Physics – I: Oscillations and Waves

Prof. S. Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology, Kharagpur

Lecture - 08
Sinusoidal Plane Waves – I

We had studied coupled oscillators in today’s class, let me show you a simulation of coupled oscillators.

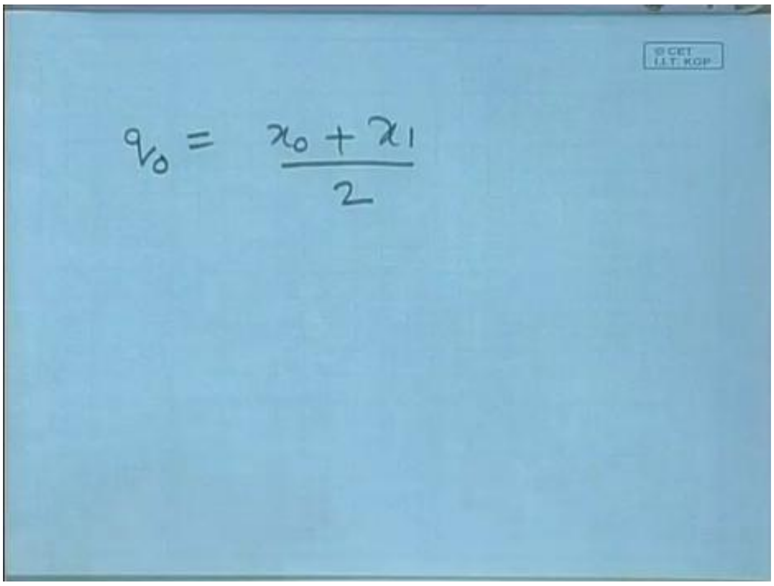
(Refer Slide Time: 00:59)

So, in the simulation we have 3 springs and these 3 springs are connected to 2 masses just like we had studied. So, we could set the value of the spring constants and the masses, let us go straight ahead and set the value of this spring constant. So, let us k1which is the first spring let us, set the value to 100. Let us set, the mass the first mass to 1, let us set the second mass to 20, the second spring to 20; the second mass again 1 and the third spring to 100. So, this spring has a spring constant 100, this spring also has a spring constant 100. Just like we had studied this mass is 1. So, these 2 springs have same spring constant.

The spring in the middle, has a lower spring constant of 20. So, it is not exactly in the very week coupling regime, but it is quite weekly coupled still. Now, let us start of by putting a displacement to the particles, such that we only excite the first the slow normal mode. That is, the only excite the motion of the centre of mass that is the first thing we shall do. So, we shall put a displacement which will excite only the centre of mass. So, if to do that we have to displace both the particles by exactly the same amount. So, let me displace this by 20 also displace and give no velocity to this particle. This again by 20 and no velocity to this particle. So, both the particles have been provided the same amount to displacement or let us see the oscillations.

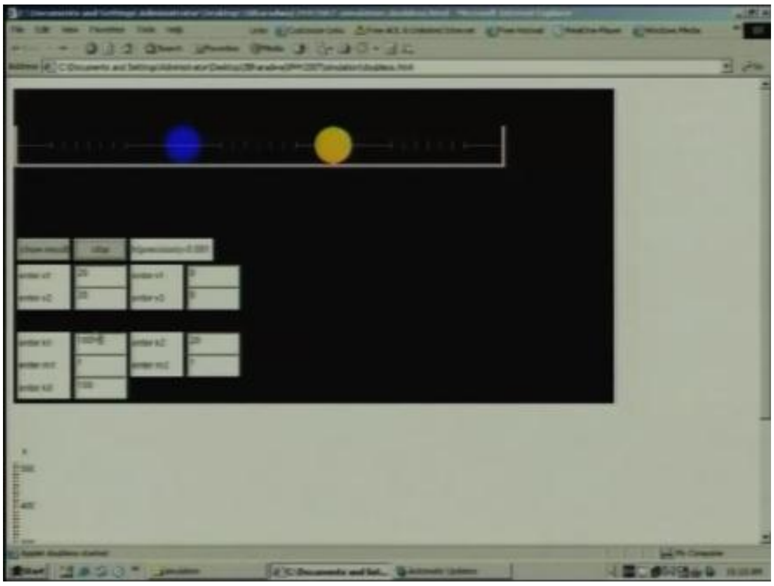
So, you can see that, this kind of displacement disturbance where i give both the particles the same displacement excites only the first normal mode.

(Refer Slide Time: 02:59)



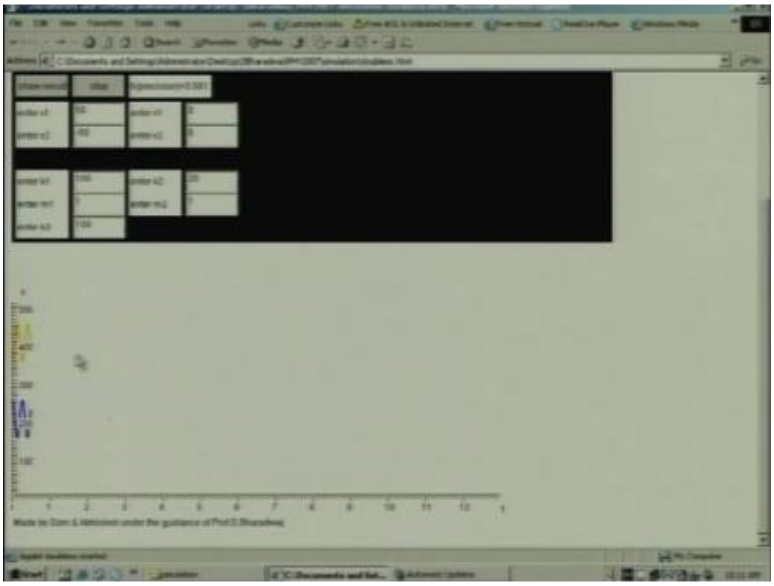
So, it only excites q nought which is x1 plus x0 plus x1 by 2. The motions of the centre of mass…

(Refer Slide Time: 03:12)

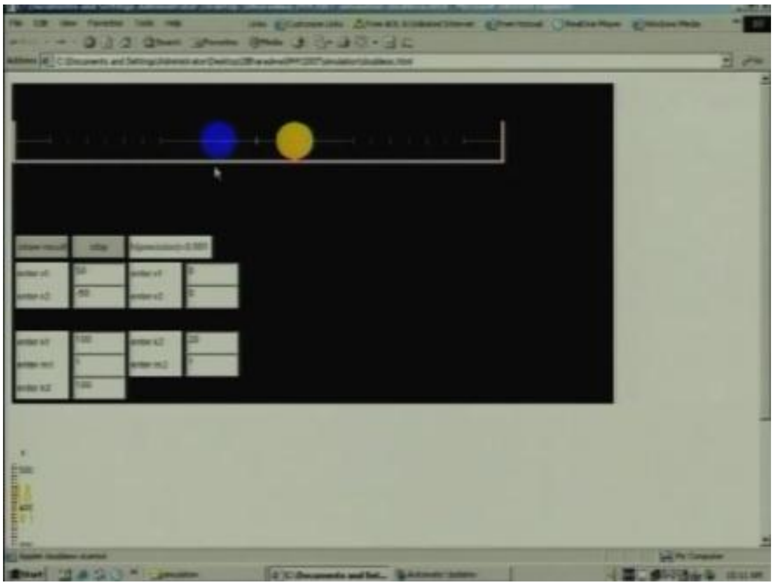


Now, let us look at a displacement a disturbance which will excite only the second mode, which is the relative motion. So, let me a magnitude of 50 and minus 50 so, they are both being given exactly opposite. And you see that, the centre of mass does not move. It causes only both the particles to do relative oscillation and the pictures over here:

(Refer Slide Time: 03:41)

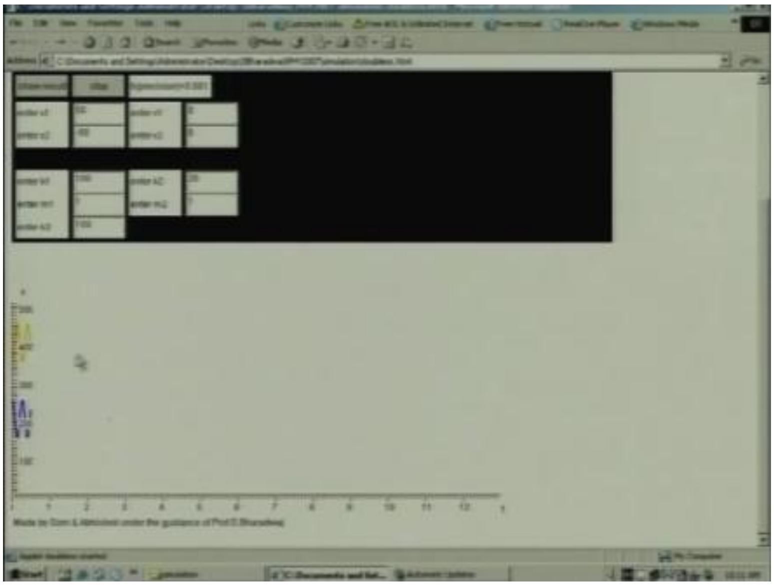


(Refer Slide Time: 03:43)



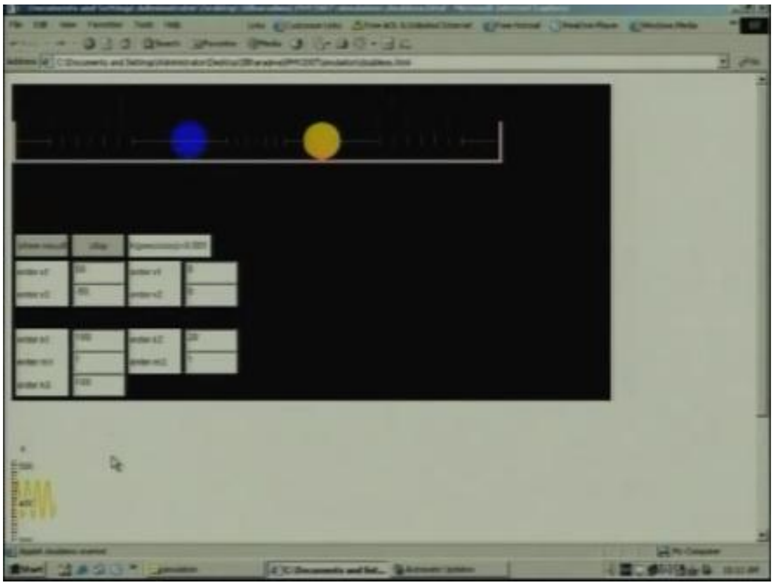
Show you the motion of the blue mass and the yellow mass respectively.

(Refer Slide Time: 03:46)



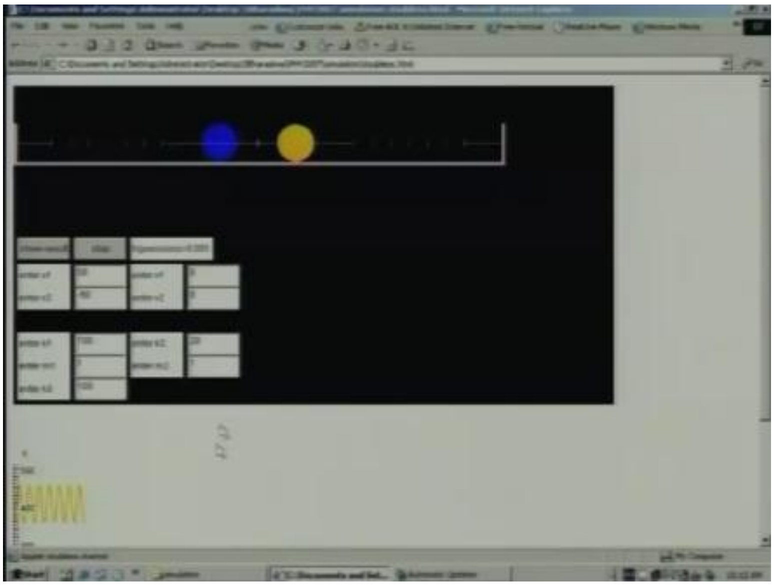
So, this shows you x as a function of time and they are moving exactly out of phase. If you had looked at it had the same thing for the q0 mode though centre of mass mode, they would have been moving exactly with the same phase.

(Refer Slide Time: 04:02)



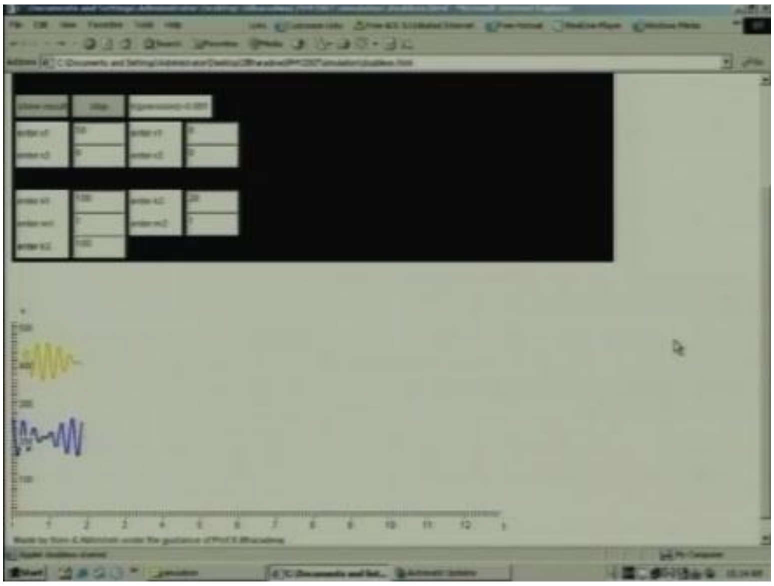
So, what we have done if we have looked up 2 different kinds of disturbances one which excites only the centre of mass mode the slow mode another which excites only the fast mode. Now, let us consider a situation where I keep 1 of the particles fixed and displace only the other particle. This should excite a combination of both these modes. So, let us which we had studied.

(Refer Slide Time: 04:25)



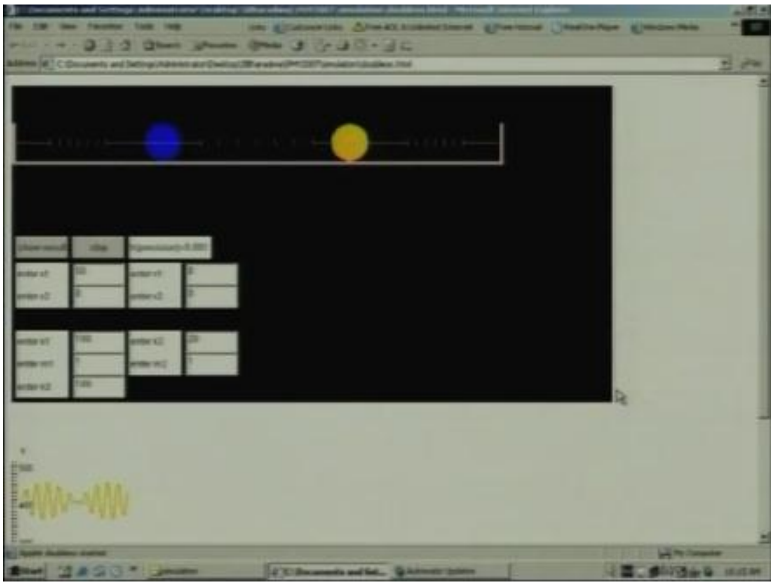
So, let us now consider this particular situation. So, I am going to apply a displacement only to the first particle and leave the second particle exactly where it is. So, notice what is happening, we had initially given a disturbance only to the blue particle. Now, the disturbance has shifted from the blue particle to the yellow particle again, it is going to shift back at then in forth. So, this is what we had studied. So now it has gone to the yellow particle, now both of them are oscillation and slowly the yellow particle will come to rest and the oscillation will be fully back in the blue particle.

(Refer Slide Time: 05:22)



So, you can see this same thing over here, the displacement is initially applied to the blue particle, a yellow particle starts from rest. And then the blue particle does this fast, both of them do this fast oscillation and it has a modulation which is modulated. The modulation causes the amplitude of the blue oscillations to die down slowly and the amplitude of the yellow oscillations to pick up. And then the amplitude of the yellow oscillation goes down the amplitude of the blue oscillation picks up.

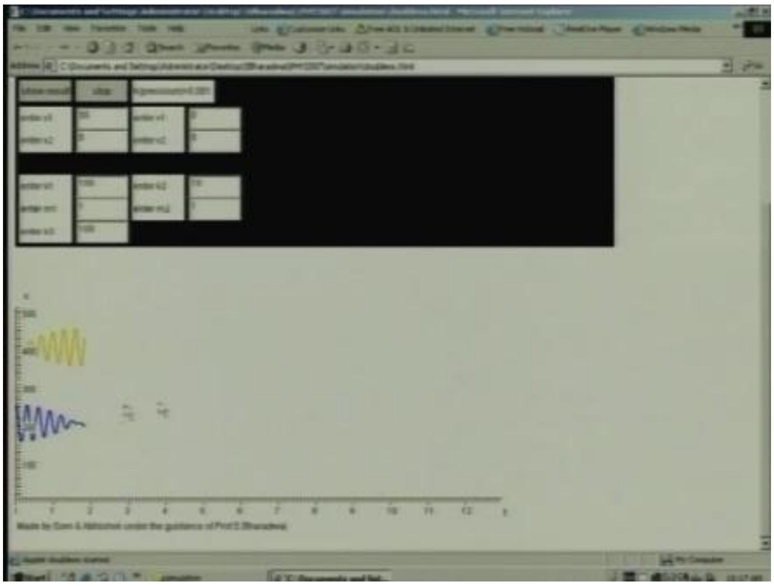
(Refer Slide Time: 05:57)



So, the oscillations go back and forth now, we could see what happens; if I vary, if I increase the coupling between the 2 oscillators, if I increase the coupling i would have to increase the spring constant of the spring in the middle. So, I am increasing it to 50 and if I increase the coupling between, I would expect the modulation to occur faster. So, I would expect the oscillations to go from here to here and come back much faster. So, let us see if it is happen, this is what our analysis leads us to believe. So, now the blue particle has nearly come to rest, the yellow particle now again, the blue one is picked up. The yellow 1 has slowed down and this will go back and forth between the blue and the yellow particle.

