Physics I : Oscillations and Waves
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Lecture - 42
Potential Well (Contd.)

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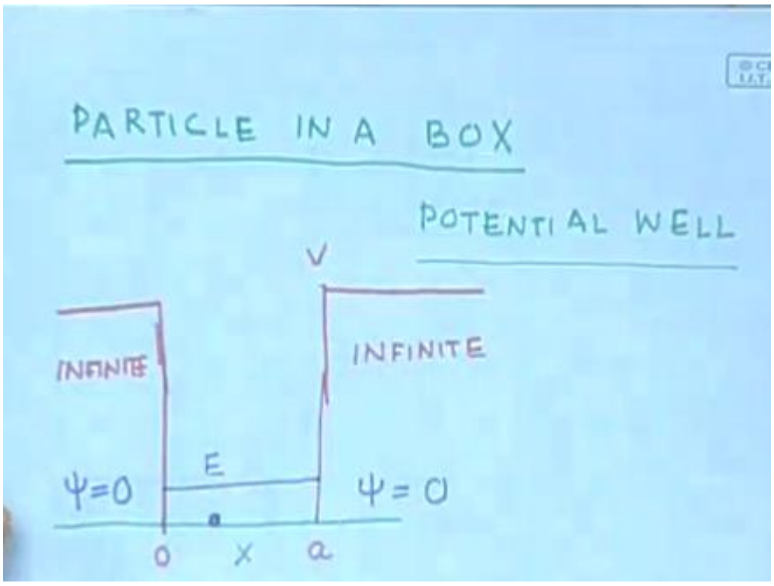

Good morning, we have been discussing a particle in a potential well, or a particle in a box, where we have a particle which is confined to a region 0 to a along the x axis, and the confinement is achieved by having to infinitely large potentials one at x equal to 0, and one at x is equal to a. So, the particle cannot move two smaller values of x, neither can it move to larger values of x. It is confined in this region. And we know that the wave function of the particle has to vanish at x equal to 0 and x equal to a.

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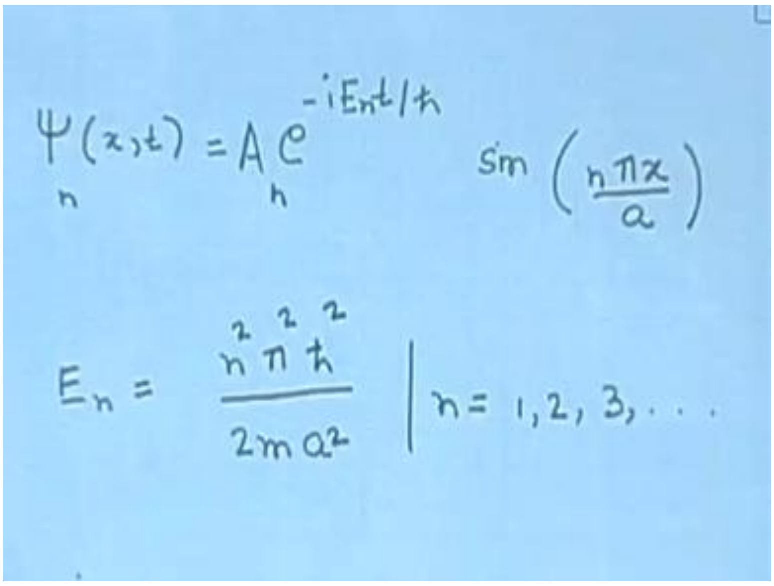

And we had in the last class worked out what the allowed wave functions are, the allowed energy levels are, and we found that there are only a discreet number of energy levels which are allowed, and these energy levels are quantised in the sense that there is one energy level corresponding to every integer; 1 2 3 4 5 6 all the way to infinity. And the energy level corresponding to any integer n is n square pi square h cross square by 2 m a square, and the corresponding wave function psi n. So, corresponding to every n there is also different wave function.

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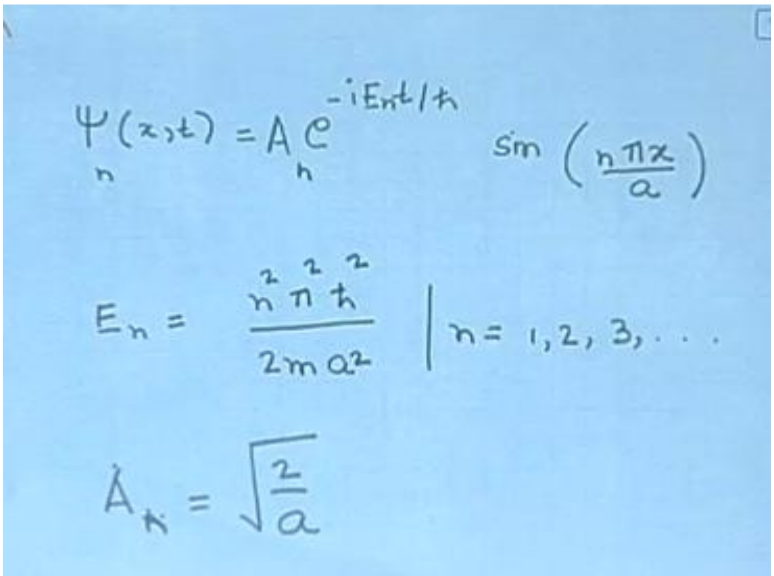

It is some normalization coefficient, which we had also worked out. The normalization

Coefficient A n is the same for all values of n, and it was we found that it is square root of 2 by a. and then we have the time dependent part e to the power minus i En t by h cross, and the spatial part sin n pi x by a. So, we had worked out these wave functions, this energies and the amplitude normalization coefficient in the last class. In today's class, let me take up a few for discussion, a few problems related to a particle in a potential well. So, we will now take up a few problems on particle in a potential well. The first problem is a situation.

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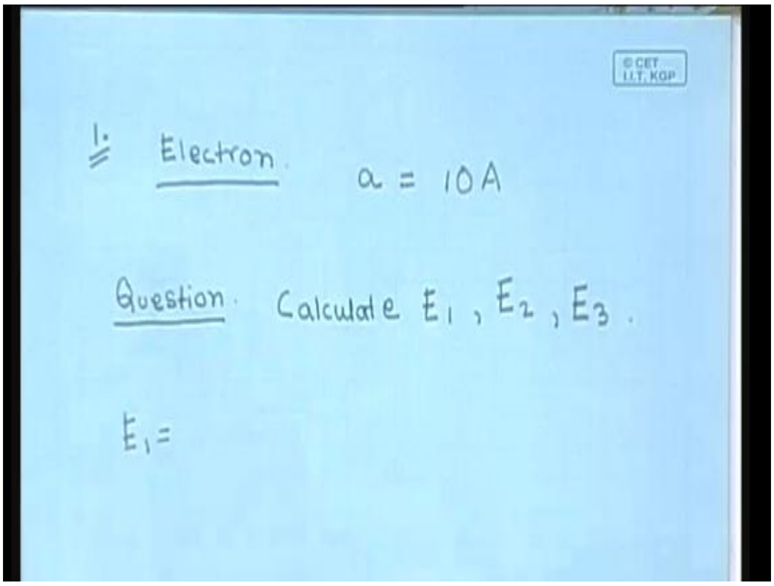

Problem number one; where we have an electron. An electron is confined in a potential well of length 10 Armstrong. And the question is, we have to find the energy of the ground state, the first excited state, and the second excited state. So, the question calculates E 1. E 1 is the energy of the ground state, E 2 the first excited state. E 3, the second excited state.

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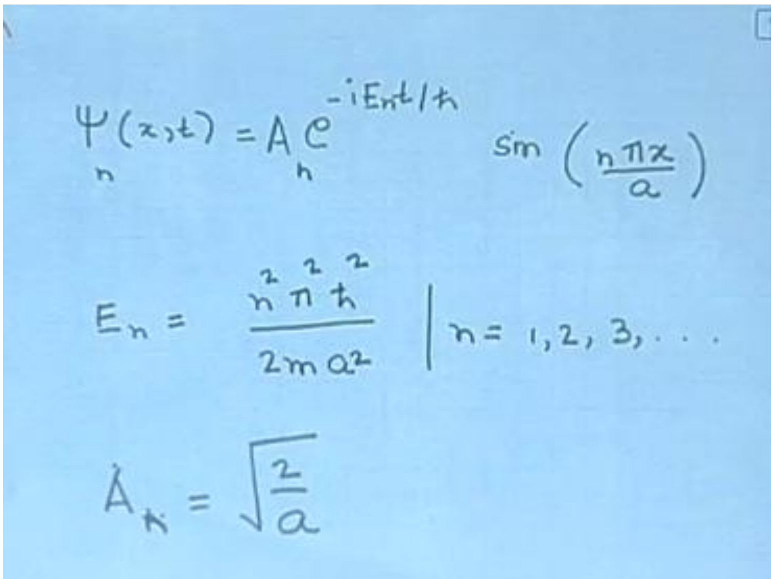

Now, the energy, the ground state energy we have. I just showed you the formula E 1 is. Let me show you the formula again, the energies corresponding to the nth energies level is, n square pi square h cross square by 2 m a square.

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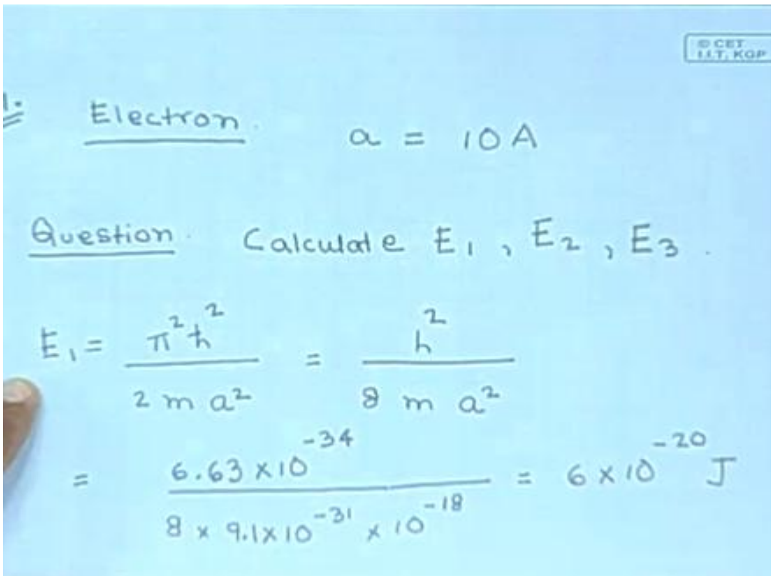

So, for n equal to 1 this is pi square h cross square by 2 m a square. And we also know that h cross is h divided by 2 pi. So, when I square it I will have h square divided by 4 pi square, the pi will cancel out. So, we can also write this as h square divided by 8 m a square. We have to put in the values. So, this h we know has a value 6.63 into 10 to the power minus 34 joules second. I have not showing the units explicitly, 8 the mass of the electron we know is 9.1 into 10 to the power minus 31 kg. I am not putting the units explicitly again, and this length the dimension of the potential well or the dimensional of the box is 0 Armstrong. 10 Armstrong is 10 to the power of minus 9 meters, 1 Armstrong is 10 to the power of minus 10 meters, so this into 10 to the power minus 10; that is 10 Armstrong square. So, we have to calculate this, and this comes out to be 6 into 10 to the power minus 20 joules, approximately 6 into 10 to the power minus 20 joules. So, this is the ground state energy the lowest energy that an electron in a potential well of dimension 10 Armstrong can have. Now, next we will calculate the energy of the first excited state.

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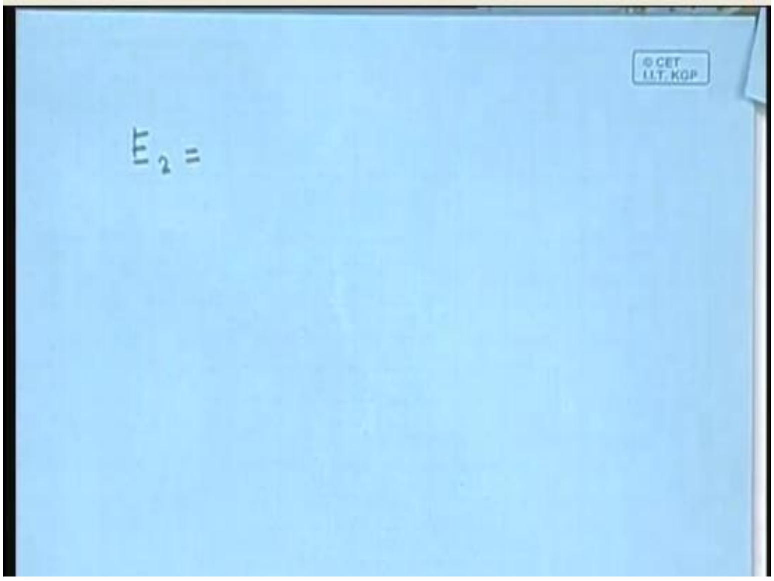

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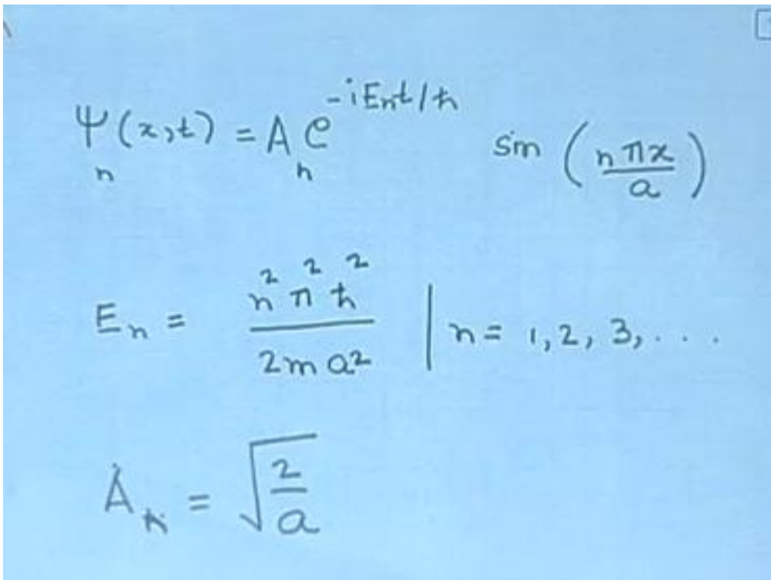

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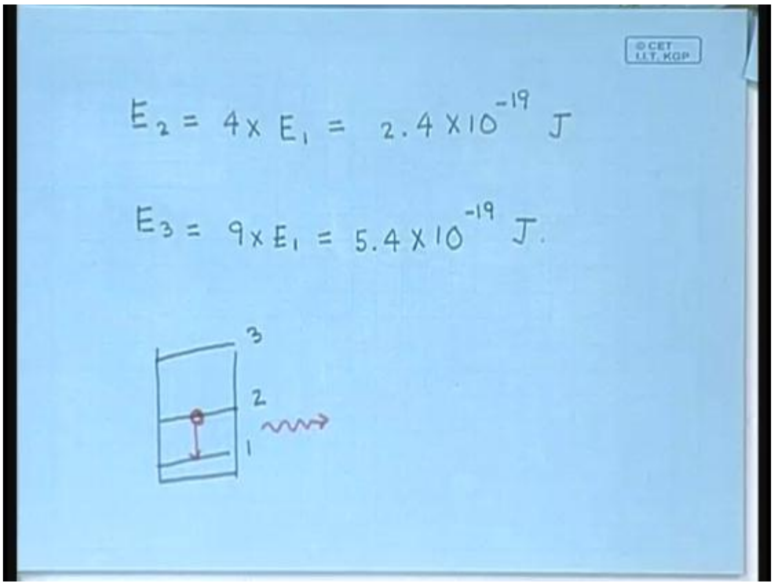

So, the first excited state E 2; first excited energy level has an energy which is n square into the ground state energy n being 2. So, we have the first excited state has an energy 4 times E 1 which is 2.4 into 10 to the power minus 19 joules. Similarly, the third energy level or the second excited state has an energy proportional to 3 squared; that is 9 times E 1. So, this is 9 times 6 is 54, so 5.4 into 10 to the power minus 19 joules. So, we see that we have these three energy levels; the first one is E 1, then we have 2 and here we have 3, and we have these higher and higher energy levels. Now, the next part of the problem, we have an electron which is in the first excited state, and the electron falls to the ground state, jumps to the ground state. And the difference, the energy, the difference in energy is radiated out as a photon. So, pictorially we can represent this as follows. There is an electron here, the electron goes from the first excited state to the ground state, and in this process the photon comes out. The question is, find the wavelength of the photon. So, the electron jumps from the first excited state to the ground state. Its goes from a higher energy to a lower energy, the excess energy comes out in the form of a photon. The problem is to calculate the wavelength of this photon.

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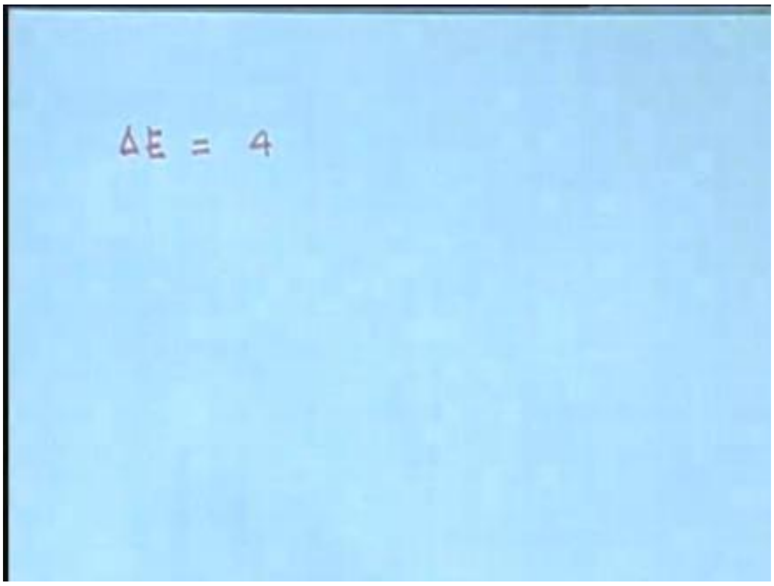

So, the difference in energies delta E is the energy of the photon that comes out. So, the difference in energy, is the energy of the first excited state which is 4 into E 1. The energy of the first exited state, remember is 4 into E 1.

