Physics I: Oscillations and Waves

Prof S. Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology, Kharagapur

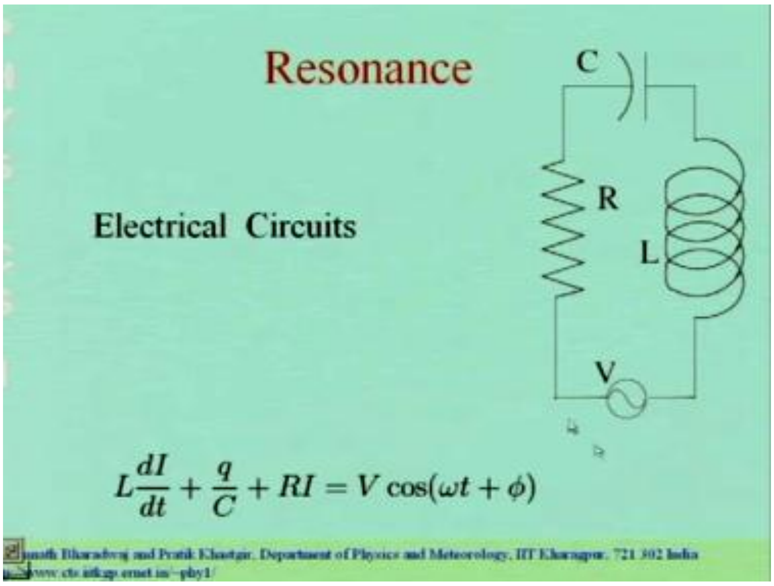
Lecture - 06
Resonance

In the last 2 lectures, we have been studying the behavior of a simple harmonic oscillator, under the influence of an external force which is time dependent. In particular, we have been looking at an external force, which has sinusoidal time dependence. And we ask the question, what happens if we change the frequency of the external force? We saw that, if the frequency of the external force matches the natural frequency of the oscillator, the amplitude of the oscillations that occur becomes very large.

If, there is no damping, you get infinitely large oscillations. If, you have damping you have finite oscillations. And if you move away, if the angular frequency of the external force is differed is moved away from the natural frequency, then the amplitude of the oscillations falls. Not only that, the energy transferred to the oscillator is also maximum, the power dissipated in the oscillator the power transferred to the oscillator, it is also maximum, very close to the natural frequency of the oscillator. And if you drive the oscillator with frequencies, which are away from the natural frequency, the power transferred or the amplitude of the oscillations false.

And this phenomenon, where the oscillations up come close to the natural frequency is called resonance.

(Refer Slide Time: 02:12)



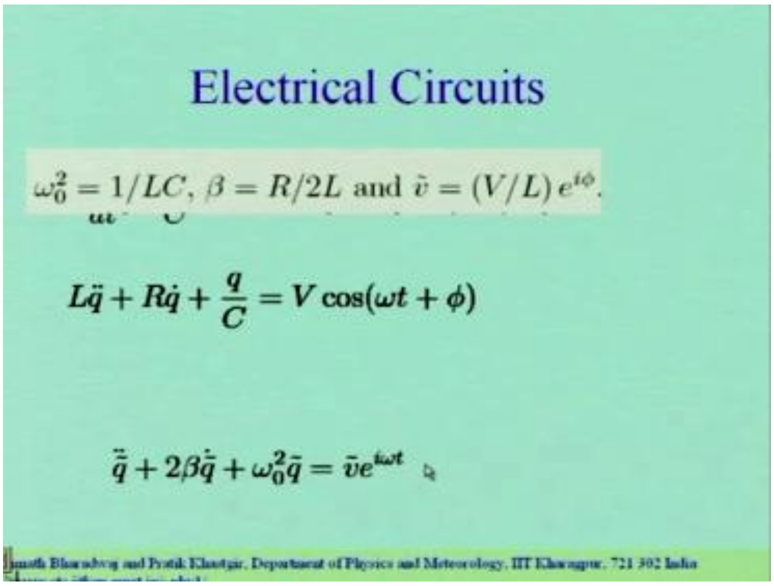
So, let us now look at, a particular situation which is familiar and where we have resonance. So, we will first look at electrical circuits. So, we consider an electrical circuit; where we have a resistance a capacitance and an inductance. All 3 in series and you also have a voltage source; the voltage source is a sine wave function generator, where you could in principle change, the frequency of the voltage. So, the voltage source is also connected in series to these 3 elements. And you could change the frequency of the voltage generator.

So, applying Kirchhoff's law to this circuit, then you find the Kirchhoff's law basically tells us; at the voltage drop across the voltage generator, should be balanced by the voltage drop across the resistance, the capacitance and the inductance. So, the voltage drop across the voltage generator is V cos omega T plus phi, V is the amplitude of the voltage that is produced here, omega is the angular frequency of this voltage that is produced here.

So, this voltage should be balanced, should be exactly be equal to, the voltage across the resistance R into I, the voltage across the capacitance q by C where, q is the charge in the capacitance, I is the current in the circuit and the voltage across the inductor, which is L into the derivative time derivative of the current. So, this is the equation which you get, when you apply Kirchhoff's law, which tells us; that the total voltage across the circuit, if I start from here and come back here the total voltage drop should be 0.

So, this consideration leads us to this particular equation for the circuit.

(Refer Slide Time: 04:11)

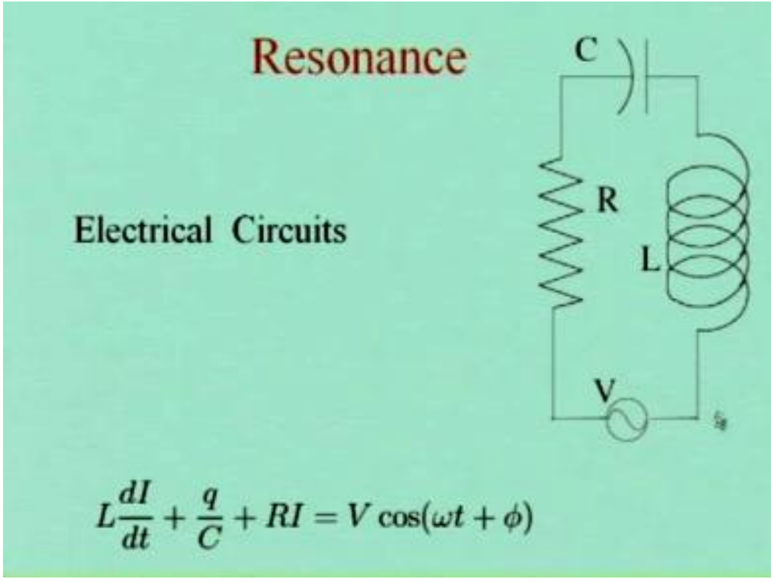


Now, this equation which governs the circuit can be written fully in terms of only the charge. So, the current we know is; the rate of change of charge, the rate at which charge flows in the circuit. So, we can replace this term over here with R into q dot where, q dot is the type derivative of the charge and we can replace this term with L into a second derivative of the charge. So, the same equation can also be written in this fashion.

Now, if we divide this equation throughout by the inductance L and we redefine the variables. So, I have divided this equation by L and then, I identify the term over here; 1 by LC with omega naught square and the term that occurs here R by L with 2 beta. So, beta is defined as R by 2 L, with this identification. And if I define v tilde as this amplitude of the voltage into e to the power i phi where, phi is the phase of the voltage generator then, this equation can be written like this; q double dot plus 2 beta into q dot plus omega naught square q is equal to v tilde e to the power of i omega t.

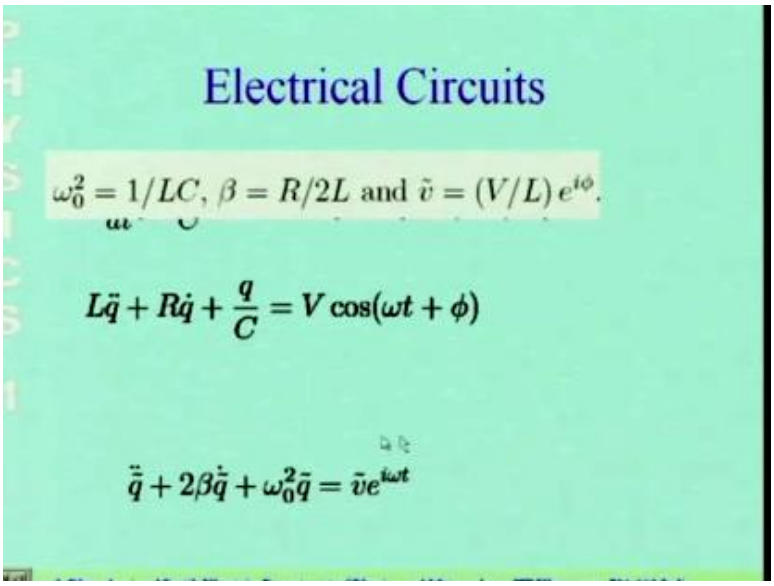
So, this is the same equation written in complex notation and using these constants, which have been redefined omega naught square is 1 by LC beta is R by 2 L. So, we see that the charge and the current in this circuit, which flows in this circuit.

(Refer Slide Time: 06:16)



In the L C R circuit is governed.

(Refer Slide Time: 06:20)



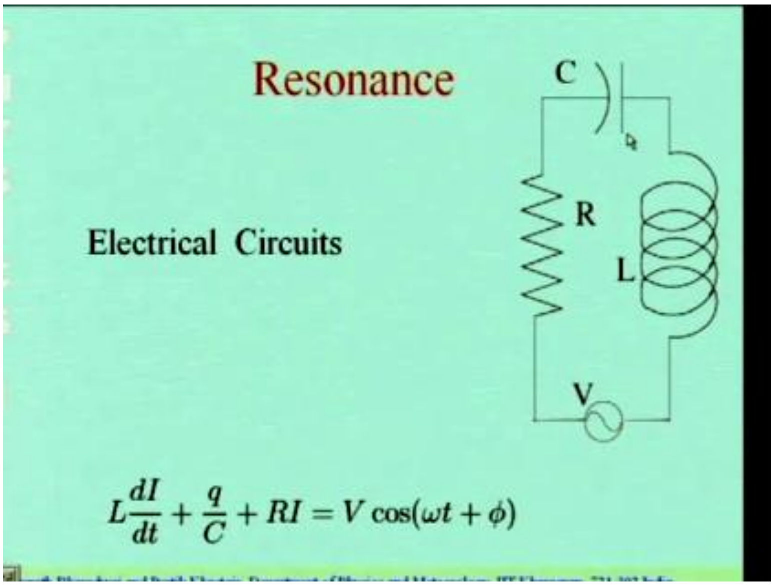
By exactly the same equation as that of a damped simple harmonic oscillator, with an external force, the external force here is the voltage that is, generated by the voltage generator. And here the resistance plays the role of the damping element, the inductance plays the role of the mass and the capacitance plays the role of the spring. So, with this identification, we can analyze the L C R circuit couple to a signal generator, in exactly the same way.

So, the analysis, which we have already carried out, for the spring mass system with an external force, the results which we have already obtained for this spring mass system, with an external force; can straight away be applied with just the variables being redefined. It can straight away be apply to this electrical circuit that, we have discussed right now. So, we already know the solution to this problem.

In this problem, if you have no external force, you will have transients, the current and the charge across the capacitance, all of them will decay exponentially. If, you have an external force, the last time behavior is very have oscillations, at the frequency of the external force which is omega and the transients die away. This is what we have already learnt.