Now, both of them yellow 1 is picking up again the blue is going to come to rest. Now, the blue is again picking up. So, the oscillations will go back and forth we could also reduce the coupling .We could also reduce the coupling of the spring in the middle and see what happens; so, we have I have reduced the coupling of the spring in the middle. So, the disturbances are going to go from the blue to yellow particle very slowly. The coupling is very week. So, the modulation is a much slower modulation.

(Refer Slide Time: 07:54)

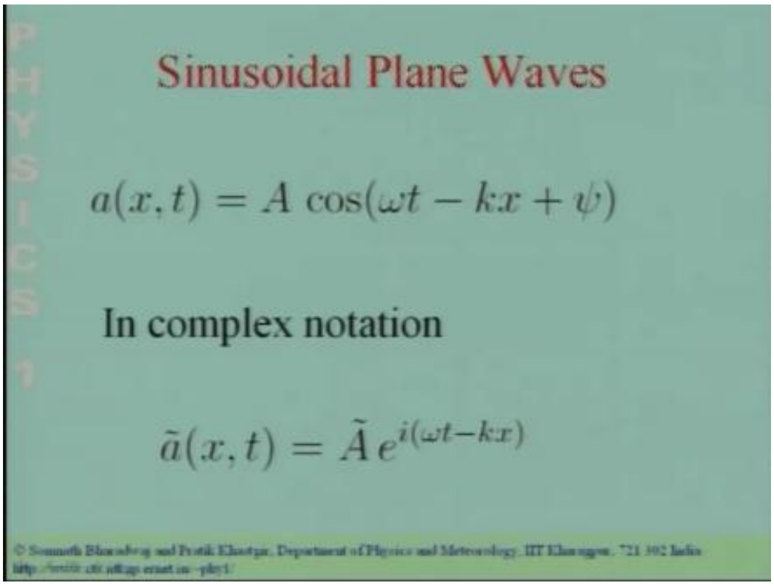


You can see it here also, the modulation is much slower, the blue particle oscillations have just died down. And the yellow one’s have picked up now, again the blue is picking up and the yellow one is dying down and its going to keep on going go back and forth between the blue and the yellow oscillations.

So, let us stop our simulation here and go forward to discussing a an entirely new topic. The sinusoidal plane waves and you shall see later on that, these waves arise in a situation when you many oscillators coupled together. So, let us start upon our discussion of sinusoidal plane waves. We have been studying, the motion of oscillators we studied a single oscillator and then we also studied, 2 oscillators which were coupled. In both situations we studied the motion of the vary of a single variables or 2 variables as a function of time.

So, time was the parameter and we studied the oscillations as a function of time. Let us now, go on to a situation where we have oscillations; which are a function of, both space and time such disturbances, such oscillations are what are referred to as waves. We shall start our discussion of waves with the simplest possible situation which is called the sinusoidal plane waves.

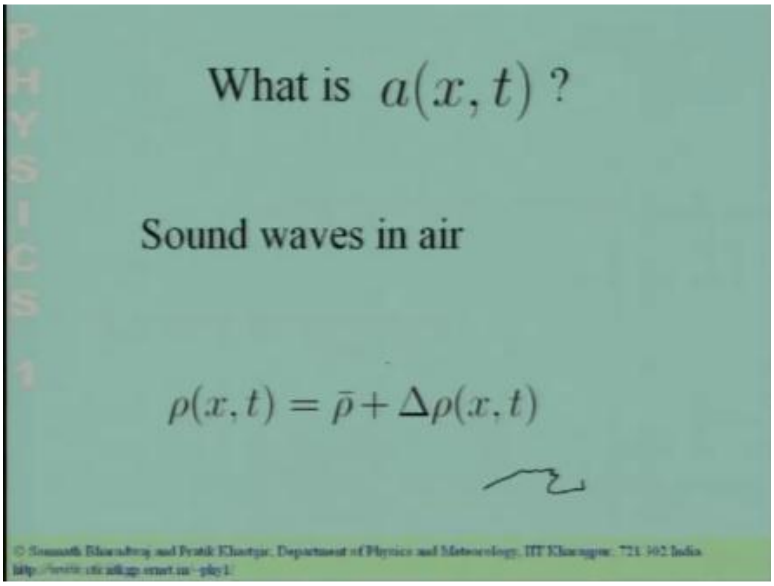
(Refer Slide Time: 09:45)



So, the equation for a sinusoidal plane waves is shown over here and we have A. A represents the oscillations is the quantity that oscillates and A is a function of x and t. So, we have oscillations that had a function of both position and time. And this is equal to some amplitude A into cos sign omega t minus kx plus psi. So, this is what we refer to as a sinusoidal plane wave. In complex notation, we can represent this as a tilde which is a function of x and t. So, you should remember that only the real part of a tilde is a the imaginary part is to be rejected when we ask what is the real thing A.

We can represent this equation, through this complex equation over here. Where a tilde which is a function of x and t is equal to A tilde e to the power i omega t minus kx. Remember, that the phase psi has been absorbed inside this complex amplitude. The complex amplitude a tilde is actually A into e to the power i psi. So, now let us discuss this particular sinusoidal plane wave in somewhat more detailed.

(Refer Slide Time: 11:29)

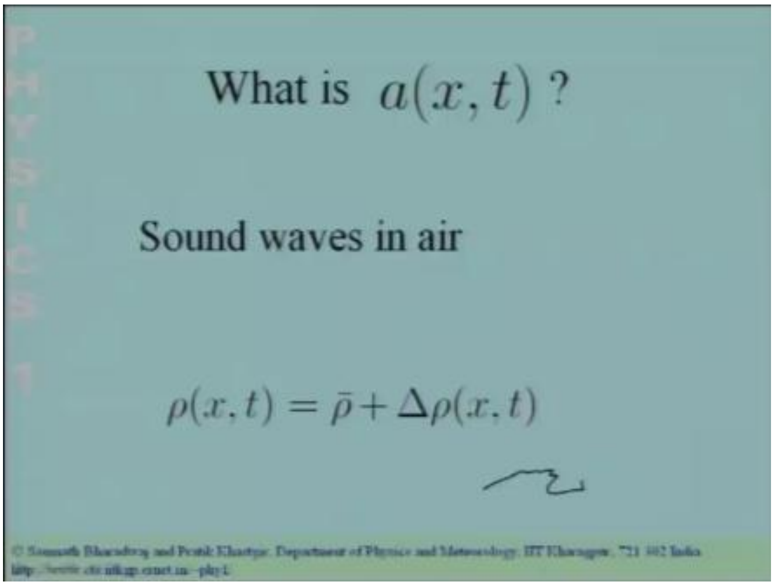


So, the first thing is this lets ask the question what do we mean by this a x which is a function of x. And t what is the physical quantity that, this A which a function of space and time which is the quantity that oscillates as a function of space and time, what does it represent? Now, the physical significance of A depends on the situation which we are analysing. So, the physical significance that you attach to this variable A depends on the physical situation that we are analysing.

For example, if we are analysing sound waves, we know that, sound is actually a disturbance it is a wave which propagates in air. So, when you have some source of sound for example, a loud speaker or the diaphragm inside our throat. These are sources of sound or when you ring a bell for example, in the loud speaker there is a diaphragm, which moves and when the diaphragm moves forward it compress the air when the diaphragm moves backward it rarefies the air. Now, when you compress air so, if you compress some air you basically increase the density of the air. If you increase adiabatically increase the density of air, the pressure of the air goes up and this causes it. When the pressure of the air goes above the surrounding pressure this will expand and it will push the air which is outside.

So, this part of the air will push the air outside and it'll cause the air next the neighbouring bits of air to get compressed which inturn will again expand and then push. So, have the disturbance which propagates, the disturbance is essentially fluctuation in the density of the air. And it is this fluctuation in the density of air which is the wave it is this quantity which oscillates.

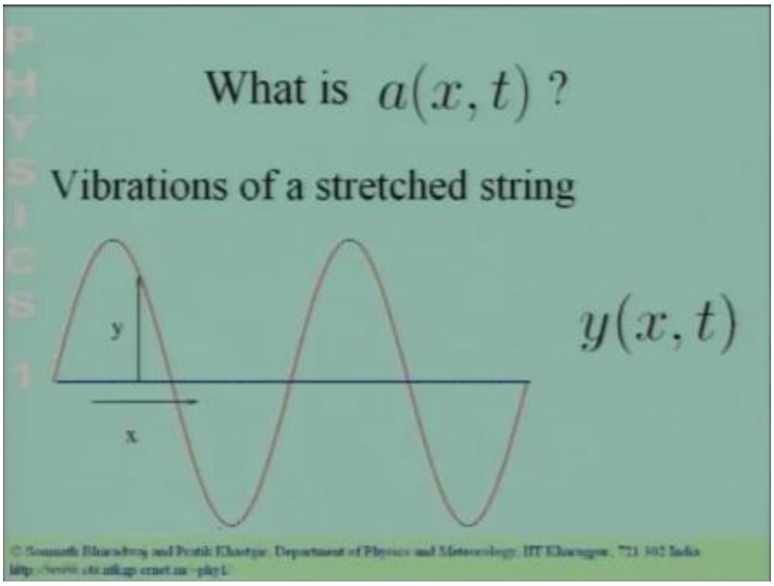
(Refer Slide Time: 13:51)



So, when you are dealing for example, Sound waves in air, then the density of the air at any point at any given time t can be written as the sum of a constant density which would be there if there were no disturbance, if there were no sound waves. And there would be a disturbance there would be a change in the density due to the sound wave. And this change in the density is a function of x and t, the density which is there when there is no disturbance is a constant. Now, when you are discussing sound waves in air, A of x and t represents the fluctuation in the density. So, when you are discussing sound waves in air, the variable a represents the variable is to be interpreted as representing the fluctuations in the density delta rho.

If rho is the density of the air then a represents delta rho the fluctuations in the density. Now, we know that in air when you have a sound wave in air the and if the disturbance is propagating in this direction, then the density change, the change in the density is also in the same direction. The density gets contracted and redefied in the same in which, the wave is propagating such a wave is called a longitudinal wave. So, sound wave is a longitudinal wave and the quantity that actually varies with x and t is the fluctuation in the density is a change in the density.

(Refer Slide Time: 15:27)



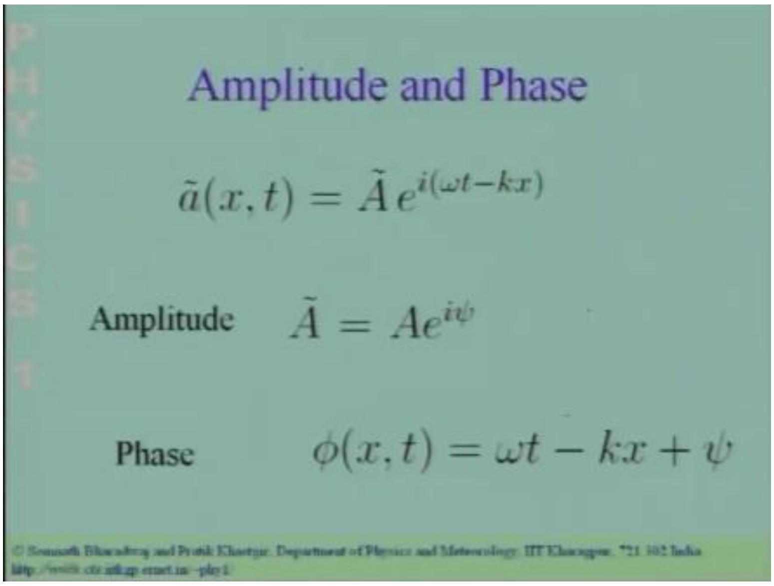
And a of t represents this change in the density, you could also discus. Another situation considers a string a long string considers, a long string which is stretched and thought it is stretched. So, that it is thought in this string if you introduce a disturbance, which is perpendicular to the string then such a disturbance will propagate along the string. So, this propagating disturbance perpendicular to the string is a wave and such a wave, where the disturbance is perpendicular to the direction in which the wave propagates. So, this is the string the blue line over here, represents the string and you have disturbed the string perpendicular in the perpendicular direction in the transverse direction. So, the wave propagates along the x axis.

The x axis is aligned with the string and you introduce a disturbance perpendicular to the x axis perpendicular to the string. So, you pull the string and leave it for example, so, you'll have a disturbance that propagates along the string. This disturbance, you represent by y. So, the y is the disturbance of the string it is the transverse displacement. So in this situation, where you have vibrations of a stretched string. The variable a represents the transverse displacement of the string. So, such a wave is called a transverse wave and in this case A the variable A, represents y the displacement of the string in the transverse direction.

So, such a wave is a transverse wave and in this situation a represents y. So, the point I am trying to make here is that, we going to discuss the whole wave phenomena in today’s lecture in terms of a variable a. The physical significance of this variable a, has to be determined will be determined by the physical situation that we are analysing. So, if you are analysing a particular situation, a sound waves a will corresponds to density fluctuations. If you are analysing vibrations in a string waves in a on a stretched string.

A will corresponds to the transverse displacement on the string and if you are considering some other physical situation the physical significance of the variable a will change.

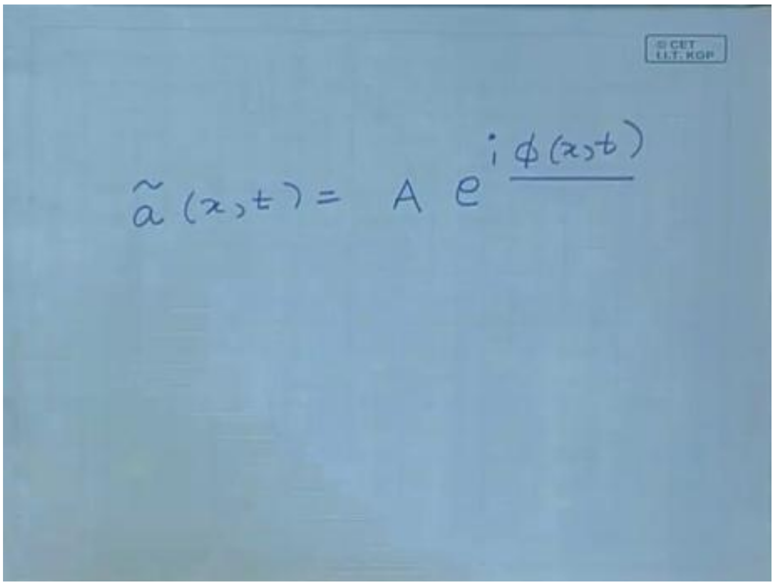
(Refer Slide Time: 17:49)



So, now let us look at the behaviour of A as a function of which we know, is a function of x and t. Let us look at, how A behaves if I fix the value of x. So, if I fix the position. So, there is a wave there is a disturbance, which is a function of both x. So, it changes with x and it also changes with time, changes with position and time. Now, we fix the position. So, we have an observer at a fix value of x and this observer, observes A, how A varies with time. So, this observer sits at a fixed place. And he observes, he or she observes how the variable A could be the density fluctuation could be the vibration in a string how this quantity varies with time.

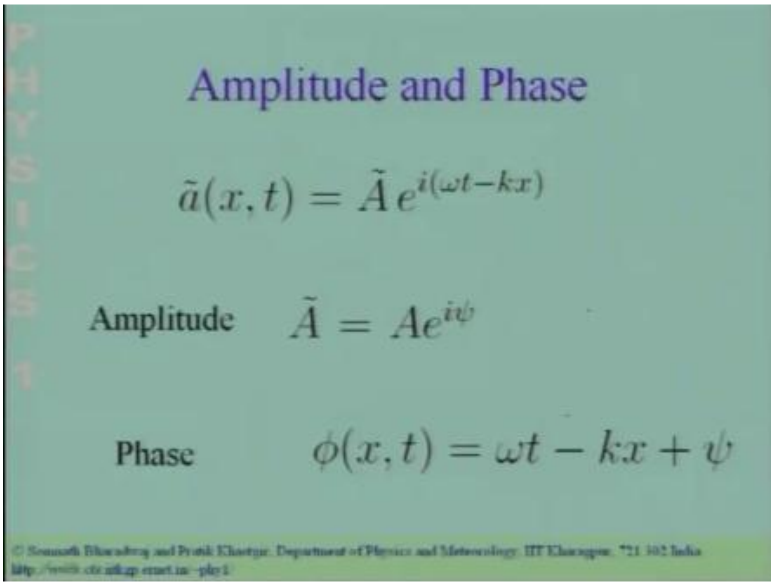
So, let us look at this, A tilde in the complex notation is before, we look at this. How it varies at a fixed position. Let me just give you a few definitions; so, this is the equation of the sinusoidal plane wave, in this equation the constant a tilde outside is the complex amplitude of the wave. The modulus of this, the modulus of a tilde which is A tells us, the real amplitude of the wave. So, it gives us the magnitude of the wave and we define omega t minus kx plus this contribution from the complex amplitude i psi this combination we define as the as the phase of the wave. So, the phase of the wave is the quantity that appears in the exponent e to the power i times the phase.

