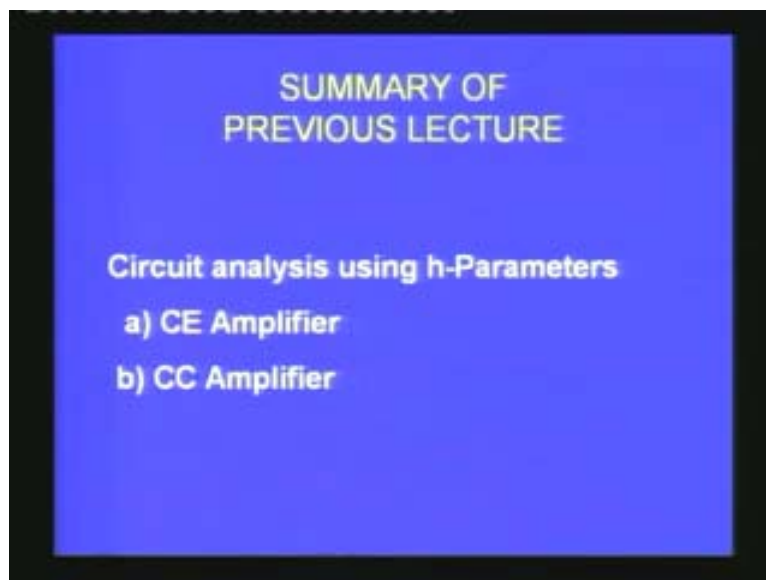


Basic electronics
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Department of Physics
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Lecture- 16

Frequency Response of Amplifiers

Hello every body! In our series of our lectures on basic electronics learning by doing we will now move on to the next. Before we do that let us quickly recapitulate what we discussed in the earlier lecture.

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You might recall that we discussed in the earlier lecture the complete circuit analysis of RC coupled amplifier for example using h parameters, the hybrid parameters. We took two examples. One is the common emitter amplifier and the other one is the common collector amplifier which is also known as the emitter follower. As a matter of fact common emitter amplifier is a standard amplifier which is used very widely for basic amplification purposes and the common collector amplifier is used for impedance matching purposes. We obtained the gain, the input resistance, the output resistance and various other parameters of the amplifier for these two cases by drawing the equivalent h parameter model of the transistor amplifier.

Now we have to go further into the actual analysis of such amplifiers. You might recall when I drew the equivalent circuit of these common emitter amplifiers for example I completely ignored the presence of the capacitors. There are number of capacitors that we used one at the input and one at the output C_1 and C_2 and we also used the capacitors to bypass across the R_E the emitter resistance. But when we took the equivalent circuit of the amplifier we ignored the contribution due to the coupling capacitor, reactance of the

coupling capacitor C_1 and C_2 etc and we actually used the short for the sake of C_E because we assume that we are going to operate the amplifier around the mid band frequencies. At the mid band frequencies you normally have a constant gain and therefore at that frequencies the reactance offered by the C_1 C_2 and C_E are very, very small and therefore we neglect it. But we cannot do it all the time. We have to analyze the circuit for the entire frequency range in order to obtain very important parameters of the amplifier which is known as the band width of the amplifier, all the bands of frequencies over which the amplifier can be reliably amplifying all the voltages, input voltages.

In order to get that you have to take into account the contributions due to C_1 , C_2 , etc and that is what we will discuss. But before we go into the frequency response of this it will be better to look at a very important aspect of how the gains are represented for amplifiers usually in terms of units of Bel after the great inventor Alexander Graham Bell. Before we go into that what is the power gain of an amplifier? The power gain of an amplifier is the ratio of output power to the input power.

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DECIBEL GAIN

POWER GAIN :

The power gain G of an amplifier is the ratio of output power to input power.

$$G = \frac{P_2}{P_1}$$

If the output power is 15 W and the input power is 0.5 W.

$$G = \frac{15 \text{ W}}{0.5 \text{ W}} = 30$$

The output power is 30 times greater than the input power.

For example G is P_2 by P_1 where P_2 is the output power and P_1 is the input power. Usually if it is a gain then P_2 should be larger than P_1 . If the output power as an example is 15 watts in a typical case and the input power is about 0.5 watt then you know what the gain is. G is 15 watt divided by 0.5 watt and that will be equal to 30. That means the output power is 30 times the input power. That is there is a reasonable gain, power gain in the amplifier. The power gain of a RD amplifier is normally measured in units of Bel in honor of Alexander Graham Bell the inventor of the telephone.

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DECIBEL GAIN

The power gain of an audio amplifier is measured in units of 'BEL' in honour of Graham Bell the inventor of Telephones.

1 Bel = 10 deciBel

As a unit, the 'Bel' was actually devised as a convenient way to represent power loss in telephone system wiring rather than gain in amplifiers. It is defined in logarithmic scale since human ear is sensitive to sound energy only in logarithmic scale.

The Decibel power gain (G') is defined in terms of the normal gain(G)

$$G' = 10 \log G \text{ dB}$$

The usual unit of power amplifier is decibel rather than Bel and 1 Bel is 10 decibel. As a unit the Bel was actually devised as a convenient way to represent power loss in telephone systems because Graham Bell is associated with telephone rather than gain of the amplifiers in general and it is also in logarithmic scale. The decibel scale is actually in logarithmic scale and that is because I already mentioned to you in earlier lectures also that the human ear is sensitive to sound energy only in logarithmic scale. So the decibel power gain G' is defined in terms of the normal gain G as G' is equal to 10 times $\log G$. That is the decibel power gain. I will give you an example. If the circuit has got a power gain of 100 what will be its power gain in decibel units?

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DECIBEL GAIN

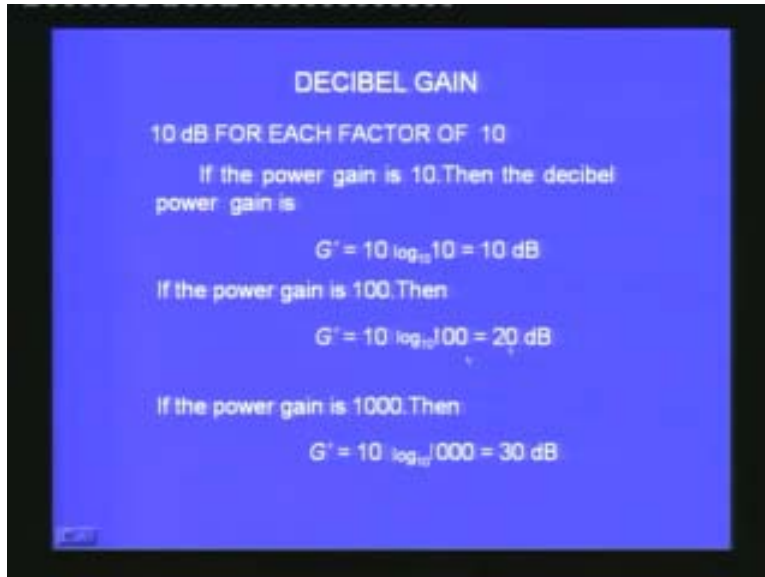
DECIBELS :

If the circuit has a power gain of 100, its decibels power gain is

$$G' = 10 \log 100 = 10 \log_{10} 10^2 = 20 \text{ dB}$$

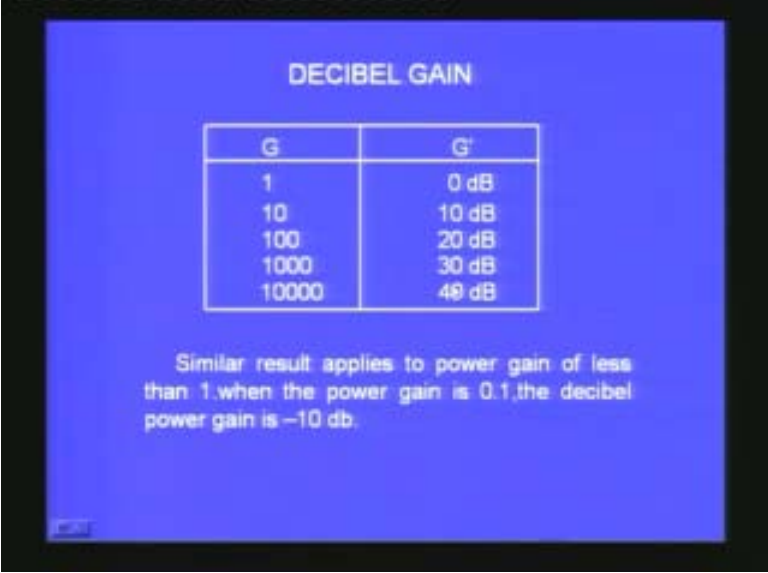
G prime is $10 \log 100$, $10 \log$ to the base 10. 100 can be written as 10 square and you can take the exponent over here. So it becomes 20 dB. Because $\log 10$ to the base 10 is 1, it becomes 20 dB. So instead of saying it has got a power gain of 100 as a ratio you can also say it has got a power gain of 20 dB, 20 decibels. When I now increase the gain by every 10 for example if the power gain is 10, the power gain in decibel unit is $10 \log 10$. That is equal to 10 dB and if I increase it 10 times the power gain is 100. Then G prime is $10 \log 100$ which is 20 dB just now we saw.

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If the power gain is 1000, 10 into 10 into 10 then the G prime becomes $10 \log 1000$ which is 30 dB. What you see is every time for a factor of 10 the power gain increases in decibel unit by 10 units; 10, 20 dB, 30 dB, 40 dB etc. That is what is shown in this table here. When the gain ratio is 1 it is 0 dB. When the gain is 10 it is 10 dB. When it is 100 it is 20 dB etc.

