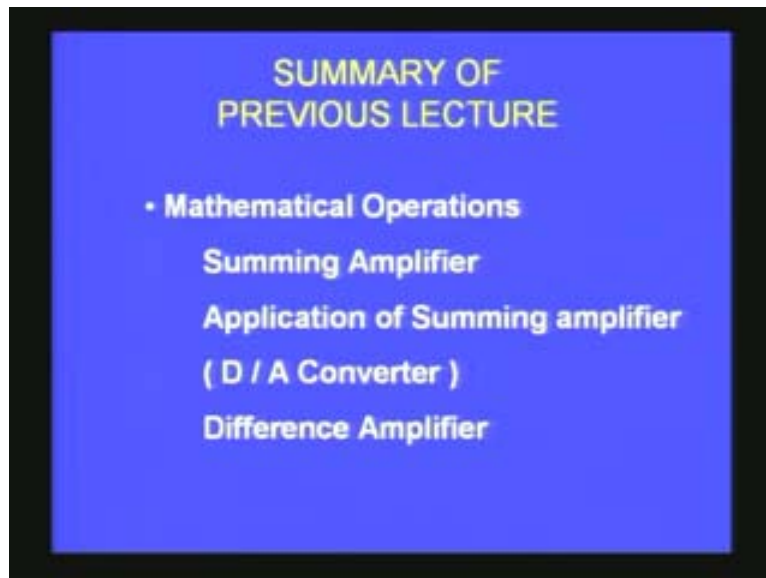


Basic electronics
Prof. T.S. Natarajan
Department of Physics
Indian Institute of Technology, Madras
Lecture- 25

Mathematical operations
(Difference Amplifier, Integration Amplifier...)

Hello everybody! In our series of lectures on basic electronics learning by doing let us move on to the next one. Before we do that let us recapitulate what we discussed in our previous lecture.

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You might recall we discussed about the applications of operational amplifier more specifically on the different mathematical operations that can be performed using the operational amplifier. I already mentioned to you that the very name operational amplifier comes from the capability of this amplifier so that I can use it for performing different types of mathematical operations on the input voltage. The examples, as you can see on the screen, are the summing amplifier, multiplication by a constant and then we also looked at some applications of the summing amplifier like the digital to analog converter and the difference amplifier. We will continue our discussion on the mathematical operation. You may recall that multiplication by a constant is basically a very simple amplifier; voltage amplifier. V output is some k times the V input where the k is a constant which we call as the gain.

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MATHEMATICAL OPERATIONS

MULTIPLICATION BY A CONSTANT $y = k \cdot x$
 $V_o = k \cdot V_m$

SUMMING AMPLIFIER $y = k(x_1 + x_2)$; with $k = 1$
 $V_o = k \cdot (V_1 + V_2)$

DIFFERENCE AMPLIFIER $y = k(x_1 - x_2)$
 $V_o = k \cdot (V_1 - V_2)$

This very simple amplifier could in principle be called a mathematical operation. It is performing a mathematical operation namely multiplying a given input with a constant value. We also saw how we can perform summing or addition of two input voltages using an operational amplifier. The output voltage will be a constant multiplied by the two inputs $V_1 + V_2$ in a typical case. You can have more than two inputs also; you can have multiple inputs in principle. Only condition being that the output voltage has got a limitation in terms of the V_{cc} supply the plus and minus power supply voltages of the operational amplifier. We also discussed another configuration which is basically a difference amplifier or differential amplifier where the output voltage is the difference between the two input voltages V_1 and V_2 . It can be proportional to $V_1 - V_2$ or $V_2 - V_1$ or it could be a with a multiplication factor k . You can have a difference amplifier in the sense the difference will be amplified also. These are the other operations.

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MATHEMATICAL OPERATIONS

INTEGRATION $y = \int x \cdot dx$
 $V_o = k \int V_{in} \cdot dt$

DIFFERENTIATION $y = \frac{dx}{dt}$
 $V_o = k \frac{dV_{in}}{dt}$

We will also see how integration and differentiation can also be performed. Before we do that I showed you one type of a difference amplifier in the pervious lecture. Type 1 is what I have shown on the screen now. We have already seen this. It has got two inputs V_1 and V_2 . You have R_{11} and R_{12} and R_1 as the resistors connected in the circuit. The output voltmeter measures the output voltage.

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OPERATIONAL AMPLIFIER

DIFFERENCE AMPLIFIER (Type1):

$$V_o = -\frac{R_2}{R_1} (V_1 - V_2)$$

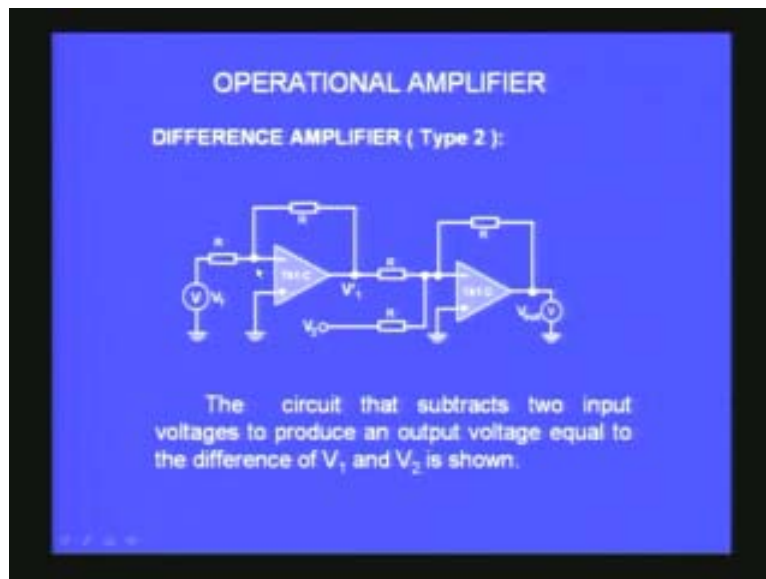
The input is difference by the two signals V_1 and V_2 .

We also derived an expression in previous lecture how the output voltage is equal to $-R_2$ by R_1 within bracket $V_1 - V_2$ or V_1 difference V_2 where R_2 here, the feedback resistor and R_1 here even though I have shown this as different R_{11} R_{12} they can be the same and this

R_2 and this can be same. You will get $-R_2$ by R_1 times V_1 difference V_2 . The input difference by the two signals V_1 and V_2 is amplified in this case by a factor given by R_2 by R_1 . If I have R_2 as 100K and R_1 as 10K I will get a gain of 10. If I have a R_2 100K and R_1 1K I will have a gain of 100 etc. It is possible for us to have a gain factor here by the choice of R_2 and R_1 and there is a minus sign which shows there will be an inversion depending upon which voltage is larger. For example if V_2 is larger V_2-V_1 will also become negative. Then you will get effectively a positive output. If V_1 is larger than V_2 you will get a negative output.

You can also implement the same difference amplifier by using two op amps where first op amp is used as a simple inverter. The subtraction can be performed as an addition. If I want A_1-A_2 what I can do is I can perform A_1 plus something where the something is actually the inverted value of A_2 . That is I will have first A_2 converted into $-A_2$ and then I will perform an addition with A_1 which is equivalent effectively to A_1-A_2 . So that is the trick I am going to play here. In the previous circuit if you go back you can see this is given to the inverting input V_1 and V_2 is given to the non-inverting input. This will be coming out as minus constant multiplied by V_1 ; this V_2 input will come out as plus constant multiplied by V_2 . What you will get effectively by the addition of these two will be something proportional to V_1-V_2 . In the present case where I am going to talk about type 2 difference amplifier what I have done here is the first one that you see on the screen is a simple inverter.