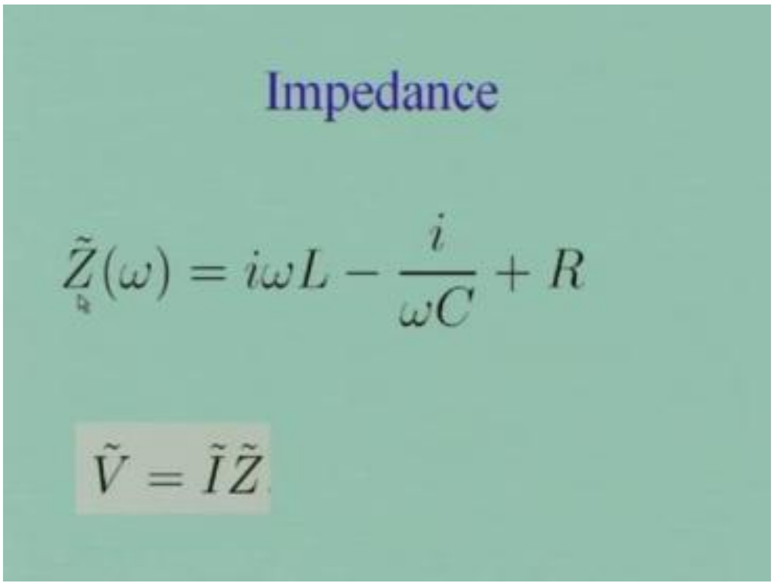
Now, there is another way, which the same situation can also be analyzed and this method. The other method which I shall very briefly we discussing here is; quite common in electrical technology or in this particularly, popular in the analysis of electrical circuits, but there is no reason why this method cannot be used also to the spring mass system, which we had discussed earlier. So, let me tell you what this method is.

(Refer Slide Time: 08:29)



So, the circuit which we are going to analyze which we are analyzing; the L C R circuit connected the L C R elements connected to an, to a voltage source this circuit can also be analyzed if, we talk in terms of the impendence. So, just like if I had only a resistance across the voltage source, I could apply Ohm's law, we could continue to use Ohm's law provided, we associate impendence with the inductance and the capacitance. So, let us calculate the impedance of this circuit.

(Refer Slide Time: 09:11)

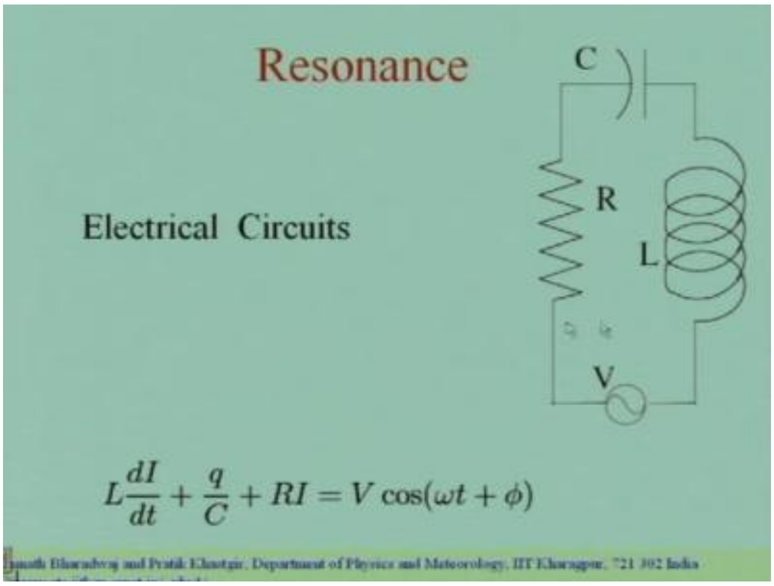


So, the impedance of the inductor is i omega L if and the impedance of the capacitor is 1 by i omega C which also written as; minus i by omega C where, omega is the angular frequency at which you are driving the circuit or at which we are studying the behavior of the circuit. So, you have the impedance, the impedance has an imaginary part, imaginary part arises from the inductance and from the capacitance. The impedance has a real part which arises due to the resistance.

So, the resistance is the real part of the impedance and the capacitance of the inductance are the imaginary parts. So, the impedance is a complex number which, depends on the value of the resistance, inductance and capacitance. It also depends on the frequency at which you are driving the circuit or the frequency, at which you are interested in the behavior of the circuit.

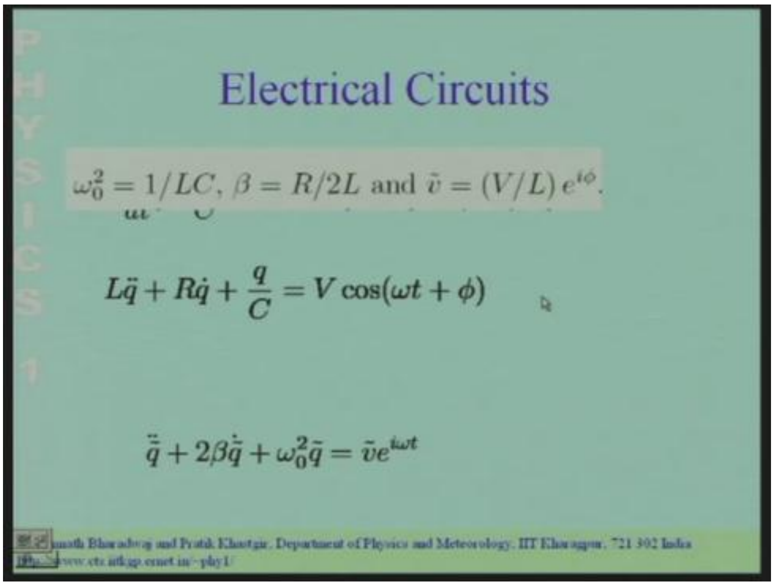
So, this impedance is a function of the angular frequency in this case omega. And once you have calculated the impedance, you can apply Ohm's law to the circuit.

(Refer Slide Time: 10:24)



So, you can apply Ohm's law to the circuit and say that the voltage across this, should be equal to the voltage across all of these elements combining together, these elements are in series. So, you can add up the impedance of all of these 3 elements. And the impedance across this is that, the current into the, voltage across these 3 elements is the current into the impedance.

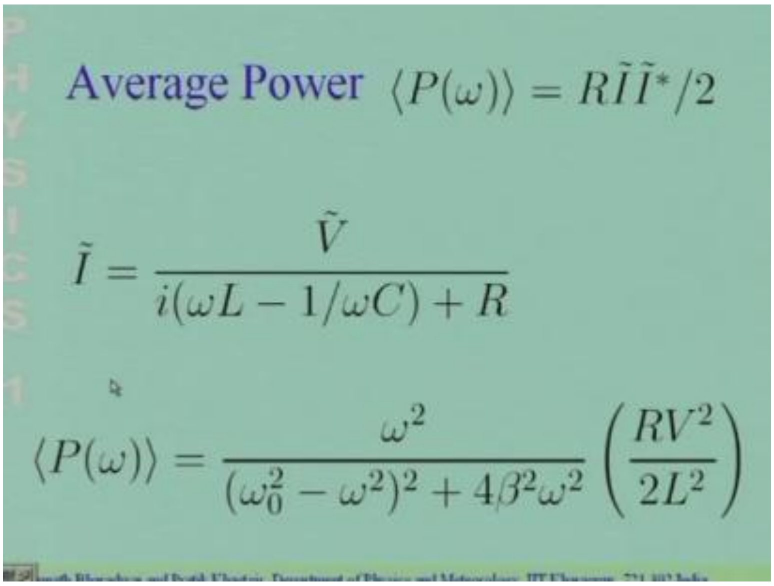
(Refer Slide Time: 10:51)



The impedance plays the role of the resistance for purely resistive circuits. So, this is like applying Ohm's law to this circuit just that, I have to replace the resistance with an impedance and the impedance is in general complex So, the voltage could be complex because, it has a phase. The current could also be complex and the impedance is also complex I have these 3 complex numbers which are related to the complex voltage. There is a complex current and there is complex impedance. The complex nature of these variables, takes into account the fact that, the voltage and the current could have phases in general and these phases need not be the same.

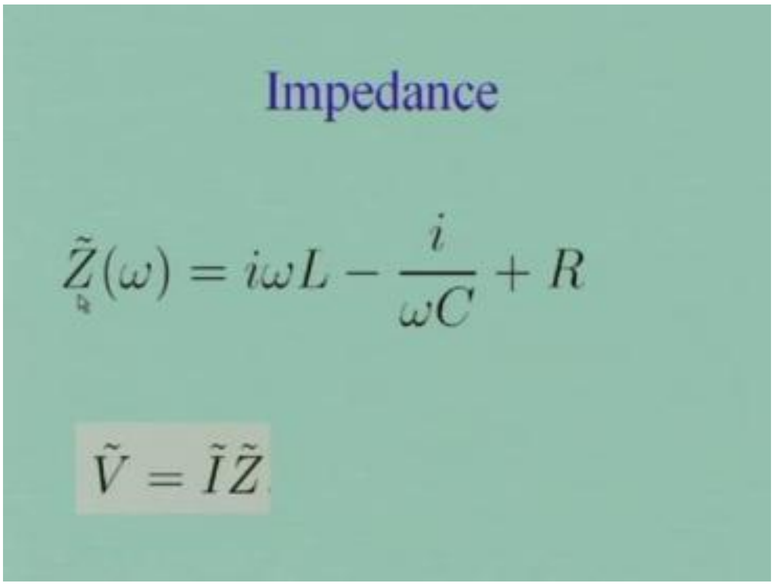
So, this is Ohm’s law which has been modified so that, it can now we applied to a situation to a circuit, which not only has resistances, but also has inductances and capacitance. Now, you could take this particular equation and re derive, all that has been derived till now for the oscillator, which is driven by an external force. We shall not be doing that.

(Refer Slide Time: 12:06)



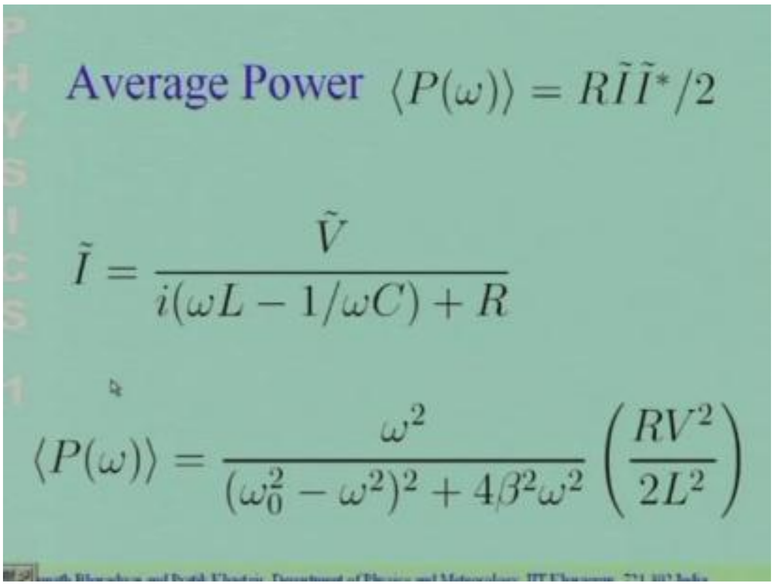
We shall only look at the behavior of the average power. So, the average power, which is dissipated in the circuit is R into I into I star by 2. Remember that, it is only the resistance which dissipates power. The capacitor and the inductor have voltages, which are pi by 2 out of phase with the current. So, there is no average power, which is dissipated in any of these elements. The voltage and the current are pi by 2 out of phase; it is only the resistance which dissipates any power and the power that is dissipated in the resistance, the average power dissipated in the resistance, can be calculated in the complex notation as R into the current I into the complex conjugate of the current divided by 2.