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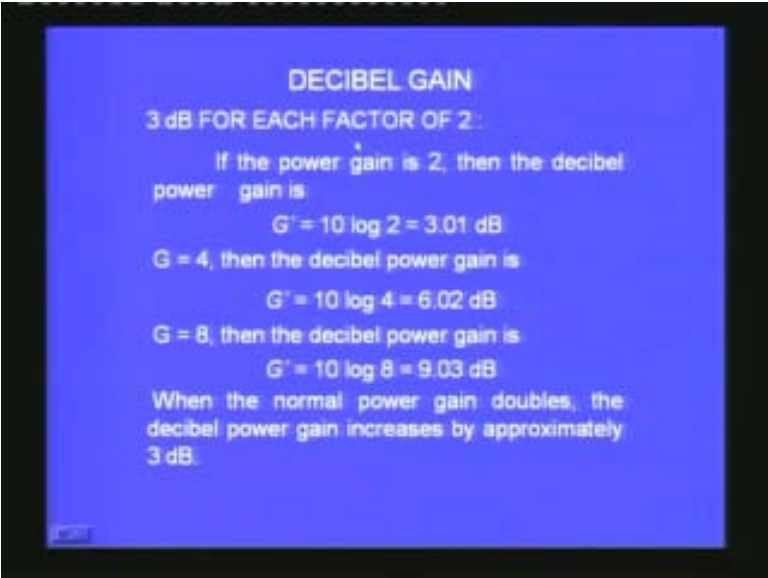
DECIBEL GAIN

G	G'
1	0 dB
10	10 dB
100	20 dB
1000	30 dB
10000	40 dB

Similar result applies to power gain of less than 1. When the power gain is 0.1, the decibel power gain is -10 dB.

Similar results will apply to power gain which is not actually the gain or loss which are less than 1. When the power gain is 0.1 the decibel power gain is -10 dB. When the power gain is -0.01, that is 100 then it will become -20 dB etc. Similarly we can also look for what happens to the power gain when it is a factor of 2.

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DECIBEL GAIN

3 dB FOR EACH FACTOR OF 2:

If the power gain is 2, then the decibel power gain is

$$G' = 10 \log 2 = 3.01 \text{ dB}$$

G = 4, then the decibel power gain is

$$G' = 10 \log 4 = 6.02 \text{ dB}$$

G = 8, then the decibel power gain is

$$G' = 10 \log 8 = 9.03 \text{ dB}$$

When the normal power gain doubles, the decibel power gain increases by approximately 3 dB.

For every factor of two the power gain will increase by a value corresponding to 3 dB. That also can be obtained from the same formula G' is equal to $10 \log 2$ that is 3 dB. If it is 4 it will be 6 dB and if it is 8 it will be 9 dB etc. For every factor of 2 the

decibel power increases by value of 3. It is not very difficult it is reasonably simple to understand.

I also mentioned to you that when there is a loss that means when the gain is less than 1 it becomes negative the dB becomes negative.

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DECIBEL GAIN

NEGATIVE DECIBELS :

If power gain is less than 1 there is a power loss(attenuation), and the decibel power gain is negative. For instance, if the output power is 1.5 W when the input power is 3 W, then

$$G = \frac{1.5 \text{ W}}{3 \text{ W}} = 0.5$$

and the decibels power gain is

$$G' = 10 \log 0.5 = 10 \log(\frac{1}{2}) = -10 \log 2 = -3.01 \text{ dB}$$

When the power gain is 0.25,

$$G' = 10 \log 0.25 = -6.02 \text{ dB}$$

If the power gain is 0.125, then

$$G' = 10 \log 0.125 = -9.03 \text{ dB}$$

When the power gain is 1 there is attenuation or loss and the decibel power gain is negative. For instance if the output power is 1.5 watt in a typical case and the input power is 3 watt then G the power gain the normal ratio is 1.5 by 3 which is actually half; 1/2 or 0.5 decibel and the decibel power gain is by applying the same formula G prime is equal to 10 log 0.5 to base 10. 10 log half, 0.5 is half, to the base 10 and that is minus because it is 1/2 I can write that as - 10 log 2 and that is -3 dB and if the power gain is 0.25 ratio then the decibel gain as you can see here becomes -6 dB. For every octave on the negative side below 1 you find the power gain decreases by 3 dB just as it was increasing on other side. So -3 dB, -6 dB, -9 dB, etc., it goes for every half.

That is what is shown in the next table that you see here .if the power gain in 1 it is in decibel 0 and if it is 0.5 it is -3 dB. If it is 0.25 that is half of this it is 0.6 dB and if it is 0.125 which is again half of the earlier one 0.25 then you have to add another -3 and it becomes -9 dB, -12 dB etc.

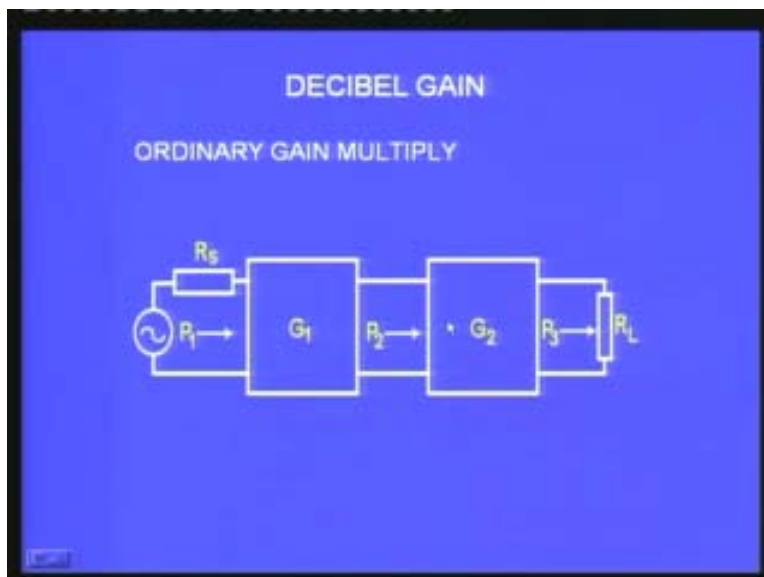
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DECIBEL GAIN

G	G'
1	0 dB
0.5	-3 dB
0.25	-6 dB
0.125	-9 dB
0.0625	-12 dB

If you understand these two simple things you would be quickly be able to convert from a normal ratio to a decibel value. What is the advantage of decibel? There are some advantages also that you get when you go into the decibel. The first thing is it corresponds more to the sensitivity of the human ear. I already mentioned to you it is in logarithmic scale. Apart from that it becomes much convenient mathematically also because a product in normal scales becomes an addition when you come to logarithms. Now I have here an example of two amplifiers. G_1 and G_2 are the gains, power gains of the two amplifiers.

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P_1 and P_2 are the input and the output power of G_1 , P_2 is input of G_2 and P_3 is the output power of G_2 . That means the G_1 output is directly connected to G_2 as the input and this input here is a sine square wave or a sine wave signal and the corresponding signal resistance R_s is shown and you have R_L here at the output. This is a very simple case of two stage amplifier with a corresponding power shown. **Let us assume what happens?** What is the gain of the first stage G_1 ? It is nothing but P_2/P_1 because P_2 is the output power of G_1 , P_1 is the input power of G_1 . Therefore P_2/P_1 is the power gain for G_1 .

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DECIBEL GAIN

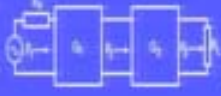
The Figure shows two stages in an amplifier. The first stage has an input power of P_1 , an output power of P_2 , and a power gain of

$$G_1 = \frac{P_2}{P_1}$$

The second stage has an input power of P_2 , an output power of P_3 , and a power gain of

$$G_2 = \frac{P_3}{P_2}$$

The total power gain of the two stages is

$$G = G_1 \times G_2 = \frac{P_2}{P_1} \times \frac{P_3}{P_2} = \frac{P_3}{P_1}$$



For the second stage the P_2 becomes the input and P_3 is the output. So G_2 is P_3/P_2 the output power by the input power of the amplifier G_2 as you can see in this figure. What is the total power gain? The total power gain is the product of the two. If the first one amplifies by 10 times and if the second amplifier amplifies by another 10 times totally you will have a gain of 100, 10 into 10. You have to multiply the gains of the two stages. I hope you see the point. G the total power gain of the entire amplifier having two stages is equal to the power gain of the first stage G_1 multiplied by the power gain of the second stage G_2 . We have already got the expression for G_1 and G_2 . P_2/P_1 multiplied by P_3/P_2 and therefore you would find that the two P_2 's will cancel because it is common to both and ultimately you are left with P_3/P_1 . So the final gain is the final power output to the initial power input.

I have taken a numerical example to make it clear to you. The first stage has got a gain of 100 and the second stage has got gain of 200. Power gain G_1 is 100 for example G_2 is 200. What will be the overall power gain of the two stage amplifier? You have to multiply these two. That means it is 20,000; 200 into 100, 20,000 or G_1 into G_2 .

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DECIBEL GAIN

Which is equal to



$G = \frac{P_2}{P_1} \cdot \frac{P_3}{P_2}$

$G = G_1 G_2$

This proves that the total power gain of cascaded stages equals the product of the stage gains.

For an example, a first-stage power gain of 100 and a second - stage power gain of 200, the total power gain is

$G = 100 \times 200 = 20,000$

If I have a cascaded stage; cascaded stage means you take one stage couple it to the next stage and then couple it to third stage etc. If you want large gains you cannot achieve it by one single amplifier may be you have to use multiple amplifiers and then amplify the amplified signal of the previous stage. That's how you can achieve very large gains. If you do that then the only way to calculate the total power gain is to multiply the power gain of the individual stages. That's what we have shown. So the total gain of the two stage amplifier example that I have taken is 20,000. Let us do the same in decibel scale.

