Basic electronics
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Lecture- 17

Frequency Analysis

Hello everybody! In our series of lectures on basic electronics learning by doing we will move on to the next topic. Before we do that it will be better to recapitulate what we learnt in some of the previous lectures.

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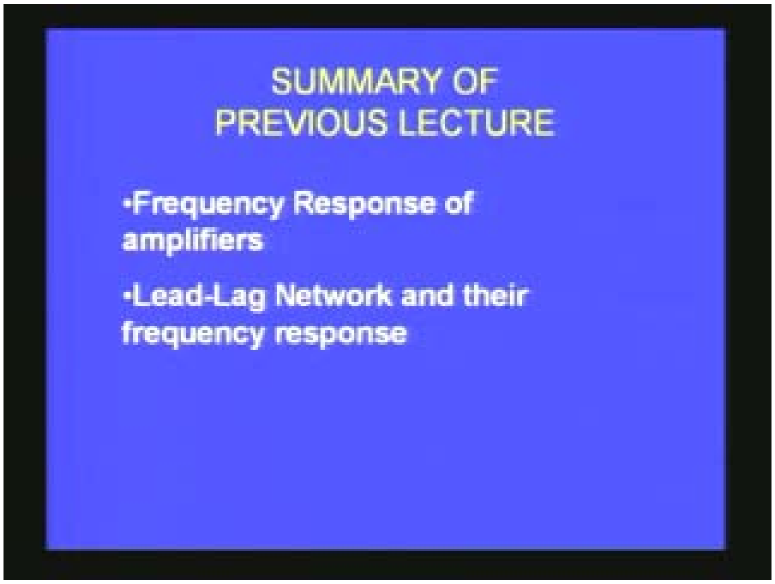

SUMMARY OF PREVIOUS LECTURE

- Frequency Response of amplifiers
- Lead-Lag Network and their frequency response

You might recall in the previous lectures we discussed about the frequency response of amplifiers. In order to do that we actually looked at some simple networks like the lead network, the lag network and then looked at the frequency response of these two circuits because we understood that the frequency response of an amplifier can be understood in a simple manner by looking at the contributions to the frequency response from different capacitive reactances that we have in a given circuit on the basis of equivalent lead-lag network. That is the reason we discussed about the frequency response of the lead network and the lag network independently in the last lecture.

Before we go on to the actual thing I will again show you the general graph of the band width of an amplifier. You can see on the x-axis the frequency scale is there in logarithmic scale, 10, 100, 1000 etc, and in the y-axis you have the gain, magnitude of the gain which is basically the output voltage by the input voltage.

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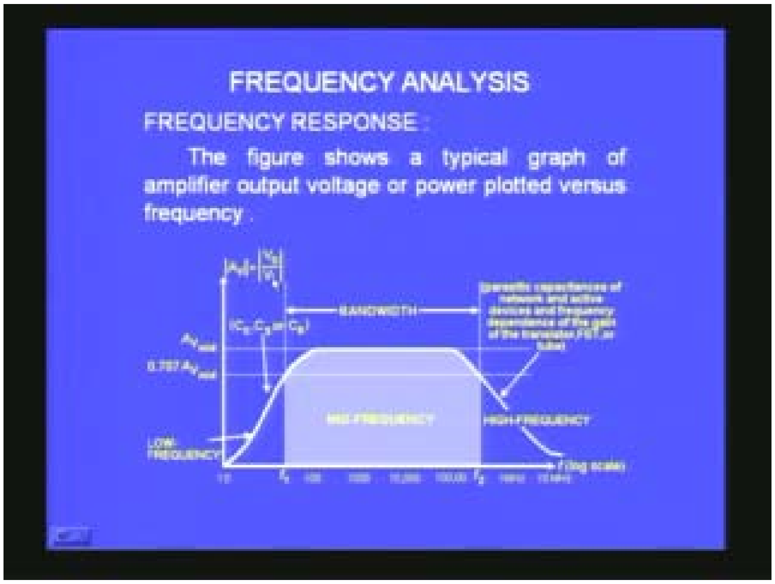

FREQUENCY ANALYSIS

FREQUENCY RESPONSE :

The figure shows a typical graph of amplifier output voltage or power plotted versus frequency .

Approximately over the mid frequency range, over a wide range of frequencies you find the gain is almost a constant straight line, flat straight line corresponding to gain of voltage at mid frequencies and as you come below you find at very low frequencies the gain starts falling of almost to zero and similarly at high frequencies also you find the gain starts falling of beyond certain frequencies. When we discussed our h parameters, equivalent circuits for the basic transistor amplifier, common emitter amplifier we ignored the contributions due to the various capacitors like the coupling capacitor, the bypass emitter capacitor, etc, because the reactance offered by these capacitors will be very, very small. We can almost take them as a short, for practical purposes, at those frequencies. But when you want to focus your attention to the frequency behavior of such amplifiers at very low frequencies then you cannot ignore any more the contributions from these capacitors and that is why here the arrow shows this fall in the frequency at low frequency regions is due to the contributions of the reactances of the various capacitances like the C_1 the coupling capacitors, the output capacitor and the emitter capacitor.

Similarly if you look at the high frequency, the contributions to high frequency fall comes from several other new contributions like the parasitic capacitances about which I will perhaps explain in a moment and the various other dependents for example of the gain of the transistor beta or the ….. This is the general scheme of the band width of amplifier which takes into the account all range of frequencies; the low frequency, the high frequency and the mid frequency.

Let us start looking at the general discussion on the frequency response of an amplifier. You see on the screen the transistor, common emitter amplifier with a voltage divider network R_1 and R_2 and you have a coupling capacitor C_1. You have the signal source V_s and R_s is actually the source resistance of this V_s and you have the R_C the collector resistor. You have R_L the load resistance and the C_2 is the coupling capacitor at the output connecting to the load and C_E is the bypass capacitor across the emitter bias R_e.

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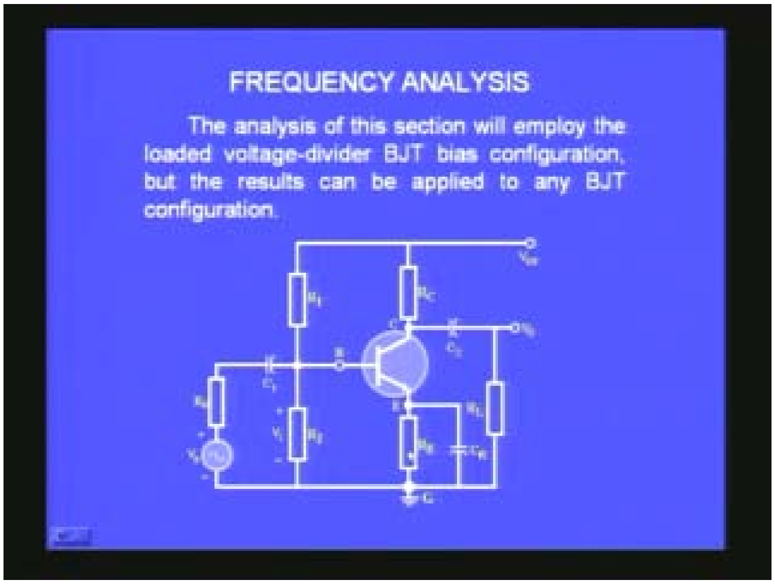

FREQUENCY ANALYSIS

The analysis of this section will employ the loaded voltage-divider BJT bias configuration, but the results can be applied to any BJT configuration.

This is a very familiar circuit. I am sure we have discussed this earlier when we discussed about the biasing circuits as well as later on and this is very familiar to you. But when we drew the equivalent circuit for this circuit earlier we ignored the contribution to C_1, C_2, C_E, etc. That we cannot do any more because we want to look at the low frequency, high frequency response also of these transistors. The best way to do that is try to find out the contribution due to each one of these capacitances alone ignoring the presence of others and then try to look at the equivalent circuit and then try to see whether it belongs to a very specific lead like network and then we have already understood the performance of the lead network and from that we can discuss the frequency response of the transistor. By this method we will be able to know the contribution to the cut off frequency. What will be the cut off frequency corresponding to each one of these contributions and finally we will take out the one which is most dominant, the one which is most responsible among these things and that will be the one which will decide the overall bandwidth of the amplifier. So this is a very standard simple scheme that we will adopt.

So we have to see the contribution to C_1, C_2 and C_E. These are the capacitors that will contribute to the low frequency response of the amplifier.

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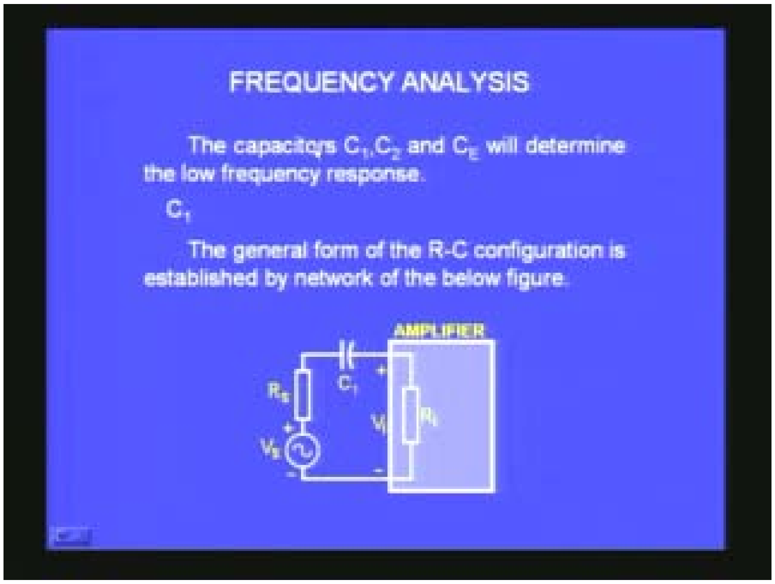

FREQUENCY ANALYSIS

The capacitors C_1, C_2 and C_E will determine the low frequency response.

C_1

The general form of the R-C configuration is established by network of the below figure.

Let us consider one by one. Let us take C_1 which is the coupling capacitor at the input. I have here drawn the equivalent circuit ignoring the rest of the information and only showing the amplifier with reference to its input resistance R_i and you have the C_1 and you have the source resistance and the signal source. This only is shown and you can see that it is a very simple circuit that we have already discussed earlier. The total resistance now is R_s plus R_i and the cut off frequency, we know already, f_1 is 1 by 2 pi R_c; here R is R_s plus R_i. So f_1 is 1 by 2 pi R_s plus R_i into C_1. So this is the contribution. This is frequency corresponding to the point at which the mid band gain will fall to 0.707 times the normal maximum gain.

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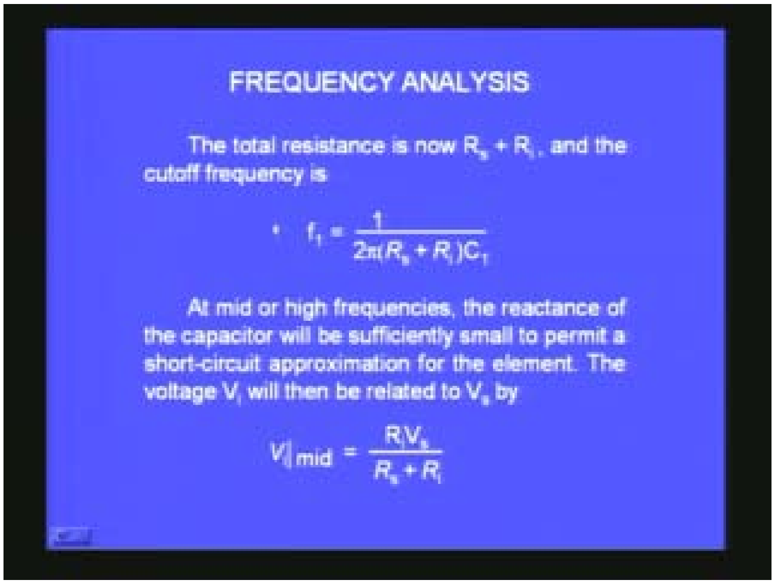

FREQUENCY ANALYSIS

The total resistance is now $R_s + R_i$, and the cutoff frequency is

- $$f_1 = \frac{1}{2\pi(R_s + R_i)C_1}$$

At mid or high frequencies, the reactance of the capacitor will be sufficiently small to permit a short-circuit approximation for the element. The voltage V_i will then be related to V_s by

$$V_i|_{mid} = \frac{R_i V_s}{R_s + R_i}$$

If I look at mid band frequencies this C_1 will be very low. The contribution from this will be very low and it can be taken as the short, the reactance component and we ignore this. But at low frequencies we have to take it into account. Similarly at the mid frequencies the actual voltage applied at the input terminals of the amplifier V_i will be equal to R_i V_s divided by R_s plus R_i because the R_s and R_i will act as a simple potential divider and the voltage across the R_i is the one which is going to be applied across the amplifier. So R_i into V_s divided by R_s plus R_i gives you the input voltage at the mid band frequencies.

If you look at the frequency response corresponding to that network it will have very low gain at very low frequencies and as you increase the frequency the gain slowly increases and finally around the mid band range it becomes a constant.

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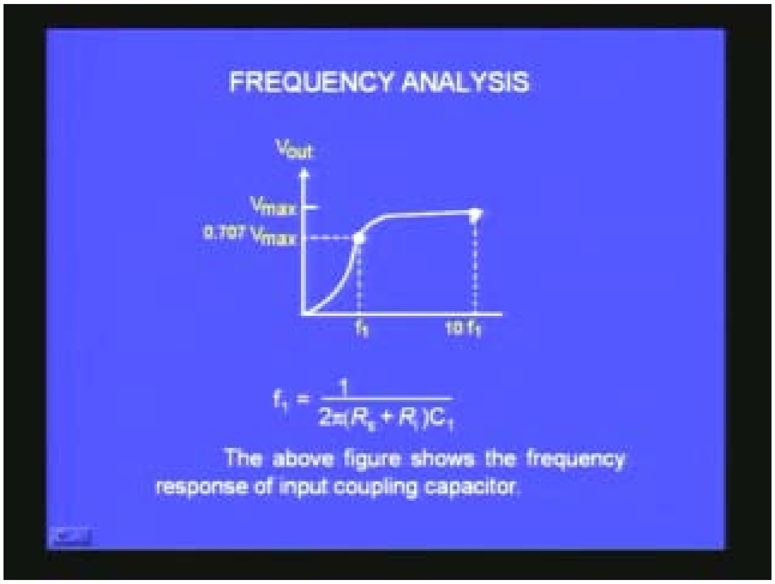

FREQUENCY ANALYSIS

$$f_1 = \frac{1}{2\pi(R_s + R_i)C_1}$$

The above figure shows the frequency response of input coupling capacitor.