(Refer Slide Time: 13:12)



And the current can be calculated using this Ohm’s law, which has now been generalized.

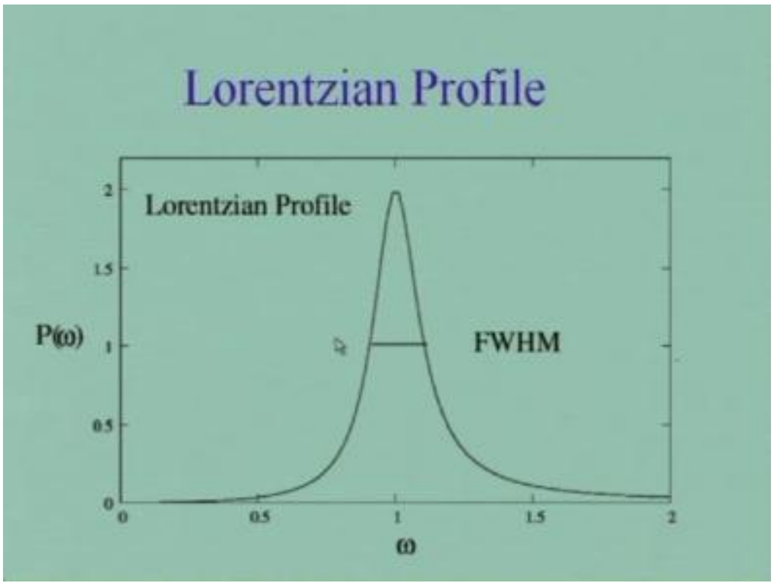
(Refer Slide Time: 13:17)



So, the current I tilde, it is complex it is not necessarily in the same phase as the voltage. So, the current is related to the voltage through the impedance. So, the voltage divided by the impedance gives us the current in the circuit. And now, if you wish to calculate the average power, you have to take the current over here given over here and you have to take its complex conjugate multiply these 2 and then multiply or essentially you have to take the modulus of the current and square it and multiplied by the resistance and divide by 2.

So, you have to, so if, you do this algebra it is a simple piece of algebra which has to be done, if you do this little bit of algebra. And you make the identifications, that omega naught the natural frequency of the circuit is omega naught square is 1 by LC 1 by LC and the damping factor beta is R by 2 L. Then you get this expression for the average power, these factors the R the voltage L square and the factor of 2 1 by 2 have all been taken common outside. And this common constant factor is multiplied by this function of the frequency or the angular frequency this particular case. And you see that, the average power as you expect is the Lorentzian profile.

(Refer Slide Time: 14:58)

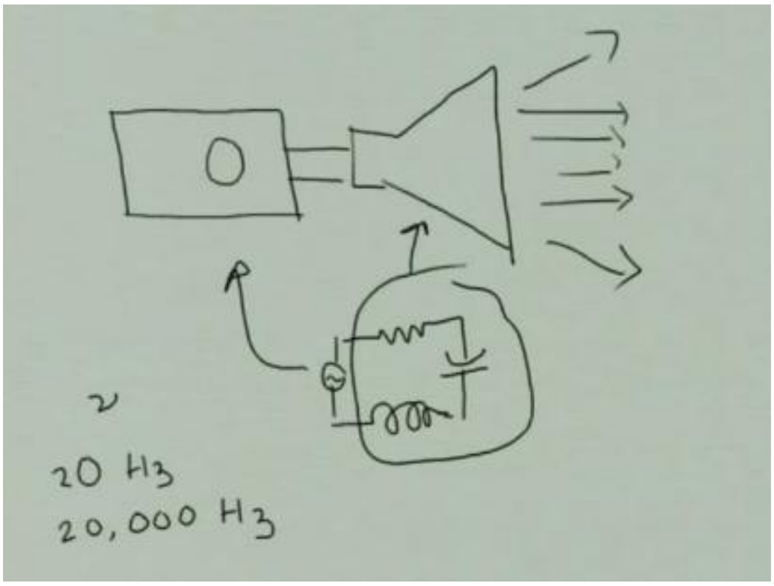


So, the average power drawn by this circuit is follows the Lorentzian profile. It has a peak at the natural frequency omega naught and then, if you drive it at other frequencies, the power that is dissipated and the resistance false off. You could calculate the full width that half maxima of this peak, for low values of the damping coefficient where, beta is much smaller than omega naught, the full width that off maxima is twice approximately twice beta.

So, we see that this circuit which we have considered dissipates the maximum power at resonant frequency. The current is also maximum at the resonant frequency; the energy stored in the circuit is also maximum at the resonant frequency. And the power dissipated, the current, the amplitude of the current, the energy all fall off as you move away as a driving frequency is shifted away from the resonant frequency.

Now, this kind of a circuit has occurs, this kind of a thing situation occurs in a large variety of situations. A large variety of electrical devices can actually be modeled, as having an inductive element or capacitive element and a resistance. For example let me just discuss 1 example very briefly.

(Refer Slide Time: 16:36)



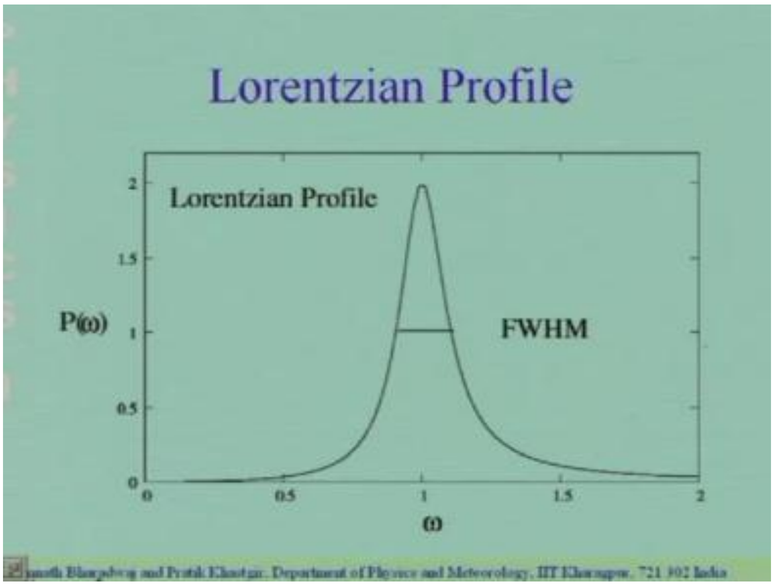
So, an example is, if I have a music system, let us consider a situation where, I have a music system. So, this is the schematic diagram, which shows you the music system and the music system is connected to a speaker. And the speaker produces a sound. So, the speaker takes the electrical signal and converts it to a sound. Now, this speaker has a diaphragm and magnetic coil etcetera. For our purposes we could model this speaker, in terms of a resistance a capacitance and an inductance and the music system in terms of a voltage source.

So, this circuit which we have analyzed could be model for a music system, which is driving a speaker. This part is the speaker and this part the voltage source, is the music system which is producing the electrical signal driving the speaker. Now, a good music system should produce should span, nearly the entire audible range. The audible range as we know; the range of frequency is that we can hear is from new, the frequency going from 20 hertz to 20000 hertz.

So, the voltage source in this case, you can the signal produce by the voltage source, you can decompose into a superposition of signals of different frequencies and the frequencies vary in this range. And we would like the speaker to produce sound, more or less of the same amplitude for signals in this large range; from 20 hertz to 20000 hertz. Now, note that this speaker, the resistive element of the speaker, models both; the power which is fed in by the music system, is a part of that power is dissipated at he had heat in the speaker because, there is some resistance. And another part of the power is actually dissipated away a sound energy. And both of these effects are there in the resistance, which I introduce to model the speaker.

So, the resistance over here has both these things. It has a sound which is coming out because that is, 1 source of that at which by which the energy is dissipated and there is also the heat. So, the resistance represents both these sources of energy being dissipated. Now, the question is how should you design a speaker? Should it be damped, highly damped or should it be under damped or should it be critically damped? Now, let us look at the Lorentzian profile.

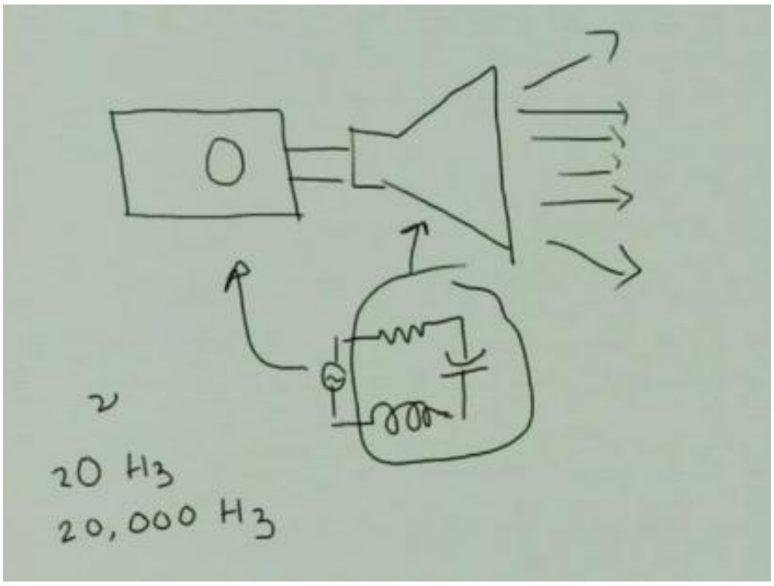
(Refer Slide Time: 19:59)



We expect the frequency response of the speaker, the power that is dissipated away and the power that is put into the sound wave by the speaker, we expect that power that is drawn from the music system, we expect that to be have a Lorentzian profile, from what we have an from analysis, we expect that to have a Lorentzian profile. And we know that, the width of the Lorentzian profile is 2 beta.

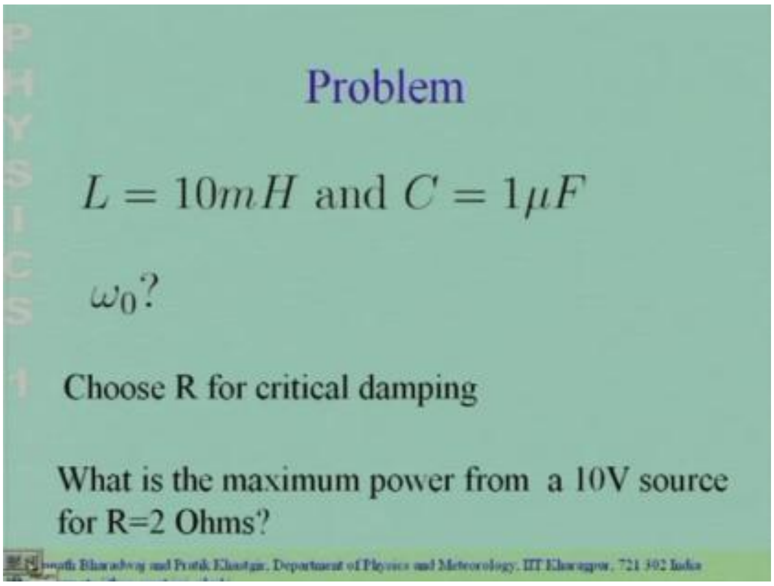
So, if have a Lorentzian profile, if you a situation where the damping is very small, you would have a very narrow peak and your speaker would respond, effectively would respond to only a very narrow range of frequencies, but this is not what we desire typically. Typically we would like our speaker to respond to a large range of frequencies and the way to achieve that would be to introduce a large damping. So, if you wish your speaker to have a broad frequency range, then what you have to do is; you have to introduce a considerable amount of damping.