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DECIBEL GAIN

DECIBEL GAINS ADD

Since the total gain of two cascaded stages is

$G = G_1 G_2$

We can take the logarithm of both sides to get

$\text{Log } G = \text{log}(G_1 G_2) = \text{log } G_1 + \text{log } G_2$

Multiplying both sides by 10 gives

$10 \text{ Log } G = 10 \text{ log } G_1 + 10 \text{ log } G_2$

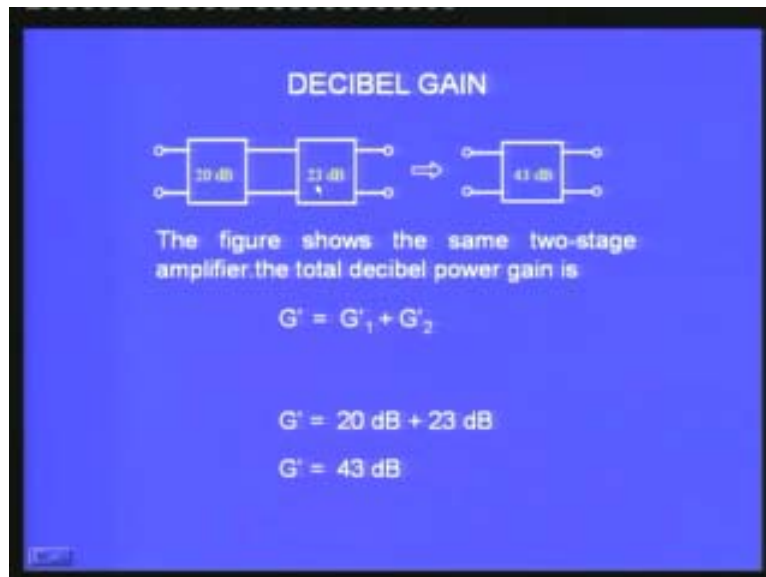
Which can be written as

$G' = G'_1 + G'_2$

In decibel scale you know G is equal to G_1 and G_2 where G , G_1 and G_2 are normal power gain. If I take the logarithm of that $\log G$ is \log of $G_1 G_2$ which is nothing but $\log G_1 + \log G_2$ from fundamentals of logarithmics and therefore if I multiply both sides of this equations by 10, $10 \log G$ is equal to $10 \log G_1 + 10 \log G_2$. You can immediately identify what is $10 \log G$. $10 \log G$ by definition is the gain in decibel scale. That is G prime. So the overall gain in decibel G prime is equal to $10 \log G_1$ that is G_1 prime which is the power gain in decibel scale of the first stage and $10 \log G_2$ which is the power gain in decibel of the second stage, G_2 prime. So you can see the power gain in decibel scale is actually sum of the two intermediate stages. The two stages that you have employed G_1 prime plus G_2 prime. It is much easier to add than to multiply numbers. So it becomes much convenient for you.

Now I have taken another simple example to show you how this happens. For example the first stage amplification is 20 dB, power gain. The second stage is let us say, 23 dB.

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Then what is the overall power gain if somebody asks all that you have to do is add. Because they are already in decibel scale you just have to add the two gains 20+23 is actually 43 dB. This is the actual gain. So that is what is shown here. G prime is G_1 prime + G_2 prime and that is equal to 43 dB. You must become familiar with converting normal power gains into decibel scale as well as the power gain given in decibel back into normal power levels. Ultimately perhaps you will understand what the amplifier is doing? How many times it is able to amplify we want to understand then you must know how to convert the decibel scale back into ratio. Then you will know the output by input as a ratio. That will be very useful. But for that you should use the earlier discussions that we had for every decade of the gain you have 10 dB to be added. So if I say 20 dB the amplification should be 100 because 10 into 10. For every 10 you have to add 10 dB. So 100 should correspond to 20 dB. Similarly if it is in steps of 2 then you have to add 3 dB for the power gains. Multiply by 2 means 3 dB; multiply by 4 means 6 dB. Using this

simple idea you will be able to approximately calculate quickly what the ratio is if somebody gives the gain in decibel scale.

There is another important idea that I want to share with you with reference to the decibel gain especially with reference to a reference. Usually you will have the input in the order of few milli watts and therefore we should like to have with reference to milli watt what is the decibel gain and if I use 1 milli watt as the reference then the power gain will have to be represented as P prime **is equal to-** seen in slide; missing in audio $10 \log P$ by 1 milli watt. So you are trying to see if it is 1 milli watt what the output gain, power gain will be? It becomes much easier to understand if I choose 1 milli watt as the input power.

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DECIBEL GAIN

1-mW REFERENCE

Although they are ordinarily used with the power gain, decibels are sometimes used to indicate the power level with respect to 1 mW. In this case, the label dBm is used instead of dB; the m at the end of dBm reminds you of *milliwatt* reference. The dBm formula is

$$P' = 10 \log \frac{P}{1 \text{ mW}}$$

Where P' = power in dBm
 P = power in watts

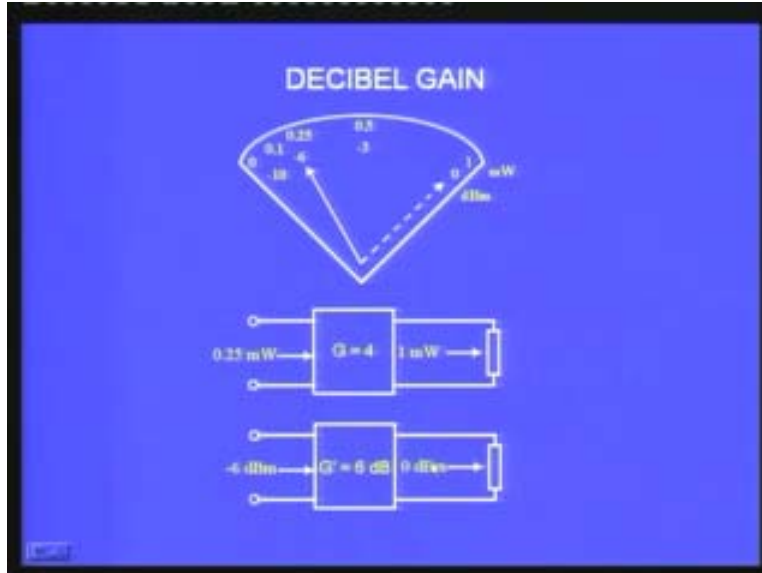
For instance, if the power is 2 W, Then

$$P' = 10 \log \frac{P}{1 \text{ mW}} = 10 \log 2000 = 33 \text{ dBm}$$

That becomes a reference. P prime is power gain in dBm. The power gain when it is referred to 1 milli watt as the reference it is called dBm where the final m in the symbol corresponds to milli watt reference. If it is P in power in watts P prime is 10 log power in watts divided by 1 milli watt and for example if it is 2 watts P prime is 10 log P by 1 milli watts that is 2/1; 2 is actually in watts, 1 is 1 milli watts. I should convert it into milli watt. That will be 2000. So 10 log 2000 that is 33. Why is it 33? I have three zeros. For every zero I should add 10 dB of gain. Therefore 10+10+10 is 30. For a factor of 2 I should add 3 dB. Therefore 30 + 3 is 33 dBm. If we understand these ideas it is possible to quickly calculate the power gain in ratio, the power gain in decibel and the power gain in dBm with a 1 milli watt reference.

Here I have another example. If the input power is 0.25 milli watt in an amplifier and the output is 1 milli watt then there is a factor of 4. If it is a factor of 4 with reference to milli watt that should correspond to 6 dB but because there is a 0.25, 6 dB is the gain in decibel scale. Similarly here if I have -6 dBm as the input because 0.25 milli watt can correspond to -6 dBm, if we do the calculation, and 1 milli watt will correspond to 0 dB and the gain will have to be 6 dB because output becomes zero.

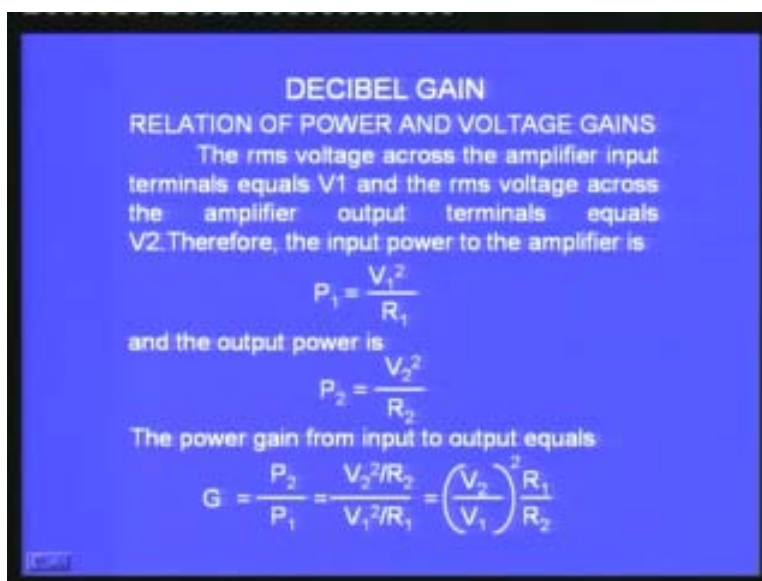
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When will it become zero? Only when I add - 6 to + 6. The gain of the amplifier in decibel is 6 dB and that is also the same as in ratio a factor of 4.

Let us move on to the gain that one normally refers to in any amplifier which is the voltage gain. Because most of the time we are operating on voltage amplifiers and all the rc coupled amplifiers that we discussed about correspond to the voltage amplifiers.

