Physics I : Oscillations and Waves

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Lecture - 44
Quantum Tunneling (Contd.)

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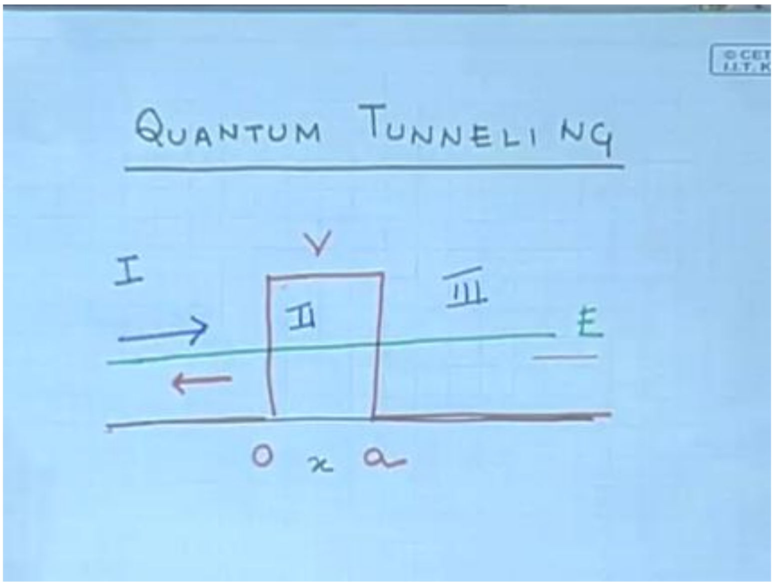

Good morning, in the last class we were discussing quantum tunneling. We have a particle of energy E incident from the left, on a potential barrier of height V, which is more than the value of the energy. The potential is zero in this region from minus infinity to x equal to 0. It is also zero from a all the way to infinity. So, if you analyze this situation using classical mechanics, you will find that all particles that are incident here, will be reflected back, because they do not have sufficient energy to overcome this barrier. The initial energy, the total energy of the particle, is not sufficient to overcome this barrier. The energy is smaller than the value of the potential. So, if you do a classical analysis, you will find that all particles are reflected back. Now, we are discussing what happens if you do a quantum analysis of this problem. And a quantum analysis is required, if you are dealing with a microscopic particle like an electron incident on a potential barrier. And in the quantum analysis you have to represent the particle using a wave. and we have three different kinds of solutions, three different solutions; One in this region, where it is a free particle, one inside the potential barrier, and the third solution again in the region where it is a free particle, on the right hand side.

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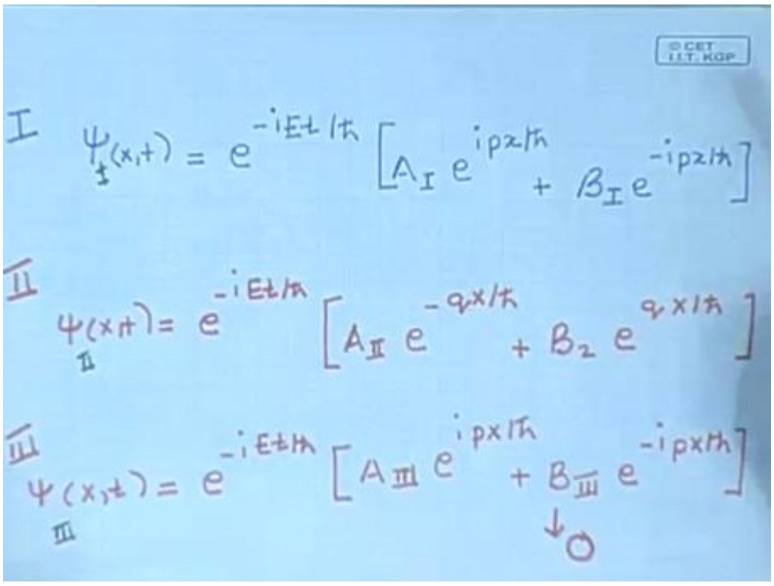

So, we had written down the three different solutions, the time part is the same in all of these, and this is the solution in region one, where it is a free particle. This is the solution inside the barrier. In the region one where it is a free particle you have an oscillating wave function. The wave function is a plane wave. It is a superposition of two plane waves; one travelling to the right, one travelling to the left. This represents the incident particle, this represents the reflected particle. This is the wave function inside the potential barrier, inside the potential barrier as the potential is greater than the energy, the wave function has two parts; one varied decays exponentially, one varied increases exponentially, and these two parts have got coefficients A 2 and B 2. Again when the particle emerges from the potential barrier, there are two possible. Thus the wave function is a superposition of two possible solutions; a right propagating solution, and a left propagating wave. This represents a particle with momentum plus p, this represents a particle with momentum minus p.

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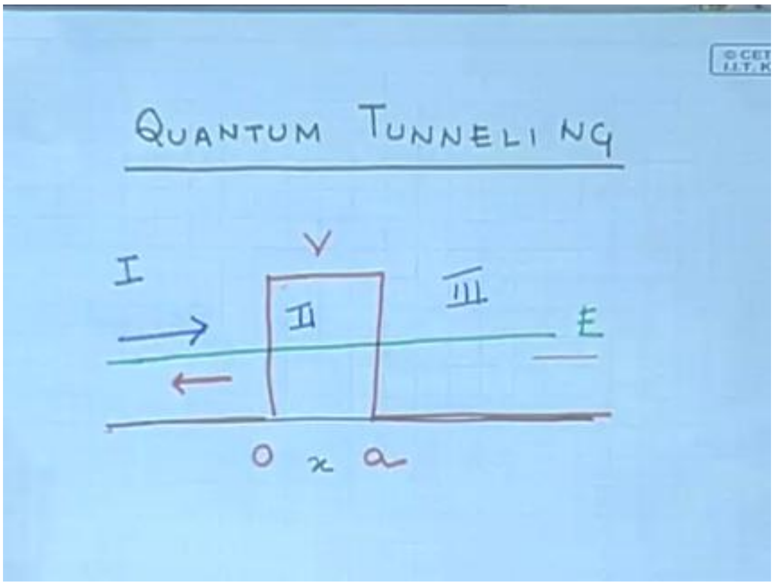

Further the situation which we are dealing with. There are particles incident only from the left. There is no particles incident from the right. There are particles incidents from the left, and we are interested in finding out, if there is a probability for the particle to be, to somehow get through the barrier, and appear on this side. So, is there a probability, of finding this particle which is incident from the left side, in this region over here, which is beyond the potential barrier. There are no particles which are being sent in like this.

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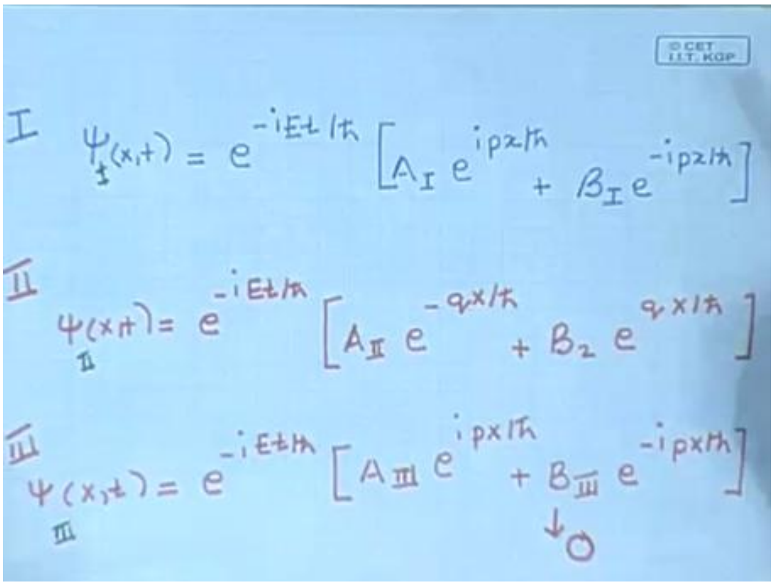

The fact that there are no particles which are being sent in from here tells us that, this part of the wave function, this part the amplitude of this part of the wave function has to be zero, because this represents particles incident from this side, and that has been. We do not have particles incident from this side, so this amplitude has to be zero.

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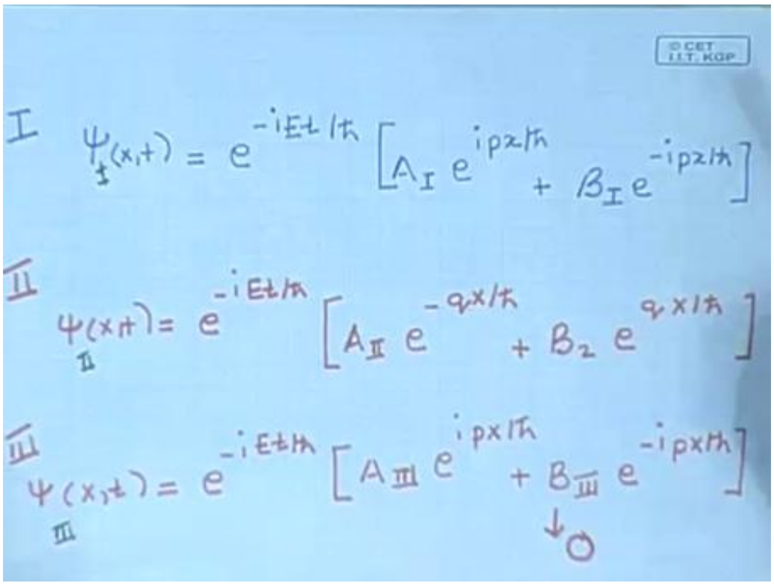

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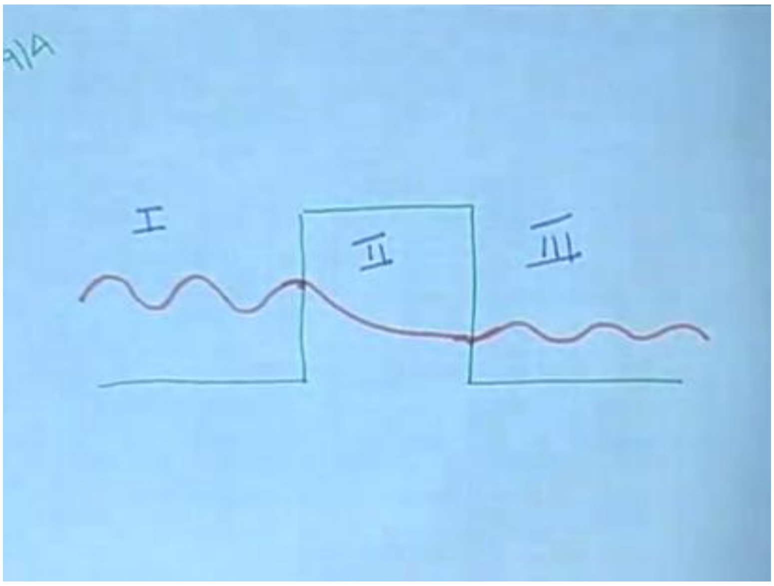

And let me show you what the wave function looks like in the three regions. So, here we have the plane wave solution. Here we have the exponentially decaying solution. And again here we have the plane wave, as we shall see shortly the amplitude of this plane wave is going to be much smaller than the amplitude of this incident plane wave. So, this has both the incident and the reflected plane wave. This is the plane wave that emerges in region three, and it is this that gives you the probability of finding the particle in this region. Particle the incident here, we were interested in the probability of finding the particle in this region, and it is this phenomenon where the particle gets through a barrier potential barrier, which is higher than its energy, which is referred to as quantum tunneling. This is a phenomenon which occurs only if you do a quantum treatment of the problem. So, what we are interested in, is basically finding the probability of calculating the probability of finding the particle here, and the probability of finding the particle here depends on the amplitude of the wave function in that region which is A 3. And the amplitude of the incident functions.

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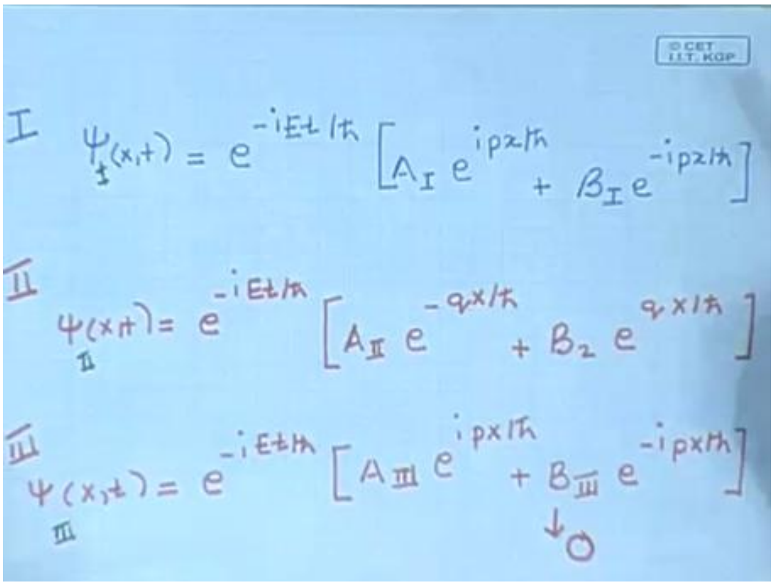

See if you look at the ratio of this amplitude to this amplitude, the amplitude of the incident wave function to the amplitude of the tunnel tunneling wave function. So, what we were doing is, we were finding relations between these amplitudes, between the amplitude here here here here and between these and these two amplitudes. And I told you that such relations can be found, by applying suitable boundary conditions. So, let me show you what are the boundary conditions that you have to apply in general.

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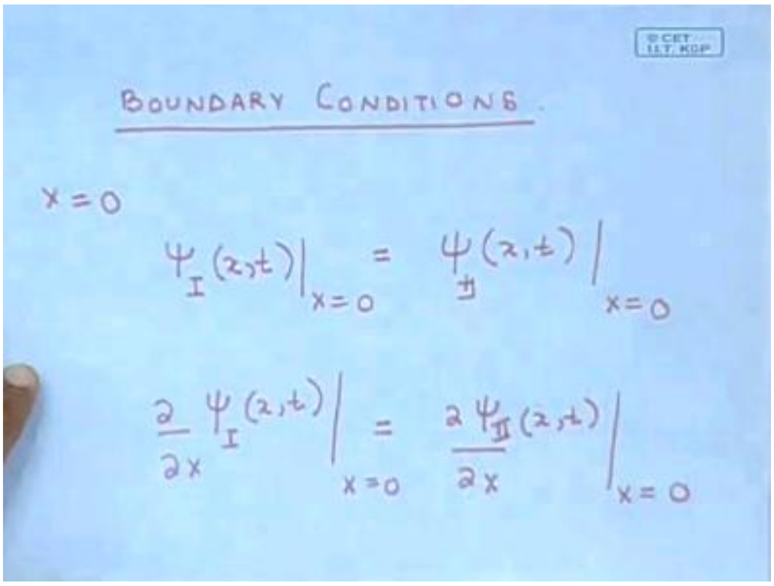

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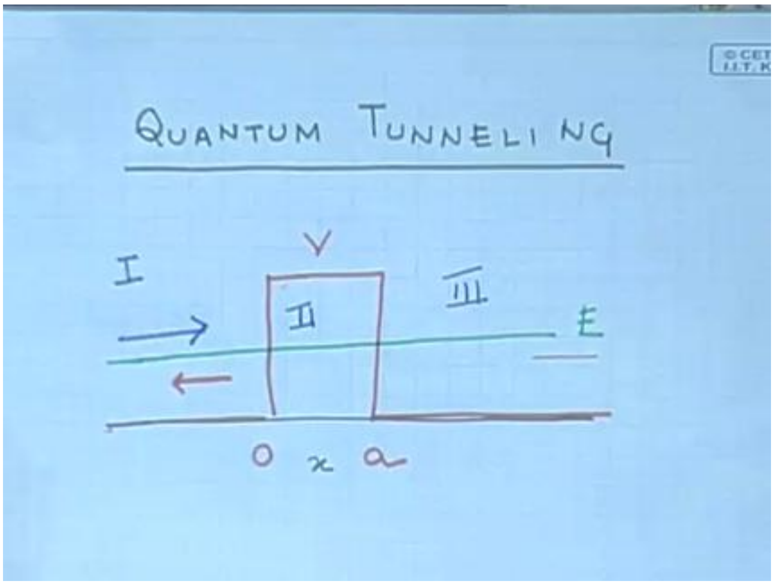

So, at any boundary the wave functions on the left hand side. This in particular refers to the boundary at x equal to 0. So, at this boundary, the wave function in region one. So, we are looking at the boundary between region one and region two. And in the last lecture, I told you that the wave function should be continuous across the boundary, and it is first derivative should also be continuous across the boundary, which is what I showed over here. So, at x equal to 0, the wave function on the left hand side should be equal to the wave function on the right hand side. Also, the spatial derivative the first spatial derivative of the wave function on the left hand side should be equal to the first spatial derivative of the wave function on the right hand side at the boundaries; boundary is x equal to 0. So, these are the two boundary conditions that we have to apply, and applying these boundary conditions to these two wave functions.