(Refer Slide Time: 21:04)



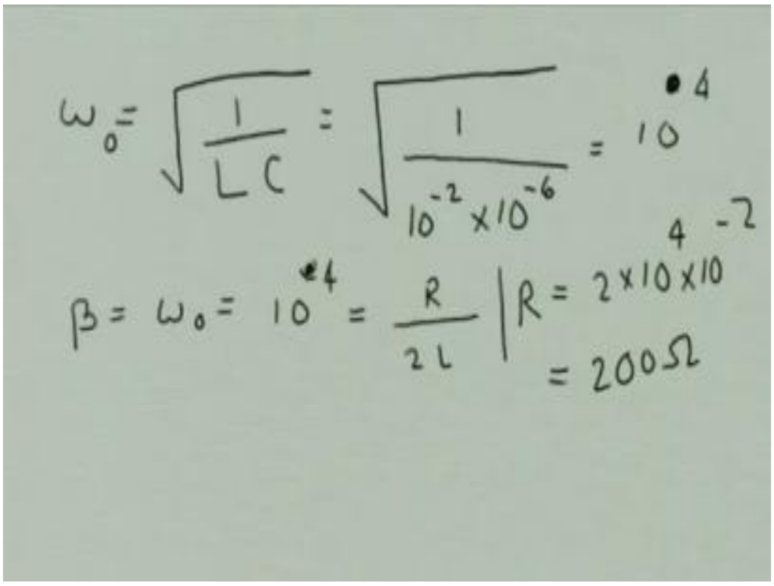
You have to ensure that, your speaker is well damped, it is reasonably well damped which is over damping region, which would make the frequency response quite broad. So, we see what we see here is 1 particular application where, we can get some inside from the things that we have learnt.

(Refer Slide Time: 21:29)



Let us next take up a brief problem, so the numerical problem. So, the numerical problem which we are going to consider is as follows: there is a L C R circuit, the type of circuit which we have just analyzed. In this circuit, the inductance has a value of 10 millihenry and the capacitance has a value of 1 microfarad. Now, the first question is; what is that natural angular frequency of this particular circuit? So, let us now calculate the natural frequency of this particular circuit.

(Refer Slide Time: 22:14)

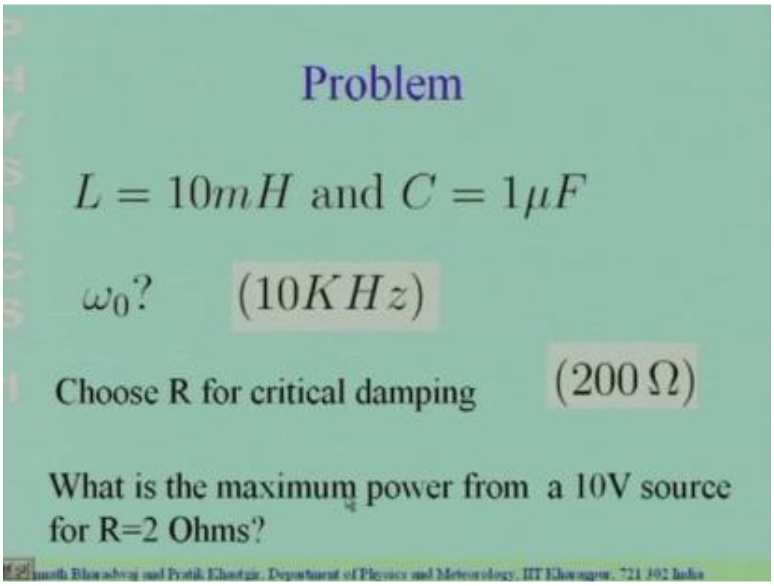


So, the natural frequency omega naught is square root of 1 by L C. So, you have to put in the values and the inductance is 10 millihenry and the capacitance is 1 microfarad. So, 10 millihenry would be 1 by 10 to the power minus 2 and the capacitance has a value of 1 micro farad which is 10 to the power minus 6. So, we get 1 by 10 to the power minus 8 and if, take this square root of this I will have 10 to the power minus 4 radians per second. And we need not bother here about the radians, which is the dimensionless.

So, we get the answer10 kilohertz; this is radians per second actually, but I am not really going to mention the radians explicitly every time. So, omega naught has a value 10 kilohertz. Now, the next question is we have to choose the resistance, so that the circuit is critically damped. So, let us now discuss this; beta for critical damping beta should be equal to omega naught.

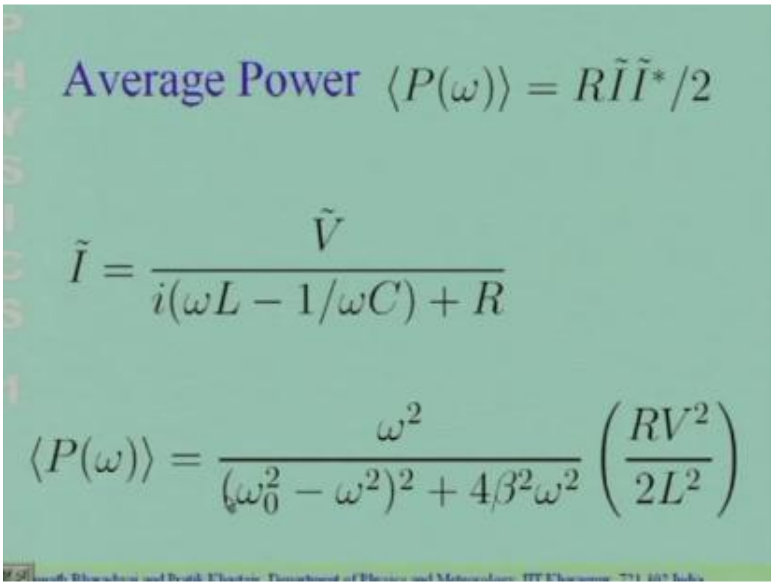
So, beta should have a value 10 to the power minus 4 and we have seen that beta is equal to R by 2 L, which gives us the value of the resistance R, R is equal to 2 into 10 to the power. So, it R is R by 2 L is beta and beta is 10 to the power minus 4 10 to the power not minus 4 10 to the power of 4, this should be power four. So, omega naught is 10 to the power 4 10 kilohertz. And R is 2 into 10 to the power 4 into L, L has a value 10 to the power minus two. So, this is 200 Ohms. So, the resistance should have a value 200 Ohms for critical damping.

(Refer Slide Time: 24:58)



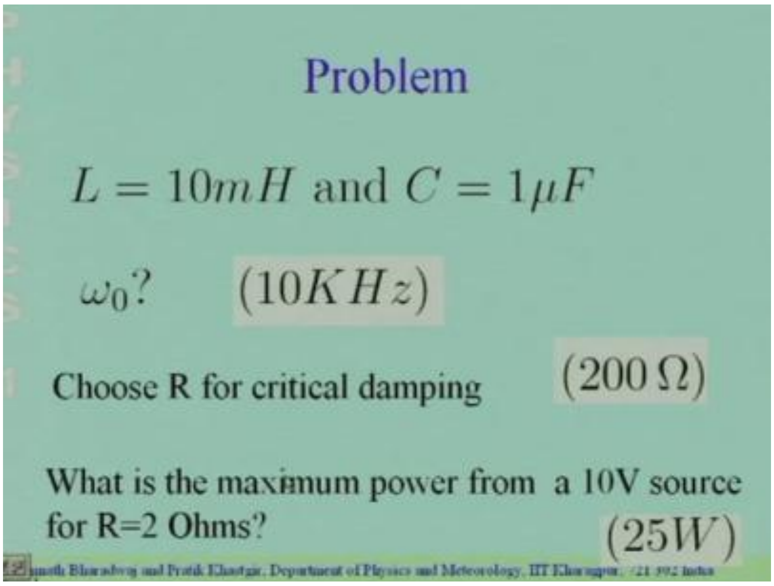
The next question is what is the maximum of value of power for a 10 volts source if the resistance is 2 Ohms? 2 Ohm's ensures that the circuit is under damped. So, if I have the under damped situation, the question is what is the maximum power that is dissipated?

(Refer Slide Time: 25:24)

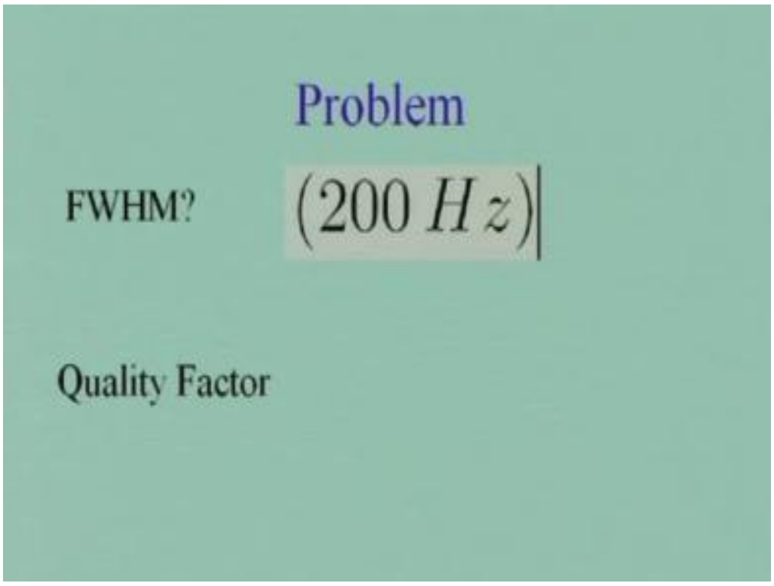


So, I will not what called the numerical value, you have to yes putting omega equal to omega naught into this equation, it will tell you the maximum power. That is where the maximum power will be dissipated and you have the answer 25 watts.

(Refer Slide Time: 25:34)

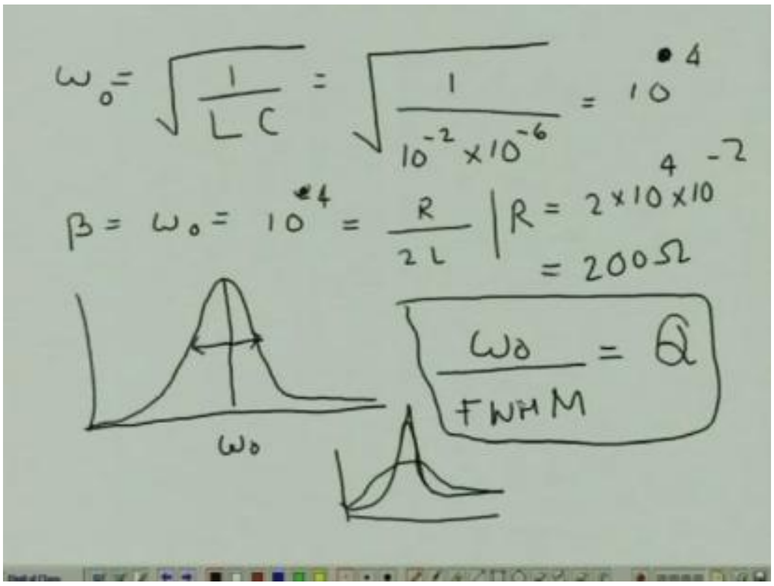


(Refer Slide Time: 25:35)



Now, next question is what is the full width that half maxima of the circuit? The full width that half maxima we know is twice beta. So, we have a value of 200 hertz. And the next question, which we shall take up is what is the quality factor of this circuit? Now, the quality factor is something which I have not discussed. So, let me spend a little while discussing it. The quality factor is defined as follows.

