Physics I: Oscillations and Waves
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Lecture - 39
Particle in a Potential

Good morning. In the last lecture, we were considering a situation where we had a particle in a state psi.

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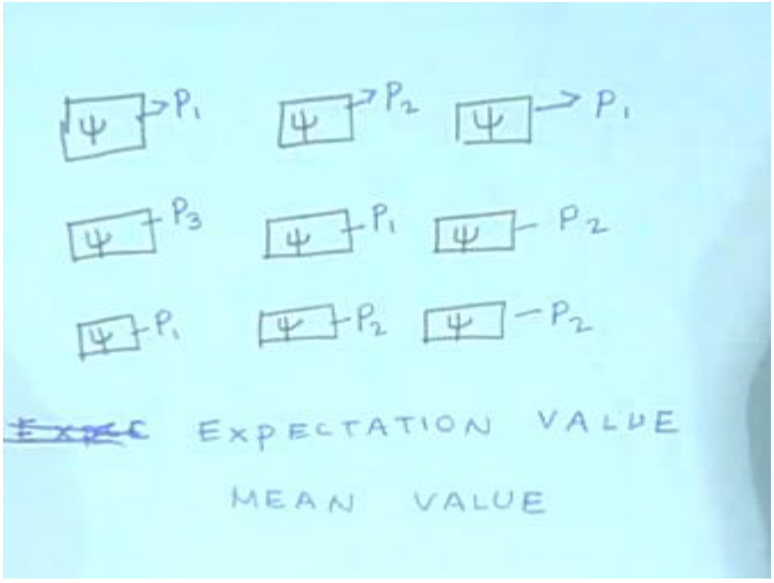

And the situation was such that we had many replicas of the same particle. So, here I have shown you schematically many particles all of them in the same state psi. So, we go and measure the momentum of the particle and the many different replicas of this particle allow us to perform the experiment independently many times. And we have learnt in quantum mechanics that you cannot in general predict the expected outcome. So, each time you repeat the experiment you will in principle get a different outcome so, if you measure the momentum each time you repeat the experiment for each of these independent experiments where you measure the momentum of the particle whose wave function is psi. You will get a different value of the momentum and these values are going to be Eigen values of the momentum operator.

So, here for this particular particle you get the momentum P 1 here you get P 2 here you get P 1 again here you get P 3 P 1 P 2 P 1 P 2 and P 2. So, this is one possible scenario and if you repeat the experiment more times you will get someone of the Eigen values of the momentum operator. And the probability of getting you can calculate you can determine from this experiment the probability of getting any particular value P 1 P 2 P 3 P 4 etcetera. So, now, we ask the question what is the mean what is the mean value of the momentum and you can calculate it from the experiment by taking the number of times you get P 1 into the value of P 1 plus the number of times you get P 2 into the value P 2 plus the number of times you get P 3 into the value of P 3 divided by the total number of times the experiment was performed now, the question we were discussing was how to predict what this expectation value or the mean value should be from the wave function.

Because the wave function tells you the state of the particle. So, once you know the wave function you should be able to predict what the mean value of the momentum should be what that mean value can also be interpreted as the expectation value that is the value I expect to get if I do the experiment once. And I told you that you calculate this by evaluating.

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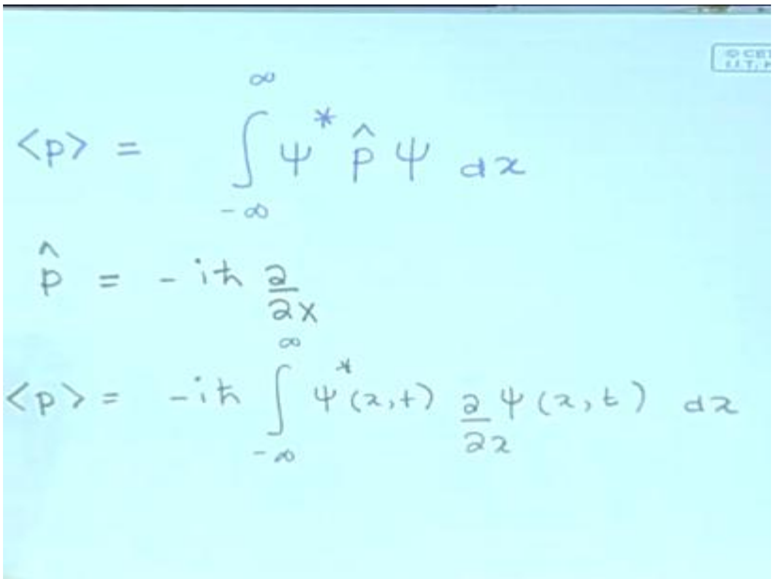

This expression the so, the expression that you have to evaluate to determine the expected value the mean value of the momentum is this is shown over here you take the momentum operator act on the wave function psi. So, the particle is in the state psi we are measuring the momentum. So, you take the momentum operator act on psi then multiply the resultant with psi star the complex conjugate of the wave function. And integrate from minus infinity to plus infinity this will give you the expected value of the momentum.

And this is something which is very simple to evaluate. So, if the momentum operator I have told you is minus i h cross del by del x So, if know the wave function psi which is a function of x and t if the particle is free to move along the x axis. Then the quantity which you have to evaluate is minus i h cross minus infinity to plus infinity psi star x t partial derivative of psi. So, if the wave function is known, it is quite straight forward to determine the expected the expectation value the mean value of the momentum the mean value that I expect for the momentum. Now, every time I do the experiment.

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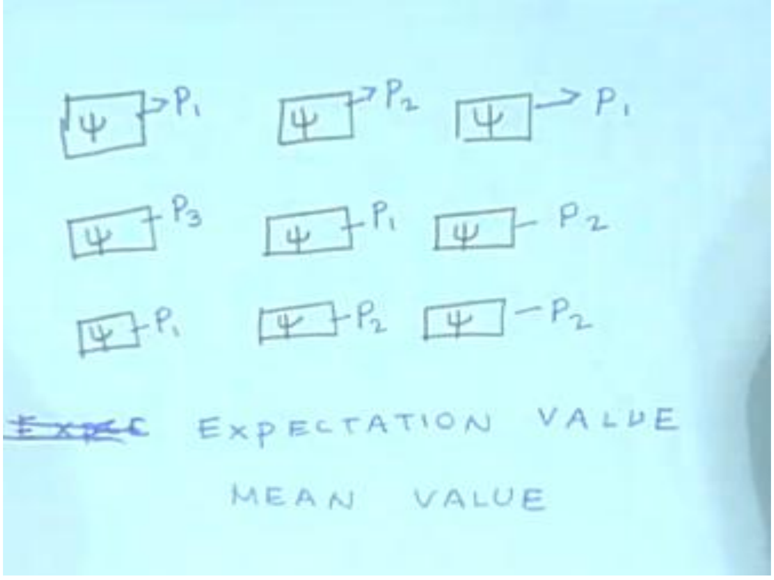

In general I will not get the mean value there will be a spread in the values and this spread in the values I have also told you that this spread in the values is quantified by the standard deviation in the momentum. So, I have to the quantity that quantifies the spread in the values.

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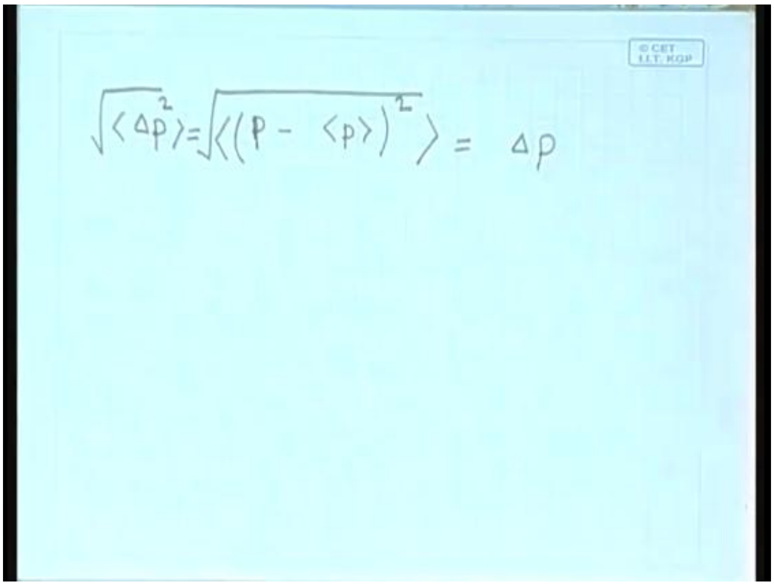

So, each time I do the experiment I will get a different value P and the difference from the mean value is what we call delta p. So, each time I do the experiment I will get a different value of P and the difference from the mean value is what I call delta P the quantity that we are interested in is the square of this difference the mean of that. This is called the variance or the mean square deviation and the square root of this gives the standard deviation that quantifies the uncertainty in the value of the momentum. And you can determine this.

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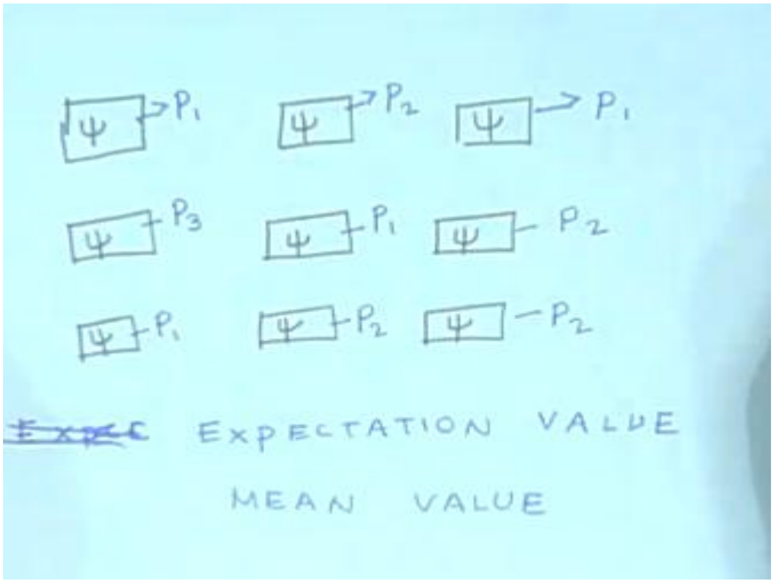

If know the values if you do the experiment each time you do it you will get a different value. So, you can determine the standard deviation we have discussed this 2 lectures ago. Now, the coefficient that we are going to address now, is how can we predict the standard deviation from the how can we predict the uncertainty in the momentum from the wave function? The wave function has all the information about the state of the particle. So, the question is how can you predict the uncertainty in the in the momentum from the wave function? And this can be done as follows.

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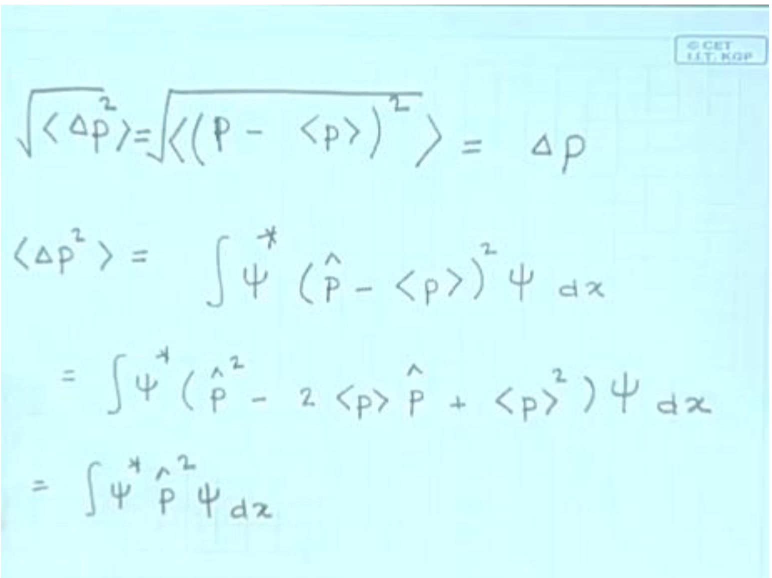

So, the mean square deviation the mean square deviation or the variance in the momentum can be calculated like this psi star P minus the mean value square into psi dx and this can be simplified a little bit. So, we can write this as psi star the square of this is going to be P the operator P squared minus twice the mean value of P into the operator P plus the mean value of P squared multiplied by psi dx. So, let us now, we can know break this up into 3 different terms the first term is going to be psi star into the operator P square psi dx. The second term you see this is a number this is the mean value of the momentum which we can evaluate like this.

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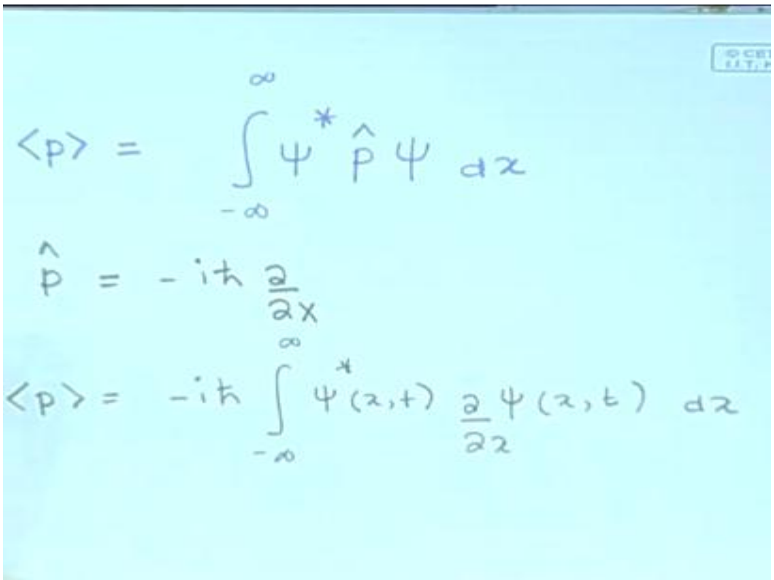

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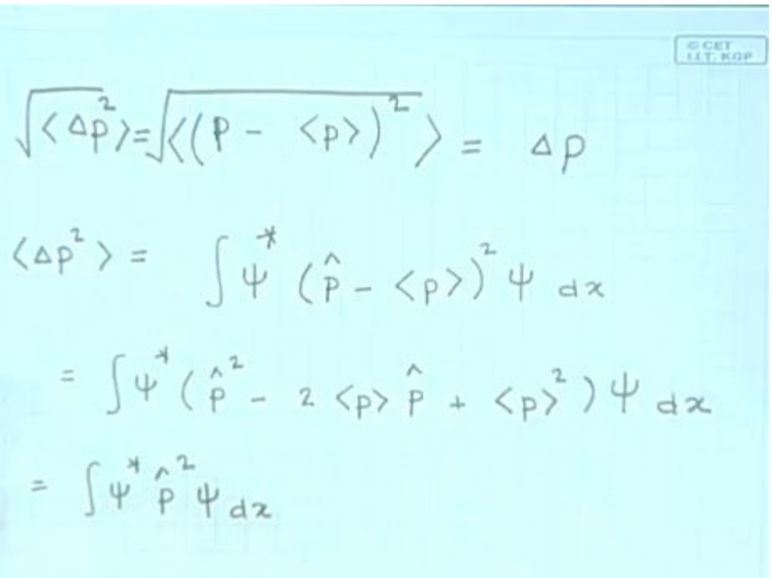

It is just a number this number Can be taken outside the integration with respect to psi psi star and dx if I take this number outside the integration then I have psi star P psi psi star P psi.