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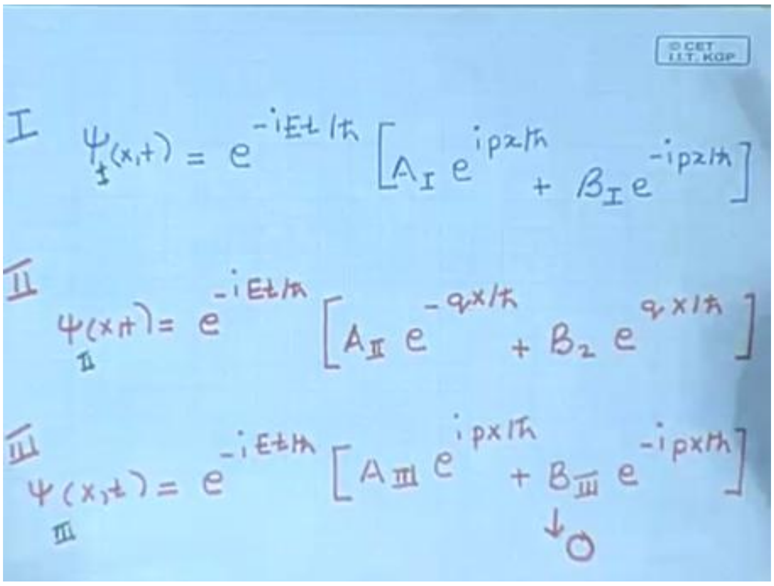

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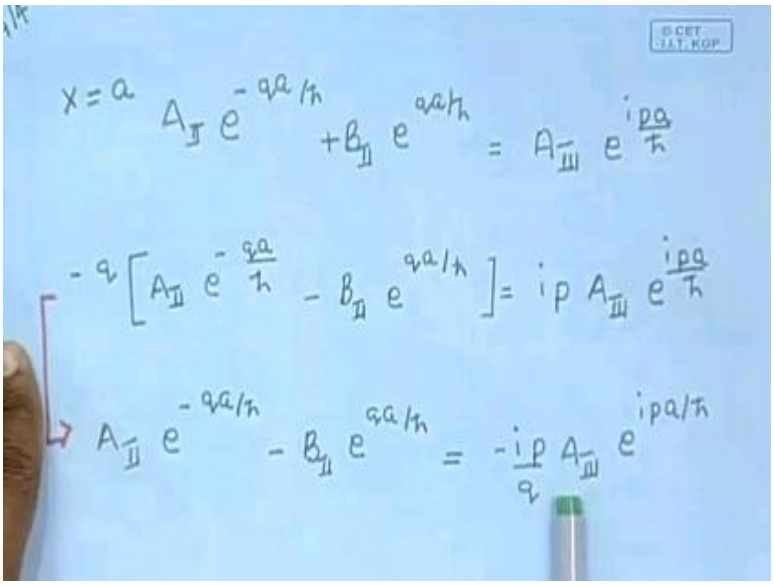

The boundary is at x is equal to 0. So, applying the two boundary conditions to these wave functions we get these relations between the coefficients. So, this is the first relation, which arises from the requirement that the wave function should be continuous.

This is the second relation which arises from the requirement at the derivative of the wave function should be continuous, which we have simplified and written in. sorry this is not the right thing; the boundary x equal to 0.

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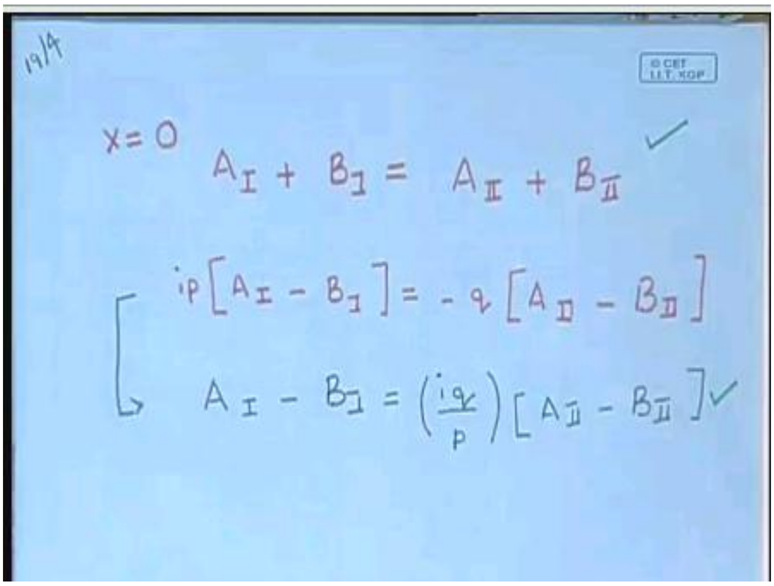

These are the conditions at the boundary x is equal to 0. So, at x equal to 0 this is the condition from the equality of the wave function on the two sides. The fact that the wave function should be continuous across the boundary. And this is the condition from the fact that the derivative; first derivative should be continuous across the boundary. We have simplified this and written it in this way. So, these are the two boundary conditions that we get at the boundary x equal to 0. These are essentially relation between the coefficients of the wave function on the two sides A 1 B 1 and A 2 B 2 A 1 B 1 are the coefficients of the wave function on the left hand side.

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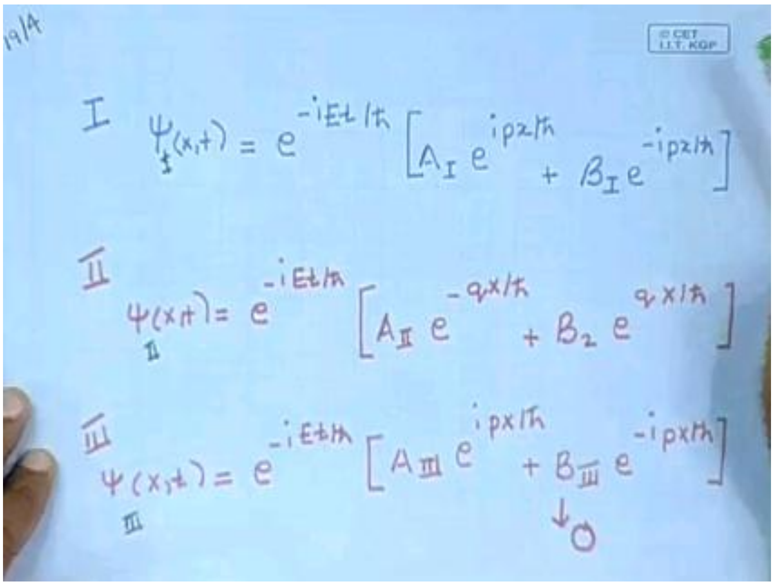

A 1 and B 1 are the coefficients of the wave function on the left hand side. This represents a right propagating wave, the incident particle this represents the reflected particle.

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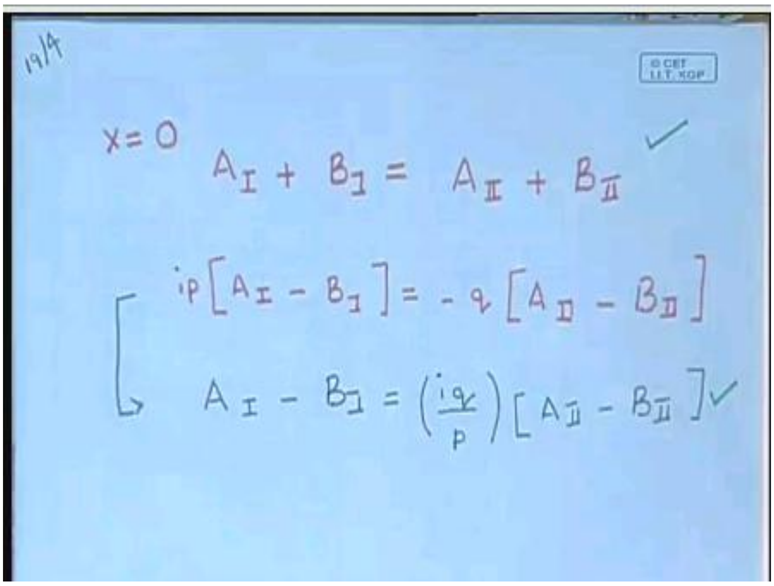

A 2 and B 2 are the amplitudes of the two different wave functions on the right hand side in region two. And what we get from the boundary condition; a relations between the A 1 B 1 and the A 2 B 2. Similarly, if we consider the boundary at x equal to A. Sorry not this one again.

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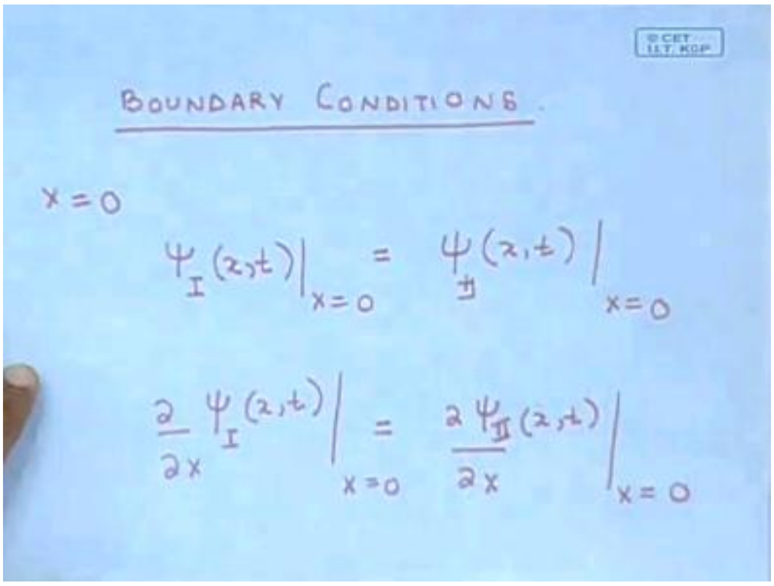

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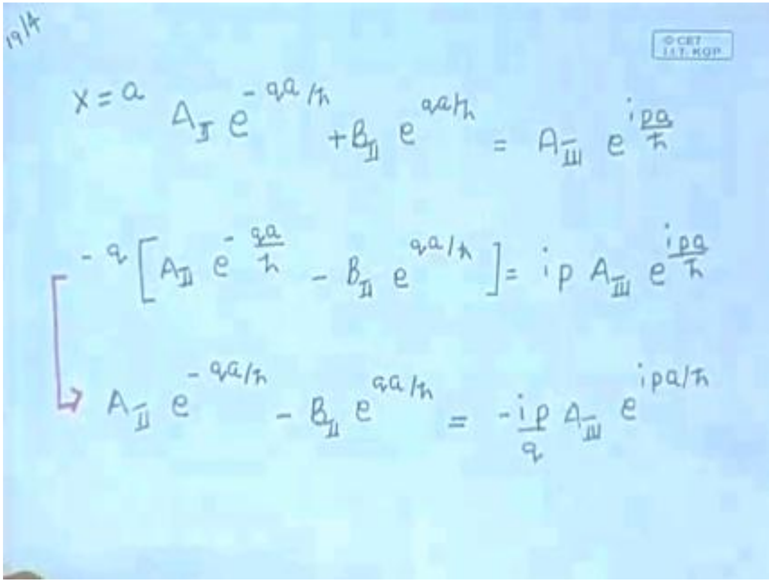

If you consider the boundary at x equal to A, the continuity of the wave function across the boundary gives us this relation. The continuity of the first derivative across the boundary gives us this relation which again we have simplified and written over here. So, this gives us relations between A 2 B 2 and A 3 B 3 is already said to 0, because there are no particles incidents from the right. So, we have two relations between A 2 B 2, and A 3. This comes from the continuity of the wave function this comes from the continuity of the derivative, first derivative.

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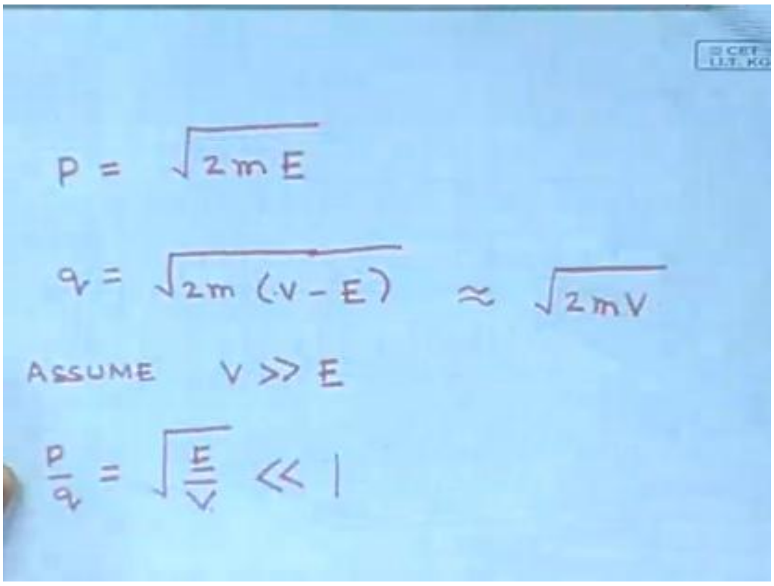

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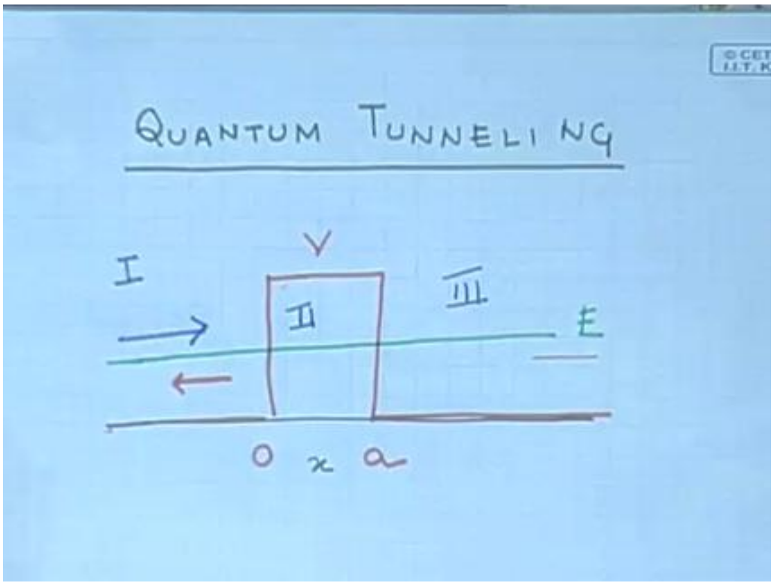

Now, we had also made an assumption, the assumption which we had made was that 'the potential is much higher than the energy of the particle. So, we had made this assumption that the height of this potential, is much larger than the energy of the particle V, is much greater than E.

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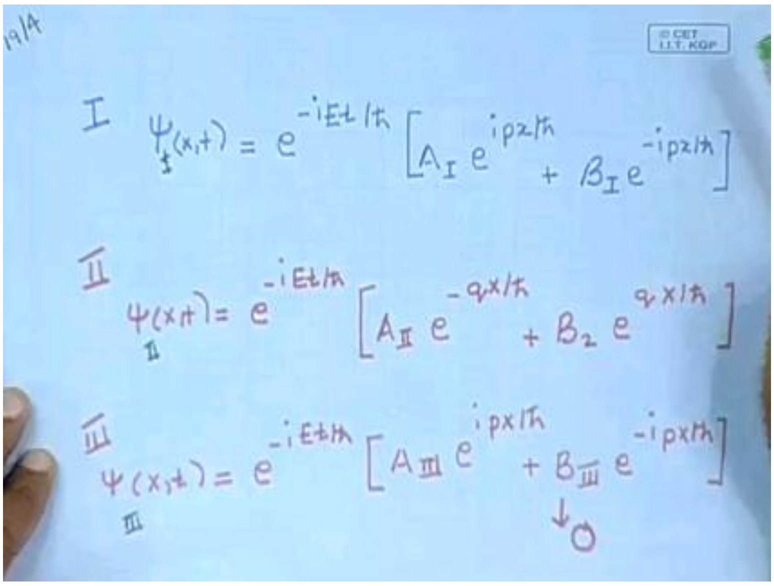

So, under this assumption the momentum of the particle p, which appears in the wave function in the region where the particle is free. So, the momentum appears here and here, here and here p. p is the square root of two m E the energy of the particle. It has nothing to do with the potential, and if you assume that the potential is much larger than the energy; the quantity q, let me remind you what q is. Q is what appears over here, it is the exponent which decides how fast the wave function decays, and how fast this increases this constant q.

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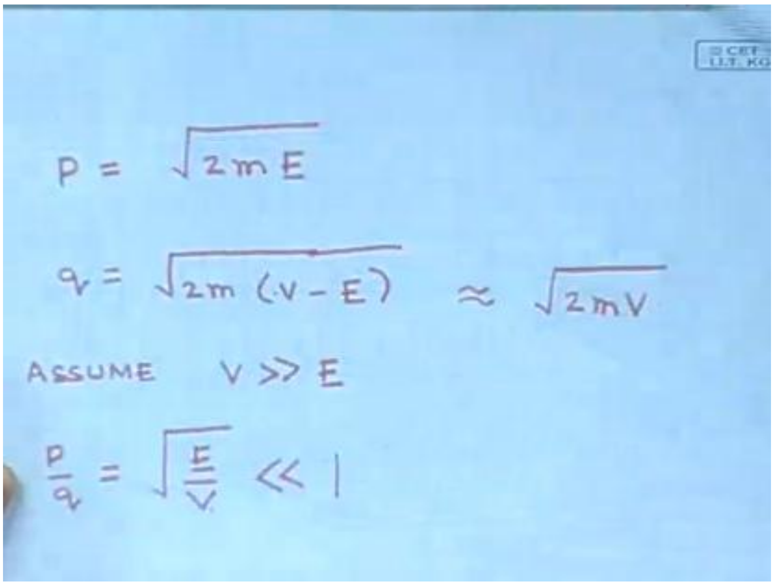

So, q is what decides the nature of the wave function inside the potential barrier, and if q is defined to be 2 m V minus E. If you assume that the potential is much greater than the energy, this is approximately equal to square root of 2 m V. And further the ratio p by q which is square root of E by V is much smaller than one. So, we are going to make this assumptions; that q is approximately square root of 2 m V p by q is much smaller than one, because the potential is much higher than the energy. So, with this assumption, let us first look at the second boundary. So, we are going to first consider the second boundary the boundary at a.

