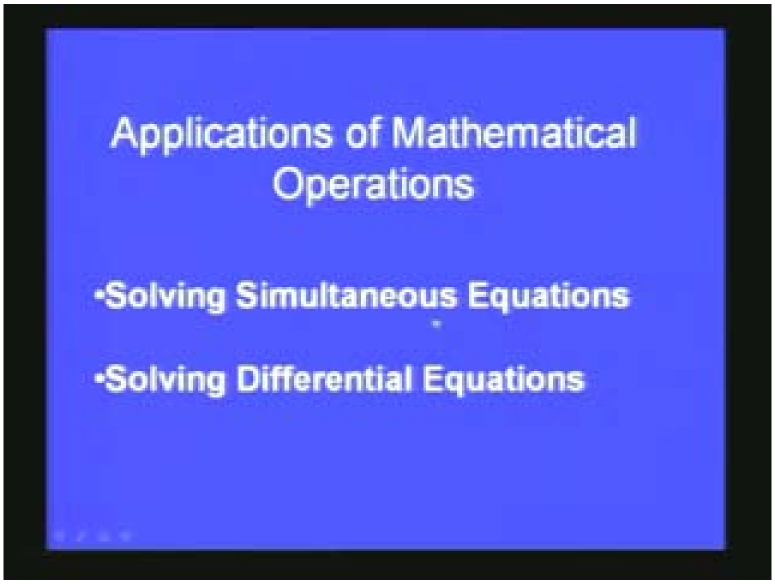

Let me take up solving the first part which is solving simultaneous equations. What do you mean by the simultaneous equation? If I have two equations as I have shown on the screen for example $a_1 x + b_1 y = c_1$ is one equation.

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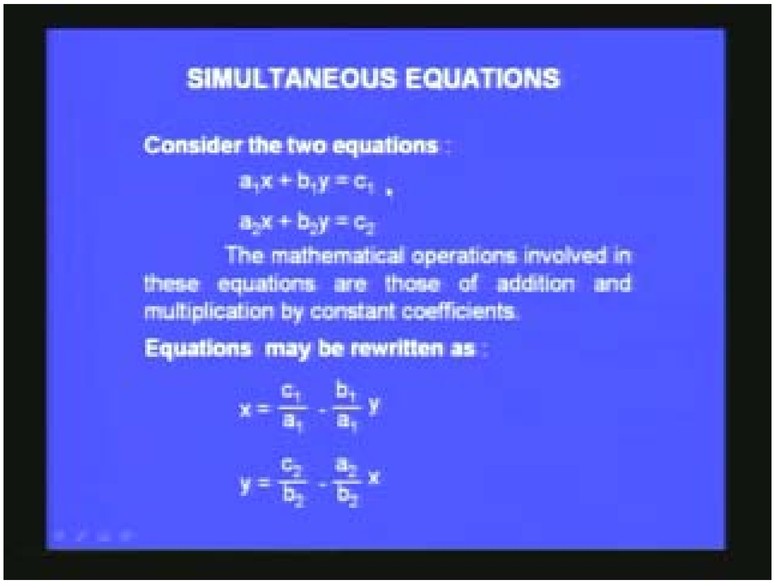

a_2 x + b_2 y = c_2 is another equation. I have two equations. From this I should solve and obtain What is meant by solving these two equations? Simultaneously means I must obtain a value of x and y which will satisfy both these equations. That is what I mean by solving the simultaneous equations. That means the x and y whatever value I assign that will be satisfied by both these equations where the a_1, b_1, c_1, a_2, b_2, c_2 are co-efficients, the constants, numbers. This mathematical operation of solving the simultaneous equation most of you know, you would have studied in your school and this can be done also electronically using operational amplifier and the solutions will be in the form of voltages. That is what I am going to know attempt to explain to you.

If I take the first equation a1x + b_1y = c_1 I can rewrite this. By retaining the first term x is equal to c_1 by a_1, I divide through out by a_1 here; so c_1 by a_1 minus b_1 by a_1 y when I take the y coefficient to the other side. So x is equal to minus - not in slide c_1 by a_1 minus b_1 by a_1 times y. Similarly the second equation can be solve for the y and y is equal to c_2 by b_2 minus a_2 by b_2 times x. These two equations were derived obtaining x from the first equation and writing the expression for y from the second equation. Once you have this then you can you understand that this involves multiplication of a constant and either summing or difference. x is some quantity, some value c_1 by a_1 minus another value b_1 by a_1 multiplied by another variable y. This is completely amenable to the all the techniques that we already discussed basically the summing amplifier and multiplication by a constant. I can use those circuits and obtain these two equations independently. That is precisely what I have done here. In the next picture this is the op amp.