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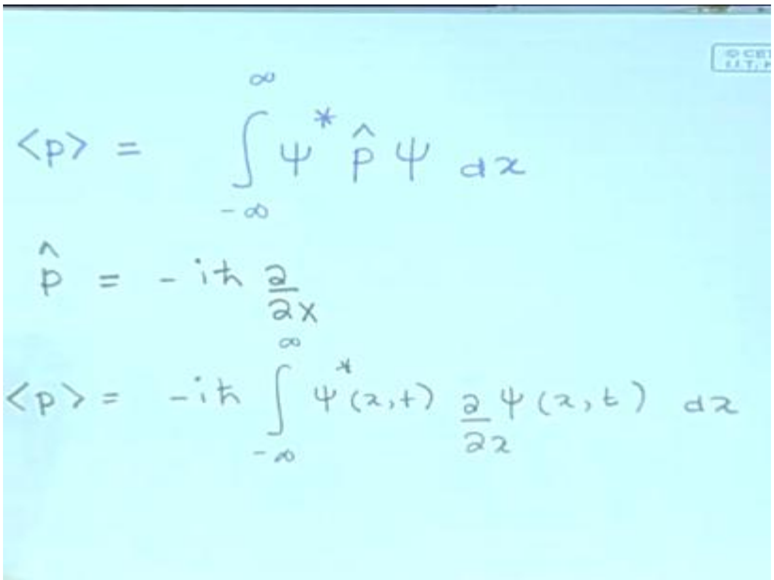

So, I have psi star P psi psi star P psi Is again the mean value.

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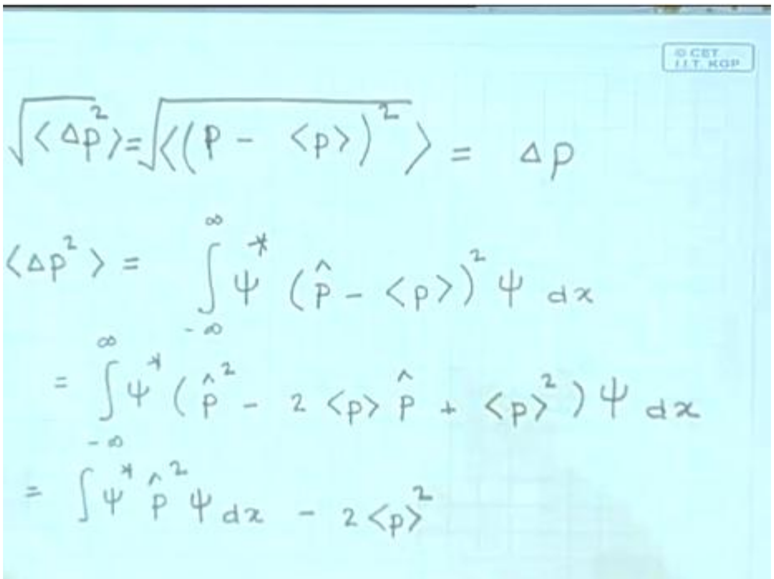

So, the second term gives me minus 2 the mean value of P squared and the third term is just a number the mean of P square I can take it out. So, I have psi star psi dx the integral from minus infinity to plus infinity.

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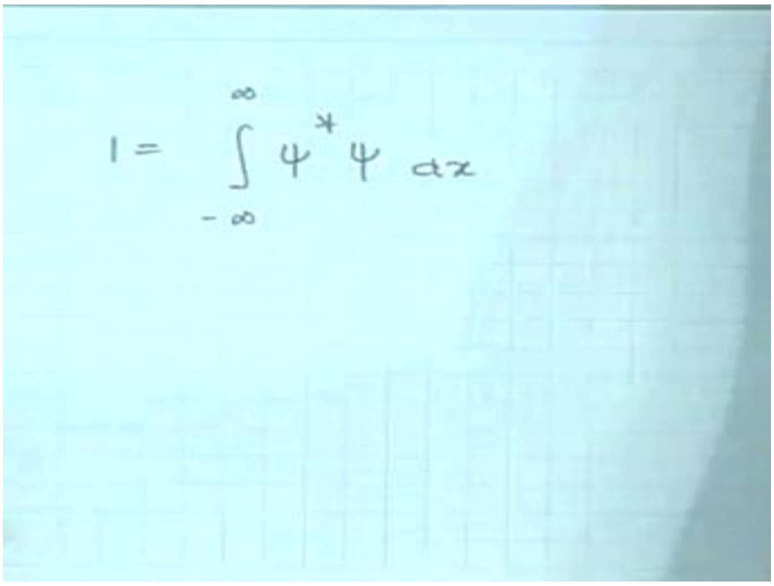

Now, the wave function is normalized so, that the total probability of finding the particle somewhere is 1.

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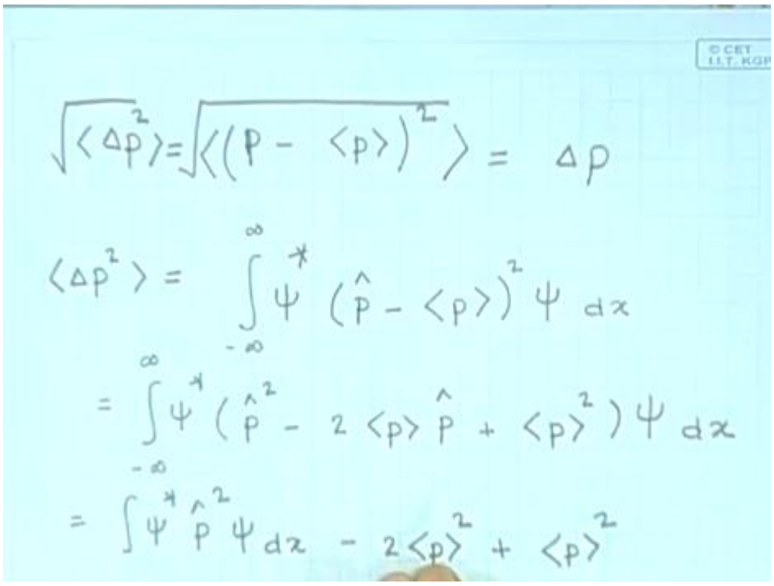

So, the third integral where this is a number... So, I can take it outside I have psi star into psi dx that gives me 1. So, I have this plus the mean value of P square. So, this these 2 terms has a cancelation and this term is the mean value of the P square of the momentum square operator. So, I can write it.

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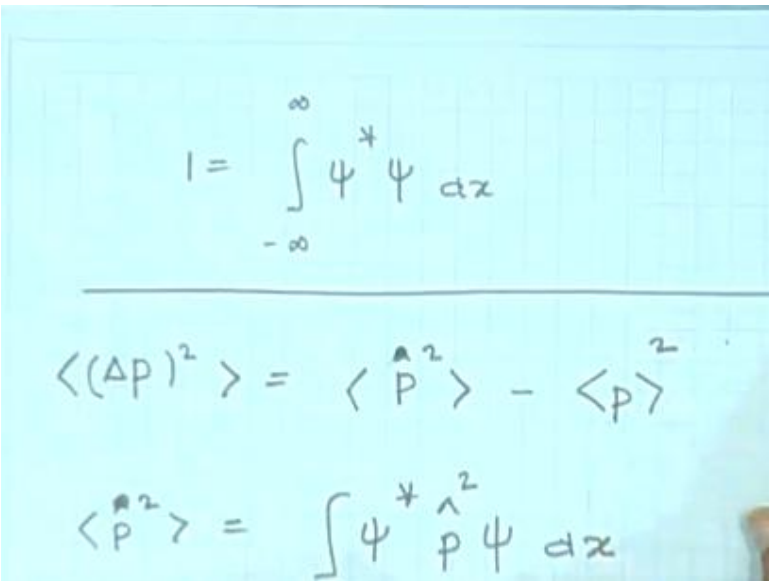

As follows that the uncertainty the dispersion in the mean square dispersion in the momentum is going to be the square the expectation value of the momentum squared minus, the square of the expectation value of the momentum where this refers. So, this where the first term has to be evaluated like this which again if you know the wave function is very simple to evaluate.

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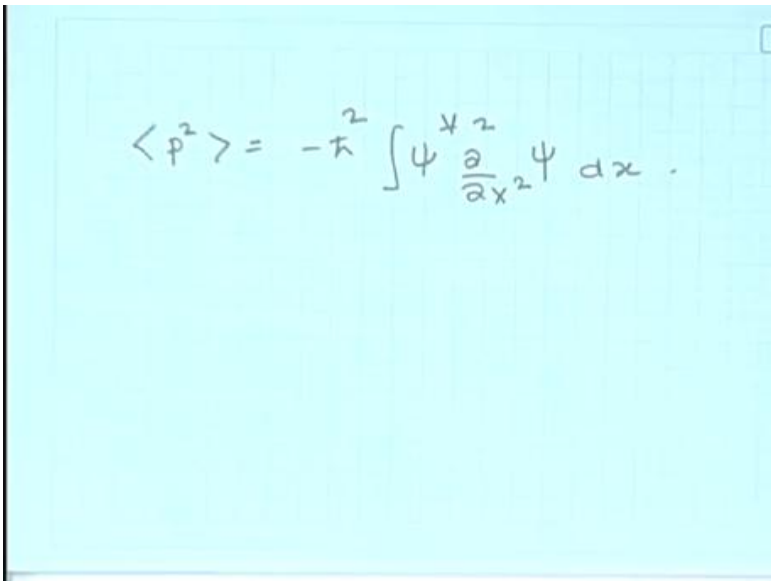

So, let we recapitulate what we have been what I have been discussing here.

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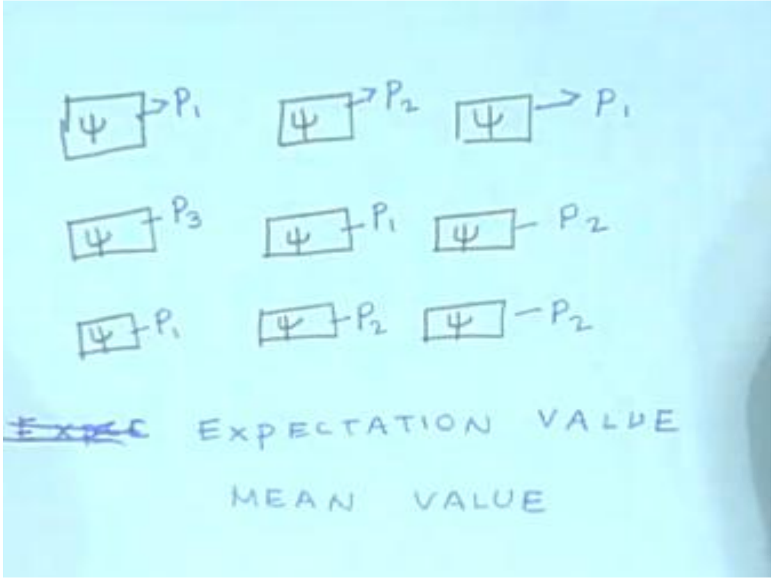

So, we were considering a situation where we have many replicas of a particle in the same state psi and each time we measure the momentum in general we will get the different value. And from these different values we can calculate what the mean value is that is the expectation values if I do the experiment only once I will expect to get that value. But when I do the experiment I will not get exactly the mean value there will be a spread around the mean values. That spread is quantified by the standard deviation in the values and from wave function once I know the wave function I can calculate both. The mean value the mean value and the expectation value the mean value can be calculated as follows the mean value can be calculated by taking psi star.

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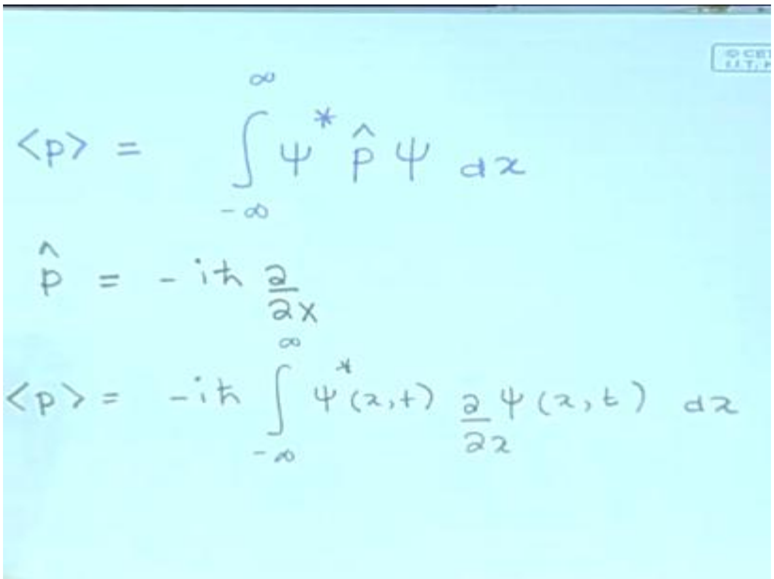

The momentum operator into psi that tells me the mean value the spread in the values can be calculated.

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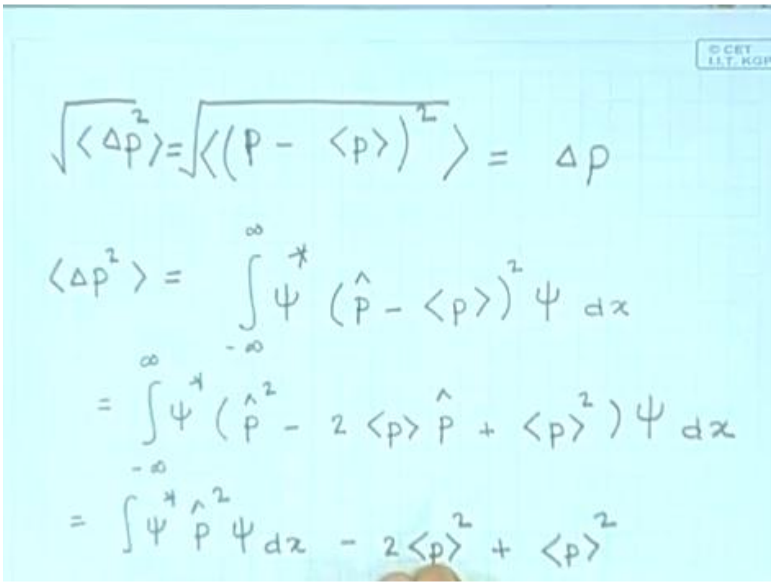

From this expression psi star the operator P minus the mean value square into psi dx we know what the operator P is the operator P is minus i h cross del by del x. This tells me the expected dispersion the spread the square root of this actually tells me the uncertainty in p. So, I have to I can calculate this by evaluating this integral. And we have simplified it little further and finally, I have shown you that this is.