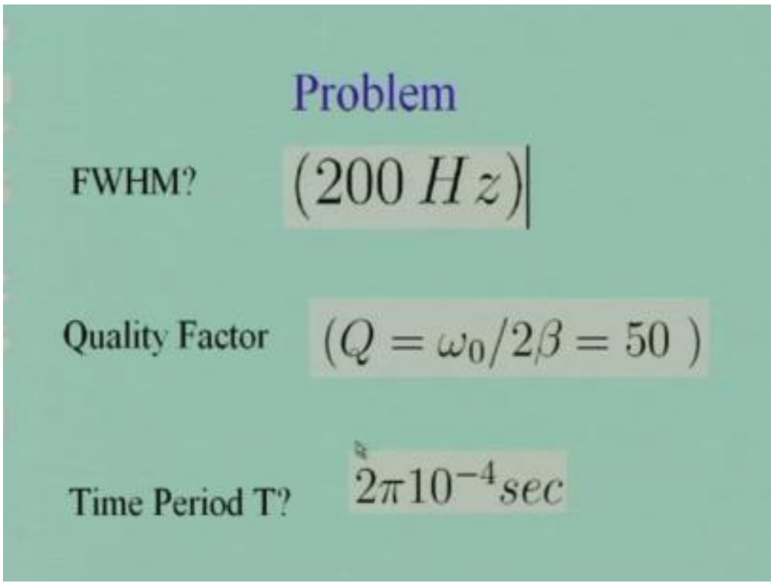
(Refer Slide Time: 26:04)



If, I have a peak which in this case is a Lorentzian. So, for this Lorentzian, the quality factor a measure how good is my oscillator how low is my damping or how much is my damping. So, this is measured through the ratio of omega naught divided by the full width at half of maxima. So, omega naught divided by the full width at half maxima gives me, an estimate of how good my oscillator is and this is what is called the quality factor. A high quality factor oscillator, for high quality factor oscillator, the peak will look will be very sharp.

Whereas, for a low quality factor oscillator the peak will look like this, it'll be much broader. So, high quality factor it will be very sharp for low quality factor will be quite broad. And this tells you how good your oscillator is. If, the damping is very small then, the quality factor is going to be quite large, because, we have seen that the full width that half maxima is twice the damping factor. Whereas, if the damping is quite large then, the quality factor is going to be quite small.

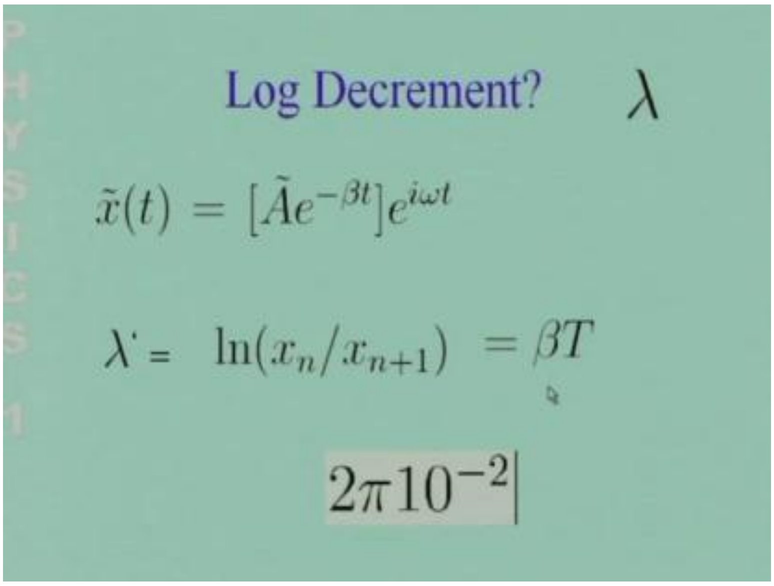
(Refer Slide Time: 27:32)



So, the quality factor is the defined as the omega naught the resonant frequency that is the value vary have the peak divided by the full width at that maxima which is twice beta in this case. And if you put in the values you get 50. So, this particular oscillator has a quality factor of 50. If, I reduce the value of the resistance, the quality factor will increase. If, I decrease the value of the resistance that quality factor will increase. If, I increase the resistance or if I increase the damping co-efficient, the quality factor will go down, it will degrade by quality of my oscillator.

The time period T if you calculate the value for this particular oscillator in the is; 2 pi 10 to the power of minus 4 seconds.

(Refer Slide Time: 28:16)

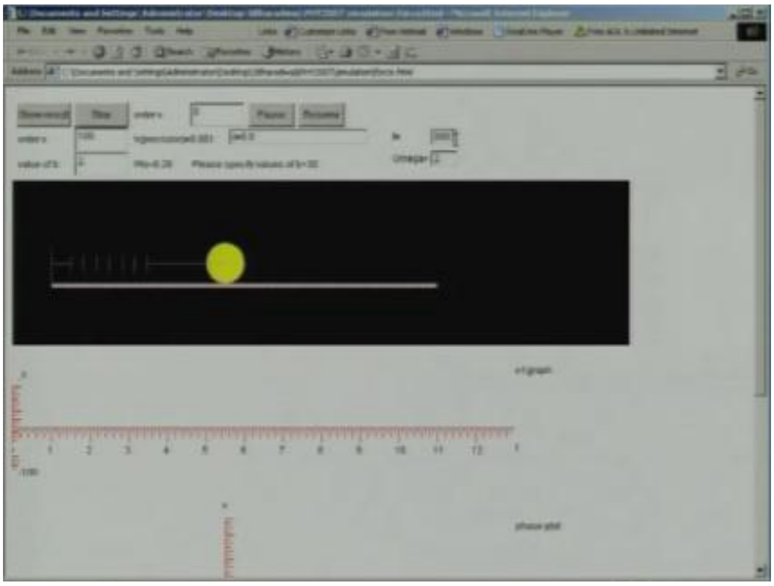


And we have already discussed the log decrement. The log decrement is a measure of how much the amplitude falls in 1 time period and it has a value beta into T, we now all the values here. So, it is 2 pi 10 to the power minus 2, you should remember. Then, the time period here is the natural time period that is, if I do not put an external force the time period here refers to that. And if you use the value here, you will get the log decrement which is 2 pi 10 to the power of minus 2.

So, in this part of the lecture, we have discussed the phenomena of resonance, in a L C R circuit which is connected to an external voltage source. So, the L C R circuit is like a damped oscillator a damped spring mass system and the external voltage source is like the external force acting on a damped spring mass system. And this circuit shows the resonance, you have resonant behavior at a particular frequency.

If, you drive the circuit at a particular frequency you get very large currents oscillating current, whose amplitude is very large. If, you drive it at other frequency is away from this resonant frequency the oscillations are smaller.

(Refer Slide Time: 29:33)



Simulation of the things that we have been discussing; so we have the same spring and mass system which we had discussed, we have been discussing. And this is the same simulation; it has the same parameters it. The spring mass system has a natural frequency omega naught equal to 2 pi. You can give the initial velocity of the particle, so we shall set the initial velocity to 0. So, let us set the initial velocity to 0. You can give the initial displacement. So, we shall set the value 200 and this parameter B allows you to set the value of 2 beta. So, let us set this value to 2.0.

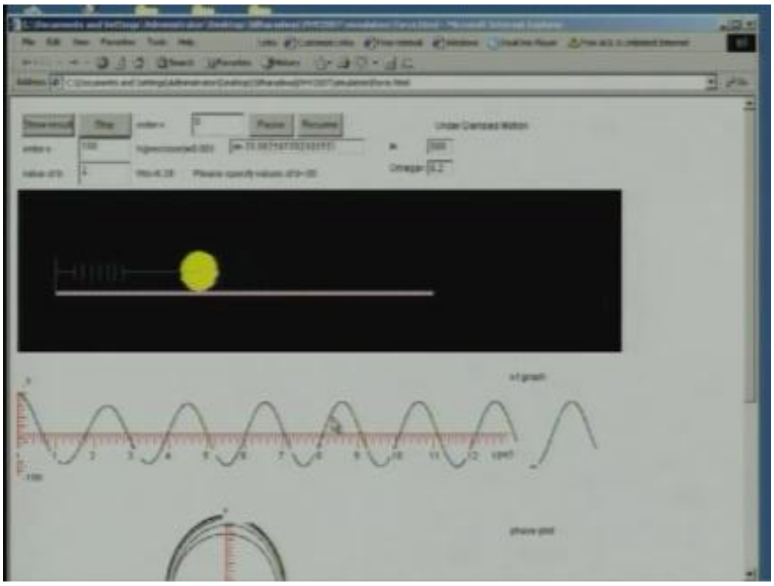
Now, this is the same simulation as that we had seen earlier for that damped oscillator. We now have 2 extra parameters which you can give. So, you can give the amplitude of the external force. Let us first see what happens, when we have no amplitude the amplitude is 0. And this is the frequency of the external force it does not matter now, but let us set a value 2. So, let us run the simulation with these parameters, there is no external force, let us see what happens.

So, you have the damped motion, it is an under damped oscillator because, the value of beta is 1 2 beta which is B is has a value 2. So, beta is 1 and omega naught is 2 pi which is 6.28, so it is under damped. So, you see this is the oscillator, the mass oscillates and the amplitude of the oscillations falls as time involves. So, these are the transients, these short lift oscillations, these are what we refer to is transient.

Now, let us see what happens if I give an external force. The external force has an angular frequency of 2. Let us set the amplitude of the external force to 1000 and run the simulation. So, notice we have the transients over here; these are the oscillations with without the external force and these oscillations decay. But, soon the oscillator settles into oscillations at the frequency of the external force, this is at the frequency of the oscillator. But this dies away and then the oscillator settles into an oscillation at the frequency of the external force. And these are the steady state oscillations, the amplitude of this oscillation is determined by the damping, by the amplitude of the external force and the frequency at which you are driving it.

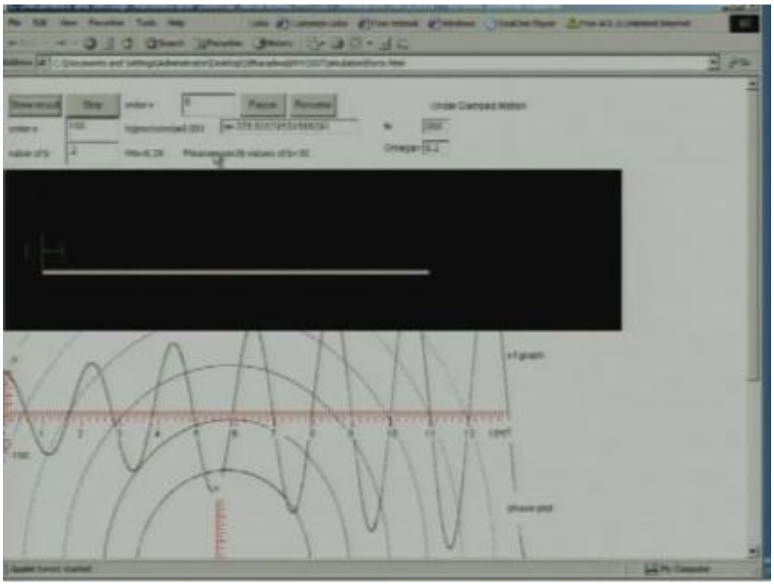
Now, at this for this particular oscillator, we are away we are quite away from the resonance. The resonance will occur when omega of the external force is equal to the natural frequency which is 2 which is equal to 2 pi 6.28. Let us make the driving frequency now, close to resonance. So, let us make it 6.2 and let us see what happens. So, we have moved to a frequency which is closer to resonance, quite close to resonance let us see what happens. So, again you have these transients.