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So you have a V_1 . You have R ; you have also R and the feedback and the other input non-inverting input is grounded. This is a very simple inverting amplifier. You have already seen it also earlier. This is R ; this is R . The gain is 1. Because it is given to the negative R , the inverting terminal the output will be minus of whatever is the input. What I get at V_1 prime will be $-V_1$. What this amplifier has done is simply to invert the input voltage V_1 into $-V_1$. That is all. Now what I do is if I look at the second configuration

using the second op amp it is just nothing but a simple summing amplifier which we are already familiar. You have one V_1 here V_2 . These two are added. That is what you get at the output depending upon the values of R. Here again I choose all the R to be equal and there will be a gain factor of 1. So V_1 prime plus V_2 prime will be the output voltage. But V_1 prime because it is an inverter it is $-V_1$. $-V_1$ plus V_2 will be the output or output will be $V_2 - V_1$. Effectively what we have done is we have just inverted one of the inputs and added using a summing amplifier the two inputs.

That is the trick that we will play here. That is exactly what I have explained here.

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OPERATIONAL AMPLIFIER

$$V_1' = (-V_1)$$

$$V_2 + V_1' = V_2 - V_1$$

$$V_{out} = V_2 - V_1$$

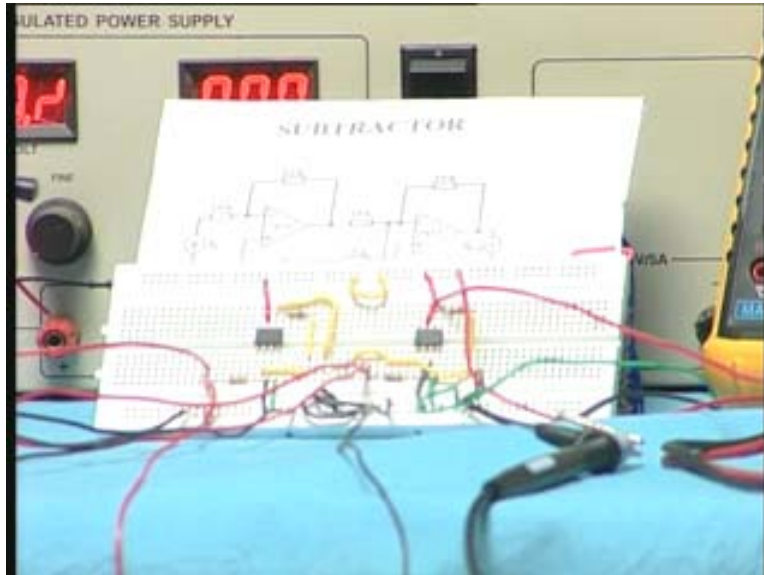
How it works ?

Input V_1 drives an inverter with a voltage gain of unity. The output of the first stage is $-V_1$. This voltage is one of the inputs to the second-stage summing circuit. The other input is V_2 . Since the gain of each channel is unity, the final output voltage equals V_1 minus V_2 .

V_1 prime is nothing but $-V_1$ and so V_2 plus V_1 prime is nothing but $V_2 - V_1$ because V_1 prime is $-V_1$ and V output will be $V_2 + V_1$ prime due to the summing amplifier that is $V_2 - V_1$. Here there is no multiplication factor because I have consciously chosen all the values of resistors to be equal and the multiplication factor or the gain becomes unity; one.

Now having seen that we can perhaps go down to the demonstration table and try to perform the We have already done in the difference amplifier corresponding to the type 1. Now we will move to the type 2 and we will try to see whether the circuit works in the way we expect it to work. Here I have the subtractor. You can see the same circuit which I just explained. This is the first operational amplifier with a normal inverter configuration and the second op amp here is in the summing mode. This V_1 is first inverted as $-V_1$ here and that $-V_1$ and V_2 are added by using the summing amplifier here. The resistors that I have used in all the case are all 10K. The gain factor becomes 1 in all the situation.

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The actual circuit you see on the bread board you have the first IC here and you have the second IC here both are 741 op amp and all the resistors that I have used are 10K from the color code and this one is wired as a simple inverter and this is wired as a summing amplifier and the output is monitored by this multimeter at the extreme right. This is the multimeter which measures the second input corresponding to an input from this power supply and this multimeter measures the input from this voltage source which is a dc voltage source which is also capable of variation. This voltmeter measures the output from this voltage source and that is given as V_1 . This output from this power supply is given as V_2 and that value is measured here and the output is measured by this multimeter.

Now let us have look at the values. I have kept here 300 millivolts and you must be able to observe a value of 313.6 millivolt in this multimeter corresponding to this input and here the input is measured by this multimeter as 220; 0.22 volts which corresponds to 220 millivolt. 220 millivolt and 313 millivolt you have to find the difference and it will be the output voltage as measured here. It's about 95 millivolts.

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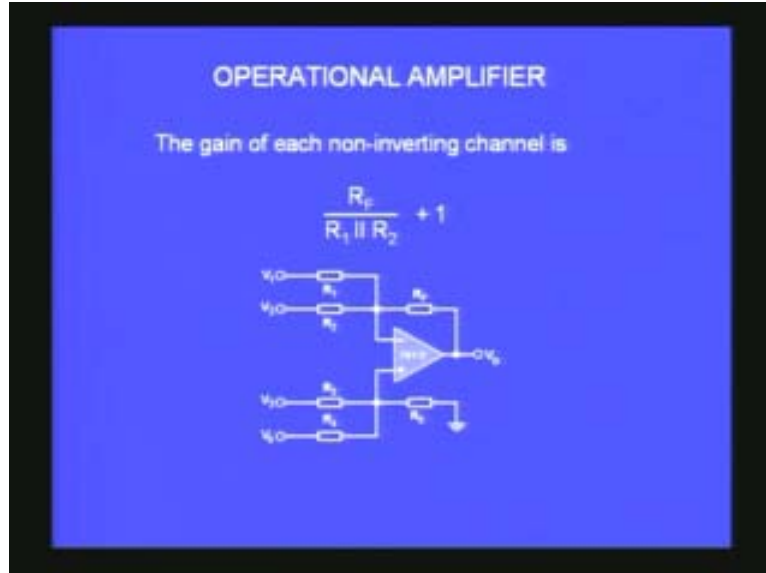


It should be around close to 100. It's about 95 milliamperes. If there are some differences in the actual magnitude that I have already explained to you is because these resistors are not 1% resistors or very precise resistors. They do have a tolerance of about 5% to 10% and there will be a slight variation in the actual value of the resistors that you have used and you see them to be not exactly equal to what you would expect because of the small variation in that but it is very close and reasonably agreeable.

Now what I am going to do is I am going to change one of the inputs. It is now presently showing 313 millivolts. I am going to increase by one more. So it becomes around 413 millivolts or 0.413 volts and this is 0.22 volts. So the difference should be around 200 millivolts and you can see the output is around 192.7. The difference between these two is what you get at the output. I again increase by one more stage. So now it becomes 500 and now I should increase the level of voltage here. So this is now 500 millivolts approximately; this is 200 millivolts. So the difference should be around 300 millivolts and you get 0.292 volt which is equivalent to nearly 300 millivolts. The two input voltages are subtracted and the result is the difference between the two inputs in this case 300 millivolts. It is possible for us to implement the subtractor using two op amps instead of one op amp as we saw in the earlier case. One op amp here is performing the inversion the second op amp is performing the summing. After inverting one of the inputs you add with the second input to obtain the difference between the two inputs. That is the trick that we play here.