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But we seem to be concentrating more on power gain. We should try to see what will happen if I want to measure voltage gain in terms of decibels. The rms voltage across the

amplifier at the input terminal let us say is V_1 and the rms voltage across the amplifier output terminal is equal to V_2 . Then what is the voltage gain? We all know voltage gain is V_2/V_1 , output voltage by input voltage. But what is the power gain in terms of voltage? The power gain you know power is V square by R . So power gain P_1 for this amplifier will be V_1 square by R_1 corresponding to the input. This is actually the input power. P_1 is equal to V_1 square by R_1 is the input power and the output power P_2 is equal to V_2 square by R_2 where V_2 is the output rms voltage. There is a square factor coming here. So what is the power gain?

Power gain is P_2/P_1 . That is V_2 square by R_2 divided by V_1 square by R_1 . That means V_2 by V_1 whole square into R_1 by R_2 . This R_1/R_2 will come inverted therefore R_1 by R_2 . The power gain G can be represented in terms of ratio of voltages. That's what we have done; V_2 by V_1 whole square into R_1 by R_2 . What is V_2 by V_1 ? That is the voltage gain. The power gain can be related to voltage gain. Actually it is the square of the voltage gain multiplied by the input resistance and the output resistance R_1 and R_2 . But usually maximum power transfer theorem tells us that power is transferred completely only when the output impedance matches with the input or the source resistance matches with the load resistance and we would always consider that case because we want maximum efficiency, maximum power delivery and we can assume in most of the cases R_1 will be equal to R_2 . We will attempt to do that and R_1 is equal to R_2 means G the power gain is equal to A square, the square of the voltage gain. That is what we get.

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DECIBEL GAIN

Since the ratio V_2 / V_1 is the voltage gain A ,

the power gain is

$$G = A^2 \frac{R_1}{R_2}$$

Usually $R_1 = R_2$ under matched load condition.

Hence $G = A^2$,

Once we know that we can immediately go on to converting it into decibel scale. Then what I have to do? G prime is $10 \log G$ and $10 \log A$ square to the base 10 and the A square term can come to the front and that will become $20 \log A$ to base 10. So the power gain is $10 \log G$ to base 10. The voltage gain is $20 \log V$ or A to base 10 where A is the voltage gain. The **premultiplier** in these two cases will be different. For power gain it is 10 for voltage gain it is 20 when you want to convert it into decibel scale. So this is very

simple to understand. For example I have a voltage gain of 40. A is 40 and what is A prime?

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DECIBEL GAIN

DECIBEL VOLTAGE GAIN

Voltage measurements are more common than power measurements. Therefore, it is not surprising that decibels are also used to specify voltage gain. *Decibel voltage (A') gain is defined in terms of normal Gain (A) as $A' = 20 \log A$*

For example,
if A is 40, then $A' = 20 \log 40 = 32 \text{ dB}$

G	G'
1	0 dB
2	6 dB
4	12 dB
8	18 dB

A	A'
1	0 dB
0.5	-6 dB
0.25	-12 dB
0.125	-18 dB

A prime is $20 \log 40$ and that is 32. Why is it 32? If you take the logarithm of 40 to the base 10 and multiply by 20 you will have this value. I have given here an example of the power gain and the corresponding voltage gain in decibel scale. For 1 it is 0 dB; for 0.5 it is -6 dB. In the power gain case it was -3 dB. In the voltage it has to be multiplied by 2. Therefore it is -6 dB, -12 dB, etc. When there is a factor of 2 there is 6 dB in the case of voltage gain.

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DECIBEL GAIN

FACTORS OF 2 AND 10

When A = 2,
 $A' = 20 \log 2 \approx 6 \text{ dB}$

When A = 4,
 $A' = 20 \log 4 \approx 12 \text{ dB}$

When A = 8,
 $A' = 20 \log 8 \approx 18 \text{ dB}$

When the voltage gain doubles, the decibel voltage gain increases by 6 dB

If the voltage gain decreases by a factor of 2, then the decibel voltage gain decreases by 6 dB. Also, when the voltage gain increases by factor of 10, the decibel voltage gain increases by 20 dB.

When there is a factor of 10 there will be a 20 dB addition in the case of voltage gain as against 10 dB addition in the case of power gain. That is what I want to show you. Here I have given examples of voltage gain; 1, 10, 100, 1000 and the corresponding decibel scale you can see is 0, 20, 40, 60.

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DECIBEL GAIN

$G' = 10 \log G \text{ dB}$

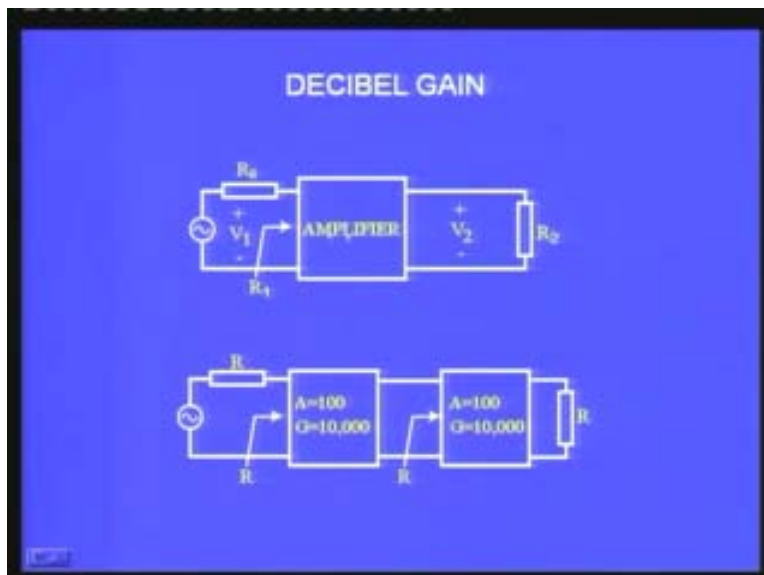
$G' = 10 \log A^2 \text{ dB}$

$= 20 \log A \text{ dB}$

A	A'
1	0 dB
10	20 dB
100	40 dB
1000	60 dB

Every time it increases by 20 dB whereas in the case of power gain it was 10 dB. That is what you should recognize. Again I got some examples here.

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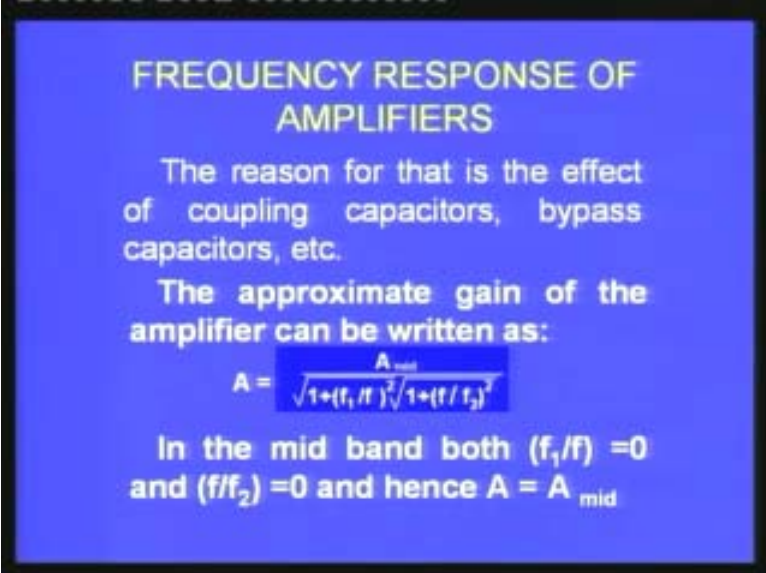


V_2/V_1 is the voltage gain. R_1 and R_2 will be matched. Usually the input resistance will be matched. When you go into the overall voltage gain you should try to add the decibel voltage gains of individual amplifiers. That's what I wanted to show here.

Let us move on to the next stage where you try to understand how to obtain the frequency response of an amplifier. What do mean by frequency response of an amplifier? I already mentioned to you earlier that an amplifier when it amplifies if I say the gain is 10 the gain should be 10 for a very wide range of frequencies. Then only it will be very useful to us. Whether it is a very low frequency or whether it is a very high frequency the gain should almost be a constant. But in practice it cannot be. No amplifier can have infinite band width. What is the meaning of the band width? I already mentioned the band width tells you how the gain is changing with reference to the frequency. Usually you would find at the low frequencies and at high frequencies the gain drops and only in the middle frequencies the gain is a constant over very large frequency range. That's why we always consider in the mid frequency range initially the amplifier performance.

We also have to understand the performance of the amplifier in the other two regions corresponding to the low frequency and the high frequency. That is what we are now attempting to do. Why does the voltage gain or the power gain fall in the case of an amplifier when there is low frequency input or high frequency input? The reason is very simple because we are using several capacitors in our circuit; the coupling capacitors, the bypass capacitors and also the capacitances due to the devices.

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FREQUENCY RESPONSE OF AMPLIFIERS

The reason for that is the effect of coupling capacitors, bypass capacitors, etc.

The approximate gain of the amplifier can be written as:

$$A = \frac{A_{mid}}{\sqrt{1+(f_1/f)^2} \sqrt{1+(f/f_2)^2}}$$

In the mid band both $(f_1/f) = 0$ and $(f/f_2) = 0$ and hence $A = A_{mid}$

For example transistor itself can have an internal capacitance parametric capacitance associated with the emitter, base, and collector terminals. All these capacitances will be dormant in the mid frequencies but when you go to the high frequencies and low frequencies they can become significant. Their reactances can become significant comparable with the other resistances that we have used in the circuit and therefore when

you are looking for low frequency response and high frequency response you should try to understand the effect of the reactances offered by these capacitances and that is what we should now look at. In general I will quickly give you a brief view, over view and then we will go into the detail.