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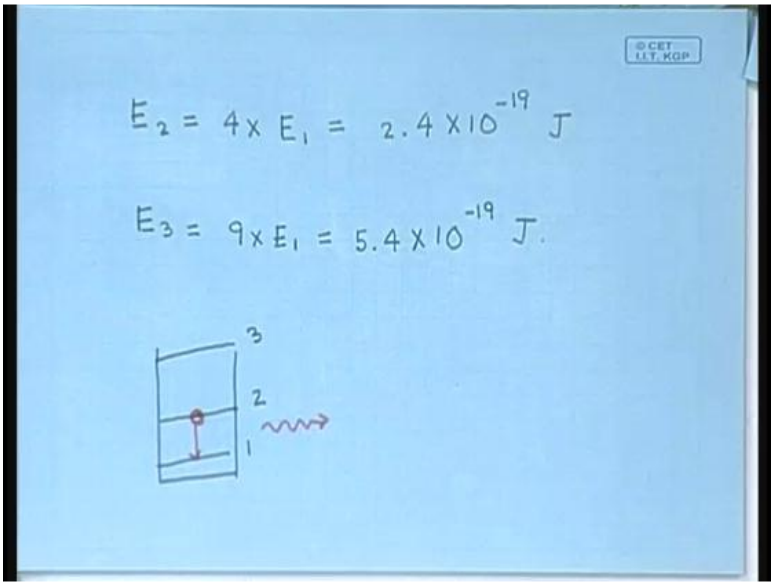

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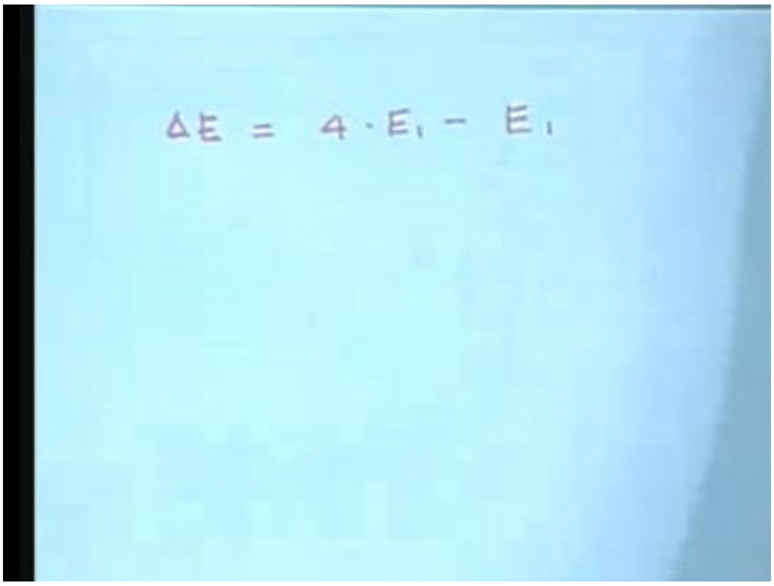

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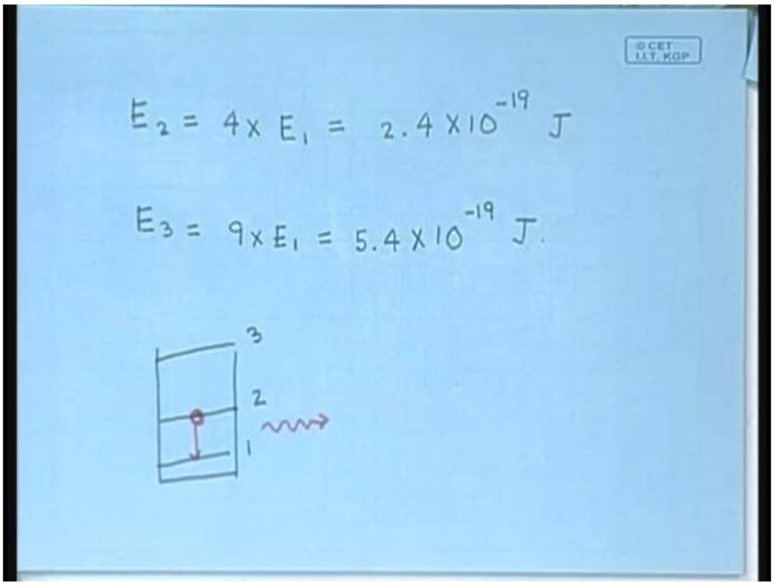

And E 1 is let me write it in terms of E 1 here, E 1 minus E 1, this is the difference in energy. This is the difference in energy between the first excited state and the ground state delta E. It is 4 E 1 and minus E 1.

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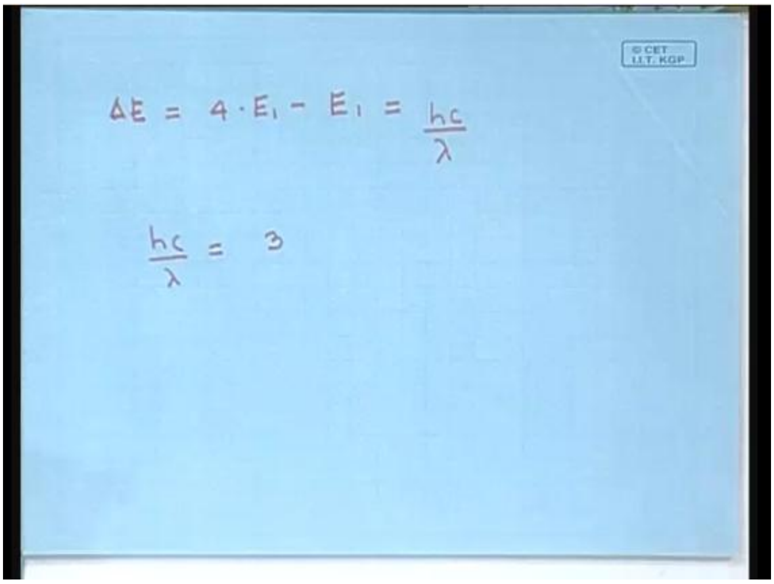

And this is the energy of the photon. We know that the energy of a photon is h nu h times the frequency, which we can also write as h c by lambda. So, the wavelength of the photon, if it is lambda that is emitted, if it is lambda, then energy of the photon is h c by lambda. So, this is the difference in the two energy levels, and this should be equal to the energy of the photon. And we know that the E 1. So, we can write this 4 minus 1 is 3. So, we can write this as h c by lambda is equal to 3 and 3 times E 1. E 1 is h square by 8 m a square.

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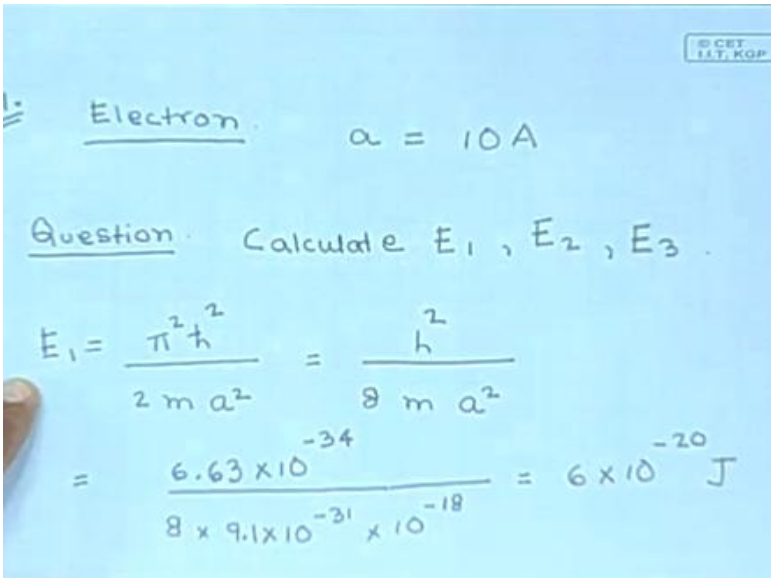

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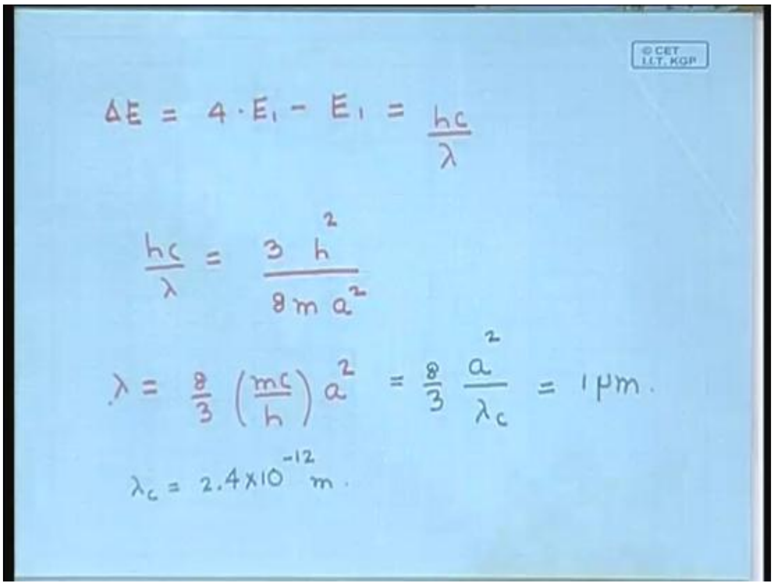

So, this is equal to 3 h square by 8 m a square. From this we see that the wavelength lambda of the photon that comes out. So, 1 h cancels out, we have 8 by 3 8 divided by 3, and then we have m c by h into a square. Now we have encountered this combination m c by h if you remember, when we discussed Compton scattering. In this Compton scattering process the shift in the wavelength, is the Compton wavelength into a factor of 1 minus cos theta, where theta is the scattering angle. And remember that the Compton wavelength the shift in the wavelength which decides the amplitude, the magnitude of the shift in the wave length was h divided by m c, where m was the mass of the electron. So, here we have the factor m c by h. So, we can identify this as the inverse of the Compton wavelength. So, we can write this as 8 by 3; the size of the potential well square. The length of the potential well square divided by the Compton wave length. And remember that the Compton wavelength was 2 point 4 into 10 to the power minus 12 meters. We had calculated this and a is 10 to the power minus 9 meters. So, we have 8 by 3, and here we have 10 to the power minus 18 divided by 2.4 into 10 to the power minus 12. So, finally, the result is the wavelength of the radiation that comes out is, of the order of one micro meter.

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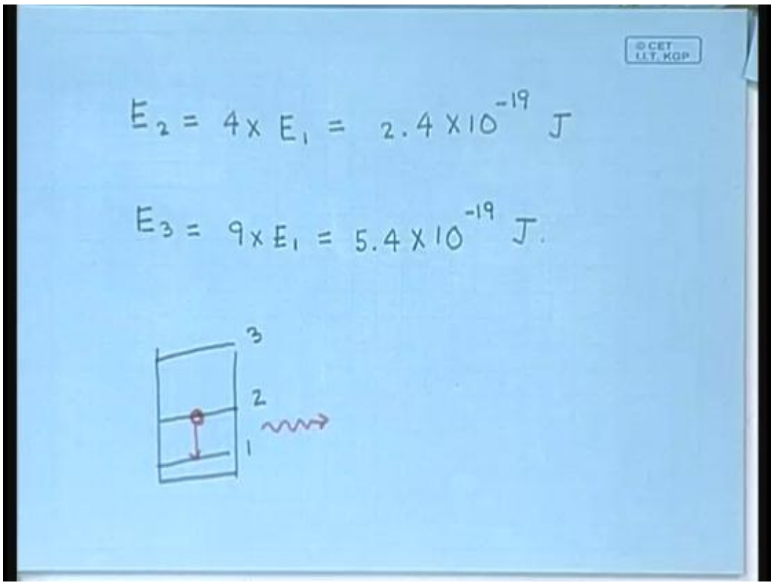

So, what we see is, that in this problem, where we have an electron that is confined in a potential well of dimension 10 Armstrong and where. So, the electron is confined to a potential well of dimension 10 Armstrong, and the electron jumps from the first excited state to the ground state. The energy that is released, the photon which carries away this energy has wavelength one micro meter. Let us ask the question, what part of the spectrum is this photon going to be. Now, one micro meter is in the infrared. So, if I have a potential well of dimension 10 Armstrong and I have an electron trapped in this, which I could possibly have in a semiconductor by quantum well in a semiconductor. The photon that is going to be emitted when the electron jumps from the first excited state to the ground state; that photon is going to be in the infrared. So, a 10 Armstrong potential well, is going to produce the first excited state to ground state transition, is going to produce a photon, which is in the infrared. Let us ask the question what should we do, if I would like to have a potential well which produces radiation in the ultraviolet. How will you manipulate the potential well, so that the radiation comes out in the ultraviolet.

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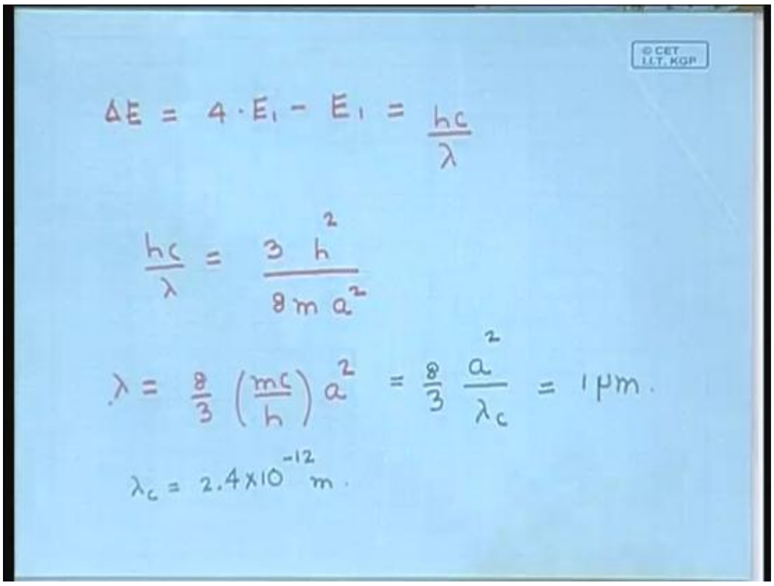

Now, if you want to increase the frequency or decrease the wave length, you have to play around. There is only one thing that you can play around here; it is the width of the potential well. So, if you reduce the width of the potential well, which if you can reduce this, then the wavelength of the emitted radiation is going down, and you can reach the optical range, provided you can make your potential well much smaller. So, in this problem, what we have done, is we have got a field for the numbers involved in these particles, when you have a particle in a potential well. And when you have an electron the number the energy levels are of the order of 10 to the power of minus 19 joules, and for a potential well 10 Armstrong’s width wide. And the first excited state to ground state transition the energy difference corresponds to a wavelength a photon of wavelength one micrometer. Let us know consider the second problem to do with potential wells. So, again we have a potential well.

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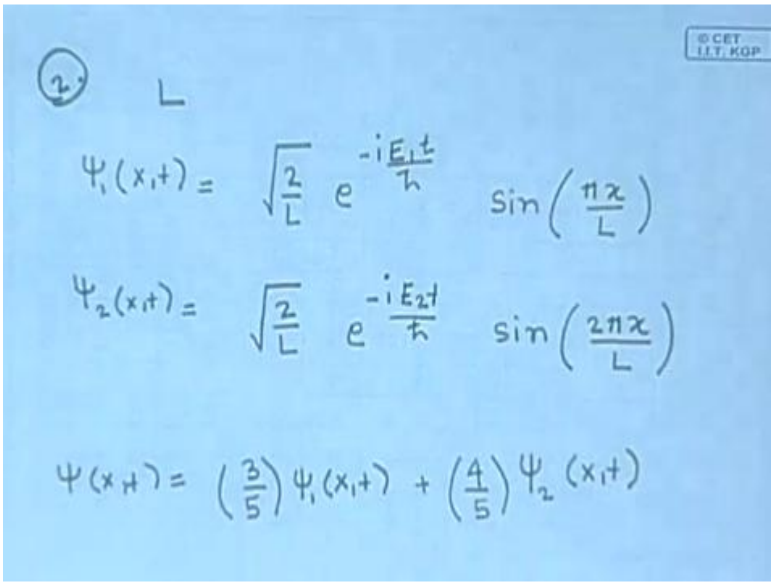

Second problem; we have a potential well of a length L, and we have already seen that in this potential well, you can have many different stationary states for a particle, so the ground state psi 1. This is the lowest energy state ,we have work this out the ground state wave function, is going to be root two by L e to the power minus i E 1, where E 1 is the energy corresponding to the ground state into t by h cross into sin pi x by L. And then we have the first excited state psi 2, this is going to be the same normalization coefficient. The time dependence is now going to be e to the power of minus i E 2 t by h cross, and we have sin 2 pi x by L. And then you have the second excited state psi 3 4 5 etcetera. So, these are the stationary states which. If a there is a particle in this state in the psi 1, then the probability density of the particle does not change with time. And if I make a measurement of the energy of the particle, I will get a value E 1. If there is a particle in this state psi 2, then again the probability density of the particle does not change with time, and if I make a measurement of the energy I will get E 2. So, these are the stationary states or the energy Eigen states. now in this problem we are going to consider a state psi function of x and t which is 3 by 5 psi 1 x t plus 4 by 5 psi 2 x t. So, this wave function is a linear superposition of the ground state, and the first excited state. And we know that this is also a solution, allowed solution of a for a particle in a potential well. Because this is a solution, this is also a solution, and since the differential equation governing psi is linear. So, any linear superposition of solutions is also a solution. So, this is also a solution for a particle in a potential well. Now, this solution is not a stationary state, if you calculate the probability density, you will find that it is not going to be independent of time, it is going to change with time.

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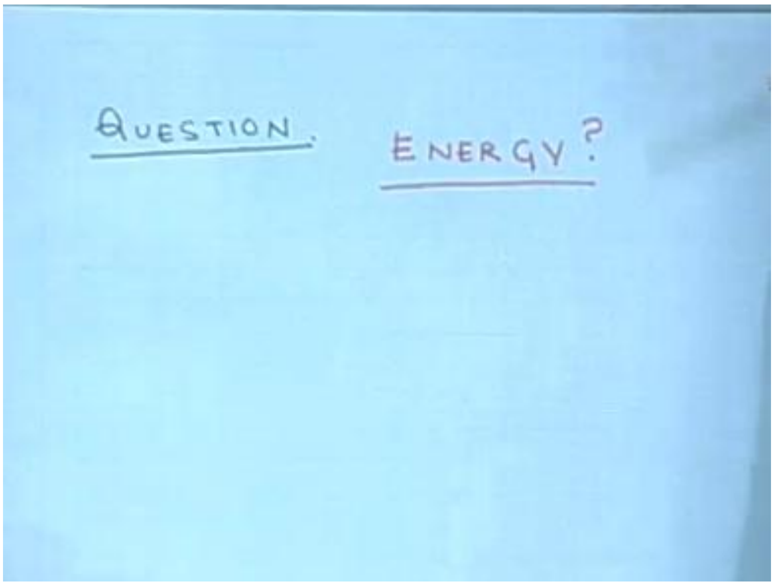

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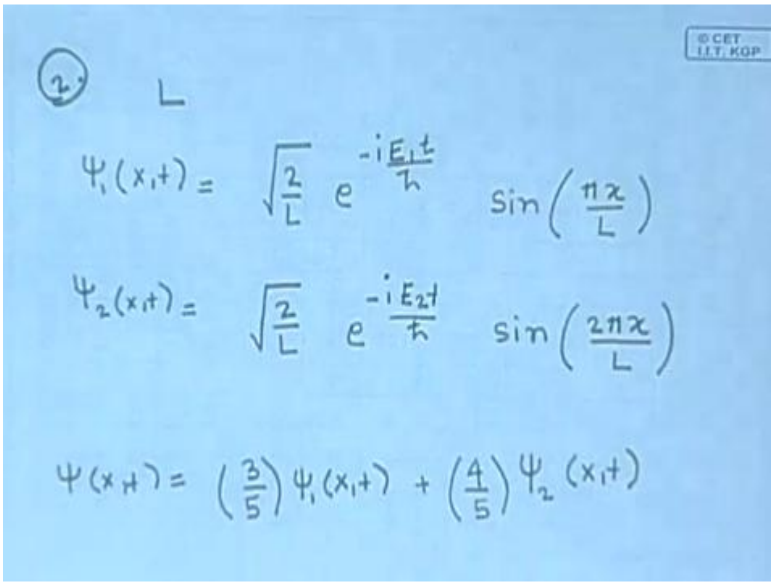

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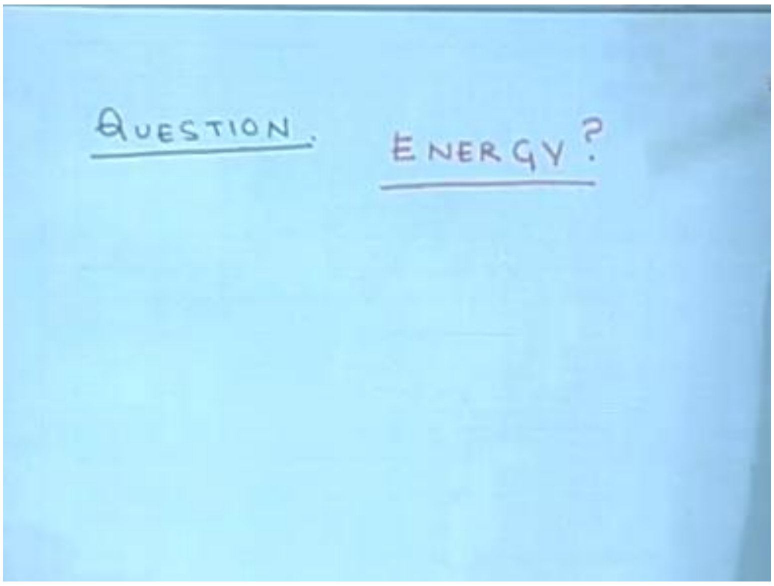

The question is, the first question is what happens if you make a measurement of the energy of the particle. So, if I measure the energy of the particle. So, there is the particle in a state which is described by this wave function. And the question is, what happens if I make a measurement of the energy of the particle.

