Physics–I: Oscillations & Waves

Prof. S. Bharadwaj
Department of Physics & Meteorology
Indian institute of Technology, Kharagpur

Lecture - 05
Oscillator with External Forcing-II

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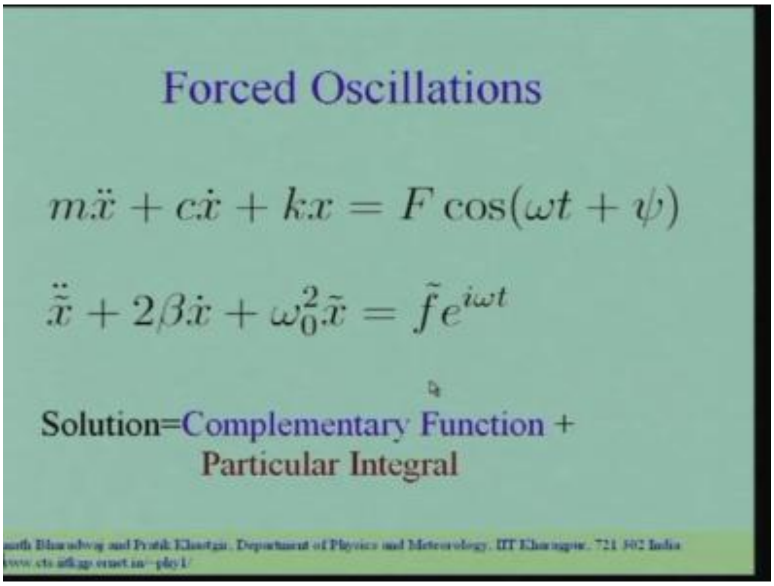

In the last class, we were discussing forced oscillations and we have started the discussion, by considering external force acting on an oscillator without damping. We then moved over to introducing damping into the problem and this is what we going to discuss in today’s class. So, this is the equation which governs the motion of an of a damped as oscillator, in the presence of an external force. The external force is itself oscillating with an angular frequency omega, which is different from the angular frequency; the natural frequency of the oscillator.

Now, that the equation over here, which governs the motion, can be written in this form, which we had discussed in the last class. Here this is the acceleration of the mass; this is the term corresponding to the damping, this is the force due to the spring and this is the external force. Now, the solution of such an equation as I have told you has two parts: there is the complementary function and there is a particular integral.

Now, the complementary function; is the solution when we ignore the term force on the right hand side. And we have studied the complementary function in some detail in the past few lectures. And we saw that, for a damped oscillator under all circumstance; if you the complementary function, the solution when you have no the external force, the solution decays exponentially with time.

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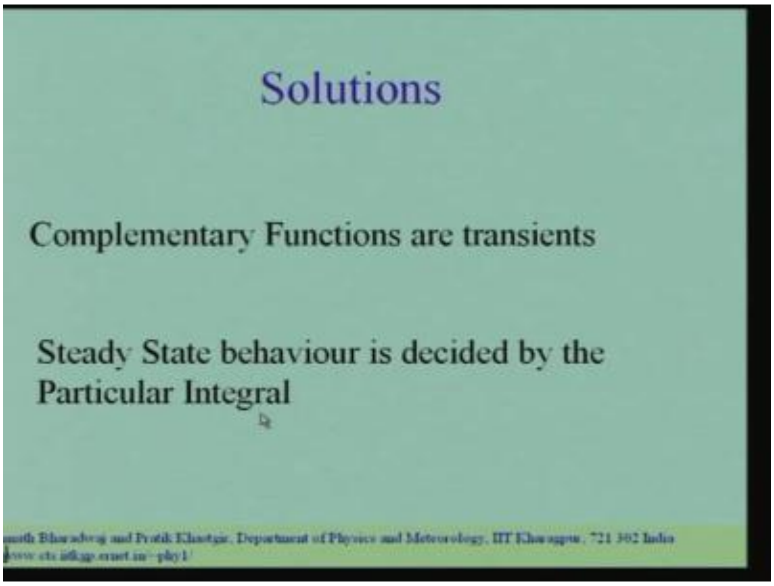

So, if you are looking at late time behavior, the complementary function gives you only transient’s short lived solutions. So, if you looking at the late time behavior, you have to look at the particular integral. This is the term; the particular integral is the part of the solution which survives at late times.

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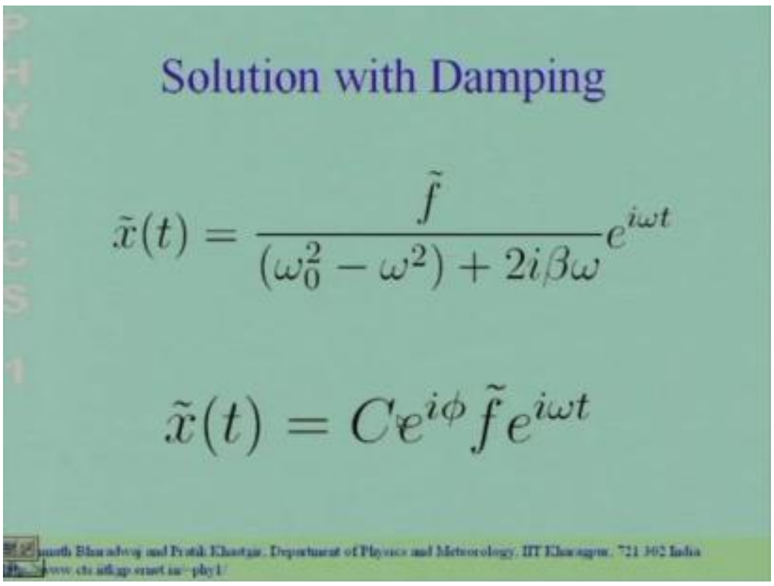

We had worked out the particular integral and this was the solution, which we had obtained in the last class. So, let us resume our discussion of this particular solution, which is the particular integral of the differential equation, which I had just shown you. So, here f tilde is the amplitude of the external force; is the complex amplitude of the external force divided by the mass of the particle and it also has the phase of the external force inside it.

So, the external force is oscillating at a frequency omega, which is what is given here and the displacement is related to the external force these; 2 terms over here are the external force, the displacement x t is related to the external force through these coefficients which occur over here. If, there is no damping present, then the coefficient is real. If you have damping then the effect of the damping is that, it introduced an imaginary part in the co-efficient, which relates the displacement to the force and now have a complex number, relating the displacement to the force. This complex number can also be written in the form of an amplitude C and a phase phi.

So, the same relation can also be written like this; the displacement is related to the force through a amplitude C; which is the mod of this number here 1 by this number here. And you have a phase which is the relative phase between the displacement and the external force.

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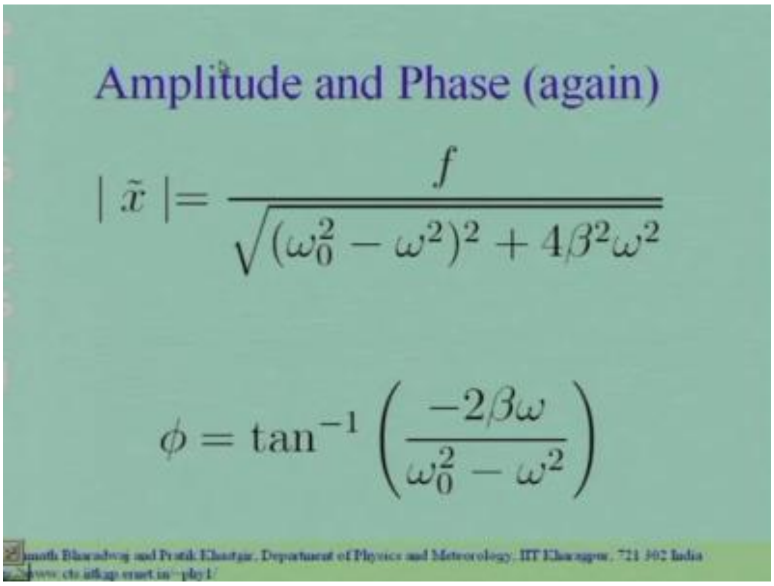

So, you can use this expression to calculate the amplitude of the oscillations, which is the magnitude of that complex number and the amplitude of the oscillations is f, where f is the amplitude of f tilde the amplitude of the force divided by this number over here; omega naught square minus omega square the whole square of this plus 4 beta square omega square. So, this gives us amplitude of the oscillations and you have the phase of the oscillations which is given over here. So, the phase of the oscillation is tan inverse minus 2 beta omega by omega naught square minus omega square.

So, let us first discuss the behavior of the relative phase of the oscillations, with respect to the phase of the force. So, the relative phase of the oscillations with respect to the force is; what is given by this phi. And the relative phi is the tan inverse of minus 2 beta omega by omega naught square minus omega square. So, let us discuss the behavior of this relative phase. The first point to note is that, in the limit of very small frequency as omega goes to 0; you have this coefficient over here vanishing.

So, the coefficient over here, whose tan inverse gives you the relative phase, also goes to 0 as the driving frequency omega tends to 0. So, if you have a very slow, slowly time varying force as a force which is oscillating very slowly, then the phase between the displacement and the force is tends to 0. So, they tend to oscillate at nearly the same phase. And this I had told you in the last class, can we interpreted in terms of the situation that occurs when omega becomes exactly 0? When omega is exactly 0, we have a constant force, if I have constant force it causes the spring to extend it and the extension of the spring is exactly as in the same direction as the constant force.

So, now if you vary the force very slowly, the extension of the spring will precisely follow the nature of the force. If, the force acts in this direction, the spring will be compressed, if it is acting in this direction, the spring will be pulled and they the motion exactly follows the force. This is what happens, when you have a very slowly time varying force. So, this is what you get when we take the limit of omega going to 0, the phase is 0.

Now, if you increase the frequency slightly, notice that the denominator is still positive, but the numerator is negative. So, if you increase the frequency slightly, you have tan inverse of a negative number, tan inverse of 0 is pi by 2 is a tan inverse of 0 is 0. Now, when you increase omega slightly, then the phase phi is the tan inverse of a negative number, which also is a negative number. So, as you increase the phase slightly, the as you increase the frequency slightly, the phase changes from 0 and it becomes negative. So, there is a negative phase relative to the, of the motion relative to the external force.

Now, when omega is equal to omega naught, if you keep on increasing the driving frequency, then omega becomes equal to omega naught and the denominator become 0. So, you have tan inverse of infinity and it approach tan inverse of minus infinity and a tan inverse of minus infinity is minus pi by 2.

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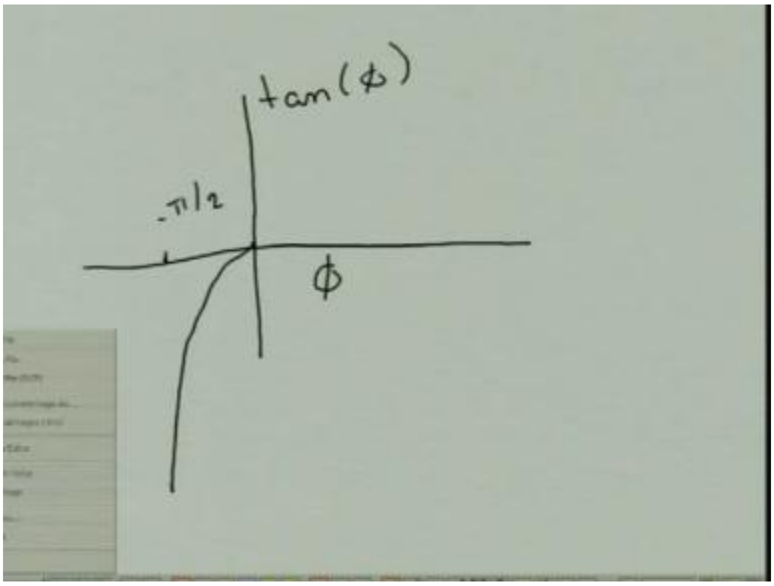

So, if you look at the behavior of tan, this is the phase phi and this tan phi. If, you look at the behavior of this in our situation, tan phi starts from 0 and then it becomes negative. So, phi starts from 0 and it becomes more and more negative and then when it reaches pi by 2, it blows up.

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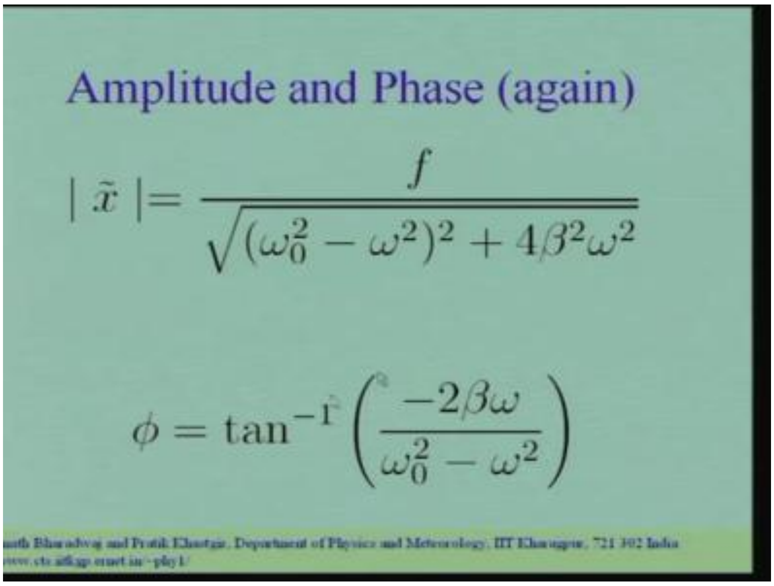

So, this is the value minus pi by 2 and then; so when omega is equal to omega naught, the phase has a value minus pi by 2, this whole thing blows up and then when omega crosses omega naught, if omega crosses omega naught it is more, then the denominator is negative the numerator is also negative. So, you have tan inverse of a positive number and you start with the very large number because, the denominator is extremely small and then if you keep on increasing omega, it slowly tends to 0 again.