(Refer Slide Time: 33:40)



And now the oscillator has gone into the steady state behavior, where it oscillates with the angular frequency of the external oscillator, with the of the external force and see that the amplitude notice that the amplitude is now larger, than the earlier situation where the frequency was away resonance. Now we are very close to resonance so, the amplitude of the oscillation are larger. And it has more or less reached us steady state; you can see here that it has gone more or less to an electrical orbit, where it oscillates at the same frequency as the external force with the steady amplitude.

(Refer Slide Time: 34:43)

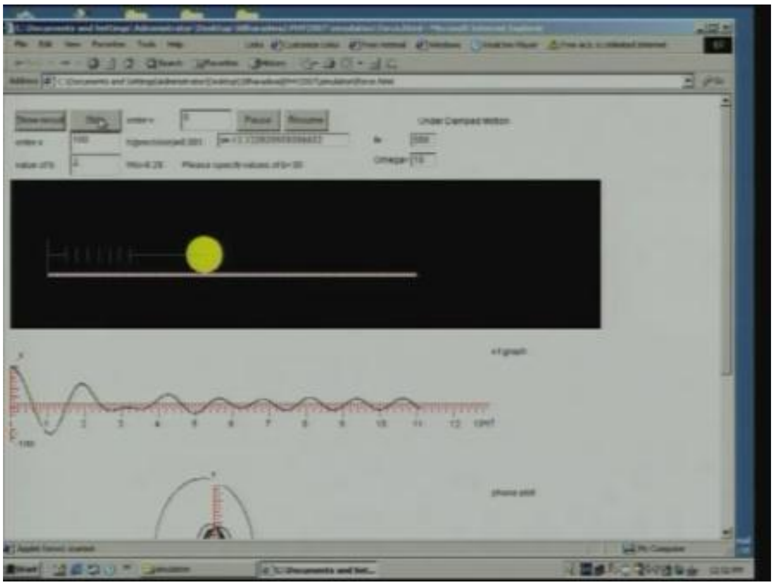


Let us next see what happens, if I reduce the damping. So, let us make the damping 0.2 instead of 2, let us make it 0.2 and see what happens. So, we have reduce the damping, the frequency is again very close to resonance, but now the damping has been brought down. So, we expect the oscillations to have a much larger amplitude because, we saw that the amplitude of the steady state of the oscillation, increase as you reduce at a damping and it becomes infinite when you have no damping at all.

So, let us see what happens, in this case where the damping is quite small. So, you see that there are still these transient, it has not gone into the steady state oscillation as yet, the amplitude is increasing. And this is what we mean by resonance, you have very large oscillations very large amplitude oscillations, when you are driving the oscillator at the resonant frequency. And these oscillations blow up as times goes on and they will reach a the go keep on increasing. Because, you have a finite damping, they will reach a the steady state, but the steady state is pretty large at this particular case and you still not reach it over here and the amplitude keeps on increasing.

So, I think we will stop it here and let us see what happens if I increase the frequency beyond the resonant value. So, I will make it 10.

(Refer Slide Time: 36:04)



So, here we have cross resonance. So, we have cross resonance and we are driving the oscillator at a force, whose frequency is considerably larger than the resonant frequency. So, you have these transient oscillations at the natural, I think we could increasing damping, the damping here is too small that, transience will continue for a long time. So, let us increase the damping to 2 B has a value 2.

So, we shall initially have these transients. The transients are as if, there was no external force, but these transients die away pretty fast. And then the oscillator goes over to the steady state oscillations, at the frequency of the external force. So, these are the transients and here the oscillator has call over to the steady state situations, where it oscillates at the frequency of the external force.

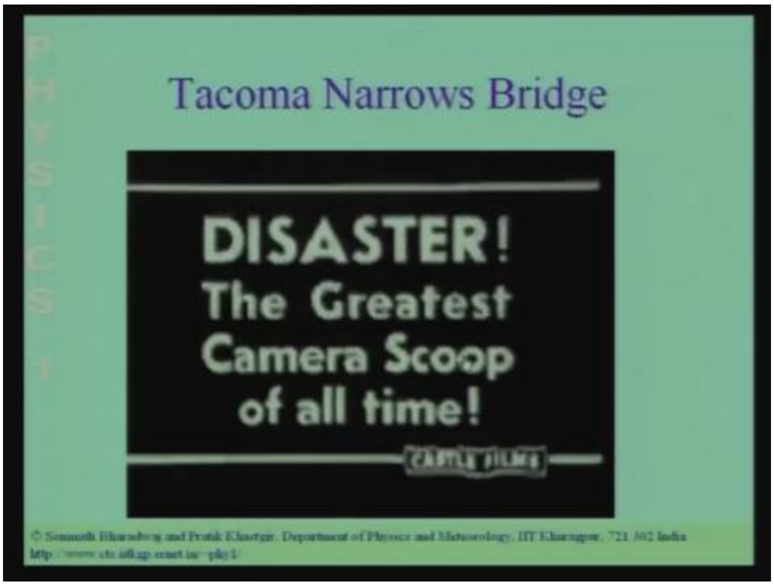
So, let me summarize what we see in the simulations. We have looked at simulations, varying the frequency of the driving force and also varying the damping. So, for low frequencies, we found that you initially have transients; when the driving force has a frequency lower than the resonance you initially have these transients. And then, the oscillator goes over; the transients die away and oscillation goes over to steady a oscillation at the frequency of the driving force. Near resonance if, you have a large damping reasonably large not over damped it is still under damped.

Then you have these transients and then you have the oscillations again at the same frequency as the external force, but the amplitude of the oscillations are somewhat larger.

And if you reduce the damping, we reduce the damping to the value B equal to 0.2 which is corresponds to beta equal to 0.1. And we found that, the amplitude you will have these transients and the amplitude keeps on increasing with time and they get larger and larger.

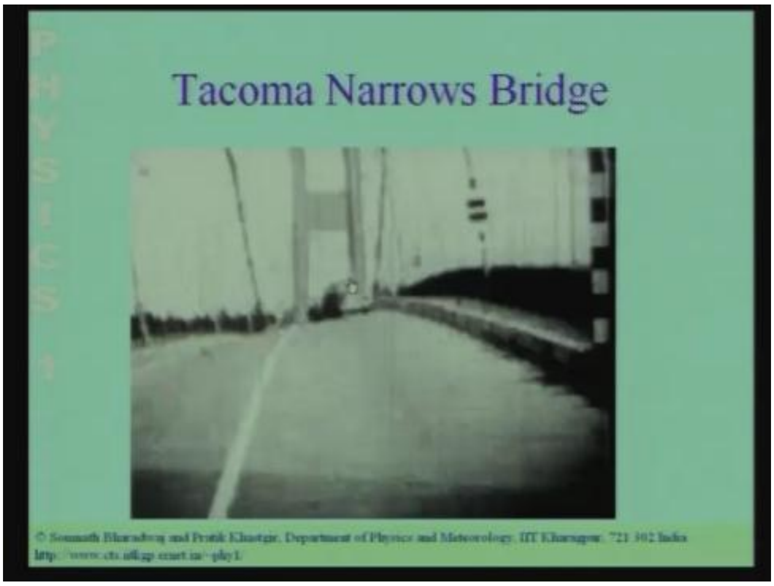
The amplitude will reaches steady state, but the amplitudes became so large that, we stop this simulations they were going outside the range of far simulation. This is the phenomena of resonance; we have these very large amplitude oscillations at then, if you drive a driving force has a natural frequency of the oscillator. And then, we went to a very large frequency which is quite larger than the natural frequency. And again there, we found that there are these transients, when we start the oscillations, but transients die away quite fast. And the oscillator settles into a steady state behavior, at the natural frequency of the external, at the frequency of the external force.

(Refer Slide Time: 39:09)



A disaster that came about because, of the phenomena of resonance.

(Refer Slide Time: 39:15)



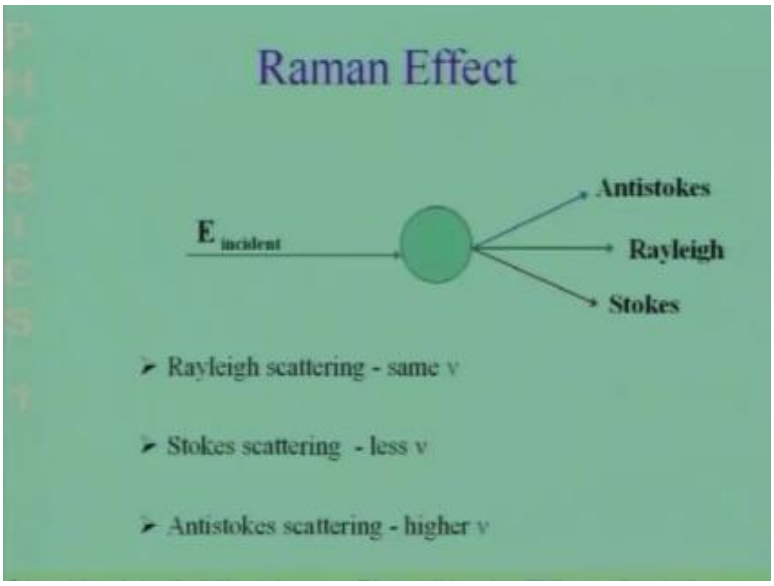
What you saw over there was a bridge; the Tacoma narrows bridge collapsing. The bridge collapsed because of resonance. There were very strong winds in the place where, the bridge was located and these winds had a time dependent oscillating part to it. So, those winds were not the wind was a not a constant wind, but it fluctuated with time. And this fluctuation in time had an oscillating component, which matched with the resonant frequency of the bridge.

The bridge had a resonance resonant frequency and a natural frequency and the time varying component of the wind match with that. And this cause a resonance in the bridge and there were oscillations in the bride and since, the forcing due to the wind matched with the natural frequency of these oscillations, the amplitude built up and finally, the bridge collapsed. Let me show you the movie again.

So, the bridge, as I have told you earlier the bridge or any other system, such complicated system for that matter, can be thought as a set of in terms of simple harmonic oscillators. And any external force like the wind can cause it, can the wind or any other external agent like that, is essentially an external force. And if the wind has a time dependent part, the wind is steady, but it oscillates as we have just, as I just showed you.

If, the oscillating part of the wind, the time dependent part of the wind matches with the natural frequency of the oscillator of the bridge then, you can have a resonance and the you will have oscillations and the bridge which will build up and then finally, the whole bridge is will collapse. So, this picture show you 1 example of where, such a thing happened. And after this people have been carefully designing bridges, to ensure that these the different modes of vibration of the bridge are all heavily damped. If, you increase the damping, then the resonant the amplitude of the resonance is can be curtailed. You can also make sure that, if the natural frequencies of the different modes of vibration of the bridge, do not match with any possible time dependent component of the wind. Let me now show you another example of resonance.