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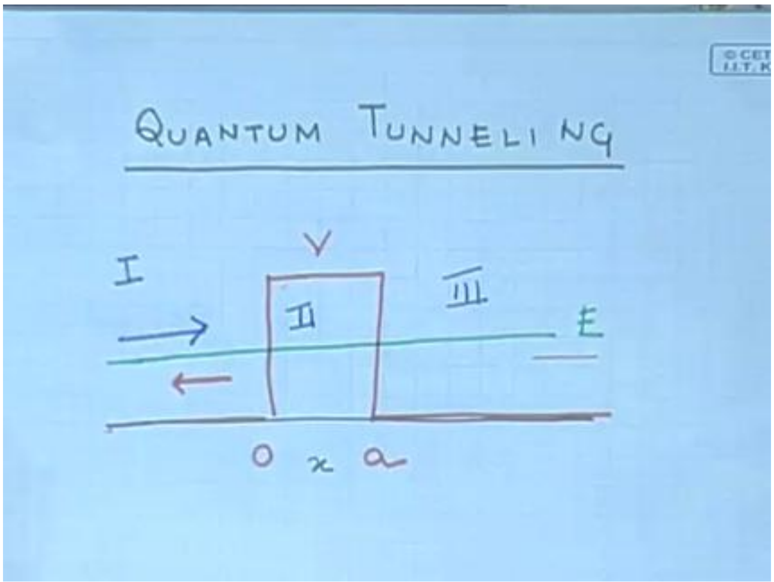

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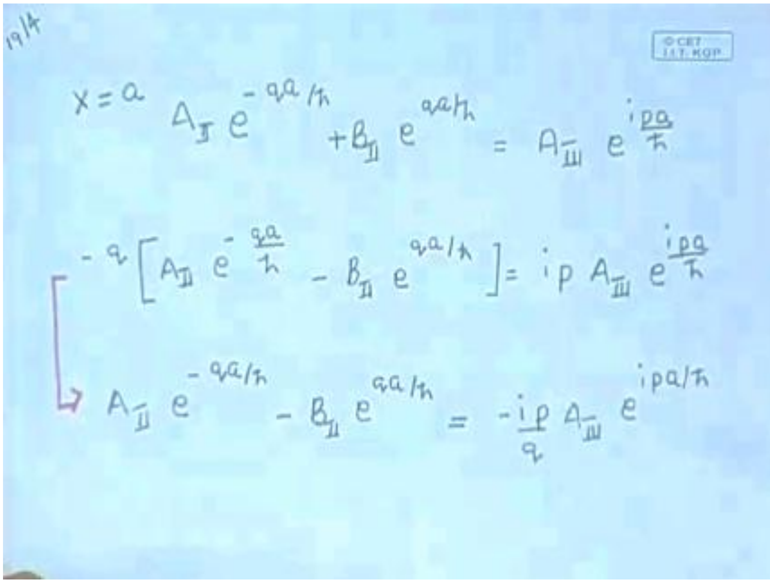

So, we are going to look at the second boundary x equal to a. The second boundary is at x equal to a. and at that boundary we have these two conditions. Now, let us first look at the second boundary condition, which comes from the continuity of the first spatial derivative across the boundary. And note that on the right hand side, we have the ratio p by q. Now we have assumed that p by q is much smaller than one, what it tells us is that the right hand side of this expression is approximately equal to zero, because this is a very small number. So, tells us that this is the difference of these quantities, has to be extremely small, because p by q has been assumed to be a very small number. So, if you make this assumption it essentially tells us, that are approximately equal to this, under the assumption that we are that this p is much smaller than q. This is approximately equal to this.

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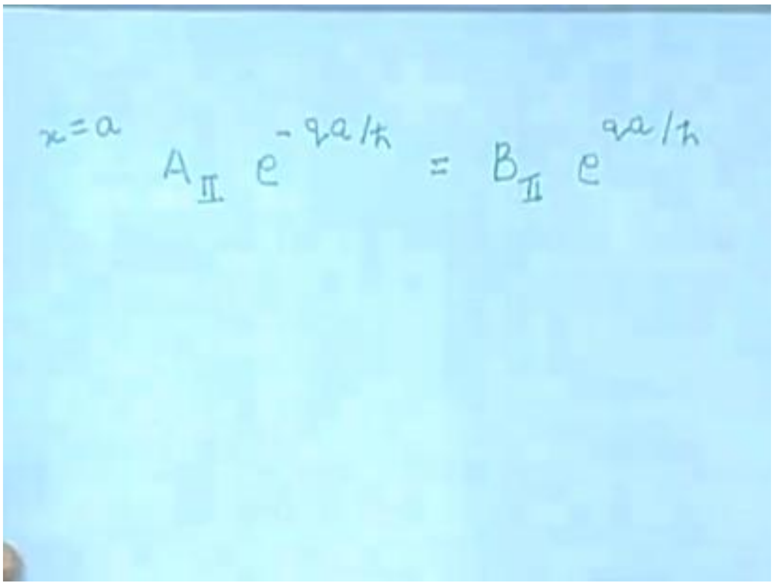

So, what it tells us, the boundary condition at x equal to a. The second boundary condition at x equal to a what it tells us is that A 2 e to the power minus q a by h cross is equal to B 2 e to the power q a by h cross. So, it tells us that these two factors have to be equal.

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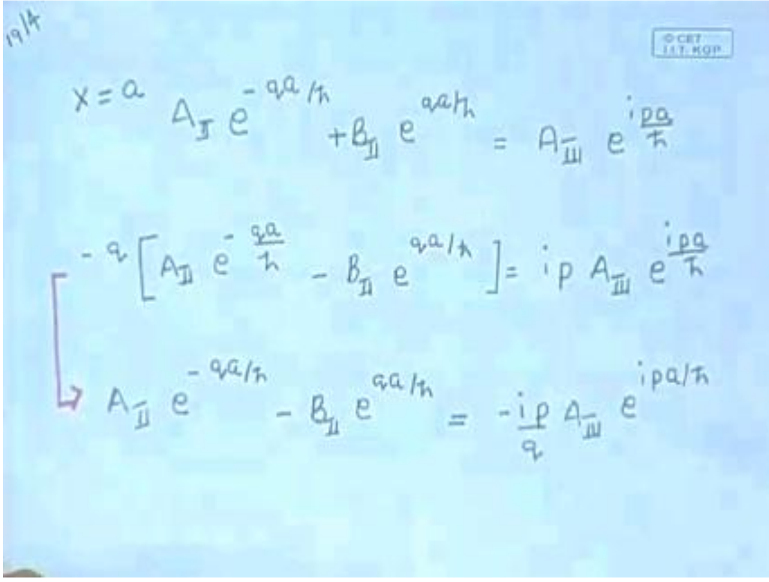

Now if you take this and use it in this expression, which comes from the requirement that the wave function has to be continuous across the boundary. So, what we have seen is that this is equal to this. So, if I take this and apply it here, what it tells us is that this is equal to this. So, I can replace this term by this term.

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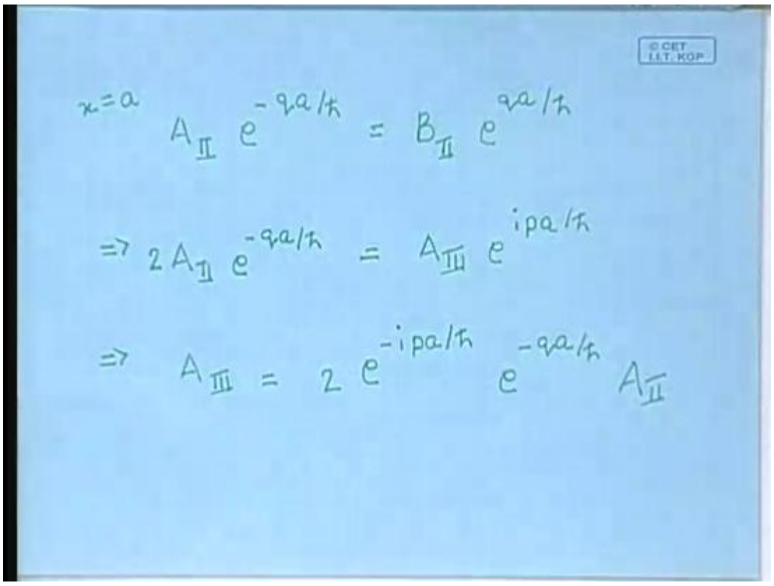

So, what it tells us, is that A 2 e to the power minus q a by h cross is twice A 2 e to the power minus q a by h cross is equal to A 3 e to the power i p a by h cross, or A 3 is equal to 2 e to the power minus i p a by h cross e to the power minus q a by h cross into A 2.So, what we have now is a relation between A 3 and A 2.

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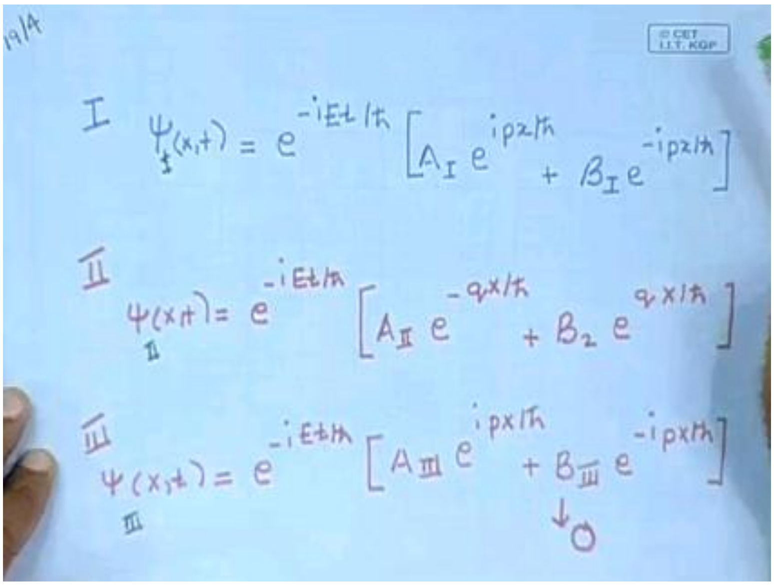

Remember that A 3 represents the wave function on the right hand side of the potential barrier, and it is the part of the wave function, which represents this part of the wave function, represents the particle going out with positive momentum p. So, this is the tunneling wave function. This represents the particle that manages to penetrate through the barrier, and we have obtained the relation between this amplitude and this amplitude. We also see in this analysis that the amplitude of A 2 of this decaying part of the wave function.

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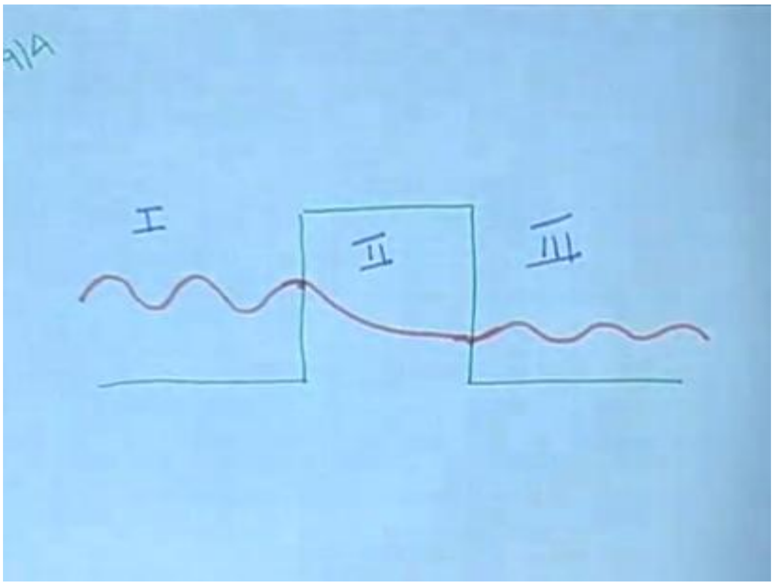

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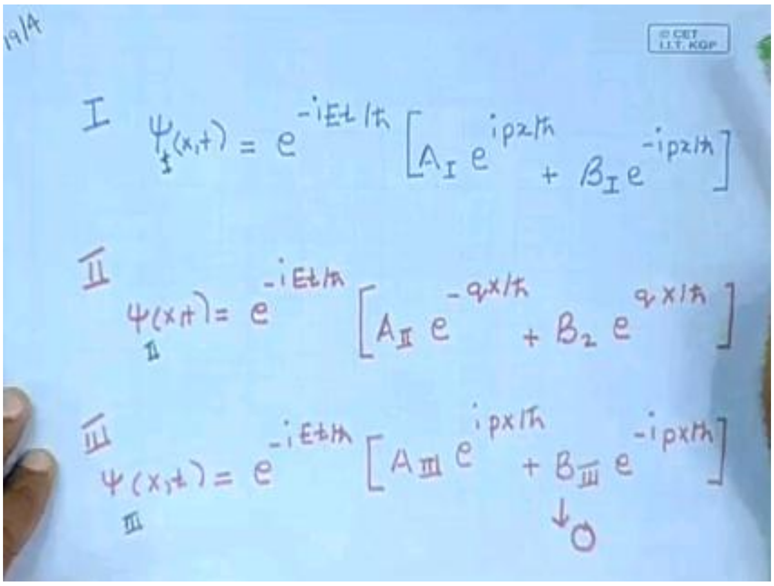

So, inside the potential barrier, the wave function is a superposition of two parts; one which decays exponentially, and one which increases exponentially. What we see here is, that B 2 is equal to A 2 into e to the power minus 2 q. So, this I can write it here also. This also tells us that B 2 is equal to e to the power minus 2 q a by h cross into A 2. So, what it tells us, is that the part of the wave function which increases exponentially has an amplitude, which is much smaller than the amplitude of the part of the wave function, that decreases exponentially. So, this is the dominant term in most of the region.

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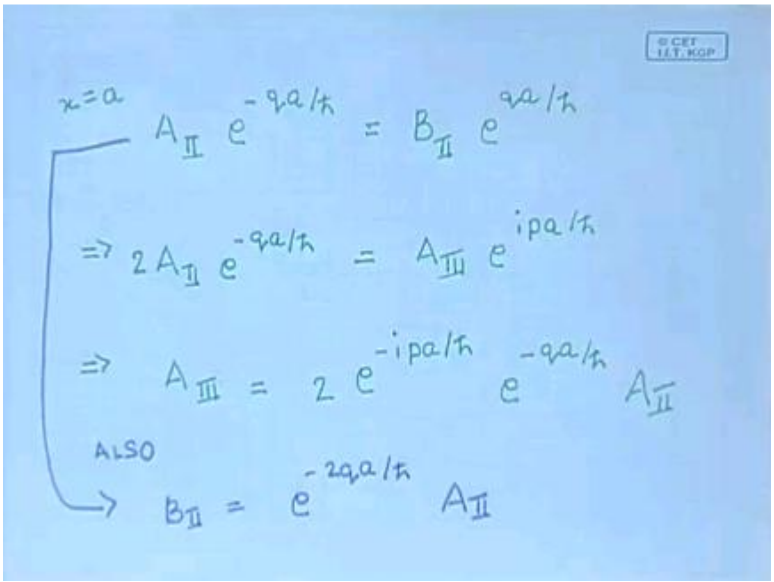

So, B 2 is much smaller than this. So, what it tells, is that the wave function is essentially decays exponentially inside the potential barrier, which is what I have drawn here.