This is the contribution due to the capacitor C_1 which is called f_1 which is given here again as 1 by 2 pi R_s plus R_i times C_1 and the R_s plus R_i are the two resistances. R_i is the input resistance of the amplifier, R_s is the signal source resistance that we have already discussed. What is that R_i? We all know what that contribution to R_i is. The Ri is contributed in the case of a rc coupled amplifier that I showed you already, a common emitter amplifier with the voltage divider bias. The R_1 parallel R_2 will be one contribution and the h_{ie} which is equal to beta times re which we have already discussed this is the other contribution.

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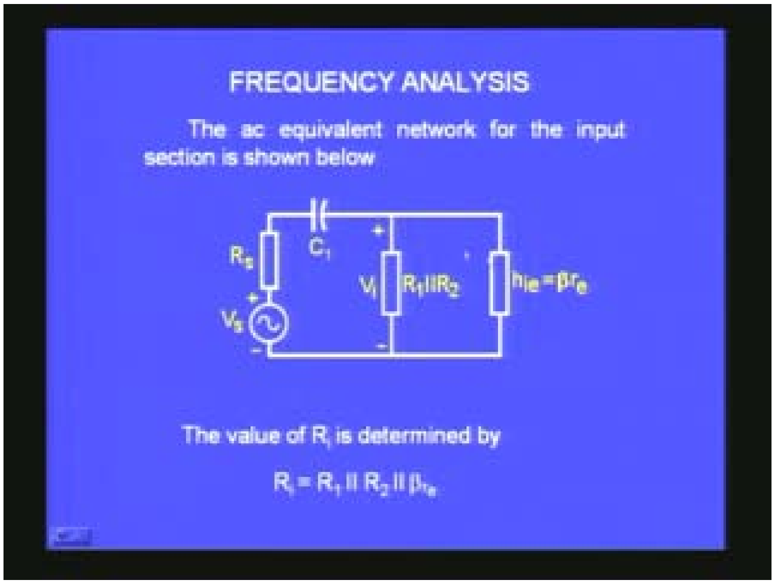

FREQUENCY ANALYSIS

The ac equivalent network for the input section is shown below

The value of R_i is determined by

$$R_i = R_1 \| R_2 \| \beta r_e$$

The resistances at the R_i will be the parallel resistance of these two resistors. So R_i is nothing but R_1 parallel R_2 parallel beta times r_e. So that plus R_s will be the total resistance in the circuit with the C_1 and the output voltage at these low frequencies will be R_i multiplied by V_s divided by the potential divider which is R_s plus R_i minus j X_{Cs}. (X_{Cs} seen in slide; audio says X_{C1}- ?)

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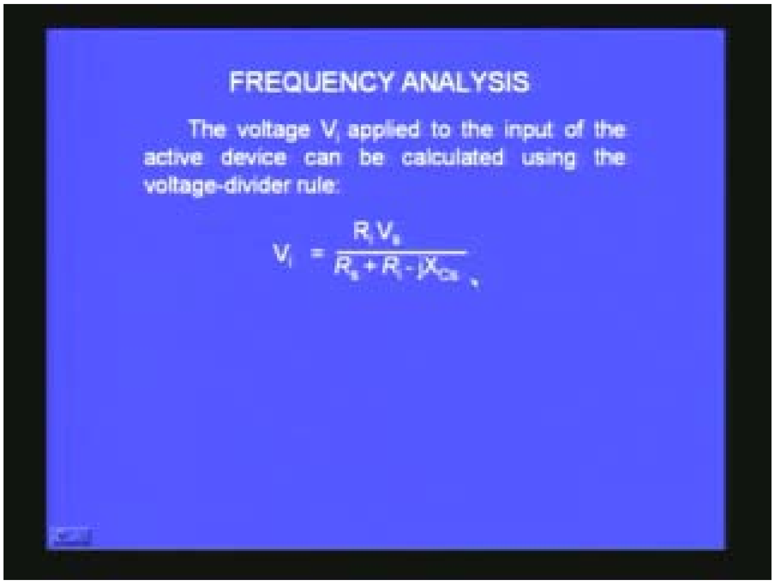

FREQUENCY ANALYSIS

The voltage V_i applied to the input of the active device can be calculated using the voltage-divider rule:

$$V_i = \frac{R_i V_s}{R_s + R_i - jX_{Cs}}$$

What is that $-jX_{C1}$? That is the contribution due to the reactance component corresponding to the capacitors. That also has to be this reactances. It cannot be ignored any more.

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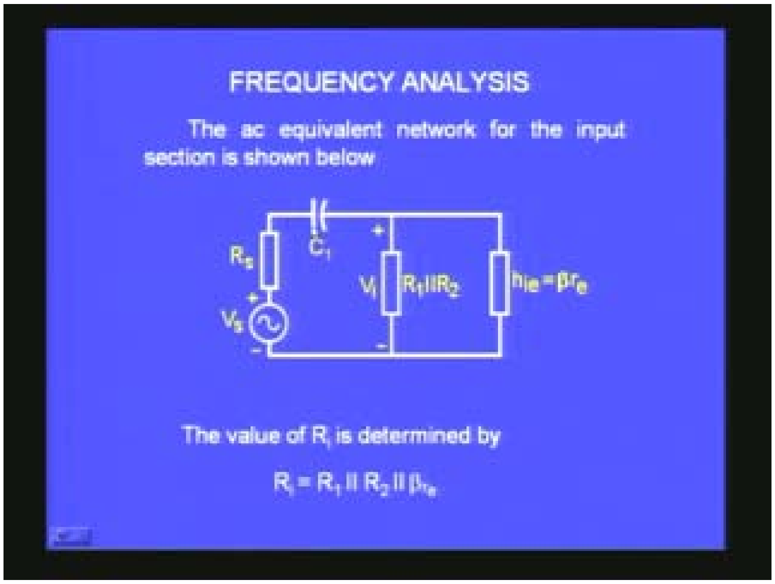

FREQUENCY ANALYSIS

The ac equivalent network for the input section is shown below

The value of R_i is determined by

$R_i = R_1 \| R_2 \| \beta r_e$

The contribution is there at low frequencies and we should take it into account. That is what we have shown here.

If we want to move on to the contribution due to the other coupling capacitor at the output which is C_2 the network can be now redrawn corresponding to that.

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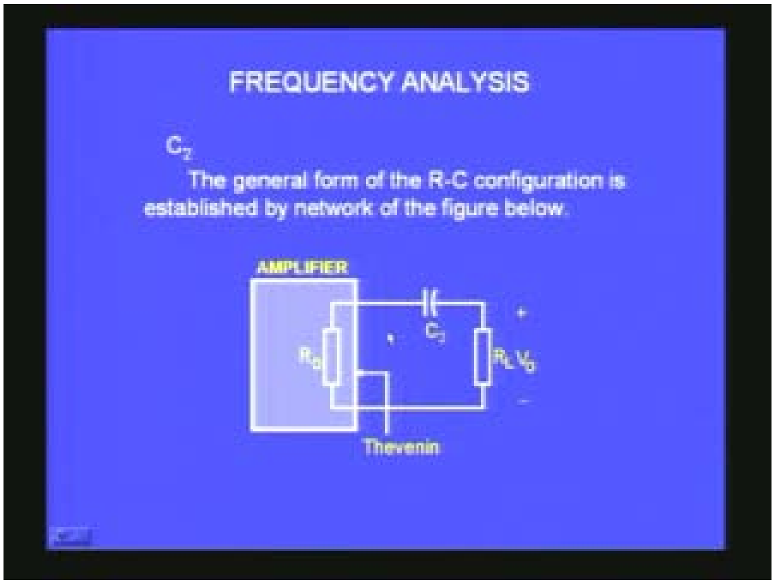

FREQUENCY ANALYSIS

C_2

The general form of the R-C configuration is established by network of the figure below.

You have R_L, the load resistance and C_2 the coupling capacitor which is of interest to us now and R_o the output Thevenin's resistance of the amplifier. This is the circuit that we should now look at to find out what will be the contribution to the low frequency response corresponding to this coupling capacitor. The total resistance now is R_0 plus R_L at the output circuit and the cut off frequency due to C_2 two is f_1 prime.

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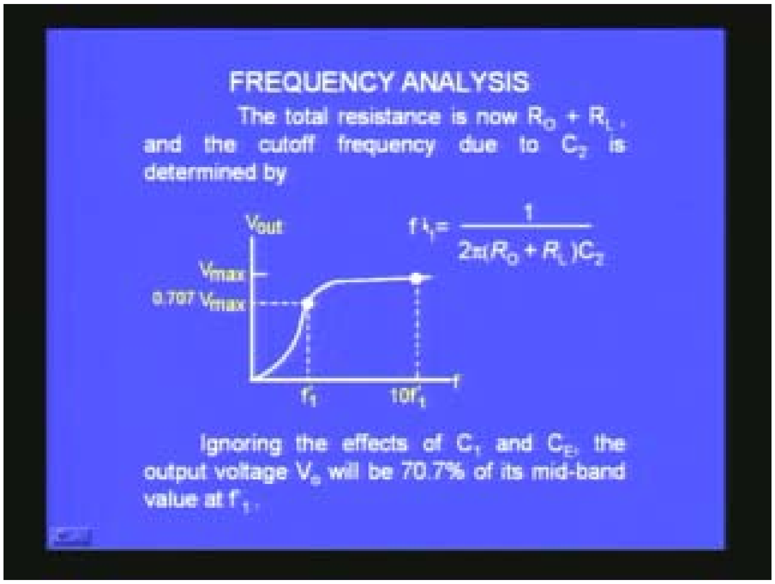

FREQUENCY ANALYSIS

The total resistance is now $R_o + R_L$, and the cutoff frequency due to C_2 is determined by

$$f'_1 = \frac{1}{2\pi(R_o + R_L)C_2}$$

Ignoring the effects of C_1 and C_E, the output voltage V_o will be 70.7% of its mid-band value at f'_1.

Why I have said f_1 prime is to distinguish from f_1 which is the contribution due to the capacitor C_1 and f_1 prime is a contribution due to the capacitor C_2. f_1 prime is 1 by 2 pi the total resistance in the circuit which is R_0 plus R_L multiplied by C_2 the coupling capacitor and the graph will almost look identical to what you saw in terms of the response. But the value of f_1 and f_1 prime can be very different depending upon the magnitude of the capacitance as well as the output impedance R_0, the input impedance R_i, etc. When you discuss this C_2 we would like to ignore the contribution of C_1 and C_E and take into account the contribution due to C_2 alone. Like that we discuss the contribution to each one of capacitances that we have in the circuit and then look at the overall response of the amplifier.

In the ac equivalent circuit for this circuit when the input voltage is zero, V_i is equal to zero. is something like that We have R_o, you have the R_c that will also come as a shunt parallel to R_0 and you have the R_L and C_2. The value of R_o now is decided by R_c and small r_o. Small r_o is the output resistance of the transistor.

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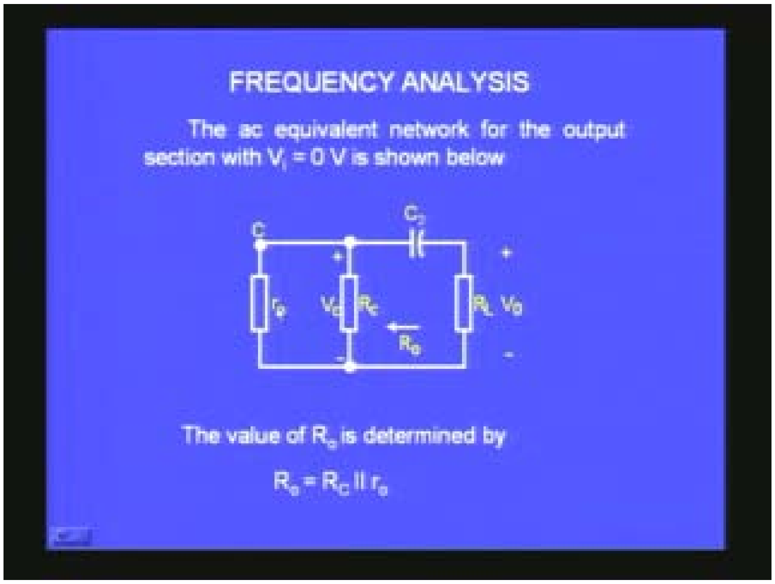

FREQUENCY ANALYSIS

The ac equivalent network for the output section with $V_i = 0$ V is shown below

The value of R_o is determined by

$R_o = R_C \parallel r_o$

R_o, capital R_o, which is the effective Thevenin's resistance, is equal to R_C parallel r_o which we can see from the circuit.

Let us move on to look at the contribution to the low frequency response due to presence of the emitter bias or the bypass capacitor in the emitter bias circuit. There is a bypass capacitor C_E.