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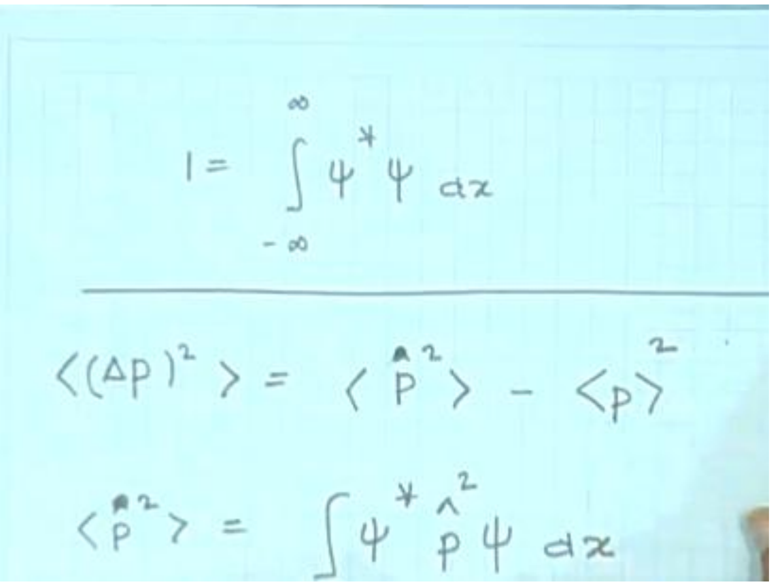

The expectation value of P square minus the expectation of P squared. So, here I have to first square P and then take the average here I have to take the average and then square it. So, this difference tells me the dispersion in P and the square root of this the uncertainty in P. And I can evaluate this by considering this integral where here I have the P square operator and putting in the expression for the momentum operator.

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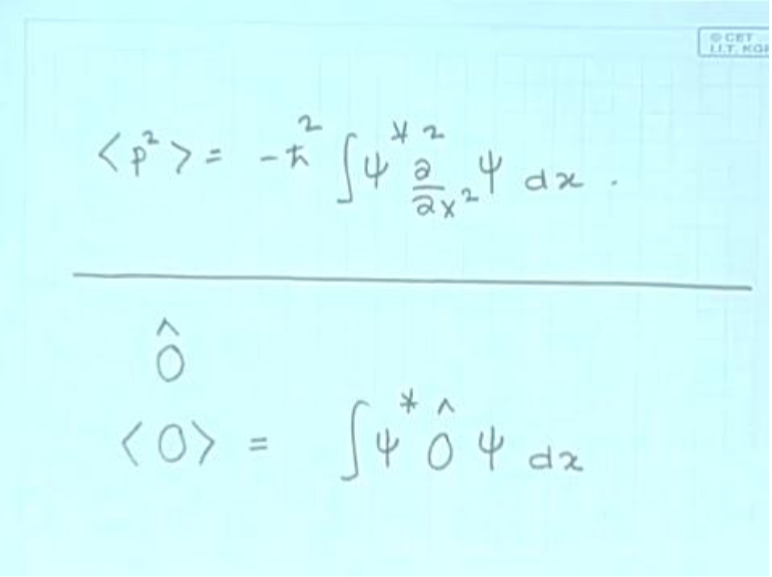

This is just this integral that I have to evaluate. So, let me finally, summarize what i is this part if I wish to calculate the expectation value for any operator o. So, the expectation value for any quantity O. O is some observable the expectation value of this quantity O can be calculated by evaluating psi star the operator corresponding to this observable into psi dx.

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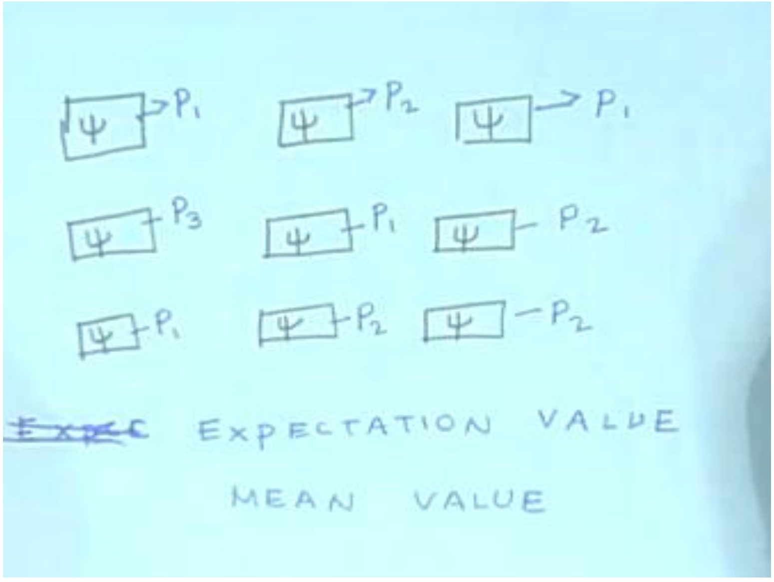

Let me repeat I have particle in a state psi I have many replicas of this same particle in the same state I may I measure some observable quantity O. And I do many independent measurements of the same observable quantity. So, each time I do the measurement I will get a different value now, I can ask the question what is the expectation value of this observable or what is the expectation value of the square of this observable and so, forth.

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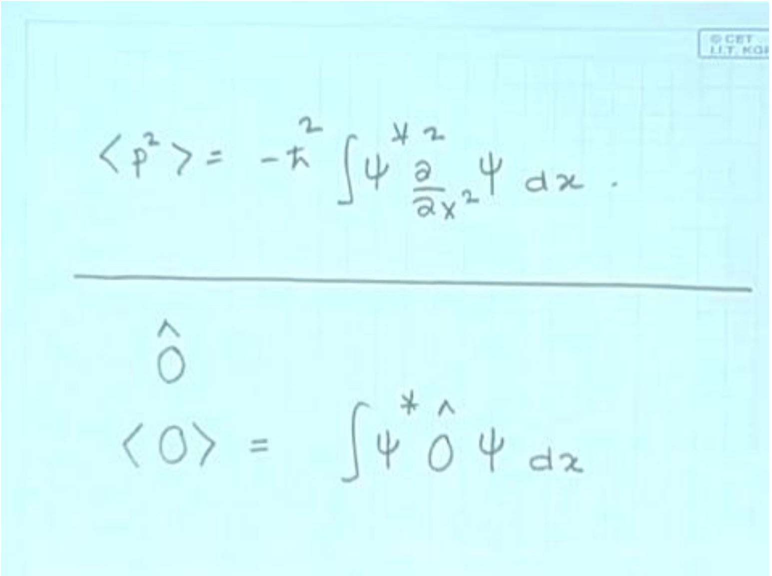

I can predict this by taking the observable quantity or what whichever quantity whose expectation value I wish to determine taking the operator corresponding to that acting on psi multiplying it by psi star and then integrating over x. So, this brings to a close our discussion of how to interpret psi and how to interpret different observations; how to make predictions for what I expect to get in different experiments where I do measure different quantities for the particle in a state psi. Now, let me know shift on move on to a different shift over to a different topic the topic that we are going to take up next is how to determine this wave function for a particle in a potential? So, the situation we are going to consider is where is as follows there is a particle which could be an electron some microscopic particle. And this microscopic particle is in is an external potential so, it is under the influence of some external force which can be represented the external force can be represented in terms of a potential.

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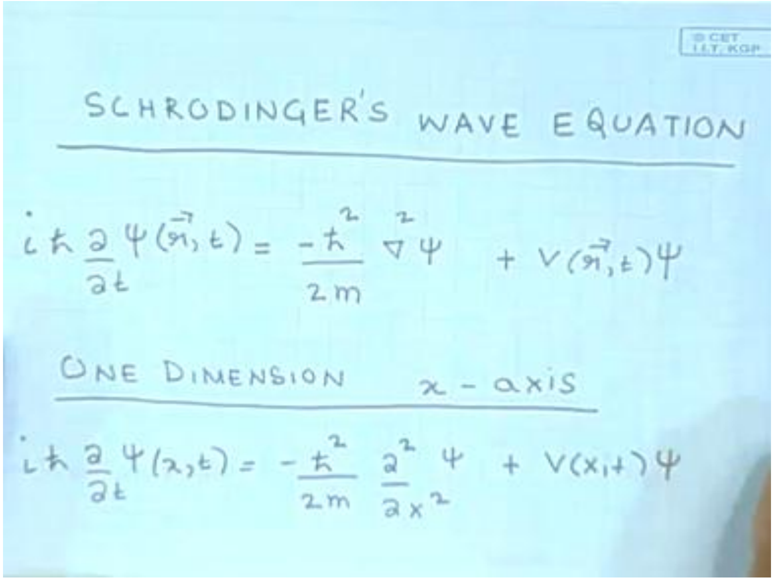

So, if it is a macroscopic particle the particle is macroscopic then we can think of the particle as a trajectory having a trajectory where it has a well-defined position and momentum at every instant of time. But for microscopic particles I have told you that we have to associate a wave with the particle and we have to think of the particle actually interns of a wave. The difference between microscopic and macroscopic particle is that the wavelength. So, a macroscopic particle in principle also has a wave associated with it, but the wavelength is extremely small that the wave effect does not become important.

It is the this differentiation is exactly analogous to the situation in optics where you have geometrical optics where you can think of the light moving in a straight line having a trajectory and wave optics where you have to think of the light as a wave. Now, suppose we send light through an aperture and ask the question when does when do we think of it as ray when can we think of it as a wave then from our earlier discussion of diffraction we have learnt that the wave effects become important when the size of the aperture is comparable to the wavelength

If the size of the aperture is much larger than the wavelength or the wavelength is much smaller than the size of the aperture the wave effects are not going to be discernable and you can think of it in terms of geometrical optics you can think of it as rays. But if the size of the aperture becomes comparable to the size of the wavelength you have to take into account the wave effects. Similarly, the in quantum mechanics if the wavelength is considerably large. So, that the so, that it is becomes comparable to the different length scales say apertures or such thing that you are in that the particle encounter then the wave effect is become important.

If the wavelength is much smaller than the wavelength the wave effects can be ignored. So, this you have these two regimes the microscopic and the macroscopic and in the microscopic world you have to think of the particle as a wave. And I have also told you that the wave is governed by the Schrodinger wave equation and if I have a particle in a potential this is the wave equation that governs the evolution of the wave. And if the particle is restricted to move only in 1 dimension along the x axis then the Laplacian over here gets replaced by a partial derivative with respect to x. So, we have del by del x squared del squared by del x square of the wave function.

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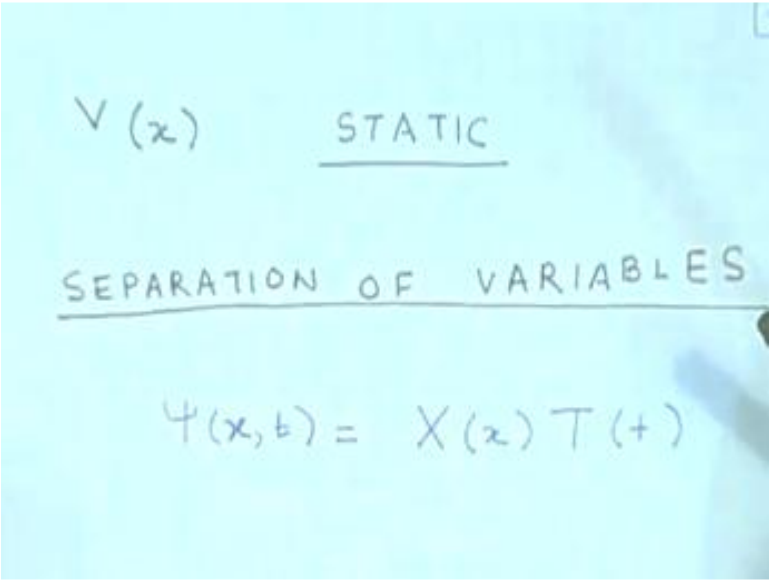

And further we are going to restrict so, we are going to make this assumption and we are going to restrict. Ourselves to a situation where the potential is static it does not have any time dependence it is a static potential. So, in this situation we have discussed how you can use the method of separation of variables where you assume that the wave function is a product of a function of position and a function of time. So, we take this trail solution and put it in the wave equation.

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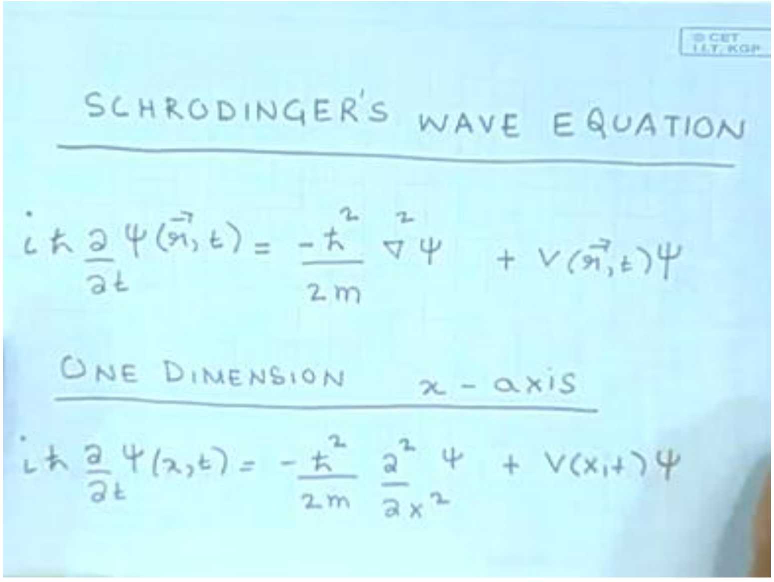

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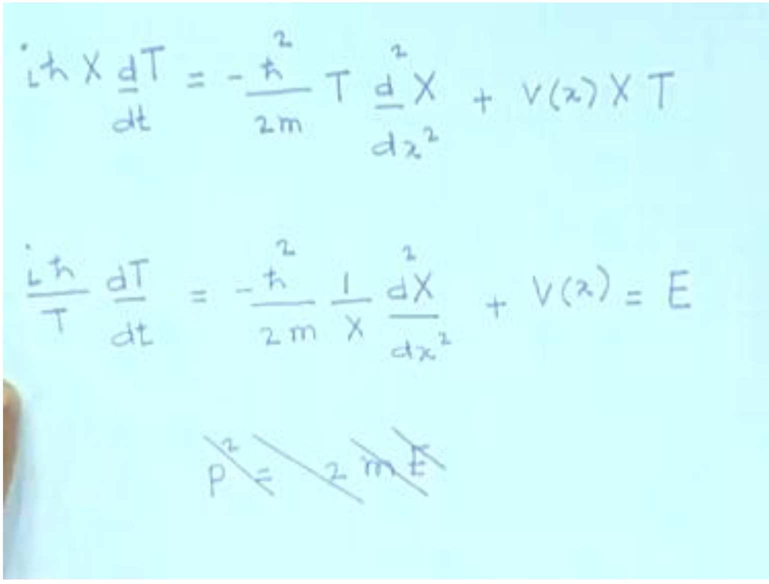

So, we put into this equation and this is the equation that it gives us we have discussed this already a few lectures ago. So, I am just going through it a little quickly. So, if you put in the trail solution which is the solution separation of variables solution trail solution like this.