(Refer Slide Time: 19:55)



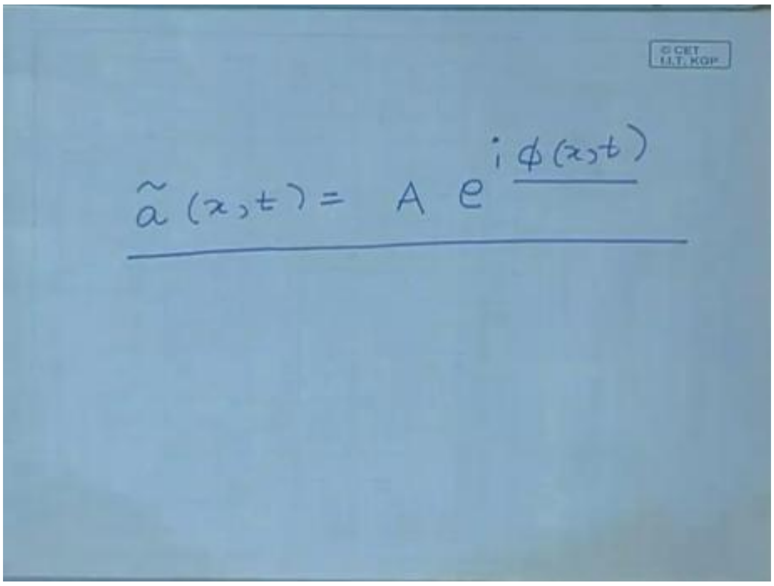
So, this wave so, we can also express the wave as a tilde as a function of xt is equal to A e to the power i psi x t. This psi we shall be referring to as the phase of the wave and this psi is the function omega t minus kx plus psi.

(Refer Slide Time: 20:23)

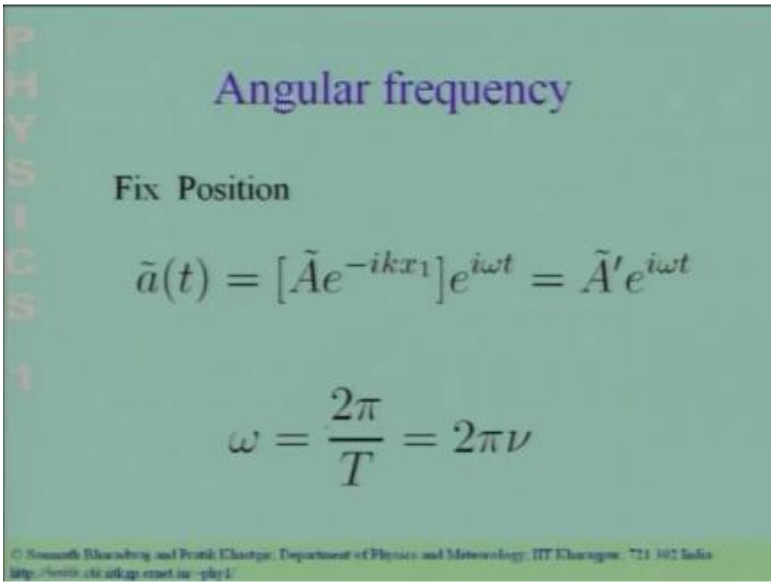


Sometimes, we shall also refer to this psi as the phase of the wave and it shall be clear from the context what we are referring to. So in general, we shall be referring to this function phi which is omega t minus kx plus the phase psi as the phase of the wave. So, the wave as an amplitude A and it has a phase phi xt and it can be written in this fashion shown over here with this prelude.

(Refer Slide Time: 20:52)

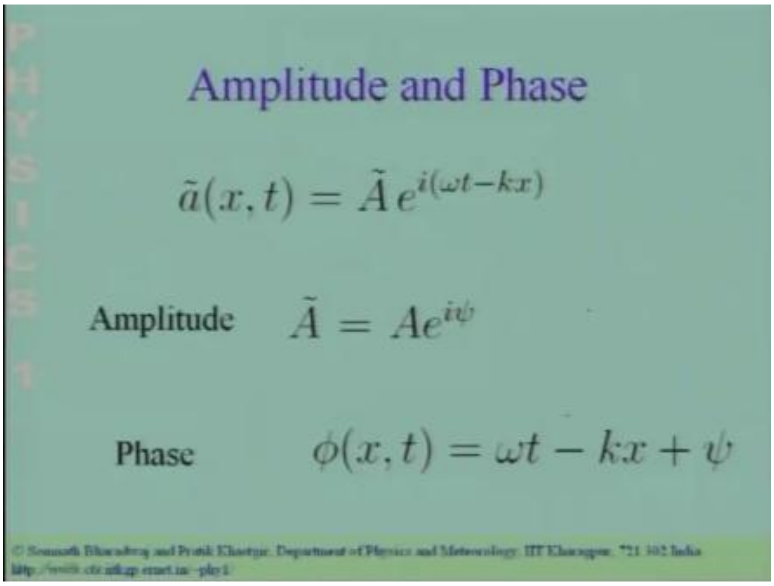


(Refer Slide Time: 21:01)



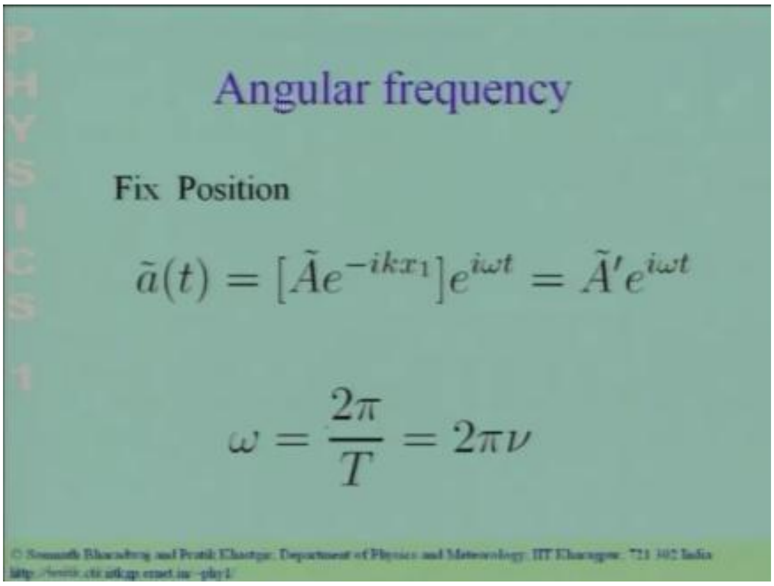
Let us go back, to the question which we were asking.

(Refer Slide Time: 21:12)



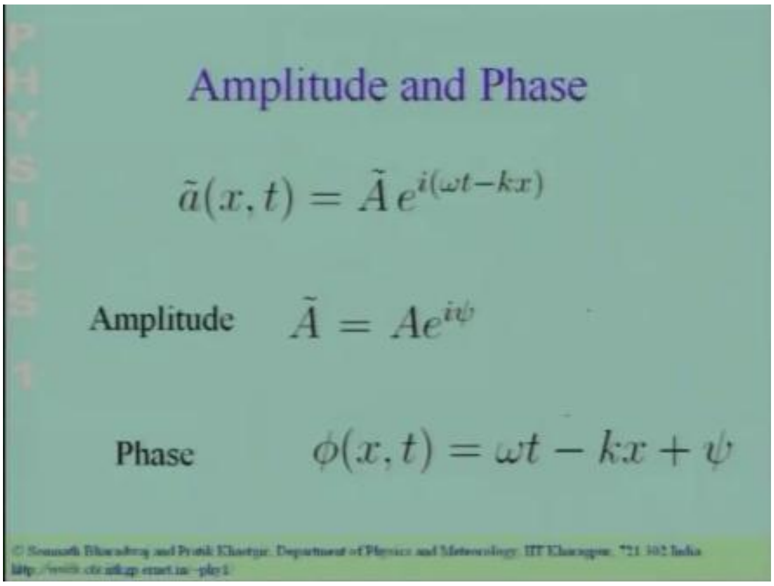
We are looking at the behaviour of this oscillation we are looking at the behaviour of this quantity A which varies with x and t we are looking at its behaviour at a fixed value of x at a fixed position.

(Refer Slide Time: 21:22)



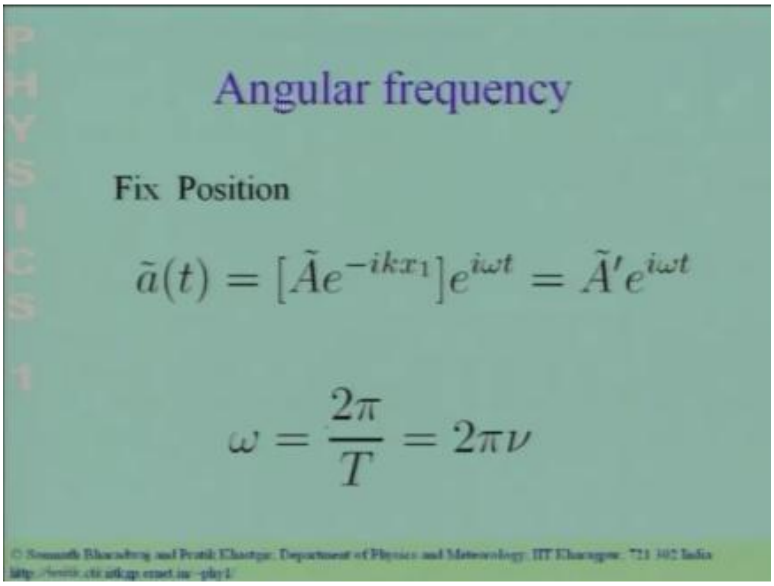
So when we fix the position e to the power at let us say x1. So, we have fixed the position at x1 A tilde a is now just a function of time.

(Refer Slide Time: 21:35)



So, A now becomes a function of time alone because we have fixed the value of x at x1.

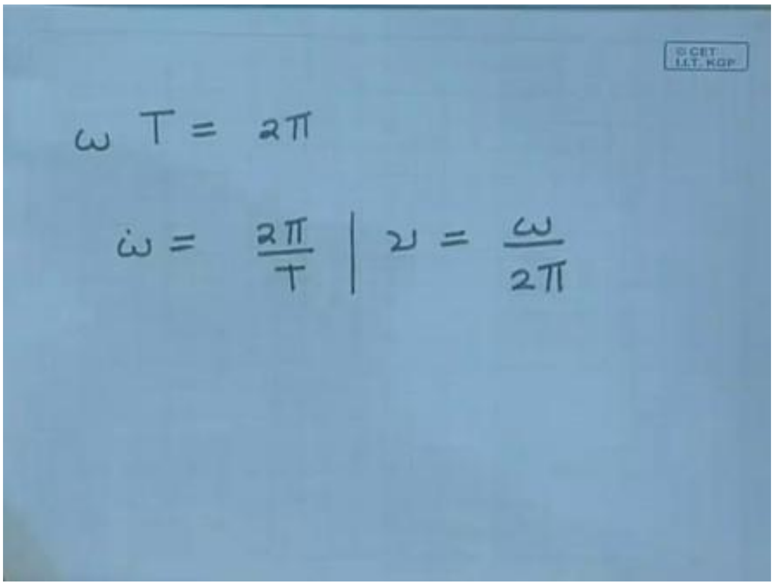
(Refer Slide Time: 21:40)



If you fix x at x1e to the power minus i kx1 becomes a constant and this constant you can absorb inside the complex amplitude and redefine a new complex amplitude A tilde prime. So, A as a function of t can be written as A tilde prime where A tilde prime has absorbed this extra phase factor which arises because of this x1 which is constant. So, A tilde it has a function of time A tilde prime is complex amplitude into e to the power i omega t. Now, recollect that this is just like the simple harmonic oscillator which we had studied.

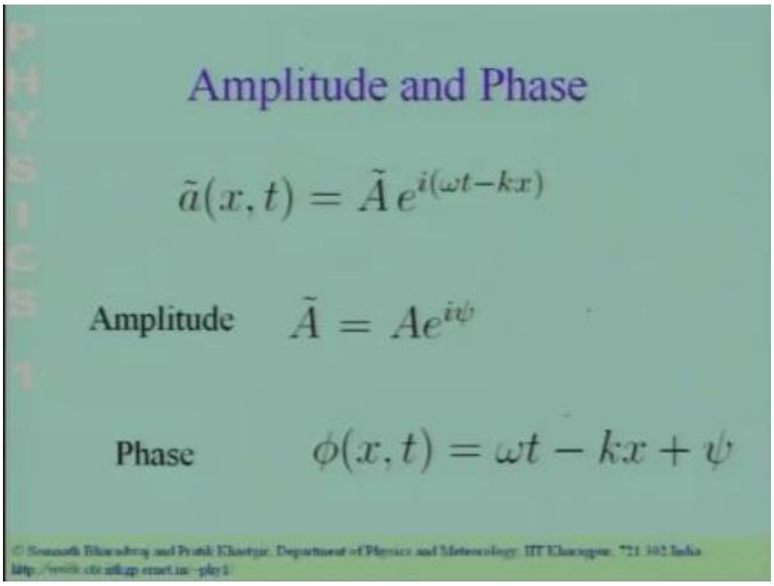
We have a time dependence so, which is e to the power i omega t. So, you should take only the real part. So, you have time dependent which is cos omega t. So, what we learn from this is that if you sit at 1 place and observe the behaviour of the wave. You will see that, it oscillates the quantity A now, which is just a function of time oscillates like a simple harmonic oscillator. So, if you sit at a fixed x and watch the time evolution of A it will oscillate like a simple harmonic oscillator. So, you could ask the question after what time periods will it repeat, it will repeat. So, let us say that t is the time period.

(Refer Slide Time: 23:17)



So if T is the time period, it will repeat when omega t is equal to 2pi which essentially tells us, that omega is the angular frequency 2pi by the time period and it is related to the frequency which is new by omega divided by 2pi.

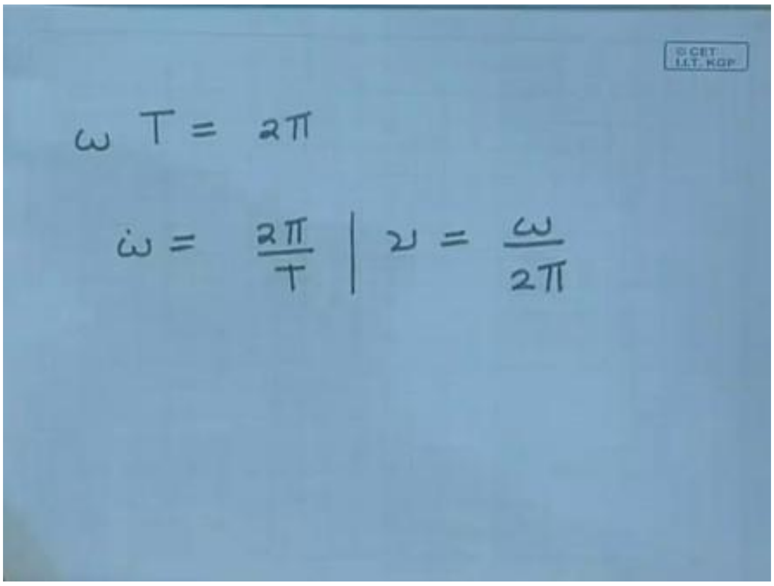
(Refer Slide Time: 23:44)



So, what we see here is that if I sit at a fixed value of x, if I have an observer whose position is fixed. Then A going to oscillate like a simple harmonic oscillator, these oscillation are determined by omega which is the angular frequency of that oscillation.

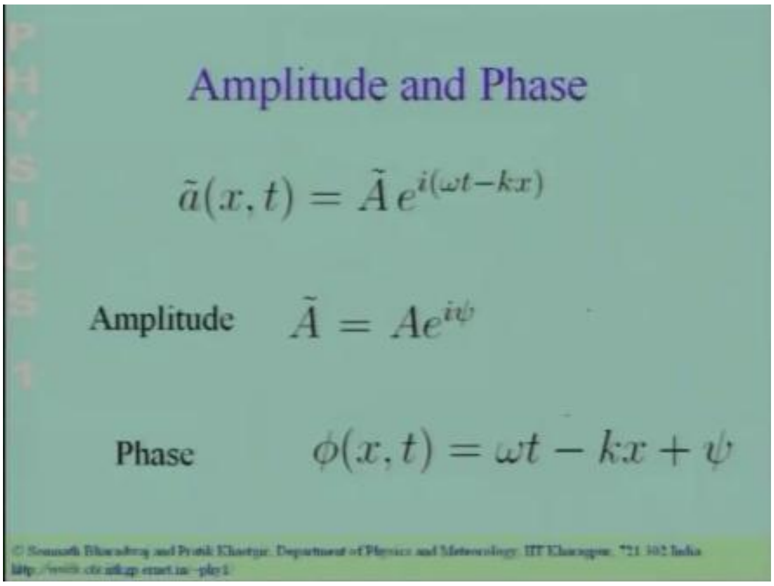
The time period of that oscillation is omega is 2pi by omega which you can see from here and the frequency is omega by 2pi.