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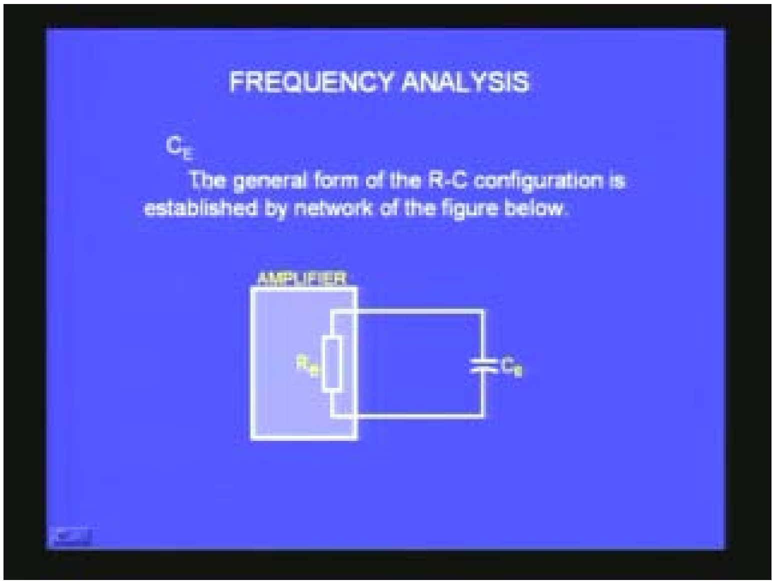

FREQUENCY ANALYSIS

C_E

The general form of the R-C configuration is established by network of the figure below.

C_E, the general form of the R-C configuration for this is very similar. We can see that C_E is there and the amplifier contribution is put in terms of the R_E which is the effective emitter resistance as seen as the Thevenin's resistance at the emitter point of the transistor, which I call capital R_E and because it is again having the same type of a resistor and capacitor the cut off due to C_E can also be determined in a similar manner and the response will look very similar.

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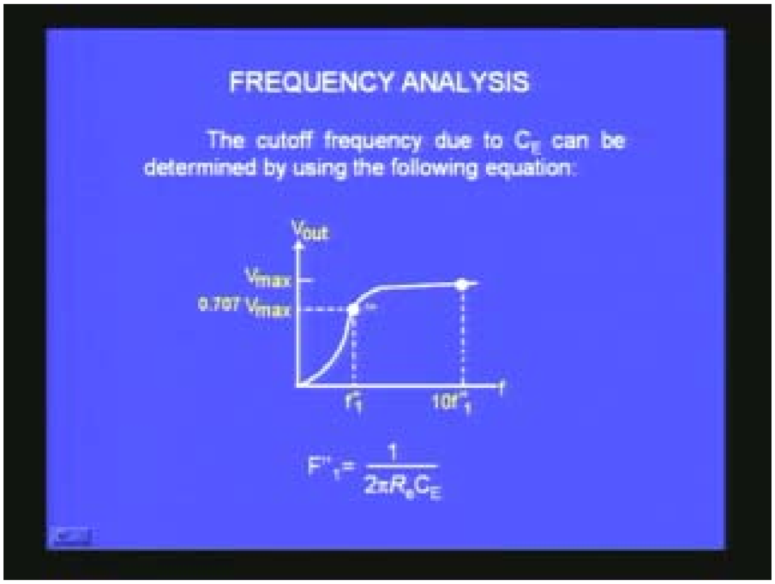

FREQUENCY ANALYSIS

The cutoff frequency due to C_E can be determined by using the following equation:

$$F''_1 = \frac{1}{2\pi R_e C_E}$$

At low frequencies the gain will fall and the F_1 double prime is the cut off frequency corresponding to the contribution from C_E which I call here F_1 double prime is equal to 1 by 2 pi R_e C_E where R_e is the effective emitter resistance that is to be considered. C_E is the emitter bypass capacitance. If you look at the equivalent circuit of this section including the transistor the r_e will be at the emitter point, the contributions due to all the resistors on the base side will be given by R_s prime by beta plus r_e. This is the basic emitter resistance. This R_s prime basically is the contribution from the R_1, R_2 and the R_s. So it is R_s parallel R_1 parallel R_2 is what I call R_s prime and R_s prime divided by beta because you are now translating that emitter base resistance contribution to the base resistance on to the emitter side. So you have to divide by the beta of the h_{fe}. R_s prime divided by beta is the contribution of those resistances at the emitter side plus the r_e the intrinsic emitter resistance plus the capital R_E which is the resistance we have introduced for biasing purposes and the C_E the bypass capacitor. So this will be the equivalent circuit of this part and we can evaluate the total R_e as this R_e and this entire thing comes in parallel. Therefore R_E parallel R_s prime divided by beta plus r_e.

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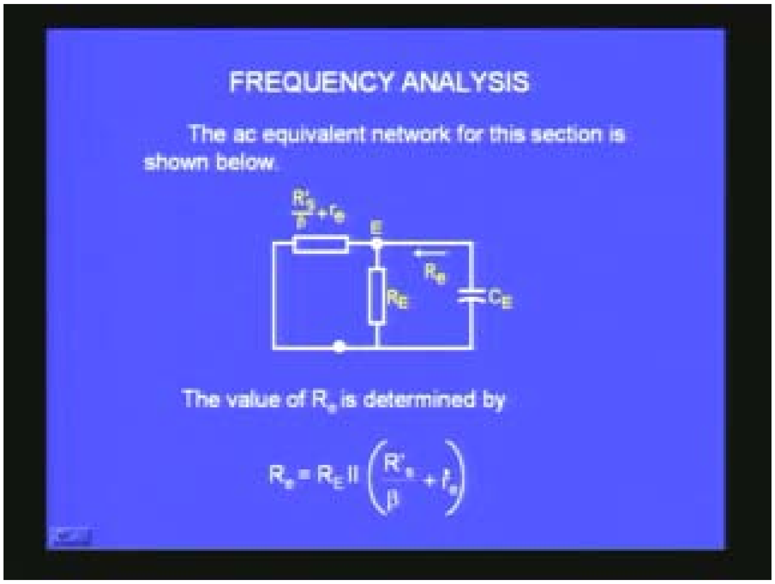
FREQUENCY ANALYSIS

The ac equivalent network for this section is shown below.

The value of R_e is determined by

$$R_e = R_E \parallel \left(\frac{R'_s}{\beta} + r_e\right)$$

This will be the contribution to r_e. Once you evaluate this thing r_e you can calculate the contribution to the frequency f_1 double prime as 1 by 2 pi R_e C_E which we have already seen. So it is possible to get the cut off frequency due to each of these contributions, the contribution due to C_1, the contribution due to C_2 and the contribution due to C_E that is what we have …. and I have also told you how it can be evaluated for a given common emitter amplifier. Now it would be good to take a very simple example and then try to work out the various frequency responses.

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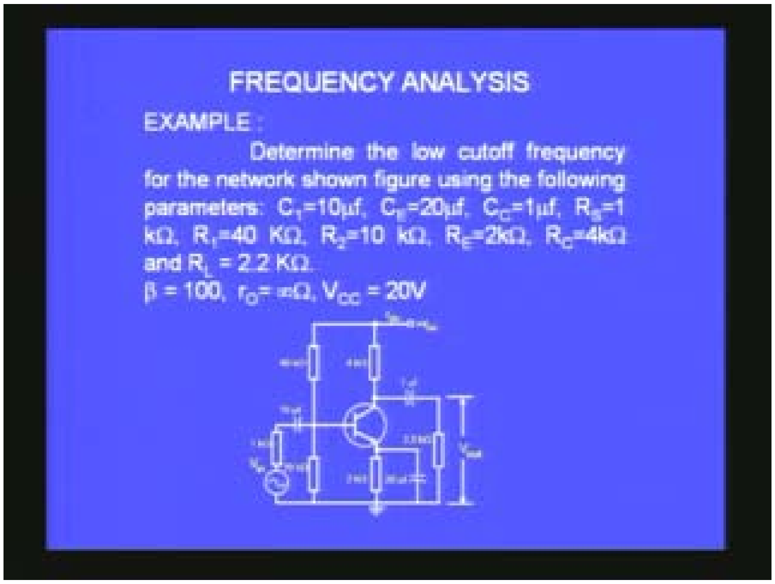
FREQUENCY ANALYSIS

EXAMPLE :

Determine the low cutoff frequency for the network shown figure using the following parameters: C_1=10µf, C_E=20µf, C_C=1µf, R_s=1 kΩ, R_1=40 KΩ, R_2=10 kΩ, R_E=2kΩ, R_C=4kΩ and R_L = 2.2 KΩ.

β = 100, r_O= ∞Ω, V_{CC} = 20V

I have a problem here. Determine the low cut off frequency for the network shown in figure below using the following parameters. There are values given C_1 is given as 10 microfarad, the emitter bypass capacitor C_E is 20 microfarad, the coupling capacitor at the output is 1 microfarad, the R_s source resistance is 1 kilo ohm and R_1 is 40 kilo ohm and R_2 is 10 kilo ohm, R_E is 2 kilo ohm, the emitter bias and R_C is 4 kilo ohm and R_L is 2.2 kilo ohm. It is also mentioned that the beta value for the transistor is 100 and the internal resistance r_o of the transistor is assumed to be infinity for simplicity. Most of the time it is true; very close to that and V_{CC} is 20 volts. All the information is given. Now we have to find out what is the low frequency cut off of this circuit? What is the procedure? The procedure as I explained to you is very simple. Take the contribution of each of the capacitors that we discussed. The 10 microfarad C_1, the 1 microfarad at the output C_2 and the contribution due to the 20 microfarad C_E and find out which one is the worst in terms of the frequency response and choose that as the dominant contribution to the low frequency. Before we do that we have to get certain parameters. First let us check R_E. How does it compare with beta times capital R_E? Beta is 100, capital R_E is 2K that comes about 200 K and that if it is very, very large compared to 10 times the R_2 value then you can ignore the contribution from beta R_E.

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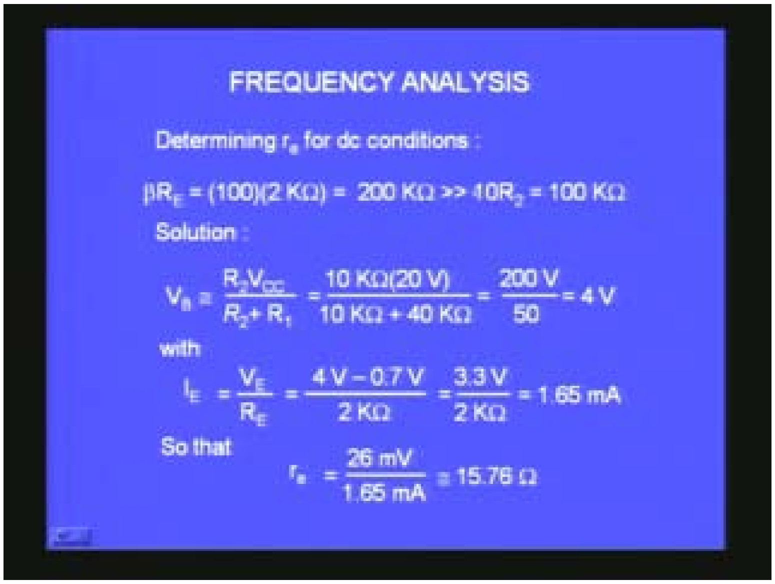

FREQUENCY ANALYSIS

Determining r_e for dc conditions :

$\beta R_E = (100)(2\ K\Omega) = 200\ K\Omega >> 10R_2 = 100\ K\Omega$

Solution :

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10\ K\Omega(20\ V)}{10\ K\Omega + 40\ K\Omega} = \frac{200\ V}{50} = 4\ V$$

with

$$I_E = \frac{V_E}{R_E} = \frac{4\ V - 0.7\ V}{2\ K\Omega} = \frac{3.3\ V}{2\ K\Omega} = 1.65\ mA$$

So that

$$r_e = \frac{26\ mV}{1.65\ mA} \cong 15.76\ \Omega$$

So that is coming to be 100 K. Let us calculate what is V_B? The voltage at the base is R_2 divided by R_1 plus R_2 times V_{CC}. We all know it is potential divider. It's a voltage divider bias and if you substitute the value of the resistors 10 K and 40 K multiplied by 20 volts the value comes to be around 4 volts. So at the base of the transistor the voltage is 4 volts. What is I_E, the emitter current that is given by V_E the voltage at the emitter divided by R_E, that is the dc current; so 4 volts. V_E we can calculate by knowing the V_{BE} drop across the base emitter junction. So 4 volts – 0.7 volts divided by 2 kilo ohm is the contribution to I_E and that is 3.3 by 2 kilo ohms. That is equal to 1.65 milli amperes. You have calculated I_E and then once you know I_E you can calculate the small r_e the intrinsic emitter resistance of the transistor because we know r_e is equal to 26 milli volts divided by 1.65 milli amperes. This is standard formula we have used number of times and the value of small r_e comes out to be around to be 15 ohms when you do this for this circuit and once you know the small r_e you can calculate what is beta r_e which is 100 times that value and that is around 1.5 K.