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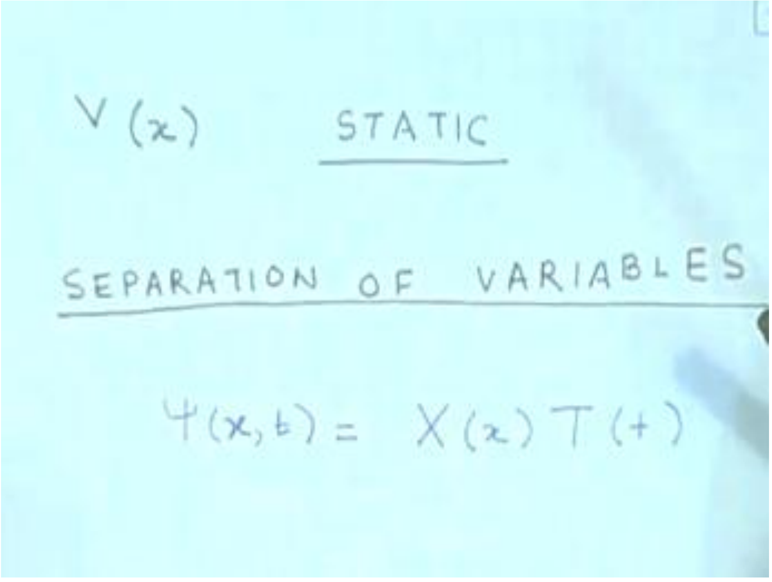

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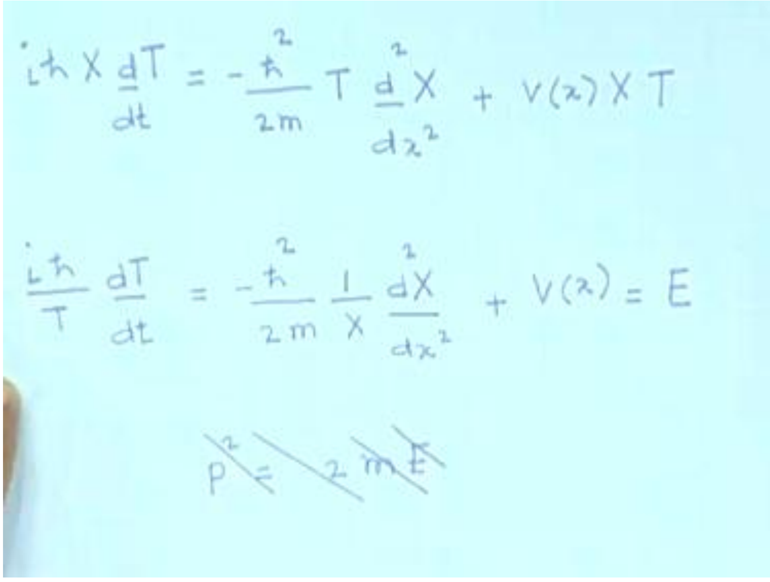

Into the Schrodinger differential equation the partial derivatives now become total derivatives and the first term in the Schrodinger equation.

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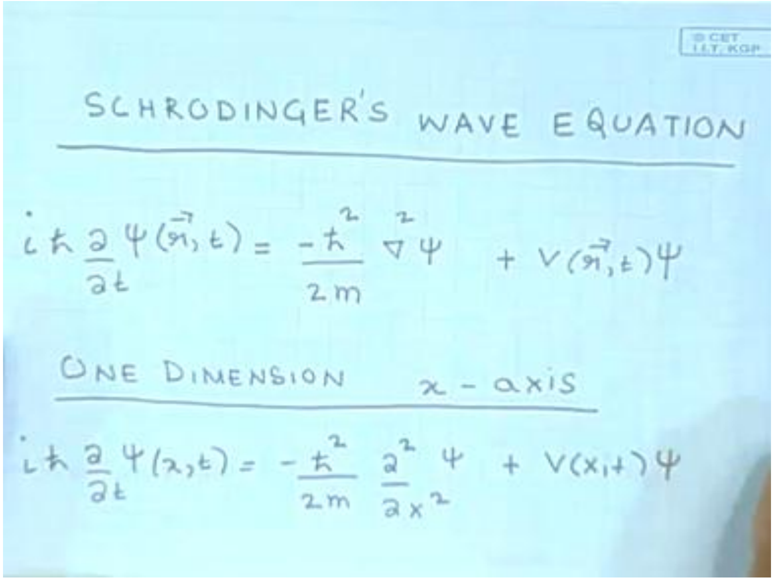

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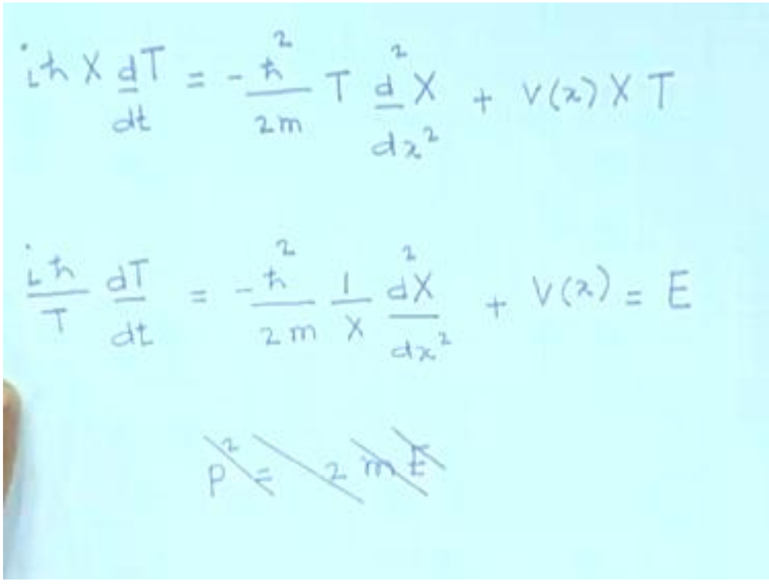

I h cross del by del t of psi now, becomes I h cross psi now, gets replaced by X into T the X comes out and I have a total derivative with respect to time.

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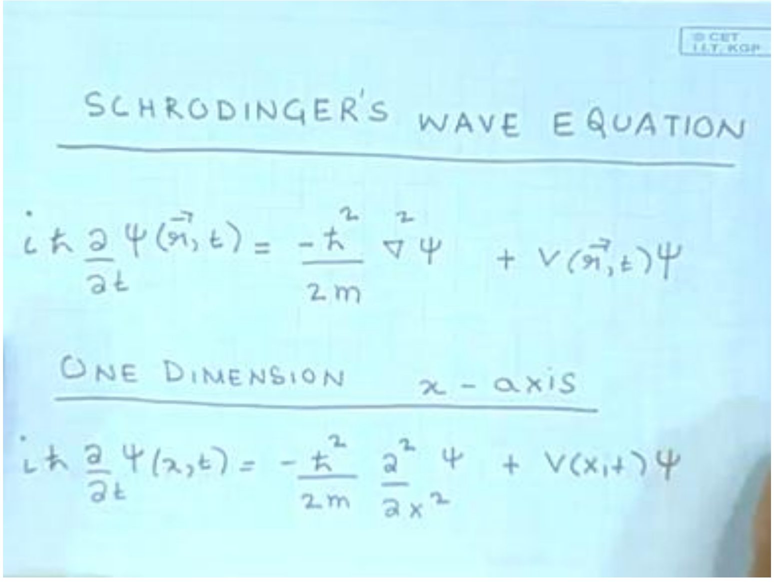

Similarly this terms over here minus h cross squared by 2 m the partial derivative of psi with respect to x the double derivative double partial derivative.

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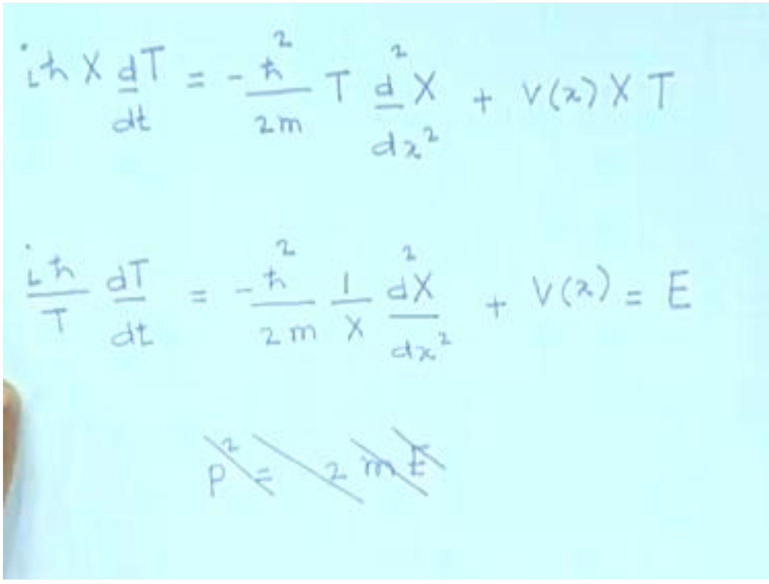

Now, becomes the double partial total derivative of X the X depended part of psi the T dependent part the time dependent part comes out and I have these the overall factors and I have v X T over here. Now, what we did remember what we do in the method of separation of variables is that we divide this whole thing by psi. If I divide this by psi the X cancels out from here T cancels out from and both of these will cancel out from here.

And what we have is i h cross by capital T the time derivative of capital T is equal to minus h cross squared by 2 m 1 by capital X the second derivative of capital X with respect to the position plus v which is the function of x only. So, this is equal to this the left hand side is just a function of time. This term over here is just a function of x if this function of time has to be equal to this function of x for all values of the time all values of x. It tells us that this and this should basically be equal to a constant. And we have worked out the solution so, the solution of the time part let us look at only the time part.

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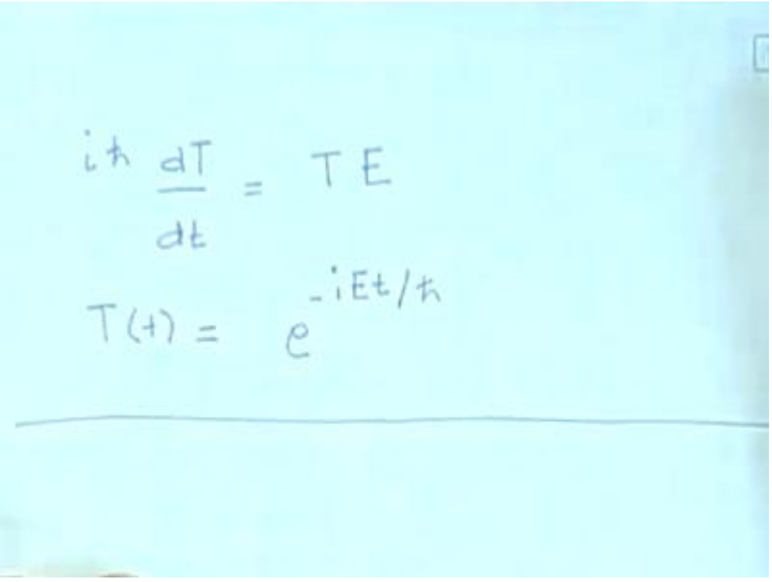

So, we are going to look at only the time part we have done this is we have already discussed this the time part just take the time.

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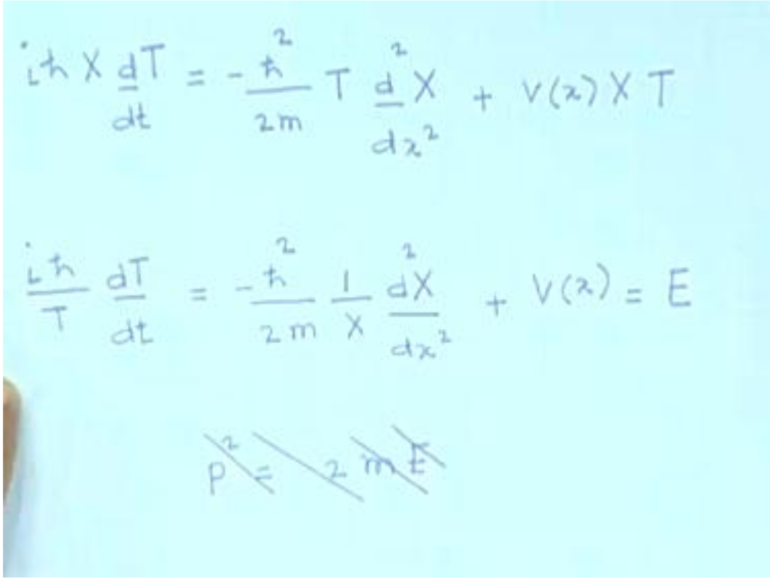

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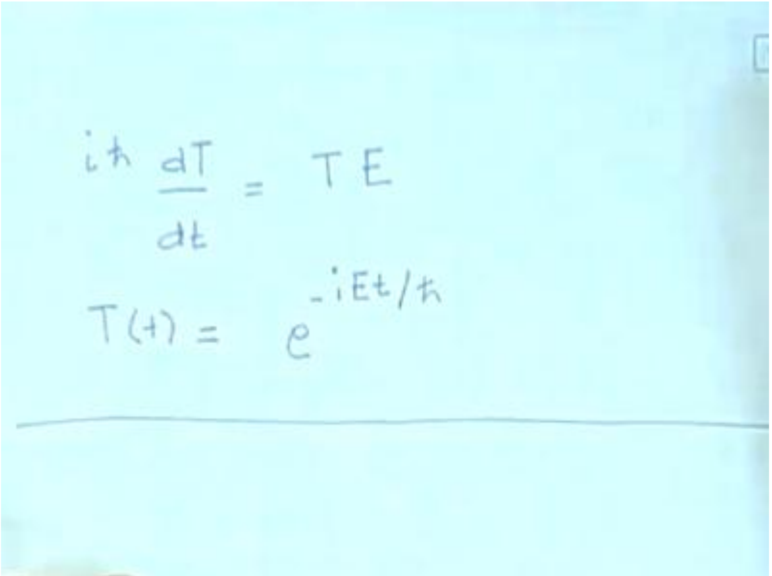

Part of this equation the time part of this equation is I h cross the derivative of capital T with respect to time is equal to capital T into the constant E and you can integrate this gives us the solution shown over here. Now, so, we have got the time part of this wave function let us now, look at the spatial part of the wave function.

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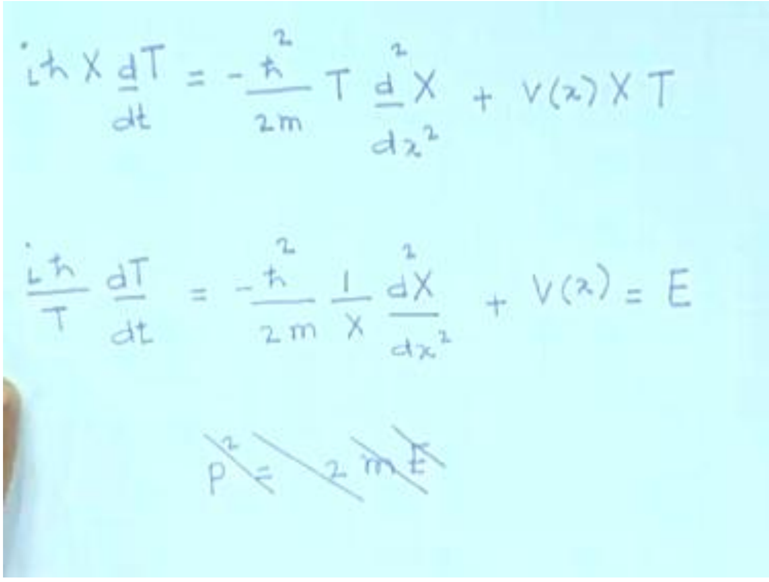

So, we had done this in an earlier lecture.