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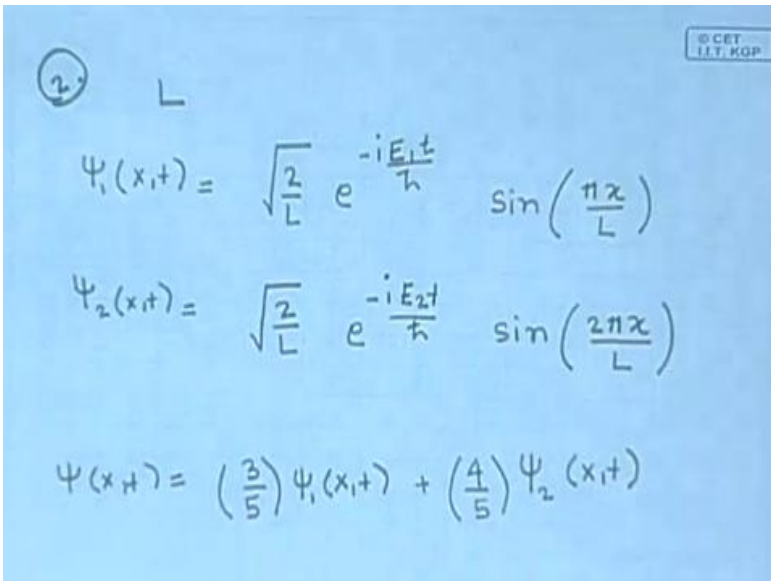

Now, remember if the particle had been in this state, I would have got an energy E 1. If the particle had been in this state I would have got energy E 2, but now the particle is in a superposition of these two states. This state, let us see what happens when we measure the energy.

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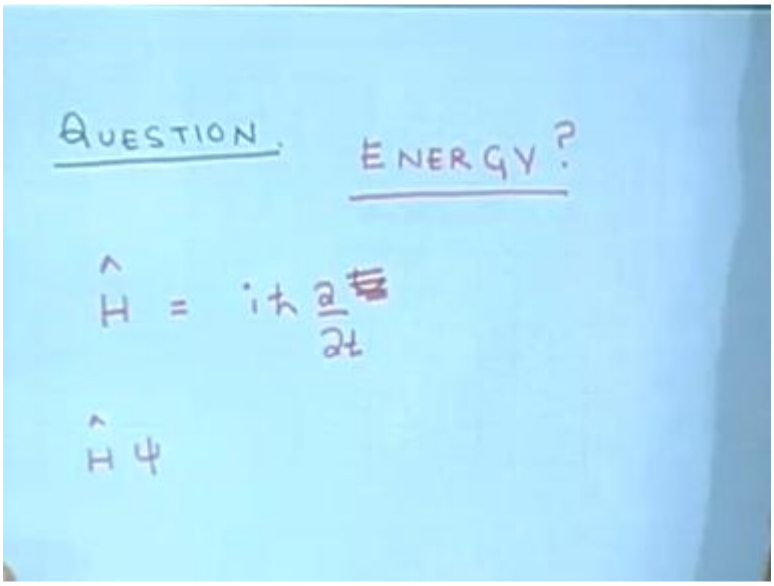

So, you have to. Corresponding to energy, there is the operator the Hamiltonian operator H i h cross Del by Del t. this is the Hamiltonian operator. So, let us see what happens when the Hamiltonian operator acts on this wave function psi.

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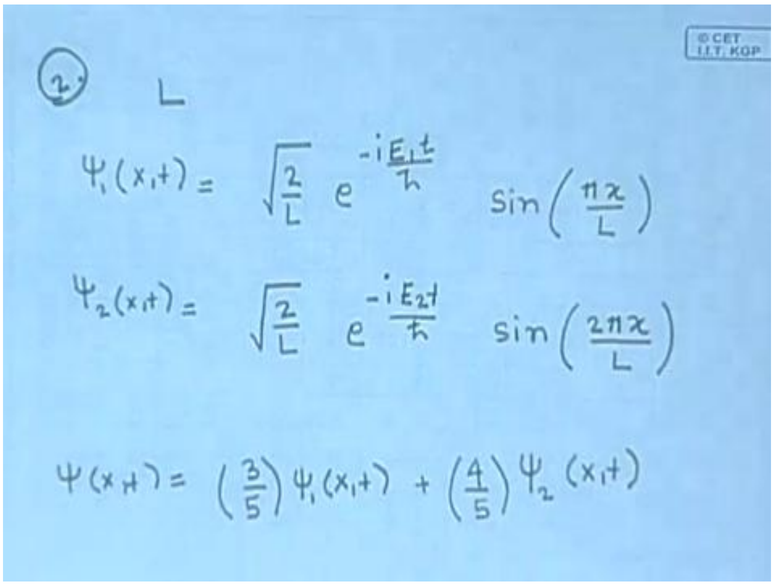

So, let us see what happens when the Hamiltonian operator acts on this wave function psi. Because corresponding to energy there is this, corresponding to every physical dynamical variable. In quantum mechanics there is an operator, and I have told you that corresponding to energy, there is the Hamiltonian operator. So, let us see what happens when the Hamiltonian operator acts on this wave function psi.

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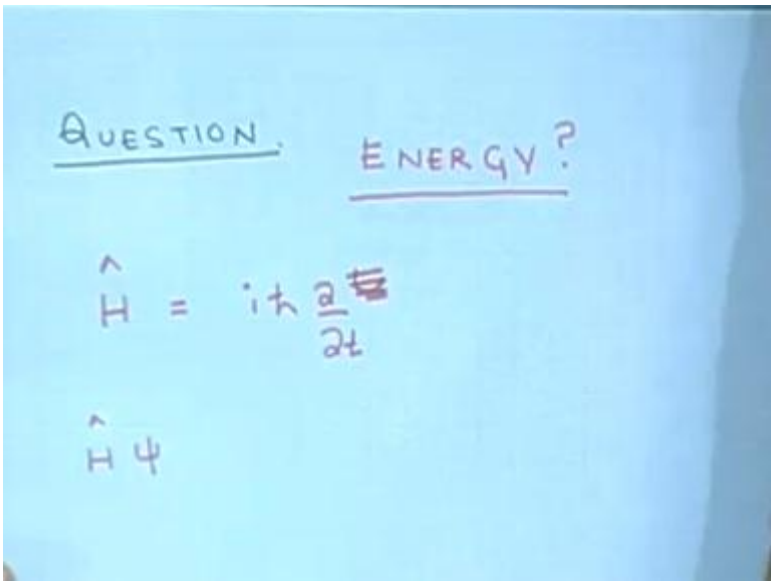

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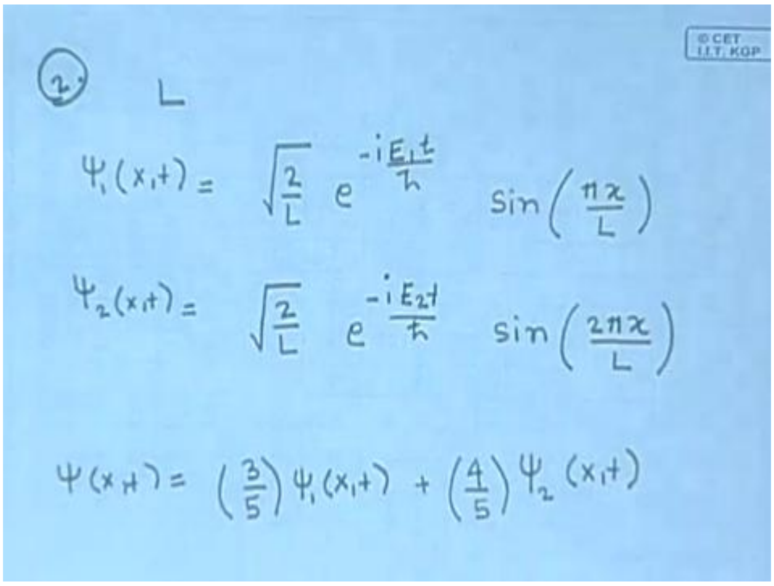

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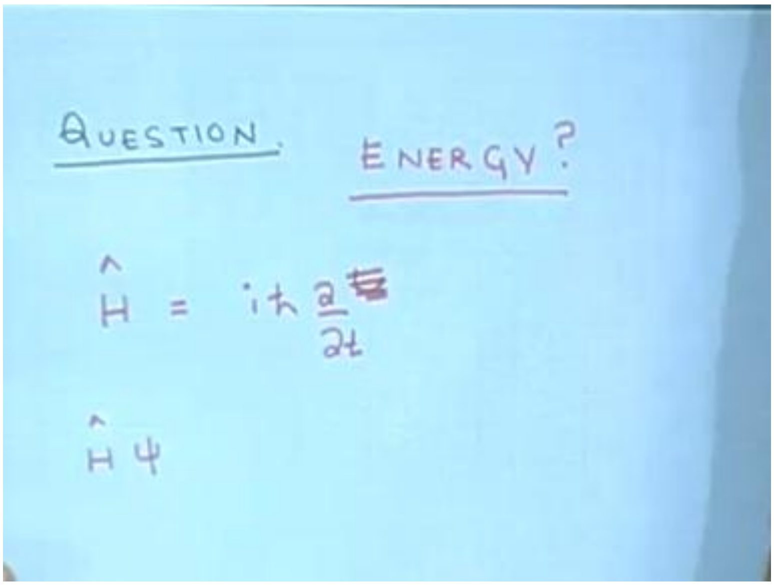

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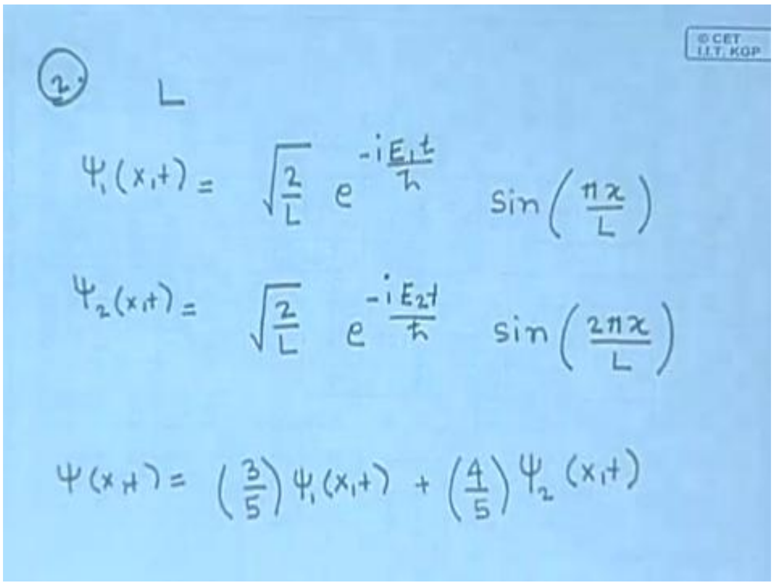

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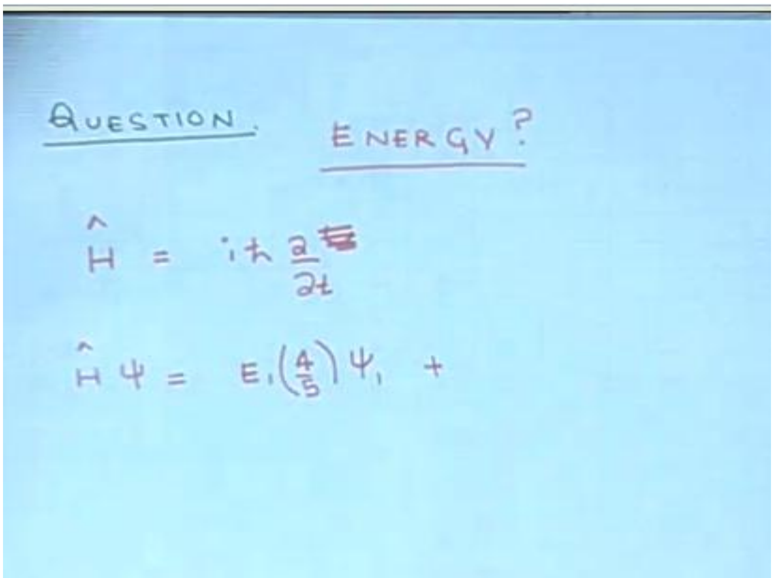

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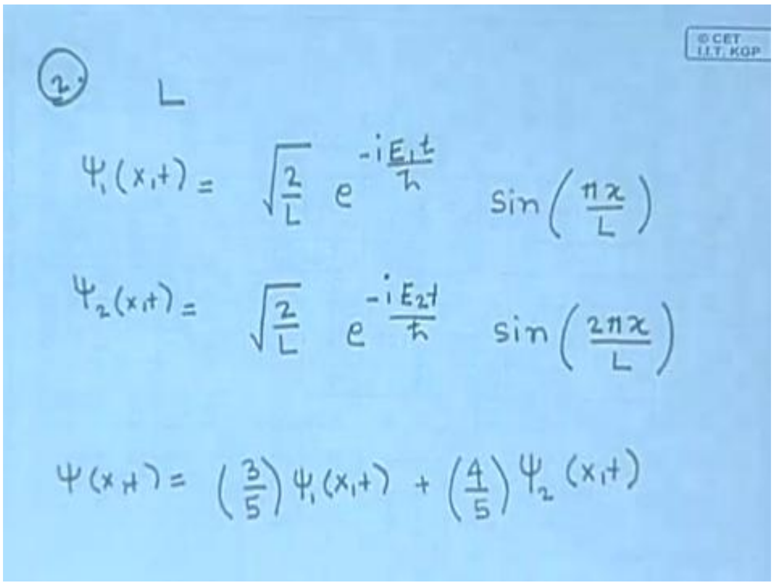

Now, the Hamiltonian operator is i h cross del by del t. When this operator acts on psi 1, the time derivative is going to give me a factor of minus i E 1 by h cross, and minus i E 1 by h cross into h cross will give me E 1. So, the Hamiltonian operator the derivative that operator acting on this, is going to give me E 1 into the same thing. This is an Eigen function of the Hamiltonian operator, and we will have E 1 into four fifth into psi 1 plus.

If the Hamiltonian operator acts on psi 2, it will give me E 2, because when I take the time derivative of this I am going to get minus i E 2 by h cross, and again multiplying with this factors I will get E 2. So, this is going to give me E 2 four fifth psi 2.

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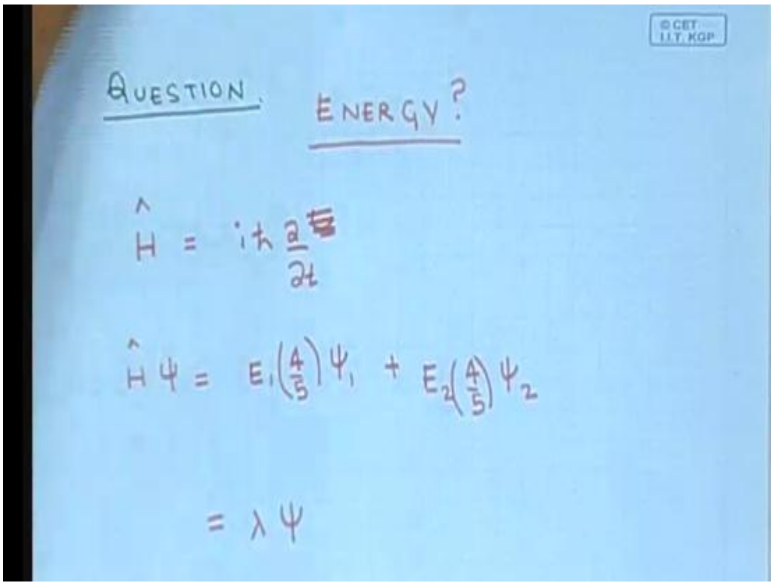

Now, let us ask the question is this wave function psi, and Eigen function of the Hamiltonian. A wave function is said to be an Eigen function, if H psi gives you a number lambda into psi itself.

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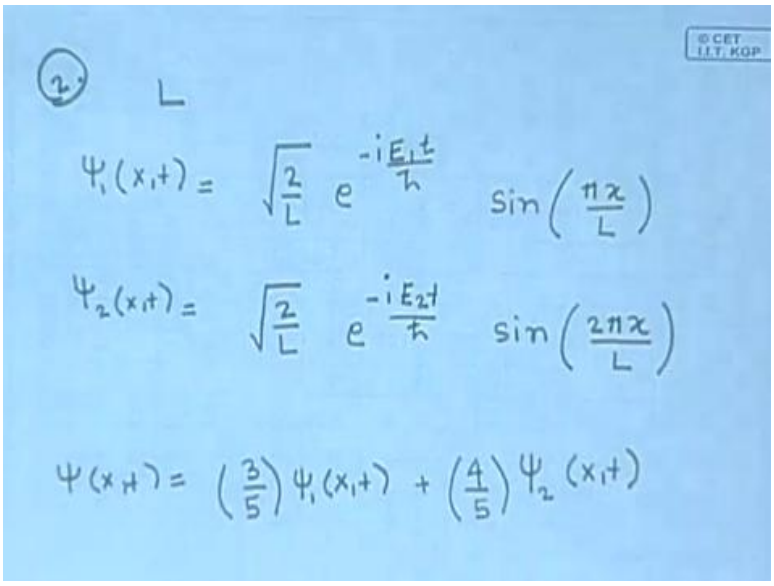

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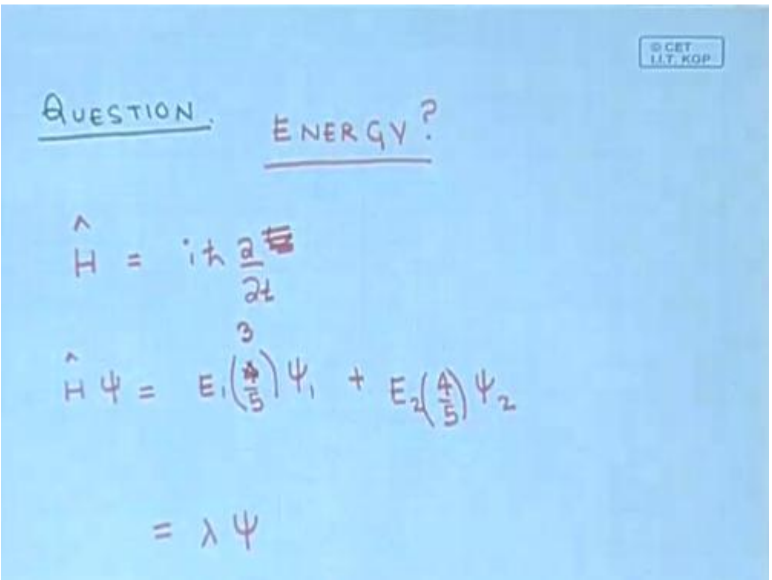

Now, psi remember is this 3 fifth psi 1 plus four fifth psi 2. H acting on psi gives us E 1 three fifth. This should be three fifth sorry; E 1 three fifth psi 1 plus E 2 four fifth psi 2.