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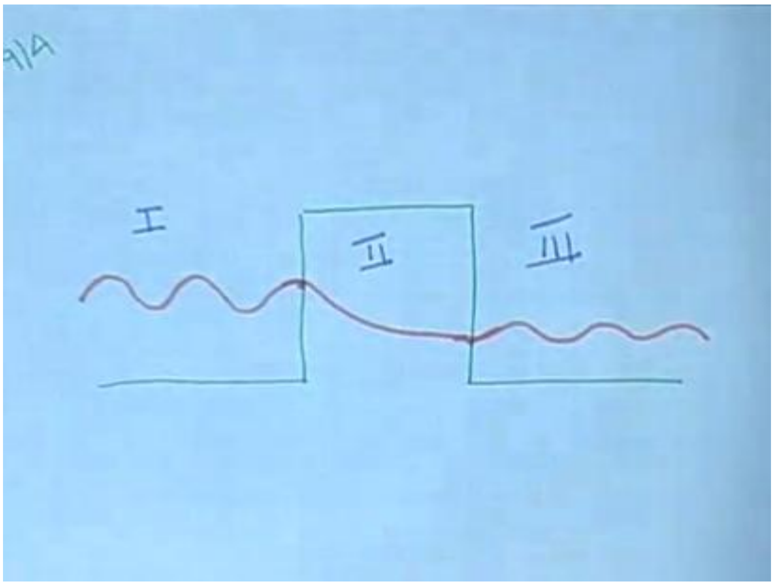

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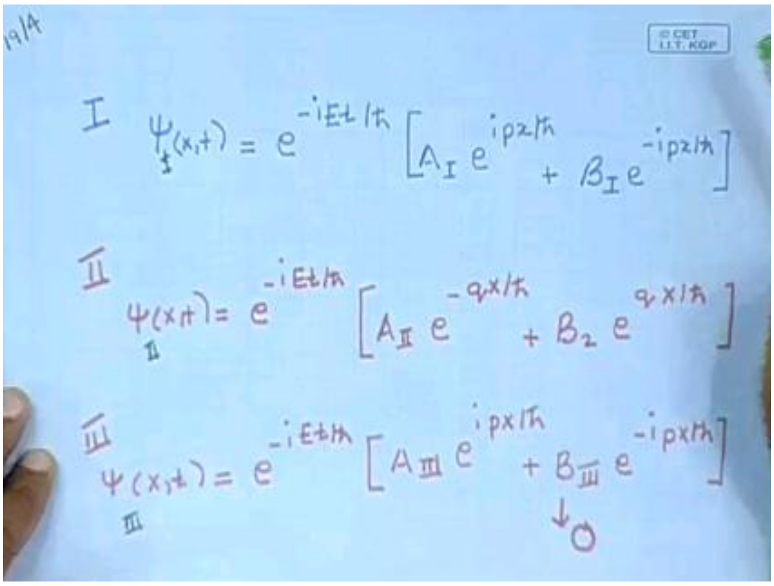

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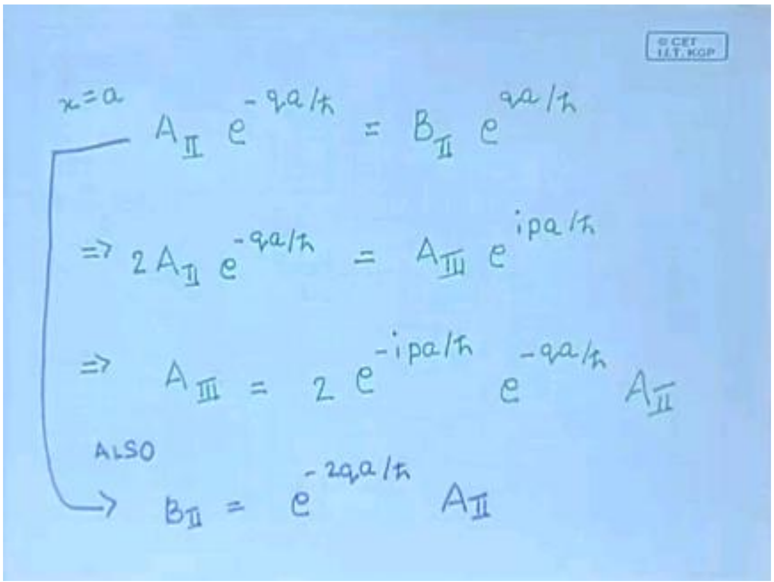

The amplitude of the part that increases are much smaller, which is what we have seen here. So, we have analyzed the boundary x equal to a, and using this we have obtained a relation between the coefficients on the right hand side of this boundary, and the coefficients on the left hand side of this boundary. Let us now analyze the boundary conditions at x equal to 0, and see what it tells us. So, x equal to 0, is what you are going to look at next.

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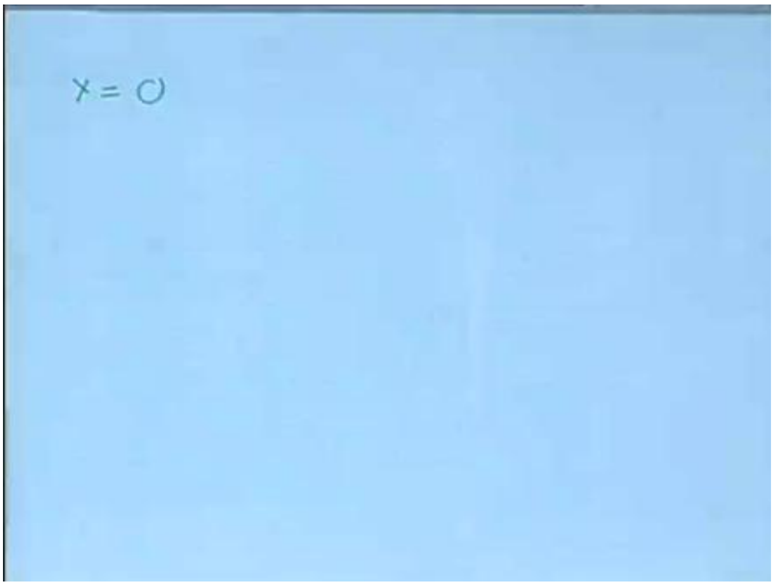

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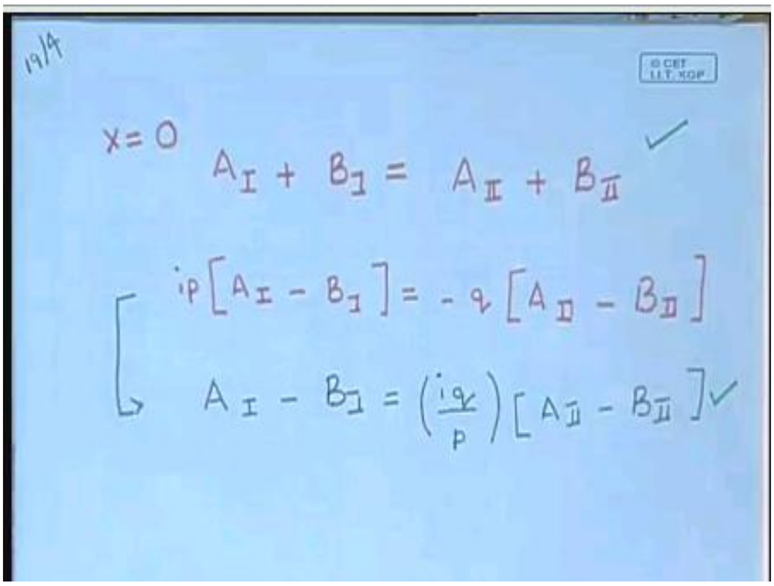

So, these are all x equal to a. I can shift it here. Now we will look at x equal to 0. So, at x equal to 0, we have these two boundary conditions. And what we can do is, we can add these two relations, and if we add these two relations, we will get 2 twice A 1 twice A 1, because B 1 is going to cancel out, twice A 1 is going to be equal to A 2 into. Let us see we have A 2 into 1 plus i q by p.

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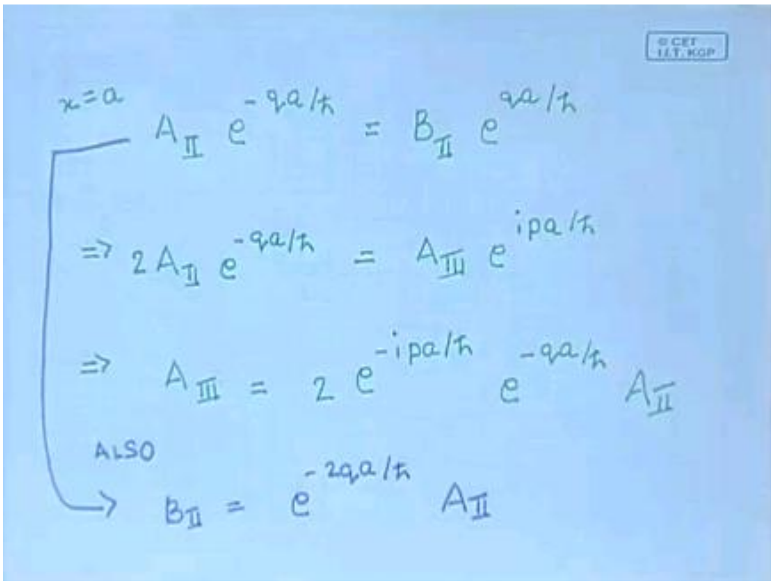

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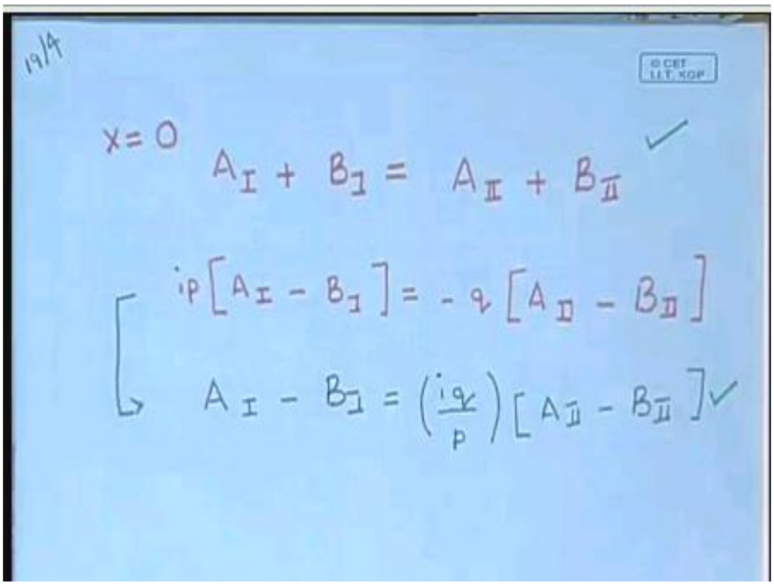

Before that, we just saw that B 2 is much smaller than A 2. So, when we deal with these two. Since B 2 is much smaller than A 1, we can ignore B 2. It is much smaller than A 2, for that reason we can ignore B 2 and deal only with A 2.

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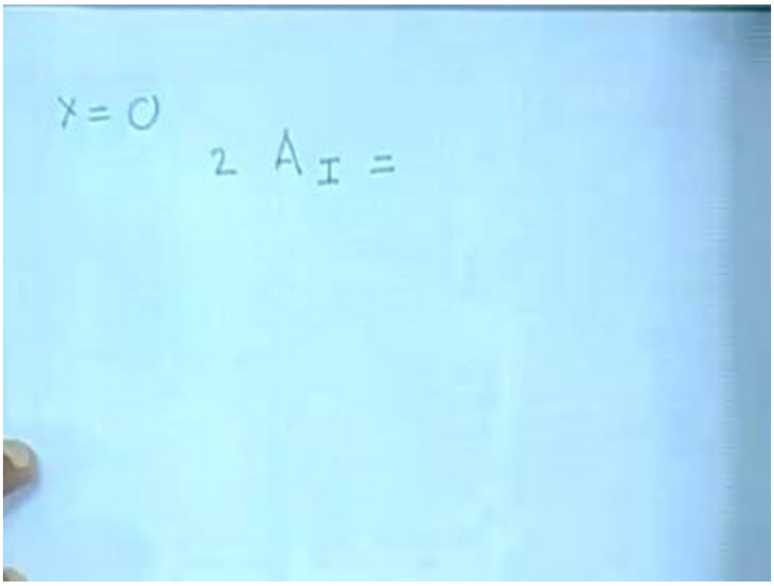

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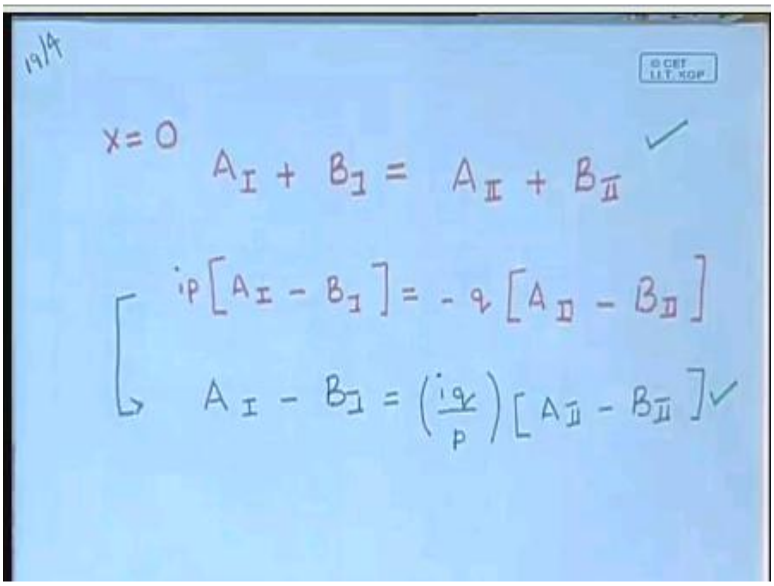

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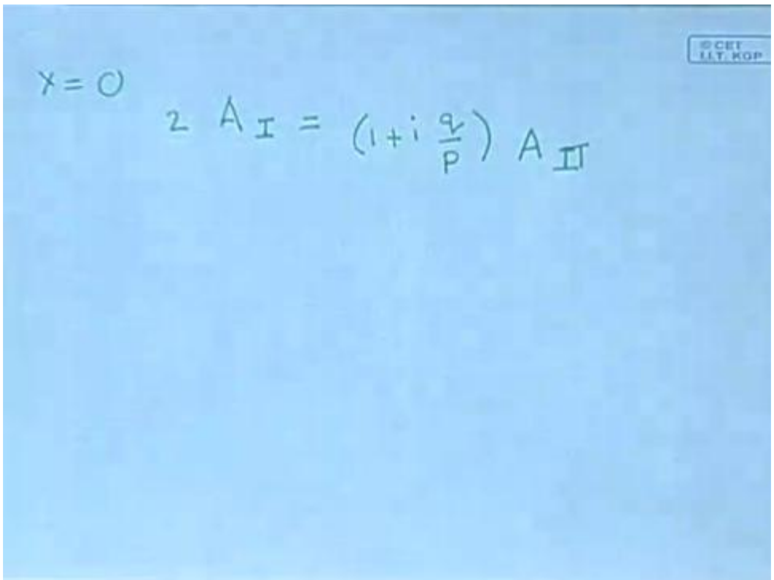

And when we add these two relations what it tells us is, twice A 1 is equal to. This gives us twice A 1 is equal to 1 plus i q by p into A 2.

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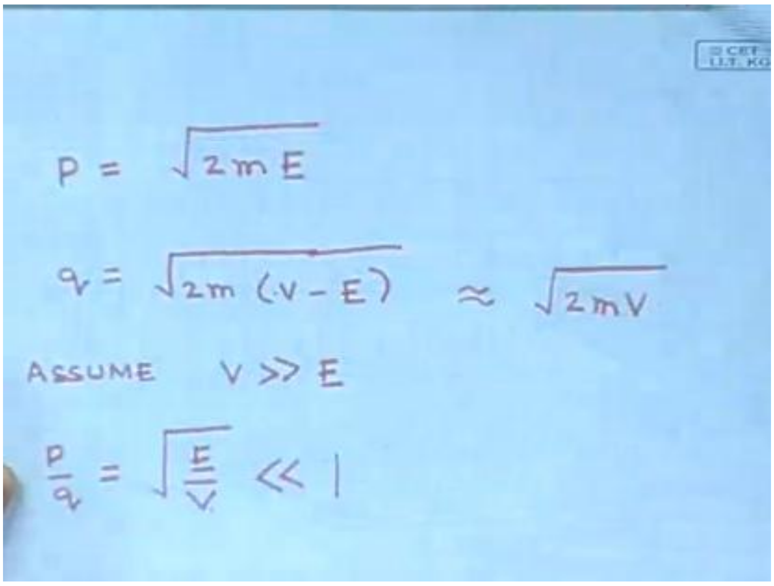

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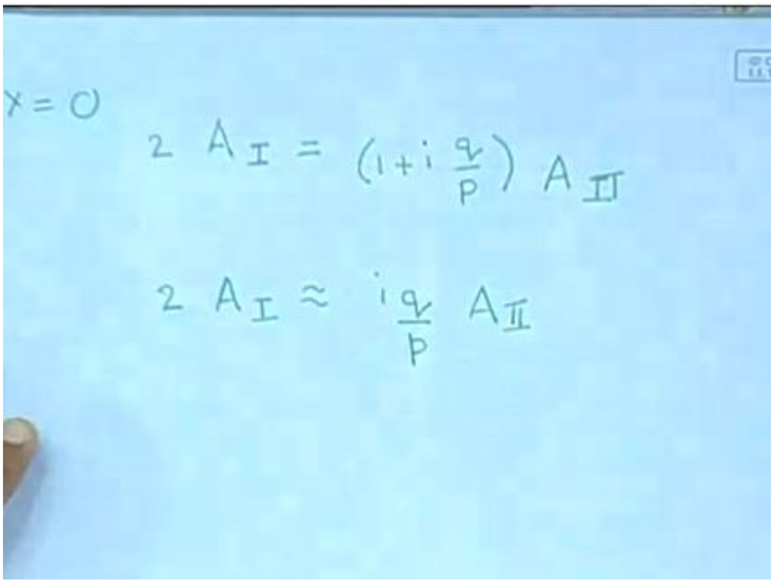

Now we have also made the assumption that p by q is much smaller than 1, or q by p is much greater than 1. So, q by p is much greater than 1. So, what we can say is that, we can ignore this one, because this term is much greater than 1. So, what we get is that 2 A 1 is approximately equal to i q by p into A 2, because this q by p is much greater than 1. We have ignored this factor 1 over here. keeping it would not have made a big difference it would just have made the algebra a little more complicated, that is all, but we can ignore it, to the order of accuracy which we are working. So, we have now a relation between A 1 and A 2.