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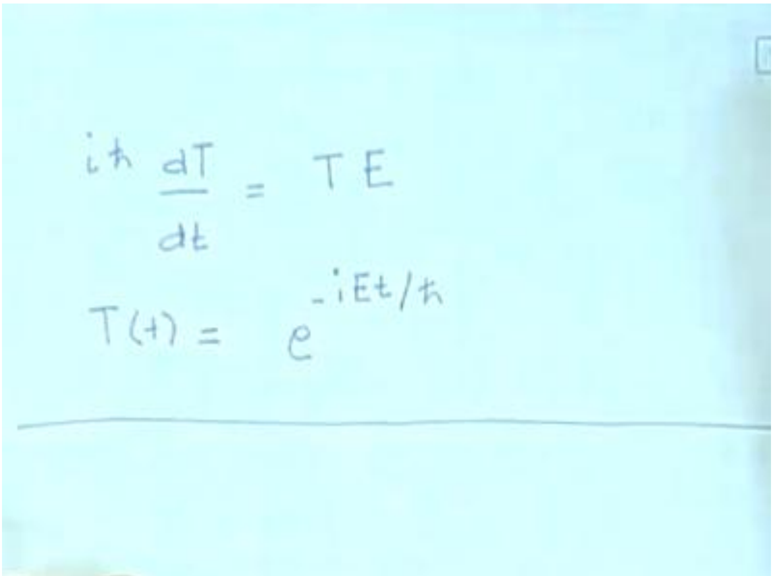

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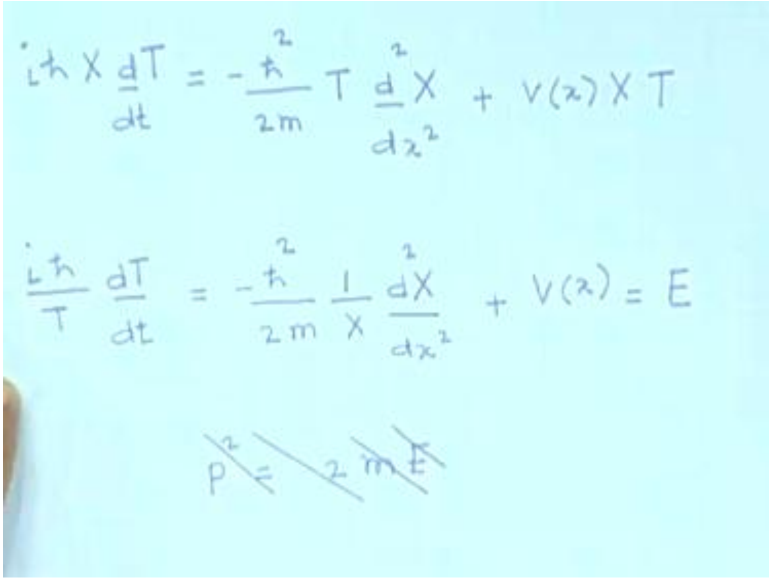

Now, we shall look at the spatial part of this equation the spatial part of this equation has to satisfy is governed so, the space part has to is governed by this equation. Let us look at this space part again the space part has to satisfy the condition.

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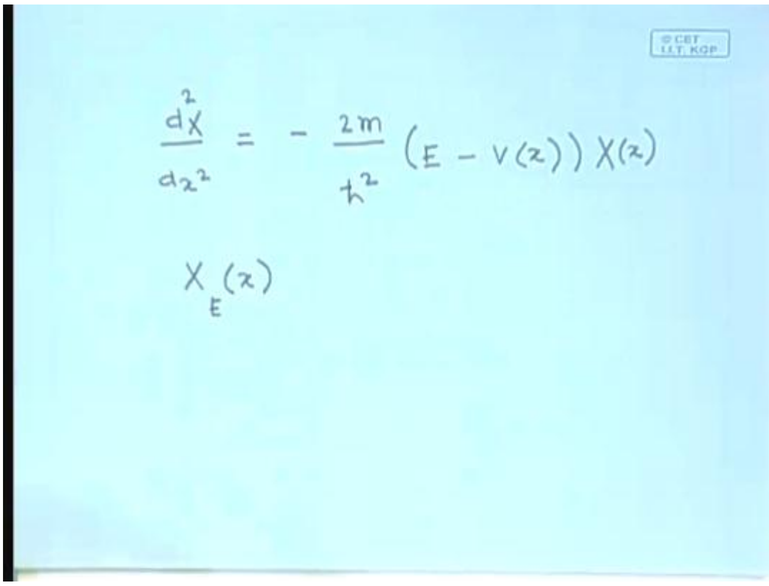

That this should be equal to E and we can write this as D square X dx square is equal to minus 2 m by h cross square E minus v x into X so, the spatial part of the wave function has to be obtaining by solving this equation. And solving this equation is a little complicated, because we have this function of x the potential we also have this x which depends on x. So, we have a product of 2 functions of x and solving it is a little difficult a little complicated we have in an earlier lecture we have considered a particular situation where this was zero which was the free particle and we have worked out the solutions when this was a free particle

For a free particle let me write down the solution for a free particle we have worked out the solution for a free particle the value of E could assume any the this constant E could assume any value and we had worked out the solutions we shall come to the solution a little later. Now, in general it is not easy to work out a solution for any arbitrary v x there are a few functional forms of the potential for which it is possible to work out solutions. There are other functional forms of the potential for which it is difficult to work out this solution, but the solution exist there is no doubt about that

So, in general one can work out a solution to the x dependence of the wave function. Now, there are situations where it turns out that solutions are possible only for certain values of E and there are situations where it is possible to obtain solutions for all values of E. Remember for a free particle we found that it was possible to have a solution only for positive values of E negative values were not permitted. So, follow free particle where the potential is zero any positive value of E will give you a valid solution. But there are situations where there are only there are restricted values of E which will give you solutions.

This is an issue we shall come to in tomorrow's lecture whichever be the case there will be some solutions which will depend on the value of E for a free particle. We have seen what the solution is and we have seen that it depends on the value of E. So, there will be a solution there will be certain solutions which will depend on the value of E which I am denoting as X subscript E and it is a function of the position. So, this schematically indicates the solution to this equation for some value of the energy E there this constant E. So, once we have this we can work out we know what the total wave function is the total wave function.

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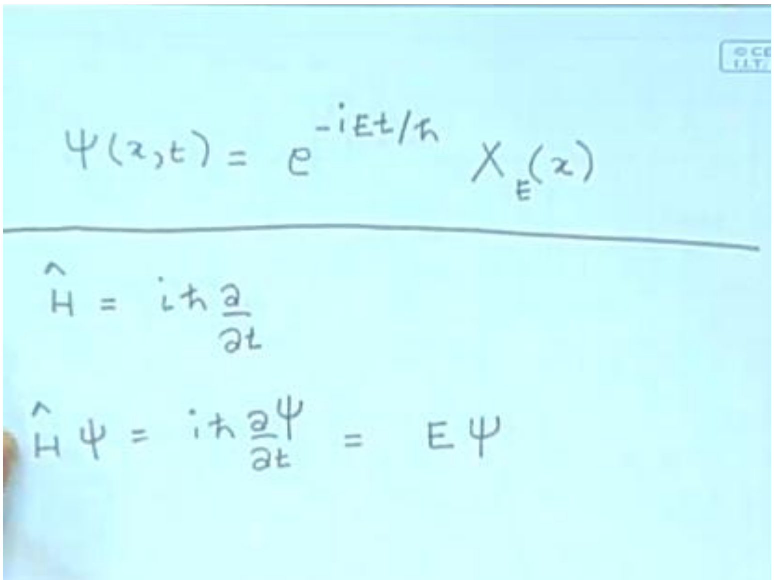

Psi x t is now, a product of the time part the time part is e to the power minus i into E into t divided by h cross that is a time part and we have the spatial part which is X which depends on the value of E that you have chosen that the constant that you have chosen in which is an capital X is the function of the position. So, this is a solution to the Schrodinger equation and the solution involves an arbitrary constant E. Now, let us ask the question first that I once I have this solution if I go and measure the particles energy what do we expect?

So, the question is that I have a solution to the Schrodinger wave equation for a particle in a potential and the solution is of this form. Now, the question is what happens when I go and measure the particles energy? So, if I make a measurement of the energy and if the state is an Eigen function of the operator corresponding to energy. So, I have also told you that corresponding to energy we have an operator call the Hamiltonian operator. And the Hamiltonian operator is i h cross del by del t. So, let us see now, if this wave function is an Eigen function of this Hamiltonian operator. So, H acting on psi is i h cross del by del t of psi.

So, if I differentiate this with respect to time I get the same function into minus i E by h cross minus i into i gives me 1 h cross divided by h cross gives me 1. So, I find that this is equal to E into psi. So, indeed this is an Eigen function of the Hamiltonian of the. So, it has so, if I make a measurement of the energy. Since it is an Eigen function of the

Hamiltonian operator I am going to get this value E which is the Eigen value of the Hamiltonian operator as my result. So, if I make a measurement of energy for a particle in this state the energy will turn out to be E and the wave function is going to be unchanged.

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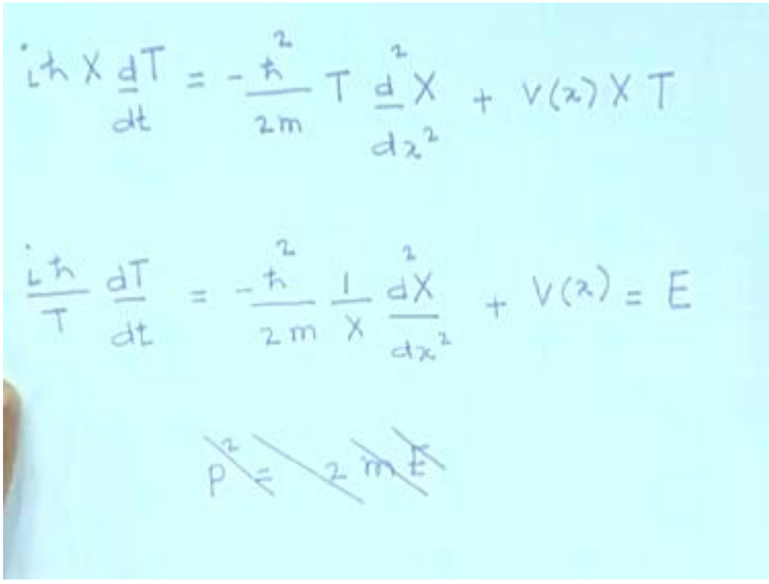

So, we now, see that this constant E which appeared when we use the method of separation of variables in the Schrodinger equation this just appeared as a constant. This constant we now, see.

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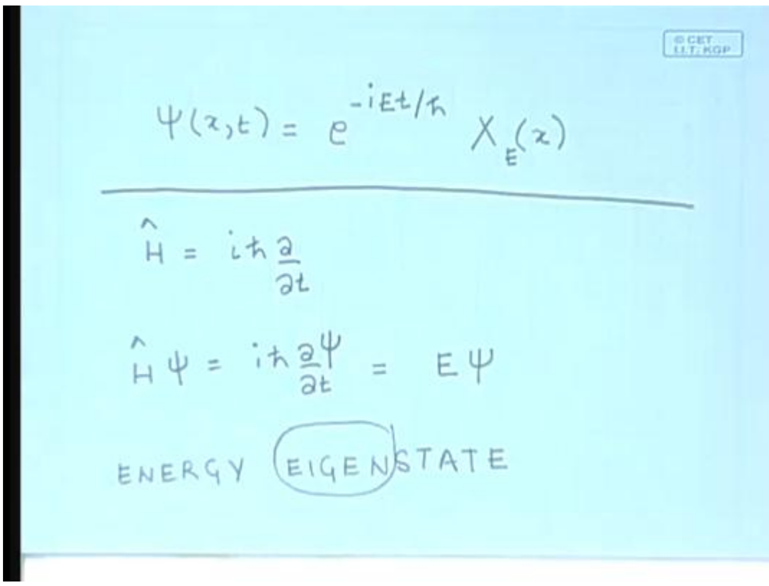

Can we interpret as being the energy of the particle in this particular state? And such a state is called energy an energy Eigen state or an energy state or an sometimes it is also refer to as energy state so, particle is an is in an energy state with value E. Now, let us calculate the probability density for a particle in this state remembers that if I ask a question.

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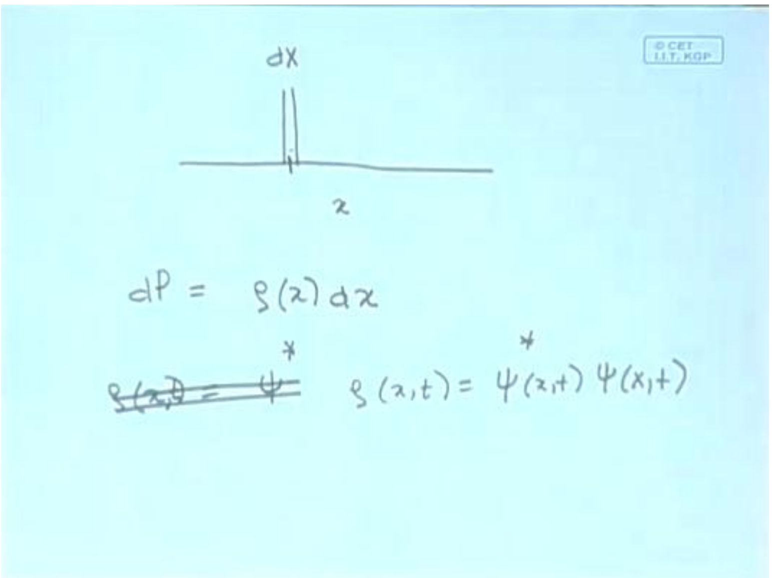

So, the particle is free to move along the x axis if I ask the question what is the probability of finding the particle in this range dx interval dx around a value x over here. Then this probability dP is given by the probability density into dx where the probability density is psi star probability density in principle could be a function of x and time both. So, rho x t is psi star x t into psi x t. So, the probability density tells me the probability of finding the particle in this interval around this point. And the way to calculate the probability density from the wave function is to take psi star and multiplied with psi.

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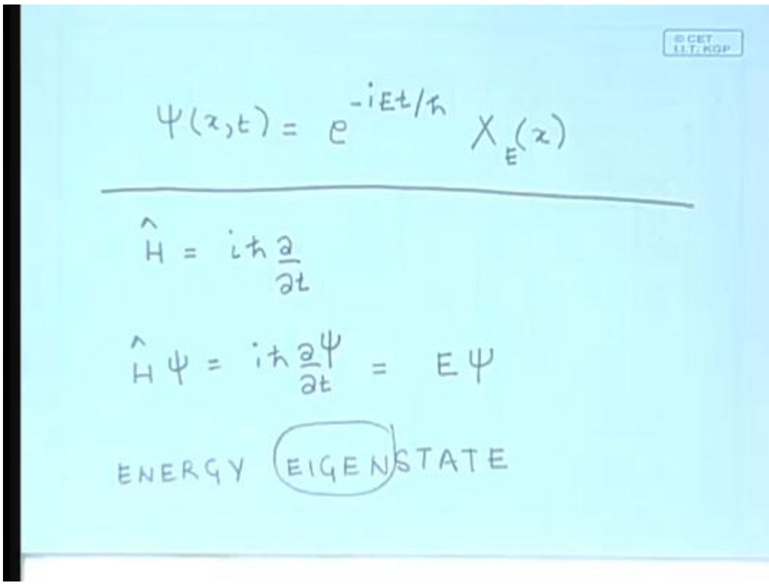

Now, for this particular kind of a solution which has a fixed value of the energy the for this energy Eigen state. If I take psi star psi star is going to be this exp1ntial with the minus sign g1, because the complex conjugate of this is going to be e to the power i E t by h cross and I am going to have the complex conjugate of this. So, when i multiply this with its complex conjugate this term and its complex conjugate both will together give me a value 1.