Having seen the demonstration now let us move on to the next circuit which is a slightly modified version of the earlier circuits that we have seen. Here what we are going to see is a new circuit in which I am using both the non-inverting input and the inverting input as I used in the first case of the subtractor that I showed you except that I have used multiple inputs on both the nodes. Both at the inverting input and at the non-inverting input I have used multiple inputs.

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(All the V's are in small case. Pl check)

There are v_1 and v_2 connected to the inverting terminal v_3 and v_4 are connected to the non-inverting terminal and this is otherwise very similar to the first difference amplifier that we discussed earlier. Now I have got different values of resistors. For v_1 it is R_1 for v_2 it is R_2 R_3 and R_4 and R_f is the feedback resistor and R_5 is the resistor which is connected to the ground from the non-inverting input. We should measure the output. What will be the gain of this amplifier with so many inputs? v_1 v_2 are connected to the inverting input and the output is going to be $-R_f$ by R_1 times v_1 or $-R_f$ by R_2 times v_2 depending upon which one is connected. That is the gain corresponding to the resistors that I connected. With reference to non-inverting input v_3 v_4 are not directly connected to the non-inverting input. If you recall the discussion that we had on the superposition theorem when v_3 alone is connected then we connect v_4 to ground v_2 to ground v_1 to ground everything else is connected to ground only v_3 is applied. In that case v_3 is going to be divided by R_3 and the parallel combination of R_4 and R_5 . What I am going to get is V_3 multiplied by this effective resistance which is R_4 parallel R_5 divided by R_3 plus R_4 parallel R_5 . That is the factor by which v_3 will be reduced and that will be applied at the non-inverting input. So that will be amplified by a factor which is given by R_f divided by 1 plus R_f by R_1 . That will be the gain factor corresponding to these. Similarly for v_4 this v_3 will be grounded. Therefore R_4 and R_3 parallel R_5 will divide the voltage v_4 before it is being applied at the non-inverting input.

I will explain to you that in one case R_4 parallel R_5 divided by R_3 plus R_4 parallel R_5 will be the factor by which the input will be multiplied before it is applied at the non-inverting input.

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OPERATIONAL AMPLIFIER

Reduced by the voltage-divider factor of the channel, either :

$$\frac{R_4 \parallel R_5}{R_3 + R_4 \parallel R_5}$$

or

$$\frac{R_3 \parallel R_5}{R_4 + R_3 \parallel R_5}$$

The figure gives the equations for the gain of each channel. After getting each channel gain, we can calculate total output voltage.

In the other case corresponding to v_3 and v_4 R_3 parallel R_5 divided by R_4 plus R_3 parallel R_5 will be the factor by which that voltage will be applied at the non-inverting input. But for that the rest of the equation is very simple.

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OPERATIONAL AMPLIFIER

$$V_{out} = A_1 v_1 + A_2 v_2 + A_3 v_3 + A_4 v_4$$
$$A_1 = \frac{-R_f}{R_1}$$
$$A_2 = \frac{-R_f}{R_2}$$
$$A_3 = \left(\frac{R_f}{R_1 \parallel R_2} + 1 \right) \left(\frac{R_4 \parallel R_5}{R_3 + R_4 \parallel R_5} \right)$$
$$A_4 = \left(\frac{R_f}{R_1 \parallel R_2} + 1 \right) \left(\frac{R_3 \parallel R_5}{R_4 + R_3 \parallel R_5} \right)$$

Output voltage is nothing but $A_1 v_1 + A_2 v_2 + A_3 v_3 + A_4 v_4$ where v_1, v_2, v_3, v_4 are the voltages applied A_1, A_2, A_3, A_4 are the gains corresponding to the voltages that alone being applied rest of the thing being grounded. By using superposition theorem we can obtain the value of A_1, A_2, A_3, A_4 and then you can obtain the output voltage. Having said that what is A_1 ? A_1 is going to be $-R_f$ by R_1 . A_2 is going to be $-R_f$ by R_2 because they

are all applied to inverting inputs. In the previous case the A_1 factor comes from R_f by R_1 factor because I am applying to the inverting input. Similarly v_2 is multiplied by R_f by R_2 and so $-R_f$ by R_2 times the input voltage is what I will get when I connect to the v_2 input. That gain corresponds to A_2 . For A_3 because it is a non-inverting input I must get a gain of 1 plus R_f by something. The something here is R_1 parallel R_2 . Because R_1 parallel R_2 both will become grounded their parallel value will have to be taken into account multiplied by the factor which is used because of the voltage divider at the input about which I already explained to you. R_4 parallel R_5 divided by R_3 plus R_4 parallel R_5 . This is the fraction of the voltage that will be applied at the non-inverting input and the gain of the non-inverting amplifier is 1 plus R_f by R_1 parallel R_2 because I give v_3 alone all the other things are grounded. v_1 is grounded v_2 is grounded; that means R_1 and R_2 will come in parallel. The effective gain is 1 plus R_f by the parallel value of R_1 and R_2 . The parallel value of R_1 R_2 is what we refer as R_1 parallel R_2 . R_f divided by R_1 plus R_2 plus 1 will be the gain due to the non-inverting amplifier and this is due to the voltage divider that I have at the input and total gain is the product of these two terms. Similarly for the next input corresponding to v_4 the A_4 gain will be identical except that here R_3 will be replaced by R_4 and R_4 will be replaced with R_3 etc. This is the gain. Once we get the gain if you know all the input voltages we can measure or calculate the output voltage by this expression which is by the principle of the superposition.

Having said that it is a simple exercise we will try to do where we have given values for the various resistors. R_1 is 1K R_2 is 2K R_3 is 3K and R_4 is 4K; 1, 2, 3, 4 are the values of resistors. The feedback resistor is 6K and this resistor is 5K.

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OPERATIONAL AMPLIFIER

EXAMPLE :
 In Fig. $R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 3\text{k}\Omega$, $R_4 = 4\text{k}\Omega$, $R_5 = 5\text{k}\Omega$ and $R_f = 6\text{k}\Omega$. what is the voltage gain of each channel?

SOLUTION

What is the gain of each of the channel? There are 4 channels here v_1 , v_2 , v_3 and v_4 . What is the total gain and what is the gain of each of the channel? Actually what they want is the voltage gain of each of the channel. We already know how to calculate by using those

expressions. A_1 is $-R_f$ by R_1 . This is my R_f . This is my R_1 . $-R_f$ by R_1 and here $-R_f$ is 6 kilo ohms R_1 is 1K minus 6 kilo ohm by 1K. Therefore A_1 is -6.

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OPERATIONAL AMPLIFIER

With the equations given in fig. the voltage gain are

$$A_1 = \frac{-R_f}{R_1} \Rightarrow A_1 = \frac{-6k\Omega}{1k\Omega} = -6$$

$$A_2 = \frac{-R_f}{R_2} \Rightarrow A_2 = \frac{-6k\Omega}{2k\Omega} = -3$$

A_2 is equal to $-R_f$ by R_2 and R_f is -6K again. The minus sign comes because of the inversion and R_2 is 2K. Therefore -6K by 2K gives me a factor -3. So the channel gain for the second is -3. The gain of the first channel is -6. This is rather straight forward no problem because $-R_f$ by R_1 . But when we come to the non-inverting case the expression becomes slightly more complicated. But it is not very difficult.