The approximate gain of an amplifier if I want the **....** irrespective of the frequencies then I should write a general expression where the gain A is given by A_{mid} ; what do you know by A_{mid} ? It is the gain at the mid frequency where it is reasonably constant divided by $1 + f_1/f$ whole square under square root multiplied by $1 + f/f_2$ whole square under square root. There are two terms additional terms which are coming here where f without any subscript, the f is the frequency of operation, f_1 is the low frequency cut off, f_2 is the high frequency cut off. In the mid band the reactances offered by the capacitances will be very, very small, insignificant. Both f_1/f and f/f_2 will become zero at mid frequencies and therefore these will become root 1 and root 1 which is just 1 and A_{mid} gain at mid frequencies is equal to A_{mid} , the mid frequency. The other two terms that I introduced will have no role to play at the mid frequencies. But if you go to the low frequency range the contributions coming from the high frequencies f/f_2 , the f_2 is the high frequency cut off that will become zero; that will be insignificant and therefore what will be left is only the effect due to the low frequency term.

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FREQUENCY RESPONSE OF AMPLIFIERS

In the low frequency ranges $(f/f_2) = 0$
and hence

$$A = \frac{A_{mid}}{\sqrt{1+(f_1/f)^2}}$$

In the high frequency ranges $(f_1/f) = 0$ hence

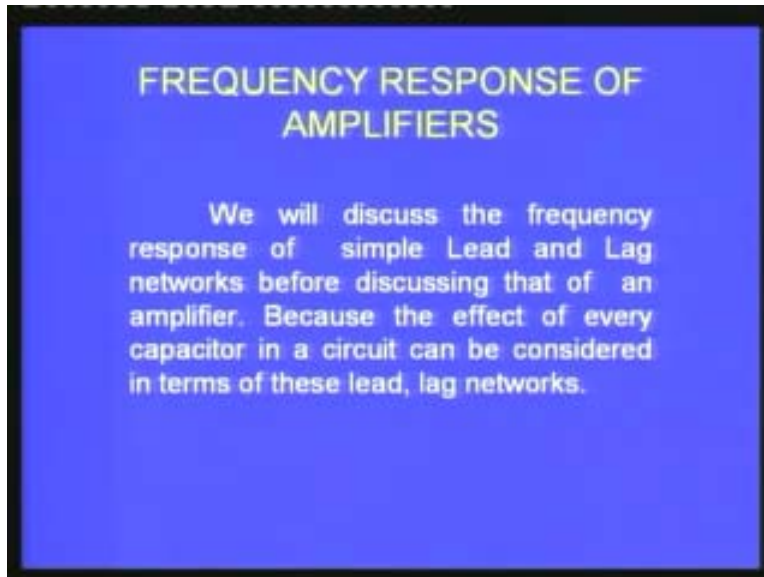
$$A = \frac{A_{mid}}{\sqrt{1+(f/f_2)^2}}$$

Here we assume only one dominant capacitor dominates both at low and high frequencies.

$1 + f_1/f$ whole square under square root is in the denominator and this becomes the gain at the low frequencies. Similarly if you go to the high frequency the f_1/f term will become zero and you will be left with f/f_2 term only in the expression. In all these cases f_1 , f_2 correspond to dominant capacitors. That means we have used several capacitors for coupling at different stages and there will be one capacitor which will give you the lowest cut off at the high frequency and the highest cut off at the low frequency. The worst case condition and therefore they will dominate. These two will dominate the rest of the capacitances. We do not have to worry about all the capacitances but we should look at

one capacitance which is dominant and calculate the bandwidth corresponding to that and that will tell you what is f_1 and f_2 ? We will go a little deeper into this and try to understand by taking a very simple example.

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Before we do that it is always good to have some background information built and the background information is to study the frequency response of two simple circuits. One is called the 'Lead' network, lead and the other is called the 'Lag' network. One of them will be having importance at low frequencies the other will have importance at high frequencies. We should look at both these contributions. Now I will move on to lead lag network discussion.

What is a lead network? A lead network is basically a very simple combination of a capacitor and a resistor connected to a signal source. If I connect a capacitor and resistor and take the output across the resistor then this becomes a potential divider. The reactance offered by the capacitance is X_C and the resistance is R . So X_C and R form a potential divider and you take the output across R . What will happen? If I now vary the frequency, keep the signal input constant, amplitude constant; V_{in} is constant for I change the frequency over a wide range starting from very low frequency to very high frequency. If you do that you would find you will get a graph as shown on the screen.


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LOW FREQUENCY RESPONSE

LEAD NETWORK :

The lead network of is the key analyzing low-frequency effects in amplifier. Capacitive reactance is given by $X_C = \frac{1}{2\pi fC}$

A capacitor is equivalent to an open circuit at very low frequencies and it is equivalent to a short circuit at very high frequencies. Incidentally, the circuit is called a lead network because the output voltage leads the input voltage.



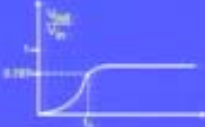
The diagram shows a lead network circuit on the left and its frequency response graph on the right. The circuit consists of an AC voltage source V_{in} in series with a capacitor C , connected to a load resistor R_L . The output voltage V_{out} is measured across the resistor. The graph plots the magnitude of the output voltage $|V_{out}|$ against frequency f . The curve starts at a low value at low frequencies, rises to a constant value at mid-frequencies, and then levels off at high frequencies. The mid-frequency gain is labeled as 1. The cut-off frequency f_c is marked on the x-axis, where the gain is 0.707. The y-axis also shows the 0.707 gain level.

Initially there is very low output and as you keep increasing the frequency, this is in the frequency range; this is the gain which is V_{out} by V_{in} corresponding to this network. Then you would find as I keep increasing the frequency slowly the gain increases and then when it reaches the mid frequencies it becomes constant really for a large range of frequencies. We talked about f_1 and f_2 earlier and this is f_c the cut off frequency which is the frequency at which the gain becomes 0.707 times the mid frequency range. If I take the mid frequency gain to be 1 this gain will have to be 0.7 or about 70%. That frequency at which the voltage gain drop to about 0.7 is called cut off frequency. So band width will always be referred with reference to this cut off frequency. This is the lower limit of band width and the higher limit will have a similar lag network which will produce a corresponding cut off frequency at the output stage. If you want to understand the frequency response of an amplifier you should understand the frequency response of a simple lead network having one capacitor and one resistor. What is the output voltage?

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LOW FREQUENCY RESPONSE

FREQUENCY RESPONSE :



The voltage gain of the lead network approaches 1 at higher frequencies.

CUTOFF FREQUENCY :

The lead network of fig. is an AC voltage divider with an output voltage of

$$V_{out} = \frac{R}{\sqrt{R^2 + X_C^2}} V_{in}$$

From our elementary knowledge of electricity you know V output is a potential divider. I said R divided by root of R square plus X_C square multiplied by V_{in} . This will be the output voltage. So what will be the gain? The gain will be V_{out} by V_{in} . That is equal to R divided by R square plus X_C square whole square and the cut off frequency is the frequency at which the reactance and the resistance equal and when that happens X_C is equal to R. What is X_C ? 1 by ωC . ω is $2\pi f$ where f is the frequency. 1 by $2\pi f$ into C is equal to R or if I want the corresponding frequency f will be equal to 1 by two πRC .

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LOW FREQUENCY RESPONSE

$$\frac{V_{out}}{V_{in}} = \frac{R}{\sqrt{R^2 + X_C^2}}$$

The *cutoff frequency* is the frequency at which X_C equals R . In symbols,

$$X_C = R$$
$$\frac{1}{2\pi fC} = R$$

solving for f gives

$$\frac{1}{2\pi RC} = f$$
$$\frac{1}{2\pi RC} = f_c$$

This will be the cut off frequency. In principle one can choose the value of R and C and precisely choose a cut off frequency that you want to design. You can have very specific values selected for giving very specific bandwidth in most of the amplifiers **if we can do that**. The f_c is $1 / 2\pi RC$. You might be wondering why I am saying the cut off frequency is corresponding to f_c is equal to R. It is actually coming from the power gain. I already mentioned to you in the earlier stages also when the power gain comes down by half by 50% then only we will be in a position to recognize. A power gain corresponding to half will correspond to a voltage gain fall of equal to 1 by root two because power gain is equal to square of the voltage gain. The voltage gain is equal to square root of the power gain. Therefore 1/2 of the power gain will correspond to 1 by root 2 and the value of 1 by root 2 is nothing but 0.707 if you calculate and therefore this all comes from the estimate of the power gain. That is why it is also called half power point. That cutoff point is also referred to as half power point because at that place the power gain drops by half.

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LOW FREQUENCY RESPONSE

HALF-POWER POINT :
 At the cutoff frequency $X_C = R$.