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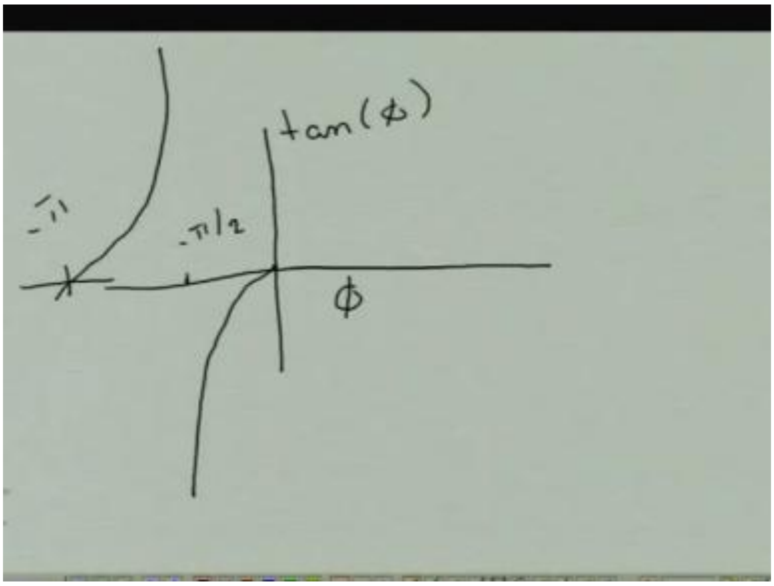

So, if you increase omega, you effectively and you when you cross omega equal to omega naught, you effectively start from here and then you go all the way to a phase of minus pi at a phase of minus pi tan of phi is 0.

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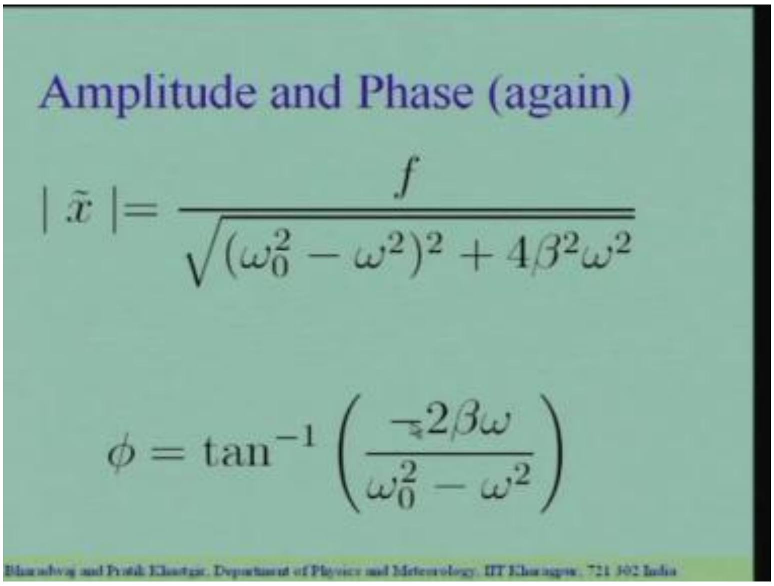

So, you see that the phase starts from 0 and extends all the way to minus phi as the frequency tends from 0 very low frequencies to very high frequencies.

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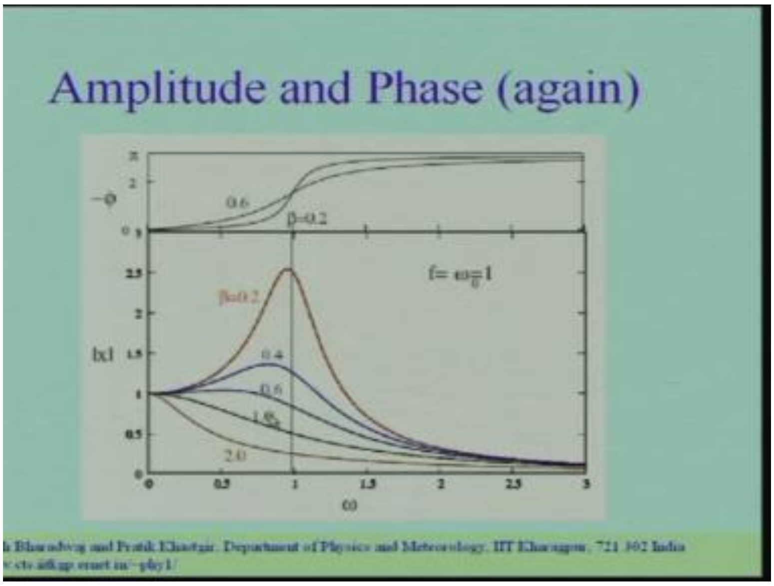

This is what is shown over here; this curve over here is the upper curve shows you minus phi that the phi is the relative phase between the oscillations and the external force. So, what I show here minus of that minus phi and I show this; as I vary the frequency of the external force. The oscillator is such that, it has a natural frequency omega naught equal to 1. So, if there was no damping and if the oscillator was left alone to oscillate, it would oscillate at a natural frequency of omega naught equal 1.

Now, we saw in the last class, that at frequencies less than omega naught, the oscillation was in the same phase as the force and at frequencies more than omega naught, the oscillation was exactly minus pi out of phase with the force. Today we have understood, why minus pi why not plus pi. And in the case of no damping, there is a sudden jump in the phase from 0 to minus pi at the natural frequency omega naught equal to 1 in this case.

So, when there no damping the phase is 0 here, here and then suddenly jumps to minus pi and then remains constant. When you introduce damping and you plot the function which I had shown you just a short file ago tan inverse of all the coefficients over there, then what you find is the phase again start from 0 at very low frequencies. But, instead of having a short jump at there is an frequency at omega naught equal to 1, the phase gradually minus phi gradually rises. It reaches a value pi by 2 over here, so minus phi is exactly pi by 2 out of phase, as the oscillations at the resonant frequency, at the frequency omega naught equal to 1 in this case. And then it crosses over and tends to pi as the frequency the driving frequency is increased. Also notice, that as you increase the damping, these transition becomes more smoother. So, if I no damping there is a very sudden transition in the phase. And as I increase the damping, the transition from 0 to pi the transition in minus phi from 0 to pi becomes more and more smoother. Let us next look at the amplitude of the oscillations.

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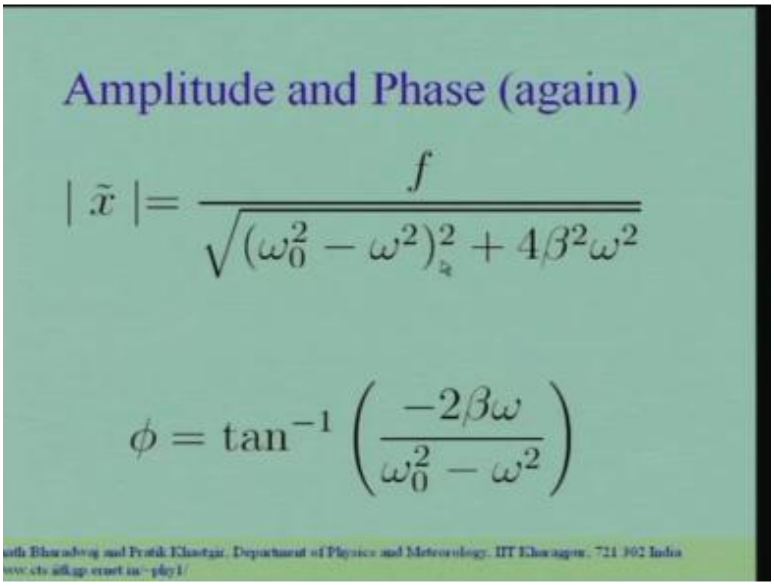

So, this expression over here, gives us the amplitude of the oscillations. I had told you in the last class and we had also seen that, in the absence of damping you could check for yourself that in the absence of the damping, if omega is equal to the natural frequency, the amplitude of the oscillation blows up, you have infinitely large oscillations. Now, the first point to note is that, if you introduce damping, the oscillation amplitude of the oscillations is finite throughout.

So, damping ensures that your oscillations are finite. And in all situations you have damping, so you do not in reality, we really do not encounter infinite oscillation of infinite amplitude, you have finite amplitude oscillations. So, damping ensures that your oscillations are finite and this is what you see here.

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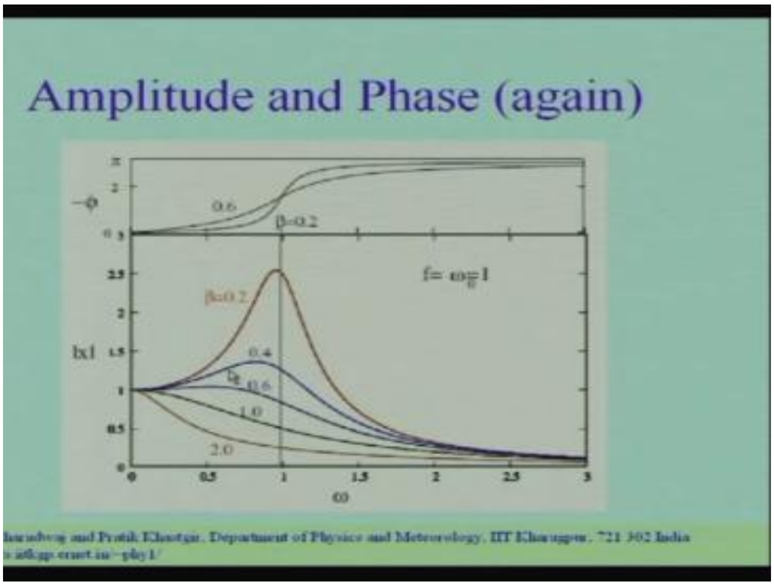

So, this shows you the amplitude of the oscillations as you vary the frequency of the driving force. The resonance occurs, when the frequency of the driving force is equal to the natural frequency of the oscillator, which is omega naught equal to 1. So, at 1 the resonance occurs. And if there was no damping, you would have infinitely large oscillations here, but damping ensures that the oscillations are finite. Another point to note is that, the more the damping the less is the maximum value the peak value of the oscillations. And you have a peak in the amplitude at the resonant value, as long as you are in the under damped regime, in the over damped regime where, beta the damping coefficient is more than omega naught.

So, you find that there no peak, there is no peak in the amplitude as you vary omega. And the amplitude falls monotonically from, the low frequency value to the values at high frequencies. So, there is a change, when you go from under damped situation to be over damped situation. In the under damp situation you have a resonance, but the resonance is now finite, the amplitude of the resonance is now finite and the amplitude falls if you increase the damping. Not only does amplitude fall, but the width of the resonance the width of the response curve, as the function of the frequency gets broader and broader as you increase the damping.

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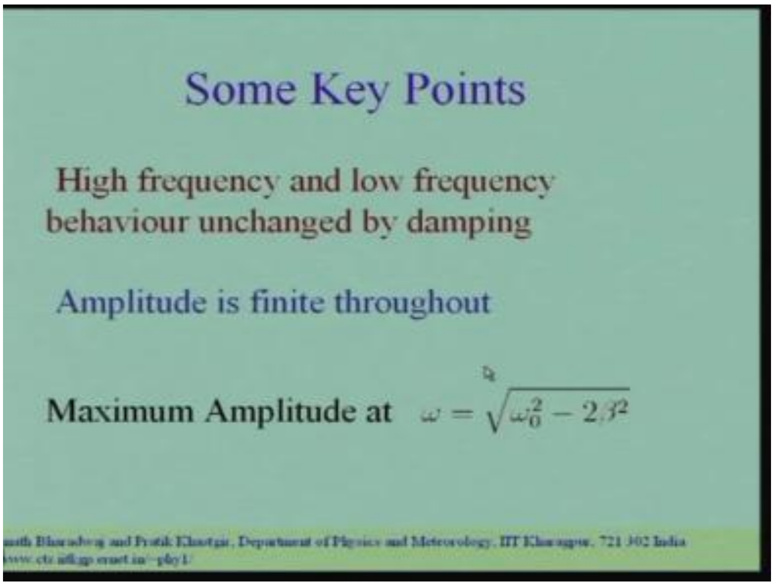

So, let me summarize the key points of what we have seen. The first key point is that, the behavior of the oscillator of a forced oscillator at low frequencies, at very low frequencies and at very high frequencies and both these extremes, the behavior is not affected if I introduce damping. So, whether there is damping or not, really does not affect the behavior at the very low frequencies where, the oscillations exactly follow the force. And you have the amplitude of the oscillation as f by the spring constant K. This is the mass; this is the stiffness dominated regime which we have discussed in the last lecture.