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OPERATIONAL AMPLIFIER

$$A_3 = \left(\frac{R_f}{R_1 \parallel R_2} + 1 \right) \left(\frac{R_4 \parallel R_5}{R_3 + R_4 \parallel R_5} \right) \Rightarrow$$

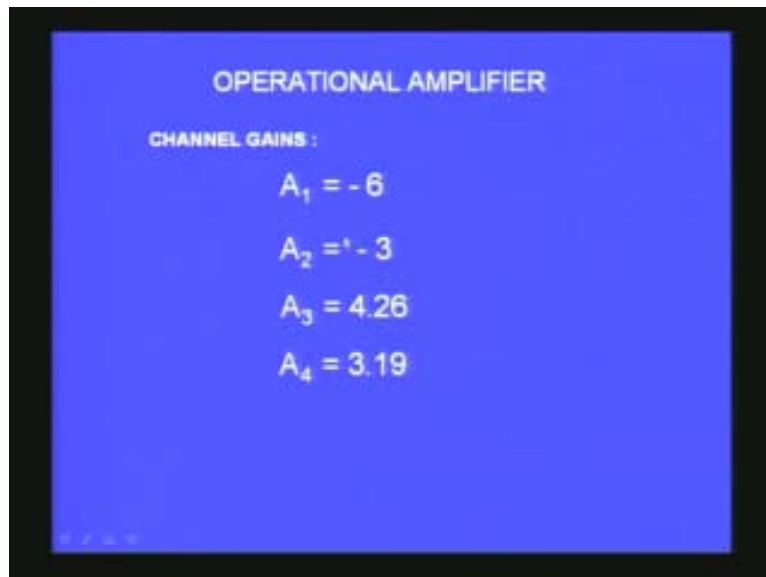
$$\left(\frac{6k\Omega}{1k\Omega \parallel 2k\Omega} + 1 \right) \frac{4k\Omega \parallel 5k\Omega}{3k\Omega + 4k\Omega \parallel 5k\Omega} = 4.26$$

$$A_4 = \left(\frac{R_f}{R_1 \parallel R_2} + 1 \right) \left(\frac{R_3 \parallel R_5}{R_4 + R_3 \parallel R_5} \right) \Rightarrow$$

$$\left(\frac{6k\Omega}{1k\Omega \parallel 2k\Omega} + 1 \right) \frac{3k\Omega \parallel 5k\Omega}{4k\Omega + 3k\Omega \parallel 5k\Omega} = 3.19$$

For A_3 R_f divided by R_1 parallel R_2 plus 1. R_f is 6K; R_1 R_2 are 1K and 2K. The parallel value of 1K and 2K you should find out and divide 6K by that value and then add 1. This will be the gain exclusively and this is the multiplication factor due to the potential divider I have and there R_4 parallel R_5 divided by R_3 plus R_4 parallel R_5 . R_4 is actually 4K R_5 is 5K. So I should find out the parallel value of 4K and 5K. The simplest method, you may recall we have discussed that in earlier lecture, is 4 into 5 divided by 4+5. 4 into 5 is 20; 4+5 is 9. 20 by 9 kilo ohm will be the answer for this and so that value I should evaluate and put it here divided by 3K plus again the same value. This will be the fraction of the voltage that will be applied at the input and that will be multiplied by the gain factor that I have here and if I do the simplification it is 4.26. The gain of the third channel is 4.26 by using the values of the resistor in the example. For the channel four it is again R_f by R_1 parallel R_2 plus 1. That part is the same because it is a non-inverting input. But the voltage divider that we have here is formed by R_4 and R_3 and R_5 are coming in parallel. Therefore R_3 parallel R_5 divided by R_4 plus R_3 parallel R_5 . This is the multiplication factor. We apply all the known values of resistors. R_f is 6K R_1 and R_2 are 1K and 2K R_3 R_5 are 3K and 5K and R_4 is 4K. When you substitute these values and calculate the effective gain, the gain factor is about 3.19. Again you can see the positive sign. These two are positive sign because they are applied at the non-inverting input of the amplifier. Totally all the channel gains are shown here.

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A_1 the channel one is -6. The channel two gain is -3 and channel three gain is 4.26 and the channel four gain is 3.19. Once we know the gains we can apply different voltages v_1 , v_2 , v_3 , v_4 and multiply appropriately by the corresponding gain factor and add them all together and that will be the voltage that we will get at the output. I would suggest that you should try this as a simple exercise and try to see whether you are able to measure the corresponding voltage in the lab when you have these values of resistors. So far what we have done is how you can have different types of a mathematical operation perform summation and subtraction. In the subtraction or the difference amplifier configuration I

discussed two different types. One involving both non-inverting and the inverting input, the other one involving two operational amplifiers; one performing the inversion and the other performing the summation. I also showed a demonstration of the working circuit.

Now let us move on to the next mathematical operation. The next mathematical operation that we would like to discuss is integration.

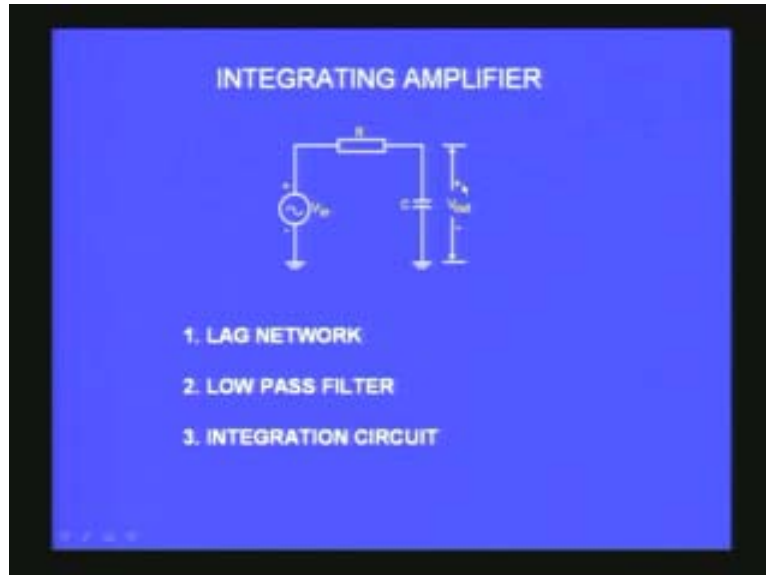
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How an operational amplifier can be used to perform a mathematical operation of integration on an input signal is what we will take up now. You know y is the integral of $x dx$ or in this case actually with reference to time. Therefore V output of an amplifier is k times integral of $V_{in} dt$ differentiation with reference to time. This is an integration performed on the input voltage and k becomes a constant factor or the gain factor if you want. The output is proportional to the time integral of the input voltage. This becomes an integration circuit. Already we have discussed a simple integrating circuit with networks. We talked about one resistor and capacitor being connected and if you take the voltage across the capacitor it becomes a lag network. When we discussed about the frequency response of amplifiers we discussed this circuit. With a simple R_C configuration if you apply the input between the R and C and take the output only across the C you will get a lag network because there is a phase lag introduced due to the R_C . This also can be looked at as a filter circuit. You are applying an ac and you know the reactance offered by a capacitor to an ac signal. The reactance offered to ac signal is $1/\omega C$ magnitude wise where ω is $2\pi f$ where f is the frequency of the signal. $1/\omega C$ the frequency factor is coming in the denominator and if you increase the frequency the denominator becomes larger and larger and effective voltage coming out of this will become smaller and smaller. When the frequency is very low the capacitor offers large resistance. When the frequency is increased the capacitor offers very low resistance and initially for low frequencies I will get enough voltage at the output because the value

of this potential divider, the value of X_C is large initially for low frequency so I will get very good voltage at the output.