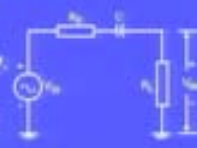
$$\frac{V_{out}}{V_{in}} = 0.707$$

This is the voltage gain at the cutoff frequency.
 The cutoff point is sometimes called the half-power point.

$$= \frac{(0.707 V)^2}{1 \Omega} = 0.5 W$$

SOURCE RESISTANCE :

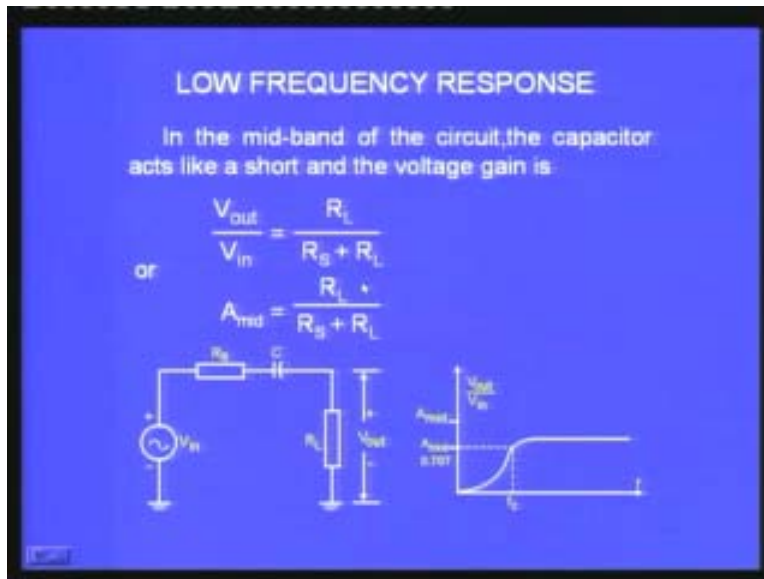
The voltage gain is given by.

$$\frac{V_{out}}{V_{in}} = \frac{R_L}{\sqrt{(R_s + R_L)^2 + X_C^2}}$$


In terms of voltage gain it is 0.7 times of the voltage gain at the mid frequencies. Corresponding to the dB scale it is -3 dB because a power gain of half will correspond to -3 dB we have already discussed. That is what I have shown here and if I now include also because I give a signal source here the signal source will also contribute through the source resistance R_s in series with this capacitor and the resistor that I have already connected in the circuit in the lead network. When this happens, the output circuit will slightly be modified. For example the gain now V_{out} by V_{in} will be R_L which is the output resistance, load resistance divided by square root of R_s plus R_L whole square plus X_C square. In the previous case we used only R but now because R_s and R_L are also there we have to add the two. R_s plus R_L whole square plus X_C square is what we have to do. That is the only thing that I have to do. So it will be slightly less; the gain will be slightly less now in this case. The half power point will act now when X_C is equal to the total resistance which is the combination of the source resistance plus the load resistance and

the $\frac{1}{2\pi f_c}$, the cutoff frequency now will be $\frac{1}{2\pi R_s + R_L} C$. So this gives you the frequency response of a lead network. Actually it is a simple problem. You keep on varying the signal source and try to measure. You will reproduce a similar graph that I have shown here without any difficulty. I will also perhaps show you a demonstration at a later stage.

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At the mid frequencies it is very simple. It is just R_L by R_s plus R_L because X_C will become insignificant, zero. The expression will simplify to R_L divided by R_s plus R_L at mid frequencies. That also we should remember. We sometimes refer to stiff coupling when I couple different amplifiers.

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LOW FREQUENCY RESPONSE

STIFF COUPLING :

Stiff coupling means that all lead networks satisfied this rule.

$$X_c = 0.1(R_s + R_L)$$

at the lowest frequency to be coupled Substituting this into above eqn. gives a voltage gain of

$$\frac{V_{out}}{V_{in}} = 0.995 A_{mid}$$

This shows, how effective stiff coupling is. At the lowest frequency to be coupled, the voltage gain is within half a percent of the mid-band gain.

$$f_{min} = 10 f_c$$

The stiff coupling means the reactance offered should be at least one tenth. When X_C is about 0.1 times R_s plus R_L , reactance offered by the resistance, then we call it as a stiff coupling and that corresponds to a decrease of gain only of about 5%. That is not even recognized; it is not much at all. It is almost equal to 1; 0.995. At the lowest frequency it will be about within half a percent of the mid band gain and the f minimum will be nearly about 10 times the cut off frequency. That is the minimum that you will have.

In terms of the decibel voltage A prime is equal to - seen in slide but missing in audio $20 \log 1$ by $1 + f_c$ by f whole square. This is the voltage gain of the lead network.

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LOW FREQUENCY RESPONSE

STIFF COUPLING :

The analysis of a lead network is similar to that for a lag network. The Decibel Voltage gain is

$$A' = 20 \log \frac{1}{\sqrt{1 + (f_c/f)^2}}$$

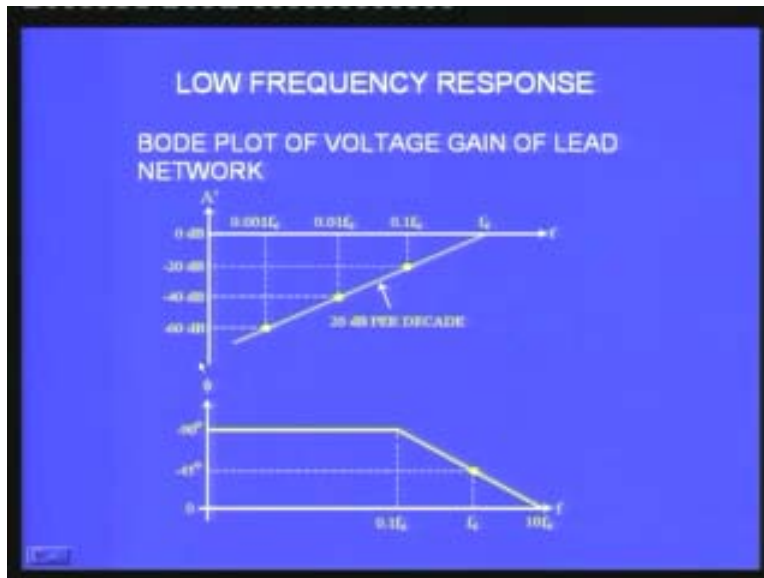
PHASE ANGLE :

The phase angle is

$$\phi = - \arctan \frac{f_c}{f}$$

There is also a phase because you have a capacitor you will also have a phase change and the phase angle is given by X_C by R . It is nothing but $\text{minus arctan } f_c \text{ by } f$ because it will be X_C by R $1 \text{ by } 2 \pi R_c$ and one by $2 \pi R_c$ is called f_c and f_c by f where f is the frequency that we consider now. You have two expressions one for the gain the other for the phase which we have to worry about and if you look at the frequencies, this is the frequency scale. It is a gain scale. At very low frequencies this will be very, very low.

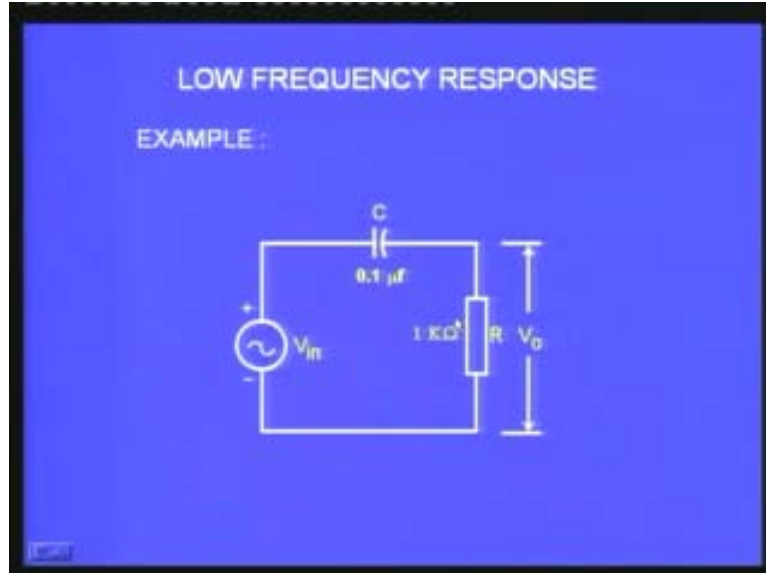
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It is in the minus. For example when it is 0.001, one thousandth of the cut off frequency the decibel gain is -60 dB. When it is 100 of the cut off frequency it is -40 dB. When it is 0.1 times that is what we discussed just now the cut off frequency is -20 dB and at cut off frequency it is 0. That means as I increase the frequency the gain starts increasing from very low values. That is what we should understand. That is what the graph also shows and this is corresponding to the phase angle. Then at the cut off frequency you have a phase angle of 45 degrees. Because X_C is equal to R , X_C by R will become 1 and at very, very low frequencies corresponding to the cut off frequency the phase angle will be almost equal to 90 degrees and very close to the cut off frequency or above the cut off frequency 10 times, 20 times it will be almost equal to 0 if the contribution will not be seen. These are the important ideas that you should remember with reference to a lead network.

A very similar discussion we will try to quickly go through in the next class about the lag network where I will just interchange the resistor and the capacitor and perform the same analysis and this will become important at the high frequencies.

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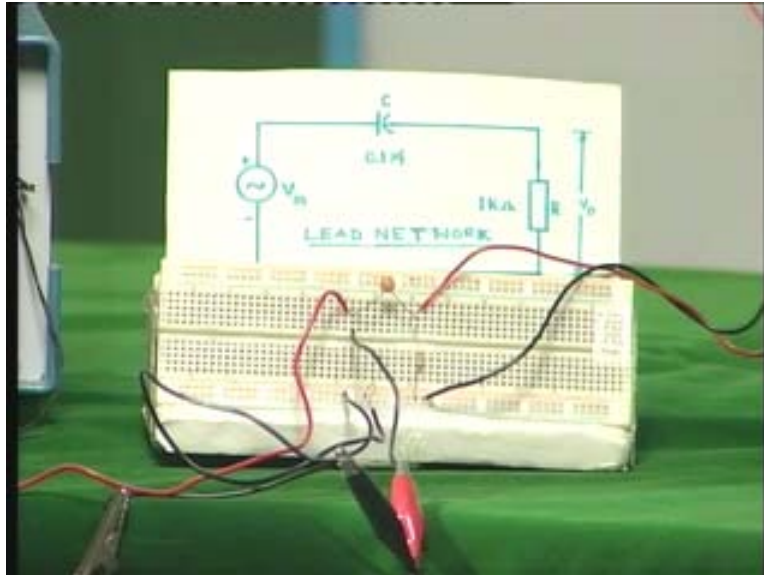


The lead network becomes important at the low frequencies and that corresponds to the coupling capacitors. The coupling capacitors are like this if they go into the base of a transistor amplifier here. The coupling capacitors are the ones which are to be very carefully analyzed for looking at a low frequency cut off amplifiers and similarly for the high frequency you will write are all the contributing factors.