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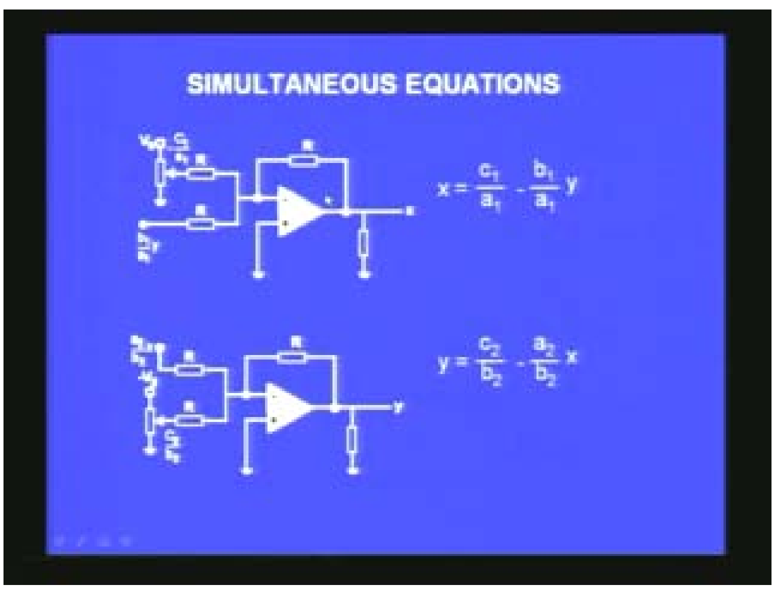

What I have done is I have taken b_1 by a_1 times y. I assume b_1 by a_1 times y is available at this input. At this input from a voltage source I obtain minus – not in slide c_1 by a_1 times this voltage V_s. This I apply. This feedback resistor is R this is also R. The summing amplifier has got a coefficient of 1; R by R. R by R in both. But I am applying it to the inverting input and I am going to get all of them with a minus sign at the output. I give minus c_1 by a_1 times some voltage into 1 volt, V_s is 1 volt. I will adjust the potentiometer here such that I get minus c_1 by a_1 voltage. If I get that when it comes here it will become plus c_1 by a_1 because the gain factor in this is 1, R by R and I will get the first term x equal to c_1 by a_1. But there is also another additional term and to obtain that I have used b_1 divided by a_1 times y, the other variable. If I apply it here that will come at the output as minus b_1 by a_1 times y and I get here minus b_1 by a_1 times y. This is what we got from the first equation x_1 equal to c_1 by a_1 minus b_1 by a_1 times y and that is precisely what I have implemented using this circuit with one op amp.

Now what I do? I do the same thing with reference to the second equation. y is equal to c_2 by b_2 minus a_2 by b_2 times x. In this case you find it is exactly similar to the earlier one. I apply a_2 by b_2 times x at this input and I apply c_2 by b_2 as a proportional voltage at this point and these two will be added at the output with a negative sign because there is an inverting input here and y becomes c_2 by b_2 minus a_2 by b_2 times x. This is what I get and this is precisely what I have here in this equation. There is x here there is y here. This y I connect here and this x I connect here. I cross couple these two outputs and that means this amplifier will have the input from the other circuit and the second amplifier will have the input from the first amplifier. Thereby it becomes consistent and once everything is settled I will measure the voltage here that will give me the value of x and if I measure the voltage at the output of the second amplifier I will get the solution in terms of y. This is what I mean. That is the complete circuit that I have shown here. That is I have combined the two equations.

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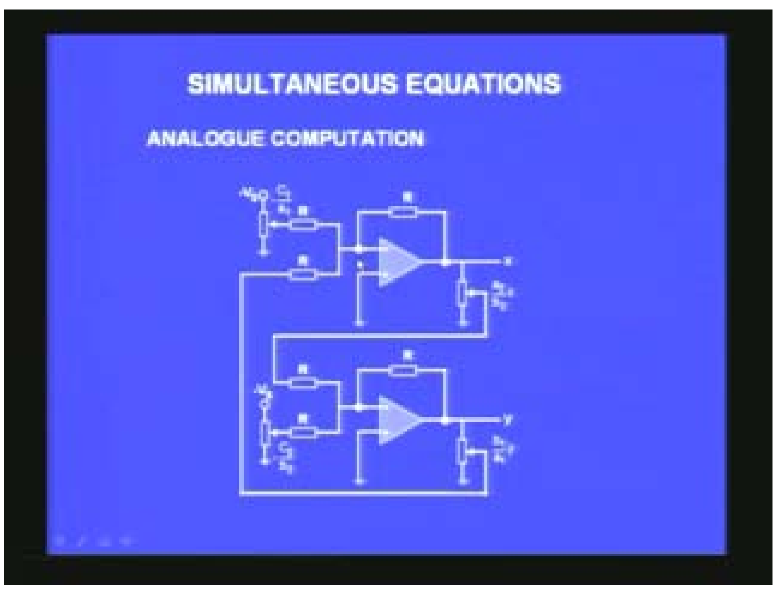

I will solve the first equation by using this circuit; solve the second equation by this circuit and then inter connect the x and y as I have shown here. This x I will take a_2 by b_2; some fraction of that x and apply at this input and the y output comes here. I will take a fraction of that and give at the input here and this becomes consistent and if I measure the voltage here it will be the value of x. If I measure the voltage here that will be value of y corresponding to the simultaneous equation solution.