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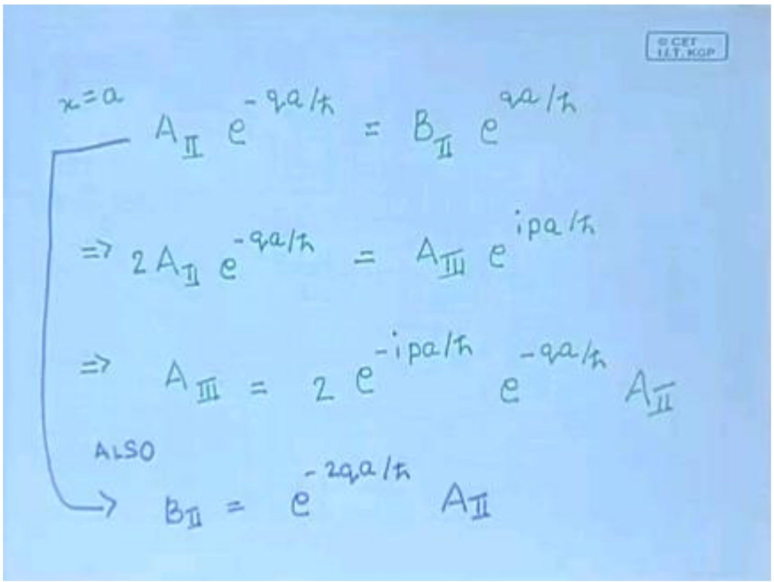

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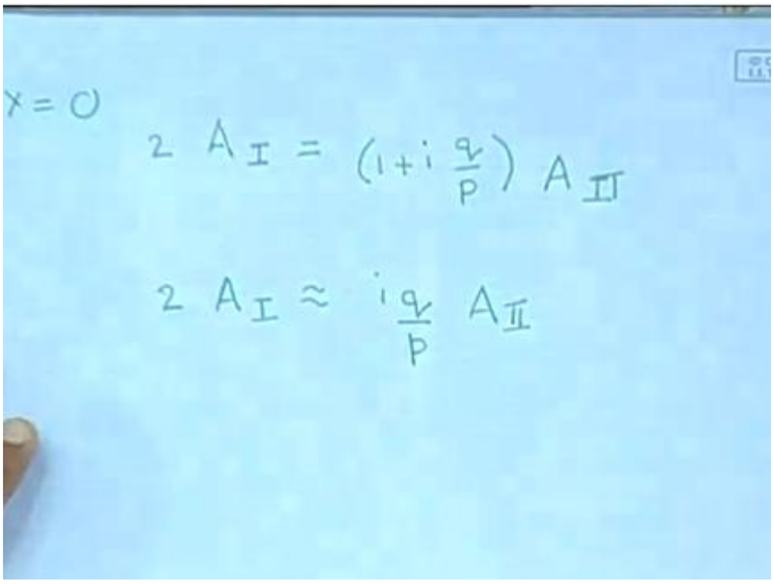

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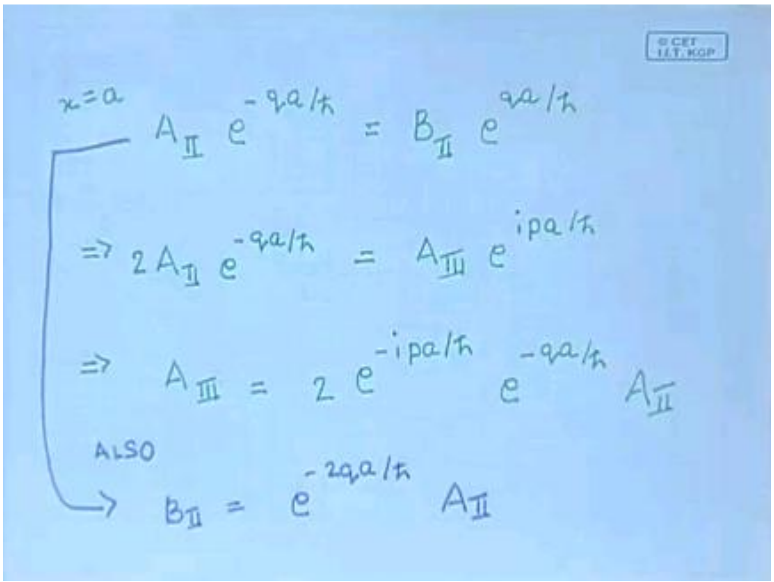

And if you use this relation between A 3 and A 2, we get a relation between A 3 and A 1. So, what we are going to do is, we are going to use the relation between A 2 and A 1. In this relation between A 3 and A 2; to finally, get a relation between A 3 and A 1.

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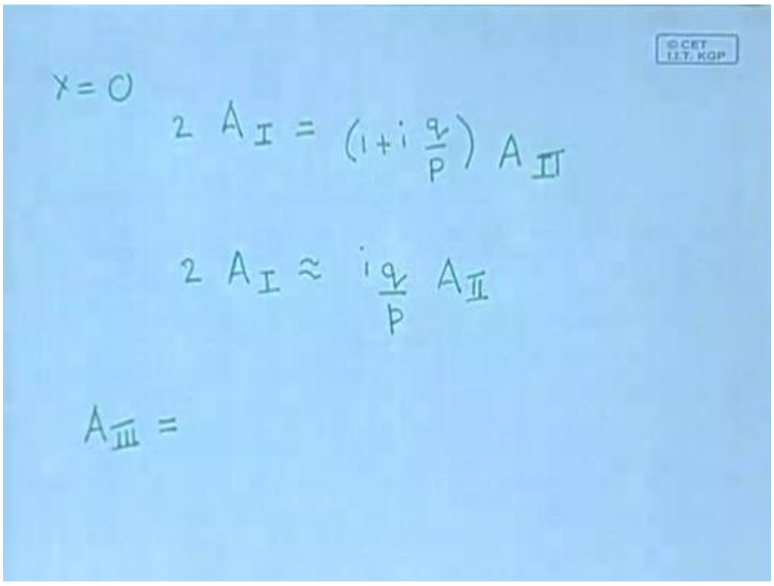

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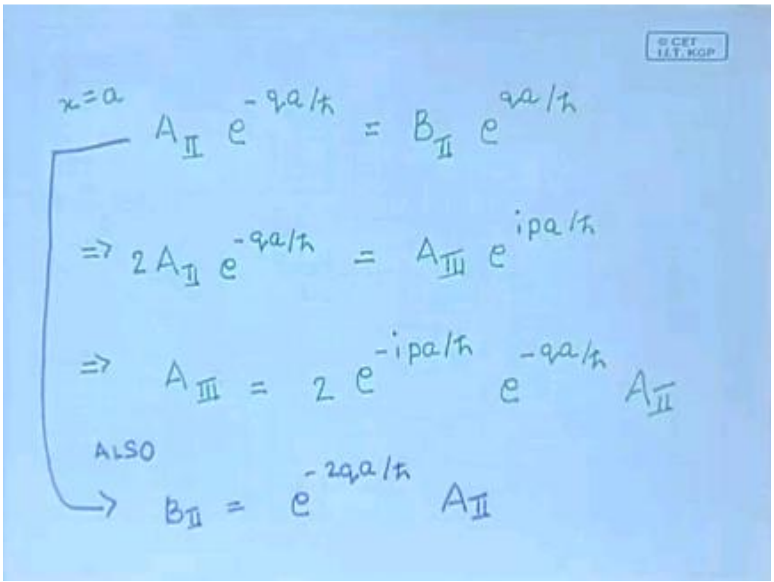

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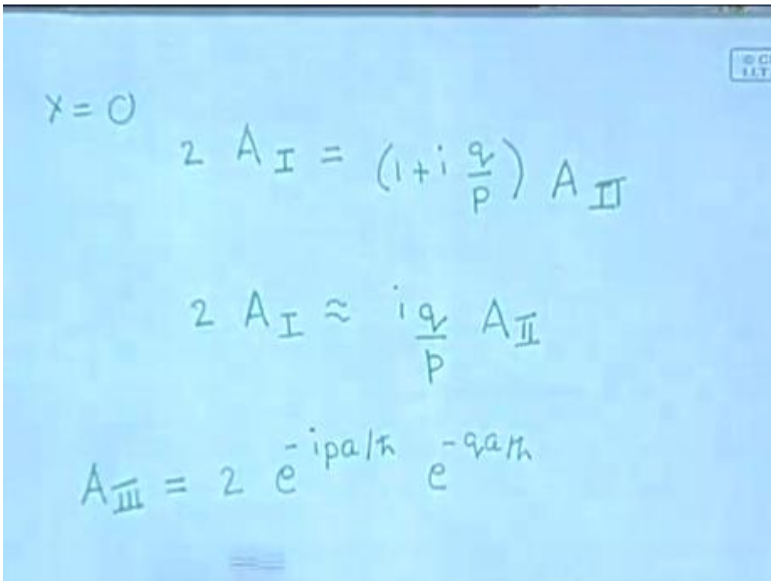

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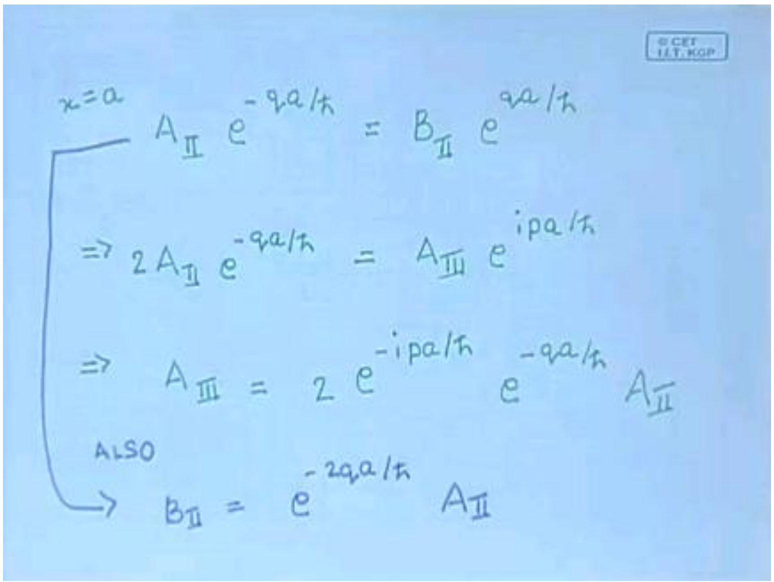

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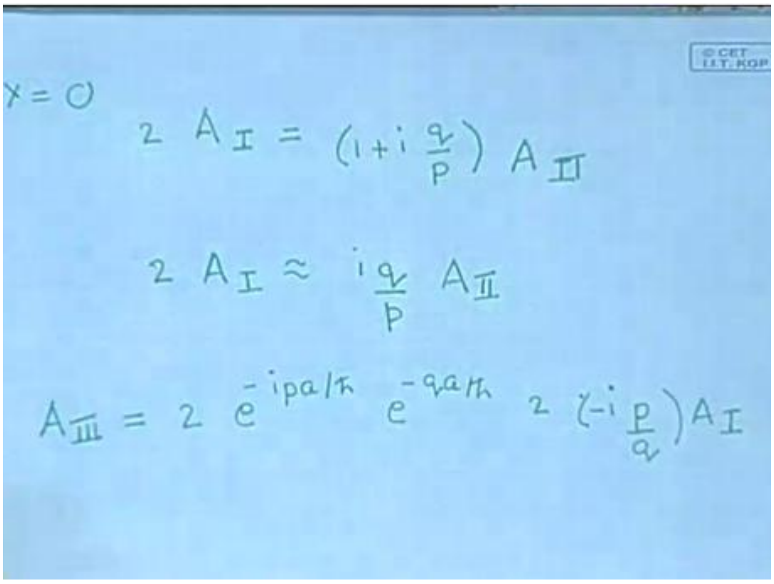

So, let us do that. So, if you do that what it tells us, is that A 3 is equal to. So, we have A 3 is equal to 2 e to the power minus p i a by h cross. So, 2 e to the power minus i p a by h cross, and then we have E to the power minus q a by h cross E to the power minus q a by h cross into A 2, and A 2 is equal to 2. Then I have minus i p by q into A 1. So, we have expressed A 2 in terms of A 1. A 2 is, I have taken the i on to the other side which gives minus i p in the numerator q in the denominator. So, finally, we obtain a relation between A 3 and A 1.

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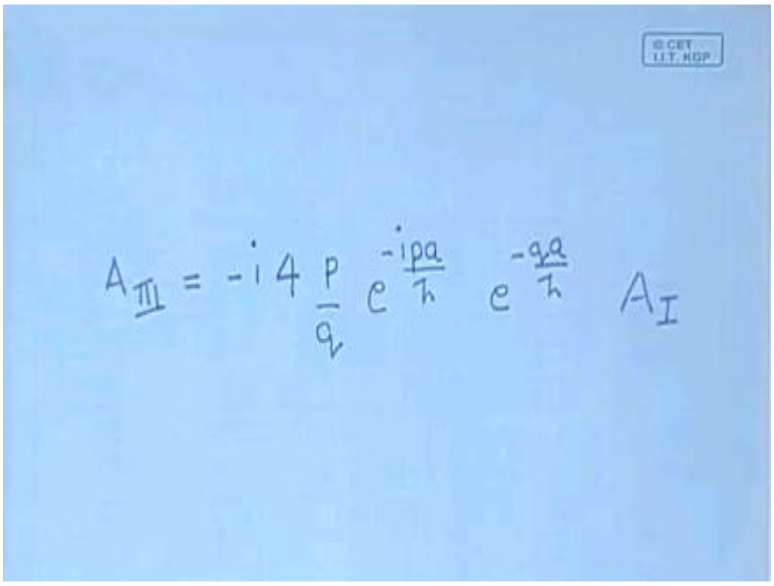

The final relation between A 3 and A 1 that we get is A 3 is equal to minus i 4 p by q p by q e to the power minus i p a by h cross e to the power minus q a by h cross into A 1. So, by applying the boundary conditions, by matching the boundary conditions across these boundaries, we finally have a relation between A 3 and A 1. A 1 is the amplitude of the wave corresponding to the incident particle. A 3 is the amplitude of the wave corresponding to the particle, which emerges on the other side of the barrier. So, let me show you these again.

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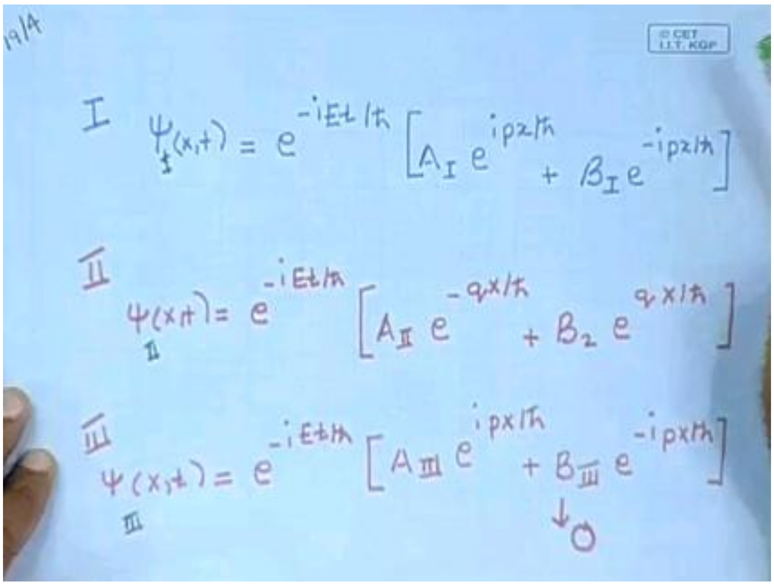

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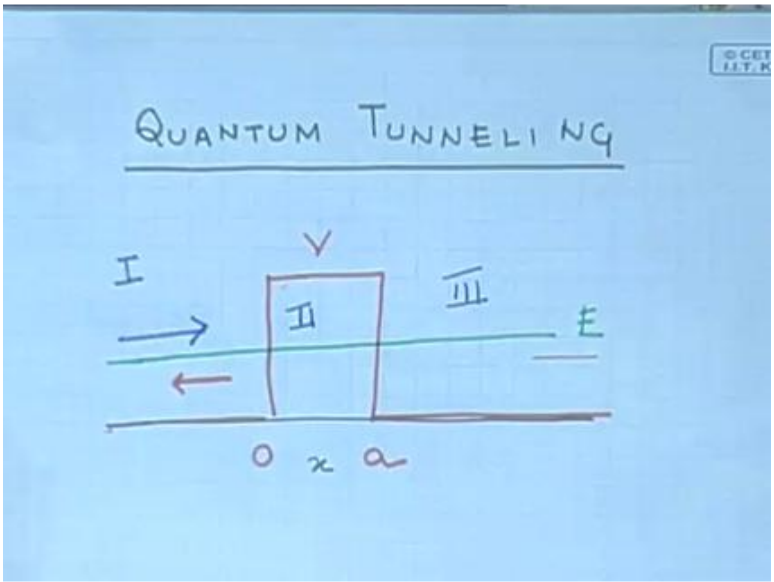

A 1 is the amplitude of the incident wave. So, A 1 represents the particle coming in. B 1 represents the reflected particle, and A 3 represent s the particle which comes out on this side, and A 1 B 1 and A 3 are the amplitudes of these corresponding waves. Now, remember how to convert amplitudes into probabilities. The probability is the modulus of the amplitude squared. So, if you ask the question, what is the tunneling probability or the transmission coefficients?