(Refer Slide Time: 25:12)

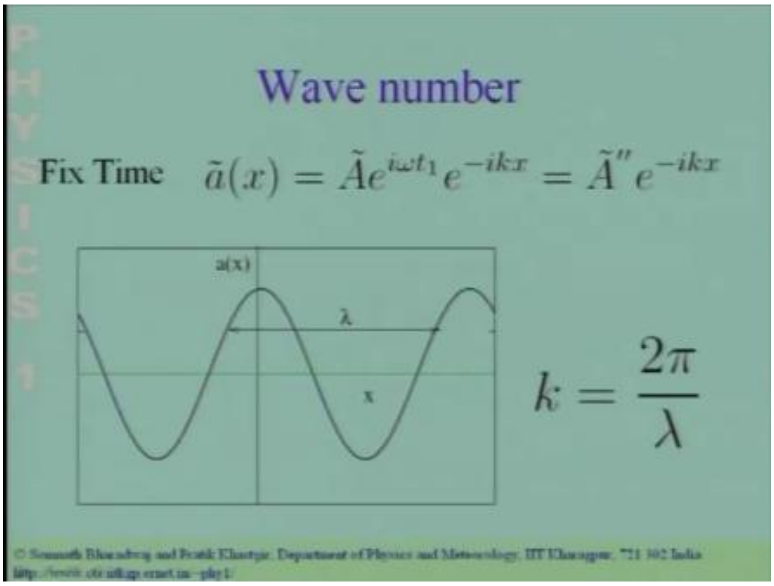


So, if you fix the position and look at a as a function of time alone you have a simple harmonic oscillation of angular frequency omega, where omega is a constant. That occurs over here.

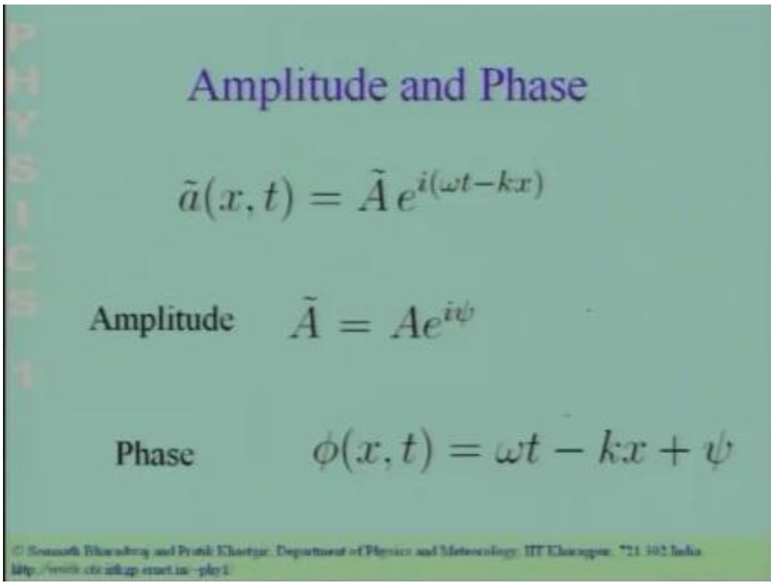
(Refer Slide Time: 24:23)



(Refer Slide Time: 24:24)

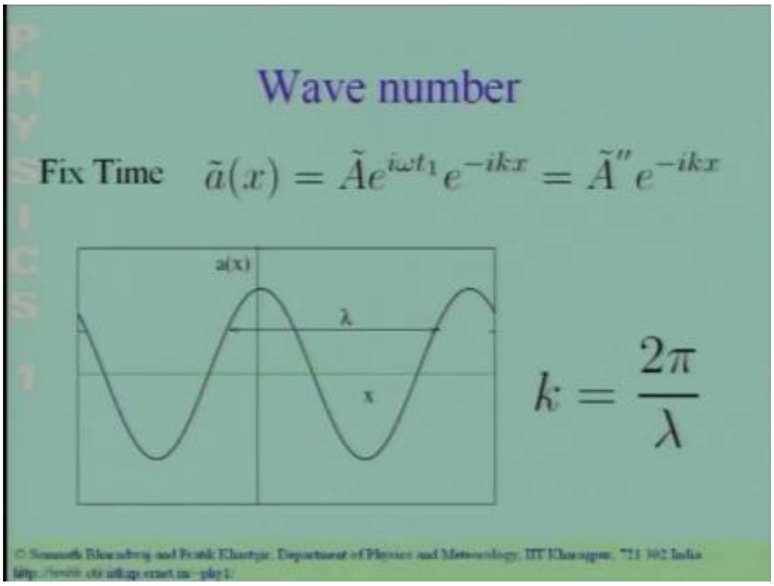


(Refer Slide Time: 24:42)



Let us, next look at the behaviour of A at a fixed value of time, if you fix the value of time if you fix the time instant over here, in the equation for A you now, have A as a function of the position alone. So, if you look at A as a function of position alone.

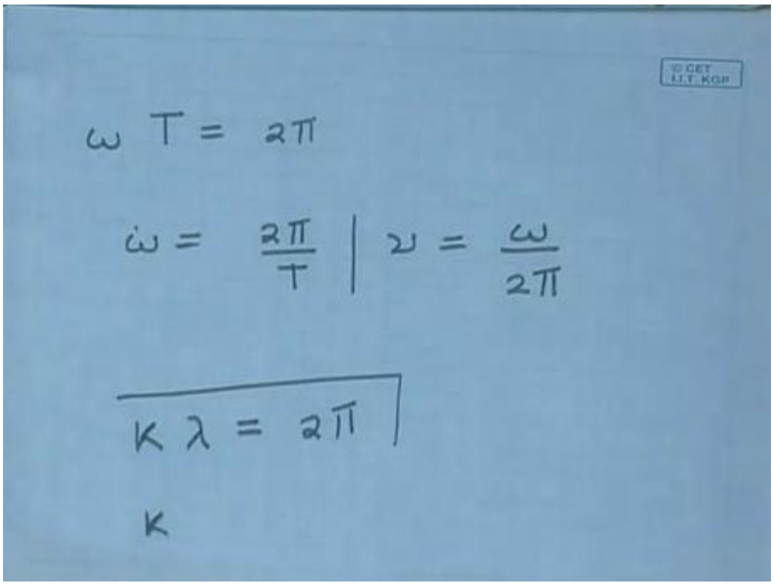
(Refer Slide Time: 24:53)



You have fixed the time instant to t 1 so, a particular time t1. You can now, absorb this factor e to the power i omega t1 inside the complex amplitude. Redefine the complex amplitude with a different phase, where you have absorbed the factor i omega t1. So, the new complex amplitude is A tilde double prime and you have this factor of e to the power minus i kx. Now, let us ask the question what does it look like? If you ask the question what it looks like, as a function of x. You should take only the real part the real part of this is a cosine for convenience we will assume that the phase of A double prime is 0.

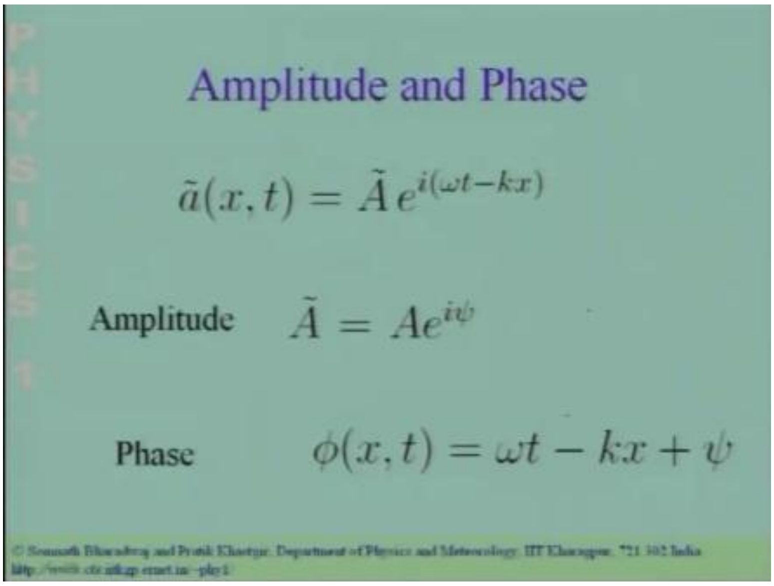
So, you have cos kx so, as a function of x A looks like cos kx, which is what i have shown here. Now, let us ask the question, after what distance does this function repeat? After what distance does a repeat, we know that, cosine whenever argument assumes a value goes from 0 to 2pi. And the period the distance after which A has a function of x repeats is called the wave length. So, here i have shown you, A as a function of x and you can see that, the value which is there over here is there again over here. And the whole function repeats after this distance which is called the wave length. So, at a fixed time the x dependence is sinusoidal, it is either cosine or sin and it repeats whenever the argument of cosine is 2pi.

(Refer Slide Time: 26:49)

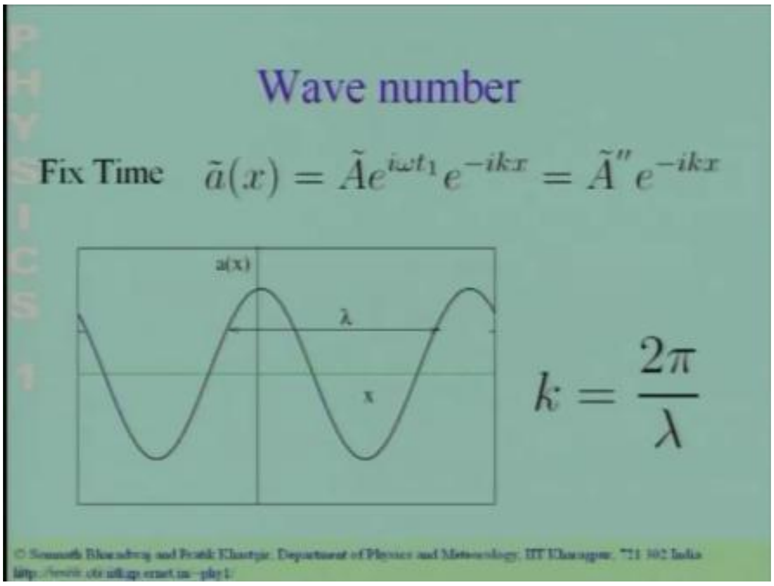


So, this gives us the condition that, K and the distance after which it repeats is called lambda is equal to 2pi. So, by definition the, A the value of a repeats after a distance lambda. So, the condition is that K into lambda should be equal to 2pi; because cosine and sin both repeat after a phase of 2pi phase difference of 2pi. So, which tells us that k which the number K, the constant K which appears in the equation for the sinusoidal plane wave is related to the wave length by K is equal to 2pi by lamda.

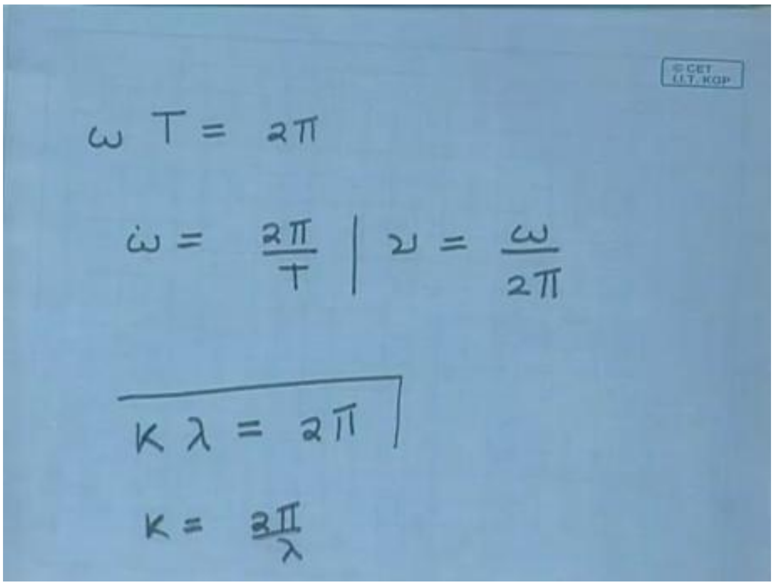
(Refer Slide Time: 27:22)



(Refer Slide Time: 27:29)

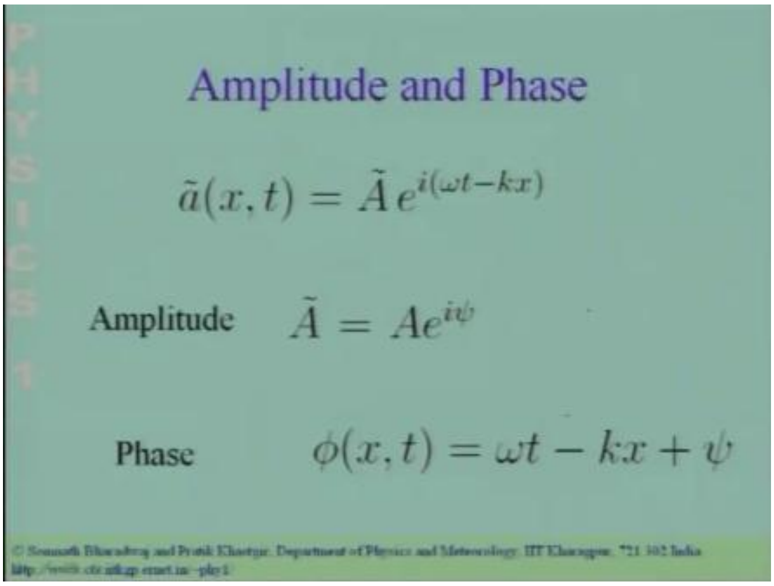


(Refer Slide Time: 27:33)



And K is called the wave number, lamda is the wave length and k is called the wave number.

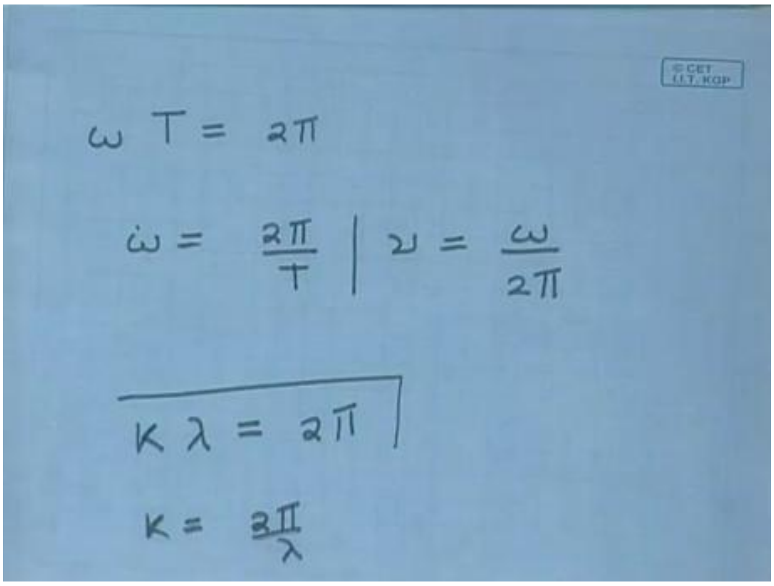
(Refer Slide Time: 27:54)



So, let me now summarise what we have just learnt, A as a function of x and t in the complex notation. If it can be written as some constant complex constant into e to the power i omega t minus kx this is sinusoidal plane wave. It has an amplitude, the amplitude is the constant which occurs outside this whole thing. In this case, it is the complex amplitude. We also have 2 constants which appear in this expression omega and K omega is the angular frequency.

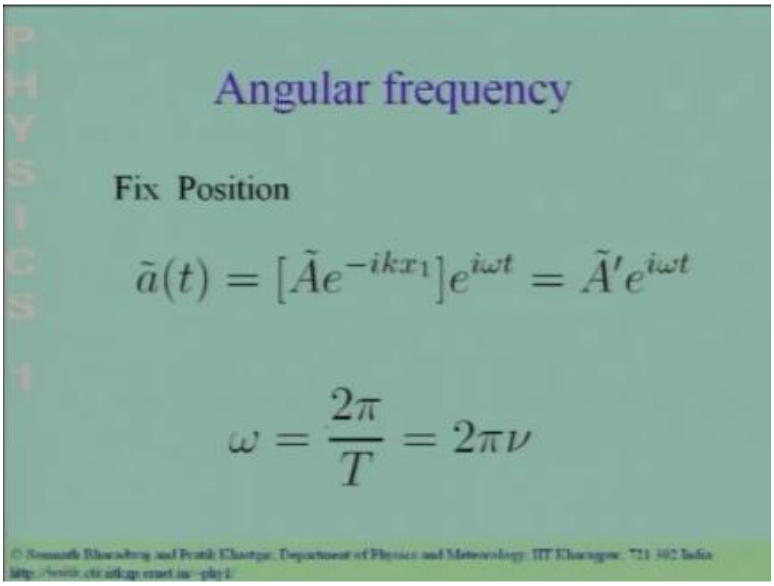
So, if i fix xi will observe oscillations just like a simple harmonic oscillator and these oscillations will have an angular frequency omega. If i fix time I will see a varying as cosine with x and the period of this cosine is called the wave length and k is related to the wave length and k is called the wave number and it is k related to the wave length through the expression given over here.