Having said that it will be good to take a very typical example and see whether we can solve it simultaneously.

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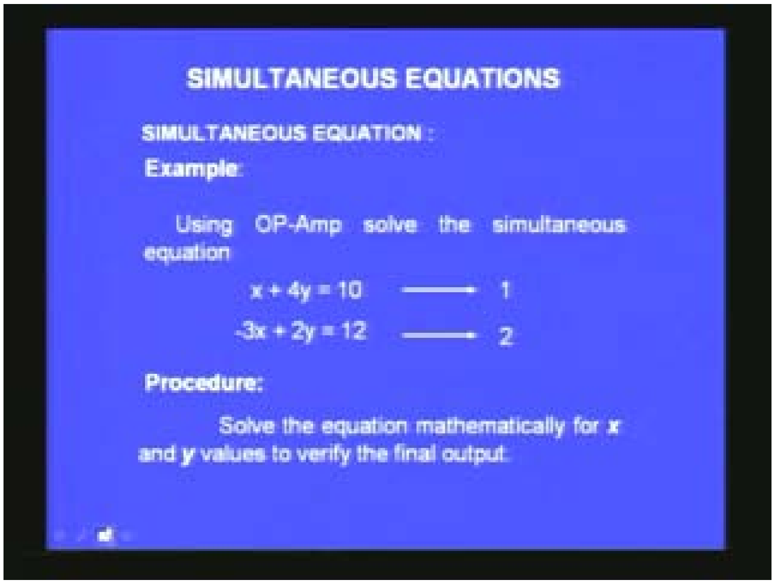

I have taken a simple numerical example x + 4y = 10 is the first equation. I previously discussed with only a_1 b_1 etc some coefficients. Now I am taking actually real numbers. x + 4y = 10, -3x + 2y = 12 are the two equations which have to be solved and the solution x and y which will satisfy both these equation simultaneously will have to be identified. For that as usual I write what is x from the first equation. x is 10 minus 4y. That is what I am writing here. x = 10 + (-4y); -4y put in a bracket. This is the first equation and similarly from the second equation I obtain the y. y is equal to this will become 12 by 2 that is 6 and this minus will become plus 3 by 2. So y = 6 + 3/2 x is the second equation.

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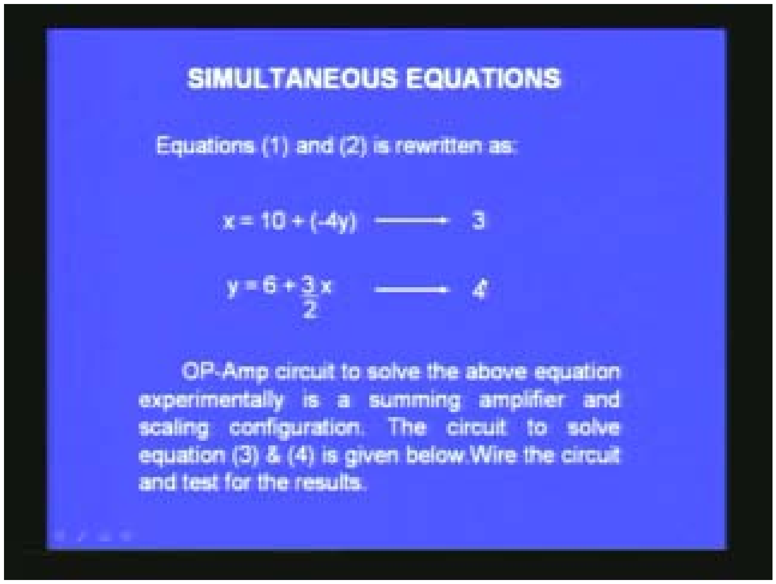

Having obtained the x from the first equation and y in terms of x from the second equation I can use an op amp to implement this in the summing mode and implement this in the summing mode and then cross connect x and y. That is all I should do. Here I have chosen some values of resistors and I have a complete circuit here in front of us. It is very similar to what I already mentioned to you.