At high frequencies, the oscillations are exactly pi out of phase minus pi out of phase with the force. And in this regime very high frequencies you can ignore the spring from the discussion and this is called the mass dominated regime. So, whether you have damping or not, these 2 features remain unchanged. The effect of damping is that, it gives you finite amplitude, at the resonant frequency; at the frequency of when the forcing frequency and the natural frequency of the oscillator match.

If, there is no damping, you have oscillations of infinite amplitude. Damping ensures that, the amplitude of these oscillations are finite. The more the damping, the smaller is the amplitude of these oscillations of the at the resonance and wider is the response to difference frequencies. Now, you can easily check by differentiating.

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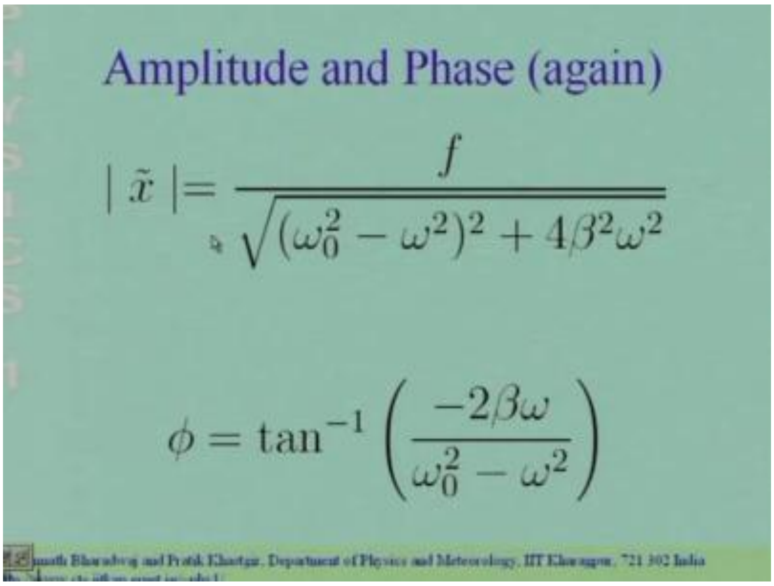

By differentiating this expression for the amplitude, it will be easily determining the value of the angular frequency omega, where you have a peak in the amplitude of the displacement.

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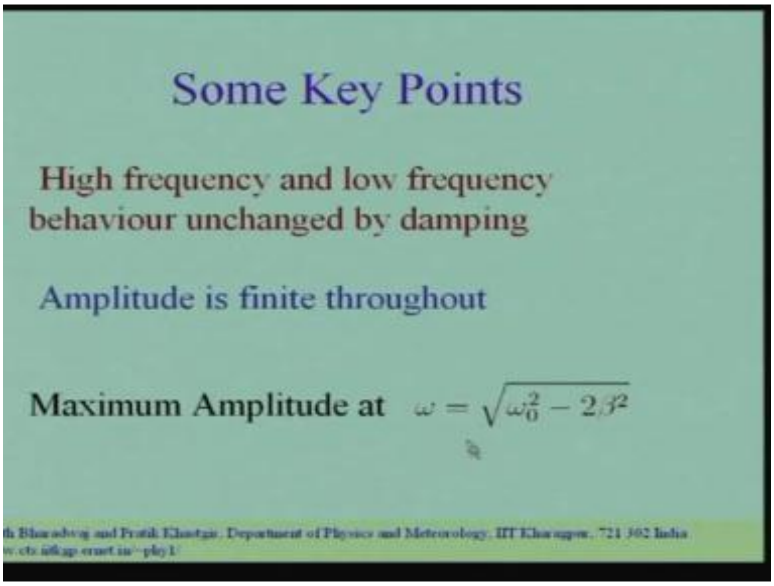

And this turns out to be at the value, this is given over here. So, you have a maximum amplitude at omega is equal to the square root of mega naught square minus 2 beta square. So, it is not exactly at the natural frequency. The maximum amplitude, the oscillations have a maximum amplitude not at exactly the natural frequency, but at a frequency which is slightly shifted; at an angular frequency which is slightly lower than the natural frequency of the oscillations. And the frequency is square root of omega naught square minus 2 beta square.

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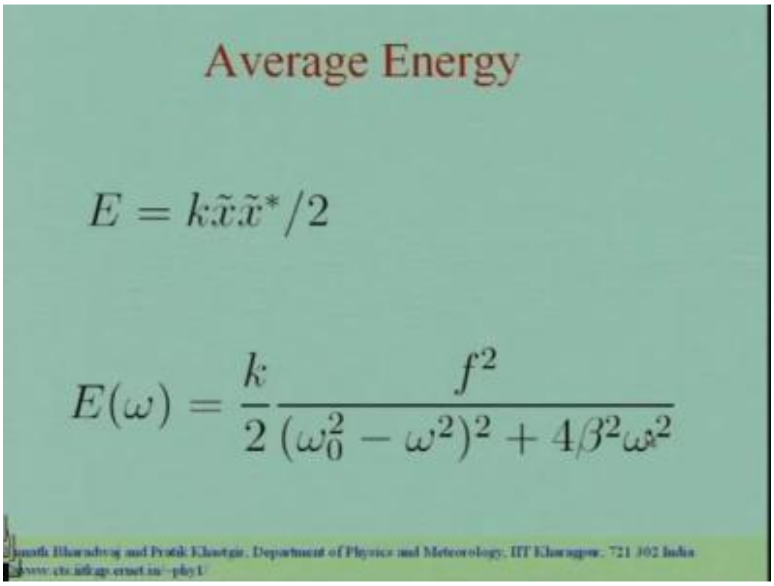

Now, let us now move over to another feature of this and of these oscillators and let us calculate the average energy in these oscillators. The average energy which is stored in these oscillators in the oscillator, when I force it can be calculated in the complex notation using k into x into the complex conjugate of x x star and dividing it by 2. So, we have seen in the first lecture, that you can calculate the average; by average we need the time average energy of the oscillator, as it oscillates at the frequency of the external forcing. So, if you calculate this that is, if you calculate k into x into x star by 2.

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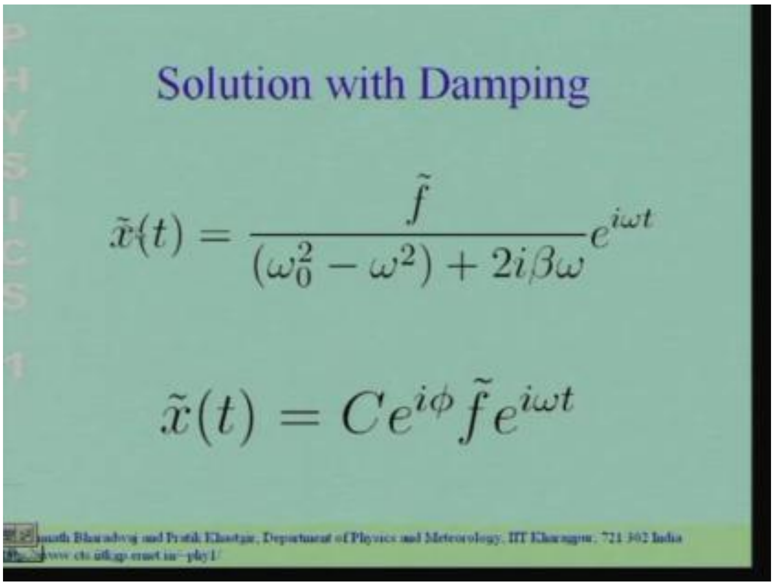

So, you have to take this expression for x; the complex x and multiplied with its complex conjugate. If, you multiplied with its complex conjugate, you will get f tilde star, this multiplied by f tilde star and this will be multiplied by its complex conjugate.

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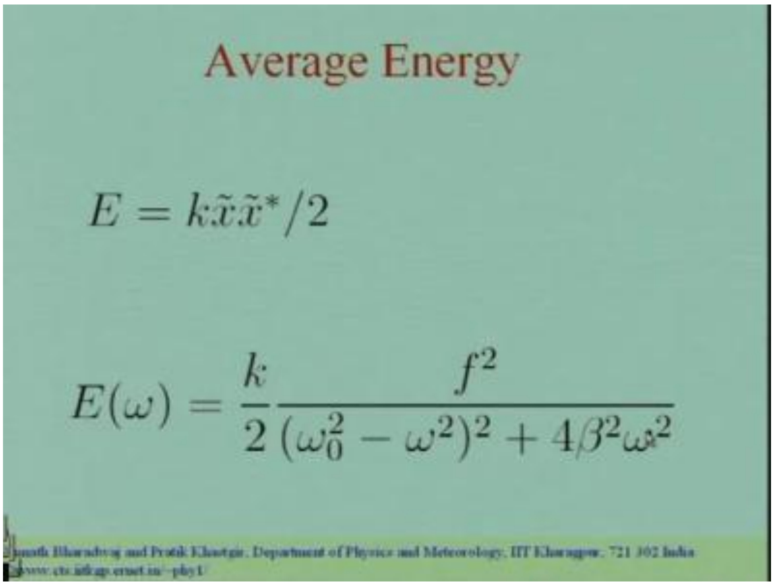

So, what you have is given over here. So, you have the amplitude of f of the force squared, remember this is the amplitude of the force divided by the mass; f is the amplitude of the force divided by the mass so, you have the square of f. And in the denominator, you have the square of the denominator that occurred, in the expression for x the amplitude x. So, this is what you get. The maximum energy is stored at exactly the same frequency where, you have the maximum displacement.

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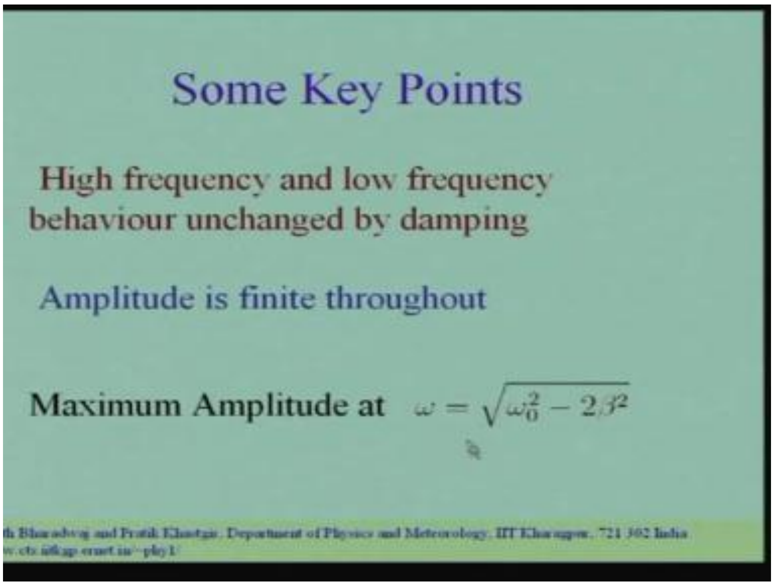

So, the maximum energy is also stored at this value of the angular frequency.

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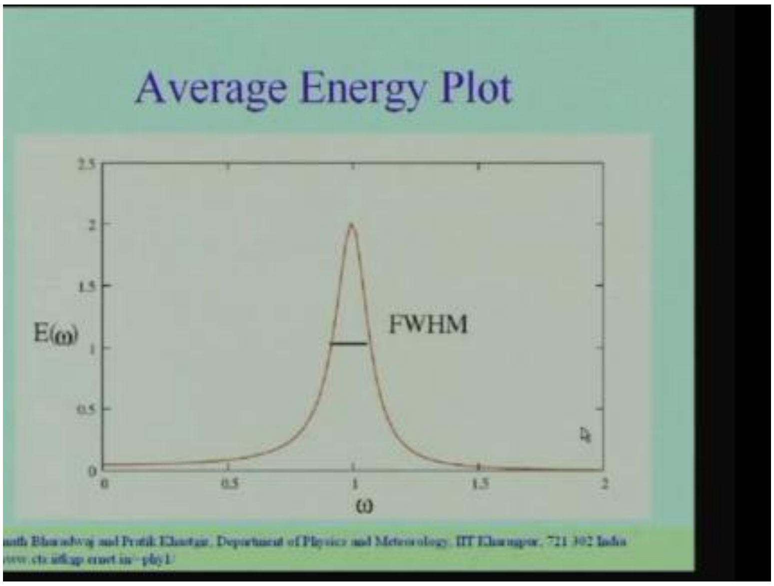

And let me show you A plot of the average energy; so this shows you a plot of the average energy stored in the oscillator. If, I work to drive the oscillator with an external force of constant amplitude, 1 with the constant amplitude and if, I were to slowly vary the frequency of the external force and for every value of the frequency, if I were to measure the energy in the oscillator, I would get a curve given which looks, I would get a curve which would be governed by this expression.