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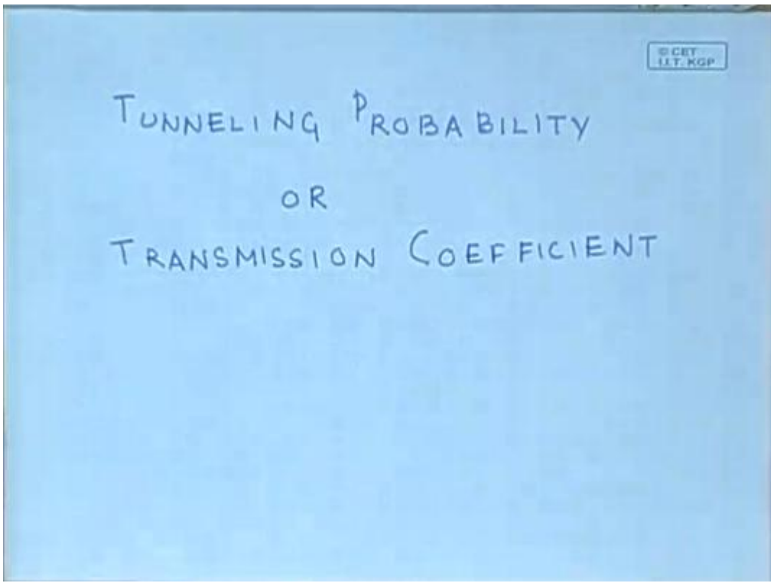

So, let me write it here, the tunneling or transmission. The tunneling probability, or the transmission coefficient T, is the probability that the particle incident from the left hand side tunnels through the potential barrier.

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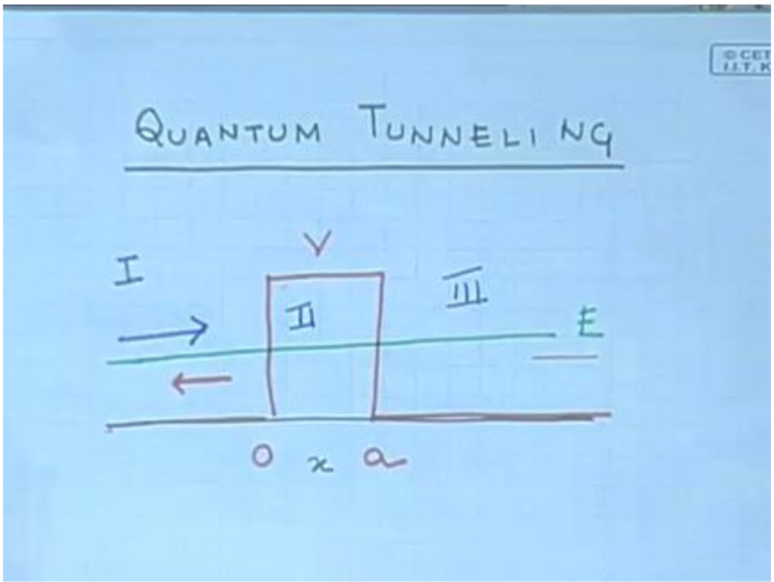

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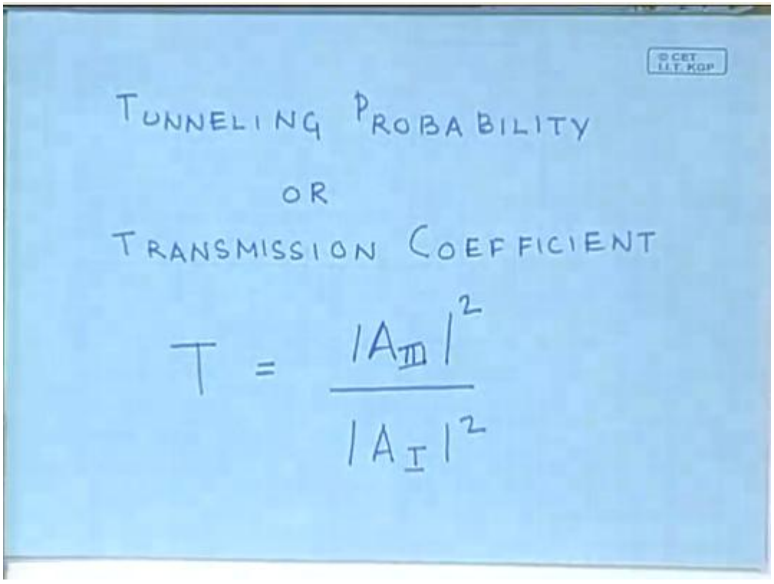

So, the tunneling probability or the transmission coefficient, is the probability that the particle that comes here, is found in this region, and this probability of finding the particle in this region is the ratio of the modulus of A 3; the amplitude of the transmitted wave divided by the amplitude of the incident wave. This is the transmission coefficient or the tunneling probability.

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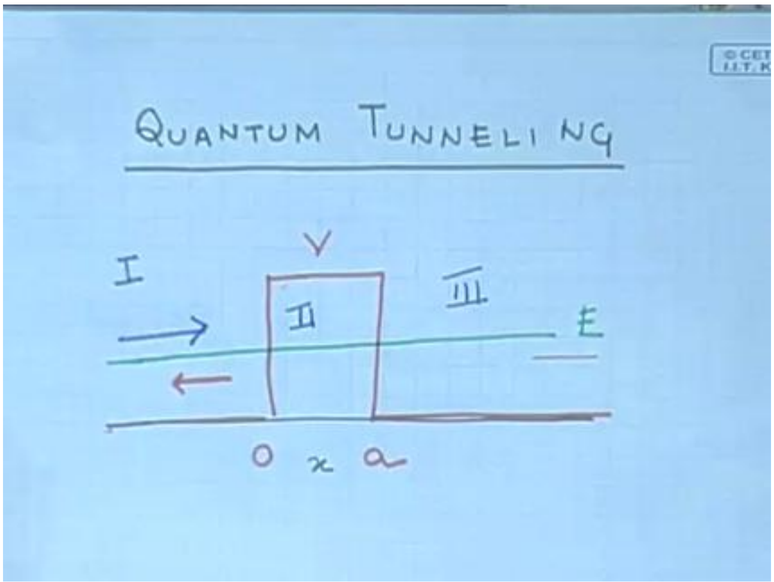

So, this tells us, what is the probability if I send the particle from the left hand side, what is the probability. I send the particle from here what is the probability of finding the particle in this region.

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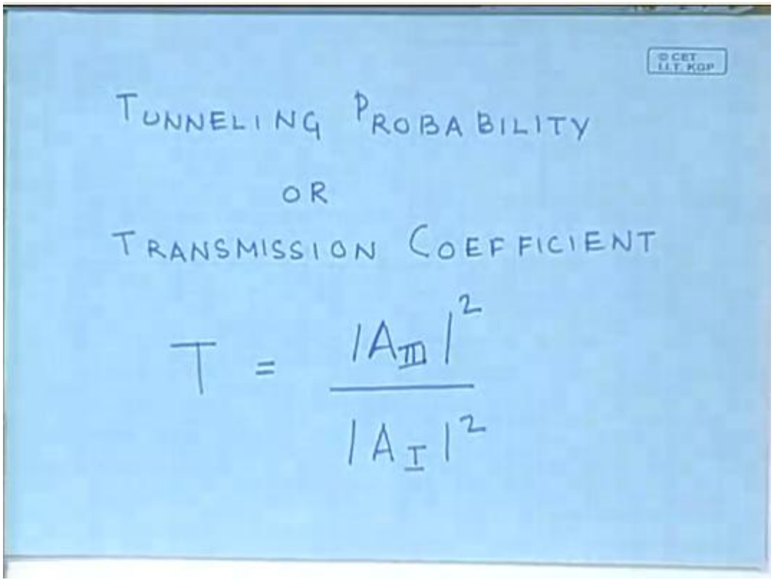

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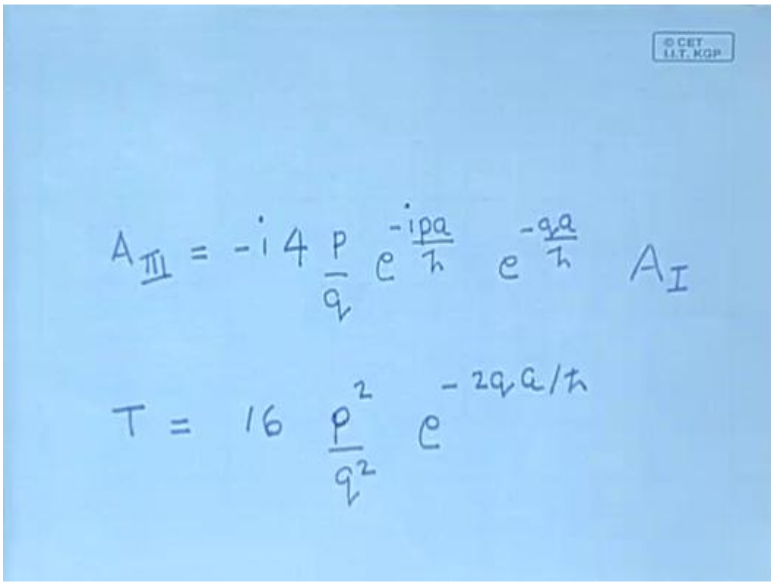

We are now in a position to calculate this. So, let us calculate it here. So, we see that the tunneling probability T is the modulus square of this. So, we haves 16 the modulus of i minus i is 1. The modulus of this factor. It is a face factor e to the power i phi that is also 1. The modulus of this squared, is going to be e to the power. I have the square of this first. So, I have p square by q square e to the power minus 2 q a by h cross. So, this is the transmission tunneling probability, or the transmission coefficient. This tells us the probability that the particle will get through the barrier. You can also ask the question, what is the probability that the incident particle, is reflected back, and this is given by the reflection coefficient.

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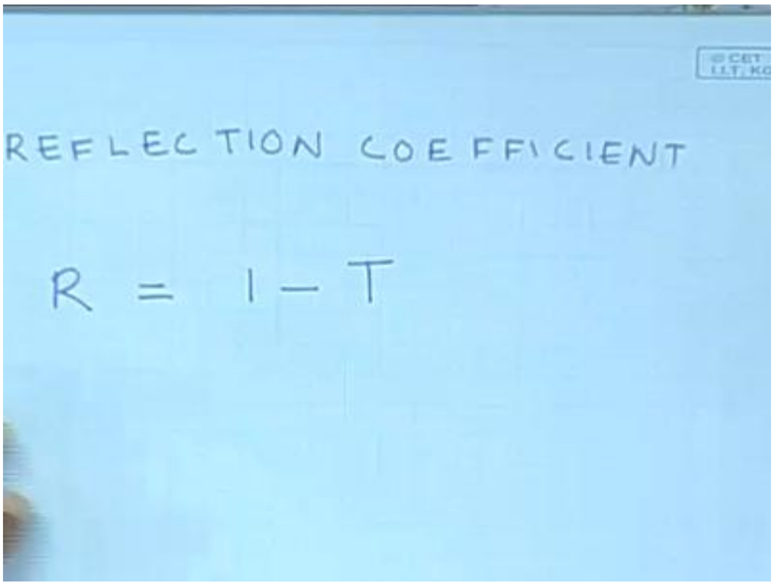

R which is 1 minus T. The total probability of the particle getting reflected and the particle tunneling through has to be 1, because these are the only two option open; the particles has to either tunnel, and get on to the other side, or it has to be reflected. So, the reflection coefficient which tells us the probability of the particle getting reflected, is 1 minus the transmission coefficient, or 1 minus the tunneling probability.

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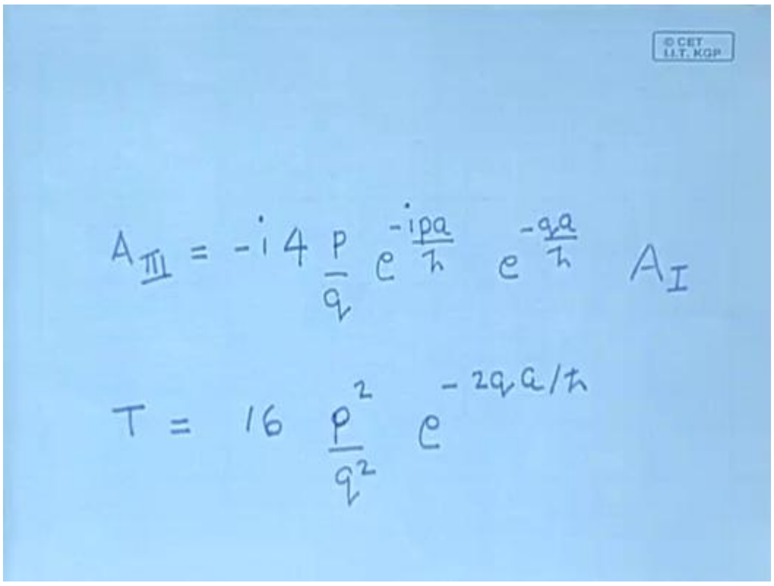

Now, let us analyze the tunneling probability. To do that let us write it in terms of the energy of the particle and the height of the potential barrier. Remember we are working in the limit, where the potential barrier is much higher than the value of the energy. So, this ratio p square by q square.

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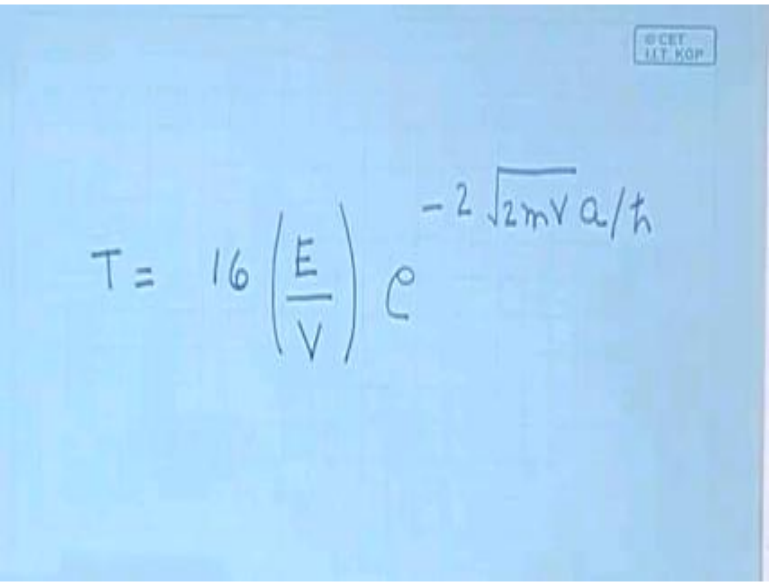

If I put in the fact that p square is 2 m e q square is 2 m V, then the tunneling amplitude T can be written as 16 p square by q square is E by V, and we have assumed that this ratio is very small. So, the tunneling amplitude is a very small. The tunneling probability is very small. Most of the particles are going to get reflected into e to the power minus 2. Then q is the square root of 2 m V into a divided by h cross.

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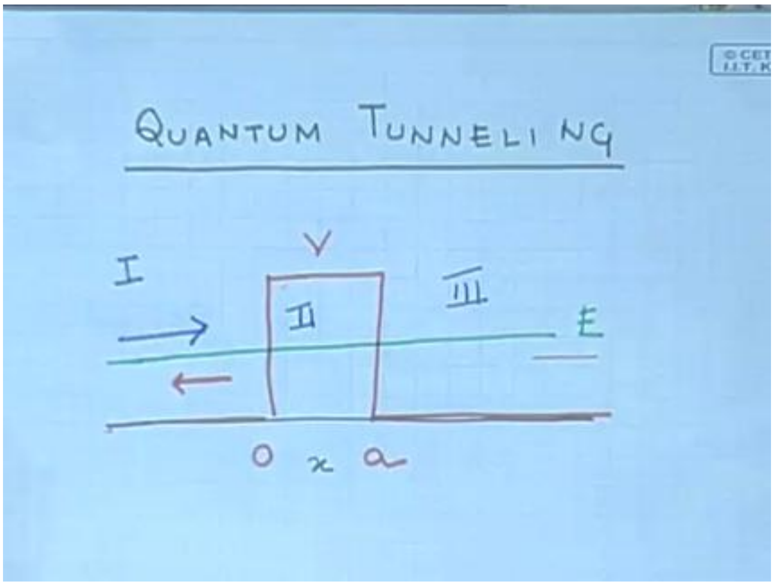

So, that is the probability that a particle incident from the left tunnels through, and is found on the right. Now let us ask the question how does this tunneling probability change if I increase or decrease the potential barrier. And it is clear over here that if I increase the potential barrier keeping the energy of the particle fixed, the tunneling probability is going to decrease on two counts. It is going to decrease, because of this one by V factor over here. So, as I increase the potential the tunneling probability falls of as 1 by V over here. It also falls off as the as through this exponential factor, because this e to the power minus 2 m square root of m V into a divided by h cross. So, this by increase V this is exponent is also going to fall, and the tunneling amplitude falls, because of both of these. So, the basic message is that the higher the potential barrier, the smaller the probability of the particle getting through the potential barrier. Now, let us ask the next question, there are two things that we can vary the other thing that can vary here, is the width of the potential barrier. How does the tunneling probability change if I vary the width of this potential barrier.