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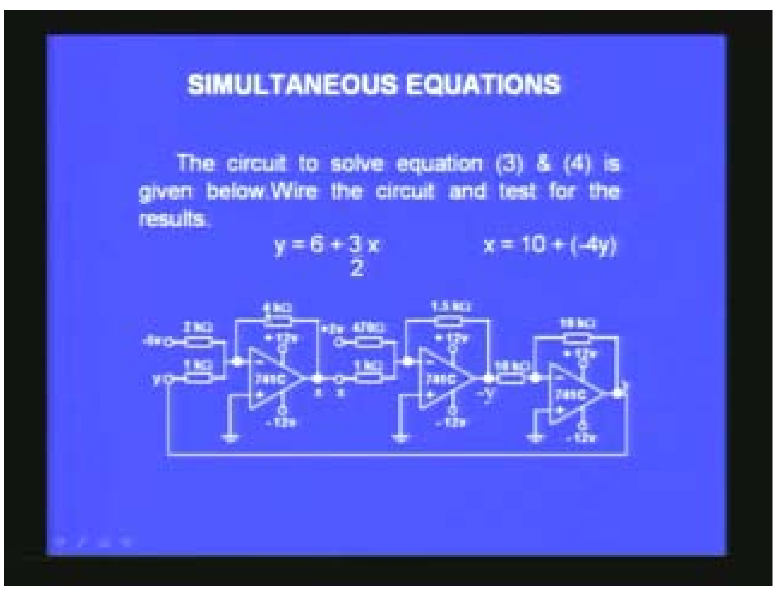

For example I have used the V_s the reference voltage to be -5 volts and I have used 2K resistor here, 4K resistor here in this first amplifier. If I have only this input what is the gain of this? The gain of this is 4 kilo ohms divided by 2 kilo ohms with a negative sign because I am applying from inverting input. -4 by 2 that is -2 is the gain and that should be multiplying the input which is -5 volts. It will become +10 volts at the output if I have only this first …… I have the +10 at the output corresponding to this and when I apply the y here this is 4K and this is 1K. The ratio is 4; 4 times y is what I get but because I am applying to the inverting input I should get -4y at the output due to this input. So I have a -4y. x which is the output of this amplifier is the combination of two terms. One term is corresponding to 10 volts the other corresponding to -4y. y is the output voltage coming from somewhere else and this is the first equation. This op amp that you see here is corresponding to the first equation.

Corresponding to the second equation I have in the next op amp. You have 1.5 kilo ohms and 1 kilo ohm and the ratio is 3/2 times x. That x is connected here; 3/2 x with a negative sign here. I will get -3/2 x here and this is 470 ohms which is close to 500 ohms and this is 1.5; it is three times. Therefore three times the input voltage plus 2 that will give me 6 volts again with the minus. Therefore it is -6 volts. What I get will be a -y. -y is what I get here because it is -6 volts; -3/2 x from this amplifier and so I get a –y. But here I have to apply +y and I have introduced one more op amp here in which I have just used simple inversion 10K here, 10K here and I give the -y and the output will become +y with the gain 1 because 10/10 is equal to 1. This becomes +y and that y I bring and interconnect at this point. Thereby I have consistently connected the whole circuit and if I measure with the multimeter the voltage here and if I measure the voltage at this point or at this point I will get the value of y and at this point I will get the value of x. The equation has been solved successfully electronically. That means by using three op amps one op amp for one circuit, another op amp for the other circuit and third op amp is for performing a simple inversion. I have used all the mathematical operations corresponding to multiplication by a constant, summing amplifier, difference amplifier and all these things put together will get the solution of a simultaneous equation.

What will be the value of x and y? If you wire the circuit and measure you will get the solution. But before we do that let us also try to do that by the normal analytical method. $x + 4y = 10$, $-3x + 2y = 12$ are the two starting equations.

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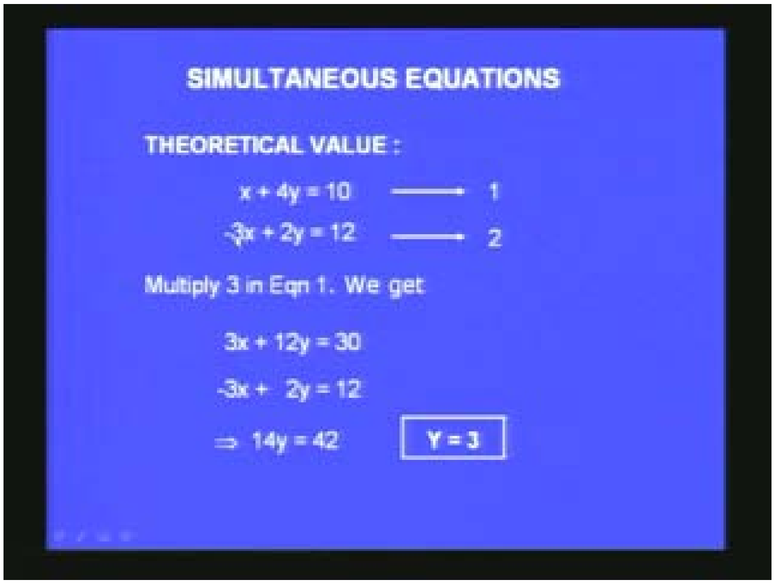

Multiply one by 3; so first one becomes 3x + 12y = 30 and this is -3x + 2y = 12. If I add them together I get 14y = 42 and y is equal to 3. y = 3 is the first solution and I use that in the next equation x + 4y = 10.