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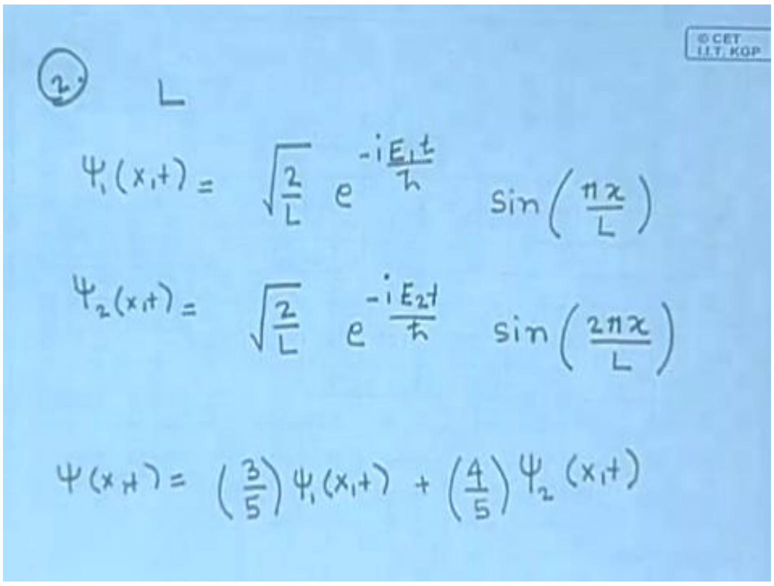

So, you see that there is no way I can write this as a number into psi again, because there is a E 1 which comes here and E 2 which comes over here.

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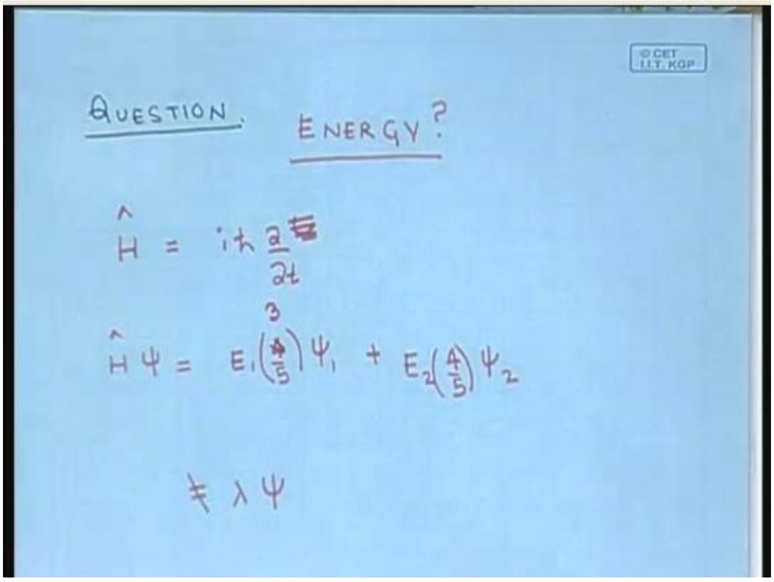

So, what we can say is that this wave function psi, which is a superposition of psi 1 and psi 2, is not an Eigen function of the Hamiltonian operator. So, it is not an Eigen function, but it is a linear super position of these two Eigen functions.

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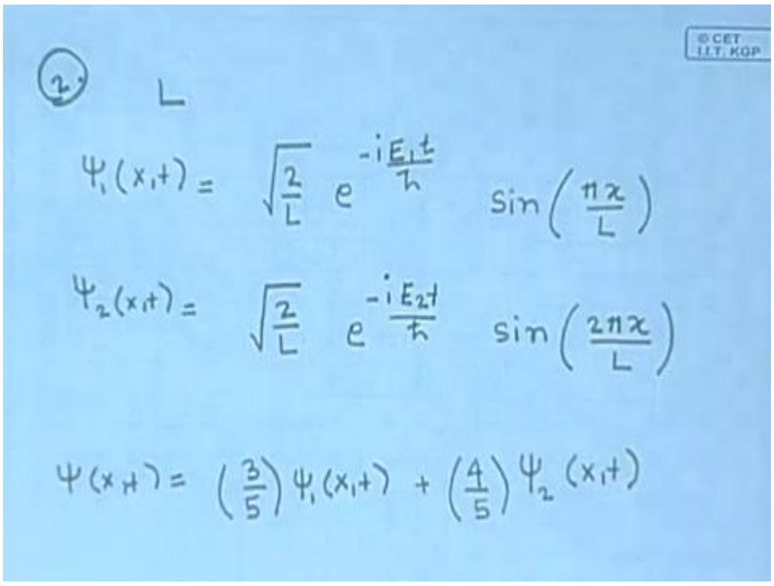

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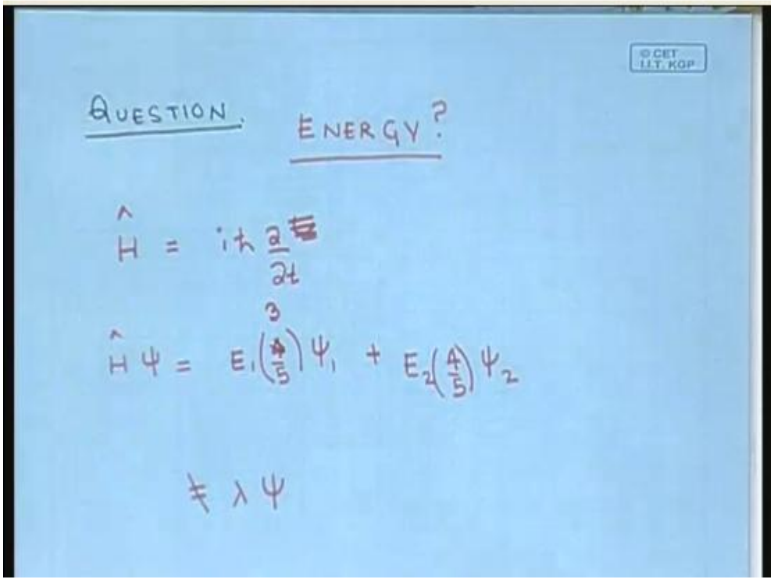

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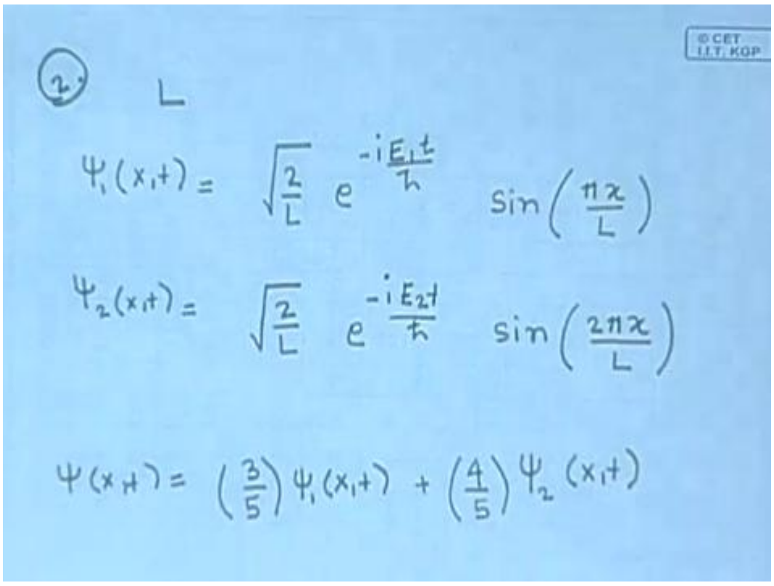

So, what we can say is that whenever we make a measurement of energy, you are going to get either E 1 or E 2, if I get E 1 as my outcome, after the measurement the wave function is going to change from here to psi 1. If i get E 2 the wave function after the measurement is going to be psi 2.

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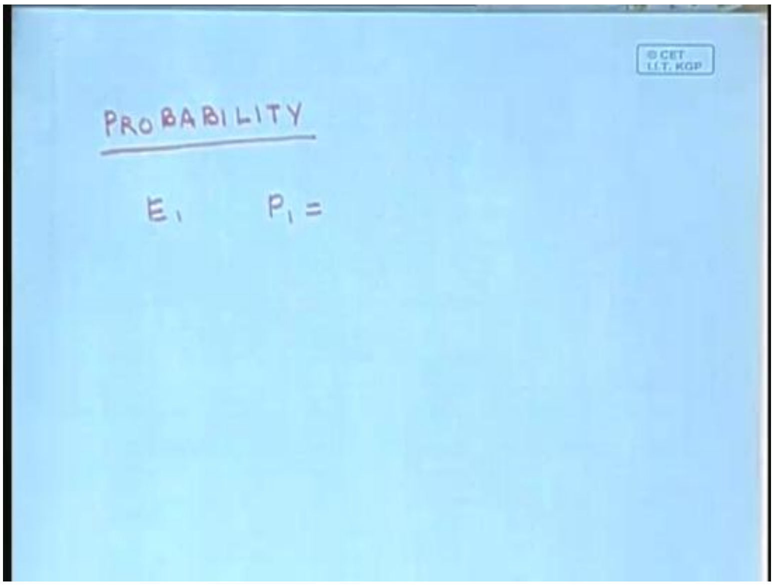

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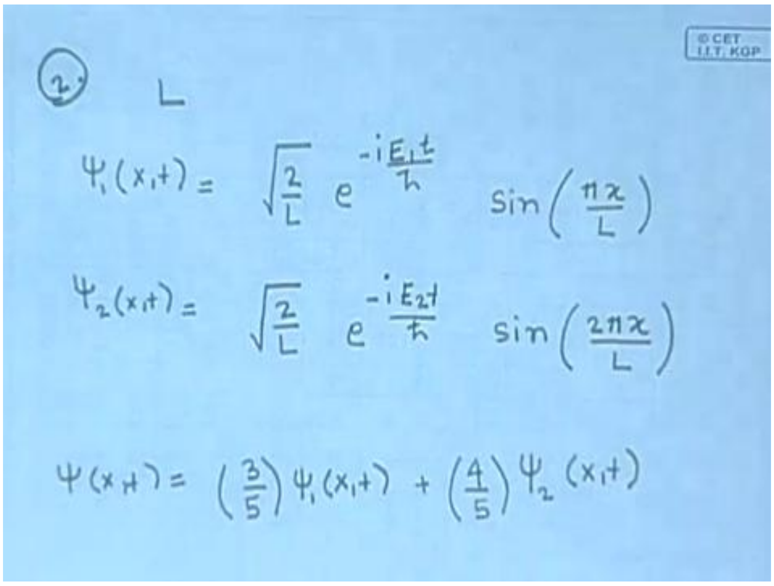

And we can also calculate the probabilities of getting E 1 and E 2. So, the probability of getting E 1. Let us call it p 1. So, we want to calculate the probability of getting the value E 1 as my outcome, if I make a measurement of the energy, for a particle in this state. Now, this state is a superposition of the Eigen function corresponding to E 1 plus the Eigen function corresponding to the E 2. So, I can have two possibilities either E 1 or E 2. And the probability of getting E 1, is the mod square of this coefficient of psi 1.

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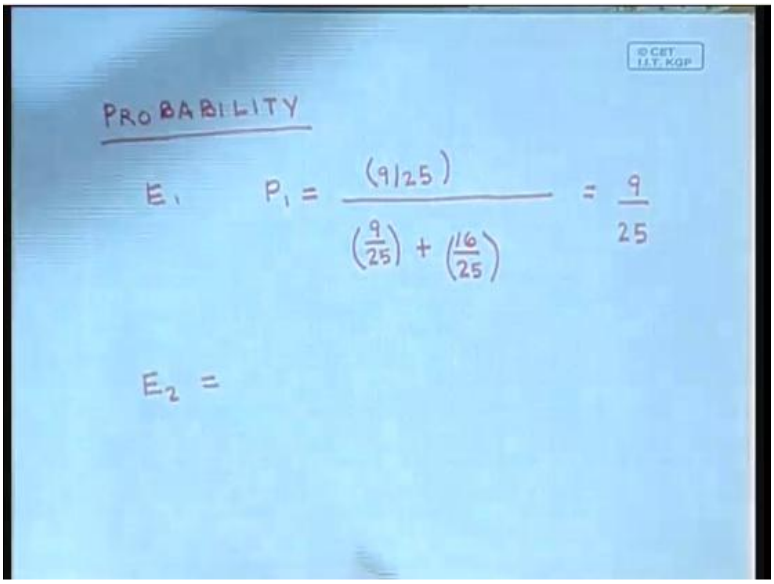

So, the mod square of this is 9 by 25 divided by the modes square of this plus the mod square of this.

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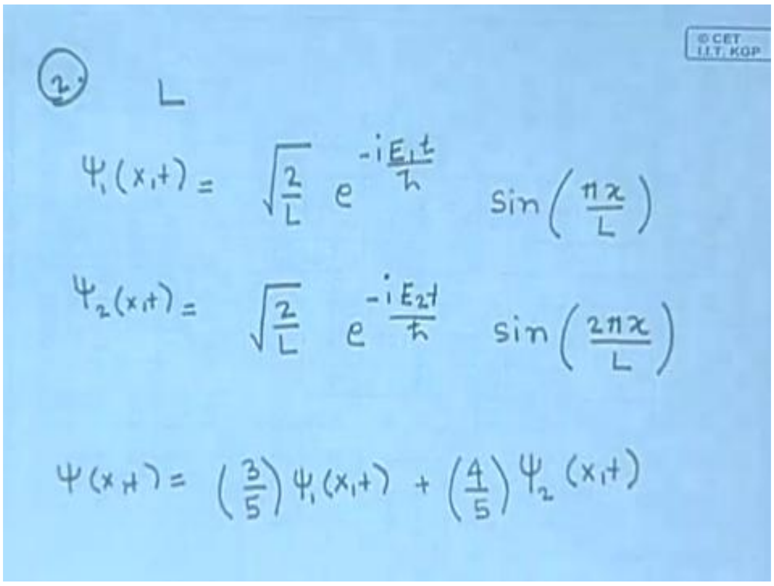

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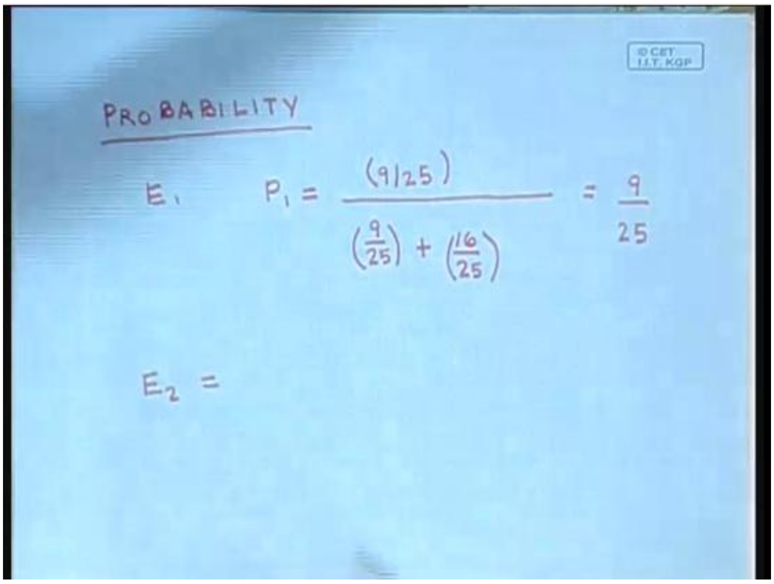

The mod square of this is 9 by 25 plus the mode square of this is 16 by 25, and the denominator is 1. So, the probability of getting p 1 is 9 by 25.

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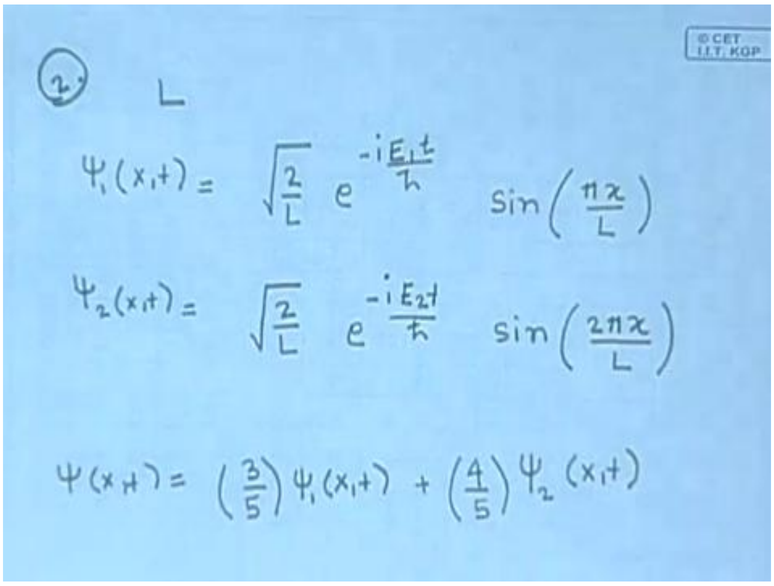

Similarly, we can ask the same question for E 2, and the probability of getting an outcome E 2. I am denoting that by p 2, is the modulus square. The modulus of this coefficient, the square of that divided by the some of the square modulus square of this plus the modulus square of this, that we have seen is 1.

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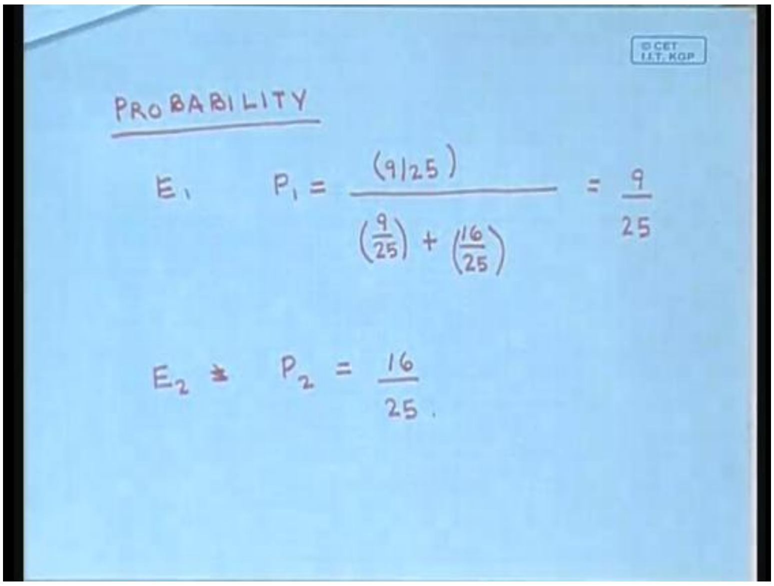

So, the probability of getting E 2 is just the modulus square of this, which is 16 by 25. So, the probability of getting E 2 is 16 by 25.