Now what I would do is I would move on to the demonstration table and show you if I just have a combination of a 0.1 microfarad with 1 kilo ohm resistor and apply the sine wave and find out how the frequency response is you would find that it is very low for very low frequencies and as I increase the frequency the gain becomes slightly higher and it almost becomes equal at a later stage as I go to high frequencies. The gain goes from very low values to high value. That is what I want to show you and this lead and lag network will come again and again at various other discussions also in electronics. It is very important to understand that. Let me move on to the demonstration.

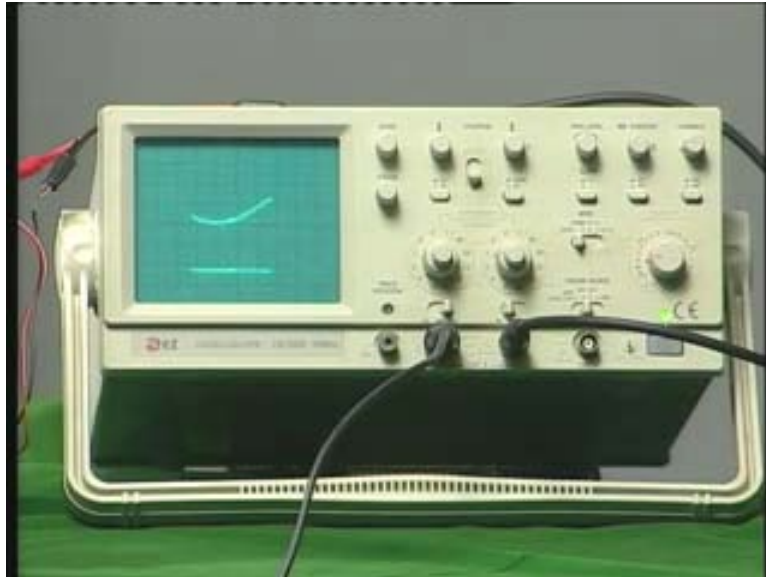
You can see here I have the bread board. This is the circuit I have wired on the bread board, a lead network. There is a function generator. This blue box is the function generator.

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The output of this I connected to the two points at the bread board and there is a capacitor here. There is a capacitor whose value is 0.1 microfarad and then you have a resistor here. I hope you can see and from the color code you can identify it is about 1 kilo ohm resistor. I have used a 0.1 micro farad capacitor and 1 kilo ohm capacitor here and I have connected the output of the function generator to the input of this network and the output of the network is taken across the 1 K resistor and connected to an oscilloscope here, CRO that you are already familiar with. In the CRO I have two channels. I want to simultaneously monitor the input sine wave as well as the output sine wave. That is what I have done here. For example we can check which one is that. The top one is actually the input the bottom one is the output. Now you can see the input amplitude is very large but the output amplitude is almost not there.

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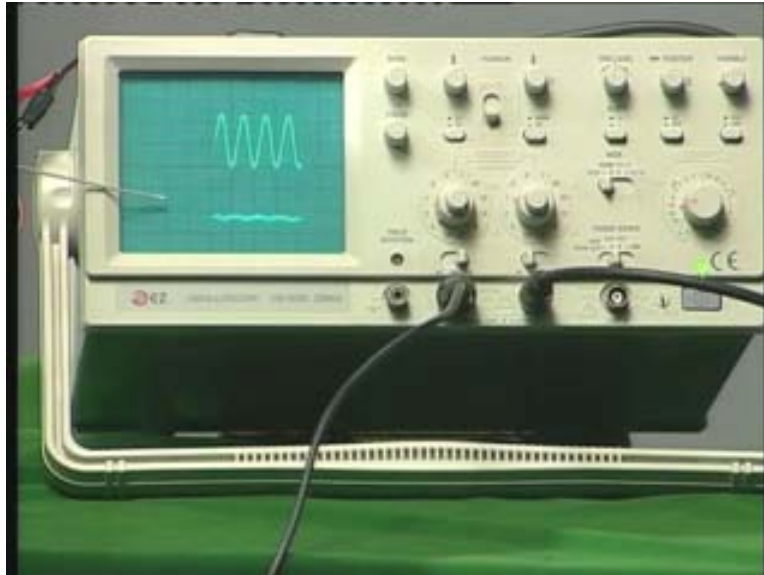
Because this is a lead network the gain of this should be very low at very low frequencies and if you look at the function generator you find I am having it in the scale 10.

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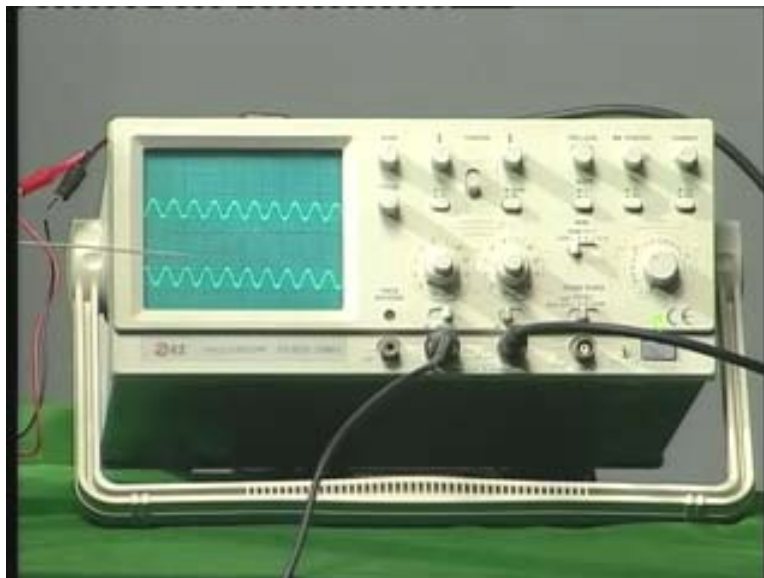
That means it is 1 into 10 about 10 hertz that is at a very low frequency. At very low frequencies the gain should be very, very low. That is what we saw. Let me quickly go to the next stage. That means I am pressing the 100. Now I am in 100 hertz frequency and now if you observe the oscilloscope you still have large amplitude for the input but the output is still very, very low.

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You see a sine wave. Previously you were seeing a straight line because the gain is too low. Now there is very small improvement here but there is no gain; still there is a loss because the input is larger than the output. Now I move on to the next range here. That means from 100 I go to 1000. It is now 1 into 1000. That means I have increased the frequency to 1000 hertz from 100 hertz and if you come back to the oscilloscope you find because of the high frequency, it is also stable on the screen; they are almost equal, the input and the output. That means the gain is better now.

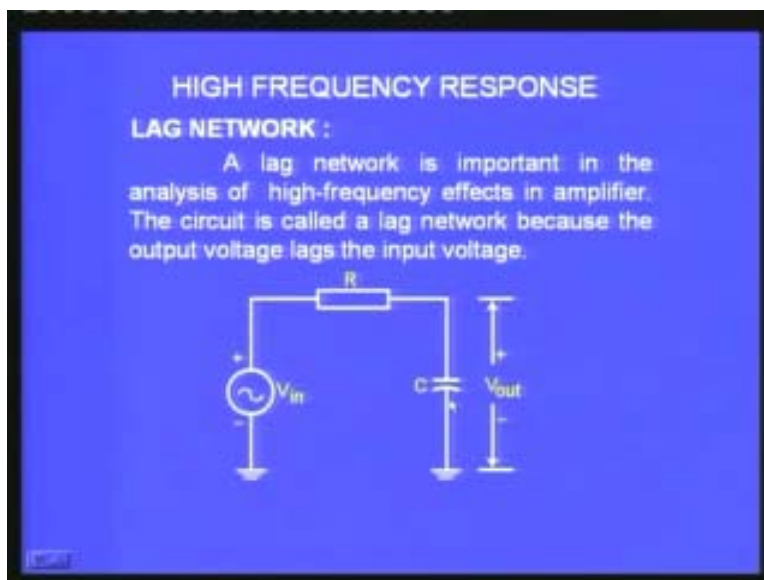
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Because the capacitance that we have in the circuit is half range some reactance which is smaller and therefore more voltage is available across the 1 K resistor and you get very high amplitude compared to the previous two cases. If I increase the frequency from very low values to very high values for a lead network which contains a capacitor and a resistor the output slowly increases and it comes very close to the **equal (or) input** value at some mid frequency range beyond about 1000 hertz. This is what I wanted to show you in this demonstration.

Now if you move on to the next network which is the lag network all that we have to do is just replace or interchange the capacitance and the resistance. I have put the resistance here, I have got the capacitance here.

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Now we are taking the output across the capacitance and not the resistance. If I have the frequency response of such a network it will just be the other way. That it means it will be high initially at low frequencies and it will fall at very high frequencies.

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HIGH FREQUENCY RESPONSE

FREQUENCY RESPONSE :
The Fig. shows the frequency response of a lag network.



CUTOFF FREQUENCY :
The voltage gain of the lag network is

$$\frac{V_{out}}{V_{in}} = \frac{X_c}{\sqrt{R^2 + X_c^2}}$$

It is the opposite of the earlier network and because you are taking the voltage across the capacitor it will be X_C divided by root of R square plus X_C square. That is the voltage gain V output by V input and the condition for cut off is still the same.