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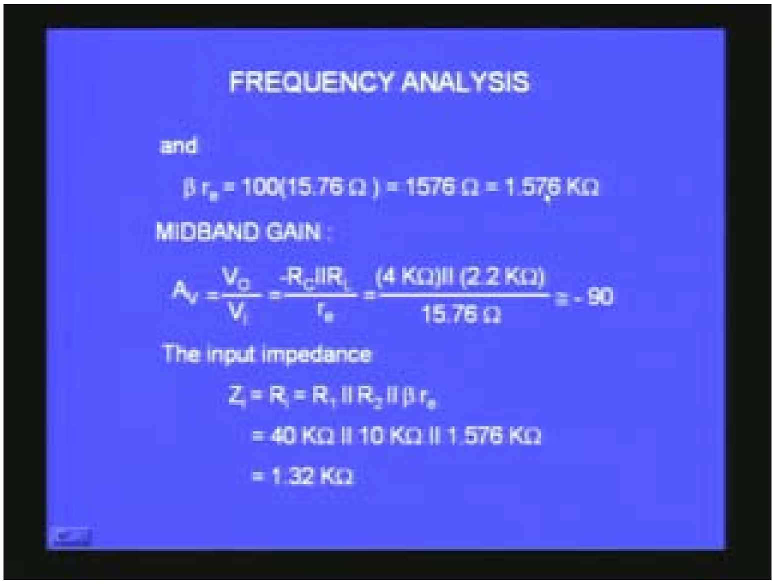

FREQUENCY ANALYSIS

and

$$\beta r_e = 100(15.76\ \Omega) = 1576\ \Omega = 1.576\ K\Omega$$

MIDBAND GAIN :

$$A_V = \frac{V_O}{V_i} = \frac{-R_C \| R_L}{r_e} = \frac{(4\ K\Omega) \| (2.2\ K\Omega)}{15.76\ \Omega} \cong -90$$

The input impedance

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$
$$= 40\ K\Omega \| 10\ K\Omega \| 1.576\ K\Omega$$
$$= 1.32\ K\Omega$$

You know beta r_e and you can also get mid frequency gain. The mid band gain is minus R_C parallel R_L divided by small r_e and R_C is 4 kilo ohm given in the problem. R_L is 2.2 kilo ohm. Therefore the parallel value of 4 kilo ohm and 2.2 kilo ohm you have to evaluate and divide that by the small value 15 ohms which is the r_e value and the answer is nearly -90 degrees. The minus indicates that there is a phase inversion between the input and the output. That is the output voltage is 180 degrees out of phase with the input. We have also discussed that earlier. What is the contribution to the input resistance Z_i or R_i is equal to R_1 parallel R_2 parallel beta r_e. We have already discussed that and if you substitute the values R_1 is 40 kilo ohm, R_2 is 10 kilo ohm, beta r_e just now we have calculated is 1.576 kilo ohms and therefore the effective resistance should be smaller than the smallest and you do them in parallel and therefore you get 1.32 kilo ohm. That is the R_i. Once I know the R_i, I know what is the actual input which is applied across the input terminal of the amplifier which is nothing but R_i divided by R_i plus R_s multiplied by V_s. We can calculate the V_i by V_s as R_i by R_i plus R_s the same thing modified 1.32 ohms divided by 1.32 kilo ohm plus 1 kilo ohm. That is 0.569 is the actual ratio of V_i and V_s.

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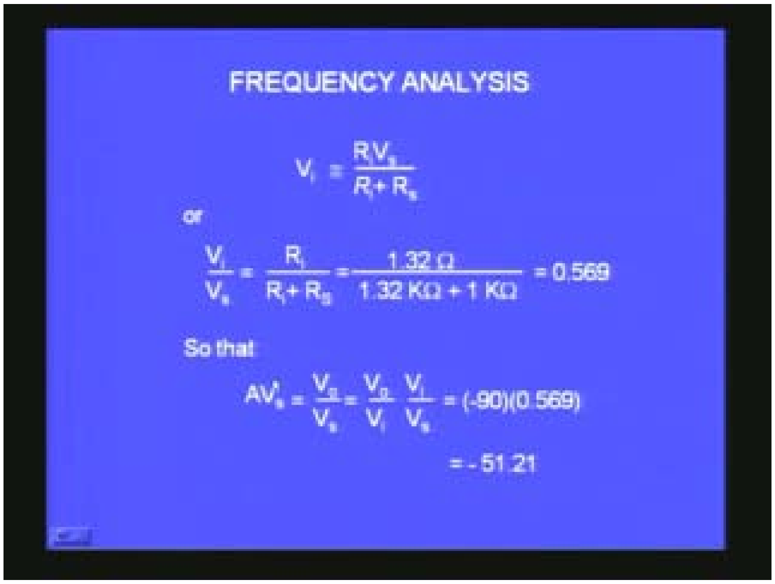

FREQUENCY ANALYSIS

$$V_i = \frac{R_i V_s}{R_i + R_s}$$

or

$$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\ \Omega}{1.32\ K\Omega + 1\ K\Omega} = 0.569$$

So that

$$AV_s' = \frac{V_o}{V_s} = \frac{V_o}{V_i}\frac{V_i}{V_s} = (-90)(0.569)$$

$$= -51.21$$

We can get AV_s the output voltage V_{out} by V_s. This is actually capital AV_s, the gain corresponding to the signal source V_s and that will be -92 into 0.69 so -51 approximately. That is the overall gain of the common emitter amplifier. Having found the mid frequency gain we can now find out what is the contribution to the frequency response at low frequency and high frequency corresponding to each of the component. I will take first R_1. R_i is equal to R_1 parallel R_2 beta r_e that is found to be 1.32. That we have already done. What is f_1? f_1 one is 1 by 2 pi R_s plus R_i into C_1.

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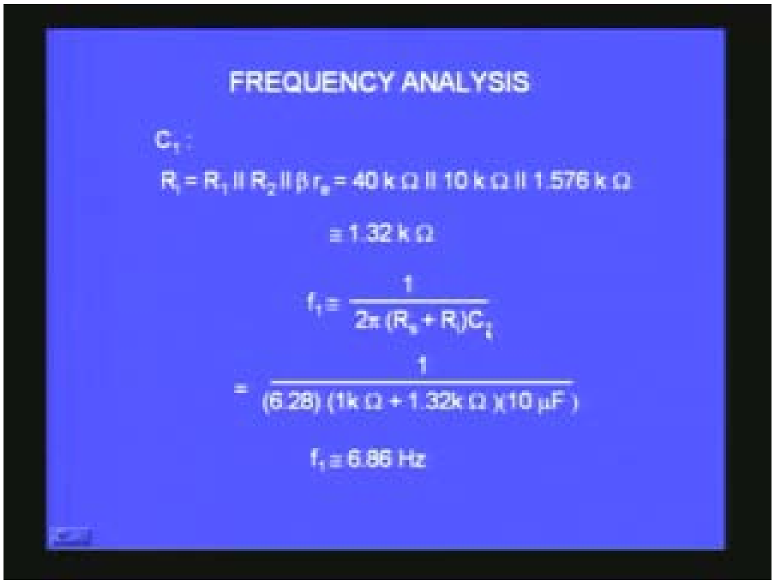

FREQUENCY ANALYSIS

C_1 :

$$R_i = R_1 \parallel R_2 \parallel \beta r_e = 40\ k\Omega \parallel 10\ k\Omega \parallel 1.576\ k\Omega$$

$$\cong 1.32\ k\Omega$$

$$f_1 \cong \frac{1}{2\pi (R_s + R_i) C_1}$$

$$= \frac{1}{(6.28)(1k\Omega + 1.32k\Omega)(10\ \mu F)}$$

$$f_1 \cong 6.86\ Hz$$

You substitute for R_s and R_i and then C_1 is 10 microfarad. You evaluate this and the frequency f_1 comes out to be 6.86 hertz that is very low frequency. So at 6.86, 70% is the mid frequency gain or voltage gain. That is what we understand from this expression f_1 is equal to 6.86. Let us calculate the contribution due to C_2.

C_2 is also f_1 prime. That is what we call C_2's contribution.

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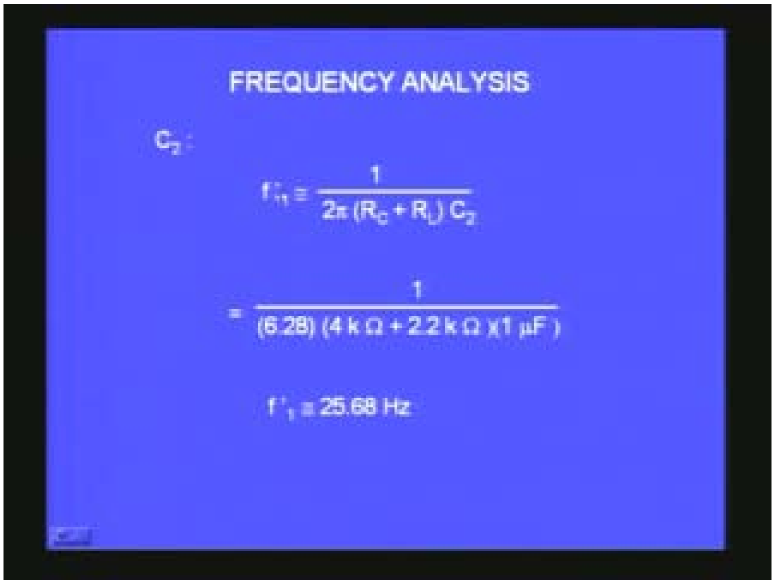

FREQUENCY ANALYSIS

C_2 :

$$f'_1 = \frac{1}{2\pi (R_C + R_L) C_2}$$

$$= \frac{1}{(6.28)(4\,k\Omega + 2.2\,k\Omega \times 1\,\mu F)}$$

$$f'_1 = 25.68\ Hz$$

1 by 2 pi R_c plus R_L times C_2. R_c is 4 kilo ohm, R_L is 2.2 kilo ohm and C_2 is 1 microfarad and if you evaluate this expression f_1 prime is found to be 25 hertz. C_1 is around 6.86 hertz and f_1 prime due to C_2 is about 25.6 hertz. This is slightly higher compared to that. Let us come to C_E. Before you calculate C_E you should find out what is R_s prime which is the combination of all those things R_s parallel R_1 parallel R_2 and that value is found to be 0.889 very low value, about 889 ohms. What is R_e, the effective emitter resistance and that will be capital R_E the one which you have connected parallel R_s prime by beta plus r_e. We have seen that earlier and r_e is 2 K given in the problem and R_s prime just now we calculated to be 0.889. Therefore 0.889 by 100 plus this small r_e is 15.76 ohms and so the effective resistance is 24.35 after simplification.

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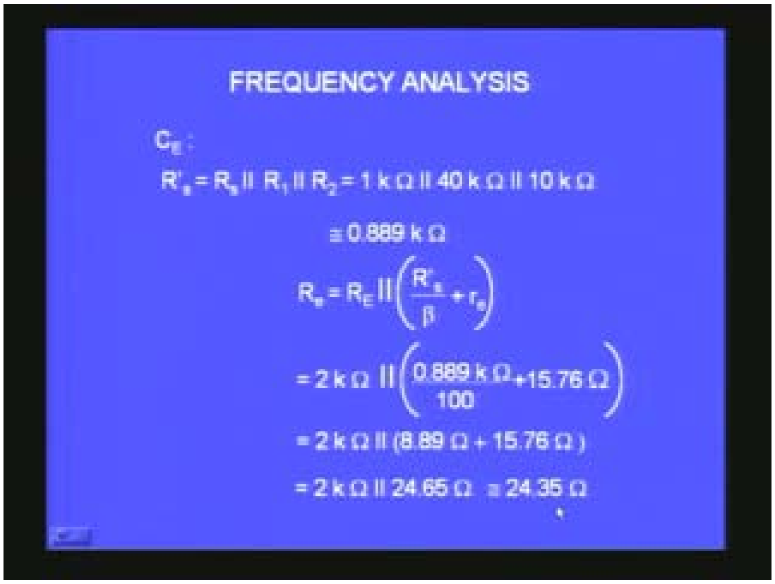

FREQUENCY ANALYSIS

C_E :

$$R'_s = R_s \,||\, R_1 \,||\, R_2 = 1\text{ k}\Omega \,||\, 40\text{ k}\Omega \,||\, 10\text{ k}\Omega$$

$$\cong 0.889\text{ k}\Omega$$

$$R_e = R_E \,||\left(\frac{R'_s}{\beta} + r_e\right)$$

$$= 2\text{ k}\Omega \,||\left(\frac{0.889\text{ k}\Omega}{100} + 15.76\ \Omega\right)$$

$$= 2\text{ k}\Omega \,||\, (8.89\ \Omega + 15.76\ \Omega)$$

$$= 2\text{ k}\Omega \,||\, 24.65\ \Omega \cong 24.35\ \Omega$$

This is my r_e. Once I know the effective emitter resistance we can calculate the f_1 double prime. f_1 double prime is actually 1 by 2 pi R_e C_E where R_e is the effective emitter resistance and 1 by 6.28 which is 2 pi multiplied by R_e which is 24.35, just now you evaluated multiplied by 20 microfarad and if you evaluate this expression you get 327 hertz.

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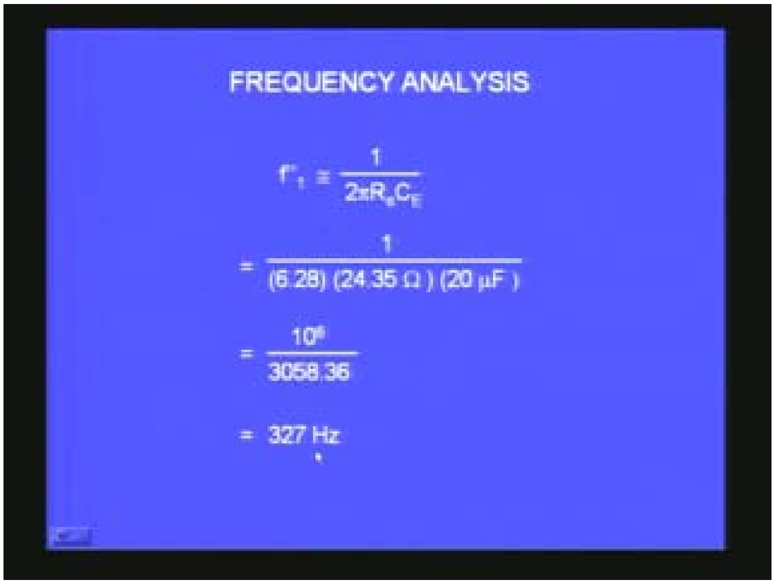

FREQUENCY ANALYSIS

$$f''_1 \cong \frac{1}{2\pi R_e C_E}$$

$$= \frac{1}{(6.28)\,(24.35\ \Omega)\,(20\ \mu\text{F})}$$

$$= \frac{10^6}{3058.36}$$

$$= 327\text{ Hz}$$

So what do you have? You now have three results. One is 6.86 the other one is about 25.8 and the third one is 327. Which one will you choose as your low frequency cut off? The best method is to choose the highest of it. The highest of the three is 327 hertz. That means the C_E capacitor is the one almost dominant among the rest of the capacitances in deciding which is the low frequency cut off, cutoff frequency. So the low frequency, the cut off frequency in this problem is 327 hertz which is basically due to the C_E, the emitter capacitor. So I hope this example will help you to understand how the various capacitances contribute to the cut off frequency at the low frequencies and how to choose the one which decides the effective low frequency response of the amplifier.

Now we will have to look at the high frequency response. Before we go into the high frequency response it is useful to learn about another theorem which is called the Miller's theorem.