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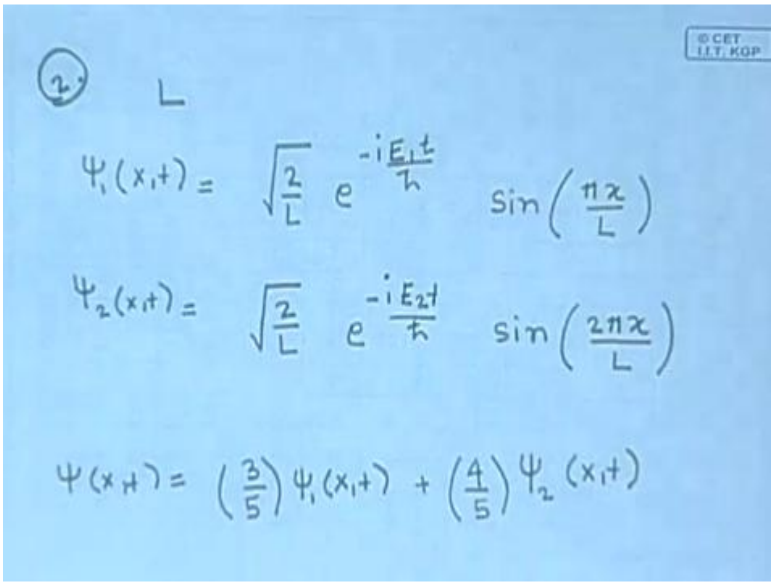

So, if I have a particle in a state which is described by this wave function, and if make a measurement of the energy, I will get either E 1 or E 2. The probability of getting E 1 is 9 by 25. The probability of getting E 2 is 16 by 25. Now, let me ask the next question. The next question is, what is the expectation value of the energy. Let me remind you what I mean by the expectation value. If I repeat the experiment many times, in the sense that I have many replicas of the same particle in the same wave function, and I measure the energies for all of these replicas, each time I do the experiment I will get either E 1 or E 2. The question is by expectation value I use the mean. So, every time I will get either E 1 or E 2 with different probabilities. The question is, what is the mean energy or if I do the experiment only once what is the value of energy that I expect to get. The mean energy, the mean over all of these different replicas, is what is going to tell us what value we expect to get. So, the question is, how can we calculate this mean value or the expectation value of the energy.

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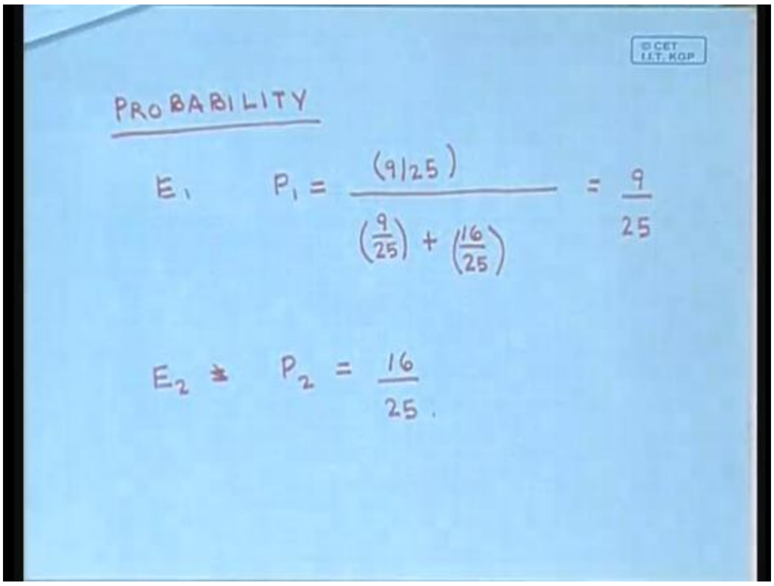

Now we know that we have already calculated this, that you have only two possible outcomes E 1 or E 2, and the probability of getting E 1 is p 1 which is 9 by 25. The probability of getting E 2 is 16 by 25.

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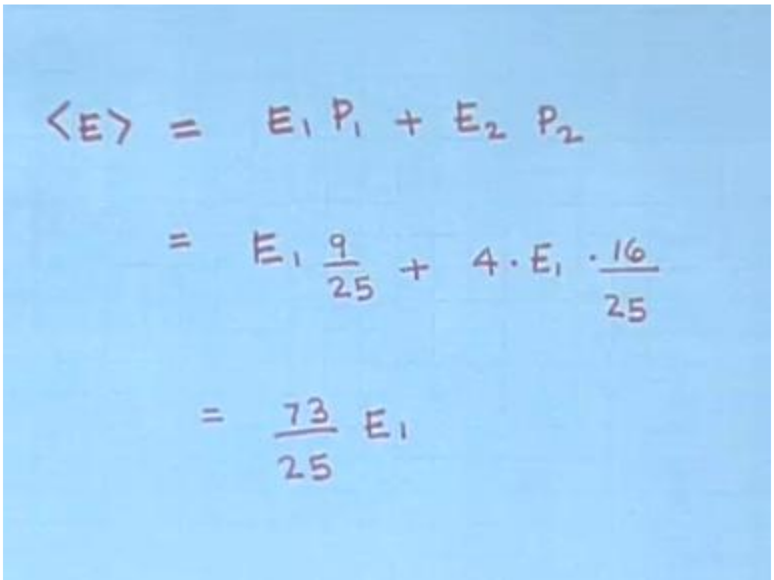

So, the average outcome, the mean outcome, the mean energy, the expectation value this can be calculated as E 1, the first out come into the probability of the first outcome plus E 2 into the probability of the second outcome. So, E 1 we know it has a value. The probability of getting that, is 9 by 25. E 2 is 4 times E 1 we have seen this already. So, 4 E 1, and a probability of getting E 2 is 16 by 25, and 4 into 16 is 64, 64 plus 9 is 73. So, the mean outcome, mean value of the energy, the expected value of the energy is 73 by 25 into E 1. So, it is the approximately three times E 1. So, we see that the mean value of the energy will be somewhere between E 1 and 4 times E 1 which is E 2, somewhere in between and this is the expectation value. This is the expected value of the energy. Now, we could calculate the expectation value of the energy in a different way also. So, we have, let me remind you of the other method in which we could do it.

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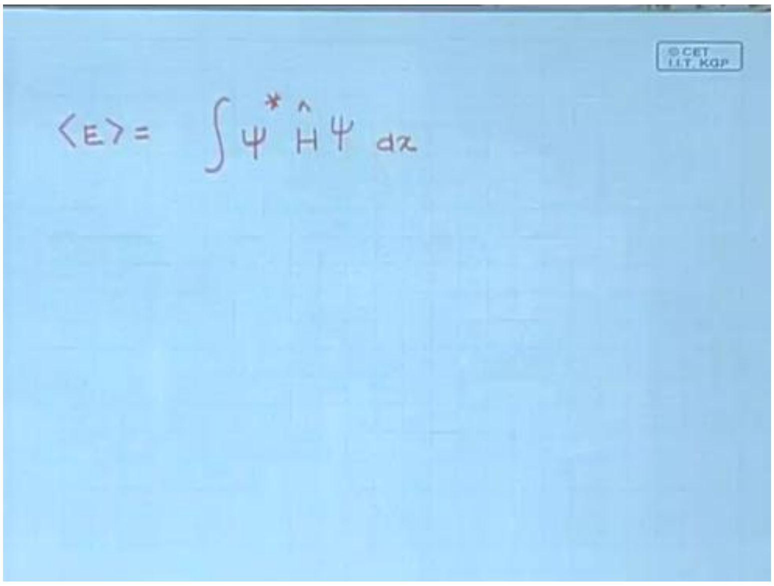

The energy, the expectation value of the energy could also be calculated in this way. Remember that for any physical variable, any dynamical variable. If I ask you the question, if I have many replicas of my particle, and I measure that quantity. For example, position or momentum, if I make a measurement of the quantity. In general each time for in each replica I will get a different value. Here I will get E 1 or E 2. I cannot predict for sure for which one I will get. So, in general say momentum position I will get some spread different values. And if I ask the question, how predict the mean outcome or the expected value. So, what we have to do is, we have to take the wave function psi corresponding to that state, take its complex conjugate. Then write the operator corresponding to those physical variables. So, here the operator is the Hamiltonian operator H, and act on psi and then integrate over d x. So, this will give me the mean value. So, let us calculate this, and see that we get back this same answer.

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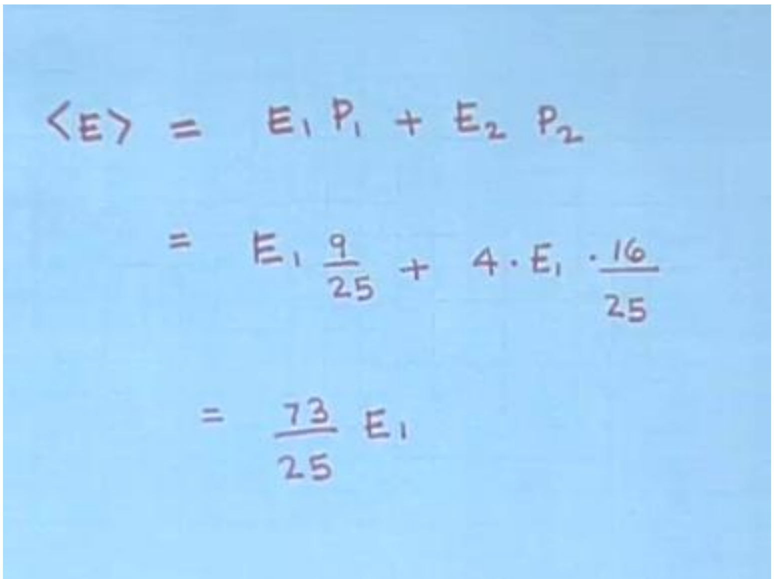

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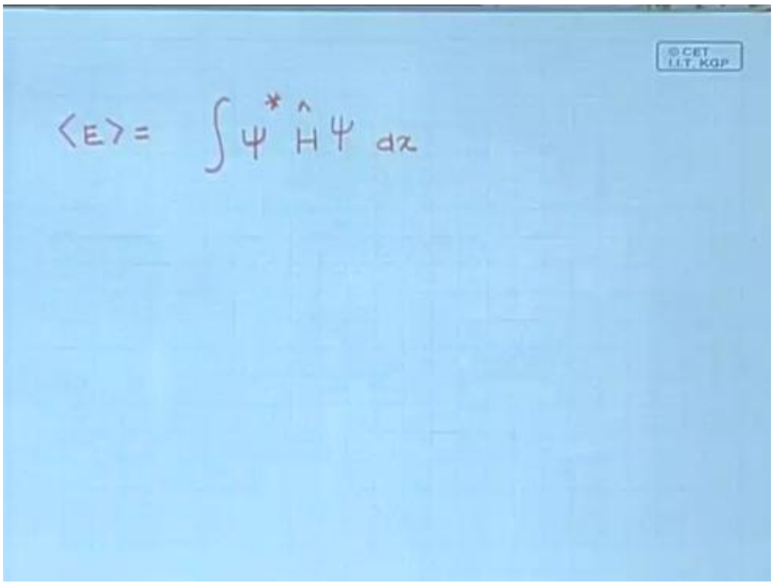

So, how much is psi star. We have to put in these values for psi star, and then act with the Hamiltonian operator on this and then do the integral.

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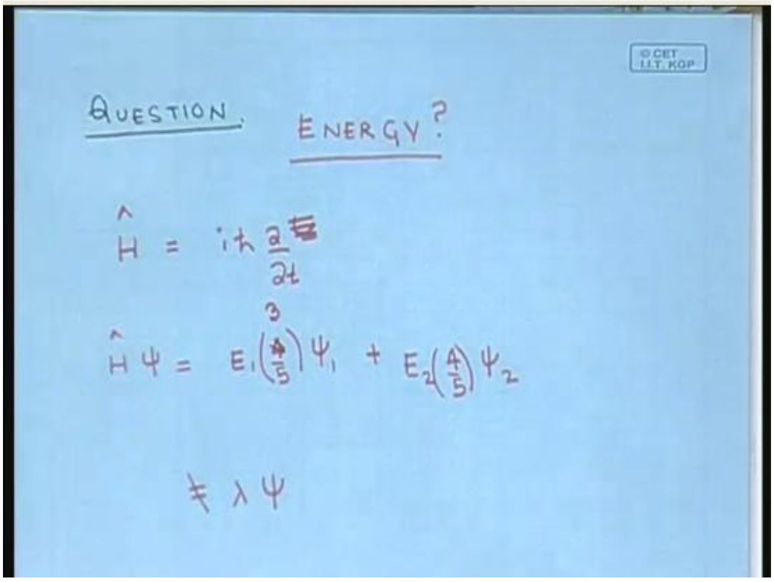

So, here we have calculated H into psi H into psi is E 1 into three fifth psi 1 plus E 2 into four fifth psi 2. So, I can put that in straight away.

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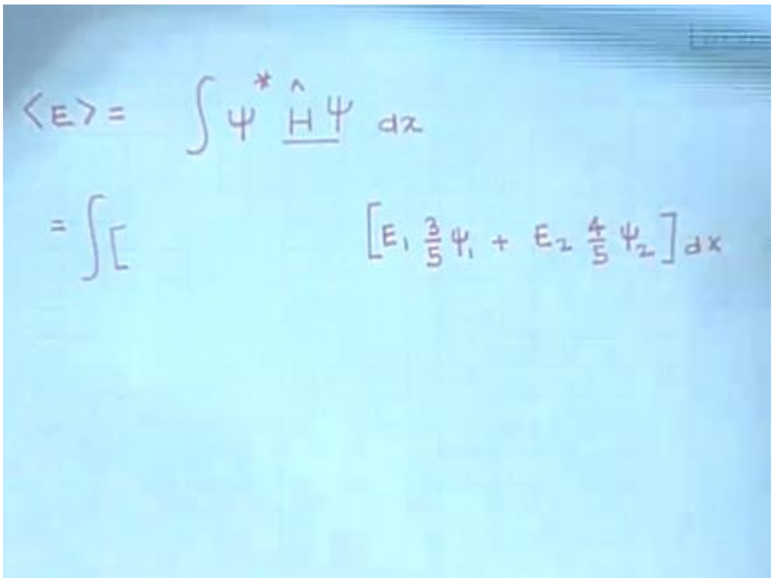

So, I have the integral, and here I will write the expression for psi star. I am now writing H psi, I am writing out this part. So, H psi is E 1 into three fifth psi 1 plus E 2 into four fifth psi 2 d x.

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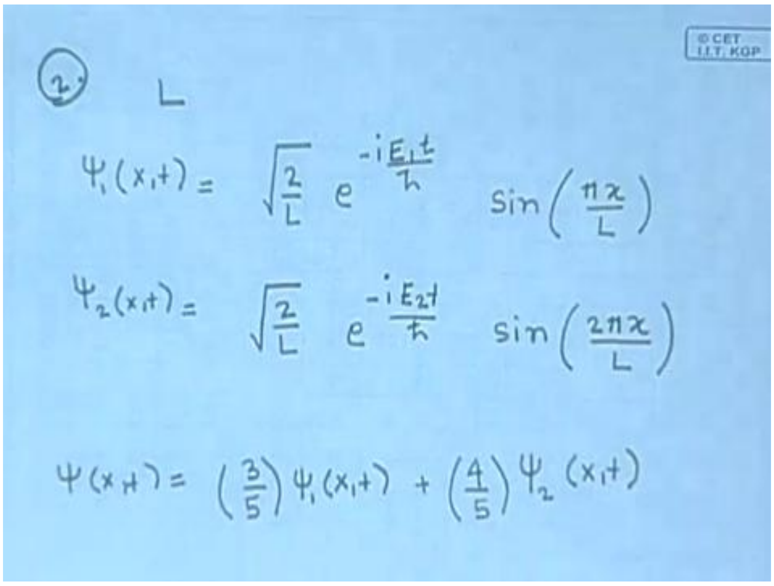

And here I have to write psi star. So, psi star, remember that psi, the wave function psi was three fifth psi 1 plus four fifth psi 2. So, if I take complex conjugate, I will get complex conjugate of this and this.

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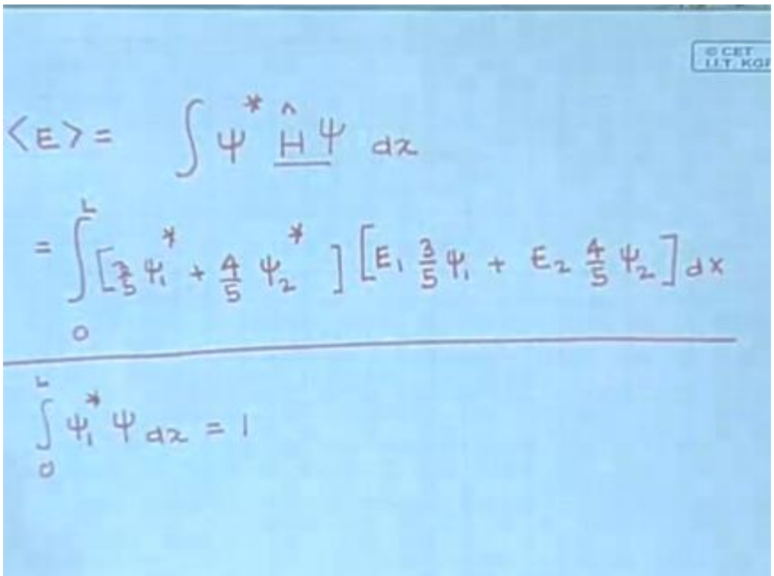

So, when I take the complex conjugate of pi I will have three fifth psi 1 star plus four fifth psi 2 star. So, I have to multiply this, this is psi star this is H the Hamiltonian operator acting on psi which I have written down here, and I have to do the integral over dx. So, I have to do this integral; the range of integral is zero to L and the wave function, and its complex conjugate both vanish for x less than zero or greater than L. So, we have to now do this integral that will give me the expectation value of the energy. Now, when I multiply these two terms, there should be a closing bracket here. When I multiply this with this, I will have one term which is psi 1 star into psi 1. Now, remember that we have normalized the wave function; that is 0 to L psi 1 star psi d x is 1, because when I multiplied psi 1with its complex conjugate, the time dependence part and its mod complex conjugate give me 1.

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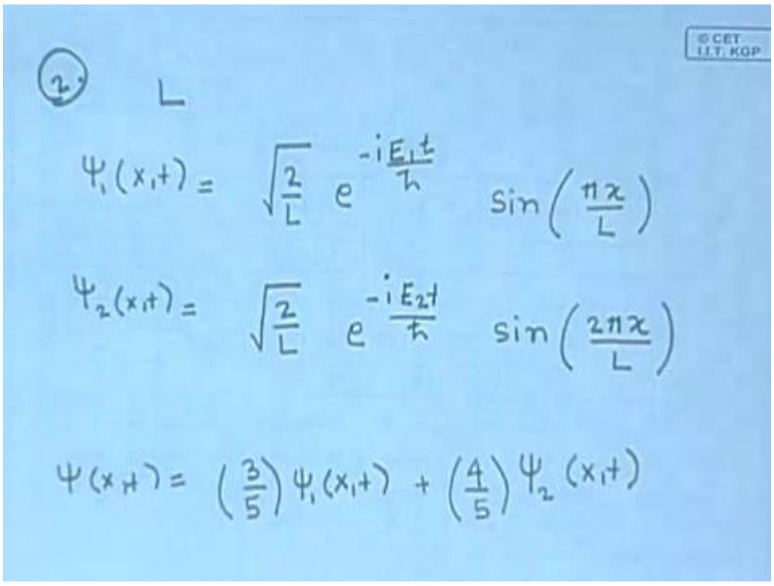

This will give me a factor of 2 by L and I will have sin square pi x by L, if I integrate sin square pi x by L from 0 to L, that integral multiplied by two by L is going to give me a factor 1, that is how we have chosen this normalization constant. So, that the integral of psi star psi gives me 1. Similarly for this also the integral is going to give you a factor of 1. This comes from the total fact that the total probability of finding the particle in this state is 1. For this the total probability of finding the particle is 1, or if the particle is in this state again the total probability of finding the particle somewhere is 1.