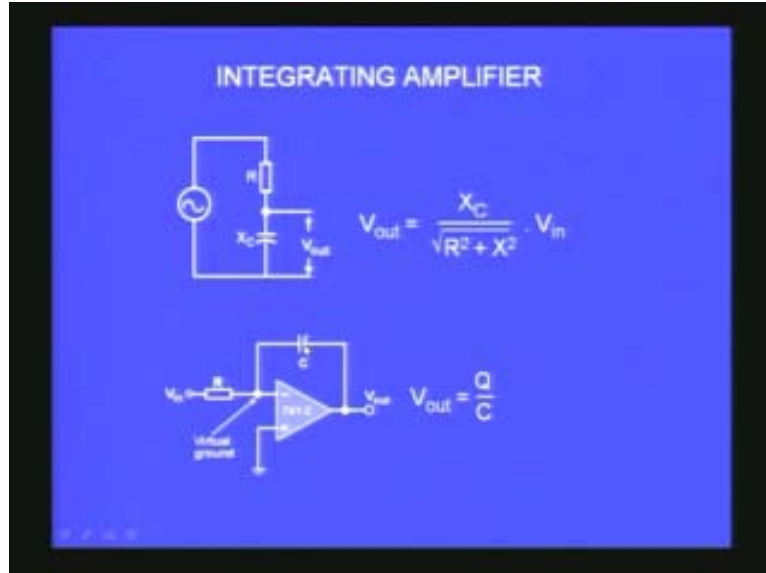
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But as I increase the frequency this reactance or the impedance offered by the capacitance will keep coming down and the output voltage will keep coming down. Beyond some limit very little output will be coming. This circuit is called a low pass filter. It is able to pass the low frequencies. Low frequencies will pass in this filter and it is called a low pass filter. If I send high frequencies very little voltage will be passed at the output. It can be understood that at high frequencies the capacitor offers very low reactance and they will short. All the voltages will go to the ground or complete the circuit using the capacitors. Nothing will come across the load whatever that I connect beyond this point. This is a lag network that we have seen and it is also a low pass filter and it is also integration circuit. In principle when I take the output from the capacitor the circuit behaves like an integrator. How is that?

Let us look at the simple mathematical expression that you already are familiar with. When I take the output using the potential divider R and C if I take the output across the C or the X_C here, the reactance factor V_{output} is equal to X_C divided by R plus X_C but in terms of magnitude it will also have the phase because of the capacitance. In terms of magnitude it will be X_C divided by root of R square plus X square times V_{in} . This is the factor by which V_{in} will be multiplied to obtain the output. This already we know. Now when I have the capacitor using an op amp how you can implement an integrating circuit? You can just implement using the configuration I have shown on the screen. You have the R which is actually at the input resistance with the V_{in} and the capacitor comes in the feedback loop you are connecting between the inverting and the output terminals.

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Now what is the output? The non-inverting input is grounded. So this is a very simple configuration of an operational amplifier integrator. You have connected the inverting input to the ground. This point is going to be the virtual ground, virtual earth. It is almost effectively equivalent to ground. If I now measure the voltage at the output how do I measure a voltage? I measure the output voltage between the output terminal and the ground. All voltages are normally measured with reference to a common ground which I call earth or ground. V_{out} is measured between this terminal and the ground terminal. But this point is almost close to the ground terminal because this is a virtual earth point and the V_{out} actually is a voltage between this terminal and this terminal at the inverting input. But what is this? That is nothing but the voltage across the capacitor due to the input signal. The output voltage is nothing but the voltage that is developed across the capacitor. What is the output voltage across the capacitor? Voltage across the capacitor is given by Q by C where Q is the charge in coulombs and C is the capacitance in farads. The V_{out} is equal to Q by C is a well known expression from basic electricity.

What is Q ? Q is the charge and what is charge? It is nothing but the total amount of current flowing for a finite time. It is nothing but integral of idt . What is current? Rate of flow of charge. I is equal to dQ by dt ; rate of flow of charge and Q is equal to integral idt . Collect the current flowing for a finite time and that gives you the total charge accumulated on the capacitor. V across the capacitor V_C is equal to Q by C from the fundamentals of electricity and that is nothing but integral idt divided by C . What is I ? In this case because you have a virtual earth here if you apply V_{in} what is the current? The current is V_{in} by R . None of the other components on the other side will affect this because this is virtual ground. The input current will be V_{in} by R . That is why here in place of the I we write V_{in} by R into dt by C we retain as such.

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INTEGRATING AMPLIFIER

Voltage across capacitor

$$V_C = \frac{Q}{C} = \int i dt$$
$$V_C = -\int \left(\frac{V_{in}}{R}\right) \cdot \frac{dt}{C}$$
$$V_{out} = V_C = -\frac{1}{RC} \int V_{in} dt$$

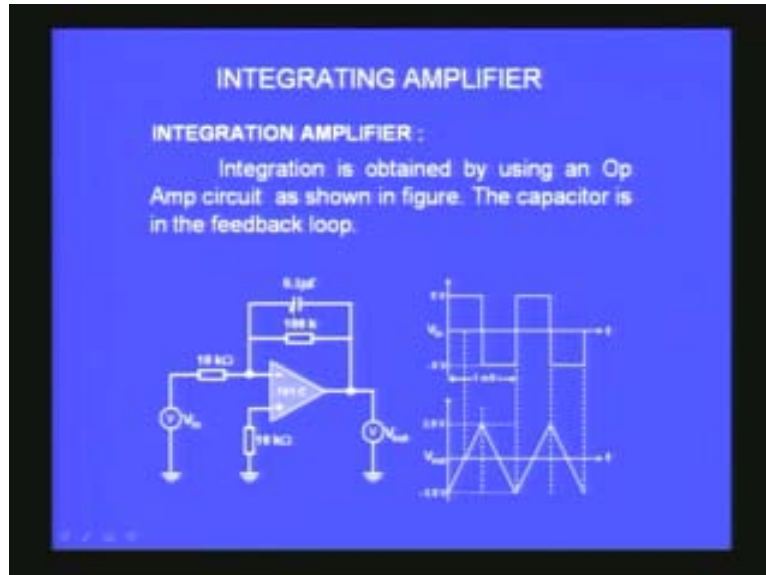
There is a R here there is a C here. These two are constants of the circuit and I can take it out. They are not going to be affected by the integration. The output voltage which is actually the voltage across the capacitor is nothing but -1 by R_C integral $V_{in} dt$. V_{in} is changing with time and it has to be kept inside the integration. So integral $V_{in} dt$. Output voltage is proportional to V_C is equal to V_C the voltage across the capacitor and that is equal to -1 by R_C integral of $V_{in} dt$. So it is precisely the type of circuit that we wanted to design where output is a time integral of the input. This circuit will perform as an integrator. How do you check whether it is performing like an integrator? If I give a sine wave what do you expect at the output?