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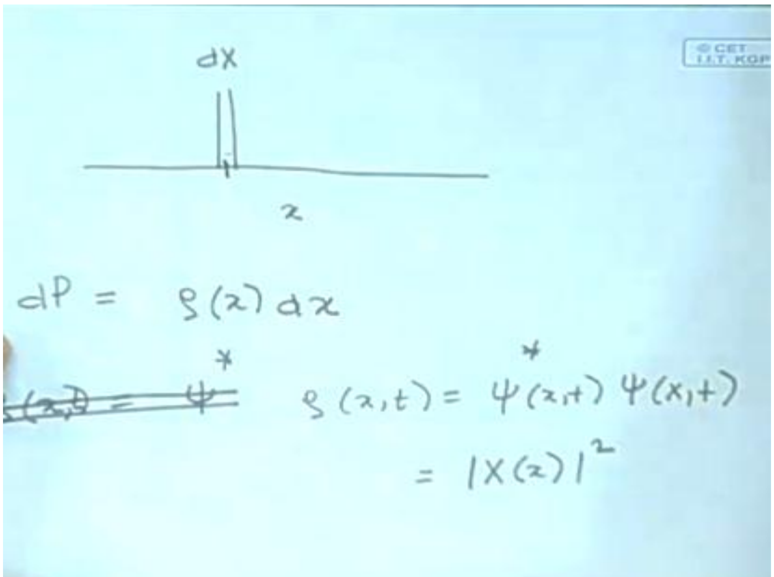

So, what I is left with is X so, what we see is that for this Energy Eigen state the probability density does not change with time..

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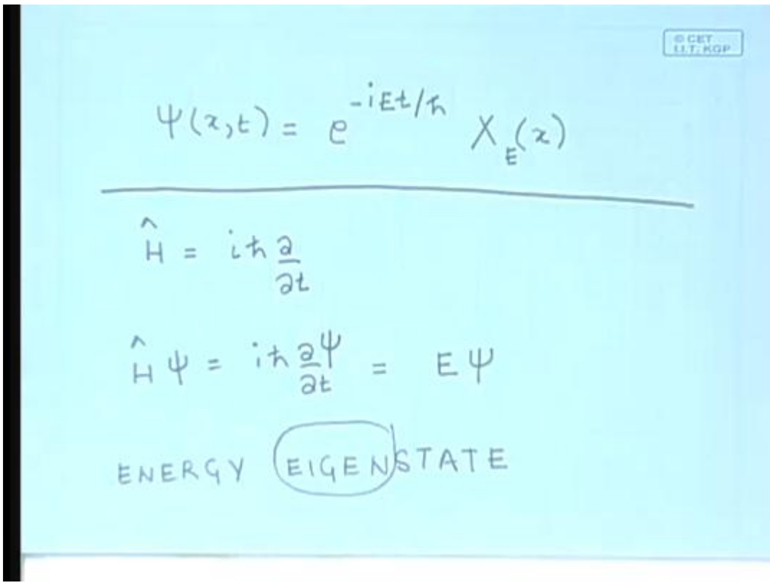

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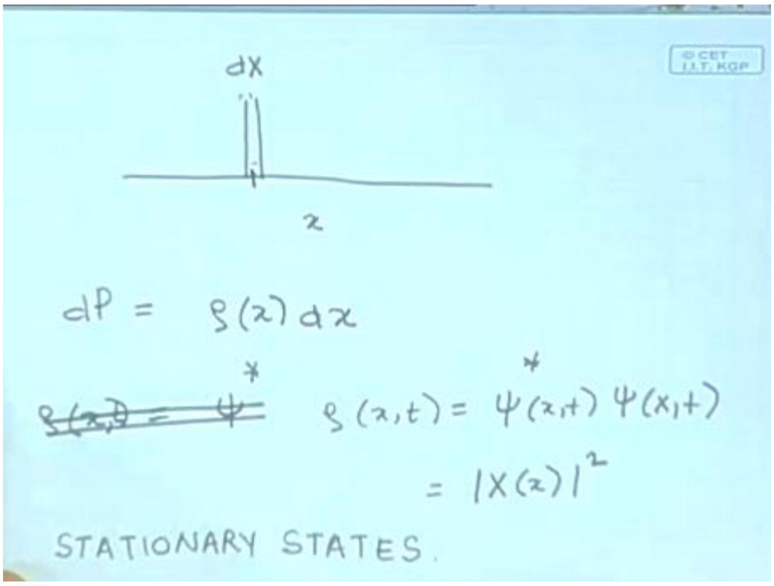

So, if the probability density the probability the finding the particle in this interval is not time dependent, it is fixed. For this reason these functions this kind of energy Eigen states these kinds of wave functions are also called stationary states so, such wave functions are also called stationary states. So, what we have been doing let me again remind you of what we have been doing we are we have been calculating.

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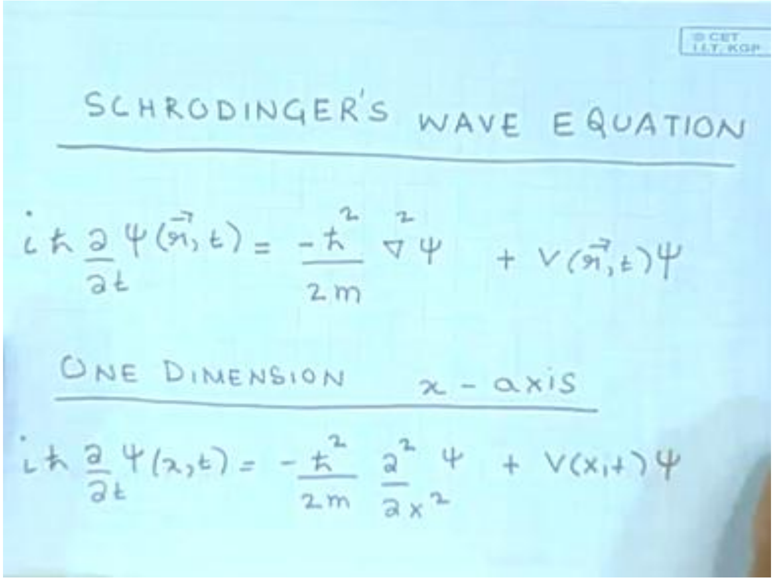

The wave function for a particle in a static potential and what I have shown you is that the wave function for a particle in such a potential can be written.

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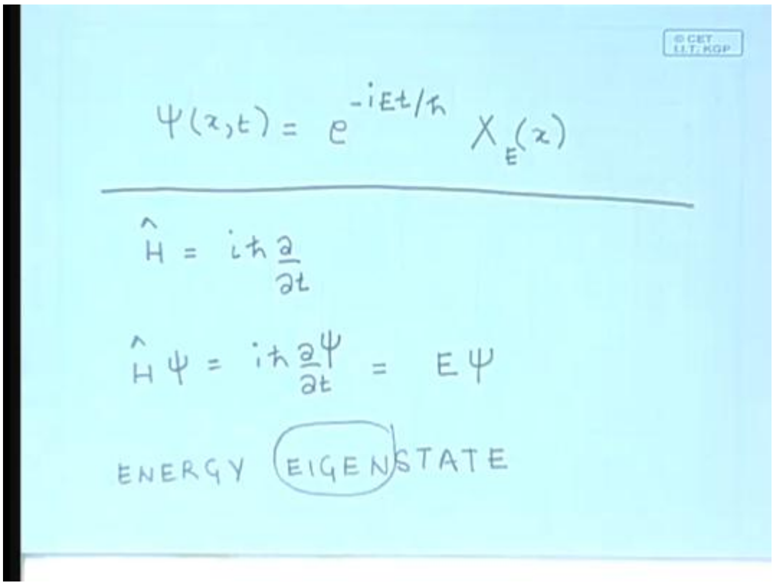

In this form where the this function X is still to be determine, but what we know is that this function X has to satisfy this differential equation.

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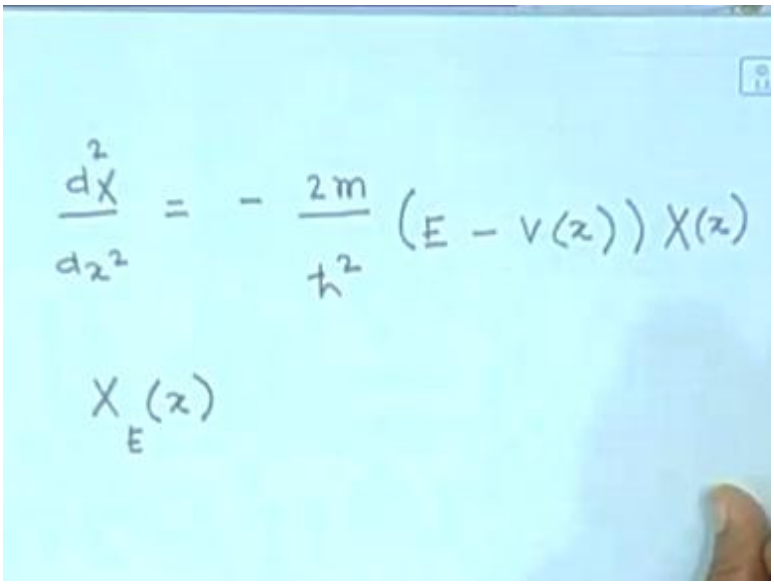

Over here and the solution will be different for different values of this constant which is why I have to noted it like this? Now, if I choose 2 different values of the constant I will get 2 different solutions. So, let me now, consider a superposition of 2 different solutions. So, I could have a wave function which is a superposition of 2 different solutions.

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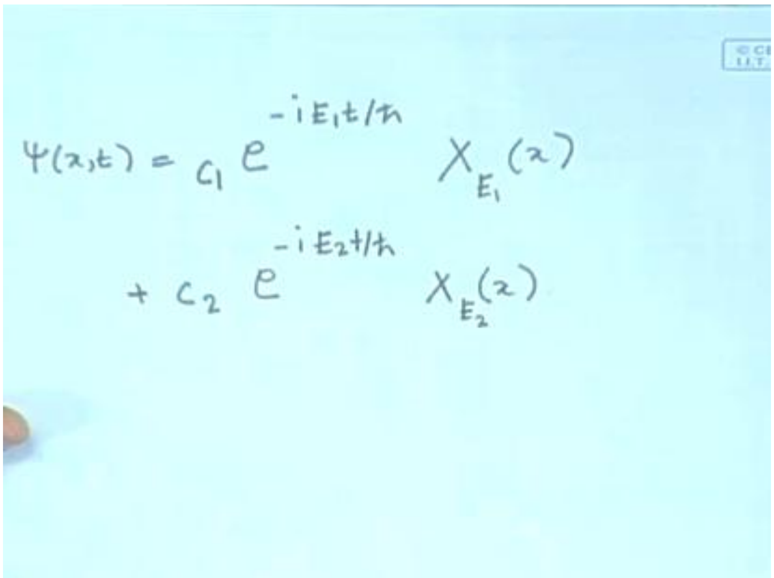

So, one solution is e to the power minus i E 1 t by h cross XE 1 I can put a constant here c 1 plus c 2 e to the power minus i E 2 t by h cross XE 2 E which is the function of x. So, what I have done now is that I have taken two different solutions of the wave functions we could have.

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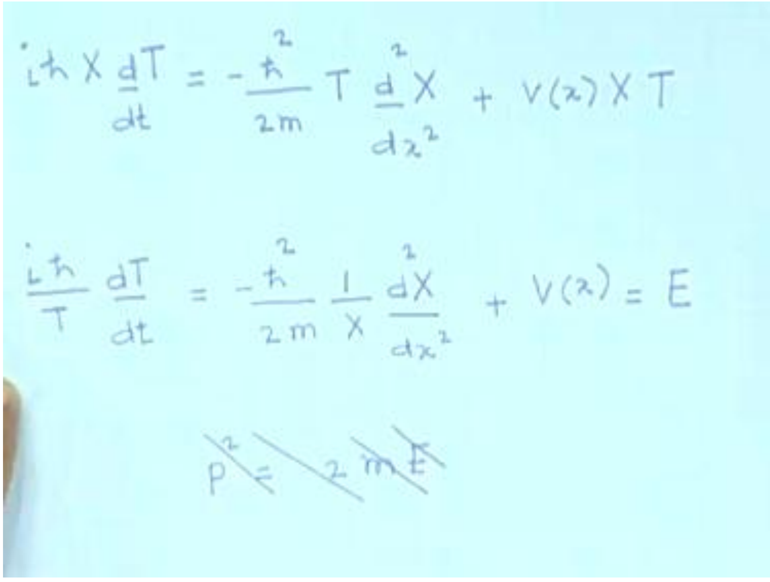

So, whenever I use the separation of variables we have this constant over here and depending on the value of the constant I will get different solution if I choose a different value of the constant.

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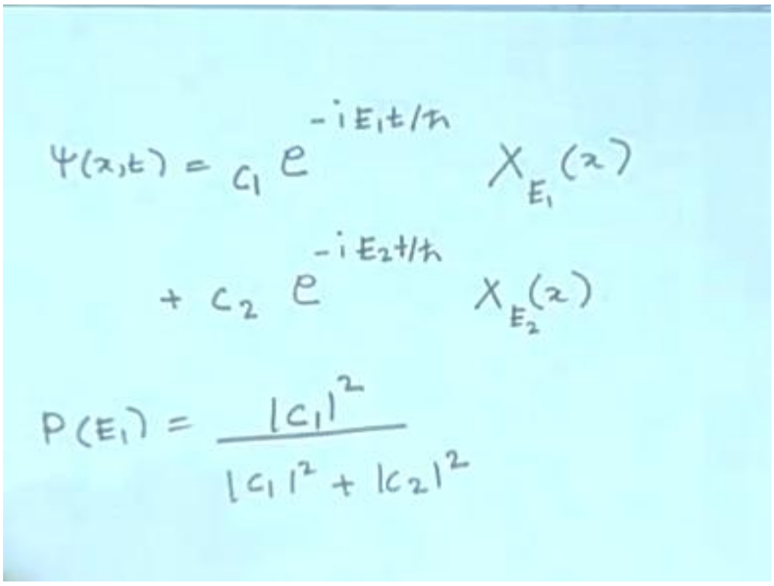

So, what I have done now is I have choose in two different values for the constant values being E 1 and E 2 and a superposition of these two solutions with some arbitrary coefficients is also going to be a solution, because the differential equation that we are dealing with is a linear differential equation. So, if I have two different solutions and I superpose them I will also get a solution. So, this is also a solution of the Schrodinger wave equation. Now, let me ask you the question what happens if I measure the momentum the measure the energy of a particle in this state what happens when I measure the energy of a particle?

In this state let us look at the wave function again notice that the wave function now, is itself no longer and Eigen state of the Hamiltonian operator. But it is a sum of two different Eigen functions the two different Eigen functions being this and this Eigen function has Eigen value E 1 this Eigen function has Eigen value E 2. So, if I measure the energy of the particle in a state like this I will get either E 1 or E 2. Now, let me ask you another question what is the probability that I will get either E 1 on what is the probability that I will get E 2.