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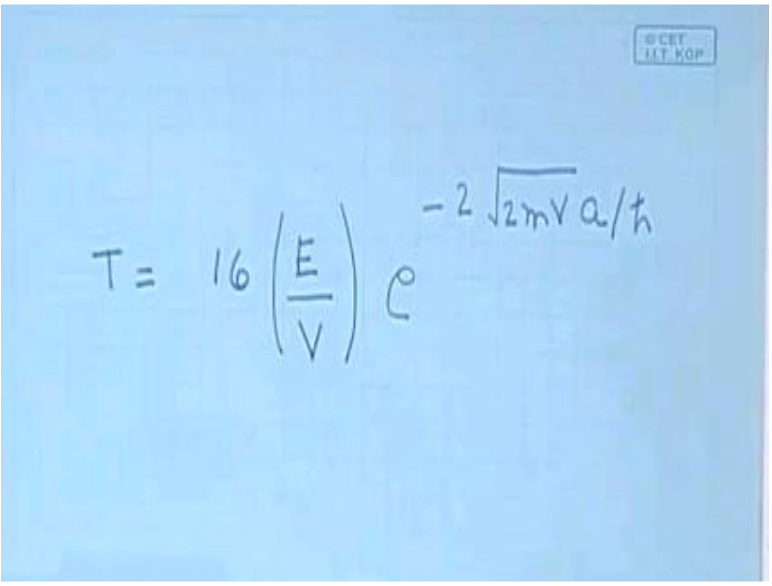

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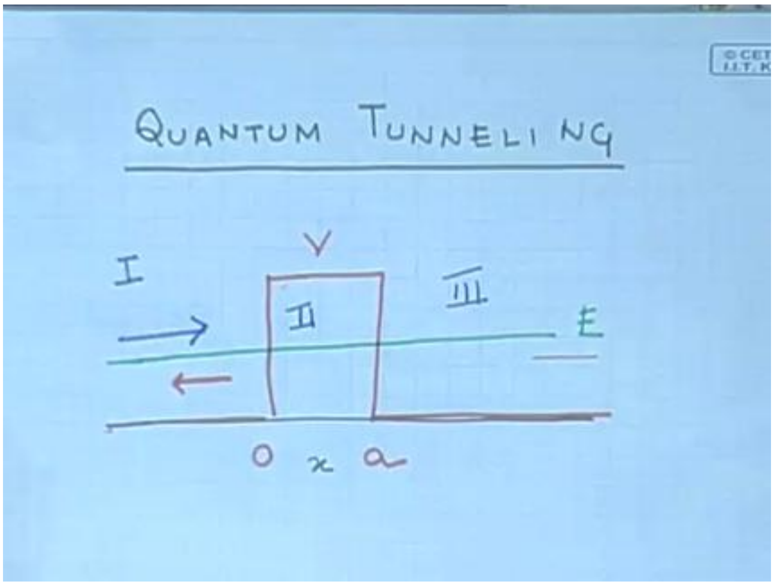

So, what we see is that, if I make the width of the potential barrier larger that tunneling probability decreases exponentially. So, this tunneling phenomenon is going to be particularly important if I have a barrier which is quite small. If the barrier is very small; the smaller the barrier. The smaller the width of the barrier, the thinner it is. The larger is the probability of the particle tunneling through and getting on to the other side. So, we seen that if you make the potential higher. The tunneling probability falls if you make the barrier wider again, the tunneling probability falls and tunneling is going to be particularly important if the barrier is quite thin.

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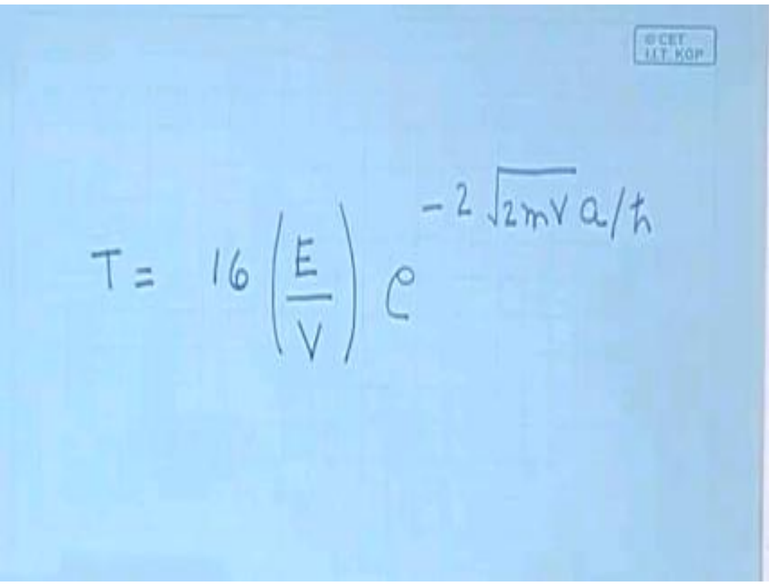

Let me also remind you that the calculation that we have done, has made an assumption. It has assumed that the potential is much higher than the energy if you do not make this assumption m. you can again repeat the same calculation matching all the boundary conditions. Just that the algebra gets a little more that is all, but you can still calculate the tunneling probability. It will have the same kind of dependence on the potential and the width, and you will recover the expressions that we have obtained if you take the limit where V is much larger than E. Another point which I wish to make is, that you can draw a some messages from this simple calculation, which you can carry over to a situation where we do not have a Step barrier, but we have a more complicated looking barrier.

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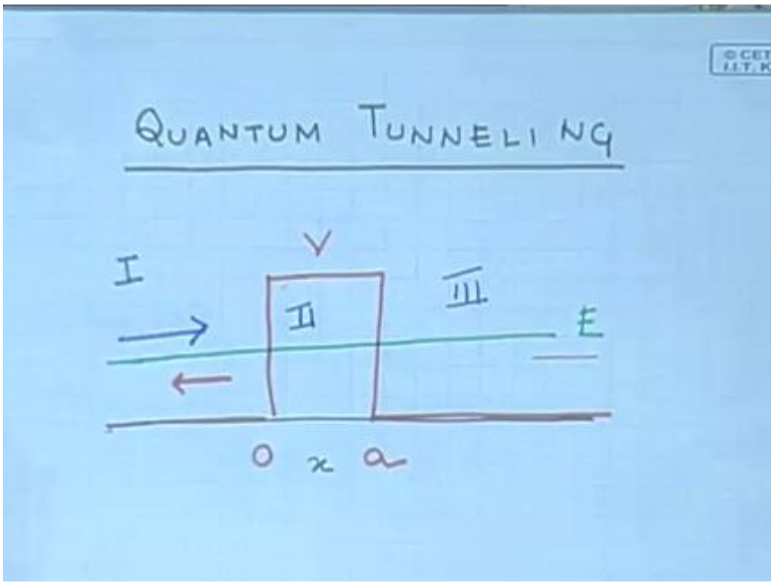

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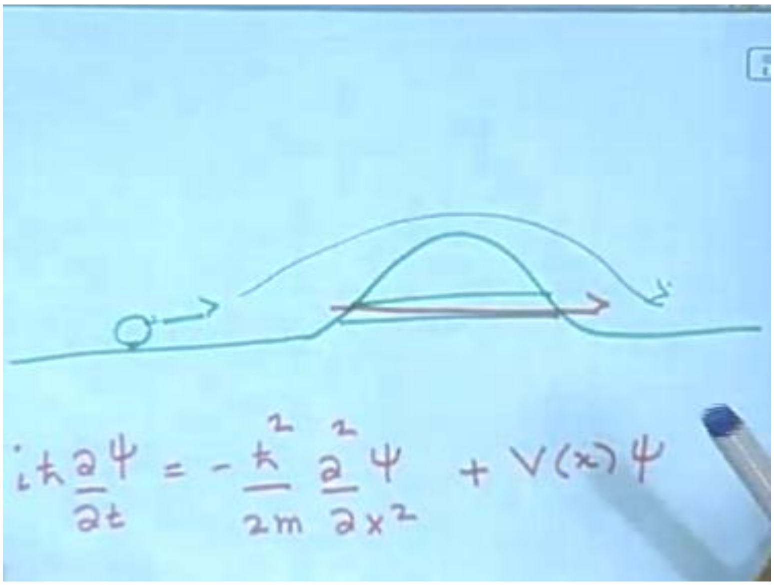

So, the barrier could in general be more complicated looking for example. I had drawn a picture like this, and you can calculate the tunneling probability, the transmission coefficient, reflection coefficient in a situation like this also.

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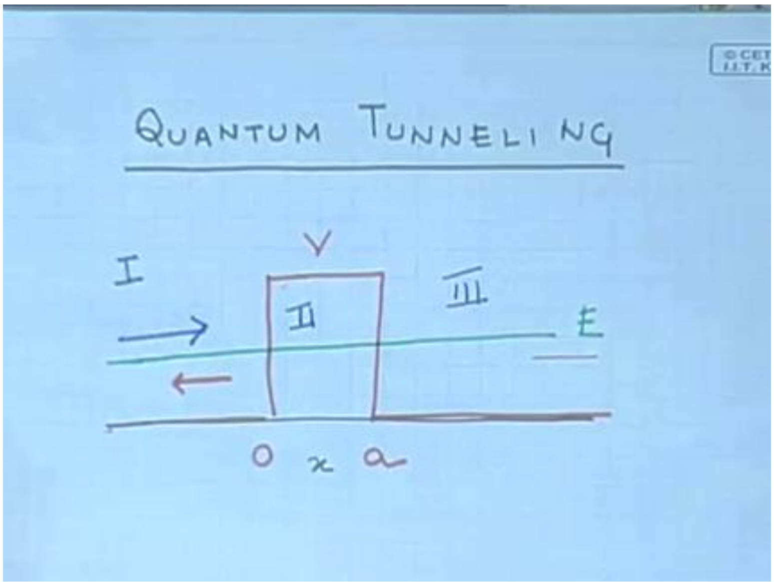

The calculation is going to be a little more complicated, but you can take the same kind of message. If you increase the height of the potential barrier, the tunneling probability is going to go down. If you increase the width of the potential barrier, the tunneling probability is going down, and you have the phenomena of tunneling. Now, the phenomenon of tunneling occurs, in a variety of situations in nature. It also has various technological applications. For example you have alpha decay, where a nucleus spontaneously emits an alpha particle. Now, you can get an understanding of this process alpha decay, in terms of the phenomena of tunneling.

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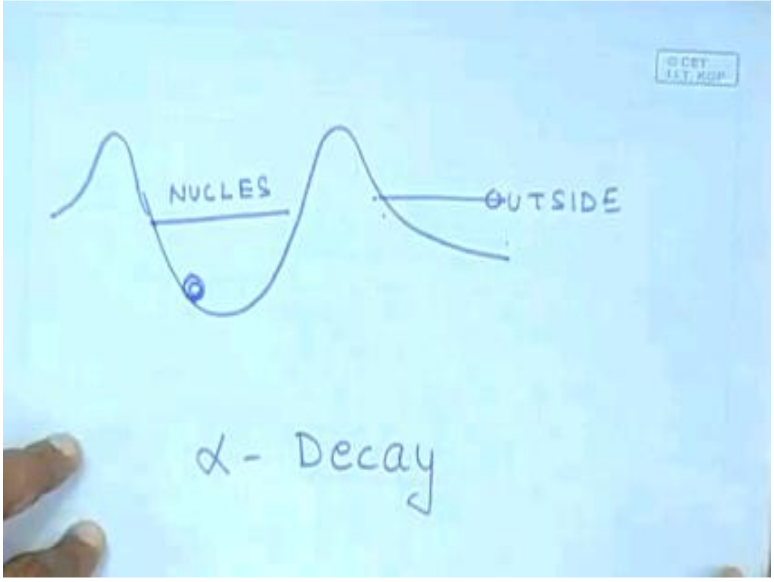

So, inside the nucleus the alpha particle, is bound. So, this is the nucleus, and this is outside of the nucleus. And an alpha particle alpha particle is a helium nucleus to neutrons and to protons. So, inside the nucleus the alpha particle experiences a potential, which is sufficient to keep it bound. To come out from the nucleus the alpha particle has to surmount a barrier, because the particle is held inside by nuclear forces. So, to come out from it, the alpha particle has to surmount a barrier. So, if you could give this energy to the alpha particle from outside, the alpha particle would be able to surmount this barrier and come out. But what happens in alpha decay is that, the particle somehow is spontaneously comes out from the nucleus, without any external energy being provided. And you can understand this as a tunneling process. So, this alpha particle which is their inside this potential well, finite potential well, it tunnels through this barrier. So, it tunnels through this barrier and it comes out. So, this is the alpha particle. It tunnels through this potential barrier, and it comes out. So, you can get an understanding of this alpha decay of nuclei in terms of tunneling, so this is alpha decay. Let me also tell you about another technological application of the phenomena of tunneling. You might have heard of the tunneling diode, but I am not going to discuss that, the application that I am going to discuss, is the scanning tunneling microscope. So, there is a technological application a device call the scanning tunneling microscope, which is a very interesting application of the phenomena of tunneling.

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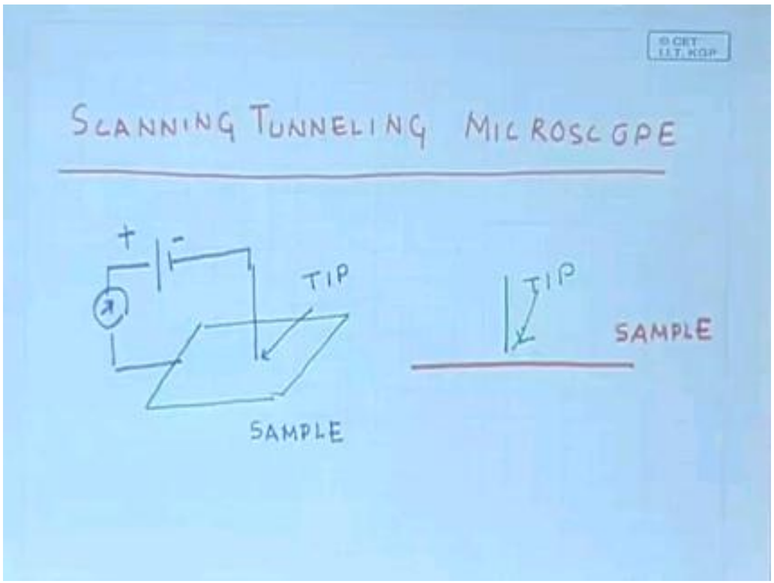

So, let me draw a schematic diagram of a scanning tunneling microscope, and then explain to you how it works. So, you have a sample in this scanning tunneling microscope, you have a sample. So, for any kind microscope you always have a sample, which you wish to image. So, this shows the sample of the scanning tunneling microscope. This is the sample, and the microscope has a tip. This is the tip which is the part of the microscope. This is the sample, which is not a part of the microscope, which is. So, this is the sample which is being imaged by the microscope, and there is a tip here, so there is a very sharp pointed tip which is used in a scanning tunneling microscope. The tip is of the order of few Armstrong, so this is the tip. The tip over here at the edge is of the order of few Armstrong. And this is put all put in a circuit.

So, there is a circuit here, where there is an ammeter and a bias, there is a bias applied, there is a positive voltage applied to the sample, and there is a negative voltage applied to the tip. If you draw a section, then the sample looks like this. I am drawing a section and the tip is held just above the sample. It does not touch the sample; there is a gap between the tip and the sample. So, the tip is over here, and there is a gap between the tip and the sample. Now, let us try to understand what will happen in a situation like this.

The circuit over here is not closed. There is a gap, this circuit is not closed, because there is a gap between the tip and the sample. And as a consequence you do not expect electrons to flow. It is like a capacitor, because there is a gap over here. So, you might think that this is like a capacitor. Now let us draw the potential which an electron experiences inside the tip, in the intervening region in between, and again inside the sample.

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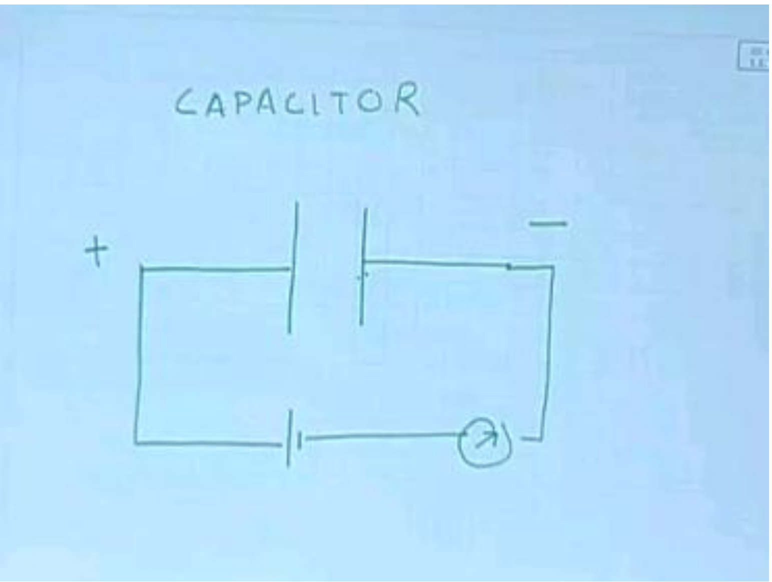

Now, if I had, let us just discuss this situation, if I have a capacitor for example, these are two capacitor plates, and I gave a positive bias voltage to one, and I gave a negative voltage to the other, why does not a current flow across the capacitor. After all we have applied a negative potential here. So, there are electrons here, if the electrons could jump from here to this plate they would go from a negative potential to a positive potential, and they would gain energy in this process, and the circuit would be closed. So, the question is, why do not the electrons jump from this metallic plate from this surface to this surface. The circuit would be closed and the electrons would gain energy in this process, and then they would, I mean lose the energy also, because of resistance, but there is a force acting on the electrons on this surface, because this side is positive. So, why do not they leave the surface and go on to that surface.

If we just think about it, you will realize that the electrons inside this surface have to be given, are at a lower potential than a free electron outside. If you take a metal surface, the electrons on that surface metal surface, are not free to leave the metal surface at will. They have to be given some energy, and if you are familiar for example, with the photoelectric effect, you will know that this energy is referring to as the work function. So, electrons inside any metal, or inside any material for that matter, are not free to leave it at will. You have to impart some energy to these electrons, so that they can overcome this potential which is holding them inside the material. So, for a metal this is called the work function, if you give that much energy to an electron on the metal surface, it will come out. And if you can give it by shining an ultraviolet ray, or you can heat the metal and cause some electrons to be emitted.