What is the integral of a sin? It's a cosine function. It will again be a sine function. You will not be able to see the difference. There will be a phase difference but unless you take great care you will not be able to detect the change in the phase. Everything will look like a sine wave. Sine wave and cosine wave will look exactly identical except for the phase factor and it becomes difficult for us to detect the integration performed on the input voltage. Therefore let us take a different wave form. One of the different wave forms that we can take up is a square wave. You can take a square wave and apply as an input for a finite frequency and finite amplitude and then see what happens to the output voltage from the circuit?

I have given you a very typical circuit of an integrator. It is very similar to the basic integrator that we already discussed except for one or two small changes. The changes are you can see I have added a high resistance 100K ohms in this case across the capacitor that I have used and the capacitor value is 0.1 micro farad. The input resistance is 10K and I have also connected a resistor equivalent to 10K which is actually the value of the other resistor R_1 , the resistor corresponding to the non-inverting terminal. I will explain to you why I do that little later. To compensate for any differences in the input bias current we add this additional resistor. Even without the resistor circuit in principle

should work well if it is a good op amp. This 100K is basically added to make sure that initially the capacitor is fully discharged.

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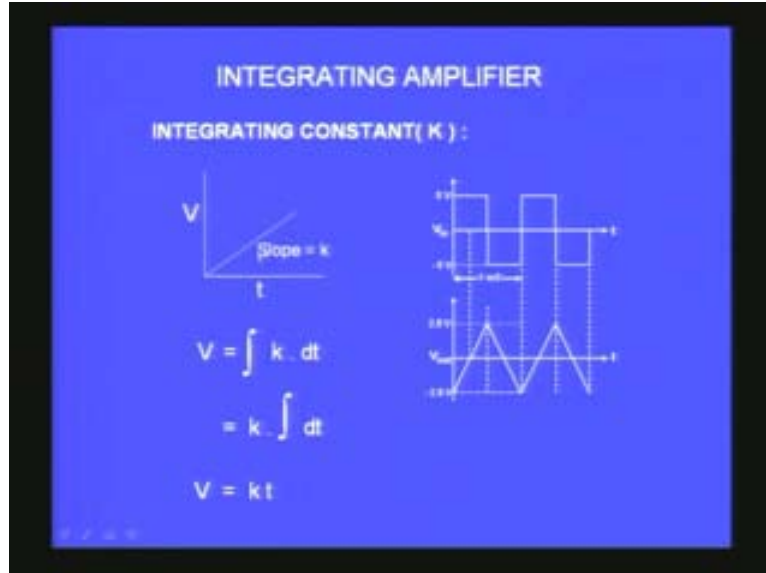


Because there is a resistor **path** available even if there is any residual charge residing on the capacitor it is discharged through the 100K and there will be no charge there when the input signal arrives at the input. That is the reason that we use the 100k. This is a circuit suggested by the manufacturer himself. If you look at the data manual of 741 op amp you would find it does have a same circuit given as an applications circuit by the manufacturer.

Now let us look at how we understand the wave form that I give. I give a square wave. This is the input wave form on the right side I have shown with the amplitude of +5 to -5. This is the +5 -5 amplitude square wave at one milli second is the period. That means it is 1 kilo hertz frequency. I apply 1 kilo hertz square wave at the input of integrator and what you get out? You get out a triangular wave and the output amplitude is + or - 2.5. Maximum amplitude is + or -2.5. When I give + or -5 volts square wave at 1 kilo hertz you get a + or -2.5 triangular wave as the output if you perform integration. But how do we know this is integration. I will briefly explain to you how it comes about.

Let us understand what is the time integral of a constant? V is equal to $\int k \cdot dt$ where k is a constant. Because k is a constant I can take the k out. It becomes $k \int dt$ and $\int dt$ is very simple if you integrated it to be t and V is equal to kt . This is the integrated value of the output. What is this? k is the constant t is the time. As time goes the voltage keeps increasing monotonically continuously with the slope equal to k .

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When I have a constant value if I integrate the constant value I get a linearly varying function at the output which is varying with reference to time. When I have a constant voltage of 5 volts at the input when I integrate it I should get a linear voltage slowly increasing from -2.5 to +2.5 with reference to time. Then once I reach this point suddenly the input voltage because it is the square wave it becomes -5 volts from +5 volts and remains constant at -5 volts. During that time the k has now become $-k$. That is all. It will start decreasing at a steady rate depending on the value of k with a negative slope it reaches -2.5 once again during that period. Then again the input voltage increases to +5 volts and the voltage starts increasing because now k has become positive the voltage starts increasing at the output. This zigzag or a triangular wave form is obtained whenever you give a square wave at the input. There is no change in the frequency. Both have got the same frequency. There is no frequency change. But there is an amplitude change; from +5 to -5 you now have +2.5 to -2.5. You must be able to understand that also.

Now let us take a typical example of the circuit that we discussed where the R which is connected parallel to capacitor is 10K; C is 0.1 micro farad. The capacitance in the feedback is 0.1 and the RC time constant is the product of these two 0.1 and the 10K that comes to around 10 millisecond which is larger than the applied frequency. I assume 1 kilo hertz square wave is applied. You should make sure that R that you use across the capacitor should have the time constant with reference to that capacitor much larger than the time period of the input signal. Substituting in this equation V output is equal to V_C is equal to $-\frac{1}{RC} \int V_{in} dt$ which is the expression for the output voltage from an integrator. What is $1/RC$? R is 100K; C is 0.1 microfarad. When you multiply them you get 1 by 10 power -3; $\int V_{in}$ is 5 volts multiplied by dt and the dt you have to integrate for half the pulse 0 to t by 2. We can also do for the whole but during the period the voltage goes from 0 to 5 volts.

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INTEGRATING AMPLIFIER

When $R = 10\text{ K}$, $C = 0.1\ \mu\text{f}$, Then
 $RC = 10\ \text{mS}$. Assume $1\ \text{kHz}$ square wave is
applied at the input.

substituting in the eqn.

$$V_{\text{out}} = V_C = -\frac{1}{RC} \int V_{\text{in}} dt$$
$$V_{\text{out}} = -\frac{1}{10^{-3}} \int_0^{T/2} 5 \times dt = -\frac{1}{10^{-3}} 5 \times 0.5 \times 10^{-3}$$
$$V_{\text{out}} = 2.5\ \text{V}$$

The voltage V_{in} at that time is +5 volt multiplied by dt and that is -1 by 10 power -3 into 5 into dt. When I integrate it becomes t by 2. t by 2 is 0.5 millisecond. t the period is 1 millisecond. Therefore it is 0.5 millisecond. 10 power -3 corresponds to seconds; 10 power -3 second is 1 millisecond. If you calculate the voltage V output will be 2.5 volts. The maximum voltage at the input when it is 5 volts the maximum voltage at the output is 2.5 volts.