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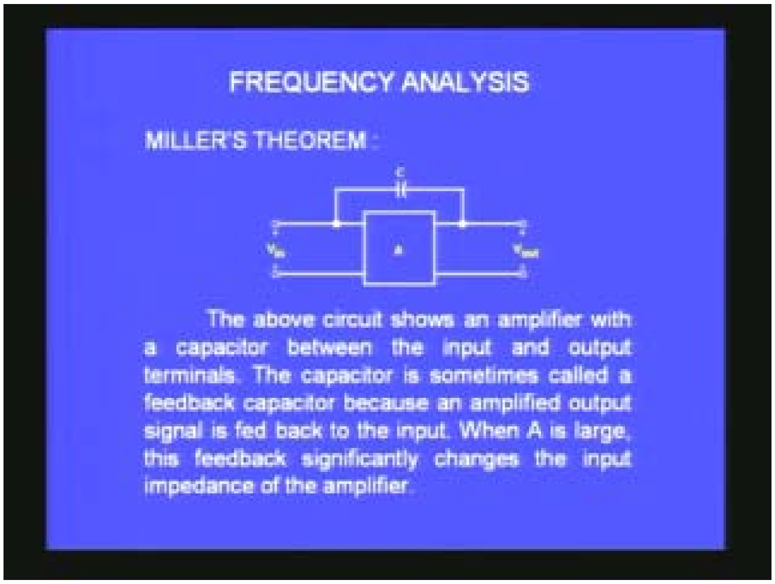

FREQUENCY ANALYSIS

MILLER'S THEOREM :

The above circuit shows an amplifier with a capacitor between the input and output terminals. The capacitor is sometimes called a feedback capacitor because an amplified output signal is fed back to the input. When A is large, this feedback significantly changes the input impedance of the amplifier.

The high frequency contribution comes from the inter electrode capacitances. That is the capacitance between the emitter and the base, the collector and the base and the base and the collector. Because of that we have to have an understanding how we can simplify some of the configurations and Miller's theorem is very useful in this context. I have taken a general amplifier 'A' that is the box at the center and you have a capacitor which is linked to the input and output. So it becomes a feedback capacitor because output will be fed back to the input through the capacitor at certain frequencies when the reactance becomes small and therefore this will have some effect on the overall performance of the amplifier or the frequency response of the amplifier. But how do we understand this contribution? How do we simplify the circuit because it seems to be having contributions both at the input and at the output? For this we try to understand the Miller's theorem and apply the Miller's theorem.

What is the Miller's equivalent circuit for this? Miller said that one single capacitor which is connected between the input and the output can be split into two capacitors one exclusively connected at the input side the other connected at the output side. That is what is shown in the picture that you see here.

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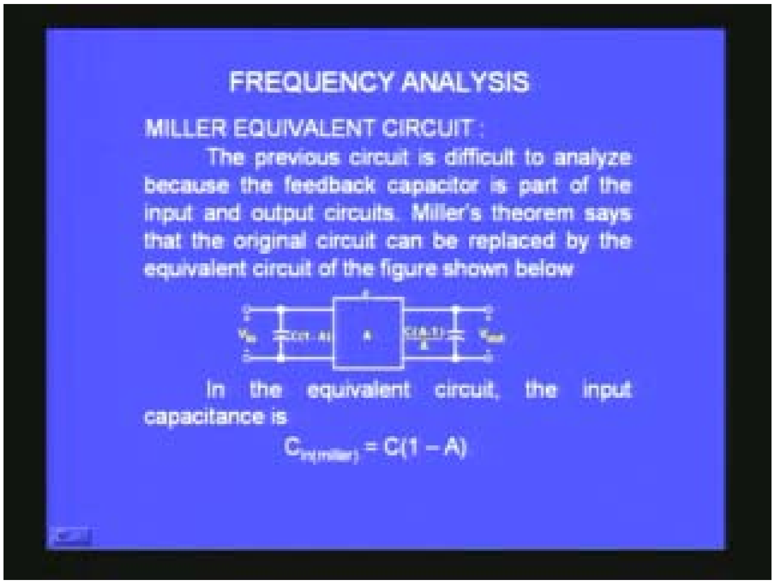

FREQUENCY ANALYSIS

MILLER EQUIVALENT CIRCUIT :

The previous circuit is difficult to analyze because the feedback capacitor is part of the input and output circuits. Miller's theorem says that the original circuit can be replaced by the equivalent circuit of the figure shown below

In the equivalent circuit, the input capacitance is

$$C_{in(miller)} = C(1-A)$$

You have got one capacitor or capacitance? coming between the input terminals and another capacitor comes at the output terminals and these are the two capacitances that you get when you apply Miller's theorem and split one feedback capacitance that is showing from the input and the output. What he says is the input capacitance the effect of the C at the input will be C into 1- A where A is the gain of the amplifier. Usually the gain of the amplifier will be very, very large. So C into 1- A is almost equal to A and that means what? A times C. So the capacitance of the input side will be a very large contribution. That is the capacitance between the input output which is the bridging capacitance multiplied by the gain of the amplifier when the gain is very large. The input capacitance will be very large in magnitude because it is multiplied by the gain whereas if you look at the output capacitance you find C into A minus one by A. That is what the Miller's theorem says and when A is very large compared to one, A minus one by A is almost equal to one and so the output capacitance will almost be the same as the Miller's capacitance which is bridging the two inputs input and the output. The C input due to the Millers theorem is C into 1-A and the C output due to the Miller's theorem is C into A minus one by A and this when A is very large compared to one is almost equal to one and C Miller is equal to C the capacitance between the input output.

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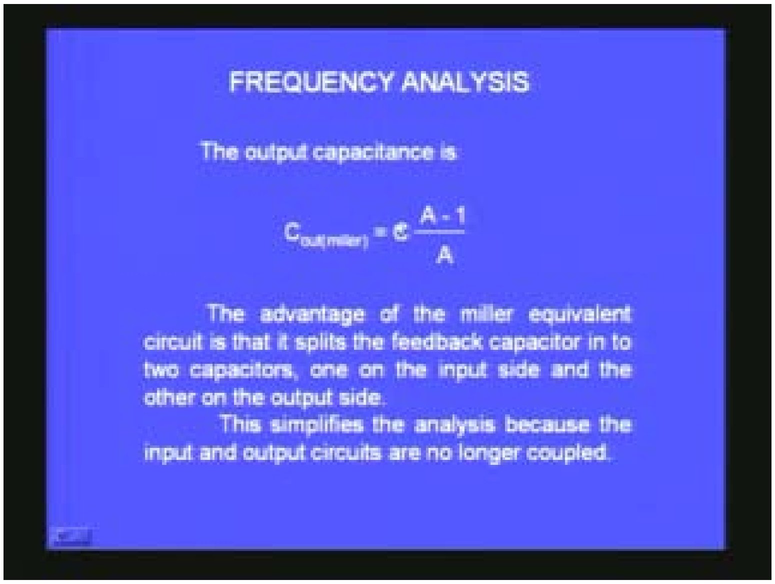

FREQUENCY ANALYSIS

The output capacitance is

$$C_{out(miller)} = C\frac{A-1}{A}$$

The advantage of the miller equivalent circuit is that it splits the feedback capacitor in to two capacitors, one on the input side and the other on the output side.

This simplifies the analysis because the input and output circuits are no longer coupled.

This theorem is very useful in understanding the high frequency response of the amplifier. If you look at the circuit that we have here you can see I have put some capacitances between the base and the collector. I call it C_{bc} the capacitance due to the base collector and similarly between the collector and the emitter C_{ce} and the base and the emitter C_{be}.

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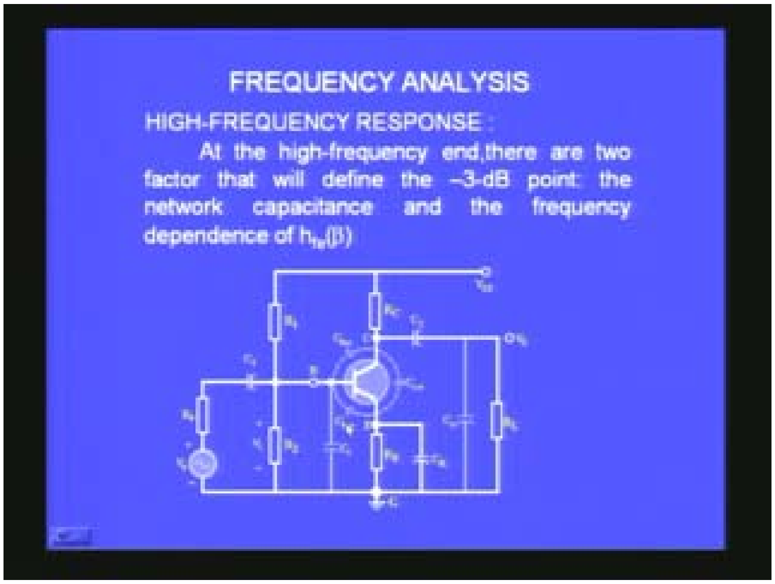

FREQUENCY ANALYSIS

HIGH-FREQUENCY RESPONSE :

At the high-frequency end,there are two factor that will define the –3-dB point: the network capacitance and the frequency dependence of $h_{fe}(\beta)$

These are new capacitances that we have introduced in the circuit. What are they? They are actually due to the device itself. The transistor itself can be looked at as a capacitor. I already mentioned to you a simple p-n junction has got capacitance. So you have a p type and n type which are conducting regions separated by a very small depletion region which is like a dielectric or an insulator and therefore this can be effectively a capacitor. You do have similarly at every junction a capacitance, the junction capacitance. In this case between the base collector junction you have a capacitance C_{bc} and you have a capacitance C_E, C_{ce} and C_{be} between the base and the emitter out of which, which one is the Miller capacitance? To which I should apply Miller's theorem? If you look at you can see the C_{bc} this collector point is the output point, base point is the input point.

The capacitor which is connected between the base and the collector which is given by C_{bc} is the one which is going to be the most crucial in deciding the frequency response at high frequency. You must also imagine that the circuit is built with number of wires and these wires can be very close to each other and due to this wiring also there could be a capacitance that is introduced into the circuit. That is called wiring capacitance. So all the capacitances due to the wiring, due to the positioning of the various devices etc., I have put all together and call them as the C_i the input capacitance due to the wiring. Similarly at the output also we can have a contribution to capacitance due to various wires. That I call C_o. What is C_i and C_o? C_i and C_o are the wiring and other stray capacitances at the input and the output side. C_{bc} is the capacitance which is actually between the output and the input. Therefore we will apply the Miller's theorem to the C_{bc} and then try to evaluate the high frequency response. Now if I drop the equivalent circuit of the network or the amplifier and taking into account the presence of the C_i, the C_o etc., you find what you get on the screen.

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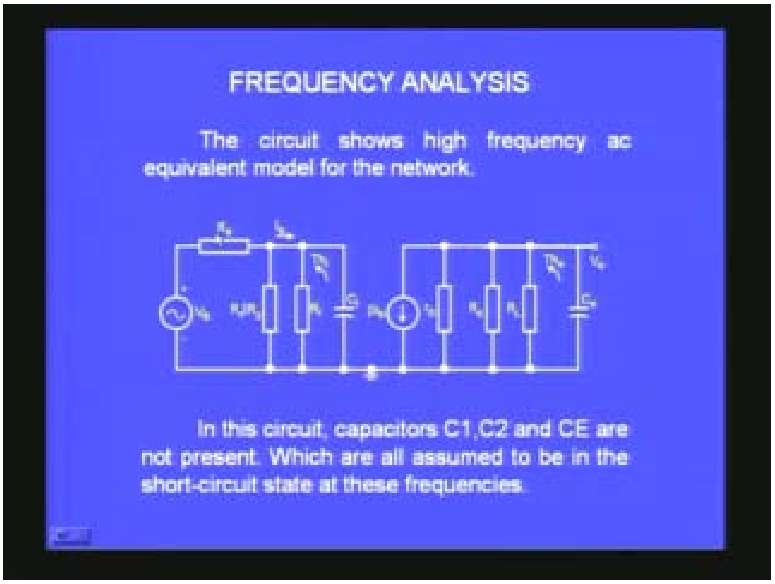

FREQUENCY ANALYSIS

The circuit shows high frequency ac equivalent model for the network.

In this circuit, capacitors C1,C2 and CE are not present. Which are all assumed to be in the short-circuit state at these frequencies.

You have C_s, R_s, V_s and R_s. You have the $R_1 + R_2$ parallel resistor coming here. You have the R_i, the input resistance the C_i the stray capacitances all of them put together. Then you have the R_o at the output R_c, R_L and C_o which is the stray capacitances of the output. Now you can see the capacitances C_1, C_2 and C_E are not shown in this picture and that is because at very high frequencies the contribution due to C_1, C_2 and C_E will be very, very small and therefore they are almost ignored or taken as short at these frequencies. What is the Thevenin's equivalent circuit in order to evaluate the frequency response of these contributions? The Thevenin's response you can see on the input and the output side, I have shown separately on the screen, it is nothing but R_s parallel R_1 parallel R_2 parallel R_i the input resistance of the transistor and you have the C_i which is the contribution due to the stray capacitances, this form the ones in full circuit.