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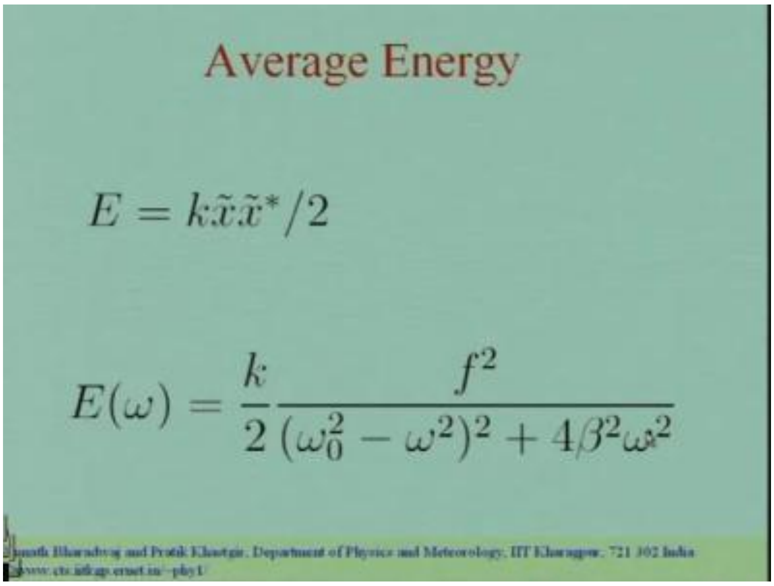

And it would look like this.

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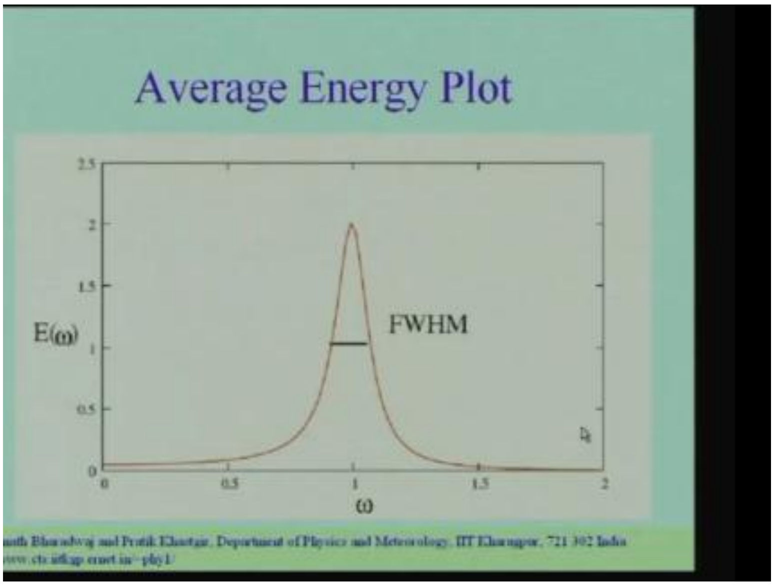

So, the maximum the oscillator would have the maximum energy when, the external force has a frequency omega which is very close to omega naught slightly less than omega naught, it would be maximum, when the frequency of the driving force is very close to omega naught. I have just showed you the value of short time ago. So, at that particular frequency, the energy of the oscillator would be maximum. For other values of energy, the energy that is stored in the oscillator would be less and you have a peak.

So, this is what you call a peak. So, you have a peak in the response, energy response of the oscillator to forces of the different frequencies. So, you have maxima of a certain value and then it falls of as you go away from that value. Such peaks in the frequency response are quiet common phenomena in nature. There also of great interest. If you are an engineer, you have such peaks occurring quiet common commonly.

Now, is it often now great interest to quantify the width of this peak. So, we use something call the full width at half maxima FWHM, to quantify the width of such peaks. So, let me explain to you, what we mean by the full width that half maxima. So, let us ask first ask the question. What is the peak value, what is the maximum value of this peak? So, the peak the maximum value which this peak has is the 2. And this occurs at frequency omega; which is the square root of omega naught square minus to beta square. So, that frequency I have the maximum value and in this case the maximum value is 2. So, the maximum energy stored in the oscillator, is 2 at that particular frequency.

Now, if I look at some other frequency, the energy stored in the oscillator falls. Now, the question is at what frequency difference, so how far I do I have to move away from the maximum frequency value so that, the energy stored in the oscillator falls by half? So, in this curve the maximum value of the energy is 2 and it occurs at this value of the frequency.

So, it occurs over here and now the, is occurs over here at this value of the frequency.

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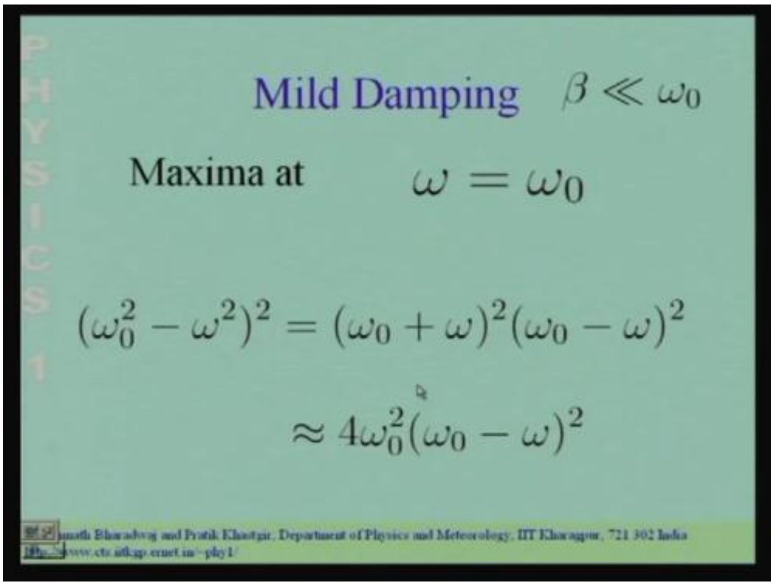

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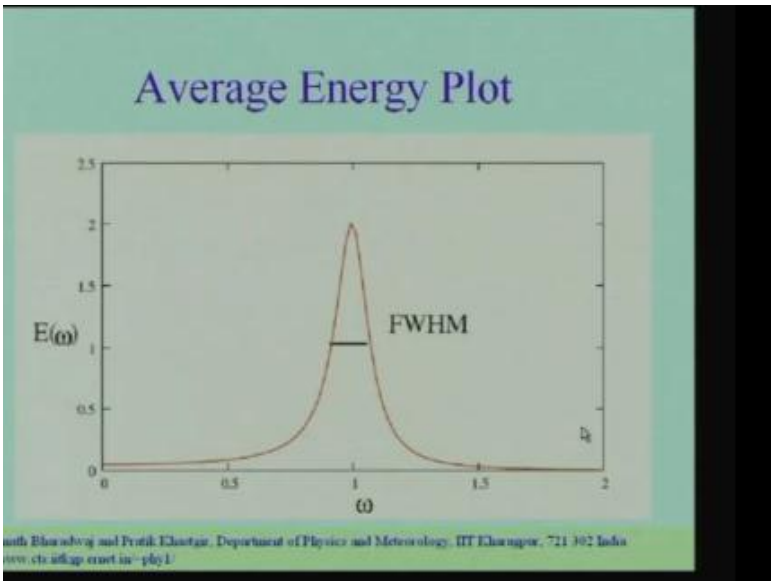

And now the question is how much you have to change the frequency, so that, the value of the curve falls to half the maximum value? So, in this case half the maximum value corresponds to 1. So, how much do I have to go away in frequency, from the position where the maxima occur, so that, the value falls 2 1? So, you can see in this curve that, you have to move away from here to here. So, it falls to half the value if you move from here to here or from here to here. So, the full width; so this is the full width of the curve, at the value, where it is half the maximum value.

So, this tells you some idea of width of the curve and this is what is called as full width. So, this is the full width, at half the maximum value at the value 1 which is, half the maximum value

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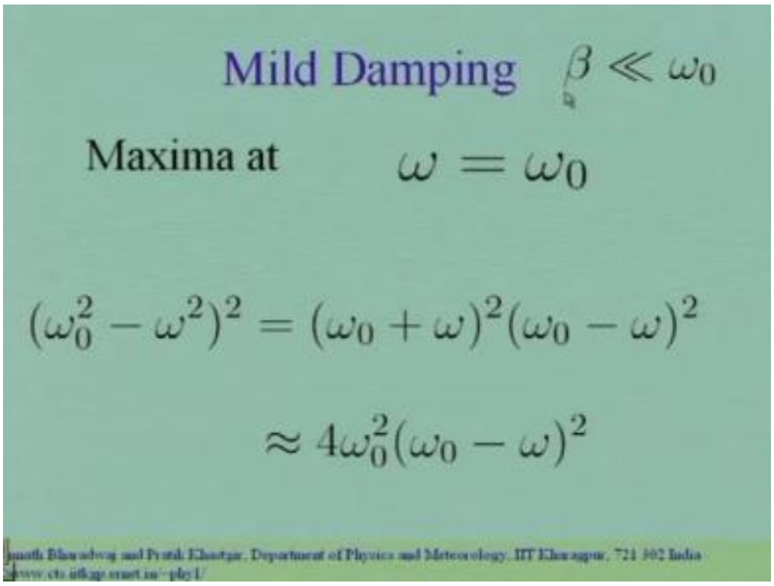

So, let us calculate the full width at half maxima, for this particular curve and we shall do it in the mild damping regime. By mild damping what we mean is; where the damping coefficient beta is much smaller than the natural frequency omega naught. If, beta is much smaller than the natural frequency omega naught, then the peak in the energy occurs at approximately the natural frequency. So, the resonant, so the omega equal to omega naught is, approximately where the peak in the energy verses omega curve occurs. We can see that from here.

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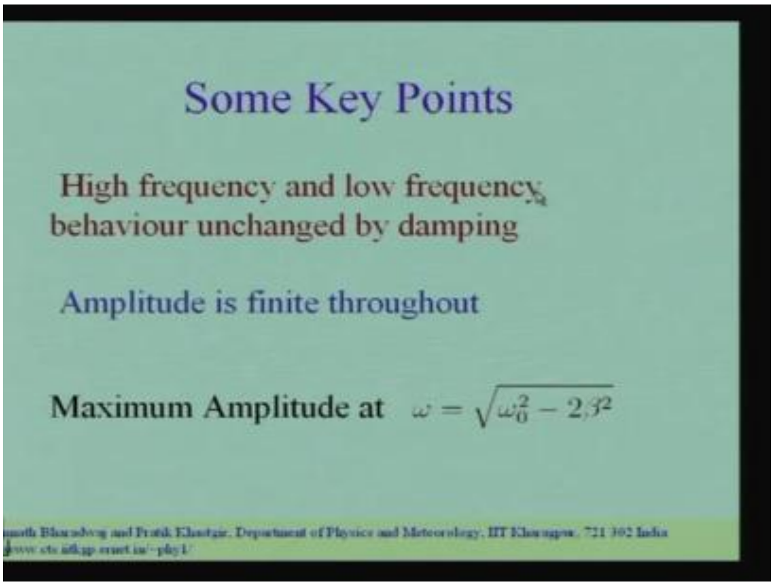

So, omega and omega naught are both approximately the same if, beta is much smaller than omega naught.

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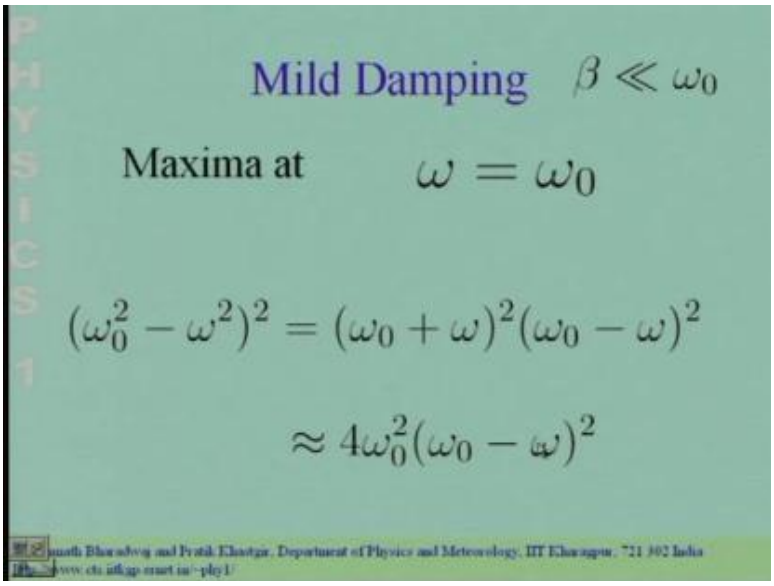

So, you can say that the maxima occurs, at omega equal to omega naught and you can approximate, you can first of first thing you can do is; you can write omega naught square minus omega square whole square as omega naught plus omega whole square omega naught minus omega whole square. And you can approximate omega not equal to omega because, that is around the peak. So, you can and the differences are quiet small around the peak. So, you can approximate omega naught equal to omega and what you get is; 4 omega naught square omega naught minus omega square.