(Refer Slide Time: 29:05)



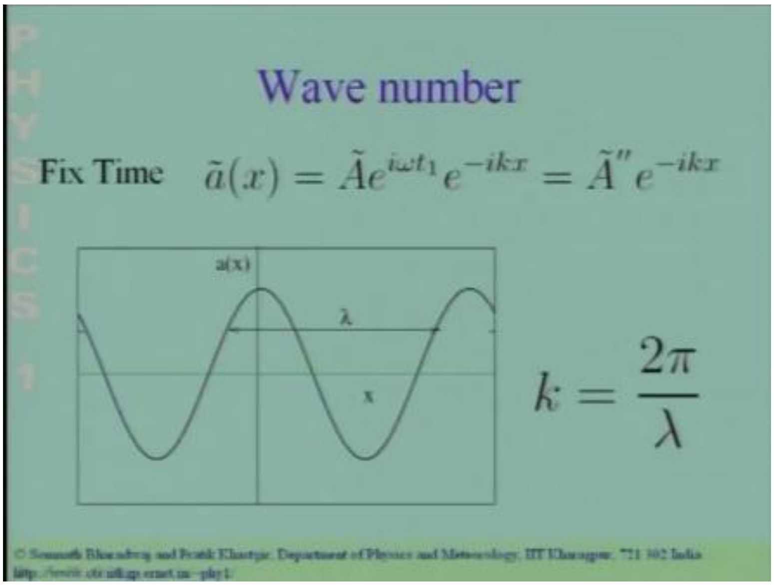
Lambda refers to the wave length and the wave number K is related to the wave length through the expression given over here.

(Refer Slide Time: 29:18)

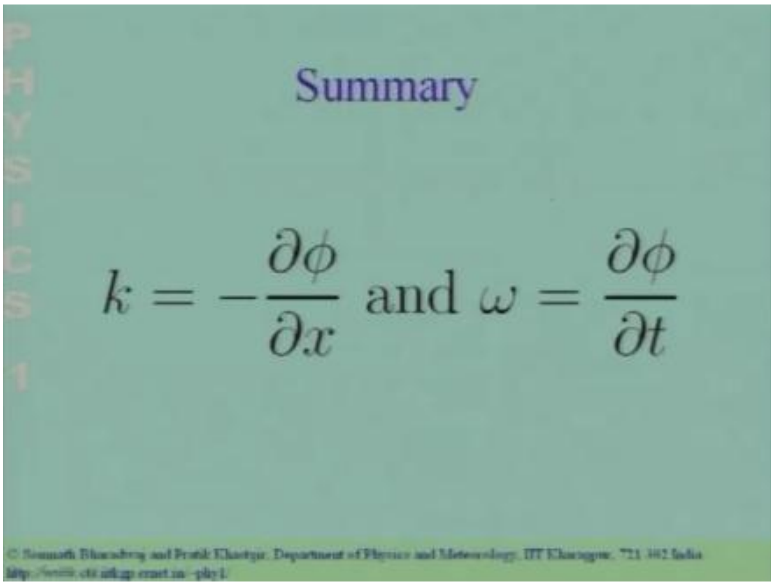


So, we have now individually studied the dependence of the behaviour of the wave. If I fix the time and then we studied if I fix, first we studied if I fix the position. How does it depend with time, we then studied what happens, if you fix the time and look at the variation with position corresponding to this we got the angular frequency Corresponding to this we got the wave number.

(Refer Slide Time: 29:37)

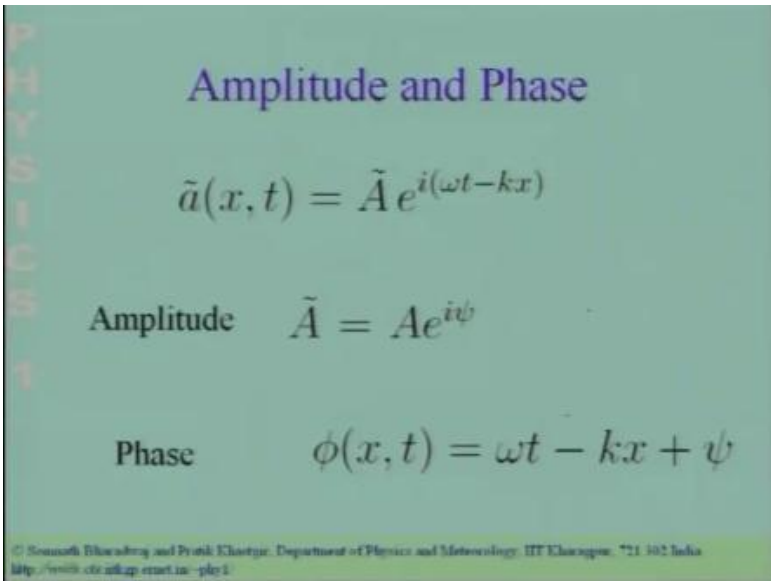


(Refer Slide Time: 29:39)



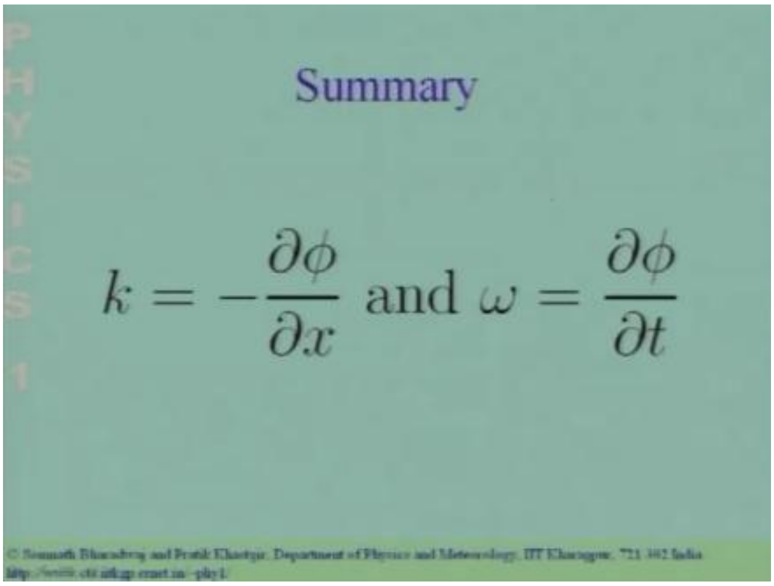
Now, in summary then we see that the wave number tells us, how the phase of the wave varies with position and it is the derivative the rate of change of phase with position. So, the wave number K tells us, the rate of change of phase with position it is it is minus del phi del x. And you can just check for yourself that if you differentiate the expression for the phase.

(Refer Slide Time: 30:10)



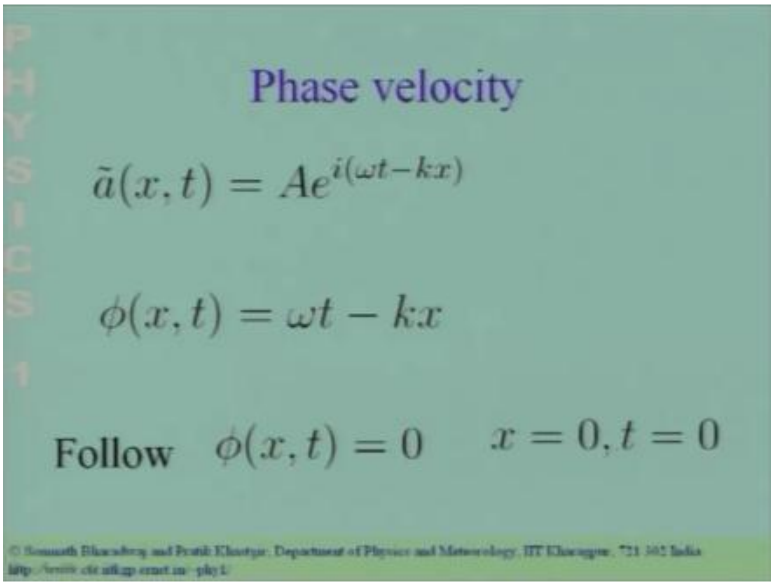
If you differentiate the expression for the phase with respect to x and put a minus sign you get the wave number.

(Refer Slide Time: 30:20)



So, the wave number tells us, the rate of change of phase with position. And the angular frequency with a negative sign the angular frequency tells us the rate of change of phase with time. So, as time evolves how the phase changes that is the angular frequency, if you move from 1 place to another, how the phase changes that is minus the wave number. So, we have learnt about a plane sinusoidal plane wave. We have seen, how it depends if I fix time, we have seen how it depends if I fix the special position. Let us now, study the combined space and time dependence of this wave.

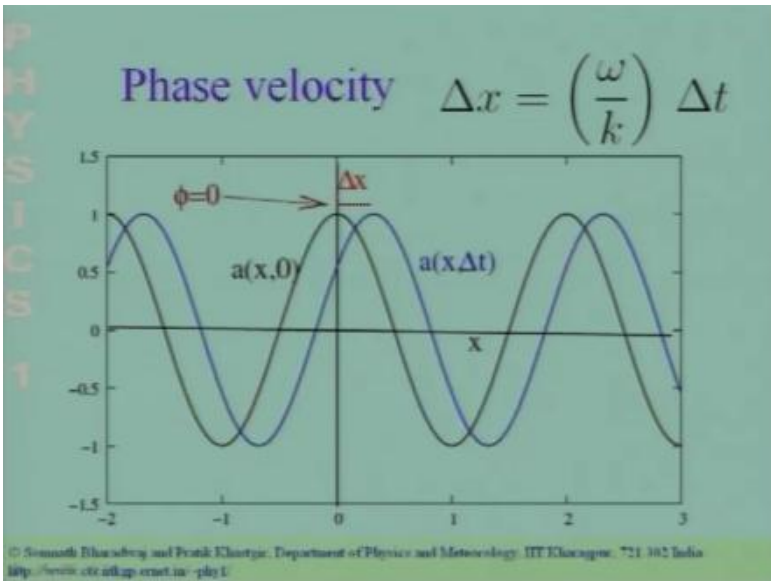
(Refer Slide Time: 31:04)



So, the thing which comes up when you study, the combined space and time dependence of the wave is the phase velocity. So, we are going to study the combined phase the space x and t dependence of this equation. Now, the value of A is essentially determined by the phase phi which is a function of x and t and the phase phi is omega t minus kx. Let us ask, the question at t equal to 0 where is the phase where does the phase have a value 0. So, you can see it is quite straight forward that at t equal to 0. The phase phi assumes a value of 0 when x is equal to 0 this is what i have shown over here.

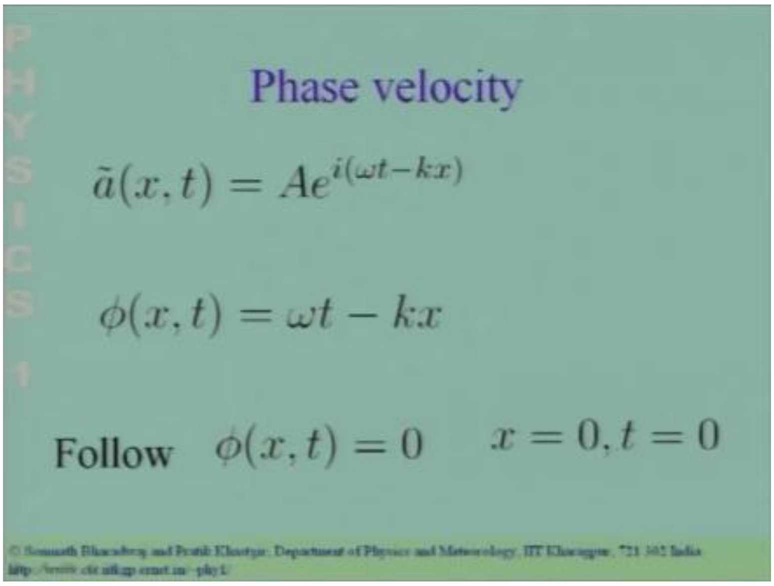
Now, the next question is as time increases as time evolve. So, time starts we start our discussion at t equal to 0. We start studying the whole thing at t equal to 0 and then we want to see what happens as time increases. And the question that we are interested in is, how does the point where the phase is 0 move as time increases. So, we will follow the point, we will follow the position x where the phase is 0. And we will follow this point as time increases from t equal to 0. We know that, this point is at x equal to 0, when t is equal to 0 we want to follow the evolution with increasing time.

(Refer Slide Time: 32:46)



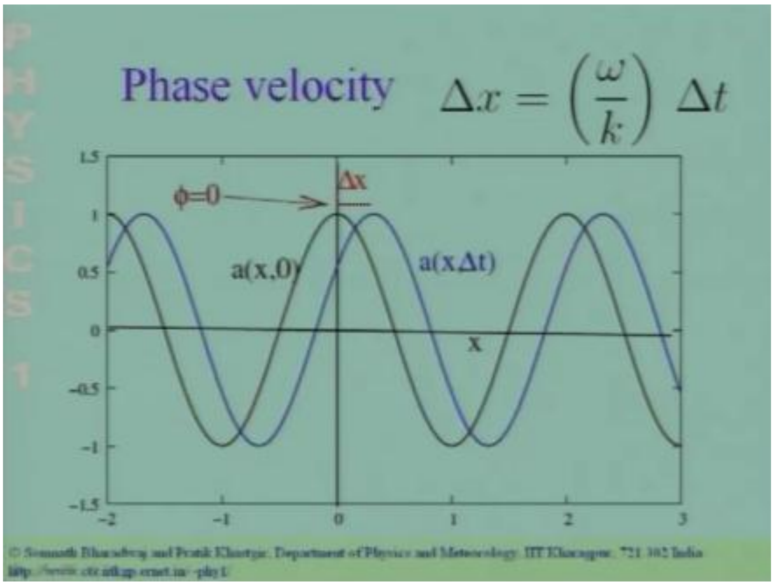
So, this graph allows us to follow the motion of the point where the phase is 0. The black curve over here, ax 0 shows you the behaviour of x at the time t equal to 0. It is a sinusoidal curve, it is actually it is cosine kx at t equal to 0, it is cos kx and the phase phi which is omega t minus kx at t equal to zero a phase phi has a value at x equal to 0. So, this is the point this is the value of x where the phase is 0 and cos omega t cos phase has a value 1 over here. So, this is the point at t equal to 0, this is the point where the phase is 0.

(Refer Slide Time: 33:48)

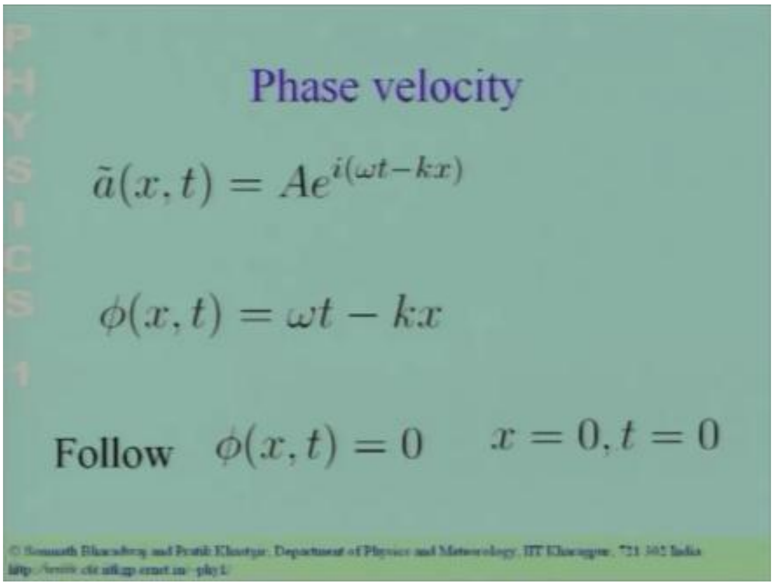


Now, as if time increases so, time increase, time has increased from t to t plus delta t. Question is how much should exchange if the phase has to still be 0. So, initially the phase is 0 where both these variables are 0 of time increases from t to t plus delta t. So, we want the phase to be 0. So, delta x the change in x has to be such. So, that it exactly balances this.

(Refer Slide Time: 34:00)

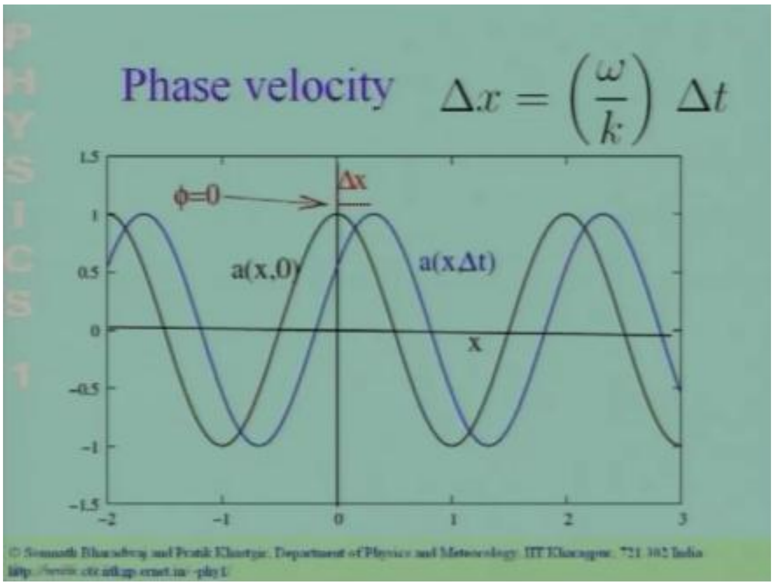


(Refer Slide Time: 34:21)



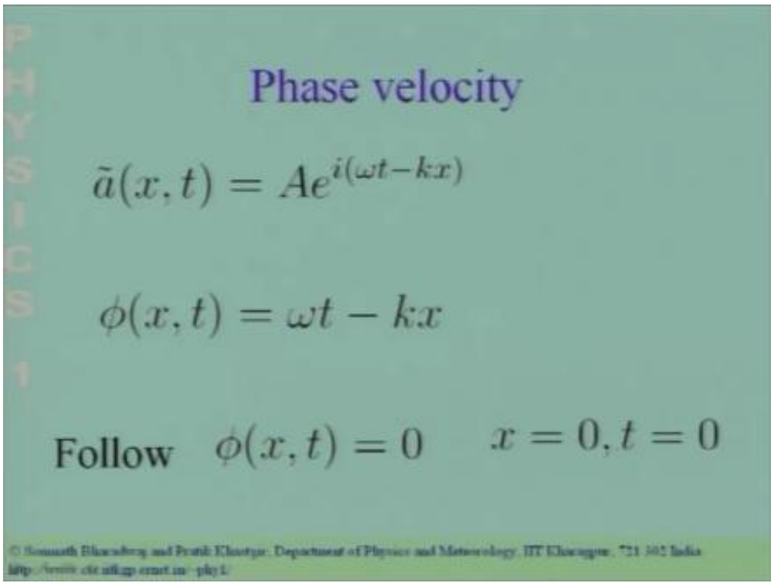
So, the change in x has to be omega by K into delta t. So, if you take this change in x put it at here, then you will find that the phase is again 0.