(Refer Slide Time: 42:27)



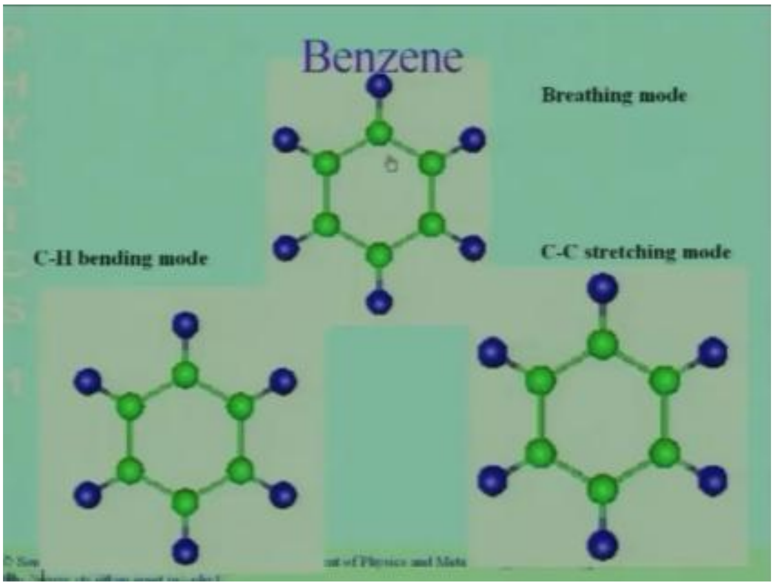
So, the next example of resonance which I shall be discussing is the Raman Effect. The Raman Effect was discovered is 1 of the most significant discoveries made by an Indian scientist Sir. C. V. Raman. It was discovered early in the 20'th century by Sir. C. V. Raman and he received the noble prize for his discovery. So, let me tell you what the Raman Effect is, what Raman noticed was as follows.

If, you shine light of high intensity. So, if you for the Raman Effect you require, light with high intensity if, you shine such a light on a liquid, on most liquids you will find this effect. Then, when the light if you look at the light that is scattered by the liquid, you will have 1 component of scattered light, which is at the same frequency as the incident light and this scattering mechanism is referred to as Rayleigh's scattering.

So, Rayleigh's scattering is the scattered light which has the same frequency as the incident light. In addition to this, there are 2 other frequency light emitted at 2 other frequencies: 1 is called as stokes and the other the antistokes. So, that the stokes scattering is; where you have light the scattered light at a lower frequency and the antistokes is where you have scattered light at a higher frequency. So, this is the Raman Effect. Raman Effect essentially is that, when you have light incident on a liquid, it can also be seen in gases, but the effect is even much smaller the effect is a small effect, which is why you need a very intense source.

So, most of the scattered light is at the same frequency as the incident light, but you also have a component at 1 at a higher frequency and another at a lower frequency. So, the lower frequency is called the stokes line the higher frequency is called the antistokes lines. So, this is the Raman Effect. And Raman Effect is a very important effect it is a very important tool, for characterizing substances because, the change in the frequency that difference in the frequency from the incident light, is a characteristic property of the material of the scattered. So, it has to do with the liquid and you can use this to characterize different substances.

(Refer Slide Time: 45:09)



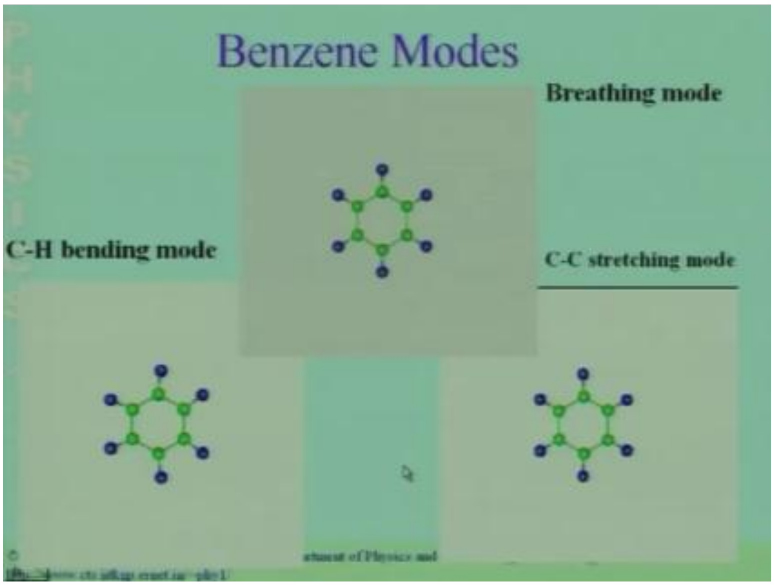
So, let me discuss a particular example; the example we are going to discuss is Raman Effect of benzene. Now, benzene I am sure you all know is C 6 8 6 and this shows you the structure of benzene molecule. So, the benzene molecule has got 6 carbon atoms and the carbon atoms form a ring like this and each carbon atom, has a hydrogen sticking out. So, the blue dots show you the hydrogen atoms. Now, you can think of the benzene molecule as a set of coupled simple harmonic oscillators. Because as a as we discussed right in the first lecture, let us just take the 1 of these bonds, this is the bond between 1 carbon and 1 hydrogen atom they are attached to each other by a bond.

Now, if I disturb the hydrogen atom or the carbon atoms slightly, then this C H bond the bond between the carbon and hydrogen, is going to get disturbed the hydrogen atom and the carbon atom and the going to get slightly disturbed. And these disturbance is from the equilibrium position are going to oscillate. So, you are going to have oscillations. So, each of these bond you can think of as a simple harmonic oscillator. If, you introduce a disturbance can think of it as a simple harmonic oscillator and in this here in this system, you have many simple harmonic oscillators, all coupled together. Because, if you disturb this you going to again disturb the hydrogen atom you going to disturb this.

So, the disturbance is going to propagate. So, it is a complicated system it is a couple set off couple set of couple simple harmonic oscillator, where each bond you can think of as a simple harmonic oscillator as a spring.

Now, many analyze such as complicated set of coupled simple harmonic oscillators we are going to discuss this in the next lecture when you analyze such a system you have something called normal modes. So, let me show you the different normal modes of the benzene molecule.

(Refer Slide Time: 47:24)



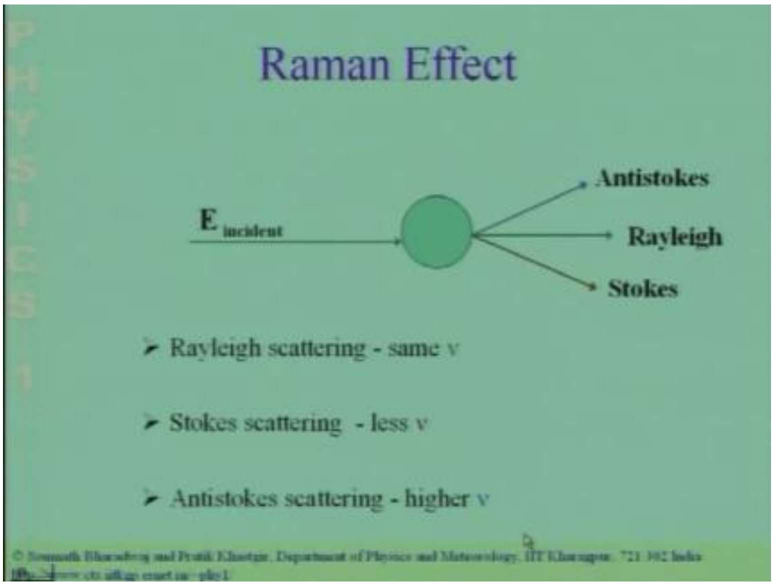
So, you have different normal mode in the benzene molecules. So, let me show you these normal modes for the benzene molecule. So, here this shows you the breathing mode of the benzene molecule. In this breathing mode, the whole molecule expands and contracts; it is like as if molecule is breathing. You can see this expansion and contraction of the benzene molecule here. Each bond gets elongated and then it gets shortened simultaneously, they all in phase.

So, let me show this to you again, this is the breathing mode of the benzene molecule. The whole molecule expands and contracts, all the bonds expand and contract simultaneously. Let me next; show you the bending mode of the benzene molecule. Noticed that, in the bending mode the carbon, carbon atoms are undisturbed, the carbon the hydrogen atom the bond between the carbon and hydrogen atom this gets bend. And when these two move together, these two also move together and these to move together and then they move a part. These two move together and this kind of can motion continues, this is another normal mode of the benzene molecule. And, then we have a third normal mode of the benzene molecule which is called the stretching mode.

So, in this in this stretching mode, the carbon bonds the bonds between the carbon atoms the yellow ones. So, the bonds between the atoms the carbon atoms, they gets stretched. And when this bond is stretched, the neighboring bonds are contracted and again this 1 is stretched, this is contracted. So, every alternative bond get stretched and very alternative bond gets contracted. And this then reverse. So, the 1 which had got the 2 atoms which came closer now, move a part and these 2 we should moved part earlier and will come closer.

So, any arbitrary disturbance of the benzene molecule can be decomposed into a some of breathing modes, bending modes and stretching modes. These are called the normal modes of the benzene molecule.

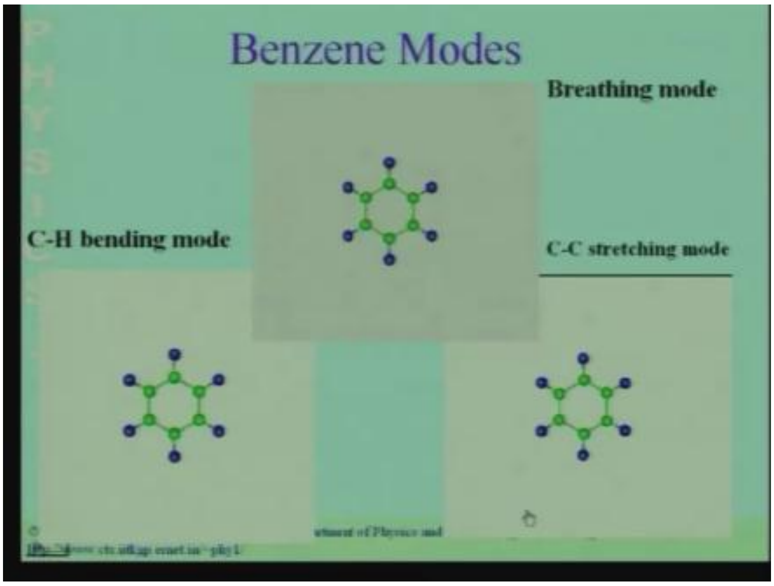
(Refer Slide Time: 50:06)



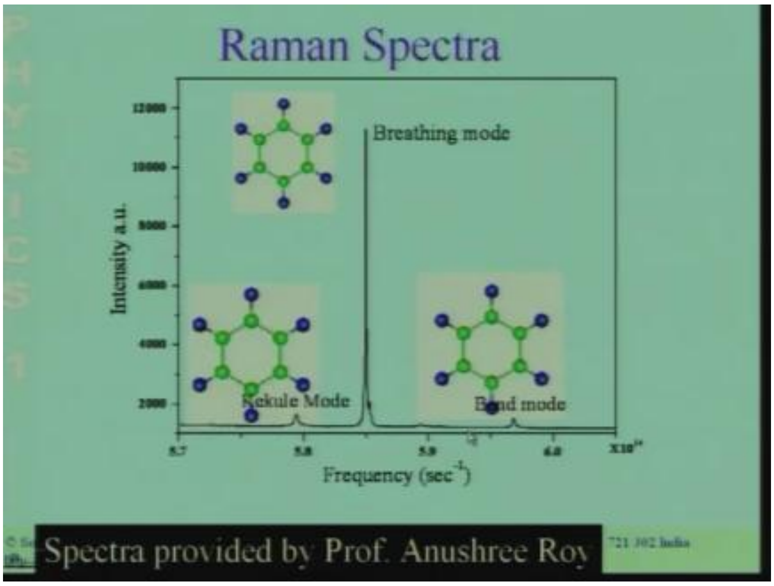
Now, when we talk about the Raman Effect the Raman spectrum of benzene molecule, the experiment that is done is as follows: you have liquid benzene in the some container over here and then on this container, you shine very intense light at some frequency nu. Let me repeat again; you have benzene liquid in a container on this container you shine some very intense light at a frequency nu and then you look at the spectrum of the light that is, scattered the light that is scattered in different directions. You do not look at the light scattered out straight because, that is going to you have large component in the same frequency, as the 1 that comes in.