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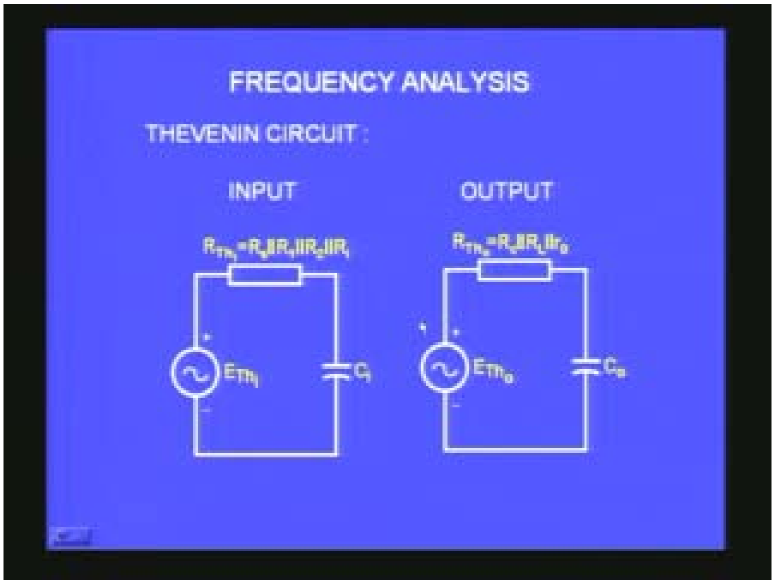

FREQUENCY ANALYSIS

THEVENIN CIRCUIT :

INPUT

OUTPUT

Similarly at the output you have the R_c parallel to R_L and also parallel to small r_o which is the transistor output resistance and C_o is the stray capacitances in the circuit. Once we know that you can immediately calculate what will be the f_1 or in this case it is f_2.

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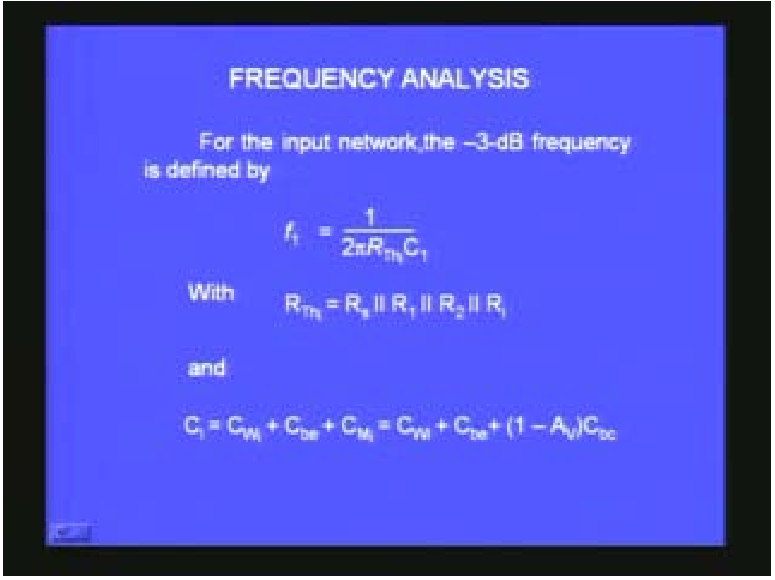

FREQUENCY ANALYSIS

For the input network,the –3-dB frequency is defined by

$$f_1 = \frac{1}{2\pi R_{Th_i} C_1}$$

With $R_{Th_i} = R_s \| R_1 \| R_2 \| R_i$

and

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{Wi} + C_{be} + (1 - A_v)C_{bc}$$

This should be C_i here. C_i is the wiring capacitance, base emitter capacitance, the Miller capacitance and that is nothing but C_{Wi} plus C_{be} plus the Miller capacitance is actually C_{bc} and when you connect it according to Miller's theorem it should be one minus A times so that is why it is written as one minus Av times C_{bc}, the base collector capacitance. For the output network the f_2 prime is 1 by 2 pi R Thevenin's at the output into C_o and R Thevenin at the output is R_C parallel R_L parallel small r_o and the C_o is nothing but the wiring capacitance, the capacitance between the collector and the emitter and also the Miller contribution to the capacitances.

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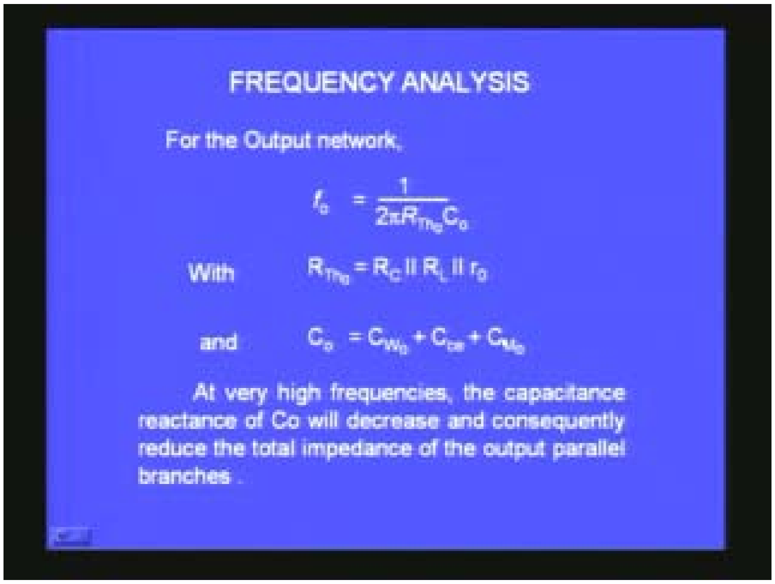

FREQUENCY ANALYSIS

For the Output network,

$$f_o = \frac{1}{2\pi R_{Th_o} C_o}$$

With $R_{Th_o} = R_C \parallel R_L \parallel r_o$

and $C_o = C_{W_o} + C_{ce} + C_{M_o}$

At very high frequencies, the capacitance reactance of Co will decrease and consequently reduce the total impedance of the output parallel branches .

As we increase the frequency the contribution due to these things will decrease and therefore the gain also will fall. I have taken another example here for looking at the high frequency response. Determine the high frequency cut off of the network shown in the figure which is basically the same amplifier using the following parameters. C_1 is 10 microfarad, C_2 20 microfarad, C_c 1 microfarad all the thing are same as this but in addition we have given the other capacitances like for example C_{be} is 36 pF; C_{bc} is 4 pF; C_{ce} the capacitance with collector emitter junction of the transistor is 1 pico farad and C_{wi} is the contribution due to wiring at the input side and C_{wo} is the contribution to the capacitance at the output side that is around 8 pF.

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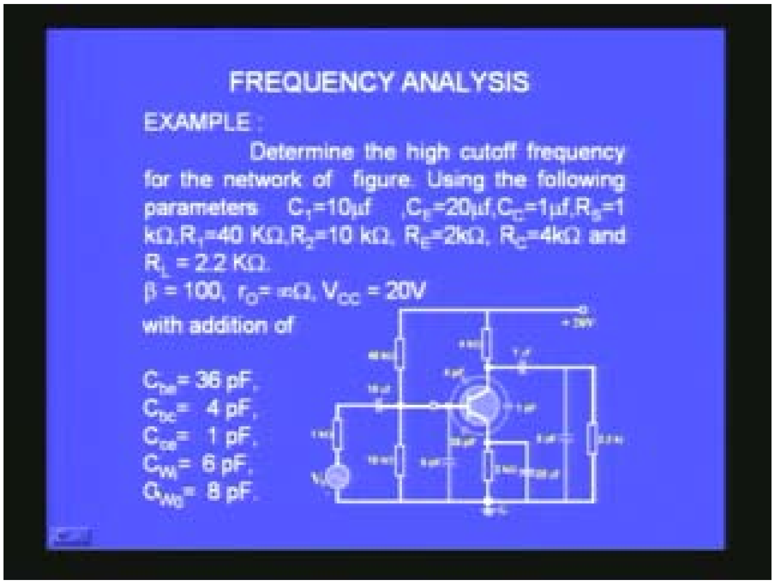

FREQUENCY ANALYSIS

EXAMPLE :

Determine the high cutoff frequency for the network of figure. Using the following parameters $C_1=10\mu f$,$C_E=20\mu f$,$C_C=1\mu f$,$R_S=1$ kΩ,$R_1=40$ KΩ,$R_2=10$ kΩ, $R_E=2$kΩ, $R_C=4$kΩ and $R_L = 2.2$ KΩ.

$\beta = 100$, $r_O= \infty\Omega$, $V_{CC} = 20V$

with addition of

$C_{be}= 36$ pF,
$C_{bc}= 4$ pF,
$C_{ce}= 1$ pF,
$C_{Wi}= 6$ pF,
$C_{Wo}= 8$ pF.

What is pF? pF means pico farad. Pico farad means 10 power -12 farads. Micro means 10 power -6; pico is 10 power -12 farads. So it's a very, very low value and the low value becomes significant only at very high frequencies otherwise it is very low. What is the solution? The solution we first calculate R_i. We have done that earlier.

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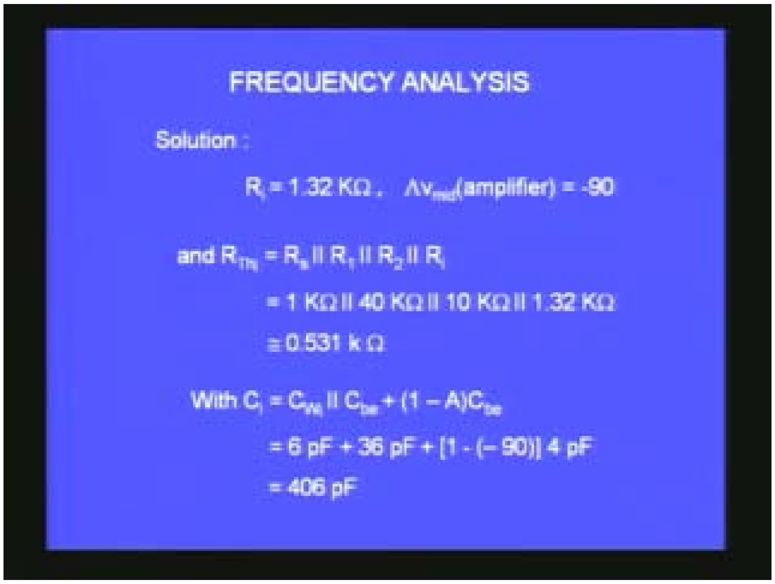

FREQUENCY ANALYSIS

Solution :

$R_i = 1.32$ KΩ , Av_{mid}(amplifier) = -90

and $R_{Th} = R_s \parallel R_1 \parallel R_2 \parallel R_i$

$= 1$ KΩ ∥ 40 KΩ ∥ 10 KΩ ∥ 1.32 KΩ

$\cong 0.531$ k Ω

With $C_i = C_{Wi} \parallel C_{be} + (1 - A)C_{be}$

$= 6$ pF + 36 pF + [1 - (– 90)] 4 pF

$= 406$ pF

R_i is 1.32 kilo ohm which is actually due to the parallel combination of R_1 parallel R_2 and parallel R_i and we will also take into account the contribution due to the source resistance R_s. What is the Thevenin's resistance at the input? R_s parallel R_1 parallel R_2 parallel R_i and if you substitute all the values 40 kilo ohm, 10 kilo ohm, etc., you get the Thevenin's equivalent resistance to be 0.531 kilo ohm and the 0.531 kilo ohm is nothing but 531 ohms. So with C_i is equal to C_{wi} plus C_{be} which is the base emitter capacitance plus the Miller capacitance at the input side which is 1-A times the C_{be}, the base emitter capacitance. If I substitute the values C_{wi} is 6 pF given in the problem. C_{be} is 36 pF given in the problem and 1-90 where 90 is the gain of the amplifier multiplied by 4 pF. The capacitance between the base and the collector that is between the input and the output is now enhanced or multiplied by the beta value and that is why you get such a huge value 406 pico farad. This is the Miller capacitance at the input side.

What is the f_i? f_i is the high frequency response due to the presence of C_i.

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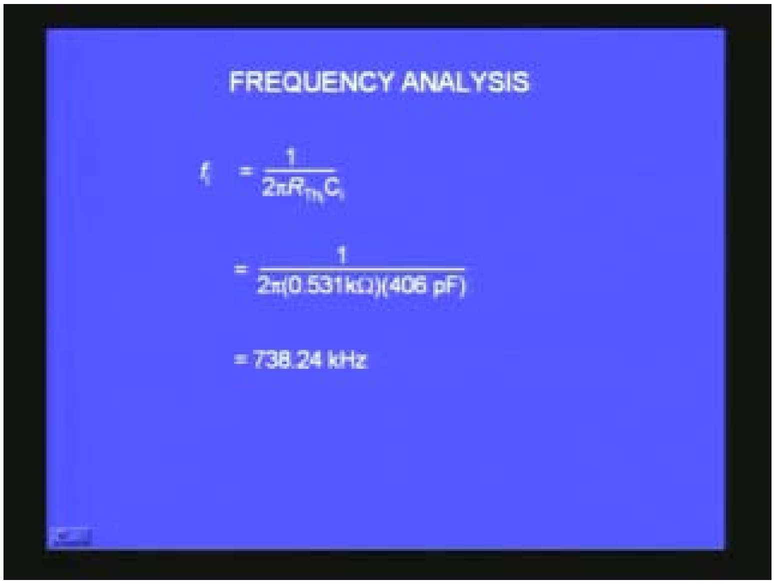

FREQUENCY ANALYSIS

$$f_i = \frac{1}{2\pi R_{Th_i} C_i}$$

$$= \frac{1}{2\pi(0.531\text{k}\Omega)(406\text{ pF})}$$

$$= 738.24\text{ kHz}$$

f_i is equal to 1 by 2 pi R Thevenin i into C_i and if you calculate the values R Thevenin is 0.531 kilo ohm we calculated; C_i is 407 pico farad. If you calculate the frequency it is 738.2 kilo hertz at the high frequency. It is in kilo hertz; very large frequency.

Now if you want to calculate the contribution to the cut off frequency due to the output side then we have to calculate R Thevenin at the output side. That is R_c parallel R_L and that is 1.419 kilo ohm and with C_o the contribution to C_o will be coming from the wiring capacitances in parallel with C_{be} in parallel with Miller capacitance. C_{Wo} in the problem is given as 8 pico farad. C_{ce} is given as 1 pico farad and the Miller capacitance is 1-A. That is 1 minus 1 by 90 degrees, 90 which is the gain into 4 pF. This is the Miller contribution and if you evaluate the total is found to be 13.04 pico farad.