(Refer Slide Time: 34:29)



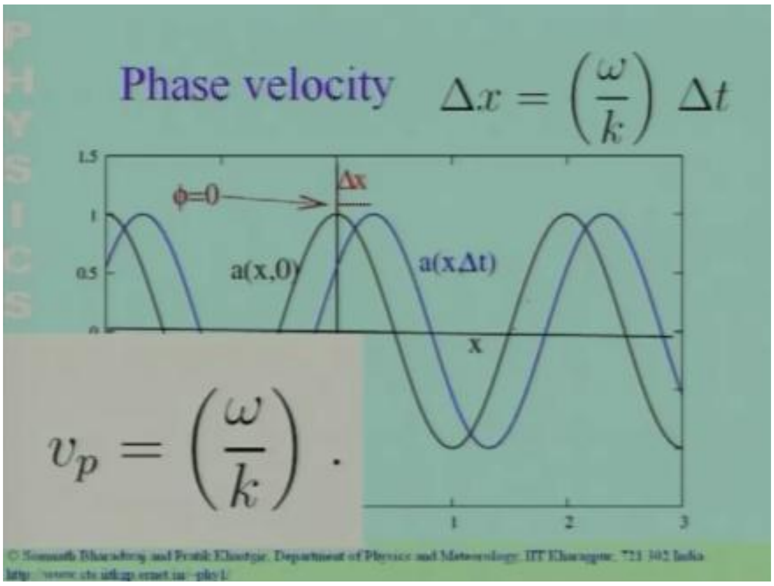
So, after a time delta t the phase is 0 at the point delta x which is omega by k into delta t, so, after a time delta t the point where the phase is 0 has moved a distance which is delta x, which is equal to omega by k into delta t. So, after a time delta t the point where the phase is 0 has moved from here to here. Now, you could have done the same thing for any other value of phase there is nothing special about the phase 0. You could have taken of value of the phase pi by 2, which is this point here. Initially, this is the point where the phase is pi by 2. So, initially the phase is pi by 2 at this point and you could ask the question after a time delta t where the phase becomes pi by 2. So, after a time delta t the phase is pi by 2 over here. So, the point where the phase is pi by 2 has moved the distance from here to here and it is quite clear.

(Refer Slide Time: 35:22)



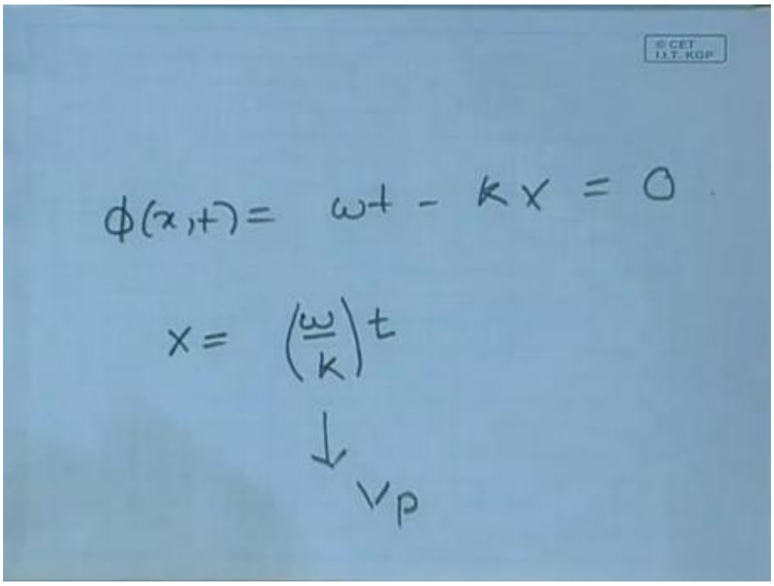
That for any value of the phase, if you follow the position, where the fixed value of the phase has shifted after a time delta t.

(Refer Slide Time: 35:29)



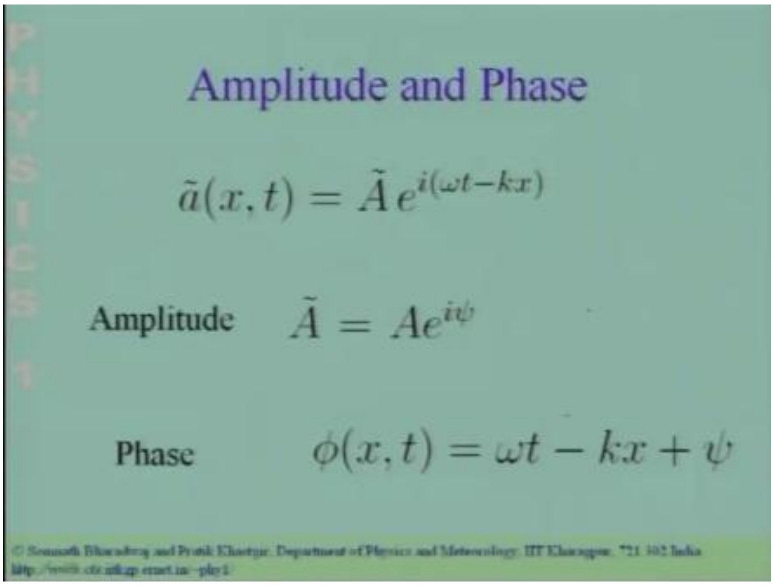
You will find that it has moved a distance x which is omega by k delta t. So, what we learn from this is; that the position where the phase if you follow the position, where the phase has a fixed value for example, if you follow the position where the phase is 0. You will find that it moves forward as time increases and it moves forward at the speed omega by K this speed is what is called the phase velocity. So, the point where the phase is 0 moves forward at a speed which is called the phase velocity and it is not only the point where the phase is equal to 0. Any fixed value of the phase moves forward at the speed omega by k.

(Refer Slide Time: 36:25)



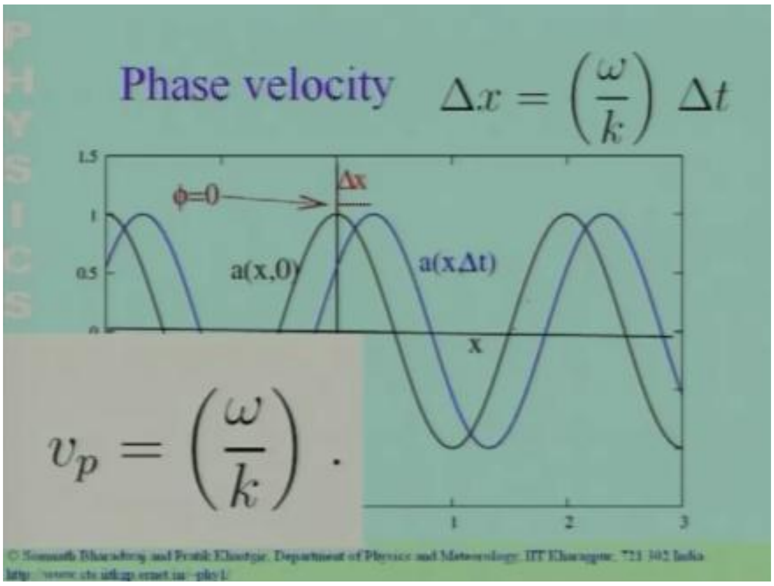
So, that is quite obvious if you are looking at the phase xt is omega t minus kx. And we are following; the point where the phase is 0 and you get this obvious from here, that this point will move along the trajectory x is equal to omega by k into t. And. So, it'll move in a straight line with the fixed velocity it will move in a velocity this velocity omega by k is what is called the phase velocity. So, the phase velocity is the speed at which the phase of the sinusoidal plane wave propagates.

(Refer Slide Time: 37:11)

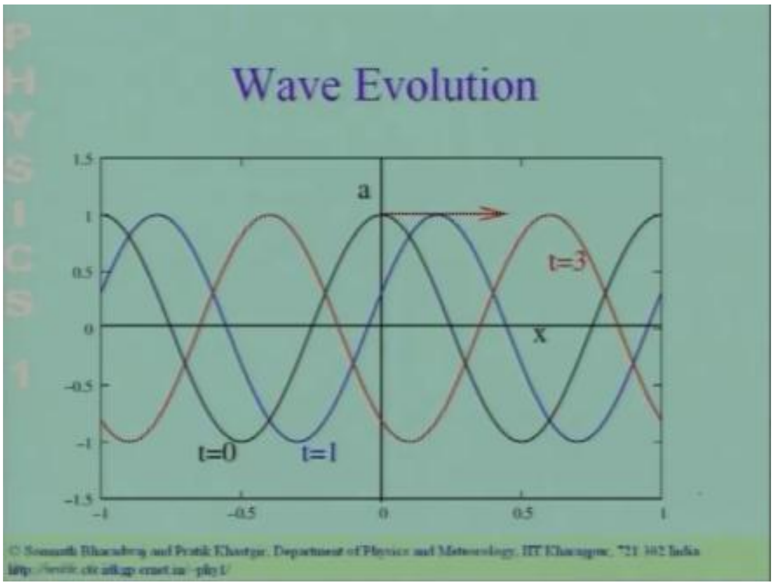


Remember that the phase had been defined in this fashion phi xt is omega t minus kx and I could have a constant psi. So, if i look at the speed at which a fixed of value phi propagates this speed is called the phase velocity. So, you should note that the phase velocity can only be defined for a sinusoidal wave. It is only for a sinusoidal wave that I have a fixed angular frequency omega and a fixed wave number K. So, you can define a phase and you can then ask the question at what speed does the phase move and this is the phase velocity omega by K.

(Refer Slide Time: 37:50)



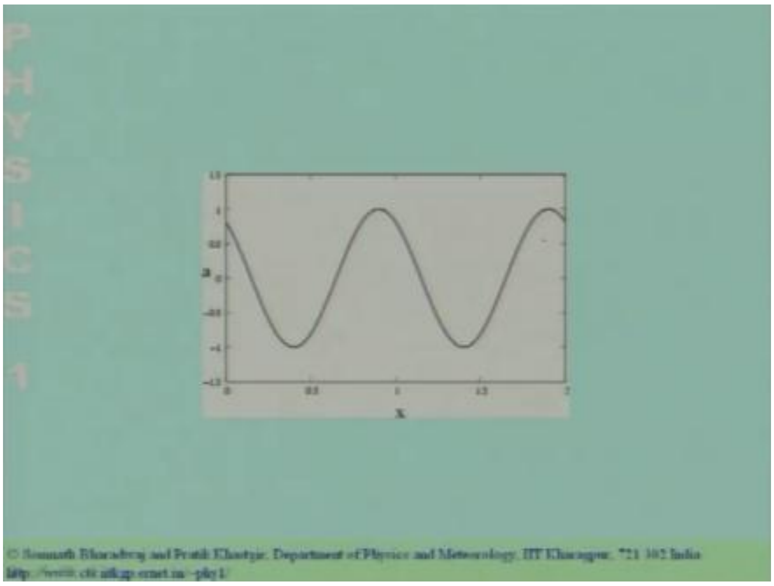
(Refer Slide Time: 37:54)



So, having told you the phase velocity let me now; again show you a picture of how the wave evolves. So, this is the wave the black curve shows you the wave at t equal to 0. After a time t 1 so, at t equal to 1 the point where the phase is 0 has shifted forward it has shifted forward by 1 into the phase velocity. So, it has moved here and the same with every other part it has moved forward by the same amount. Because, the phase has moved the same amount. So, the value the point where the phase was pi by 2 has shifted by the same amount. And all intermediate points have shifted by the same amount.

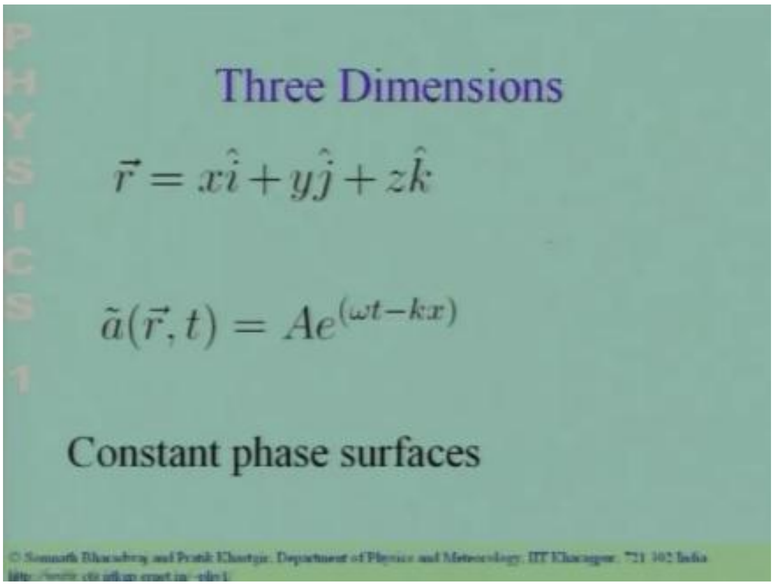
So, the whole curve moves forward and then after some time t at some time t equal to 2. It moves forward again further and t equal to three and moves even further. So, this is how the whole thing evolves in time in time the whole wave the form of the wave does not change it just moves forward in time.

(Refer Slide Time: 38:53)



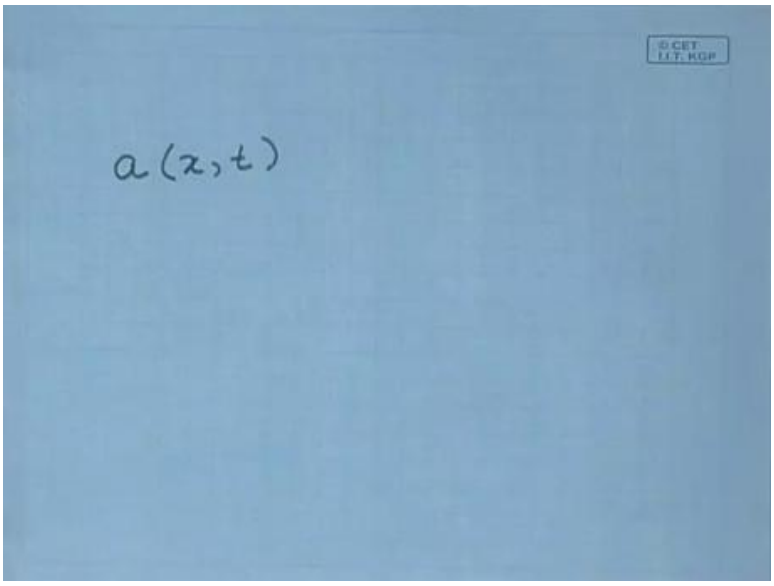
So, this shows you the wave propagation as a function of time. So, you can see that the whole sin wave or cosine wave just moves forward in x as time propagates.

(Refer Slide Time: 39:17)



So, let me show it to you again as time propagates the whole pattern just moves forward. So, we have studied wave's disturbances which are a function of 1 variable x and time.

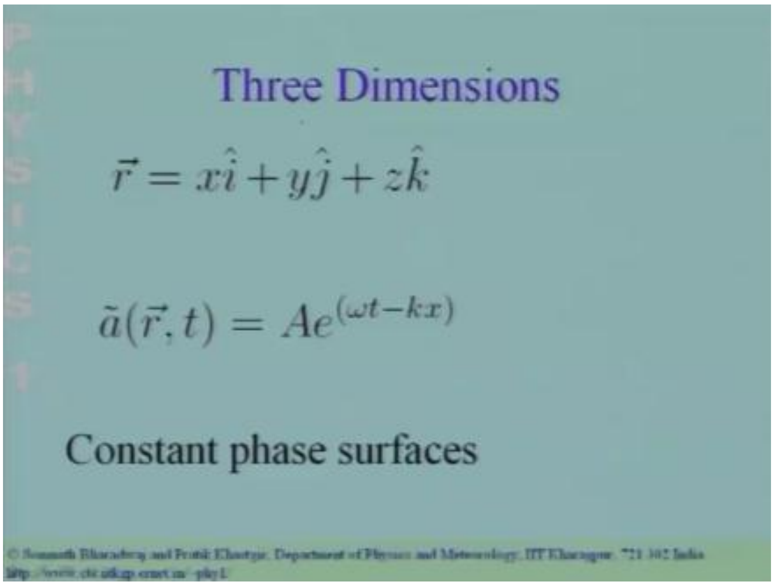
(Refer Slide Time: 39:36)



So, a thing that we have been studying until now, is a as a function of x and t. Now, such a discussion may be adequate is adequate, if you are dealing with disturbances for example, in a string on a string, if I put a disturbance it will propagate along the direction of the string. 1 variable x is sufficient to parameterise to label the different points on the string because the string is effectively 1 dimensional. So, such a description is adequate for that purpose, but if i have for example, sound waves in this room can propagate in this room is three dimensional. So, the sound wave in this room is actually a function of 3 coordinates.