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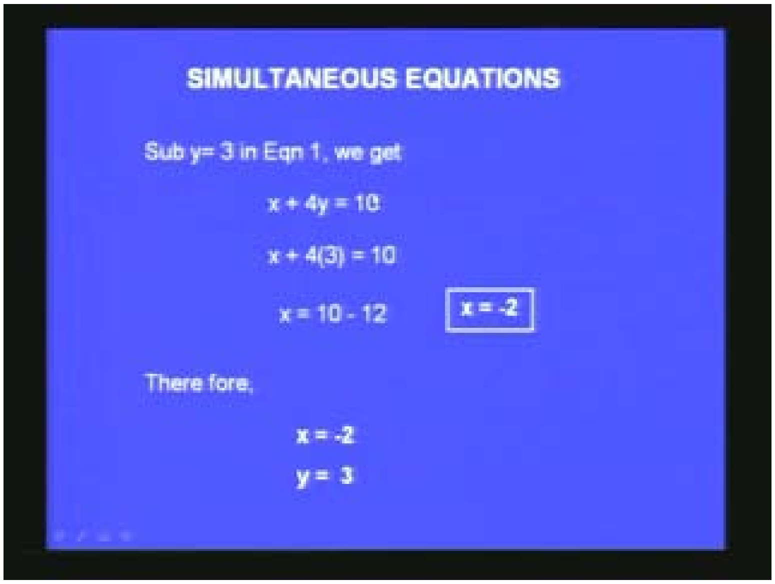

x plus 4 into 3 12 = 10. Therefore x = -2 is the second. The solution if I do analytically I get x = -2 and y = 3. If I wire the circuit which I have already shown to you and measure the voltage here it should be -2 volts and the voltage here should be 3 volts because x= -2 and y = 3 are the solutions of the simultaneous equations.

Having see that let me show you a demonstration o this simultaneous equation. I have actually taken the same two equations and we have wired the same circuit which I have shown here and then we have tried to measure the two voltages. Let me show you the actual demo. Here you can see the same circuit is shown. There is -5 volts here; the 2K resistor, 4K resistor and y is coming all the way from the output for or to? the third amplifier and this is the first amplifier which gives me the value of x and this is the second amplifier which has got 470 ohms with a +2 volts and 1 kilo ohm and 1.5 kilo ohm as the feedback resistor and this is the inverter with a 10K resistor here and the 10K resistor here so that the -y becomes +y and this +y is again fed back to this circuit so that once I wire this circuit with all these values if I measure the voltage here that will be the value of x and if I measure the voltage at this point I will get the voltage y corresponding to the solution x and y of this equation.

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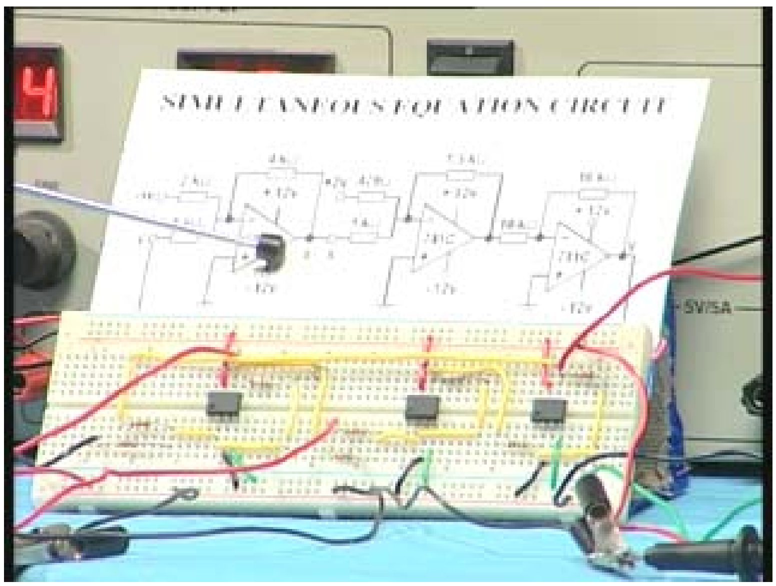

This is the actual circuit wired with the three op amp as seen here. This is the first op amp that you have here. You have got the 1K and the 2K and the feedback resistor and this is the second op amp here and you have got the 470 ohms and the 1K and the feedback resistor 1.5K and the last one with the feedback resistor 10K is the inverter and the output of that is all the way brought and given at the input corresponding to y. This is precisely the same circuit which I have wired and I wanted to measure the voltage here. I use this multimeter and connect it to measure the value of x and this multimeter I use to measure the value of y. What is the value of x? I don’t know whether you are able to see that. It is actually -2.264.