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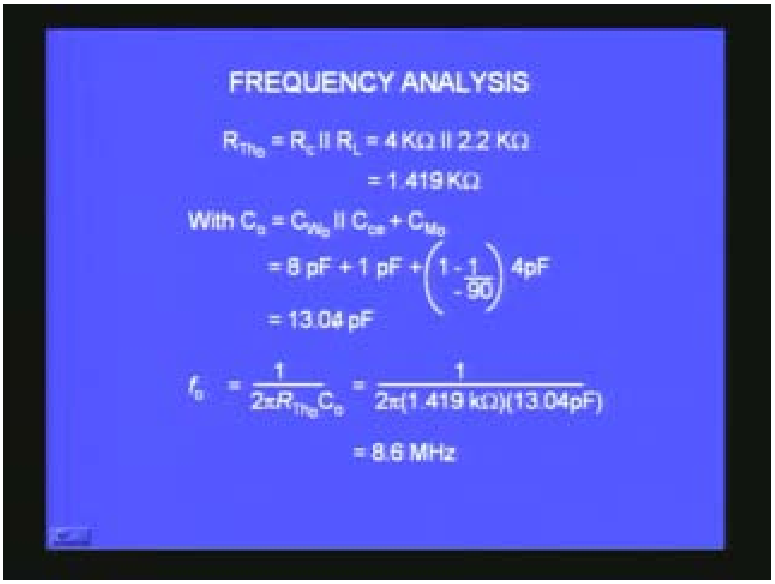

FREQUENCY ANALYSIS

$$R_{Th_o} = R_c \parallel R_L = 4\,K\Omega \parallel 2.2\,K\Omega$$
$$= 1.419\,K\Omega$$

With $C_o = C_{W_o} \parallel C_{ce} + C_{M_o}$

$$= 8\,pF + 1\,pF + \left(1 - \frac{1}{-90}\right) 4pF$$
$$= 13.04\,pF$$

$$f_o = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi(1.419\,k\Omega)(13.04pF)}$$
$$= 8.6\,MHz$$

What will be the cut off frequency? The cut off frequency now due to the contribution from these capacitances is 1 by 2 pi R Thevenin’s at the output stage multiplied by C_o and if you substitute the values you get that to be 8.6 mega hertz. Mega means 10 power 6 hertz. So the high frequency cut off is really very high corresponding to nearly 8 to10 mega hertz. I have put all these values in the form of a table. The cut off frequencies for low frequencies the contribution comes from C_1, C_2 and C_E and the high frequency contribution is 738 kilo hertz corresponding to f_i and f_o is 8.6mega hertz.

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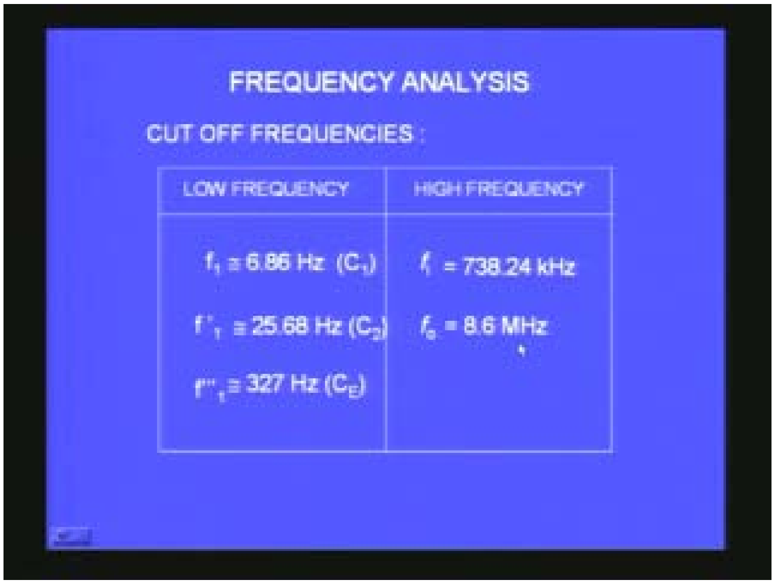

FREQUENCY ANALYSIS

CUT OFF FREQUENCIES :

LOW FREQUENCY	HIGH FREQUENCY
$f_1 \cong 6.86\,Hz$ (C_1)	$f_i = 738.24\,kHz$
$f'_1 \cong 25.68\,Hz$ (C_2)	$f_o = 8.6\,MHz$
$f''_1 \cong 327\,Hz$ (C_E)	

If you look at this table you can tell the worst case condition corresponding to the input and the output. What is the overall f_1 and the overall f_2? If I call the high frequency response as f_2 what is the overall input and the output cutoff frequencies? I must take the largest value in this; 6.86 hertz, 25.68 hertz, 327 hertz. The cut off frequency which is most important, dominant is given by 327 hertz which is the largest frequency among all and that is the worst case. If you look at the high frequency it should be the other way. In a high frequency the contribution due to the input capacitances are 738.24 kilo hertz and the contribution to the output capacitances is 8.6 mega hertz and the worst thing here is 738.2 kilo hertz. Therefore you can take that as the upper cut off frequency. So in this problem we know what is the lower cut off frequency? What is the upper cut off frequency? The lower cut off frequency is 327 hertz and the upper cut off frequency is around 738 kilo hertz. So these two are the ones which will decide what is the band width of the amplifier? Actually the band width of the amplifier is the difference between these two frequencies. That is 738.24 kilo hertz minus 327 hertz gives you the band width. If you are working with the dc coupled amplifier then there is no low frequency coupling capacitor involved and the amplifier response will start almost from 0 frequency that is dc and you don't have to worry about the presence of f_1 and if you don't have f_1 then the band width is given by the upper cut off frequency itself. That is if I don't have the input low frequency contributions if it is a dc coupled amplifier then the bandwidth of this amplifier, we have worked out the problem, is 738 kilo hertz. So that is the bandwidth.

Incidentally when we discuss bandwidth there is a related thing that we have to worry about.

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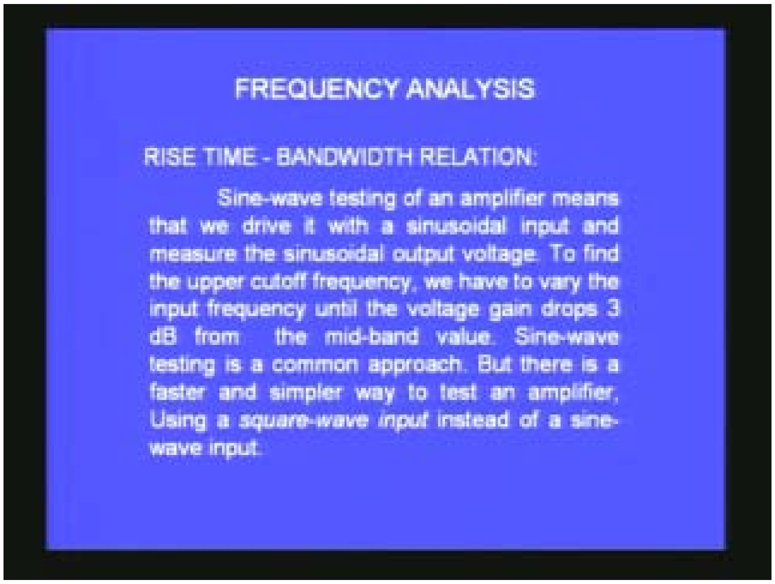

FREQUENCY ANALYSIS

RISE TIME - BANDWIDTH RELATION:

Sine-wave testing of an amplifier means that we drive it with a sinusoidal input and measure the sinusoidal output voltage. To find the upper cutoff frequency, we have to vary the input frequency until the voltage gain drops 3 dB from the mid-band value. Sine-wave testing is a common approach. But there is a faster and simpler way to test an amplifier, Using a *square-wave input* instead of a sine-wave input.

Is there a short cut method of measuring the frequency response of an amplifier? What is the normal procedure? The normal procedure is you construct the amplifier, you apply the signal source and keep the amplitude low because it is a small signal amplifier and monitor the output. You keep on increasing the frequency from very low value to very high value and every time you record the output voltage from which you can get the gain and at very low frequencies the gain is very small and as you increase the frequency the gain increases and it comes to a maximum value and it maintains the maximum value constant over wide range of frequencies and if you further increase the input frequency it starts falling off and that is how the response curve that I showed comes about.

This will take long time. If I start from very low frequency to very high frequency checking at every frequency what is the gain and what is the input output voltage etc., it will take a long time. Is there a quick method by which I can measure the frequency response or get an idea about the response of the amplifier? The answer is yes and that is by using what is known as a square wave testing of the amplifier. Before I go into that I will briefly explain to you the concept of the rise time and the fall time and the corresponding relationship between the rise time and the bandwidth.

Here I have shown a very simple circuit with a voltage source, a switch and a resistor and a capacitor; something like our equivalent circuit.

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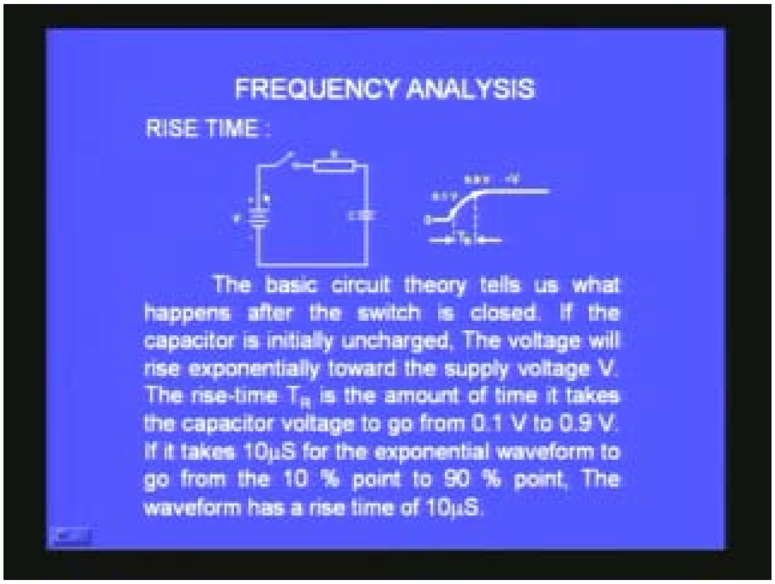

FREQUENCY ANALYSIS

RISE TIME :

The basic circuit theory tells us what happens after the switch is closed. If the capacitor is initially uncharged, The voltage will rise exponentially toward the supply voltage V. The rise-time T_R is the amount of time it takes the capacitor voltage to go from 0.1 V to 0.9 V. If it takes 10µS for the exponential waveform to go from the 10 % point to 90 % point, The waveform has a rise time of 10µS.

The only difference this is a dc voltage. If I now switch it on by moving this switch then suddenly there will be a large current flowing trying to charge the capacitor. Initially the charge on the capacitor will be zero. Once I switch on then the voltage will start building up very quickly to very high value and once it is completely charged it will be charged to the full voltage V and it will reach the voltage V and then it will remain constant if I don’t discharge for long time. What is the rise time? The rise time is shown here. It is defined as the time taken between the point when the voltage comes to about 10% of the maximum V plus and 90% of V plus. So the time between 0.1 times to 0.9 times the total voltage applied is what is called T_R the rise time. That is by definition. You can also do that by using a square wave instead of having a dc source.

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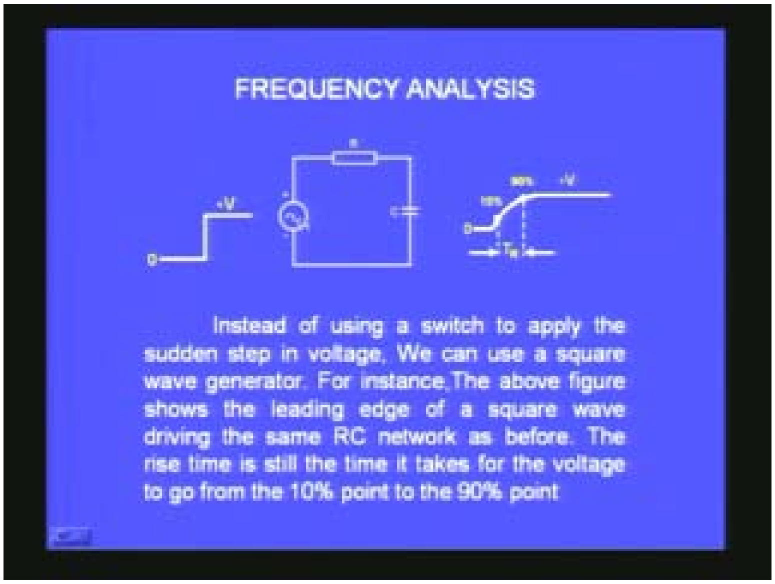

FREQUENCY ANALYSIS

Instead of using a switch to apply the sudden step in voltage, We can use a square wave generator. For instance,The above figure shows the leading edge of a square wave driving the same RC network as before. The rise time is still the time it takes for the voltage to go from the 10% point to the 90% point

You can have a square wave source that is a function generator with a square wave output. You connect the RC and you switch on you would find this increase from zero to plus V will not be instantaneous at the output depending upon the bandwidth and so there will be a finite time delay in reaching the full V value, voltage value and that can be defined as the rise time, the time taken to rise from 10% to 90% of the total amplitude V of the square wave; I can take it as the rise time. If I have a simple RC circuit and give a square wave you would find the output will not be exactly square.

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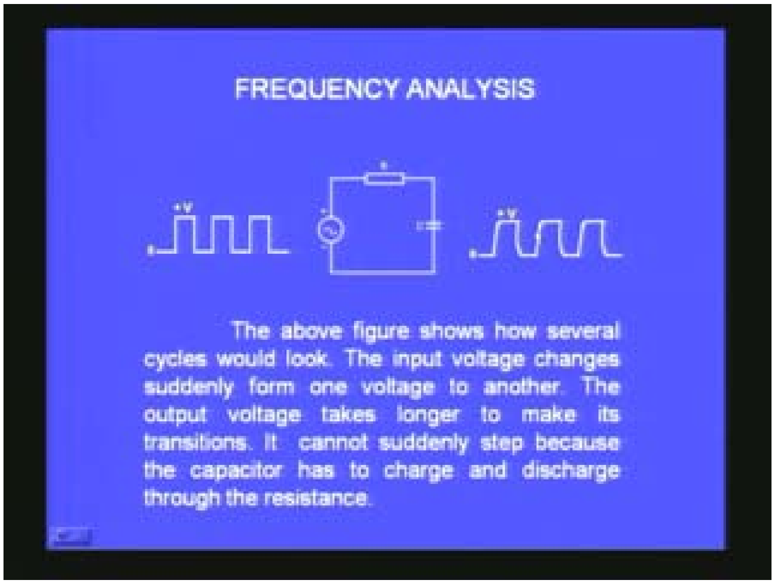

FREQUENCY ANALYSIS

The above figure shows how several cycles would look. The input voltage changes suddenly form one voltage to another. The output voltage takes longer to make its transitions. It cannot suddenly step because the capacitor has to charge and discharge through the resistance.