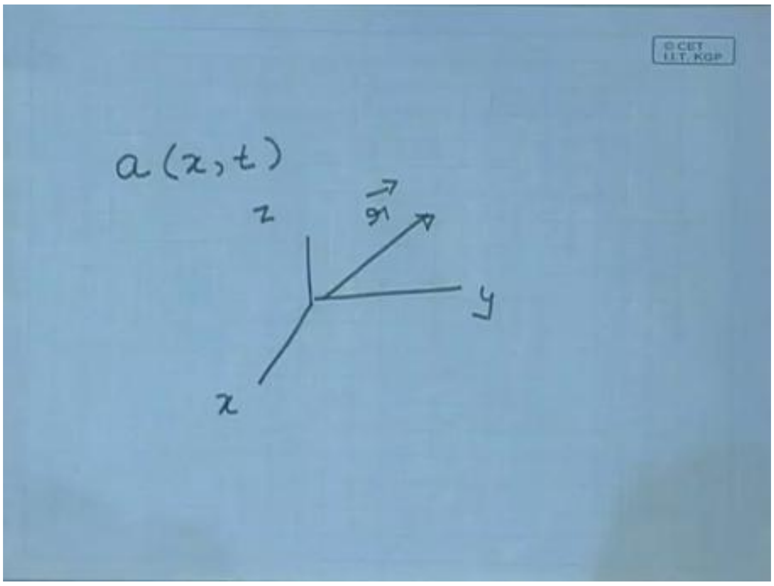
So, we need three coordinates to describe different points on the room and the sound wave in this room will be a function of three independent coordinates. So, in general we shall be dealing with we have to deal with waves in three dimensions the space that we live in is three dimensional. So, for like this room, so, in three dimensions you require 3 coordinates.

(Refer Slide Time: 40:59)



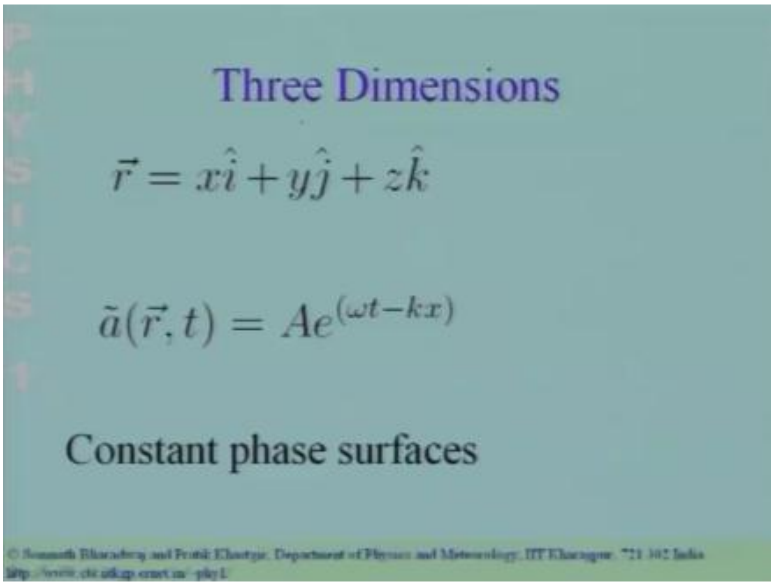
So, here we have three different coordinates x y and z. And any arbitrary point in the room could be the with reference to an origin, I can, I could refer to any arbitrary point using the vector r.

(Refer Slide Time: 41:15)



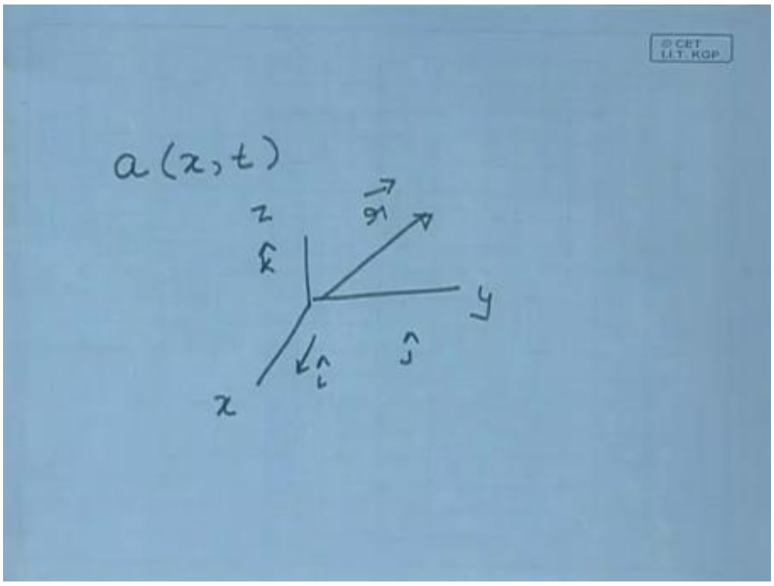
So, with in this room or in any 3 dimensional space I could choose 3 axis like this; x y and z. And I could have a vector to any point, this vector r corresponding to different points the vector r will have different coordinates x y and z.

(Refer Slide Time: 41:40)



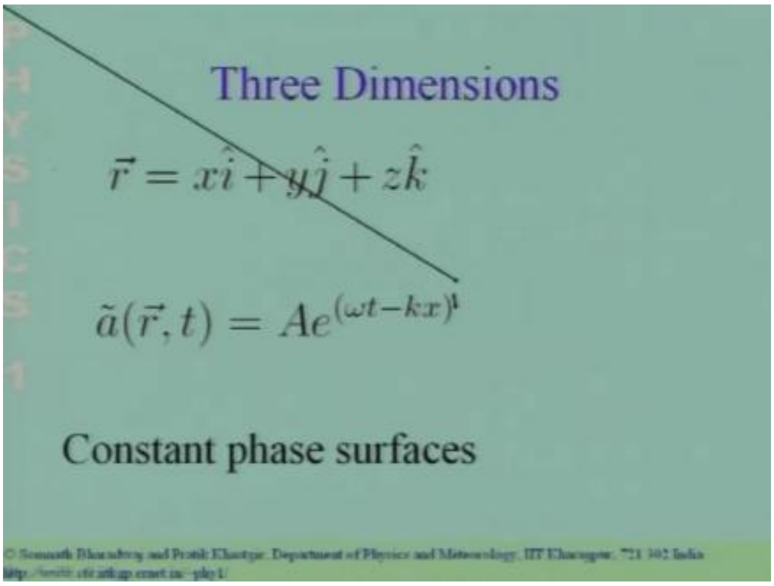
So, in 3 dimensional space any arbitrary corresponding to any arbitrary point. I have a vector which goes from the origin of my coordinate system to that point. And this vector is described is completely specified in terms of 3 coordinates x y and z i j k here, are unit vectors along the x axis y axis and z axis.

(Refer Slide Time: 42:04)



So, I have i j and k these are unit vectors along the x y and z axis. Now, so, we are working in 3 dimensions, let us first interpret the wave which we have been studying until now in a 3 dimensional context.

(Refer Slide Time: 42:29)

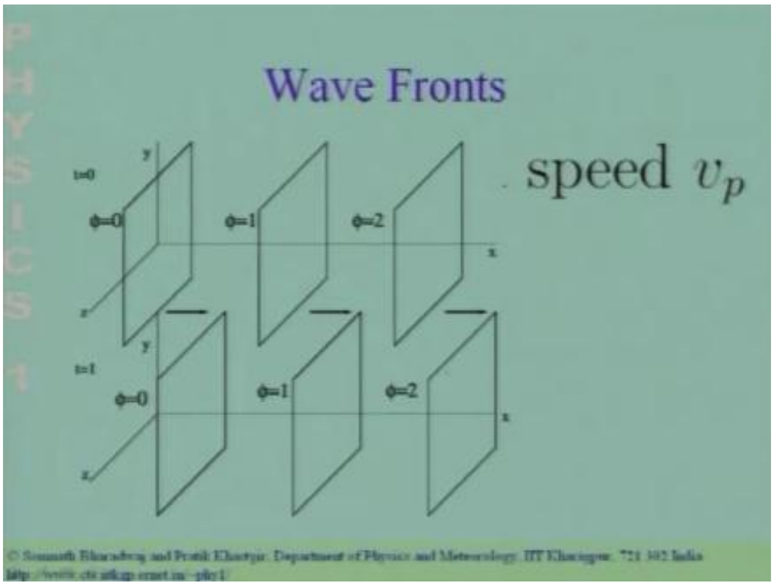


So, the wave which we have been studying until now, is a which is a function of r by r we mean all 3 x y and z. So, a is a function of r it could vary from point to point in the room. And we will be using, we will be analysing the expression which we had been dealing studying until now. So, what we had been studying until now, there was an amplitude here and we have e to the power there should be a i here. So, there should be a i over here. So, there is a i which is missing here you can put it in here. So, we have e to the power i omega t minus kx.

So, this is the wave that we had been studying. So, it only depends on x it does not depend on y and z and we are going to interpret this in a 3 dimensional context. Now, let us ask the question, does what does the phase let us look at the phase of this wave? The point to note is that the phase of this wave depends only on x, it does not depend on y and z. So, if you have a displacement in 3 dimensions; which is along the y axis or along the z axis are a combination of these. The phase the phase of the wave does not change. The phase only changes if i move along the x direction.

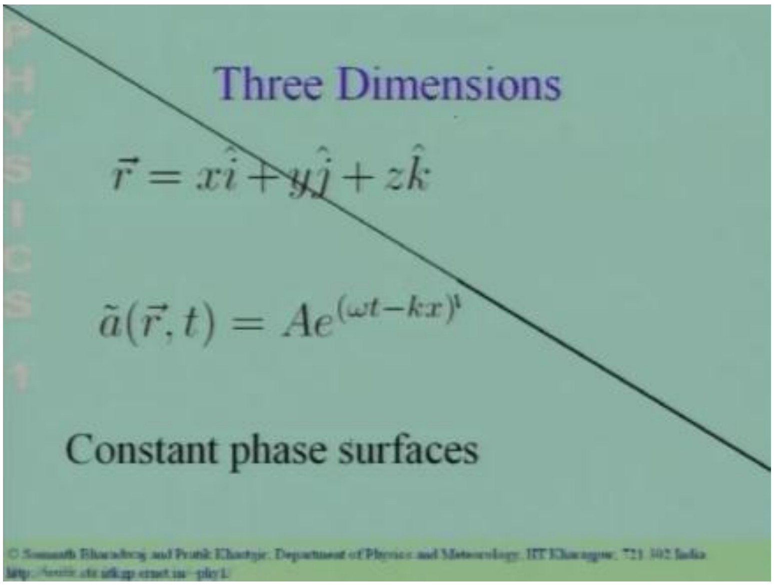
So, this tells us that there are surfaces in 3 dimensions, there are surfaces on which the phase is a constant. These surfaces in 3 dimensions on which the phase is constant are parallel to the y and z axis. So, if I move along either the y z anywhere along in the y z plane, the phase does not change. So, there are surfaces these surfaces are parallel to the y z axis y z plane and the phase is a constant on these surfaces. The phase only changes if I move along the x axis, it does not change if I move in the y z plane.

(Refer Slide Time: 44:43)



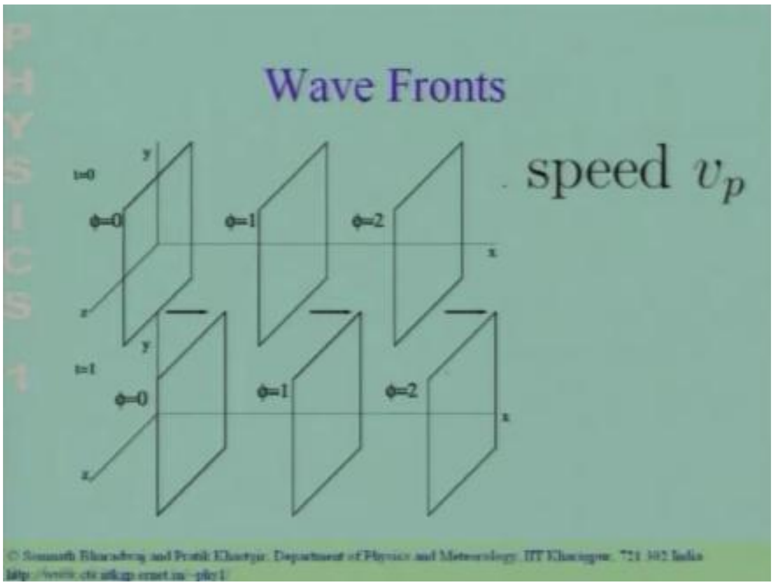
So, these there are these constant phase surfaces these constant phase surfaces, what are called wave fronts. So, there will be a surface which is x equal to 0 where the phase is 0. Then I will have another surface where the phase would have increased. And here I show you the surfaces on which the phase has a constant value. These phases are perpendicular to the x axis, because the phase changes on along the x axis. If I move perpendicular to the x axis the phase does not change. So, the constant phase surfaces are perpendicular to the x axis and they are parallel to the y z plane. So, these are the constant phase surfaces these are what are called wave fronts.

(Refer Slide Time: 45:38)



So, the point to note is that in this situation where i have a wave which looks like this, A e to the power i omega t minus kx.

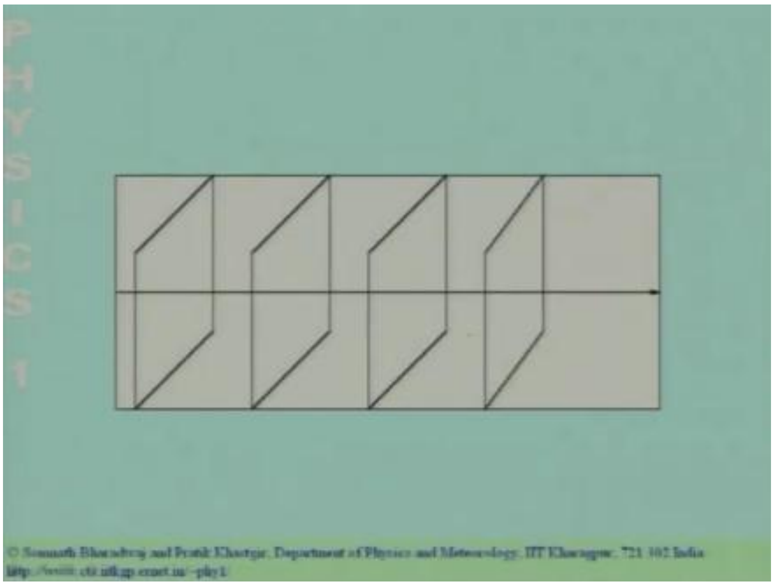
(Refer Slide Time: 45:43)



The wave fronts or planes, there are planes on which the phase is a constant. Which is why this is called a plane wave? I could have other kinds of wave fronts also which are not planes and then [Laughter-they] would not be plane waves. But in this situation which is the simplest situation, the wave fronts the constant phase's surfaces are planes and these planes are shown over here. These planes are perpendicular to the direction of propagation of the wave. In this case, they are perpendicular to the x axis. So, we have these wave fronts on which the surface the wave fronts and which the phase has constant values. Now, what happens to these constant phase values as time evolves. So, we have seen just a short time ago, that at a later time the value of x where the phase is 0 moves forward it moves forward at the phase velocity.

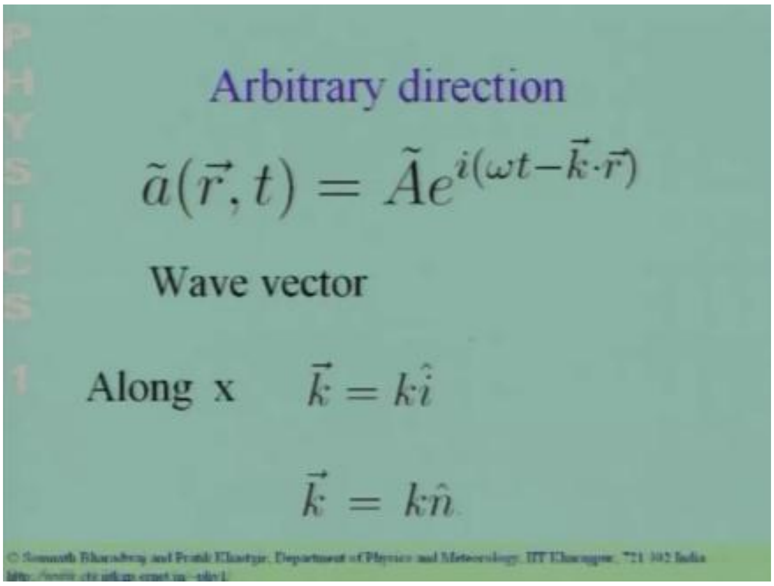
So, this is what happens here also. So, at a later time the value of x where the phase is 0 has moved forward. So, this whole plane where the phase is 0 has moved forward. And so, with time what happens is that these wave fronts the constant phase surfaces move forward along the x axis. So, in this case the constant phase surfaces move along the x axis. So, as time evolves a constant phase surfaces move along the x axis.

(Refer Slide Time: 47:26)



Let me so, this picture shows you, how the wave fronts evolve in time. As time evolves the constant phase surfaces move along the x axis.