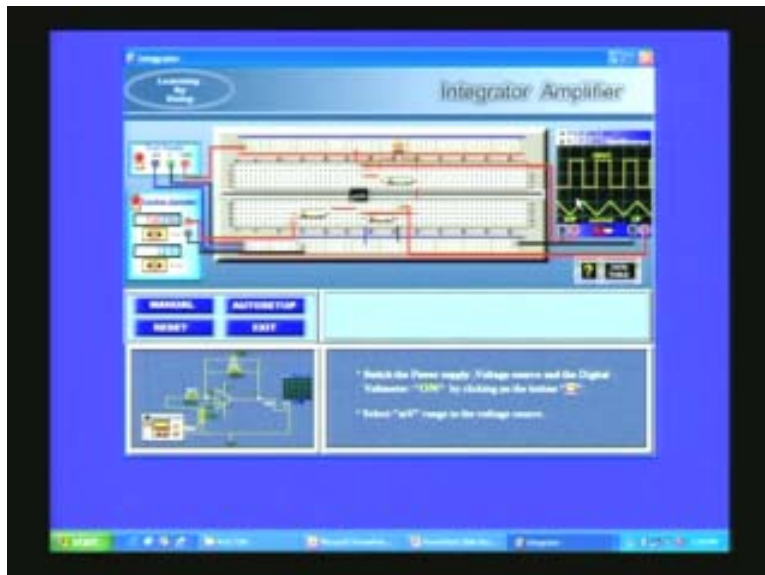
You can perform an actual experiment also. Right now we will perform a simulation. You have a bread board, you have a dual supply and you have a function generator which will generate different frequencies as well as different output voltages and on the right side I have an oscilloscope simulated of the screen. This is a very familiar scheme. You have already seen such demos earlier. Let me do auto set up. When I do auto set up the op amp and the various components will go automatically and position themselves on the bread board and the rest of the wiring will be completed very quickly on its own automatically. The circuit is completed. The feedback resistor is 10K here 10K here and that is 100K.

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Now I will switch on the power supply. I switch on the function generator and I switch on the oscilloscope. Let me give some frequency. Let us say it is about 500 hertz and the output is around 1 volt. I give 1 volt and 500 hertz input. The output is actually a triangular wave. With the square wave as the input you get a triangular wave at the output. You can also see approximately the amplitude is decreased by half.

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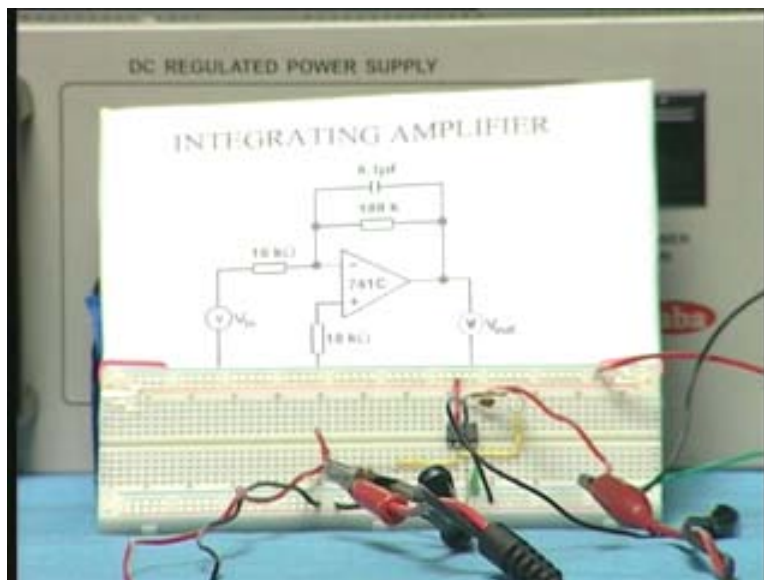


Now if I increase the frequency to 1 kilo hertz you can see the corresponding input output wave form. A simple square wave at 1 kilo hertz becomes a triangular wave at half the

amplitude. This is a demonstration or simulation of the integrating circuit. We will close this and move on to the demo.

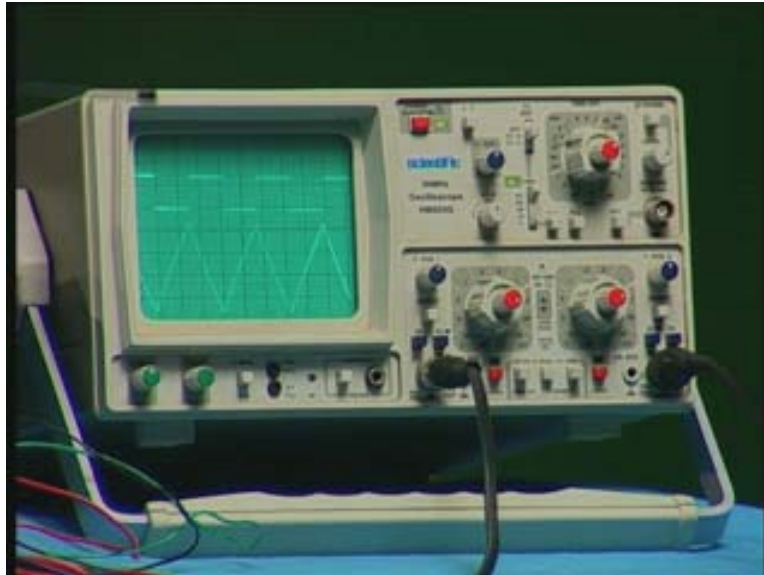
We will now go to the demonstration table and show you the working circuit of an integrating circuit. Here I have the integrating amplifier. The circuit is seen here. It is exactly the same circuit which I showed you already. You have an op amp, you have a 10K input resistor and you have 100K and a 0.1 microfarad connected in parallel along the feedback path and you have another 10K connected to the positive input or the non-inverting input. I have already told you this is connected in order to take care of the input bias current variations and this is to provide a time for the charges across the capacitor to discharge and be ready to receive the next pulse. This is going to be a function generator.

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You see here a function generator. This output can be used for measuring either the amplitude or the frequency. So you have the amplitude variation and the frequency variation knobs here. The output of this is connected to the input of this op amp circuit shown on the bread board. You have the 741 IC here and this 100K, 10K all the resistors are connected and the output is connected to the oscilloscope. Both the input and the output are connected to the two channels of the oscilloscope. I hope you are able to see the square wave which is generated from this function generator. I slightly change the output amplitude. The output is changing when I change. I have selected square wave here. We have square wave, triangular wave, saw tooth wave, different types of wave functions can be generated. I have now pressed the square wave. It is generating square wave and here I can select the frequency range. I have pressed the 1K here. It is generating 1 kilo hertz square wave approximately and that is what is seen on the oscilloscope here. You only see two dashed lines top and bottom and the vertical line is not seen because it is too fast.

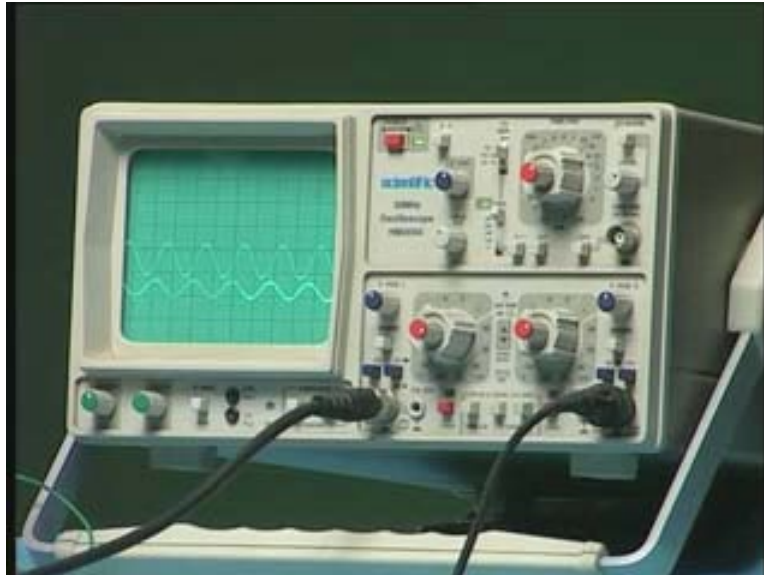
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It is basically a square wave continuously moving up and down. That is the input signal. The output signal is connected to the other channel and you can see the triangular wave coming on the screen. This is the output because it is an integrated output of the input voltage and when I apply a square wave I must get a triangular wave at the output and that is what I get here. Now I will try to reduce the gain. Now you can see the triangular wave and the square wave separately very well and if I change the frequency you can see there is a change. For example I will go to 10 kilo hertz that is too fast. When I change the frequency for a very large range of frequencies I still have the triangular wave as such without any distortion and as I increase the frequency the amplitude of the square wave is almost constant. The amplitude of the square wave is almost constant but the amplitude of the triangular wave keeps decreasing. Why is it happening like that? When I increase the frequency 1 by ωC , the ω is in the denominator and the gain starts coming down and the output becomes smaller and smaller as I increase the frequency. Even here it is almost triangular over a wide range of frequencies. But you cannot go to very high frequency. If I go to very high frequencies it will not be all that nice because this is basically a filter circuit. It is a low pass filter. Low frequencies it will be able to pass and high frequencies it will not be able to pass well. The reactance becomes very very small and the output voltage becomes too small and you will not be able to operate the circuit at very high frequencies. But right now this circuit is used for performing integration and when I give a square pulse I get the triangular wave.