So, what is the probability that I will get E 1 as my energy if I measure the energy of the particle what is the probability that I will get an outcome E 1? And I have told you that you can determine this the probability of getting E 1 is the square of this coefficient of this wave function c 1 and the probability of getting E 1 is the mod of c 1. So, the probability of getting E 1 is the mod of c 1 square divided by the mod of c 1 square plus the mod of c 2 square probability of getting E 2 is the mod of c 2 square I have to replace 1 by 2 divided by the same factor. And if I get E 1 then I know that after their measurement the wave function has now, changed to this Eigen function corresponding to E 1. If in my measurement I get E 2 then after the measurement the wave function would have changed to this Eigen function corresponding to E 2.

Now, let me ask you the third question? The third question is this state a stationary state what do we mean by a stationary state? A stationary state I just told you is the state where the probability density does not change with time. Now, if you calculate the probability density for this state you have to take this psi and multiply it with its complex conjugate when you multiply it with its complex conjugate. This is going to get multiplied by its complex conjugate which is going to be independent of time. This is going to get multiplied by is its complex conjugate which again is going to be independent of time. But when you multiply this with its complex conjugate there will be cross terms.

So, this will get multiplied with the complex conjugate of this and there will be another term where this gets multiplied with the complex conjugate of this these 2 terms is going to be time dependent. So, the probability density is also going to be time dependent. Now, let me ask you a third fourth question what is going to be the time dependent of the probability density that again is going to be should be cleared. The time dependents of the probability density is going to come from the product of the complex conjugate of this the complex conjugate of this. So, that is going to oscillate with an angular frequency which is going to be E 1 minus E 2 divided by h cross. Because when I multiply the complex conjugates of these 2 functions I will get e to the power i E 1 minus E 2 divided by h cross into t.

So, a superposition of 2 stationary states is not a stationary state the probability density is going to change with time it is going to oscillate with time. So, until now, we have been discussing in general what happens? When I have a particle in a potential and what we learnt is that when I have a particle in a static potential we have these Eigen states energy Eigen states which are also stationary states. And you can have many such energy Eigen states I also told you that in some situations the energy values can be continuous. For example for a free particle any positive number is a possible energy value there are other situations where the energy values will be discreet I shall show you an example as we go along. So, both these possibilities are there now, let us move on to discussing the we still have not discussed the spatial part.

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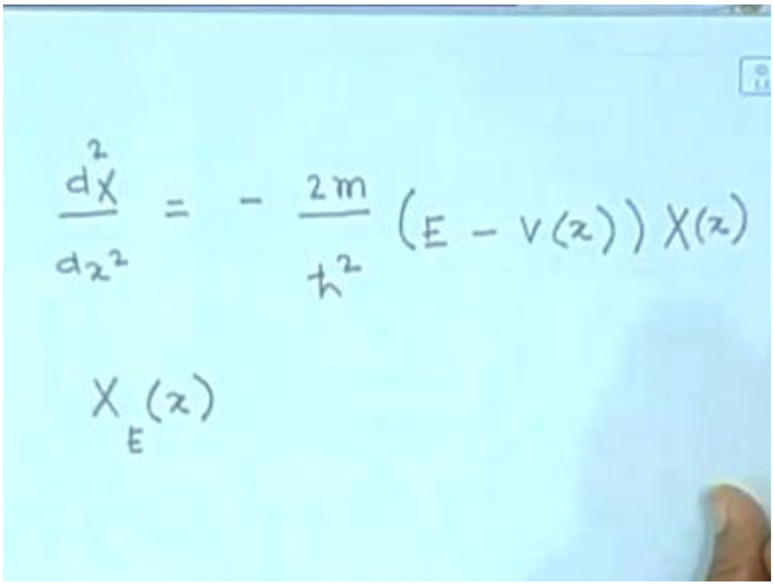

So, let us now move on to discussing the spatial part this part of the wave function we still have not discuss all that I have told you is that there will be a solution and the solutions will be different depending on the value of the energy. But before we move on to this let me briefly remand you what we expect from classical mechanics when we have a particle in a potential.

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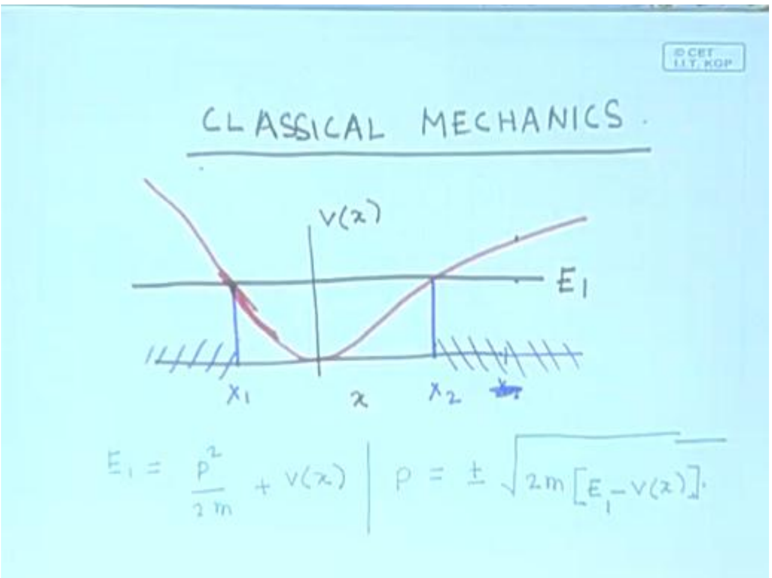

So, we have a particle in a potential and what does classical mechanics tell us? So, let me draw a potential this is v as the function of x and this is x and we have some kind of a potential in which the particle is moving. So, let me draw a potential could some arbitrary potential so, this is the potential in which the particle is moving. And the particle has energy E 1 so, this is the energy of the particle E 1 now, we know that the energy of the particle if the particle moves in a static potential the total energy of the particle is going to be conserved.

The total energy is a sum of 2 parts the kinetic energy and the potential energy. So, E 1 the total energy is P square by E by 2 m plus v x that is the total energy it is conserved it is a constant. This is a sum of 2 parts the kinetic energy plus the potential energy we can use this to calculate the momentum. The kinetic energy is momentum square by 2 m we can use this to calculate the momentum. So, the momentum P is plus minus square roots of 2 m E minus v x the square root of this whole thing. Now, let us ask the question do we expect what happens when the particle is here? Do we expect to get the particle at this position?

At this position at this value of x the potential energy v x is more than the kinetic energy the than the total energy of the particle. At this point the potential energy is more than the total energy of the particle. So, v x at this point is more than E if v x is more than E this difference is going to be negative. So, the square root of 2 m into E minus V is going to be square root of a negative number. The square root of a negative number we know is imaginary

So, if the particle is located at this position its momentum turns out to be imaginary. Now, momentum is a physical quantity which we can measure and we know that it is real it is mass into the velocity of the particle. And if the momentum is predicted to be imaginary it tells us that we the particle will not come to this part this value of x. So, what this tells us is that? The particles motion is restricted to the region where the potential is less than the energy or at most to the point where the potential energy is equal to the energy. So, the particle will move in between these 2 regions X 1 and X 2 sorry X 1 and X 2 at these 2 points the particles potential energy is equal to the total energy.

So, the particle will move through values of X which lie in between. It will not cross this value of X 1 and go to smaller values it will not cross this value of X 2 and go to lager values. When so, the particle is going to oscillate between X 1 and X 2 if this were a quadratic potential it would be a simple harmonic oscillator. If it is some arbitrary potential the oscillation is going to be different it is not going to be simple harmonic, but it is going to oscillate between X 1 and X 2. When the particle comes either to X 1 or to X 2 the energy and the total energy and the potential energy are exactly balanced.

The total energy E 1 and the potential energy v x are exactly balanced and the particle comes to res. So, the particle which is moving this way has positive momentum is going to arrive move all the way to X 2 where it will come to rest, because these 2 are exactly balanced. And then it is going to go backward the momentum is going to be negative and it will come all the way till here till X 1 where again these 2 are balanced and again it is going to go back and forth.

So, in classical mechanics the region where the potential energy is more than the total energy is forbidden to the particles. So, this region is forbidden the particle is never going to be found here neither is it going to be found over here. And it is only this region in between where we will find the particle. Now, let us go on to quantum mechanics mc what happens when we have a potential like this now in quantum mechanics we will have?

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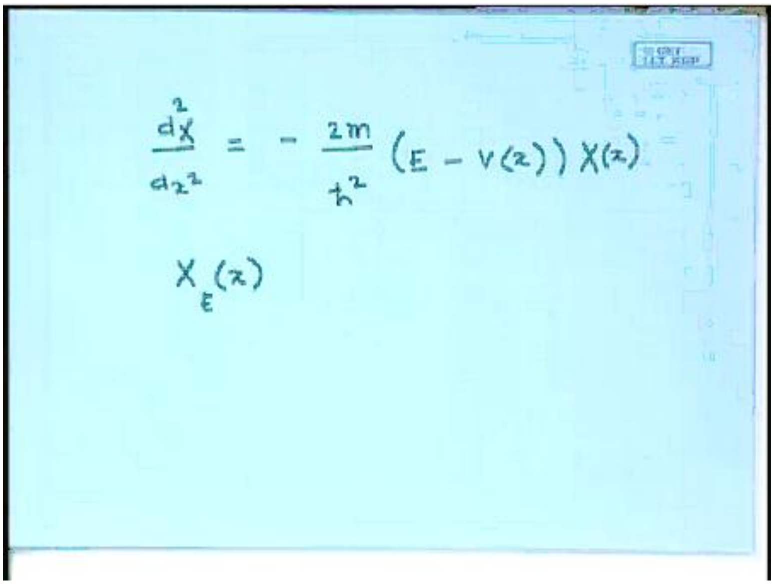

To solve this differential equation for a potential for a v x which is the potential now a potential like this.

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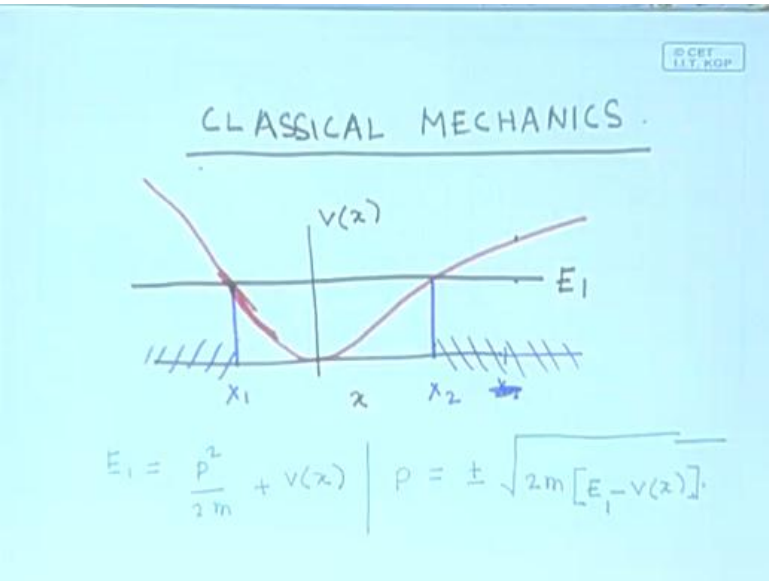

In principle has a v x which is a quite complicated and it is not it is quite difficult.

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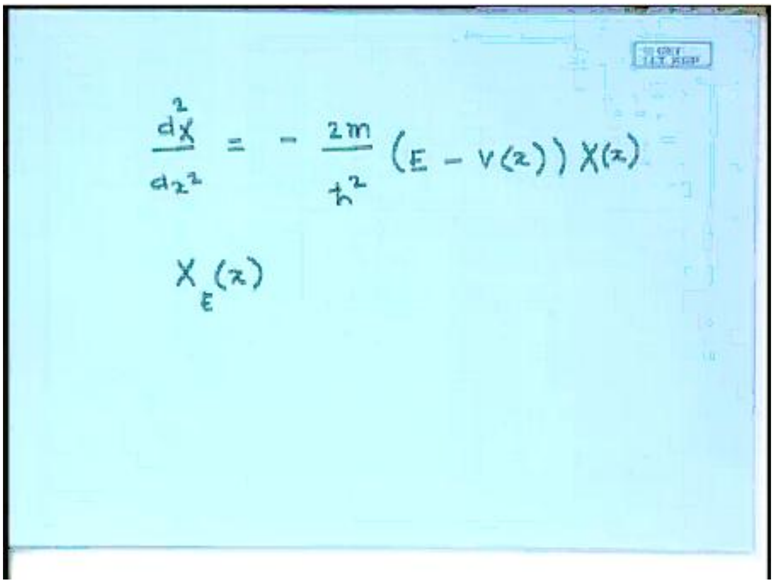

To solve such an equation so, to simplify the mathematics we are going to make a simplification. So, I have going to simplify matters. So, in we are going to.

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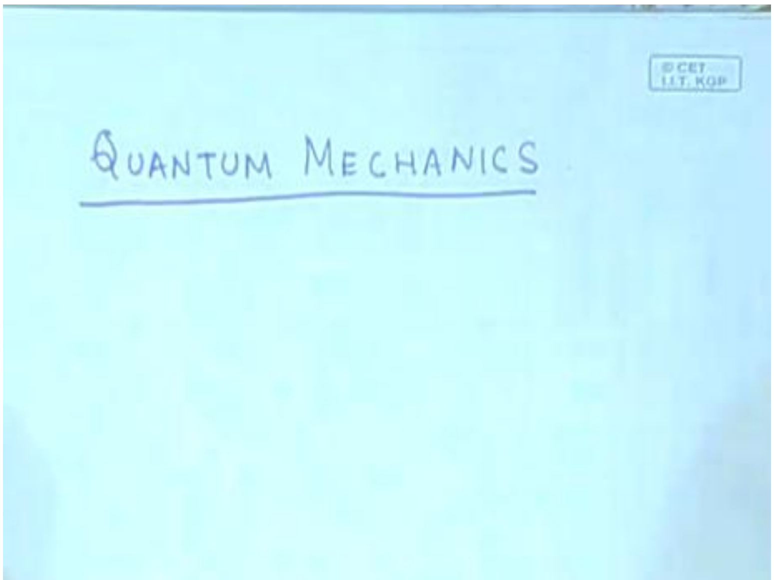

Take simpler situation which we are going to handle using quantum mechanics we are going to assume that the potential is not of this type we will make a simplification and assume that the potential is a step.