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It should be -2; you are getting -2.2 and the value of y is + 3 and that is what you read here 3 volts.

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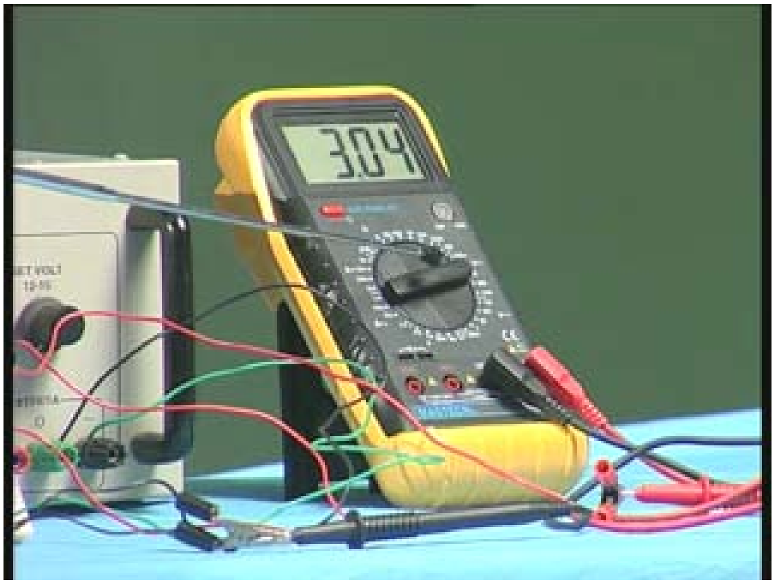

x is -2.268; that there is a slight variation here. It should be precisely -2 volts. It is not because I have not used very precise resistors here. For example I have used 470 ohms instead of 500 ohms and all these resistors will have some small variations because of the tolerance and there is some residual extra voltage that comes in terms of 2.2. If I use very precise values of resistors at every point and measure I will get very precise value of -2 volts. I don't know whether you are able to see the minus sign here. It is – 2.2 volts. This is -2.2 volts that gives the solution with reference to x and this multimeter gives the value of y and this is what we have actually done as an application of the operational amplifier mathematical operations. I have actually shown you a real circuit which is implemented using these two equations and the value of x and y will be corresponding to the voltages that will be appearing when I measure x voltage and the y voltage. That will correspondingly give me a solution of the equation.

It is more for an interest that I have shown you this circuit. But it is not for some simple applications that we use operational amplifier. It can be used for much more complicated physical phenomena which can be represented by differential equations. You can on your own try other simultaneous equations; you can write and then try to formulate your own circuit design a circuit which will correspondingly solve that particular simultaneous equation and it is much simpler. Quickly you can wire it and you can get the solution. What will happen if I go for a 3 variable simultaneous equation not only x and y but xyz. You can almost in a similar fashion solve. That means instead of having two amplifiers you will have to have three amplifiers corresponding to x, y and z and then provide the necessary feedback. You can write from the first equation what is x equal to in terms of y and z; from the second equation you can write what is y equal to in terms of z and x and the third equation you can obtain z is equal to in terms of x y and a constant term and then implement a three input summing amplifier where the corresponding terms will come at the input and you get the x and the y and the z. Three amplifiers I will do and then I will interconnect the x to x and y to y. Thereby you can in principle solve even a three variable simultaneous equation.

The three variable simultaneous equations will be slightly more involved than a two variable simultaneous equation. If I give an example of a two variable simultaneous equation to be solved electrically you may laugh at me that it can be very quickly done by an adept mathematician in a very, very short time whereas if you go to three variable simultaneous equation it involves much more involved calculation and therefore it might perhaps be simpler if I can wire a simple circuit and I provide appropriate input voltages and that I will able to solve faster. Analog computers in reality there are not many of them available in these days because most of the analog computations can now be done much more efficiently at higher speeds by using digital computers and not many applications are now being implemented using analog computers. But still because of the advent of operational amplifiers, the analog computers, various mathematical operations have become very, very handy and useful in several different types of applications that we come across. Beyond the basic electrical engineers there are plenty of other engineers from other branches and scientists who are also making use of these types of circuits for different application purposes. I only showed you a simple example of a simultaneous equation but we can also go further and discuss about the solution of some of the differential equations. I want to give couple of examples.

On the screen you see there is a differential equation which is a very well known differential equation corresponding to the radio active decay law.