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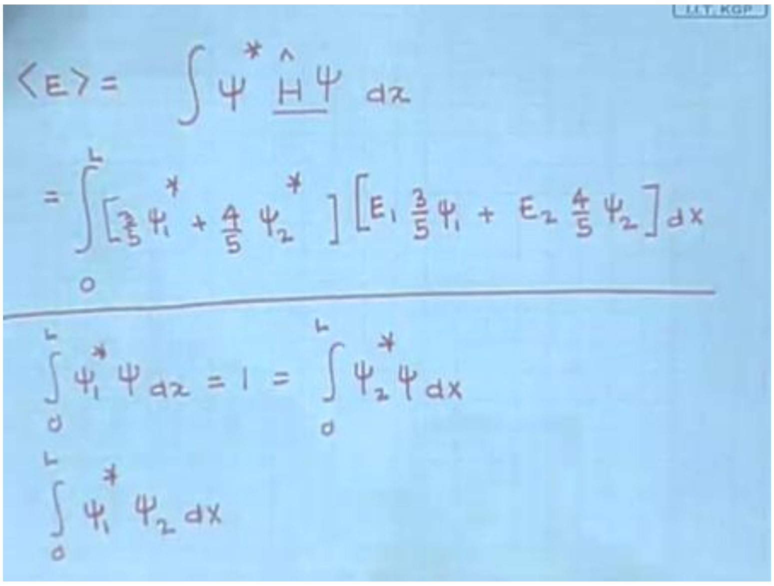

So, both of these functions have been normalized in the sense that the coefficient, the amplitude has been chosen, so that this is satisfied. So, these are the two terms when I have which arise when I multiply these 2 psi 1 star into psi 1 psi 2 star into psi 2, but there will also be cross terms. So, there will be a term of this type 0 to L psi 1 star psi 2 d x. now remember what psi 1 and psi 2 are again.

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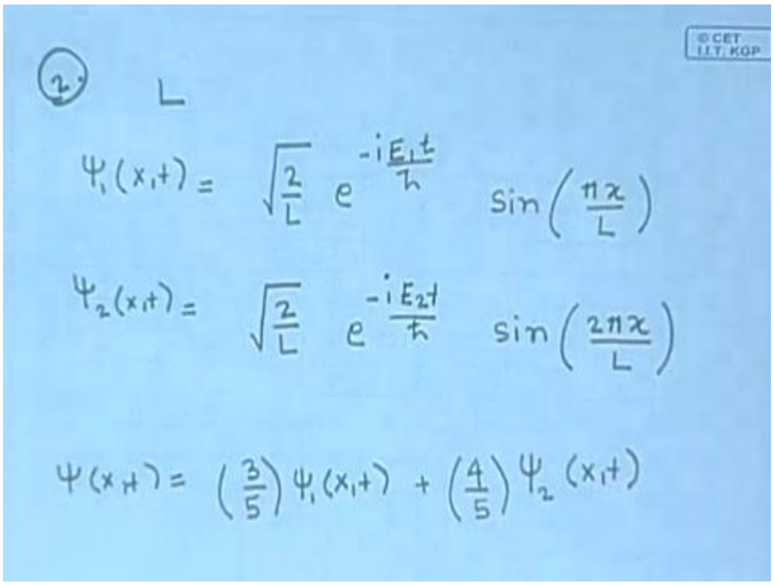

So, psi 1 the spatial dependence is sin pi x by L psi 2 is sin 2 pi x by L. So, the product of this sin and this sin term, can be written as a some of cosines. I can write this as a, some of two cosines, and both of these cosines when I integrate from 0 to L are going give me 0. So, this is equal to 0 and so is the other cross term.

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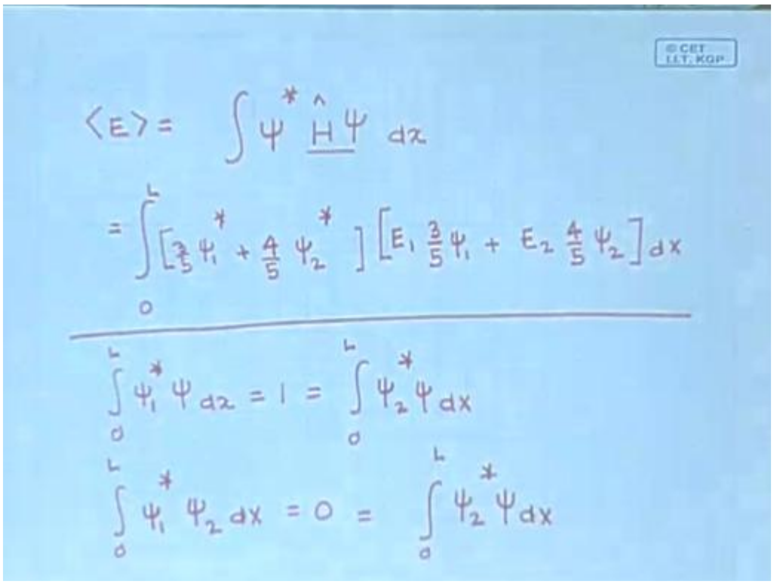

So, all the cross terms are going to give me 0, and only the terms where I have multiplying a wave function with its own complex conjugate gives me 1. This is very similar to what you have, when you deal with matrices and vectors. If you have a matrix, and work out its different Eigen values, corresponding to each different Eigen value, you will have an Eigen vector. And you can check that the Eigen vectors corresponding to two different Eigen values will be orthogonal. Exactly the same thing happens here. the Eigen functions corresponding to two different Eigen values, are orthogonal in the sense that psi 1 star psi 2 d x is going to be 0, and so is 0 to L psi 2 star psi 1 d x. So, now, when I do this integral, and multiply these two and do the integral I will get one term, which is E 1 into 9 by 25. The integral psi 1 star psi 1 d x is going to give me 1, so that is going to be one term. So, let me write it down here.

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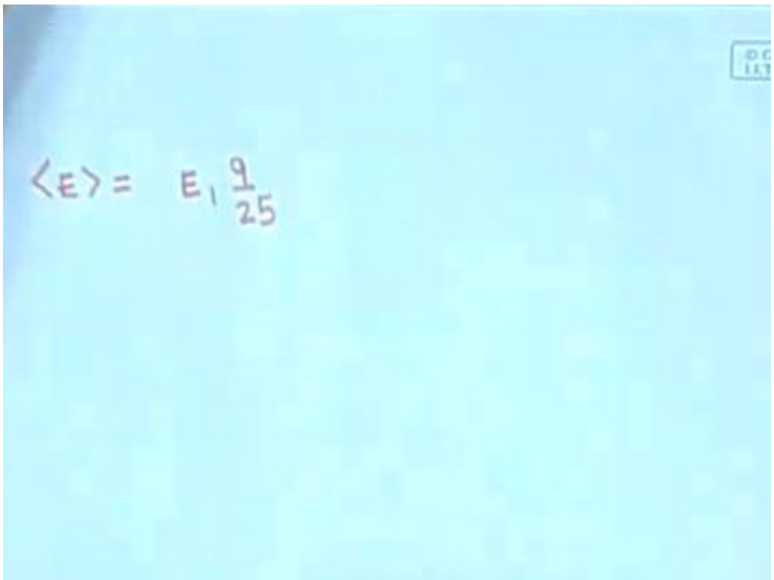

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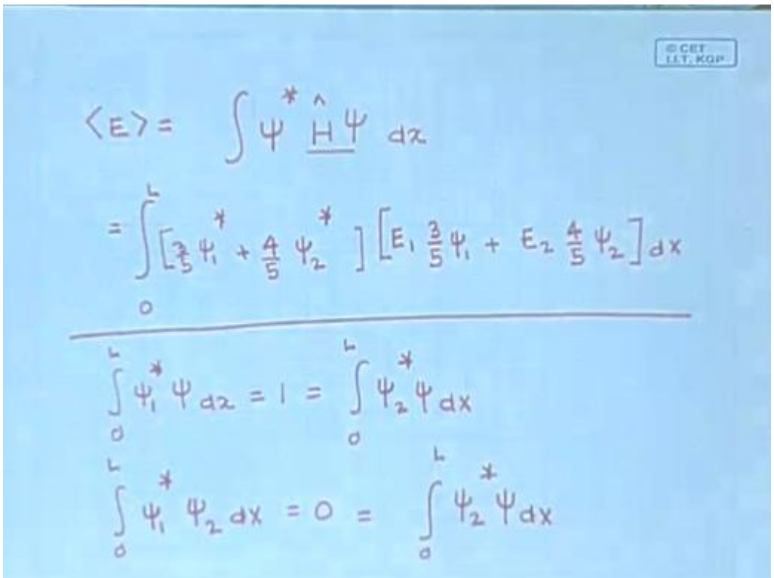

So, E 1 9 by 25 is what arises when i multiply this and this and do the x integral. When I multiply this with this I will get E 2 16 by 25. E 2 comes from here, and I have 4 by 5 4 by 5 let this integral is 1. So, what I get is this plus E 2 16 by 25.

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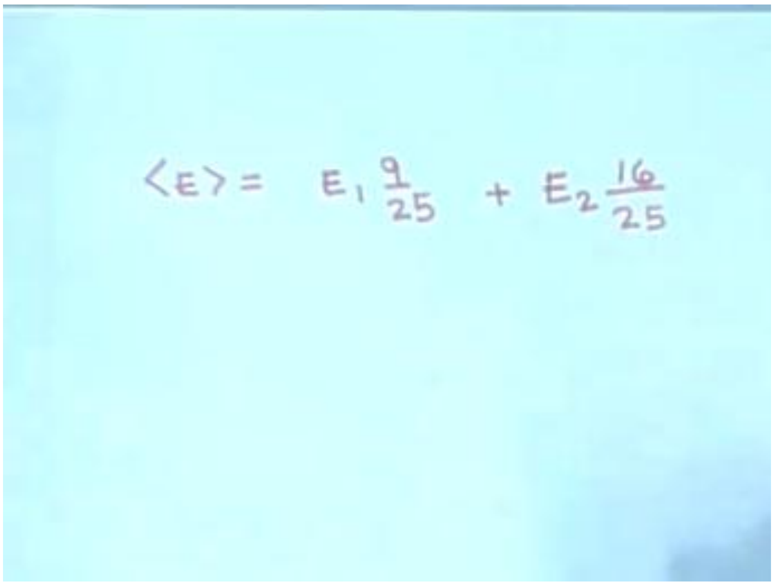

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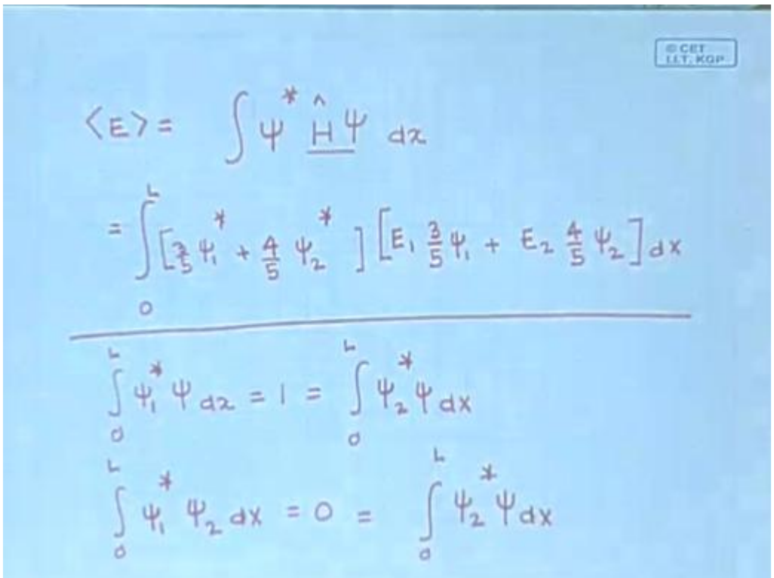

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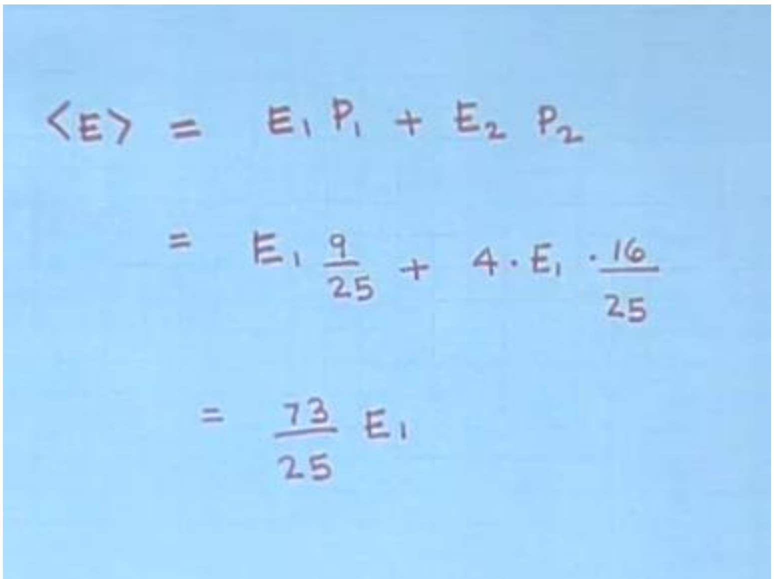

And the other two terms, where I have E 1 psi 1 and psi 2 psi 1 star psi 2 or psi 2 star psi 1, the integrals are 0. So, this is the expectation value of the energy, and this is exactly the same expression, which you had obtained when you have done this calculation earlier.

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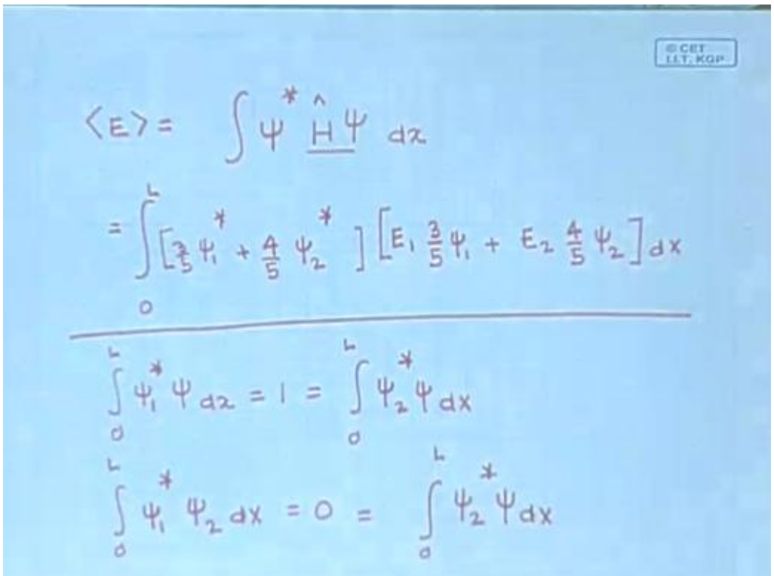

So, we see that whichever way calculate the expectation value of the energy, either you put in the operator psi act on psi psi star, or if you calculate the probability is of getting both these two values, and calculate the expectation value. Either way you get exactly the same answer. Let me next take up the question, what happens if I make a measurement of the momentum of the particle. Let us ask the same question for the momentum of the particle.

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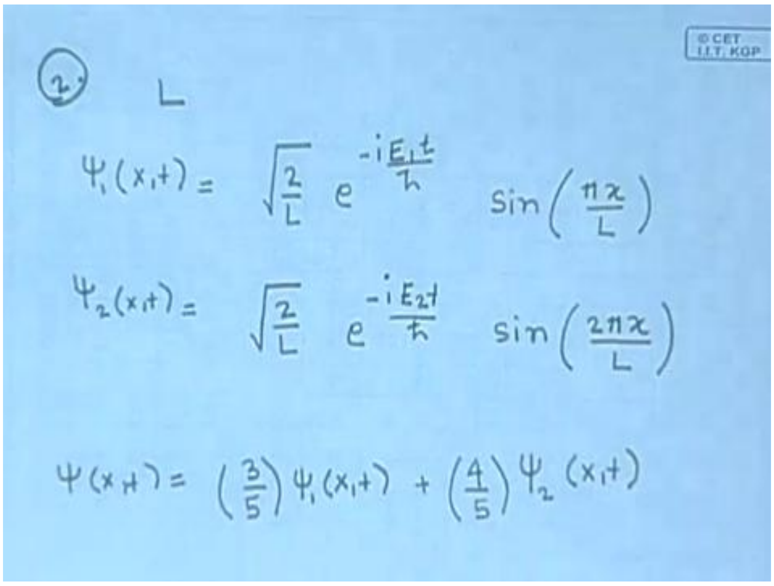

So, let us look at each of these states individually, and ask the question, is this state an Eigen function of the momentum operator. So, the spatial dependence of this function is sin pi x by L.

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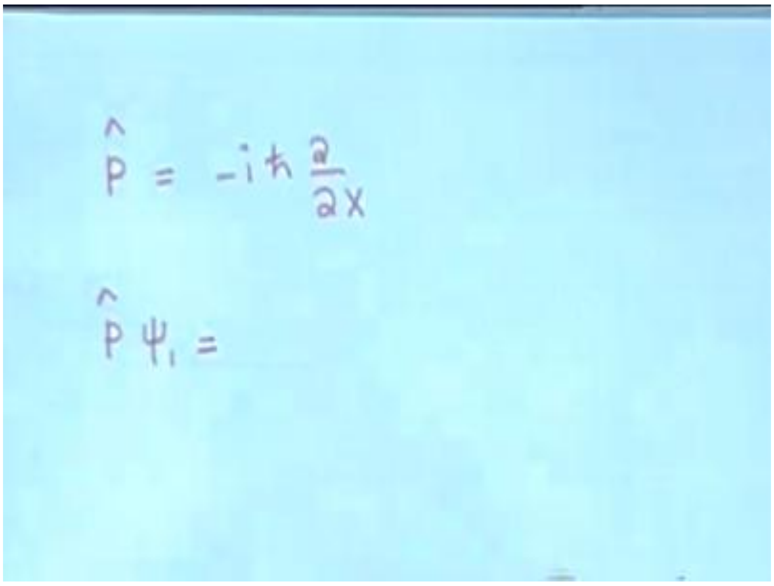

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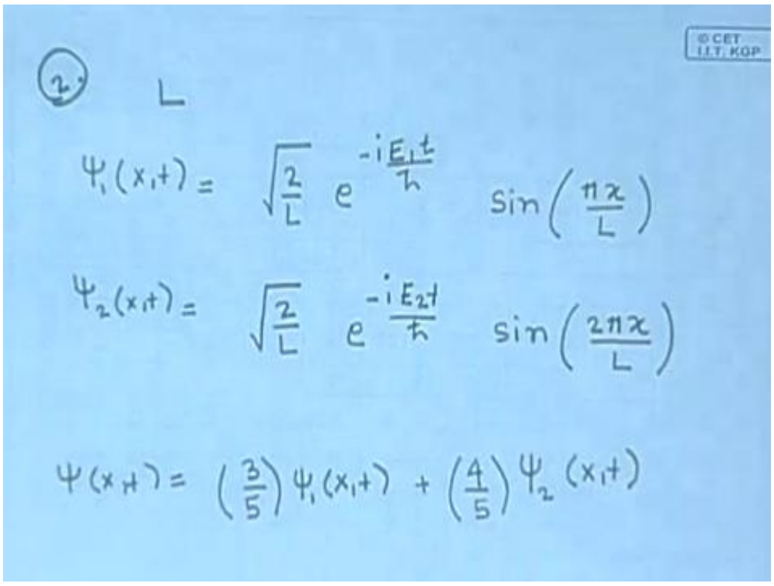

And the momentum operator p is minus i h cross del by del x. and let us calculate this p acting on psi 1. So, we are considering a situation where there is a particle in the ground state, and we want to ask the question is the ground state an Eigen state of the momentum operator of momentum. So, if there is a particle in the ground state and I measure its momentum, what is the result going be. If it is an Eigen state, you can make a definite predication, if not you can only predict expectation values probabilities etcetera. So, what we doing now, is we are checking if it is a Eigen state or not.