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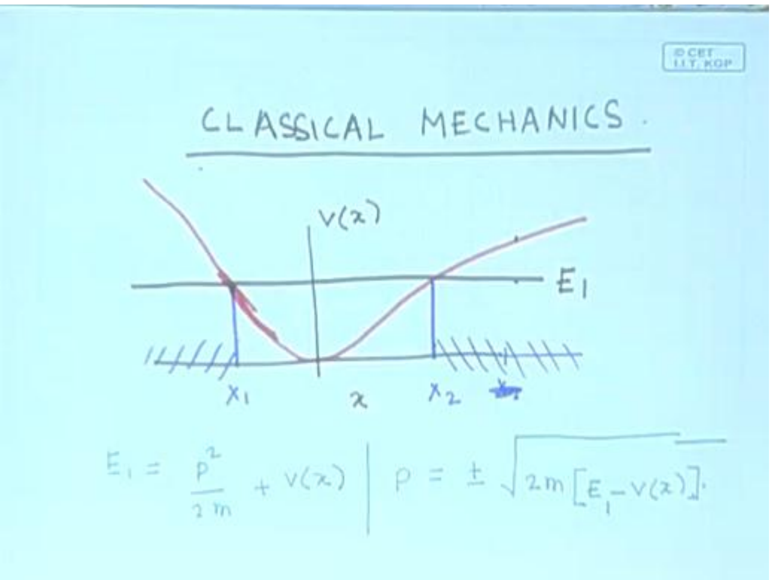

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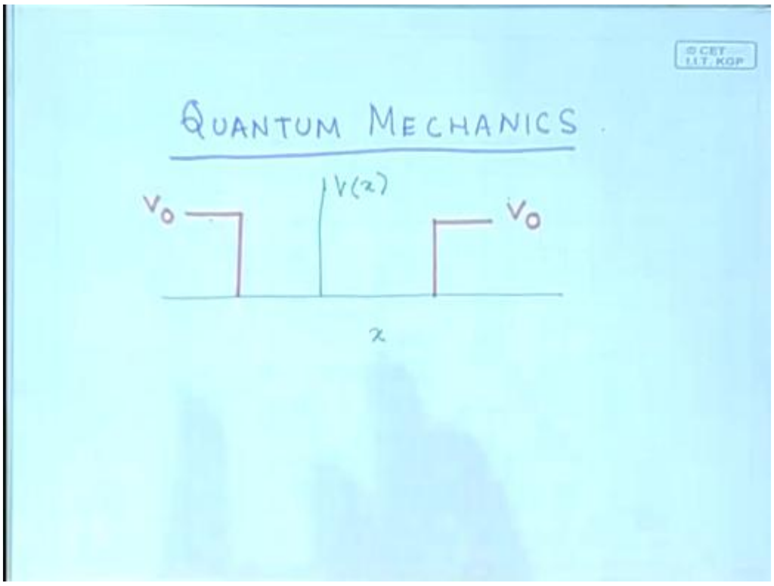

So, the potential that we are going to assume is of this kind. We are going to assume that the potential is a step like potential where it has fixed value v naught and then it goes to zero and again at another value of x the potential again arises in a step to some of the 2 value v naught. The fact that the two values are same on both the sides is not very important, but for simplicity I have assume that it is the same. So, we are going to deal with a simple situation where the potential changes in a step go to 0 and then again rises in a step. And we are going to solve for a particle in such a potential the solution in such a situation is going to give us.

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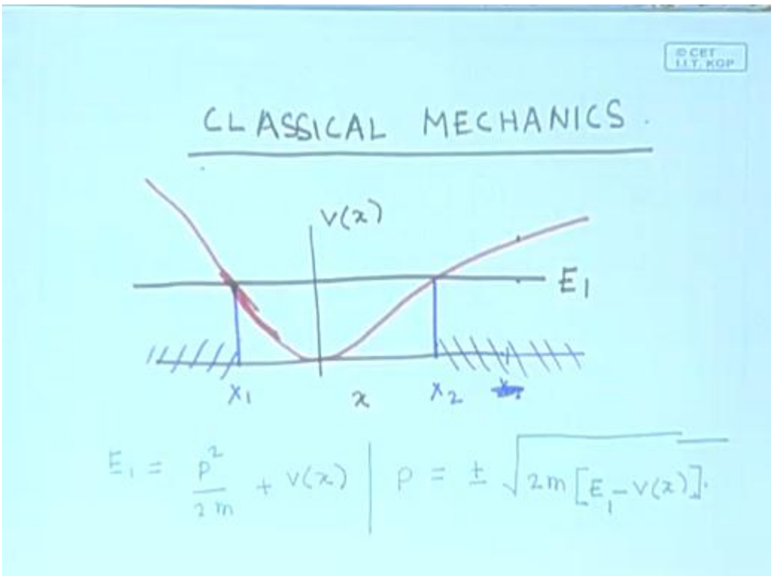

Some idea of what is going to happen in a situation which is like this where the potential vary slowly. But this situation being more difficult to handle we are going to take a simpler situation where the potential is a constant and then changes in a step with the idea that this is going to tell us certain things which can which are also be valid when the potential is more general.

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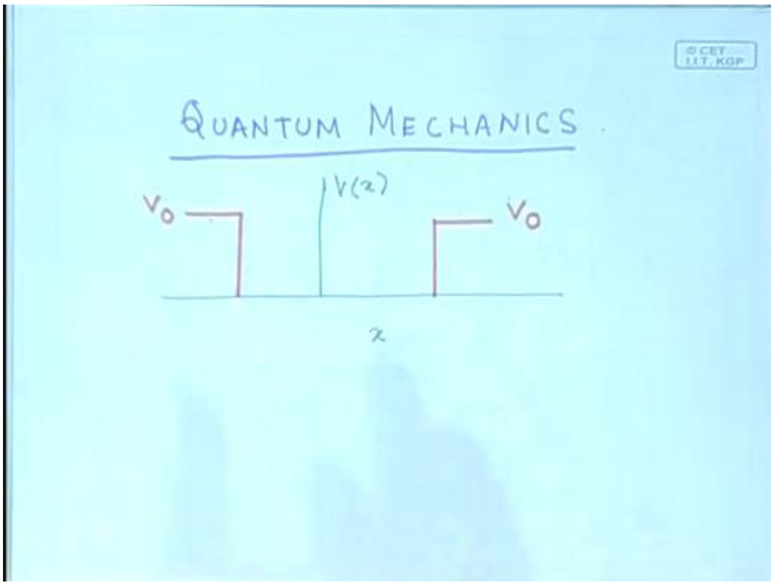

So, here we have 3 different regions 1 where potential is 0 and on the left hand side and the right hand side we have 2 regions where the potential is a constant. Now, we have already studied the wave function in the region where the potential is 0 so, this region is the particle behaves like a free particle in the region over here. And let me write down the energy so, so, in the region let we write down the solution first in the region in between which is quite simple.

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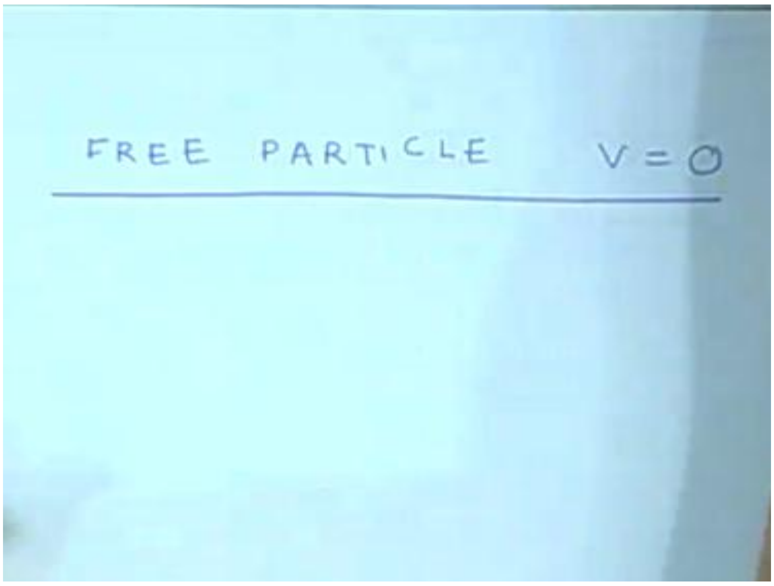

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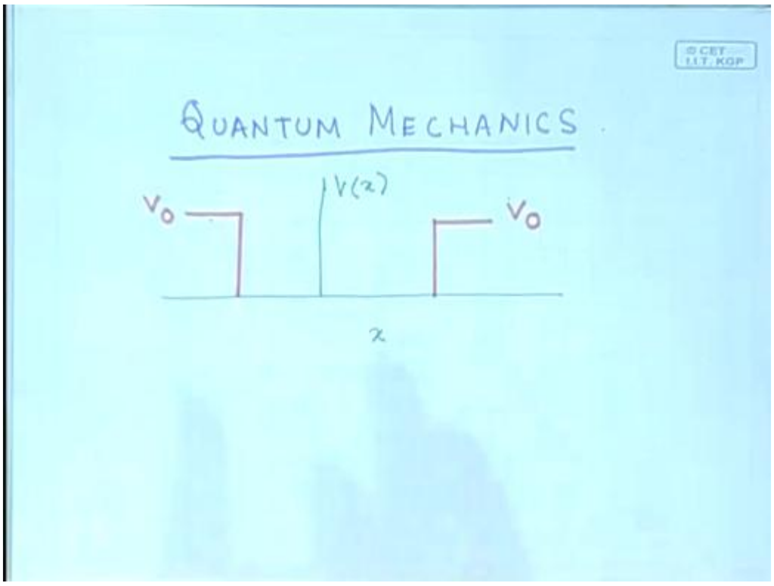

So, the free particle the part where the potential is 0 so, we are first considering the region in between where the potential is 0 and we have already discussed this situation.

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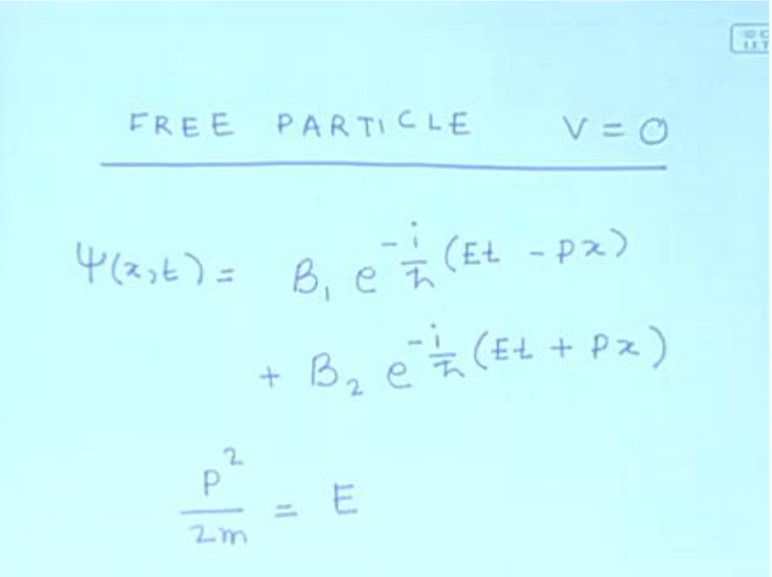

This is where what is refer to as a free particle there is no potential v is equal to 0. The solution in such a region we have already worked it out it is psi x t is equal to b 1 some constant e to the power minus i by h cross Et minus px plus B 2 e to the power minus i by h cross Et plus px. So, this is the solution to the wave function in the region where the potential is 0. And we have worked this out earlier we have also discussed the interpretation it is a superposition of a 2 parts both of which has the same energy E. This represents a particle moving to the right this wave represents a particle with momentum plus p. This wave represents a particle with momentum minus P this is a forward travelling wave this is a backward travelling wave. And the momentum in both the situations P is related to the energy like this for a free particle. So, for a free particle we have two possible solutions, one representing a particle going to the right.

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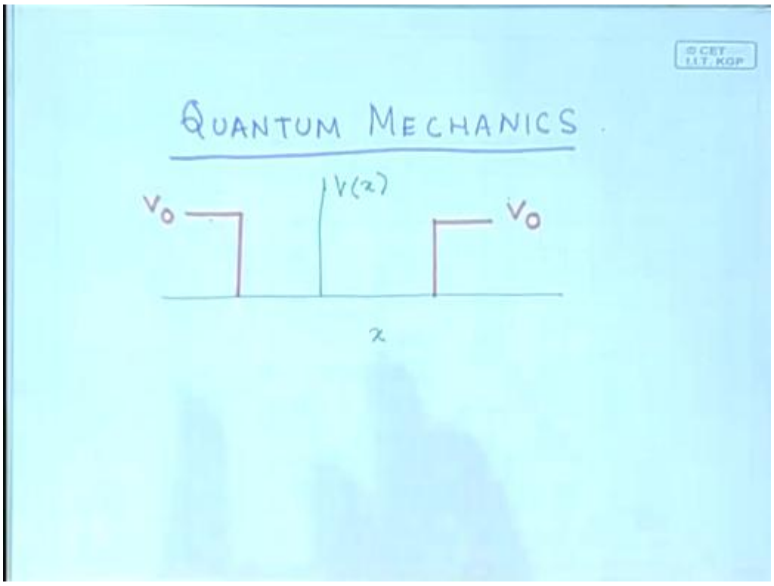

And another representing a particle going to the left both of these are plane waves with the same energy same angular frequency.

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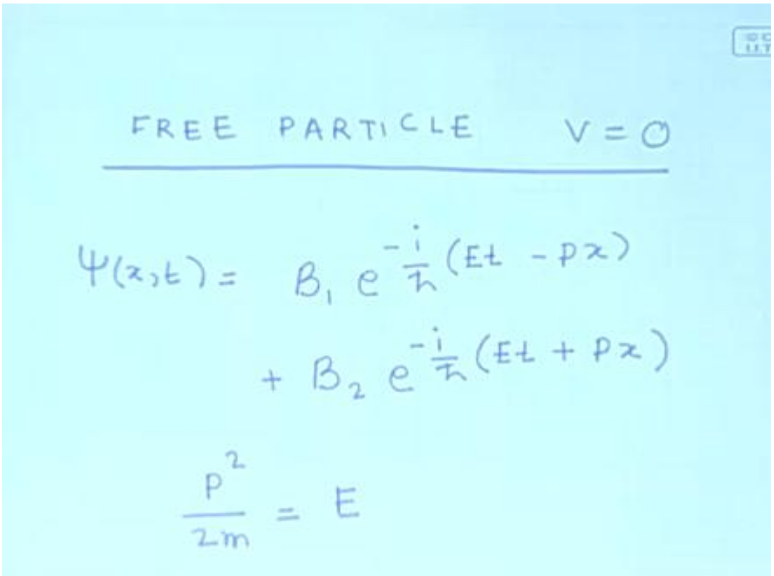

Also the same number wave number just that the wave vector are oppositely oriented. And we can have super position's of these two solutions with arbitrary coefficients B 1 B 2. I am going to take up the solution in these two regions.

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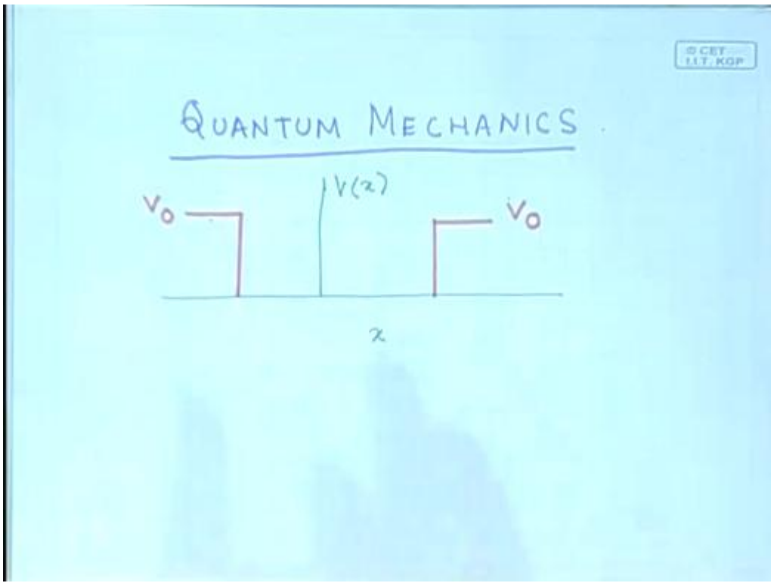

In the next class, so, let me stop today's lecture over here.