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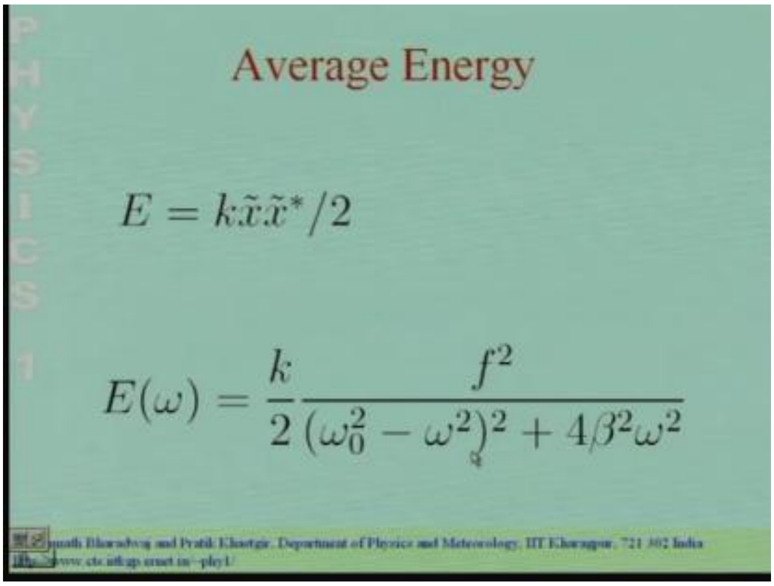

So, we can take this and put it in to the expression.

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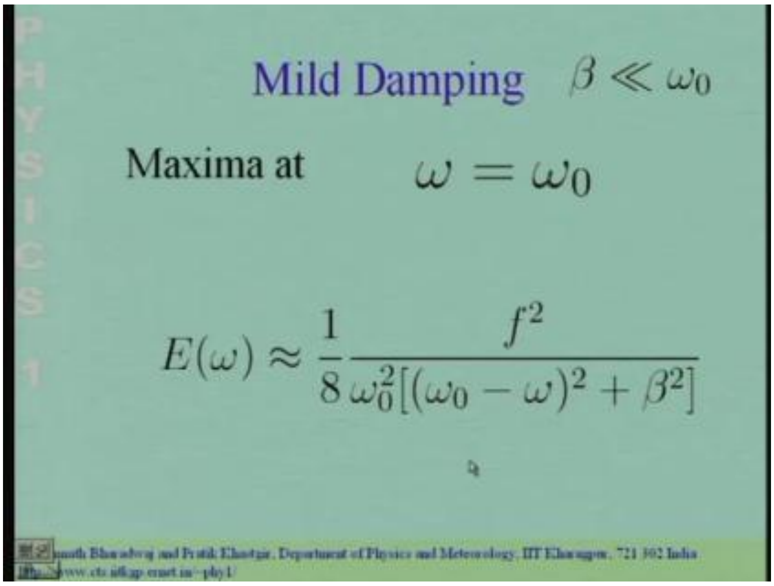

And what you see is that, the expression for the energy I have, note that I have set the spring constant K equal to 1 here. So, the expression for the energy as a function of the frequency now, becomes this. So, very close to the peak very close to omega equal to omega naught, the expression for the energy as a function of the angular frequency, of the forcing oscillator of the force can be expressed like this.

Now, let us ask the question. Where does, the maxima of this curve occur? So, the maxima occurs the omega is equal to omega naught.

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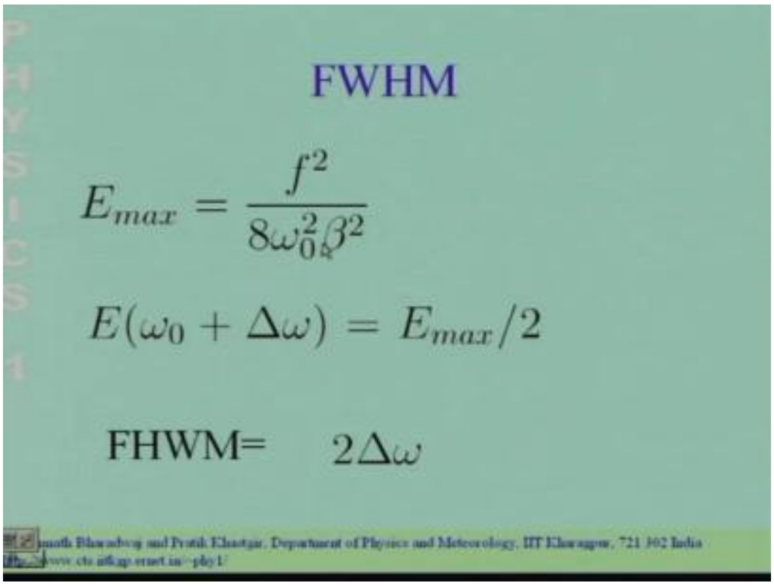

And the maximum value, the maximum value of the energy is; f square by 8 omega naught square beta square.

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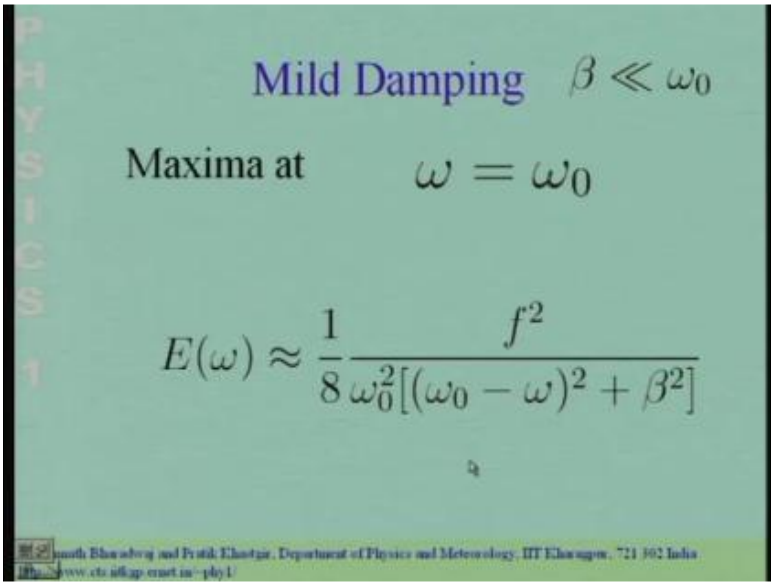

So, when you set omega equal to omega naught, you get the maximum value which is this.

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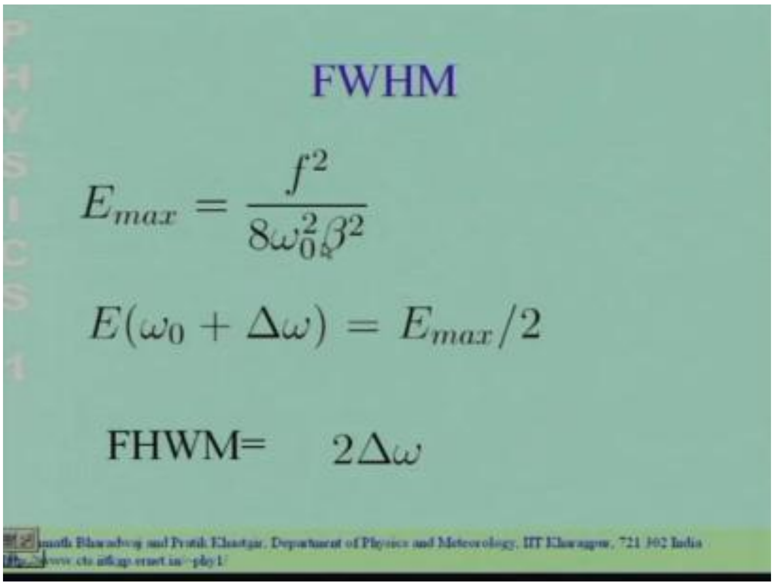

Now, the question is that, how much do you have to shift from the maxima? So, we are going to shift from the maxima by an amount delta omega. And the question is, how much should delta omega b so that the, energy at omega naught plus delta omega falls to half E maximum energy? So, we want to shift from the maximum value of the angular frequency by an amount delta omega so that, the energy falls to half the maximum energy.

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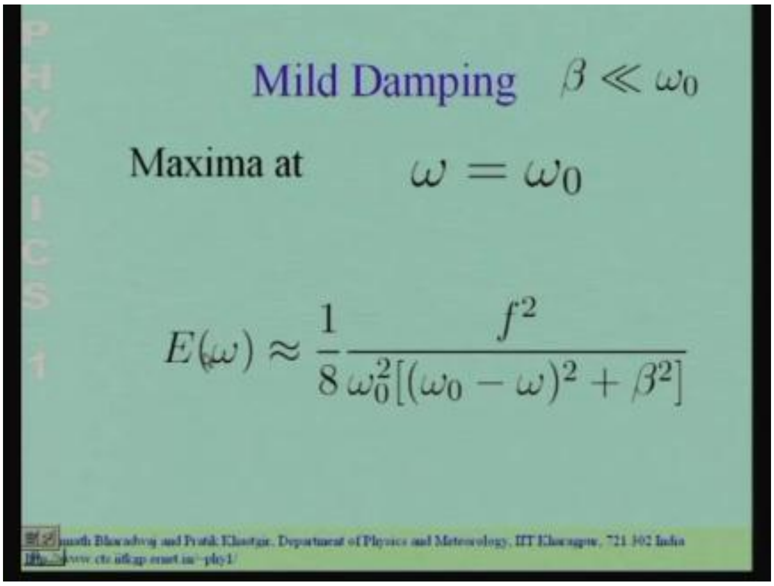

So, the maximum value is; where this equals to this and this whole thing is 0. Now, if you want this to fall to half the maximum value, you can have to basically shift by an amount which is equal to beta squared. So, if this is equal to beta squared, then the energy falls to half the maximum value.

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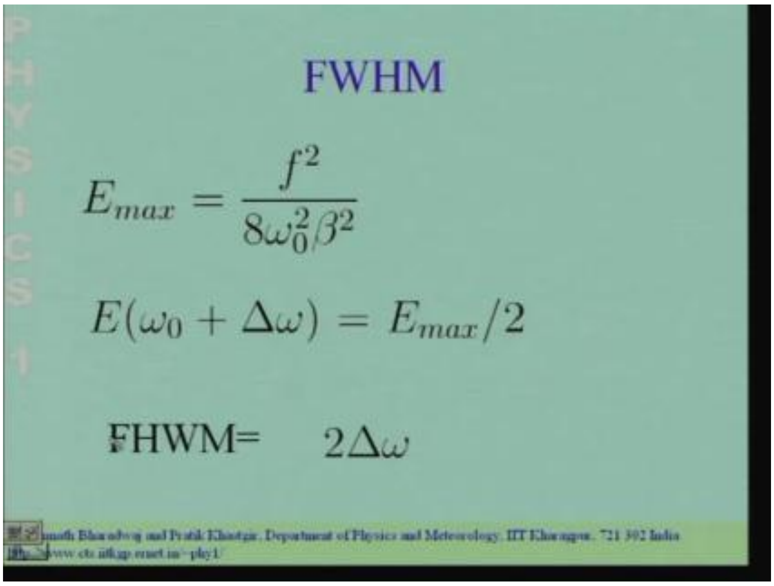

So, the delta omega should essentially delta omega.

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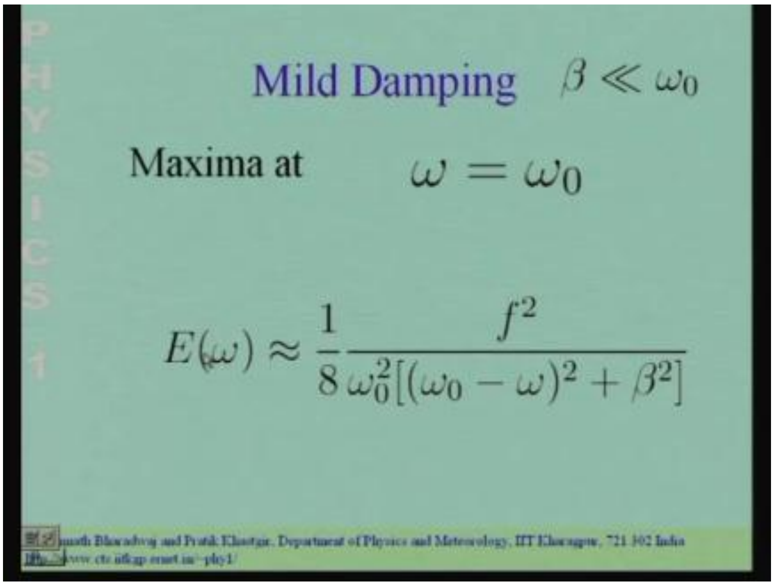

The difference from the maximum should essentially be beta.