It will show some distortions from the input square wave. These distortions give us lot of ideas regarding the overall frequency response of the amplifier. That is what we should try to understand. For example I have here a square wave which goes from V_m as the maximum voltage to $-V_m$ as the minimum voltage and has a period T.

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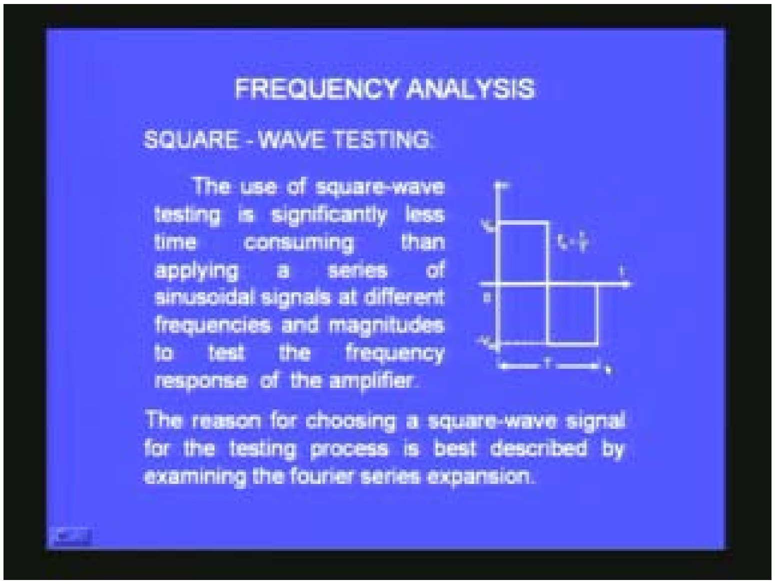

FREQUENCY ANALYSIS

SQUARE - WAVE TESTING:

The use of square-wave testing is significantly less time consuming than applying a series of sinusoidal signals at different frequencies and magnitudes to test the frequency response of the amplifier.

The reason for choosing a square-wave signal for the testing process is best described by examining the fourier series expansion.

What I do is I give this as the input to an amplifier whose bandwidth I want to evaluate and monitor the output wave form on an oscilloscope. From the type of waveform you get if it is a very wide band width amplifier what would you get? You will get an exact reproduction of the same square wave at the output. In that case the bandwidth is very, very high; reasonably high. But you will always get some distortions in the square wave and those distortions will indicate to you quickly what will be the bandwidth; whether it is having good low frequency response or good high frequency response etc., one can get by quickly analyzing using a square wave input. Why is it so? What is special about the square wave? You may recall there is a Fourier theorem or the Fourier series of expansion for a square wave. I have given it on the screen.

It is having contributions due to several frequencies sin 2 pi f_s t which is the fundamental frequency; f_s is contribution due to the fundamental frequency and 1/3 sin 2 pi 3 f_s which is the third harmonic, 3 times f_s, contribution; similarly the fifth harmonics 5 f_s, n^{th} harmonic, etc.

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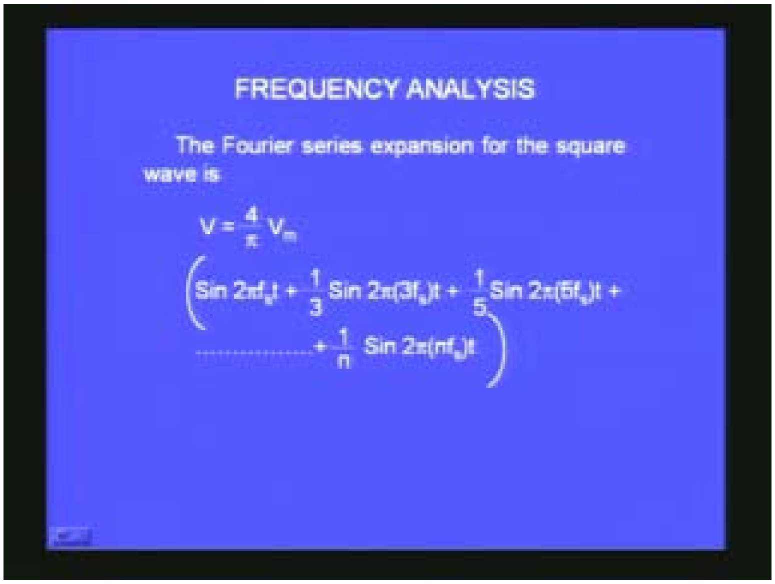

FREQUENCY ANALYSIS

The Fourier series expansion for the square wave is

$$V = \frac{4}{\pi} V_m \left(\sin 2\pi f_s t + \frac{1}{3} \sin 2\pi (3f_s) t + \frac{1}{5} \sin 2\pi (5f_s) t + \ldots\ldots + \frac{1}{n} \sin 2\pi (n f_s) t \right)$$

If I now take all these sine waves separately one at f_s, one at $3f_s$, one at $5f_s$ and monitor the amplitude to be 1/3rd, 1/5th etc and then superpose all of them together what will I get? If I keep on doing it up to infinity I should not stop with n but I should keep on doing it up to infinity. Then I would get a perfect square wave which means to say a perfect square wave can be considered as due to several sine wave contributions which are basically odd harmonics like f_s, $3f_s$, $5f_s$, $7f_s$, etc up to infinity and there is a gradual decrease in the amplitude also of these components and if you superpose all of them what you get will be the total. Instead of doing spot frequencies you are actually doing it by using a square wave and because the square wave is contributed by number of frequencies you will be able to understand the contribution. That is what is shown in this figure here.

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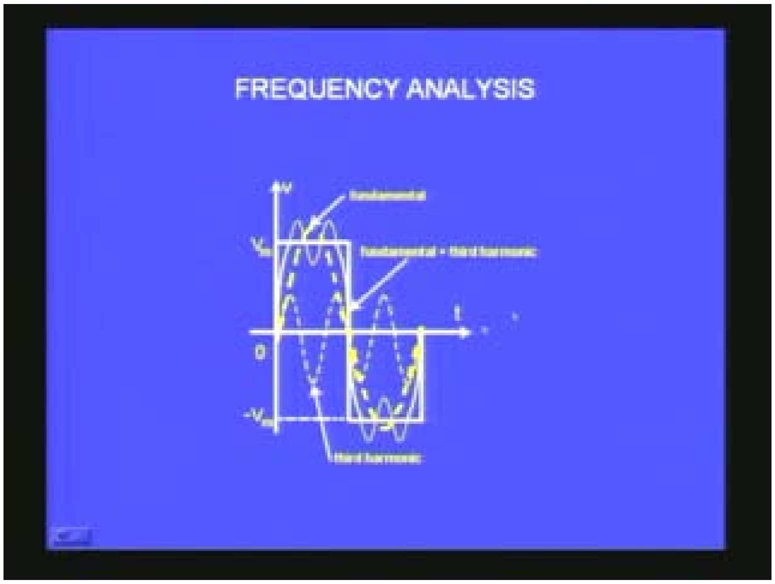

FREQUENCY ANALYSIS

You can see there are number of sine waves coming in. This is the fundamental one, large frequency. Then you have the third harmonics that is coming here and the contribution due to all of them if you add you will get a perfect square wave if it is for infinite sequence. In the next picture I have shown you the various types of square wave that we will get at the output.

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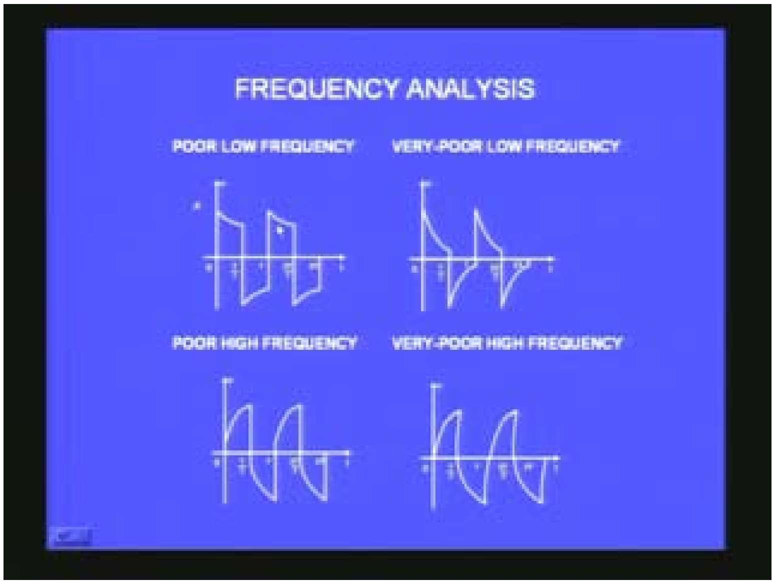

FREQUENCY ANALYSIS

For a clean square wave that you give at the input you will get different types of square wave at the output and by looking at the wave form you will be in a position to say what type of an amplifier it is. What is its frequency response? For example a poor low frequency response of an amplifier will be shown by this shallow region that you see here. There will be a curved portion at the top which should normally be flat. For a good square wave it will be curved and this shows that it has got poor low frequency response if you have an output like this. If you have output still bad very poor, low frequency it would fall by a very large extent. This fall will be very large. When that happens the low frequency response of that amplifier is very, very poor. What about the high frequency? If the high frequency response of an amplifier is very poor then you get a curve of this type. This will be corresponding to the poor high frequency and if it is very poor it will still be curved on both sides and this shows very poor high frequency response. If I just want to get quickly information regarding the performance or the bandwidth or the frequency response of an amplifier the quickest way is to just give a square wave at the input and look at the wave form that you get at the output. If it is any one of these you would able to decide whether it is having a poor or very poor low frequency response or high frequency response.

There is also a relationship with reference to the bandwidth and the rise time and that relationship is 0.35 by the rise time; t_r is rise time.

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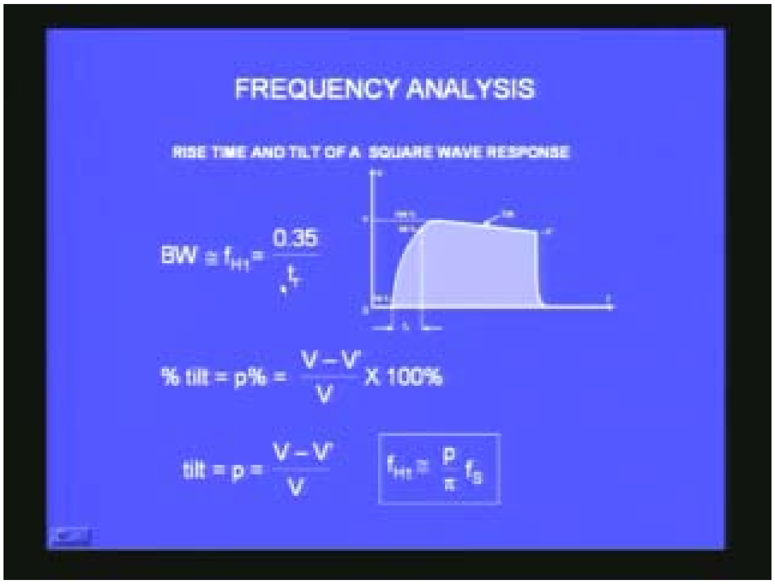

FREQUENCY ANALYSIS

RISE TIME AND TILT OF A SQUARE WAVE RESPONSE

$$BW \cong f_{H1} = \frac{0.35}{t_r}$$

$$\% \text{ tilt} = p\% = \frac{V - V'}{V} \times 100\%$$

$$\text{tilt} = p = \frac{V - V'}{V}$$

$$\boxed{f_{H1} \cong \frac{p}{\pi} f_s}$$

0.35 by t_r is actually the bandwidth f_{H1} which is actually the cut off frequency at the high frequency and for the dc amplifier there is no low frequency cut off and this itself becomes the bandwidth. In general to get a quick idea of the bandwidth people will just do 0.35 by rise time by giving a simple square wave and looking at the rise time which is actually the time taken between 10% to 90% of the maximum you can find out the value of t_r and you will be able to get the estimate of the bandwidth also. That tilt that you see here in the graph for a square wave can be percentage tilt which can be obtained as p% is equal to - missing in audio V minus V prime by V where V is this voltage at the top and V prime is the lowest value that you got for the horizontal portion multiplied by100. This also can be used and from that also the P the tilt divided by pi into f_s the frequency of a square wave will give me the bandwidth idea. These are the two different ways in which you can quickly get, by using a square wave, the frequency response of an amplifier. What we have so for seen in this lecture is to look at the contributions from the coupling capacitors, the bypass capacitors which are all contributing to the low frequency response of the amplifier and also the inter electrode capacitance between the base and the collector and by applying Miller's theorem you can split them into two contributions at the input and the output and then evaluate their performance that will correspond to the high frequency cut off and from that we will be in a position to evaluate the bandwidth of the amplifier. We also saw how in a very simple manner we can use a square wave to get a quick idea about the bandwidth of a given amplifier from the rise time as well as from the type of wave form you get because the square wave due to the Fourier theorem is basically contributed by number of fundamental, 3rd harmonic, 5th harmonic, odd harmonic. All of them together if you combine you will get a square wave therefore is equivalent to performing the frequency response at different spot frequencies like f_3, f_5, f, etc and what you get is the result of all those things and so you can get a quick idea about the frequency response of amplifier. Thank you!