But, you look at the scattered light in different directions and you will find that, in addition to the light at this incident frequency nu, you will have these stokes and antistokes line at different frequencies; 1 at the higher frequencies and 1 at a lower frequency. So, let me now show you the stokes and antistokes line corresponding to benzene.

(Refer Slide Time: 51:12)



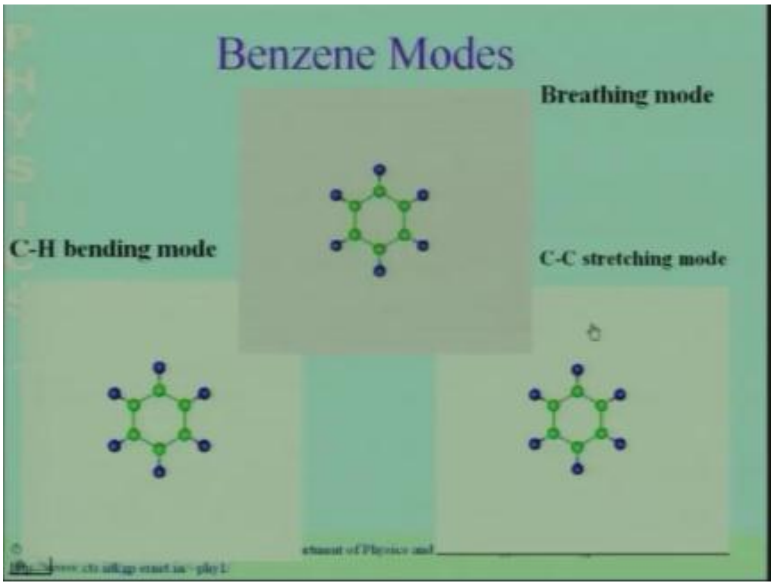
(Refer Slide Time: 51:19)



So, in this situation, that I am discussing you have light of frequency 6 into 10 to the power of 16 hertz. You have electromagnetic radiation of 6 frequency 6 into 10 to the power of 16 hertz incident on benzene. So, this shows you the strokes the strokes line, that comes out the Raman spectrum the of benzene. So, these are lines at frequency which is lower than the incident light, there incident light is at 6 into 10 to the power of 16 hertz.

If, you look at the spectrum of light that comes out from the benzene, you will find that in addition to that radiation and 6 into 10 to the power of 16. You will have radiation at these 3 frequencies 1 2 and 3 and you have the bending mode. So, what happens let me explain to you now, what the Raman Effect really, what happens in the Raman Effect.

(Refer Slide Time: 52:27)

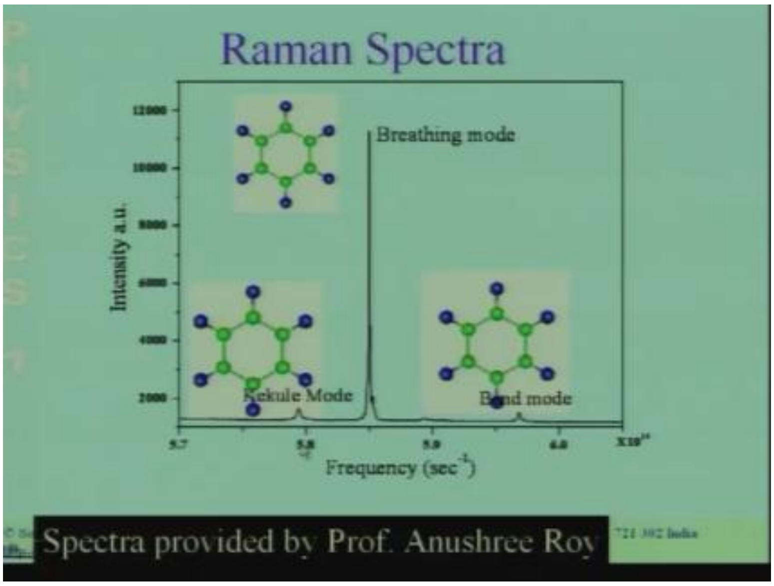


Each of these modes of vibrations, each of these modes of oscillation of the benzene molecule, have some energy associated with it. So, the bending mode, the breathing mode and the stretching mode have different energies because, these are oscillator and if you have an oscillator, they can they have energy we have learnt this. So, when the molecule is oscillating they will have energy.

Now, when I, when there is radiation incident on this benzene, some of the energy in the incident radiation can be transferred into the vibration of these molecules. So, the incident radiation can impart some of its energy into the vibration of the of the benzene molecule. And they are three possible vibrations; the breathing mode the bending mode and the stretching mode.

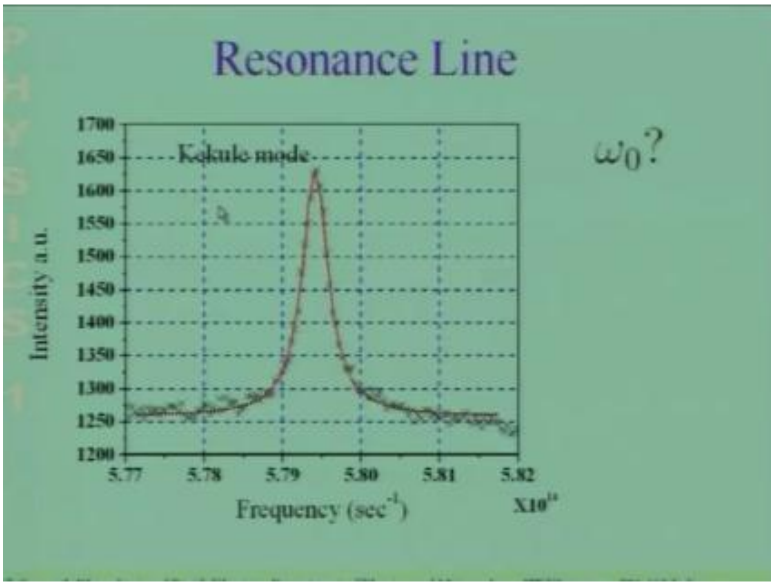
So, each and of them will take up different amounts of energy and energy, we know corresponds to a change difference, in energy corresponds to a difference in frequency. So, you get the stokes line, when some of the energy in the incident radiation goes into the energy vibrational energy of these molecules. So, if the incident light transfers some of its energy and sets the molecule into vibration, it will it frequency will go down and you will get the strokes line. The revere process could also happen; the molecule could also already be vibrating. And then, when the light is incident on it may reduce its vibration and impart some of its energy to the light to the radiation. And you will get an increase in the frequency or the anti stokes line.

(Refer Slide Time: 54:15)



So, here I am showing you the stokes part of the spectrum and you have a different frequency corresponding, to each of these bending the stretching and the breathing modes.

(Refer Slide Time: 54:21)



Now, let me show you 1 of these spectra in detail. So, this shows you the Kekule line. The Kekule line refers to the stretching mode of benzene, here the carbon atoms move away and from 1 another and then they come towards 1 another. This is called as stretching mode or the Kekule mode. And this shows you the spectrum in detail. The point which I wish to make here is that, if you look at the spectrum you can immediately recognize that, it is a Lorentzian. It is not very surprising that it is a Lorentzian because; you see that the light that this line originates from an oscillator. And in oscillators, the response of the oscillator is Lorentzian of a forced damped oscillator the Lorentzian.

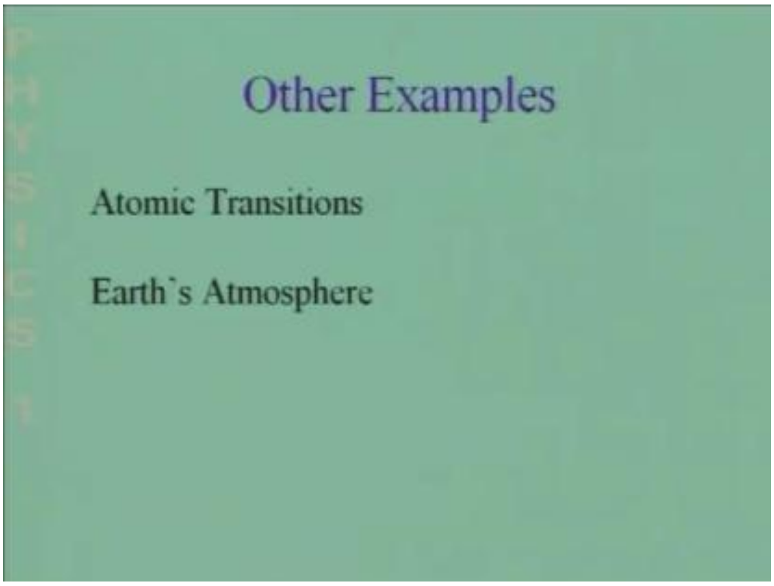
So, you think of this Raman spectrum as arising. You can think of this benzene, the each mode of the benzene as being damped oscillator. And the radiation being the external, the transfer of a energy from the radiation being the external forcing. So, the response is going to be Lorentzian, the response at the different frequency is going to be a Lorentzian, so it is not surprising. And you find this kind of Lorentzian line profile, all such spectral lines, whether it be from a atomic transition or the Raman spectrum that you see over here or any other spectral line. You always find this kind of a Lorentzian profile. There could be other effects which could change the shape of the profile, but this Lorentzian nature is always going to be there.

Now, let me put some problem in front of you. The first problem is that, determine omega naught the natural frequency of this oscillator, what is the natural frequency of this line? What is the where does peak? What is the resonant frequency of this Raman spectrum? The next question is to determine, the full width at half maxima of this particular spectral line. Now, point which you should note when determining the full width that half maxima is that, you should not take the value just the value at this point and find out where it falls to half and then take the width over there, that will not give you an estimate of the full width that half maxima. That is because, the curve does not spectrum does not start from 0, it has an offset which could due to some back ground radiation which might be there.

So, you should take the difference between this value and this value peak value and find the value of where find place, where this difference becomes half where you have half of this difference and that will give us the full width that half maxima. And then from this you can also determine the quality factor. So, these are problems which I am going to leave for you to solve.

So, in today's lecture the main point that, I have try to make is that, resonance is the very important is a very important phenomena. It occurs in a large variety of situations. And I have shown you a few examples, electrical circuits. Then I showed you the example of the bridge, where the bridge collapsed due to a resonance. And then I showed you spectral lines and I told you that spectral lines have a Lorentzian profile which is characteristic of a resonance.

(Refer Slide Time: 58:14)



There are various other situations. So, all atomic transactions, I have already told you this have a Lorentzian profile; the earth's atmosphere. The earth's atmosphere can be thought of as a simple harmonic oscillator, under the if you have an external force acting on the earth's atmosphere as a whole, you can think of the response of the earth's atmosphere as that of a simple harmonic oscillator. And again there, you will find that there is a natural frequency associated with the earth atmosphere.

The earth atmosphere is constantly being driven by an external force. This is tidal force due to the sun and the moon and you can think of the response of earth's atmosphere, that of a simple harmonic oscillator. And again you can have a phenomena of resonance, if the external frequency matches with the natural frequency of this atmosphere.

So, let me bring today's lecture on resonance, to a close over here and continue on a new topic tomorrow.