(Refer Slide Time: 47:45)

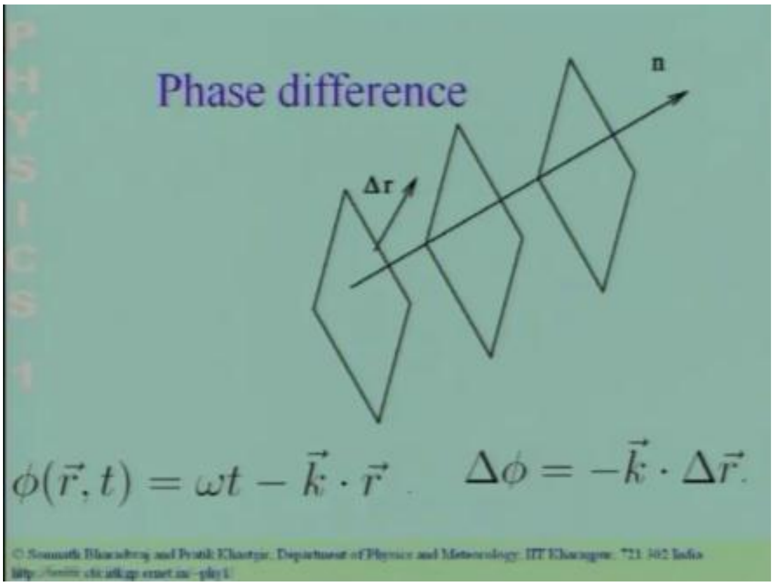


Let us now, discuss a wave in an arbitrary direction. Until now, we have been discussing a wave in 3 dimensions, but it was the wave was propagating only along the x axis. The next question that we shall discuss is how do you represent a wave propagating in an arbitrary direction. To understand this let us. So, let me first tell you the answer and then i shall explain to you why it is so. So, you can represent a wave in any arbitrary direction, as a it is now, a function of r and t r is a vector to in 3 dimensions. So, a is a function of r and t and a wave in any arbitrary direction can be represent as the complex amplitude into e to the power i omega t minus k dot r.

So, we earlier had k into x we now, have replaced that by k dot r. Now, the first thing is that this vector k is called the wave vector. We have replaced the wave number by a wave vector. So, when you are dealing with waves propagating in 3 dimensions, you have to replace the wave number by a wave vector. If the wave is propagating along the x axis you have to choose the wave vector to be the wave number k into the unit vector along the x axis. So, if you put this into this expression; you will recover K into x where K is the modulus of the wave vector K is the amplitude of this vector and A is the unit vector along the x axis.

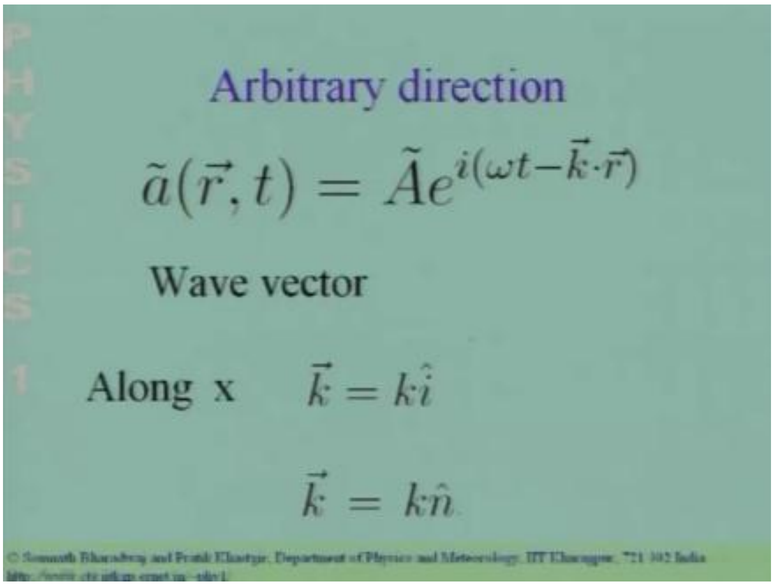
So, when you put this in here you get k into x. So, if you put this over here you get kx, you recover the wave propagating along the x axis. Now, if you want the wave to propagate not along the x axis, but in some arbitrary direction with unit vector n. So, n over here is the unit vector in which you want the wave to propagate not the x axis, but some other direction. Then the wave vector should be the wave number K into the unit vector n. So, if you use this wave vector in this expression this wave will now, propagate along the direction of the unit vector n. It will have a wave length which is 2pi by K. This wave number is continuous to be related to the wave length, by with a fact by a fact through a factor of 2pi and then you take the inverse of the wave length you will get the wave number.

(Refer Slide Time: 50:46)



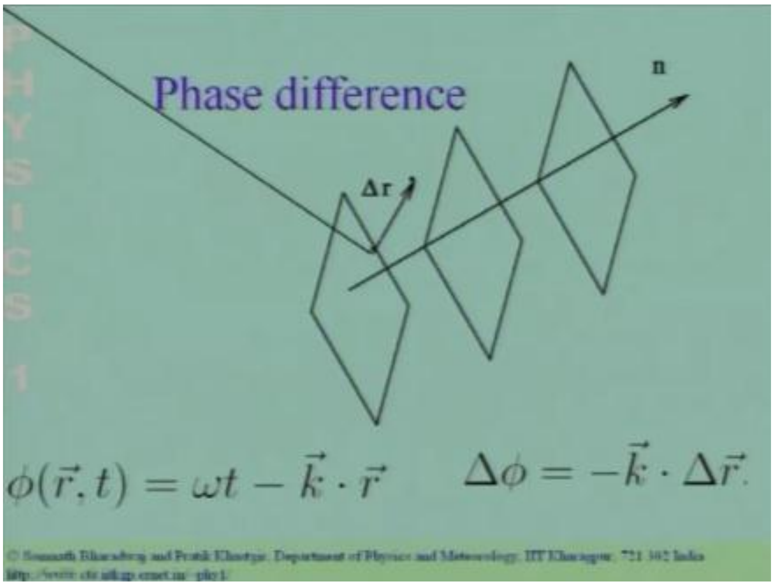
So, this is how we can represent waves in arbitrary directions. So, let me now show you, what this looks like; I have a wave in 3 dimensions the wave is propagating in this direction n. So, this n over here show is the unit vector in along the direction in which the wave is propagating. So, I have a wave which is propagating along the direction of the unit vector n shown in the slide.

(Refer Slide Time: 51:15)



Now, let us just look at the equation, representing this wave. So, the equation representing the wave is shown over here, let us ask the question, in which direction does the phase change? So, it is quite clear that the phase changes only along the unit vector n. So, if you move in the direction perpendicular to the unit vector n the phase does not change. So, from this you conclude that the wave fronts are orthogonal. So, you have a wave direction n along which the wave is propagating. If you move in a direction perpendicular to that the phase does not change. So, the directions perpendicular to the unit vector n are the wave front.

(Refer Slide Time: 52:01)

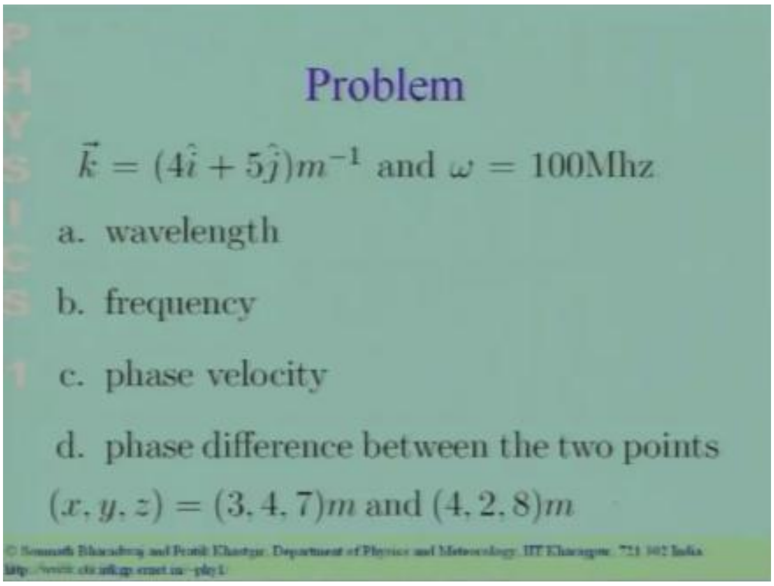


So, this is what is shown over here, these planes are perpendicular to the direction n and these are the wave fronts. So, if you move from here to here phase does not change and these are the wave fronts. So, as time evolves these wave fronts will propagate forward, the whole wave fronts these whole set of wave fronts will propagate forward. So, this is what the wave fronts look like; If I have waves propagating in an arbitrary direction.

Now, let us ask another question, how much does the phase change, how much does the phase change if at a fixed time, I look at 2 different points. So, the question we are asking is what is phase difference between this point and this point? So, the question we are asking let me repeat it again is at a given time t, what is the phase difference between this point and this point. Now, remember that, if I move in the direction perpendicular to the n there is no change in phase. So, it is only the component of the displacement delta along the direction of the wave, along a propagation of the wave, along n, that gives rise to a change in phase.

So, we go back to the expression for the phase, the phase as a function of the position and time is omega t minus k dot r. So, we are interested in the phase difference between 2points, 1point being r. The vector r the other point at the position r plus delta r. So, the difference in phase between these 2 points delta 5 is minus k dot delta r. So, the phase difference is arises only due to the component of the displacement in the direction of propagation of the wave and it is minus K the wave vector dot delta r. This is something that is going to play an important role later on, when we discuss interference and diffraction various such phenomena where we super pose waves.

(Refer Slide Time: 54:18)

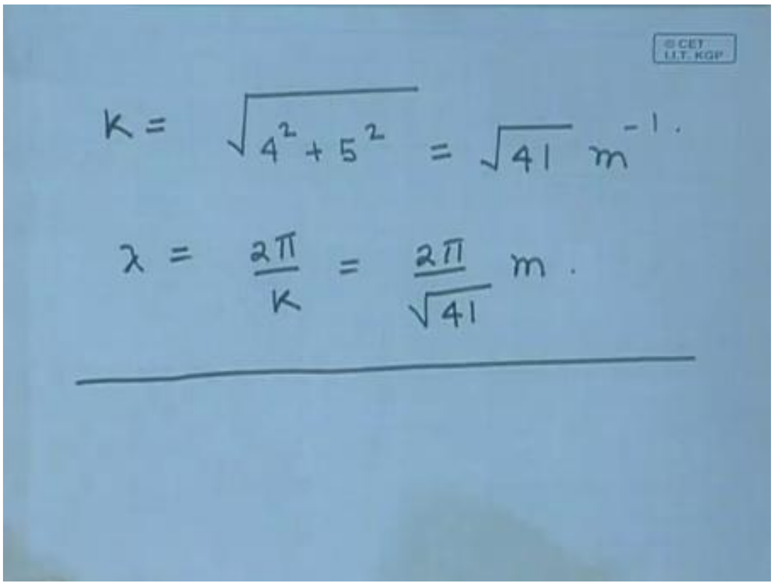


So, it is quite an important relation so, this brings us to an there is to the end to an to the end of our of our discussion of a plane sinusoidal waves. Plane sinusoidal waves are essentially if I sit at a fixed place, I will see and sinusoidal simple harmonic oscillation it. I look at the variation with position at a fixed time; I will see again a sin wave sin a function which varies with position. So, a position dependence which is sin or cosine and with time the whole pattern moves forward in the along the direction of propagation.

So, having completed our discussion of sinusoidal plane waves, let us take up a simple problem; where we can apply some of the things which we have learnt. So, in this problem, that we are going to discuss; we have a wave and the wave is described by a wave vector the wave vector is 4 i plus 5 j meter inverse. The wave also has an angular frequency which is hundred mega hertz. Or remember that it should, we could have written radiance per mega radiance mega hertz radiance per I mean radiance per 10 to the power 6 second 10 to the power of 6 per second. So, there should have there could have been a radiance here, but we are not mentioning it explicitly. So, omega has a value hundred mega hertz . So, the first question that we are going to ask is what is the wave length of this wave?

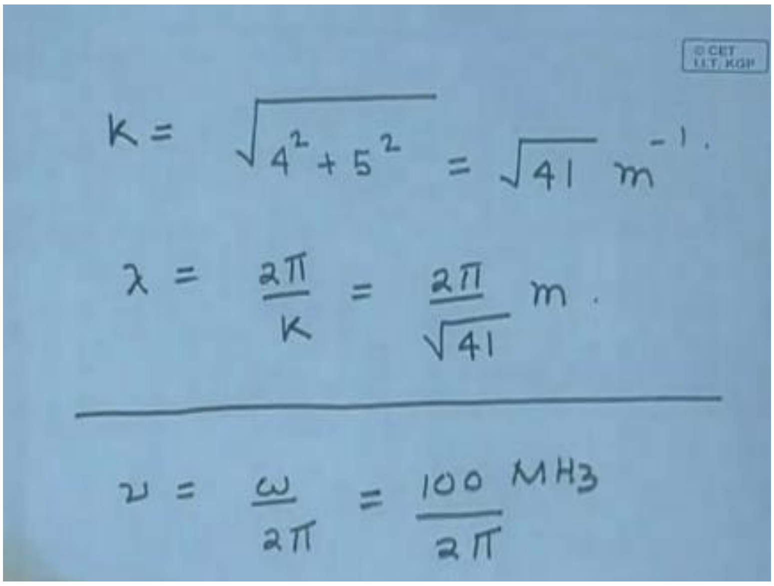
How will you determine the wave length of this wave? To determine the wave length of this wave you have to first calculate the wave number.

(Refer Slide Time: 56:20)



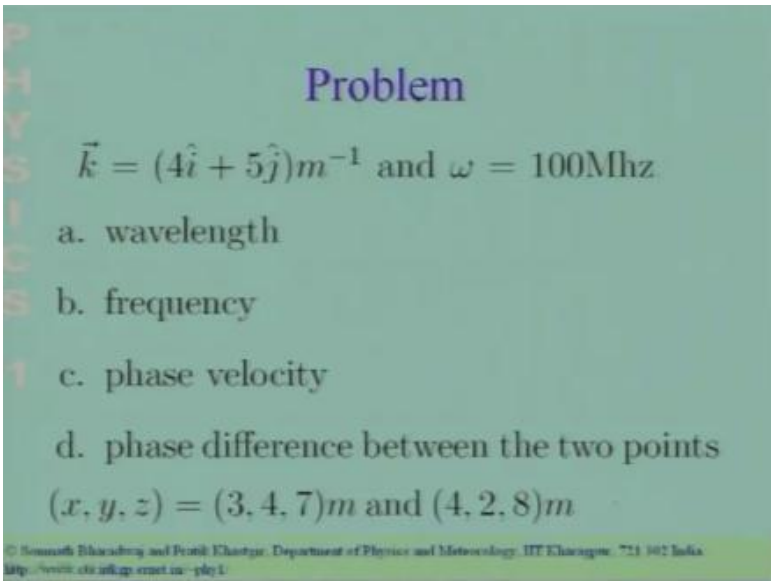
So, the wave number is the modulus of this vector. So, the modulus of this vector this vector has x component and y component. So, the modulus of this vector is the square root of 4 square plus 5 square 4 being the componential on the x axis, 5 being the componential on the y axis. So, the modulus of this vector is the square root of 4 square plus 5 square, this is 16 plus 25, is 41. So, this is a square root of 41. And the units are meter inverse and the wave length lambda is 2pi by the wave number K. It is 2pi by the square root of 41 meters.

(Refer Slide Time: 57:14)

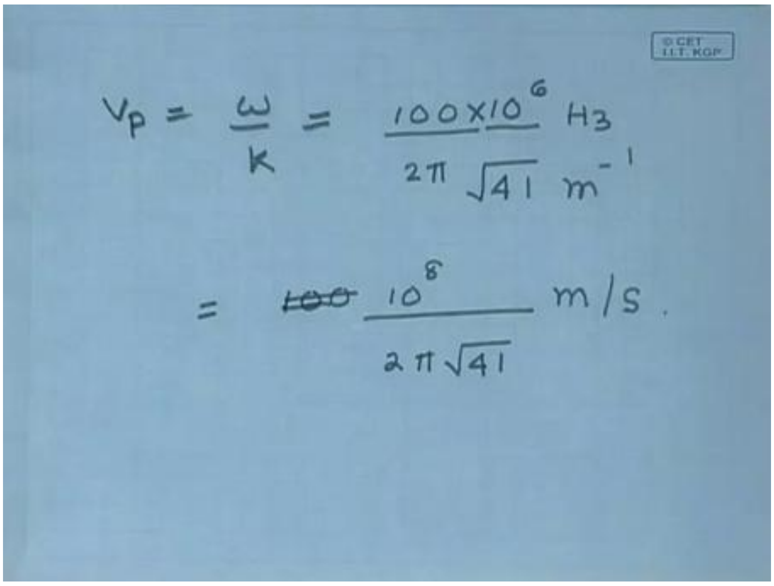


The next part is to calculate the frequency; we know that the frequency of the wave is related to the angular frequency through this. So, you have to just plug in the value 100, this will be 100 mega hertz divided by 2pi. So, we have worked out the first 2 parts of the problem we have calculated the wave length and the frequency. Let us now calculate the phase velocity of the wave.

(Refer Slide Time: 57:41)



(Refer Slide Time: 43:00)



The phase velocity is omega by K and omega has a value 100 by 2pi. And we can go directly to hertz. So, this is the value hertz and K has a value which is root 41 meter inverse. So, the phase velocity has a value 100 rather 10 to the power 8 divided by 2pi root 41 meters per second. So, this is the phase velocity of the wave.