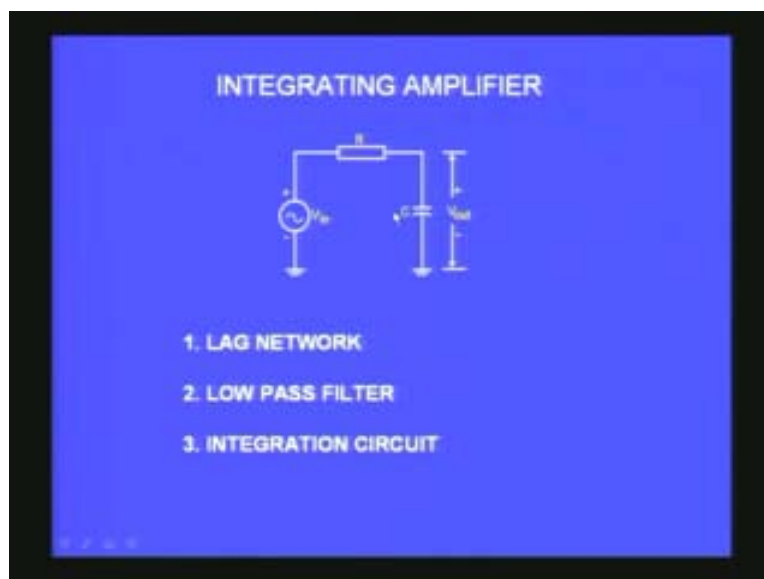
If I change this to sine wave you also get a sine wave because integration of a sine wave is also a cosine function but that cannot be seen unless I lock them and see the phase difference.

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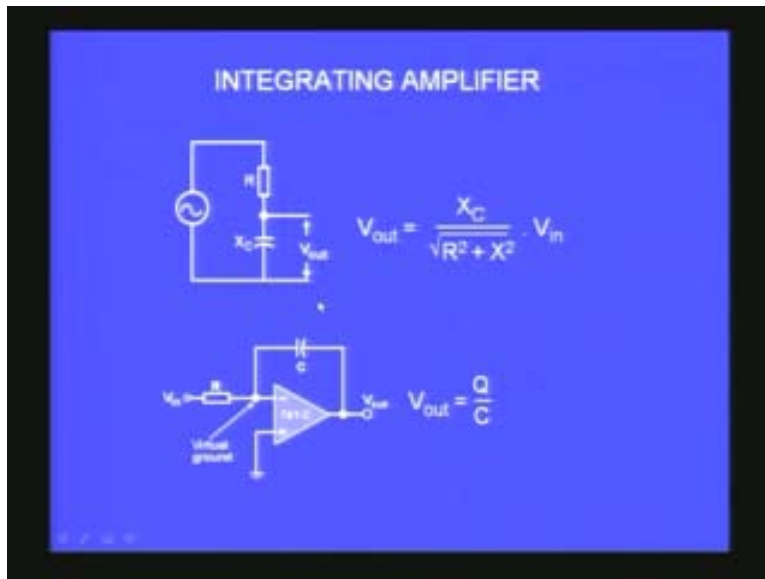
You do see a slight shift in the phase but it is not completely clear because unless you lock them or synchronize the two signals you will not be able to measure the exact phase difference that is produced. But in principle you can see both of them are sine wave. When I go to square wave you find one of them is a square wave. The output becomes a triangular wave. This corresponds to the integration of the input voltage. We can obtain the function of integration by using simple RC without the op amp. I have already told you in the earlier case. For example this circuit which you see on the screen is basically a simple integrating circuit only with R and C. Then why do we require op amp?

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If I have only this circuit the output can never be larger than the input. Because it becomes a potential divider and I am obtaining the output from one of the reactance's that I have here; two reactance's. It will always be a fraction of the input. It can never be larger. Whereas when I go to an op amp I had several advantages over that. What is the advantage that I have here?

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First I get the concept of the virtual earth coming here. The input and output are perfectly isolated. The input generates V_{in} by R as a current because this is a ground point. The output is measured between this point and this point. This is also a ground, virtual ground. That is the voltage developed across the capacitor. I can choose any current here. Without being affected by the presence of capacitor I can change the voltage here or the resistor here and obtain different currents. This is a great advantage compared to a simple RC circuit. Operational amplifier enormously improves the performance of a normal integrator. The other point I want a mention to you is I can choose my R and C appropriately. For example if I choose R as 1 megohm and C as 1 microfarad what is the value of RC ? R is 1 megohm C is 1 microfarad. 1 megohm means 1 into 10 power 6 ohm. 1 microfarad means 1 into 10 power -6 farad. When I multiply this, the 10 power -6 and 10 power +6 will cancel each other and I will just get 1 second as the time constant. In this expression V_{out} is equal to V_C is equal to 1 by RC integral, the multiplication factor becomes 1 because RC is now 1 for this choice of R and C and output is integral of input directly.

But I can choose R and C in such a way that I will get a finite value for 1 by RC . For example instead of 1 megohm I choose 10 megohm. 10 megohm and 1 microfarad will give me a factor of 10 at the denominator. It will be 0.1 times; the integrator will be multiplied by a factor 0.1. So there is attenuation.

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INTEGRATING AMPLIFIER

Voltage across capacitor

$$V_C = \frac{Q}{C} = \frac{\int i dt}{C}$$
$$V_C = -\int \left(\frac{V_{in}}{R}\right) \cdot \frac{dt}{C}$$
$$V_{out} = V_C = -\frac{1}{RC} \int V_{in} dt$$

But instead if I use 100K in place of 1 megohm it will be 10 power 5. Then I will get a factor 10 in the numerator. That means it will be 10 times the integrated value that I will get at the output. In principle I can increase the gain of the integrator by the choice of RC values and I can get here gain as well as attenuation in this case which is not possible when I use discrete R and C circuit.

In conclusion we have done a simple difference amplifier and an integrating circuit. We discussed working of the integrating circuit amplifier and also showed a demonstration of an operational amplifier based integrator. In the next lecture perhaps we will take up a differentiating circuit and show you how a differentiating circuit can be constructed. Thank you!