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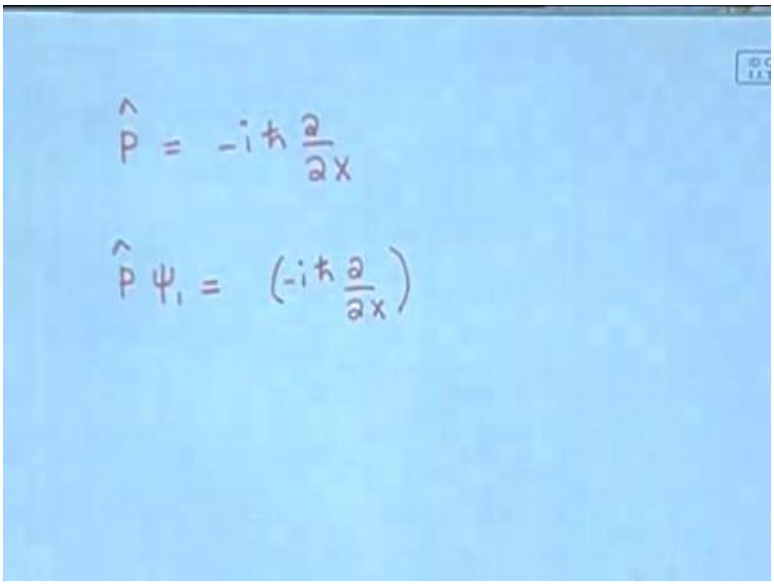

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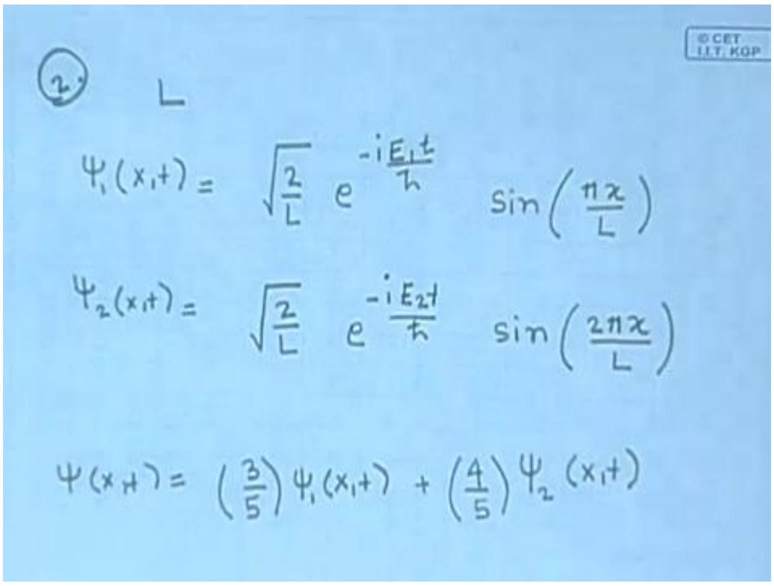

So, momentum acting on psi one1 is what we wish to calculate. The momentum operator is minus i h cross del by del x. the wave function psi 1 has these two things which have no x dependence.

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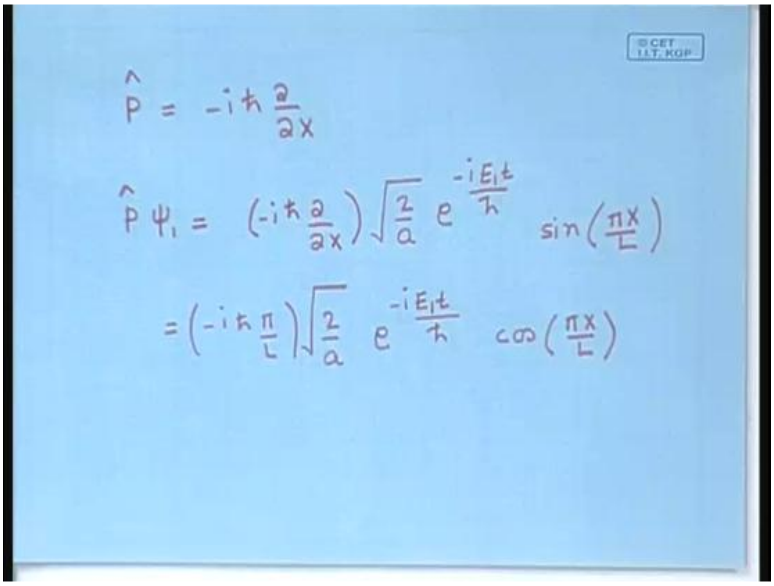

So, I can write them, let me write them any way, instead of being lazy e to the power minus i E 1 t by h cross acting on sin pi x by L. Now, when I differentiate with respect to x, these are as good as constants, so I have to only differentiate this. And when I differentiate it what I get is going to be minus i h cross pi by L is going to come out. So, I have minus i h cross pi by L into root 2 by a into e to the power minus i E 1 t by h cross cos pi x by L. So, what we see is, that when the momentum operator acts on the ground state wave function, for a particle in a potential well, the ground state wave function is sin pi x by L; that is the x dependence. When the momentum operator acts on this, it changes it to cosine pi x by L. So, the ground state is not an Eigen state of the momentum operator. So, you cannot make a definite predication as to what the momentum, is going to be, if I make a measurement of the momentum for a particle in the ground state, of a particle in a box, ground state of the energy states for a particle in a box. Now let us ask the question what is the expectation value of the momentum. We cannot predict a the definite value. Can we predict what the expectation value is going to be. If I repeat the experiment many times for the same state, so I have many replicas of particle in the ground state of a potential well of the same potential well. And I measure the momentum, what is the expectation value. So, we have discussed how to calculate the expectation value.

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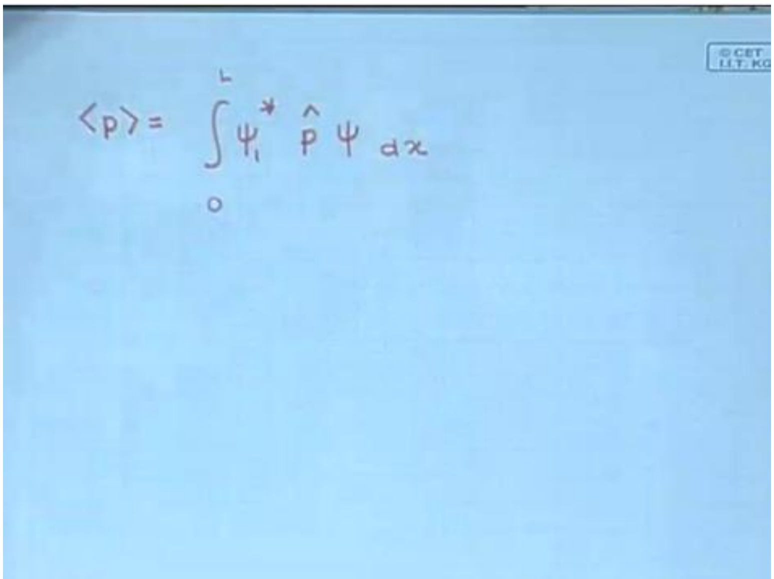

So, the expectation value of the momentum P for the ground state has to be calculated like this psi 1 star the p operator into psi d x. The integral has to be done from 0 to L. So, the psi 1 wave function, remember it has this constant.

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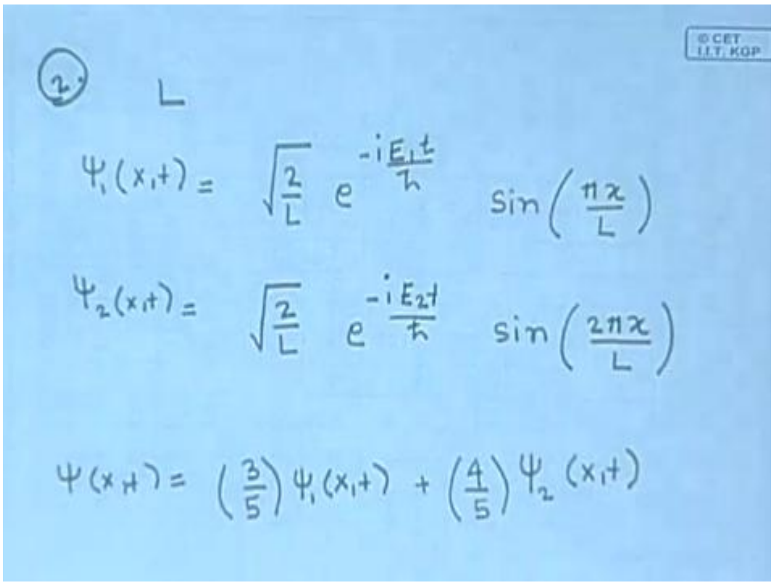

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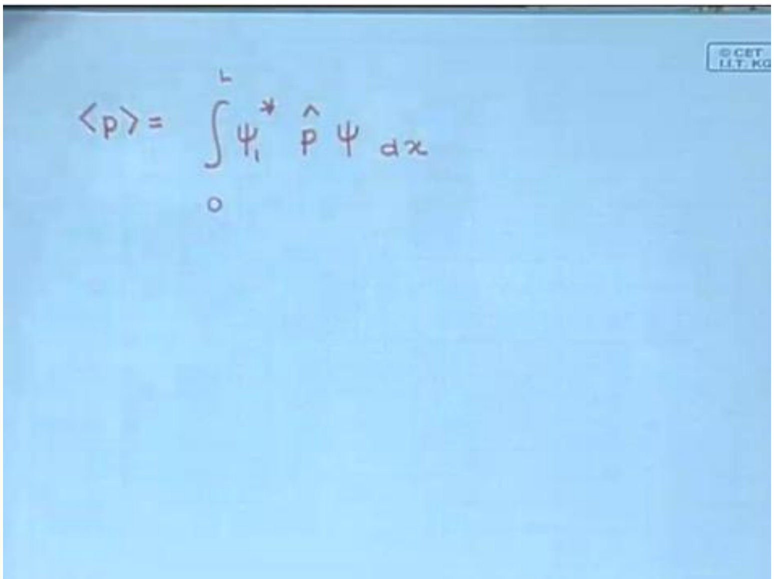

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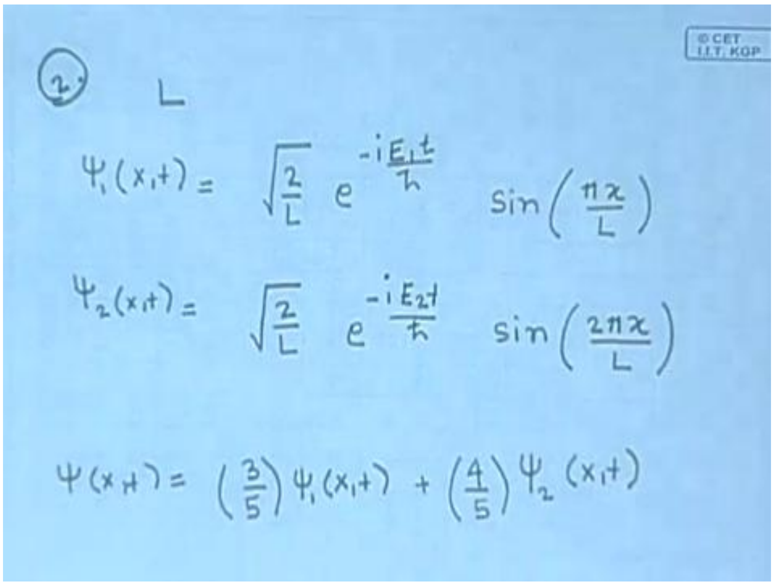

It has this time dependent part, and when i multiply psi with psi star the constant is not going to change, and psi psi star is going to give me 2 by L, is going to get squared. When I multiplied this with complex conjugate, I am going get 1, and I have sin pi x by L over here.

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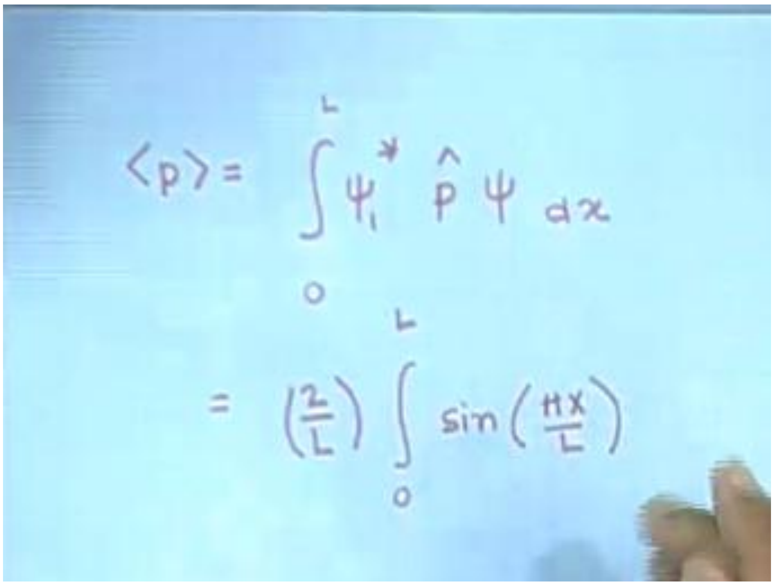

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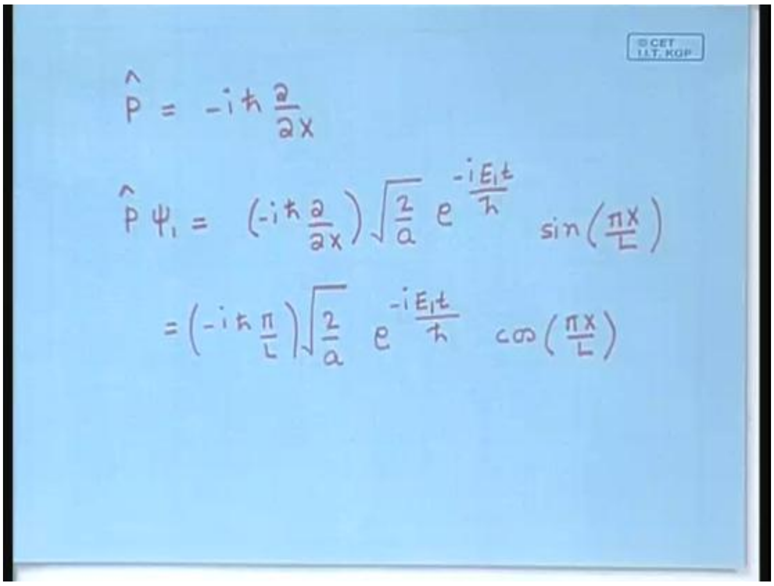

So, this is going to give me 2 by L 0 to L psi star is going to give me sin pi x by L, and the momentum operator acting on psi 1, is going to give me this extra factor into cos pi x by L.

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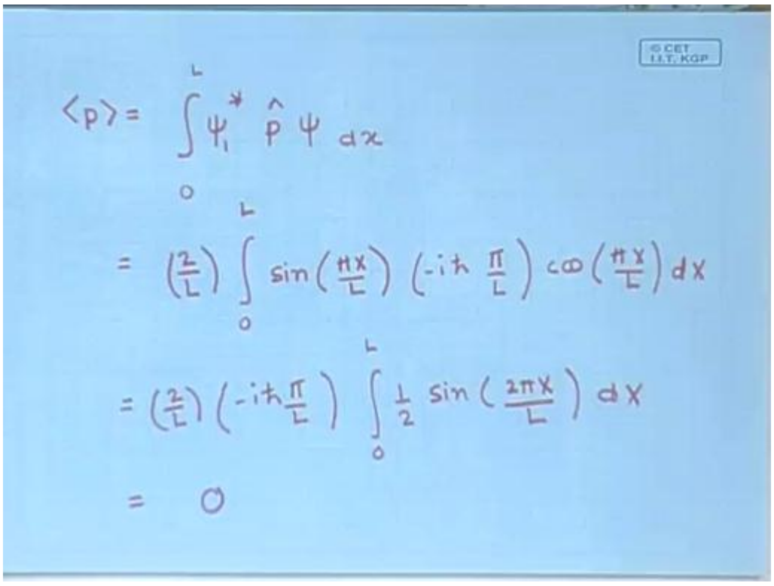

And the extra factor is, so constant minus i h cross pi by L cos pi x by L d x. now note that sin of pi x by L into cos of pi x by L this integral dx is 0, because this is going to give me sin 2 pi x by L half of that. So, this is going to give me sin and the cosine term both. And if I do this integral I will get 0, because is going to give me cos, and this is one whole period of this function if I integrate either sin or cos over a whole period I get 0.

So, this the expectation value of the momentum turns out to be 0.

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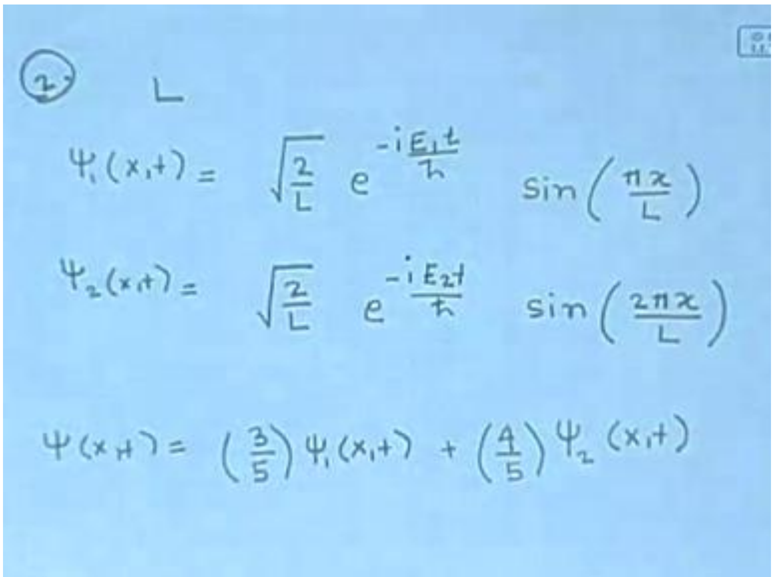

So, what we have been calculating now is that if I have a particle in the ground state. I am not bothering about the superposition or this. If there is a particle in the ground state, and if I measure its momentum what is the expectation value, and what we see here is, that the expectation value of the momentum for a particle in the ground state, the expectation value of the momentum is 0. Each time I repeat the experiment I will get different values. They will occur with equal probability for positive and negative, and the average is going to be 0. Now, you can check that also for psi 2, again even I act with momentum, I have a sin here. If I act with the momentum operator it will give me cos, so it is not an Eigen state .and if I calculate the expectation value of the momentum I will have again cos into sig the integral is 0. So, for psi two also the expectation value of the momentum is 0. So, from there we can say that for the superposition also, the expectation value of the momentum is 0. For that matter for any state of a particle, any superposition of such states of a particle, in a potential well, the expectation value of the momentum is always going to be 0. Now, the next thing that we can calculate. Let me tell you what that is, so I have a particle we will again restrict our attention to the ground state. I have a particle in the ground state, in a potential well. And we have seen that the ground state is not an Eigen function of the momentum operator. So, whenever I make a measurement of the momentum, I will not get the same answer every time, I will get different values, but the expectation value of the momentum is going to be 0. So, I will get positive and negative values with equal probability. And the outcome the outcomes are going to be such that the mean is going to be 0. The expected value is 0. But I am not going to zero at every time when make a measurement. So, this basically tells us that the momentum is going to have a spread in the values. So, what we would now like to calculate, is the uncertainty in the momentum, and I have told you, how to calculate the uncertainty in any quantity. So, the way to calculate the uncertainty in any quantity, is you have to basically calculate the variance.