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HIGH FREQUENCY RESPONSE

The Cutoff frequency is defined as the frequency at which

$$X_c = R$$

and is given by

$$f_c = \frac{1}{2\pi RC}$$

LOAD RESISTANCE :
When a capacitor is in parallel with the load resistor. At low frequencies, where the capacitor appears to be open, the circuit acts like a voltage divider with a mid-band gain of

$$A_{mid} = \frac{R_L}{R_B + R_L}$$

When the power gain falls to half or when the voltage gain drops to 0.7 times the mid frequency and that corresponds to X_C is equal to R and therefore the cut off frequency here is also equal to 1 by $2\pi R_C$. There is no change. But then we should remember that this C will be different from that C . So if I include the load resistance the mid frequency

will not change much; R_L by R_s plus R_L as you got in the previous case. But in the discussion you should also take into account R_s along with R_L .


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HIGH FREQUENCY RESPONSE

LOAD RESISTANCE :
 When a capacitor is in parallel with the load resistor. At low frequencies, where the capacitor appears to be open, the circuit acts like a voltage divider with a mid-band gain of

$$A_{mid} = \frac{R_L}{R_s + R_L}$$

At higher frequencies, the capacitor begins to shunt alternating current away from the load. This causes the load voltage to drop off.

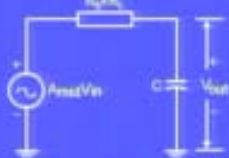


That is all the difference. With the R_L and R_s combined in the actual circuit the R_s and R_L will come in parallel.

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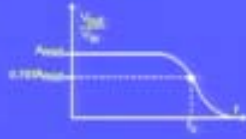
HIGH FREQUENCY RESPONSE

The simplest way to find the cutoff frequency is by thevenizing the circuit driving the capacitor. The thevenin voltage is



$$V_{TH} = \frac{R_L}{R_s + R_L} V_{in} \quad \text{or} \quad V_{TH} = A_{mid} V_{in}$$

and the Thevenin resistance is

$$R_{TH} = R_s \parallel R_L$$


The Thevenin's equivalent circuit you will find the effective resistance in the circuit will be R_s parallel R_L and therefore the cut off frequency when I also include the source

resistance will be 1 by $2\pi R_s \parallel R_L C$ times and the voltage gain is about 0.7 times the mid **band gain** corresponding to that.

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HIGH FREQUENCY RESPONSE

Therefore, it has a cutoff frequency of

$$f_c = \frac{1}{2\pi(R_s \parallel R_L)C}$$

The voltage gain is $0.707 A_{mid}$ at this cutoff frequency is shown.

$$G = \frac{P_2}{P_1}$$

The frequency response of an amplifier is normally represented on a logarithmic scale.

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HIGH FREQUENCY RESPONSE

BODE PLOTS

In complex numbers, the voltage gain of a lag network is

$$\frac{V_{out}}{V_{in}} = \frac{-jX_C}{R_1 - jX_C}$$

This can be converted to a magnitude of

$$\frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$

and a phase angle of

$$\phi = -\arctan \frac{R}{X_C}$$

We already saw the gain itself is represented in decibel unit which is also a logarithmic scale and because it is very difficult to go in a linear scale when you vary the frequency from nearly 1 hertz to 1 mega hertz. If you want to show everything then it is better to go in steps of 10 ; 1 , 10 , 100 , 1000 , $10,000$ etc., if you go then it becomes a logarithmic scale.

If I take the frequency in the logarithmic scale and the gain also in the logarithmic scale then that type of a plot is called the Bode plot. That is a Bode plot and the gain A is equal to, in the decibel scale, 1 by root of 1 plus R by X_C whole square and this we have already seen. You have again the expression corresponding to voltage gain in decibel as A prime is equal to $20 \log 1$ by 1 plus f by f_c .

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HIGH FREQUENCY RESPONSE

DECIBEL VOLTAGE GAIN

$$A = \frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{1}{\sqrt{1 + (R/X_C)^2}}$$

$$\frac{R}{X_C} = 2\pi fRC = \frac{f}{f_c}$$

Voltage gain can be written as

$$A = \frac{1}{\sqrt{1 + (f/f_c)^2}}$$

Decibel Voltage gain is

$$A' = 20 \log \frac{1}{\sqrt{1 + (f/f_c)^2}}$$

This is what previously I called f_1 and f_2 . This is actually corresponding to f_2 the previous lead network corresponds to f_1 . Here I generally call them as cut off frequency therefore f_c . The rest of the discussions are very similar. If I go in decade in voltage gain there is a 20 dB variation in decibel.

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HIGH FREQUENCY RESPONSE

Decibel Voltage gain of a lag network

For instance, when $f/f_c = 0.1$ gives

$$A' = 20 \log \frac{1}{\sqrt{1+(0.1)^2}} = 0.0432 \text{ dB} \approx 0 \text{ dB}$$

when $f/f_c = 1$, the decibels voltage gain

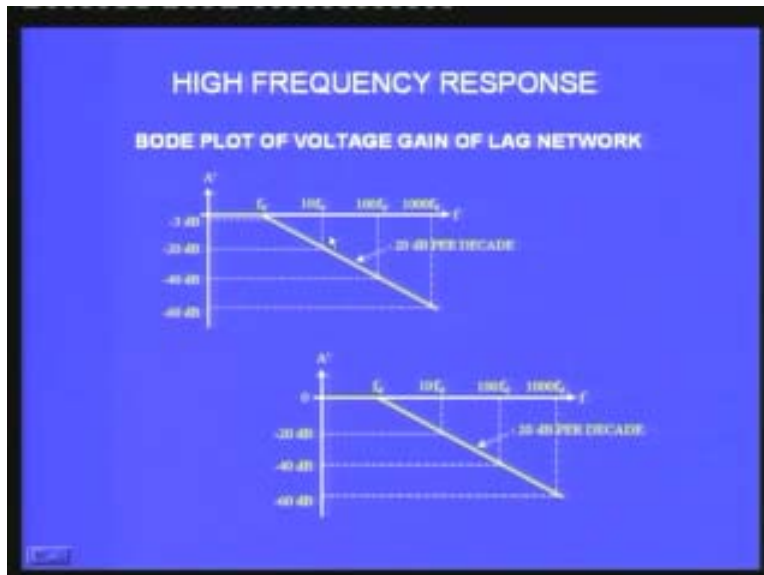
$$A' = 20 \log \frac{1}{\sqrt{1+(1)^2}} = -3.01 \text{ dB} \approx -3 \text{ dB}$$

when $f/f_c = 10$,

$$A' = 20 \log \frac{1}{\sqrt{1+(10)^2}} \approx -20 \text{ dB}$$

If I go in multiples or octaves there is a change of about 6 dB in voltage gain. So these are exactly same. I have given some examples here and then I will show you the graph corresponding to the Bode plot, log log scale of the voltage versus frequency of the lag network. At f_c the gain will be zero. Then if I go beyond f_c by 10 times you would find you have about -20 dB. It starts falling; minus means loss not gain.

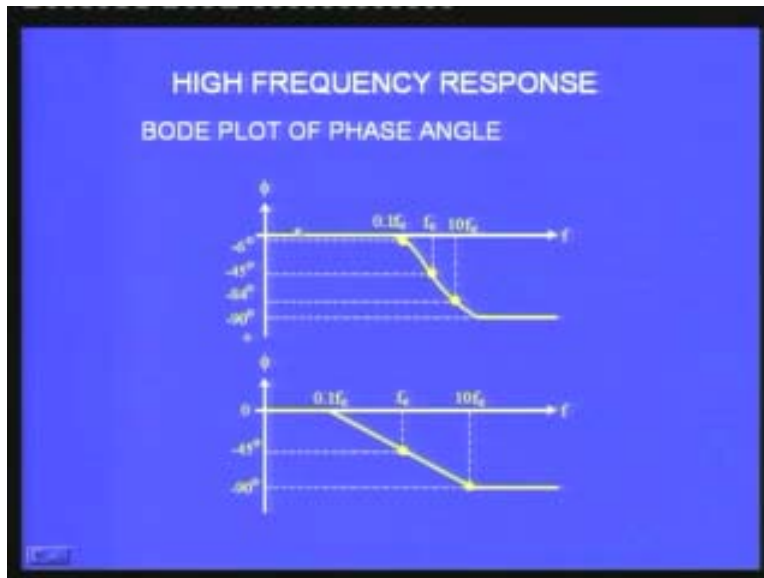
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If it is hundred times the cut off frequency it is -40 dB. If it is 1000 times the cut off frequency it is -60 dB. It is constant for sometime and then it starts falling below. This slope is 20 dB per decade. That is what we call this. This variation it is falling by 20 dB

for every 1 decade variation. This is actually the same graph which is shown. Both are same but if you look at the phase change at 45 degrees there is a cut off and the 0 degree and the 90 degree will be the other two corresponding to the very large frequencies and mid frequencies. The discussion is almost exactly similar to whatever that we have seen in the other case. You can see here on the screen the corresponding phase variation on the Bode plot.

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When it is mid frequencies you get 0 phase and if you come to the cut off that corresponds to nearly -6 degrees, corresponds to 1/2 actually. At the cut off frequency it is 45 degree phase angle and 10 times above the cut off frequency it is -84 degree phase change. If I go to very large frequencies it is almost 90 degrees. There is a large variation in phase angle as you increase the frequency because of the capacitance effect and this becomes very significant especially when you go to oscillators. This phase change can create complications in certain circuits and it can also be an advantage. We can use it to produce very specific feedback that we require to make an amplifier into an oscillator for example. There are several applications of understanding of lead and lag network and once you understand lead lag network we are now in a position to go into an actual discussion of a R_c coupled amplifier and analyze the circuit including the contributions that are coming from the various capacitors like the coupling capacitor, the bypass capacitor and the parasitic capacitor of the transistor, etc.

In the next lecture I would like to take the example of an R_c coupled amplifier, single stage R_c coupled amplifier and analyze it with reference to frequency response of the amplifier in the light of the lead lag network that we already studied. We will try to use it and try to measure how one can obtain the band width of an R_c coupled amplifier from the discussion of lead lag network. Thank you!