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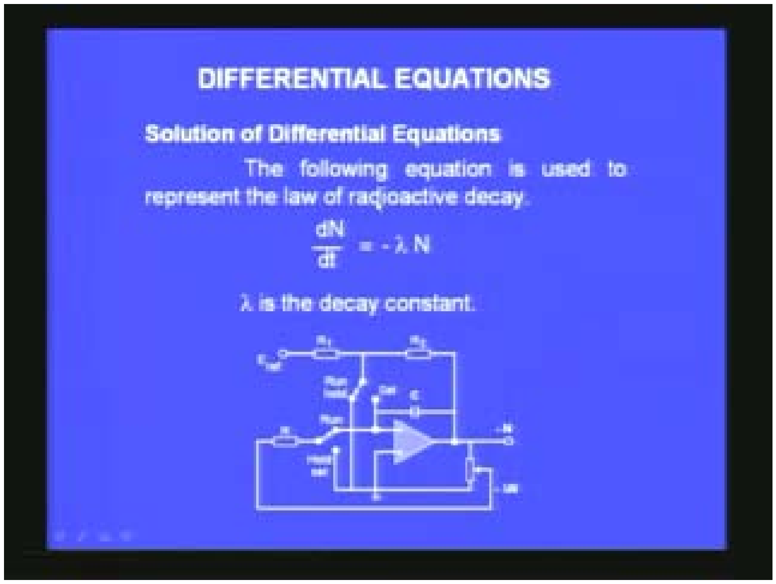

dN by dt is equal to minus lambda N. I am sure it is a very familiar equation to most of you where the N is the number of nuclei that is under going decay and the rate of decay dN by dt, the number of nucleus that is under going decay per time is dN by dt. That will be proportional to the number of nuclei available at the first instance. This is written in the form of a simple equation dN by dt is equal to minus lambda N where lambda is the decay constant a proportionality factor which comes into the game; minus lambda N. If I want to solve this what is that I should do? I will actually use a very simple integrating circuit and apply dN by dt as the input voltage, whatever is that and what I get will be the corresponding lambda N. That is what is shown here. I have an integrating circuit here. I connect the rate dN by dt here and then after integration it becomes -N with a proportionality constant which comes about because of the RC time constant, the product of RC as well as some initial voltage on the capacitor which can be adjusted using a R reference, e? reference using the R_1 R_2 potential divider. I can apply this and thereby I will get minus if I use a potentiometer and take some minus lambda times the output voltage minus lambda N and apply it to the input here. The equation becomes satisfied. Minus lambda N given at the output and that will be what we get out of the differentiator and dN by dt is equal to lambda N is implemented here in a very, very simple fashion using one single op amp. You can also complicate the scheme by having the multiple decay.

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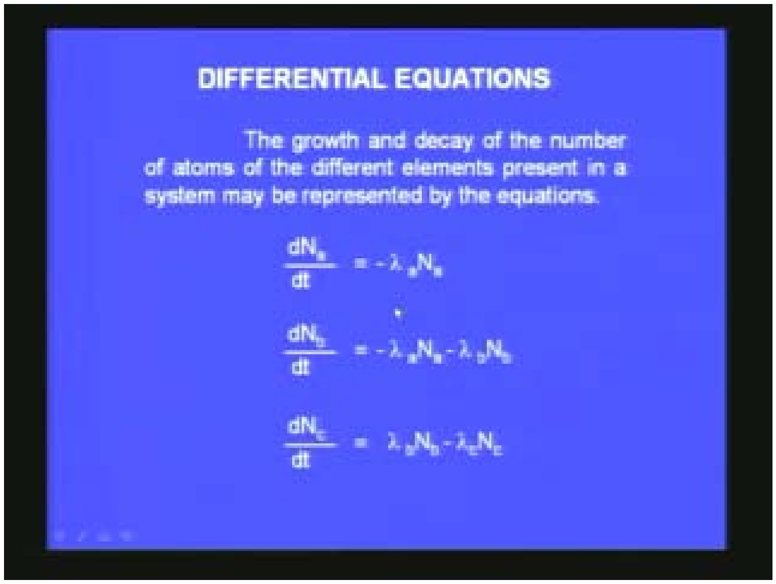

First N_a decays to become N_b and N_b decays again to go into N_c. The equation becomes much more complicated. We have to make use of the presence of N_a also here in the second equation and N_b and the third equation N_b and N_c; the mother and the daughter. You have more number of variables here and this also can be implemented by using a similar operational amplifier circuit. We can also discuss differential equations of a more complicated manner.

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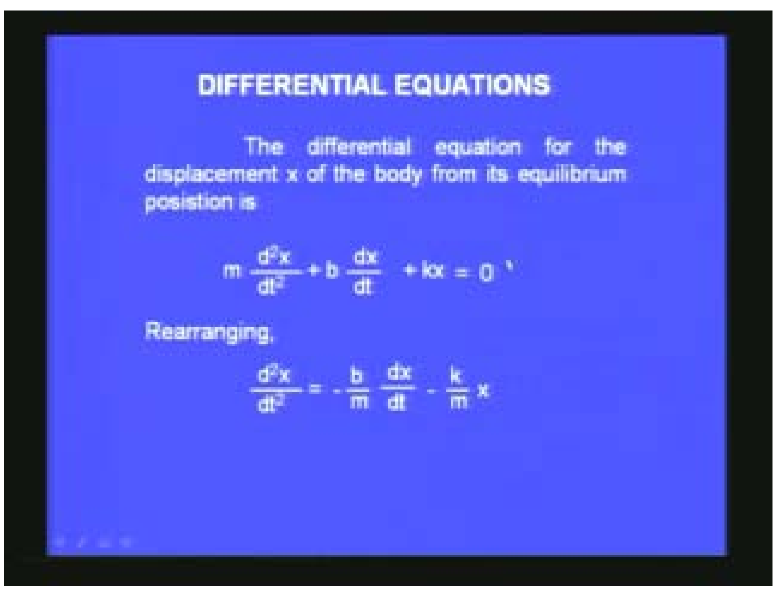