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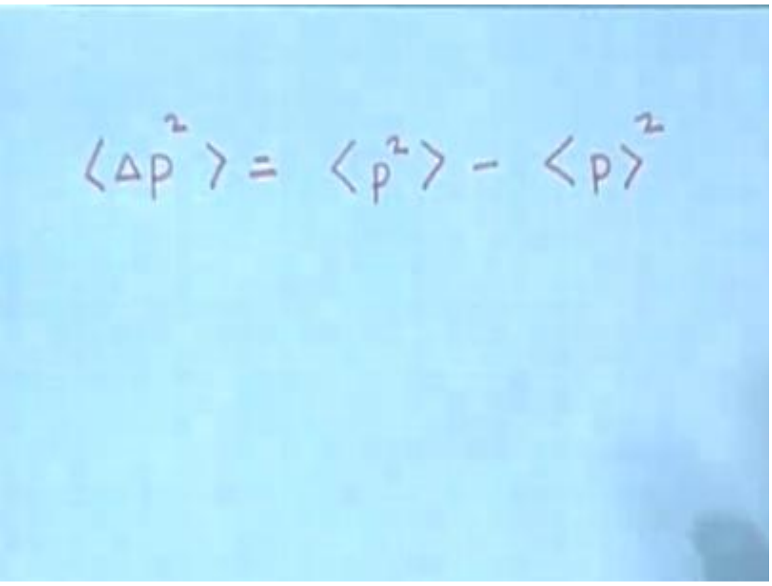

So, what you have to do is, you have to calculate the deviation from the mean, take the square of that and calculate its average. So, for the momentum you have to calculate the deviation every time you make a measurement, you will get a different value which is not the average value. So, take the square of the deviation, means square deviation. And I have shown you that this can be also written as the expectation value of p square minus the square of the expectation value of p. This I have shown you that this can be written in this way. Now, in this particular problem, where there is an electron a particle in the ground state. We have already calculated the expectation value of the momentum, and the expectation value of the momentum we have seen turns out to be 0.

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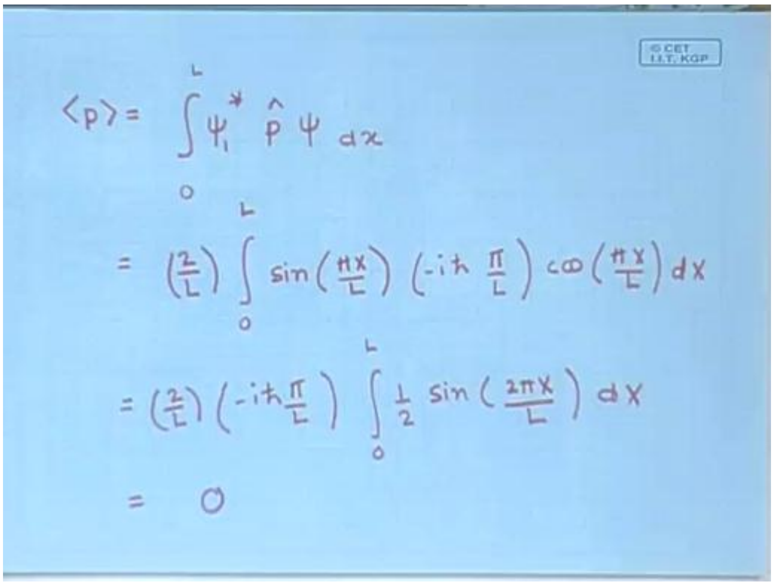

The expectation value of the momentum for a particle in the ground state, is turns out to be 0 for that potential well problem.

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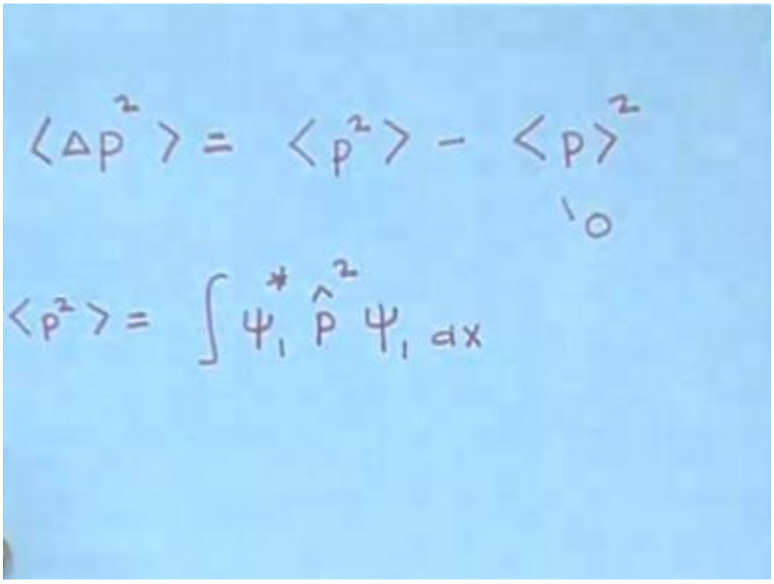

So, we have already calculated this, this is 0. So, we now have to just calculate the expectation value of p square. So, the expectation value of p square can be calculated like this; I take psi star, I take the operator p and square it, and then act on psi and do this integral. So, we now have to calculate p square acting on psi, where we are dealing with the ground state wave function, so I can write psi 1.

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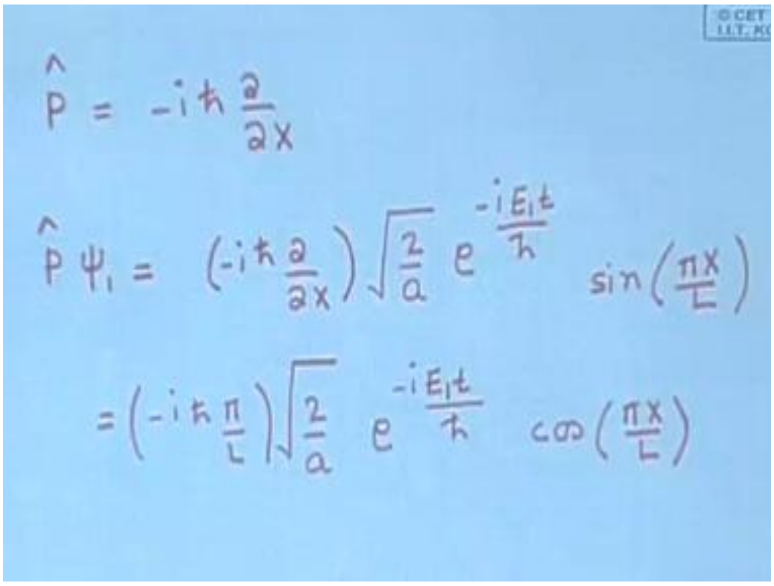

Now, I have already calculated p acting on psi 1 and p acting on psi 1 is minus i h cross pi by L into these factors; this is a constant, this is the time dependant part of cosine pi x by L. Now, this is the momentum operator. So, if I act with the momentum operator; that is, if I differentiate this.

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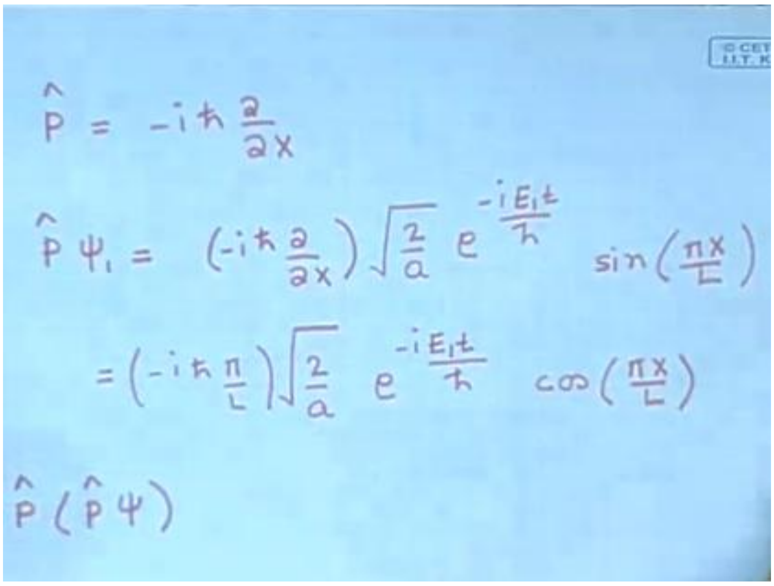

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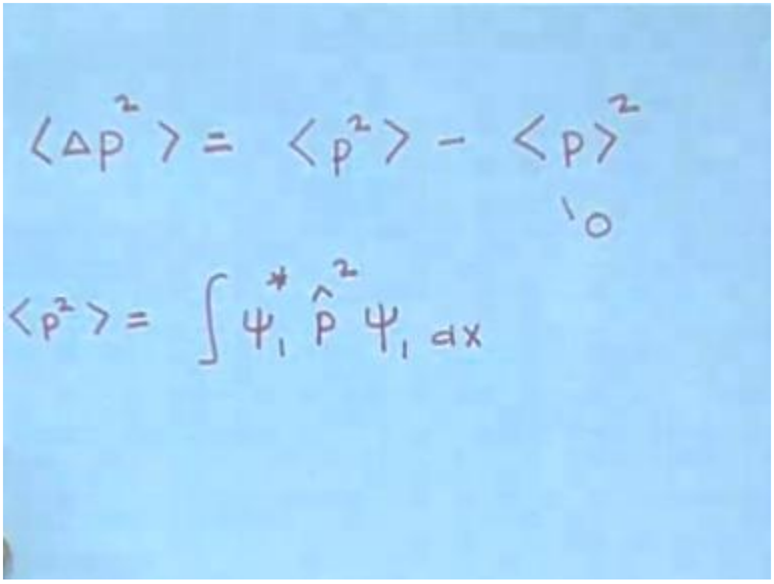

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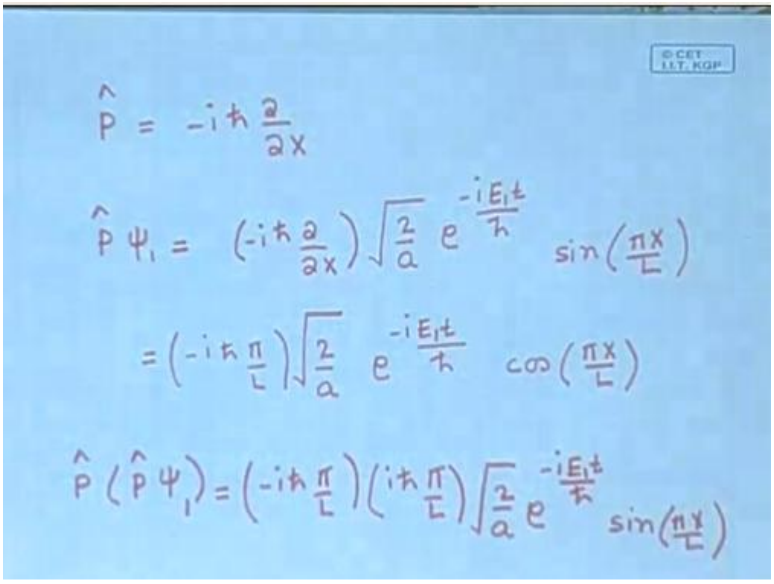

So, what I have to do now is, calculate p of p psi, this is what we mean by p square psi 1. So, I have to calculate act with the momentum operator twice on psi 1. So, I act with it once, and then act with it again. So, if I differentiate cosine with respect to x, I will get minus pi by L into sin, and that minus pi by L, if I multiply with minus i h cross, so what I will get is minus i h cross pi by L. This is already there here. And then I have one more factor of minus i h cross, and I have minus pi by L now, because when I differentiate cosine, I will pick up a minus sign. So, this into i h cross pi by L, and then I have these factors root of 2 by a e to the power of minus i E 1 t by h cross. And I have the sin pi x by L, and minus i h cross pi by L into i h cross pi by L this into this. So, minus i into i gives me factor of one, and I have h cross square pi square by L.

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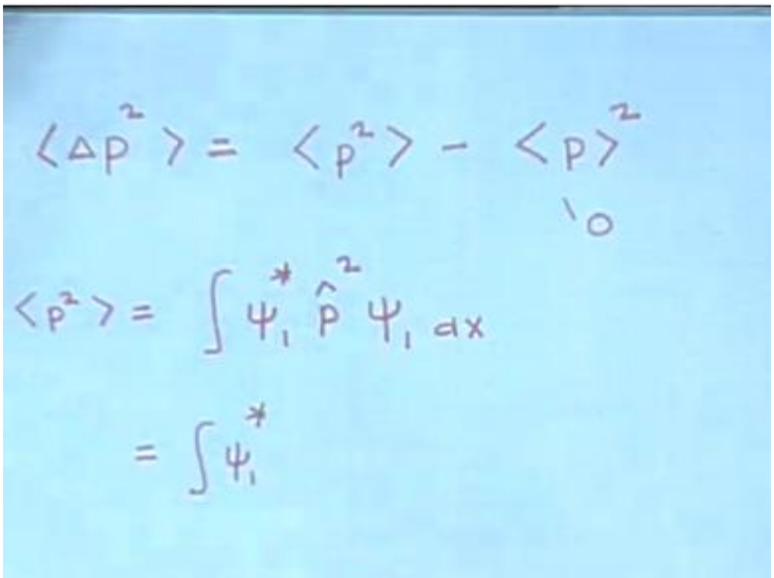

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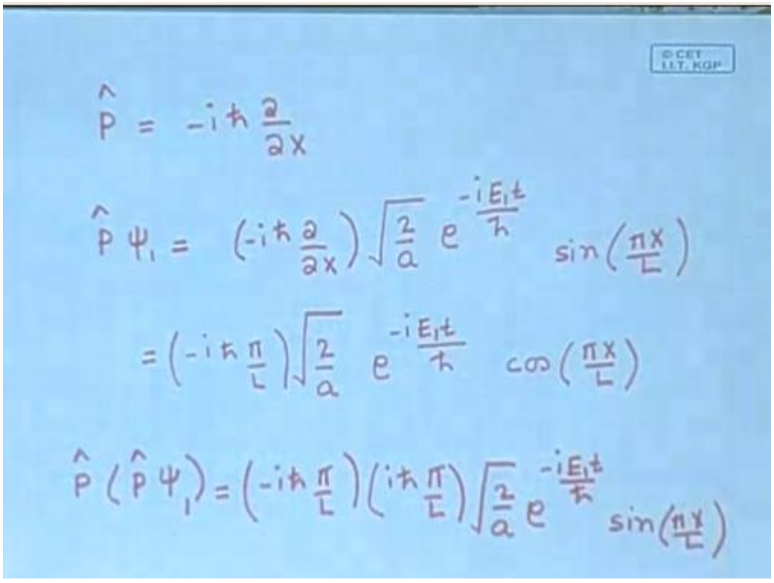

So, what we see is, that this can be written as psi one star and P square acting on psi 1 is essentially pi square h cross square by L square. This minus i and i give you factor of one into psi 1 itself, this thing is psi 1 is itself.

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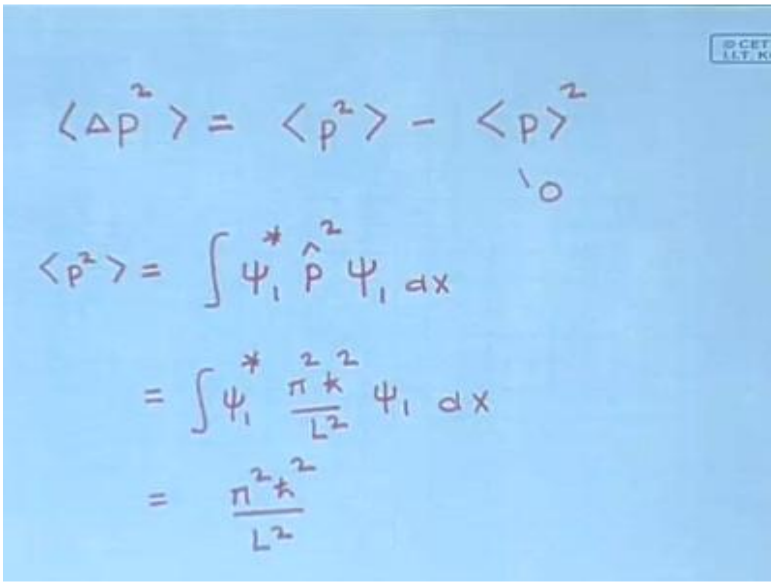

So, this is pi square h cross square by L square psi 1. So, p square acting on psi 1 gives me the same. So, same wave function psi 1 again multiplied by this number pi square h cross square by L square, and I have to do this integral d x. now this is a constant and we know that psi one star into psi 1 dx is 1. So, what we can say is that, this will give me pi square h cross square by L square. So, the uncertainty in the momentum, is the square root of this; that is the uncertainty into in the momentum, and the square root of this is the square of this.

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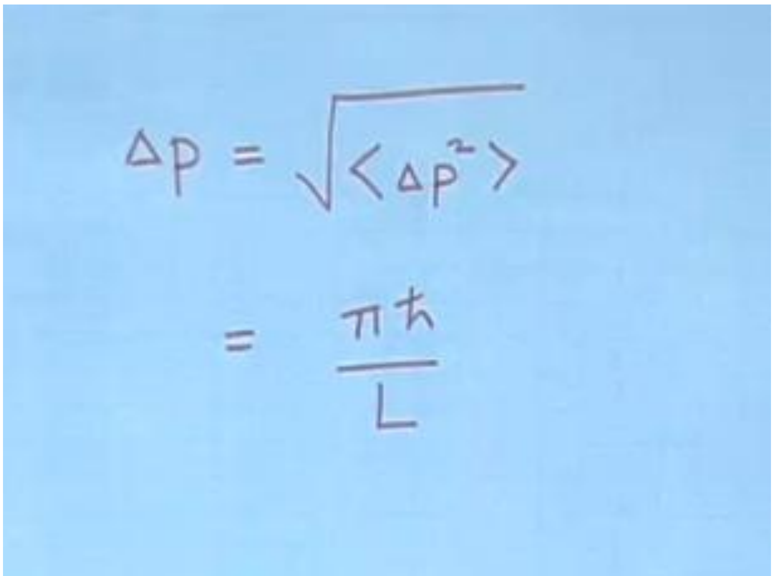

So, what we see is that the uncertainty in the momentum delta p is equal to the square root of this which is pi h cross by L. So, in today's lecture we have solved a few problems, and I have shown you how to manipulate with some of these ideas, and formulas, which we have derived in the last few lectures.