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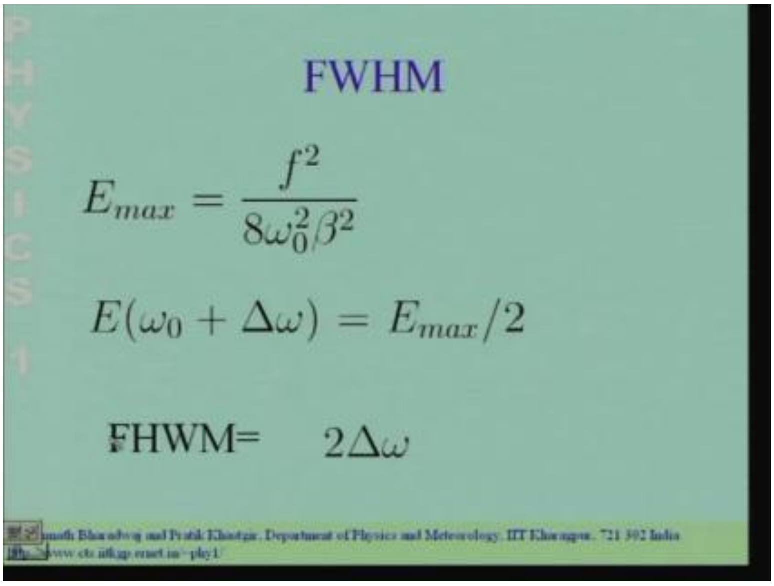

And the full width that half maxima is define, as to 2 delta omega that is; twice the shift and in this case it is equal to twice beta. So, the damping co-efficient, directly in this case the damping co-efficient directly determines the full width and half maxima. So, what see is that, in the low damping regime, the damping co-efficient directly determines the full with that half maxima. If, the full with the half maxima is twice the damping co-efficient and if you can measure 1, if you then you can directly predict the other.

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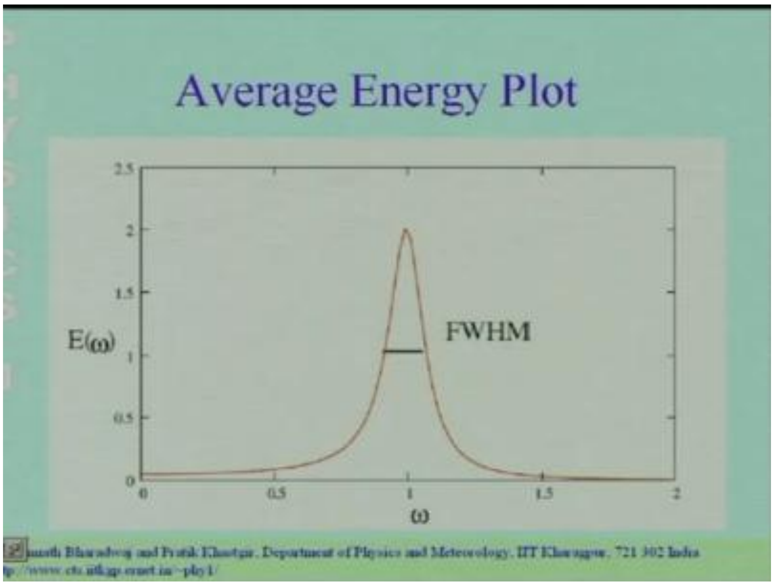

So, in this curve if you could measure the full width at half maxima, you would directly straight away know the value of the damping coefficient.

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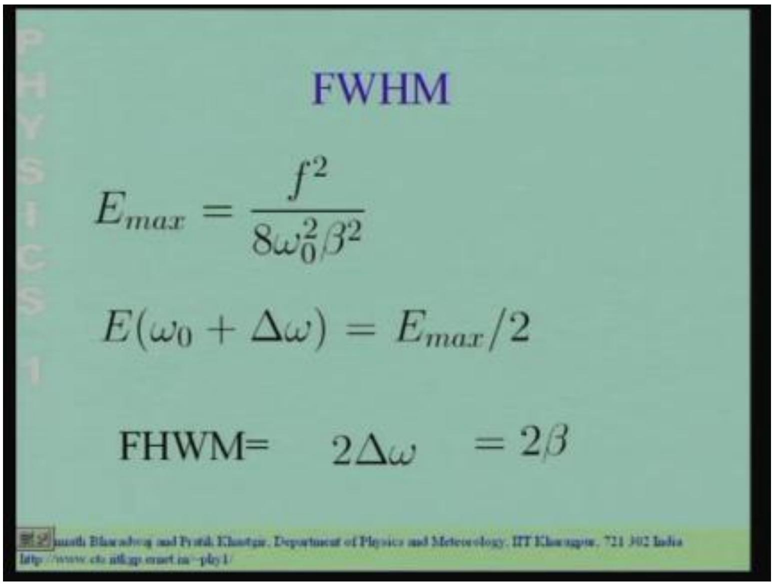

So, you could also, from this you could also predict what would happen, if you increase the damping co-efficient. If, you increase the damping co-efficient, the width of the of the resonant curve of the energy as a function of the frequency, would get broader.

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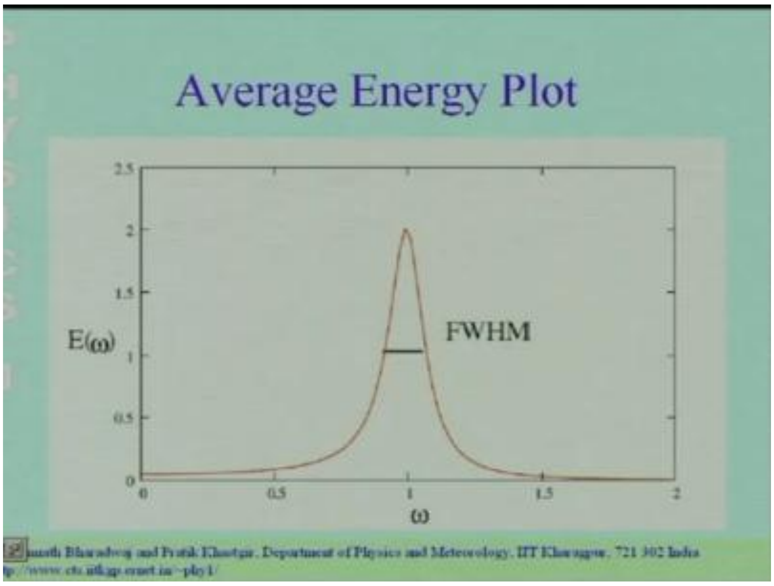

So, if I had a broader, if I had a larger damping co-efficient, then the curve I would get. If, I had a larger damping coefficient, then the curve that I would get would be broader. So, if I had a larger damping coefficient, the curve which I would get would be broader and if I had a narrow damping, if I had a smaller damping co-efficient, I would have a very narrow resonance very narrow or the very narrow peak over here.

Now, this intuition is very useful, there are many situations. For example, if you wish to design a filter and a spring mass system or effect or equivalently capacitance inductance system along with the resistance, could be used as a filter. Now, if you want to filter, will allow only a very narrow band of frequencies to pass through, so the circuit which you have should respond only to a very narrow range of frequencies and reject everything beyond you should then. So, basically you would like to have a situation where, this curve is very narrow, you should then choose a very small value of damping.

Whereas if, you would like to have a filter or a device; which response to an external signal over a broad range of frequencies, then you should have a very large value of damping. As an example you could think of, the design of a loudspeaker which response to sound. A loudspeaker which response to sound, you would it typically like it to response to sound over a large range large band of frequencies, not just to a single frequency.

Now, if you would like to some device, which response to a large range of frequencies and if you think of this device, if have if you make a simple model for the device in terms of spring mass system, then if you like it respond to a large range of frequencies, then you should a put in what you learn from today's exercise that; you should put in a large amount of damping. Whereas on the contrary, if you wanted to respond to a very narrow range of frequencies, then you should ensure that, it has a very low level of damping.

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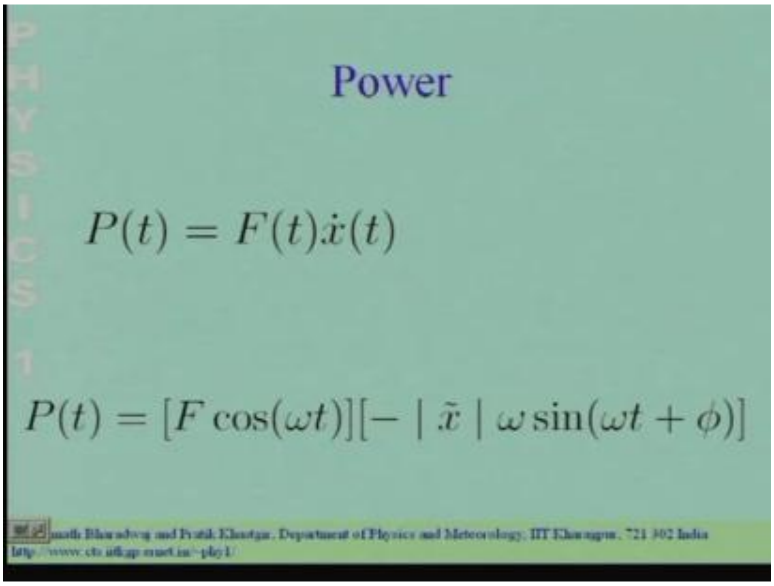

So, let us now move over another thing of interest which is; the power that is transmitted to the oscillator. So, we have this oscillator which is being given by an external force and the external force, it moves the oscillator and it transmits power to the oscillator in this process. So, we instantaneous power which is there in the oscillator at any instant, which is being transfer to the oscillator at any instant of time is; the instantaneous force F t into the instantaneous velocity of the oscillator x dot t.

Now, recollect that the external force is. So, the external force, let us just recollect the external force.

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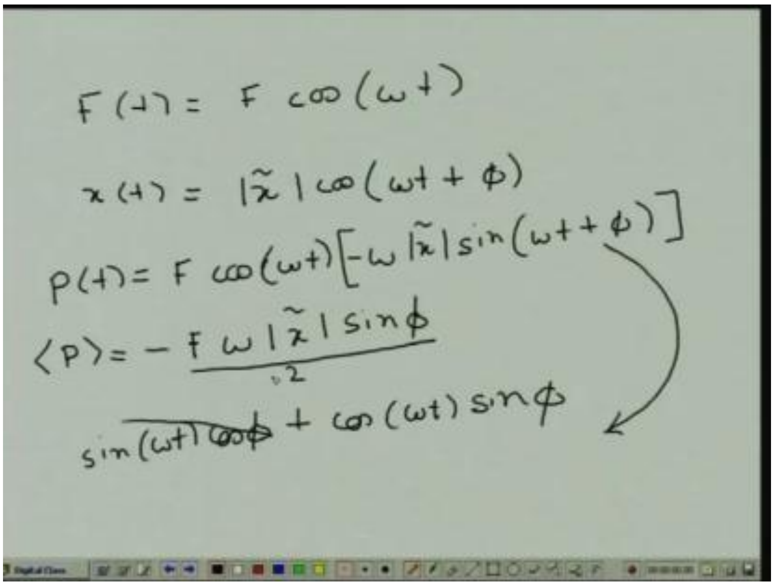

F t is equal to F cos omega t and the displacement which it causes x t is equal to the mod of the complex variable x tilde into cos omega t plus phi. So, the displacement and the force are not at exactly the same phase, we have all we have short while ago discuss, this at some detail. There is a phase gap; there is the phase lag between the displacement and the force. And we saw that the phase phi is negative, it varies from 0 to minus pi depending on, the frequency at which we are driving oscillator.

So, there is a phase difference between the oscillation and the force and that phase difference is what is there in the phi. Now, when you wish to calculate the instantaneous power P t, you have to put in this F and now you have cos omega t and then you have to multiply this with the derivative of this displacement x t. So, let us calculate the derivative of this displacement x t and put it in here.

So, you have to multiply this whether the derivative of x t. So, when you differentiate x t, you will get basically you have to take a derivative of cos omega t plus phi it you give a factor minus omega. And then you have x tilde mod of that and you have sin omega t plus phi. So, this gives us the instantaneous power. Now, the instantaneous power is not the thing of interest always. Quiet often we are interested in the average power.

So, what is on the average? This is my force, it is driving my oscillator on the average does the, how much power does the force transfer to the oscillator. So, we would like to calculate the average power, this is the quantity which is quite often of interest and so, we have to take time average of this whole thing over here. Now, we have discussed earlier, how to calculate the time average of oscillating quantities.

So, let us apply those things over here. And the way to the, this calculation in this particular situation is calculation would be simplified if, I were to write; this sin omega t plus phi as sin omega t cos phi plus cos omega t sin phi. So, this sin this term over here sin omega t plus phi can be broken up into 2 terms; 1 is sin omega t into cos phi, the other is cos omega t into sin phi.

Now, when I take the time average of the power, of the instant when I time average the power P t, I will have to take the time average of this term, on the right hand side over here and I will get 2 terms; 1 will be the time average of cosine omega t into sin omega t and this term we know, cosine omega t into sin omega is sin 2 omega t with the factor of half coming in. So, this the time average of sin to omega t is 0, we have already studied this. So, this term, this term essentially does not contribute to the time average power, it is only this term which contributes.

So, we have the time average of cos omega t into cos omega, the time average of cos square omega t we have seen is half. So, what we have here is that, the time average power is F minus sin coming from here, then I have omega, then I have the mod of a x tilde and I have a factor of sin phi divided by 2. So, that is the time average power.