Some of you may be familiar with the equations that I have written here; a second order differential equation md square x by dt square plus b into dx by dt plus kx is equal to zero. If I don't have this term it becomes a simple harmonic function; differential equation corresponding to an oscillator. md square x by dt square is equal to minus kx is corresponding to vibrating system. This b dx by dt which is corresponding to a velocity term, dx by dt will provide some damping with reference to the vibrating system and this is for a damped harmonic oscillator, a simple harmonic oscillator and this equation can be solved by simply rearranging d square x by dt square is equal to minus b by m dx by dt minus k by m x. I rewrite and then I implement using the op amp integrators. Integrators are much simpler to work with than the differentiators. I already mentioned to you; you may get into problems of noise when you introduce a differentiator. It is always preferable to use a integrator. You use an integrator. Assume that d square x by dt square is available at the input.

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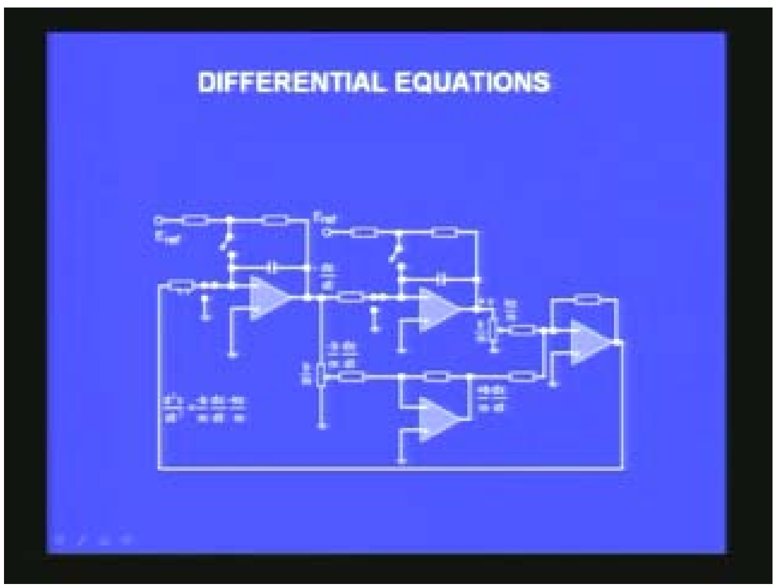

Then you can integrate it to get dx by dt at the output. You again integrate it using this circuit and you get the next. Then you provide the corresponding k by m x proportionality here and here what you have done is the dx by dt is multiplied by proportionality constant minus b by m using this potentiometer and that output is just inverted because this is minus here this will become plus and these two terms is now added that is what is d square x by dt square. So that is given at the input. This becomes a consistent circuit with the input and the output conforming to each other and you get the complete differential circuit implemented using this op amp integrators mostly and one inverter, two inverters. This is the summing amplifier, this is the inverter and these two are integrators. The initial condition can be set by using V reference and the greatest advantage of this is that you can understand a real situation if you know the corresponding damping factor and the frequency factor. You can modify this and try to implement the corresponding system over here and study that very quickly. You find the solution mathematically. Analytically if I do I will get the solution after performing certain mathematical manipulations and if I want to change some of the variables or the coefficients then I have to almost start all over again and solve the problem and obtain the solution once more. Here all that I have to do is modify some of the voltages or the coefficients b by m or k by m factors. By using potential divider I can quickly change them for a new equation and then implement that in a moments notice and then try to understand the stability of the oscillator and the damping that is ultimately needed. You can try to understand the vibrating systems in a very, very simple way by introducing several different modifications very quickly and that is one of the greatest advantages of using an analog computer in place of an actual physical model that you want to study. You can make an analogous electrical circuit and understand the behavior of the electrical circuit by changing the different parameters and that way you will get better insight into the actual working principle of the physical phenomena that you want to understand.

What we have today seen is how differentiation can be performed using an op amp and as an application I have taken two simple applications one is solving a simultaneous equation using operational amplifier and I also discussed how this can be used for solving more complicated differential equations by using simple integrators, summing amplifiers and inverters and how a physical phenomena can be understood in real situations by using analog computation. Thank you very much.