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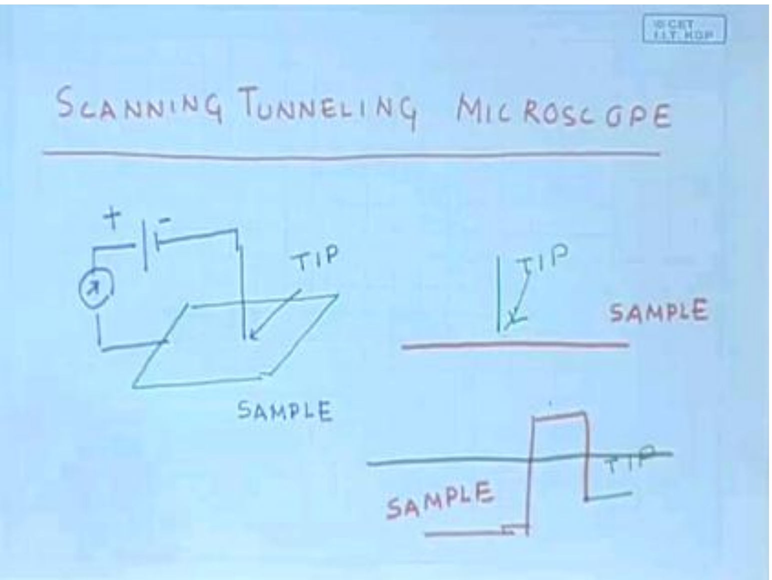

So, the reason why you do not have a current here, is because the electron has to be is inside a potential well in this region. It is inside a potential well in this region, and the two regions are separated by a potential barrier, because the free electron has a higher energy than the electron which is inside the tip, or the electron which is inside the sample. So, let me draw the potential of an electron inside the tip, in the vacuum in between and again in the sample. So, an electron inside the tip has experiences some potential. This is in the tip. Now, a free electron does not experience a potential. So, outside the tip, you can think of the electron is being free. So, the potential outside, inside the tip is going to be lower than the free electron. So, a free electron is going to be at a higher. So this is vacuum, and this is again the sample. The sample is at a lower potential than the tip, because it has been positively biased. So, an electron inside the tip is at a lower potential than a free electron. If the tip were metallic for example, and you wanted to get the electron out you would have to give this much energy, and this is what is called the work function. Similarly an electron inside the sample is also at a lower energy than an electron outside. And you would have to impart this energy difference, if you wanted the sample to, an electron to come out form the sample. So, if you had a large gap between the tip and the sample, the electrons here. Let us say an electron here has this energy. Electron in the tip let us say, has an energy which is this much. So, the electron inside the tip would not be able to come out, because it does not have sufficient energy, and it would not be able to go into the sample. So, you would have no current flowing over here.

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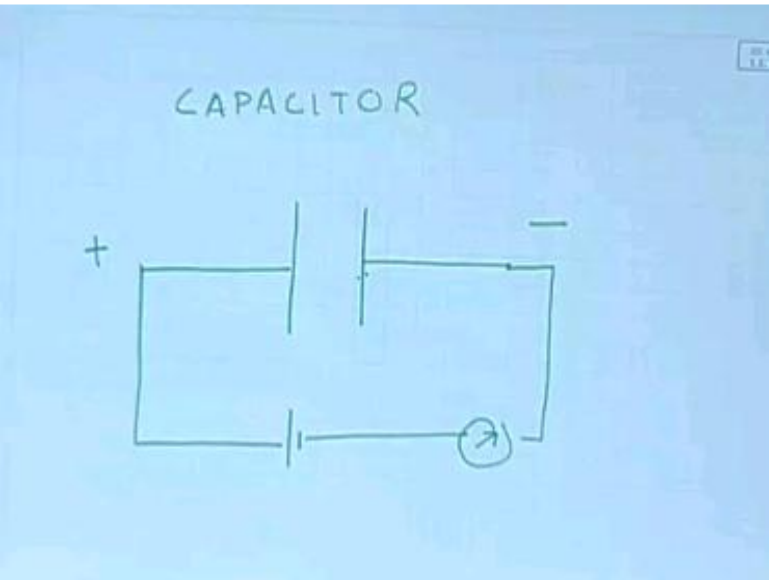

If it is exactly the same reason why you do not have current in a circuit like this, because the electron inside this capacitor plate, is at lower potential, and it does not have sufficient energy to come out. The energy outside is more. The potential energy outside is zero, whereas, inside it is negative. here also it is negative.

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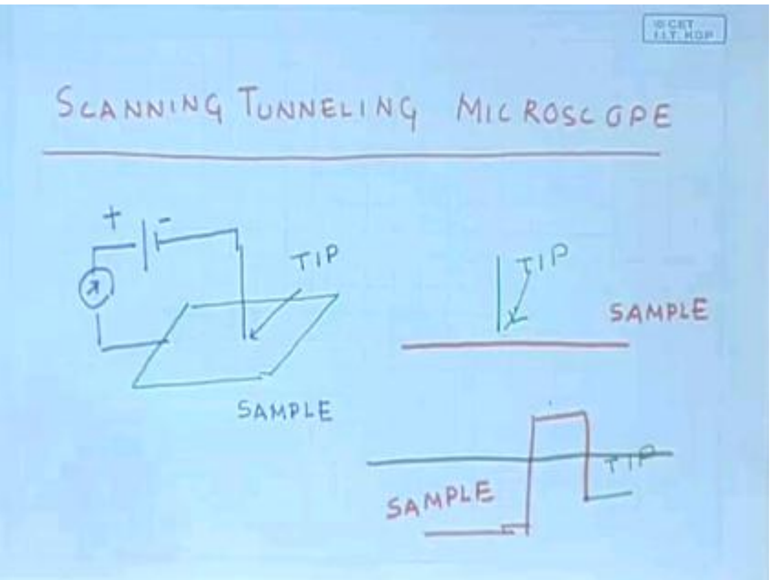

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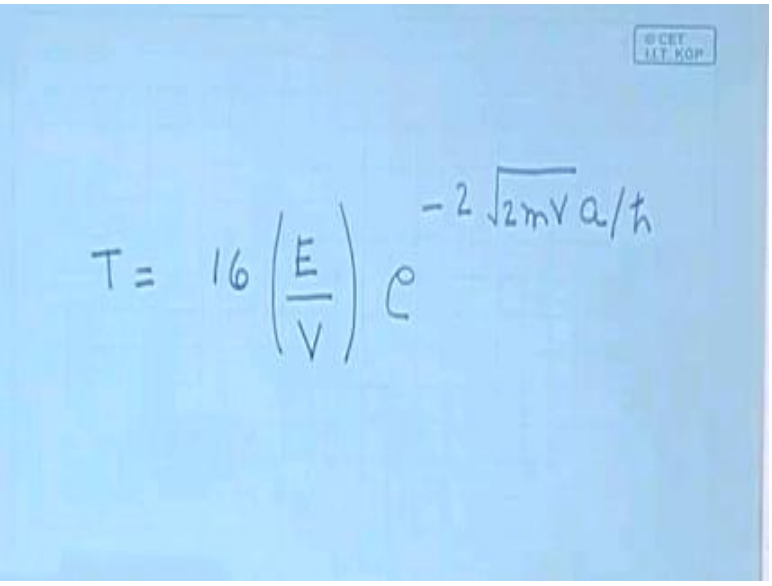

So, exactly the same thing happens over here, but if you bring this tip sufficiently close to the sample, so that the gap between them becomes extremely small. Then what happen is that you can have tunneling between the tip and the sample, and the probability tunneling probability is what I have shown, what we have calculated. So, if you make this a very small, there is a tunneling probability. So, there is a probability of the electron going from the tip to the sample, even though its energy is not sufficient to overcome this barrier.

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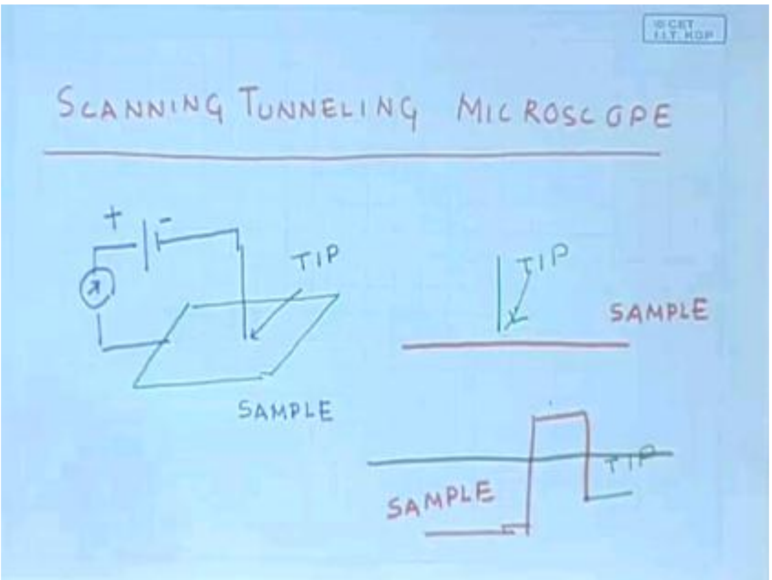

So, the electron does not have sufficient energy to leave the tip and go into the vacuum outside, and then from there go into the sample, but even then I can have an electron going from the tip to the sample, provided this gap is sufficiently small. So, in this scanning tunneling microscope what is done is, that the tip is brought very close to the sample, and you then have a small current which flows, because of tunneling. And the small current that flows, this is a small current which flows. Now, what is done in the scanning tunneling microscope, is that the tip is slowly moved across the sample. It scans the sample.

So, you can think of the scanning being like this, it will first move in this direction then shift a little bit, and then again move back, again move a little bit, and again shift back. So, it will slowly scan the sample. And as the tip scans the sample, the tip, the sample itself is not uniform, because inside the sample you have atoms, and then there is a region between the atoms. The electrons inside the sample are bound closely around the atoms. So, the electron density is more near the atoms, and then it is less outside the atom etcetera. So, inside the sample there are potential variations.

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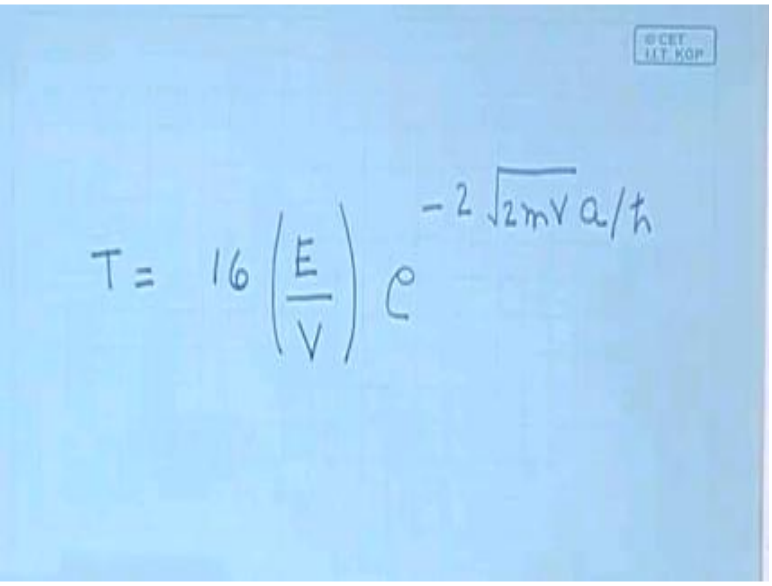

So, because of these potentials variations, the tunneling probability will change as the tip scans the sample. The potential is not exactly the same everywhere on the sample, because the electron distribution inside the sample is not exactly the same. There are some nuclei, there are atoms nuclei etcetera inside the sample. So, the potential is going to vary. So, because of this the tunneling current is going to vary.

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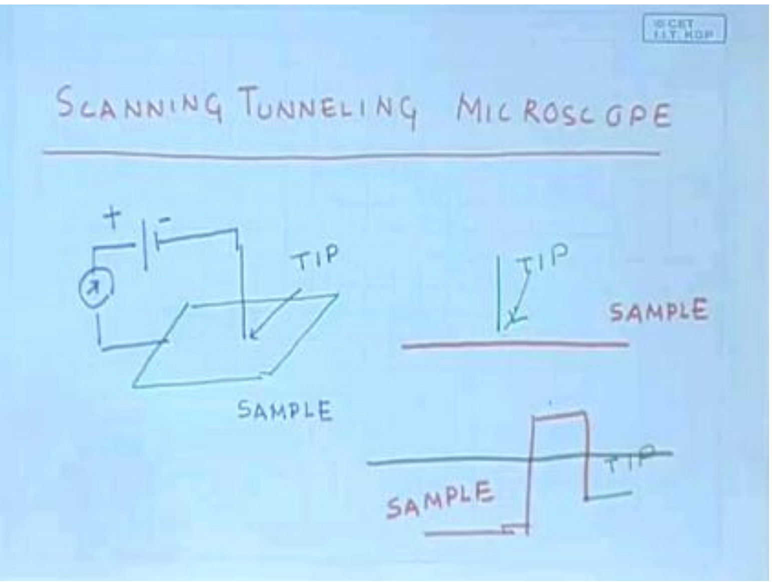

Now what is done in the scanning tunneling microscope is that, the tip is moved up and down, as it scans the tip is moved up and down automatically by a feedback loops, so as to maintain the fixed current. And these up and down motions that are required, to maintain the fixed currents are recorded, as the tip scans across this whole sample.

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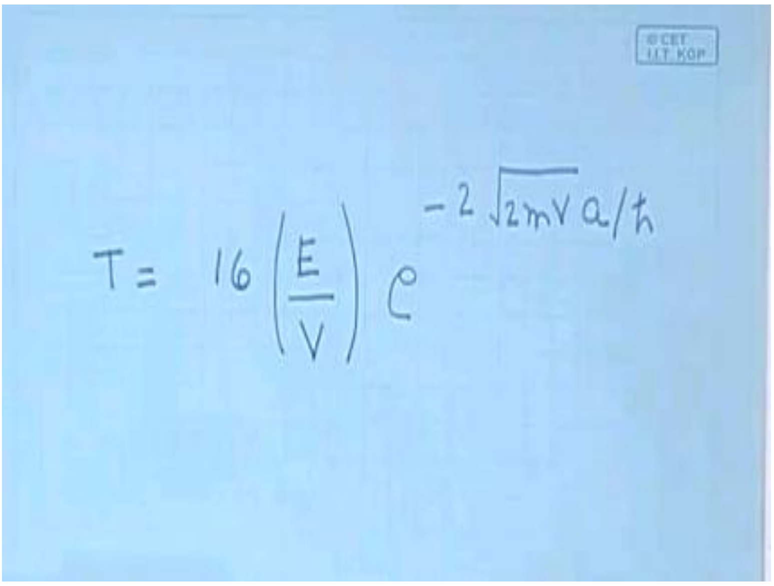

And it is this record of these up and down moments which are made, so as to maintain the same current, or the same tunneling probability, which give us a picture of the electron distribution inside the sample. So, let me show you a picture, which has been made by a scanning tunneling microscope.

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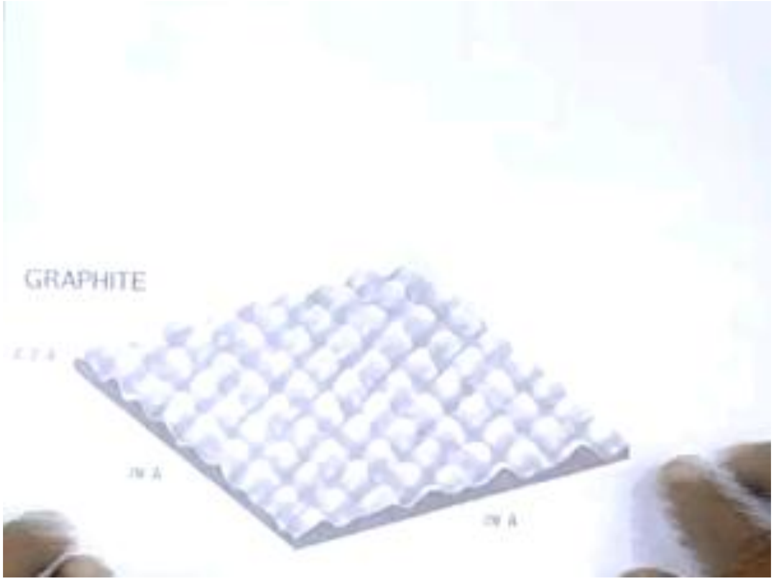

So, this shows you an image of a graphite sample, made using a scanning tunneling microscope. This image is 20 Armstrong wide in this direction. It is 20 Armstrong in this direction, and it is 2.2 Armstrong wide thick. The thickness the small dimension over here is 2.2Armstrong. And what you see in this picture, is the location of the carbon atoms. The undulations that you see, are the locations of the carbon atoms in the graphite sample. So, using a scanning tunneling microscope, a very remarkable device indeed, you can actually map out the positions of the atoms, on the surface of the sample that you are imaging. It is a very remarkable device, you can image, you can actually determine where the atoms on surface are located, and you can use it for large variety of studies. Unfortunately it does not tell us much about the interior of the sample.

So, in this last lecture, final lecture, I have told you about the phenomena of tunneling, an important consequence of quantum mechanics, where a particle gets through a potential barrier. Even though it does not have sufficient energy to cross the barrier, it can somehow tunnel through it. So, I have told you about tunneling, and I finally showed you one technological application, where people have put tunneling, to image the surface of materials.