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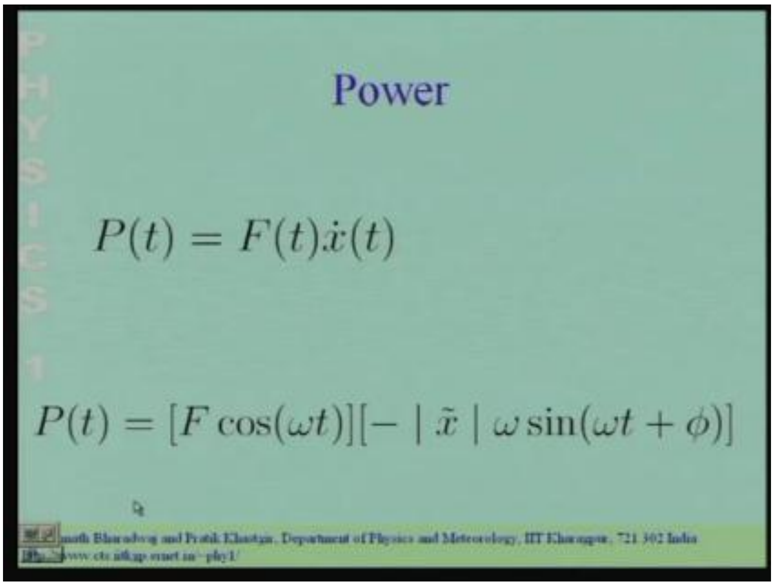

So, let me go back to the, so when I take the time average of this, as I have just showed you.

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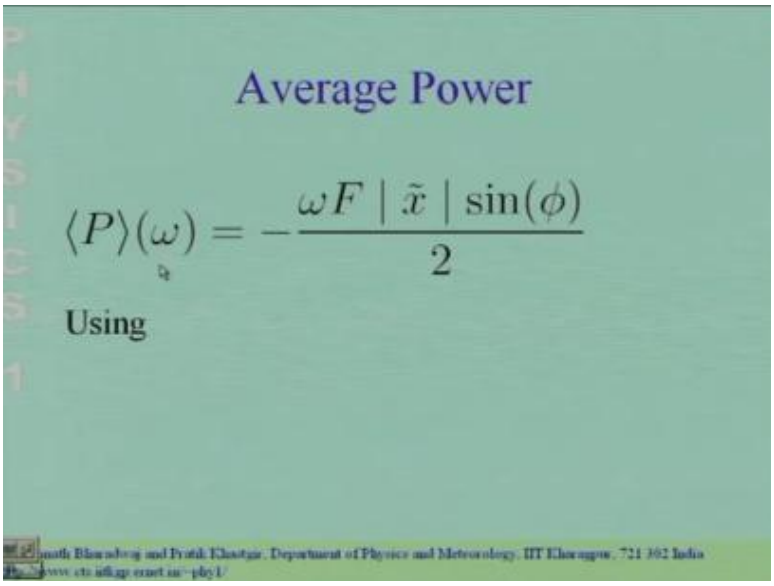

The time average of the power gives us this expression. Now, in this expression, we have to calculate the, this term over here that is, the mod of the displacement into sin phi. Now, let us just go back to the expression for complex displacement.

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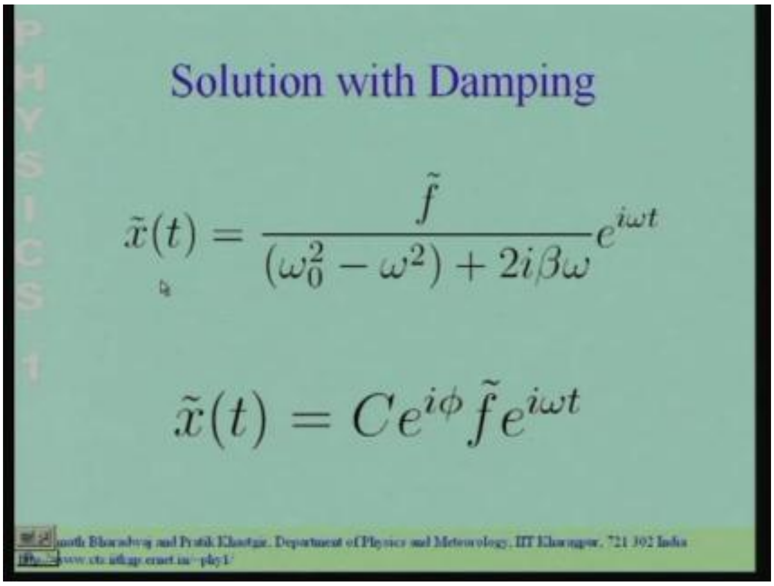

This is the expression for the complex displacement. And let us ask the question. What do we mean by the mod of the x in to sin phi? Now, sin phi the phase phi is the relative phase between this x and phi. With this e to the power and omega t occurs in both the force and the displacement. So, we need not bother about this. Sin phi is the relative phase between this and this. So, if you wish to calculate x tilde sin phi, it is basically the imaginary part of this x tilde because, x tilde you can write as has the mod of x into cos phi plus i sin phi.

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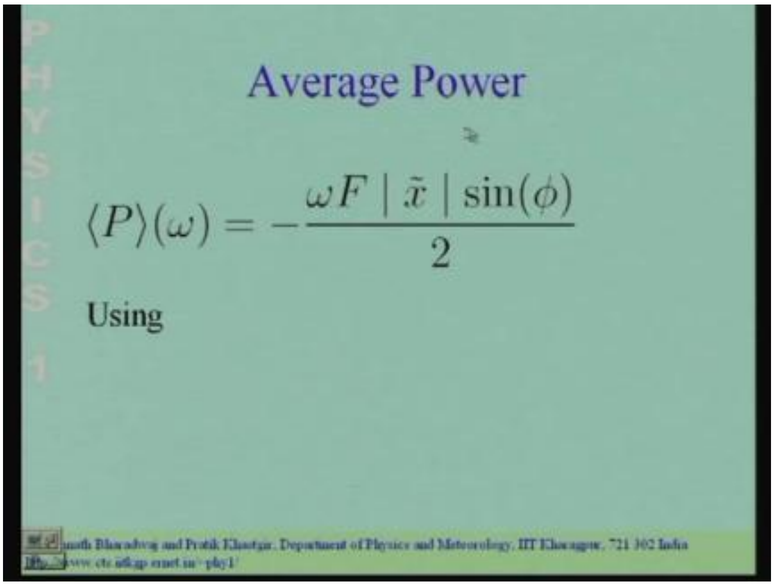

So, the imaginary part of this gives us this quantity over here which we wish to calculate.

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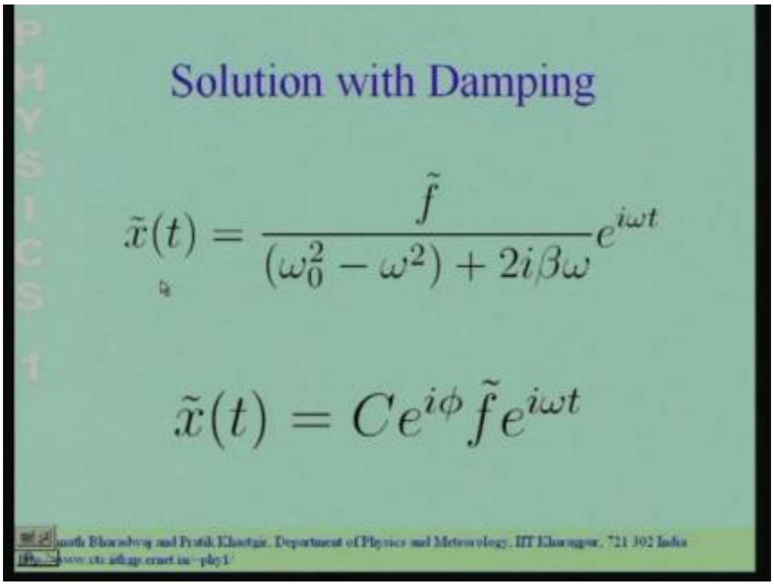

So, let us calculate the imaginary part of this expression and when calculating the imaginary part, it should there in mind that, we are not interest in this term because, this occurs both for the force as well as for the displacement, we are only interested in relative phases. Further any phase in this also would occur here, so we are not interested in that. What we are interested in is the relative phase; phi is the relative phase between the displacement and this.

So, we have to calculate, so let me write down, what we have to calculate.

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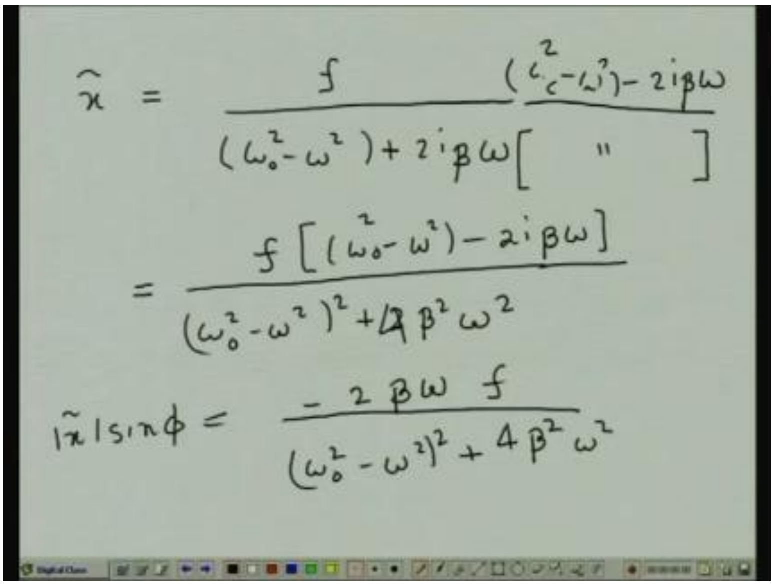

So, we would like to calculate the imaginary part of this. We have to calculate the imaginary part of this. Now, to break this number into a real and imaginary part, we have to essentially multiply it with the complex conjugate of the denominator. So, let us do that. We will put a factor of omega naught square minus omega square and then I put in a factor of 2 minus 2 i beta omega and here I also put in the same thing, which is there in numerator. If, I multiply this and this what I get is and in the numerator I have omega naught square minus omega square minus 2 i beta omega.

So, this has a real and imaginary part, this is the real part, we are not interested in that. We are interested in the imaginary part. So, if I identify the imaginary part of this, with mod of x tilde sin phi, then what we can say is that, this is equal to minus 2 beta omega into f divided by omega naught square minus omega square the whole square of this plus this should be 4 2 4 plus 4 beta square omega square.

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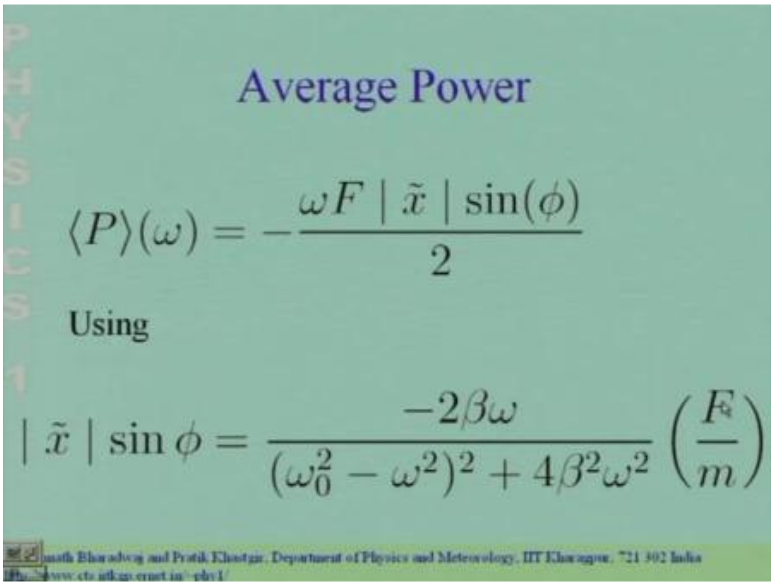

So, using this and here we have also put in the fact that, the f is the amplitude of the force divided by the mass. So, putting this in, in the expression for the average power…

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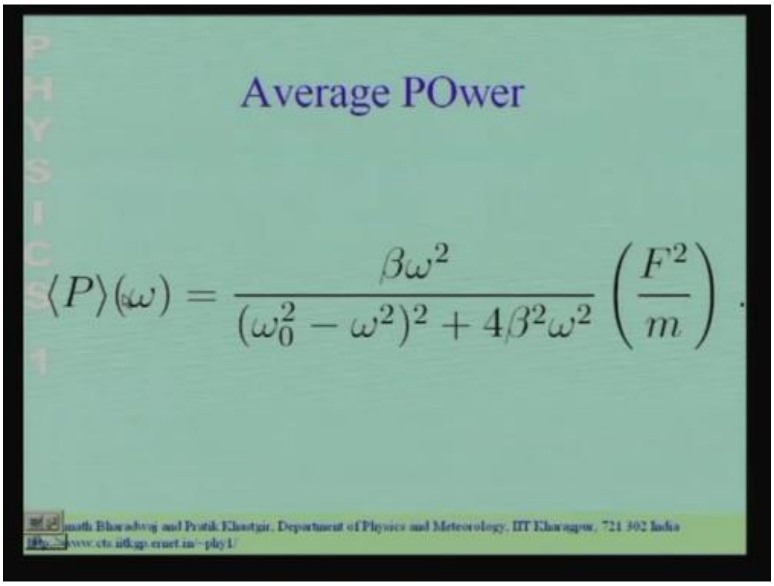

We have the average power, as a function of the angular frequency of the driving force. So, the average power we see, that is transmitted to the oscillator as a function of the frequency of the driving force, is given by this expression over here. This expression is a very important expression; it appears such an expression appears in large parts of a large verity of situations, in physics this expression is called a Lorentzian profile.

So, such a function of omega, such a function of angular frequency is called a Lorentzian profile. And in this case it tells you, the power that is transmitted to the oscillator. Such a thing occurs in a large verity of situations and we shall discuss this in more detail in the next lecture.

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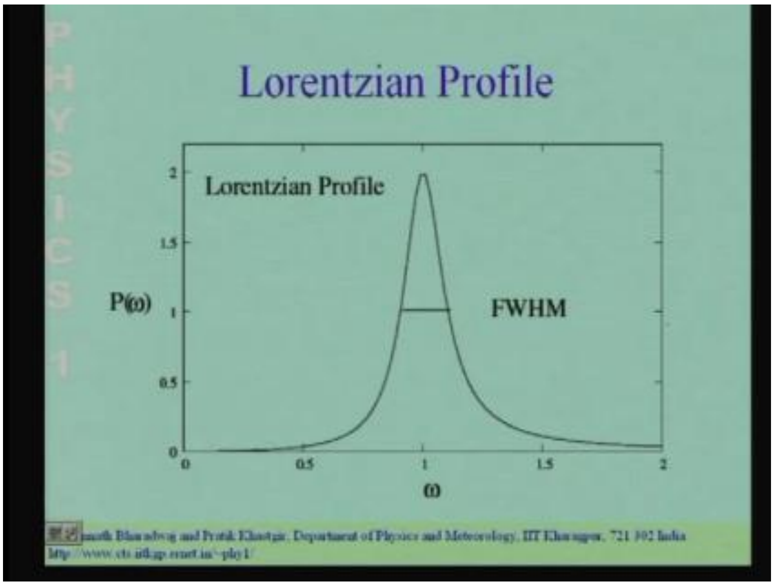

This curve shows you the Lorentzian profile which is, the power that is transferred to the oscillator, as a function of the angular frequency omega. So, let us determine the peak value of the, let us determine the value of the angular frequency, where you have the peak of the curve.

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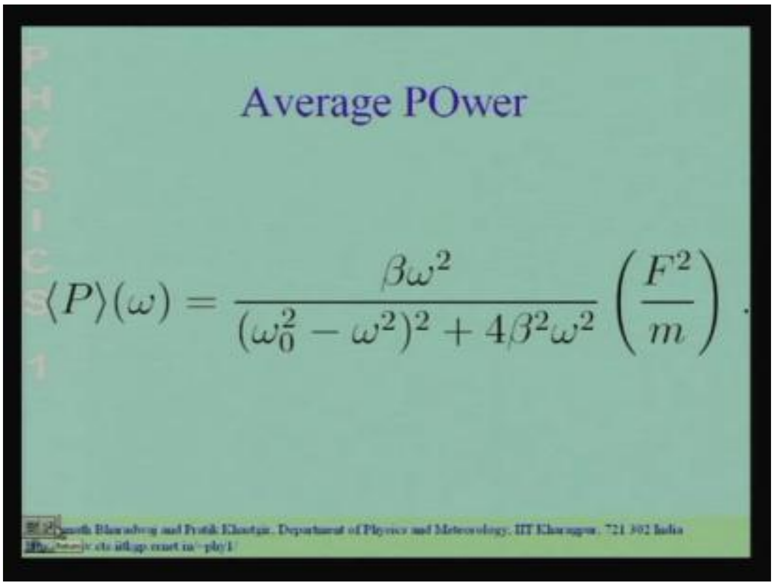

So, to do this, we have to differentiate the omega the dependence of the average power and then set it equal to do 0. So, let us do this calculation.

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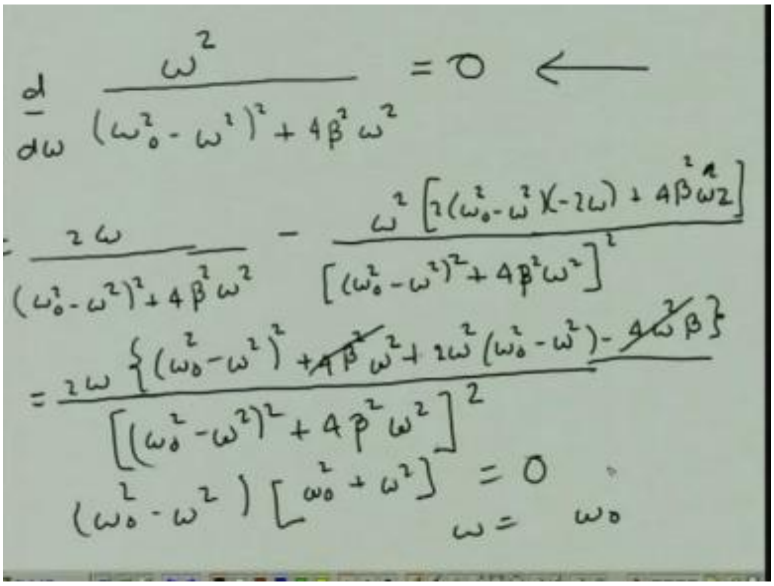

So, the quantity that we have to differentiate is; omega square divided by omega naught square minus omega square the square of this plus 4 beta square omega square. This is the angular frequency dependence of the average power, of the Lorentzian profile. And we wish to find, the value of the frequency where the Lorentzian has a peak. So, we need to differentiate this and set the derivative equal to 0.

If, you differentiate the numerator, we get 2 omega and this is divided by omega naught square minus omega square whole square plus 4 beta square omega square. And if, I differentiate the denominator then, we get the same old omega square on top. If I, differential the denominator I will have a; minus sin and then I have to square the denominator. So, if I square the denominator I get omega naught square minus omega square square plus four 4 square omega square. And then I have to differentiate the, I have the whole square of this and I have to differentiate the quantity inside. So, let me write down the derivative of the quantity inside.

If, I differentiate the first term over here, what I have is 2 omega naught square minus omega square and then I have to differential minus omega square if, I differentiate minus omega square, I will get minus 2 omega. And then if I differentiate the second term then, I get the factor of plus 4 beta square omega square and into 2. So, let me now combine both of these and I can forget a combine both of these terms.

So, I can take a factor of 2 omega, this is equal to there is no minus sign here it should be equal to this is equal to, I could take 2 omega common here and in the denominator I have omega naught square minus omega square plus 4 beta square omega square the whole square. So, I have to multiplied this particular term with 1 factor of this, so I will have I have taken 2 omega common. So, I will have omega naught square minus omega square whole square plus 4 beta square omega square and I have to and I this term has to also be added. Now, when I have take 2 omega common, if it 2 omega common then, the first term over here; there is still a factor of 2 which remains, there is a omega square which remains and there is omega naught square minus omega square.

So, the terms, the first term gives us 2 omega square omega naught square minus omega square and the second term from here, if I take 2 omega there should be no omega square, there should square should not be here differentiate it. So, if I take 2 omegas common then, I will have minus 4 omega square beta square. So, this is my numerator, notice that this term cancels out to this term and I have to find the solution of this equal to 0. So, I need not bother about the denominator. I have to find the solution where, this plus this is equal to 0.

Now, if I look at this plus this, I can take omega naught square minus omega square common. So, effectively if I want a solution to this., I should have satisfy the condition that, omega square omega naught square minus omega square, I have take it common from both of these terms. And the first term still gives another factor of omega naught square minus omega square and this gives me a factor of 2 omegas square. So, when I add this to this, what I get is omega naught square plus omega square. This should be equal to 0. And this only the only possible solution is when omega is equal to omega naught.

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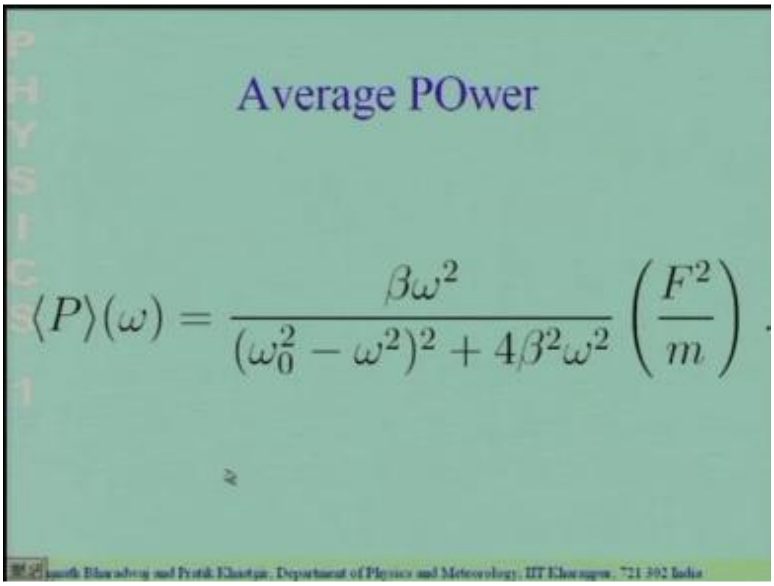

So, what we see here is that, the peak of this curve, the peak of the Lorentzian occurs at omega is equal to omega naught and you can check for yourself that, the full width at half maxima is 2 beta, it could easily check that in the weak damp, this true only in the weak damping regime. So, in the weak damping regime, the full with that half maximum of the average power of the Lorentzian is 2 beta. So, let me summarize what we have done today.

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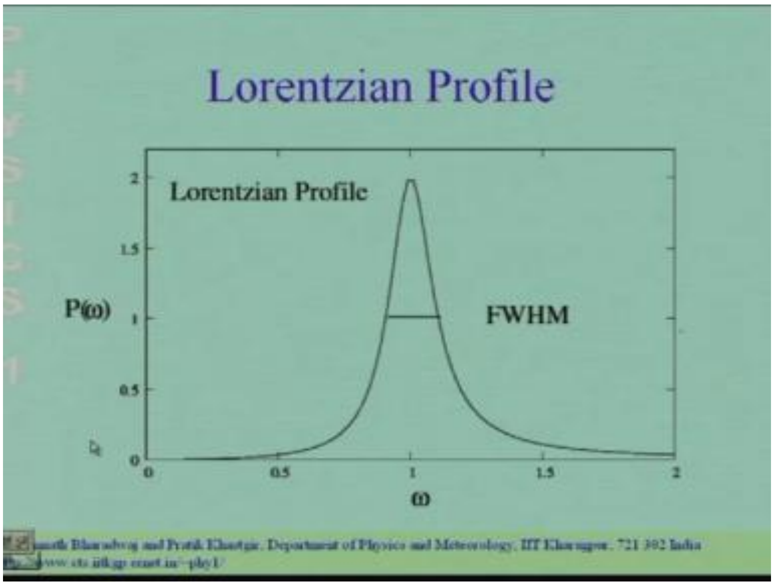

Today, we calculated the average energy and the average power that is, that have external force on oscillating external force, pumps into an into the oscillator which is also damped. We calculated the average power and the average energy we found that the average energy, has a peak at omega is equal to omega naught square minus 2 beta square.

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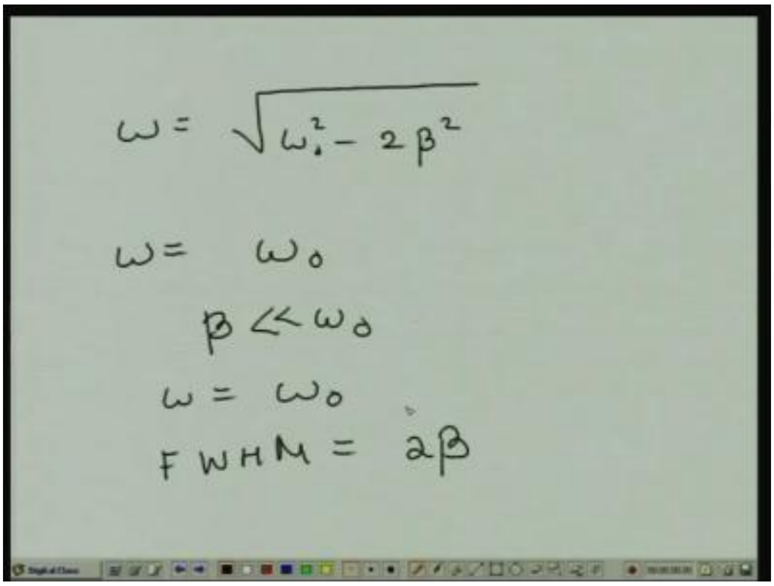

And the average energy falls of; as if you have a driving force, which differs from this value of omega, the average power is governed by a Lorentzian which has a peak at omega equal to omega naught; the natural frequency of the oscillator. And in the both cases; in the weak damping regime, the peak occurs at omega approximately equal to omega naught and the peak has a width of full width at half maxima FWHM approximately equal to 2 beta in the weak damping regime. If, you are not in the weak damping regime then, you have to numerically calculate the full width at half maxima.

So, let us bring today's class to an end. In tomorrow's class we shall discuss, several applications of this Lorentzian profile and the phenomena of resonance.