Physics I : Oscillations and Waves
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Lecture - 38
Measurements

Good morning. In the last few lectures I have told you that to describe the behaviour of microscopic particles it is necessary to think of them as waves. And the evolution the dynamics the rules governing this wave is what is known as quantum mechanics. And we were discussing the rules of quantum mechanics. So, what I told you in the last class was the first thing that I told you in the last class was that we have to represent the different states of a system or a particle.

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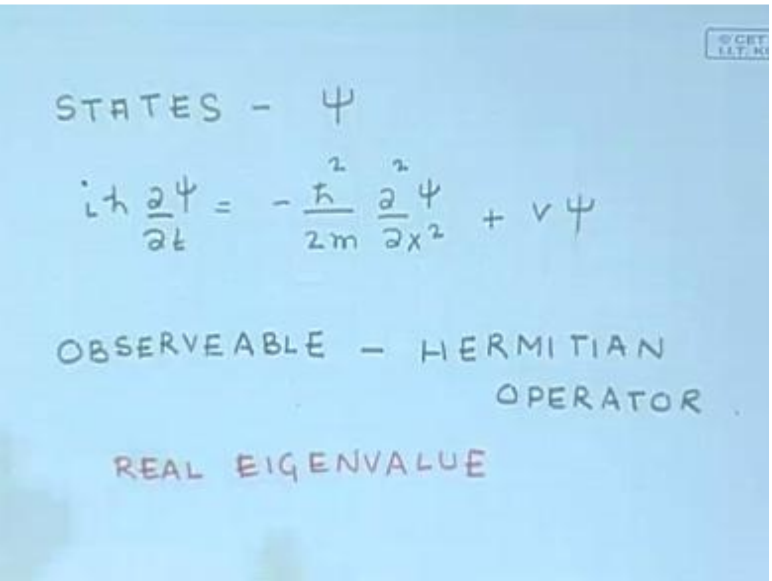

So, states are represented by different wave function psi so, corresponding to every possible state of a particle. There is a wave function psi if I have a different state then I am going to have a different wave function. And I also told you that the wave function psi is governed by the Schrodinger wave equation. The wave equation governing the evolution of this wave function is the Schrodinger wave equation. So, let me again write down the Schrodinger wave equation just to remained you i h cross del by del t psi is equal to minus h cross square by 2 m del del x square del square x by del x square psi plus v psi.

So, this the wave equation governing the wave psi the wave function psi in a situation where the particle is restricted to move only in 1 dimension 1 direction which is the x direction. And where the particle has a, is in a potential which is represented by v so, the external influences on the particle are there in this potential v. And in the last lecture, we had considered the situation where the potential v is 0. That is v have a free particle on which there is no external influence and for this particular situation. We had used the method of separation of variables defined a solution to this find a solution for the wave function.

And I also told you that any linear superposition of these solutions of different such solutions is also a solution of the Schrodinger equation. Then I told you that corresponding to every dynamical observable there is a Hermitian operator. So, for every different possible state of the particle you have a different wave functions psi. Now, given a particle there are various possible dynamical quantities that you can observe for example the position is one of them. The momentum is another the energy is a third possibility so, these are all dynamical observables. And in quantum mechanics where the state of e the system is represented by a wave function every dynamical observable is represented by a Hermitian operator.

And I also told you what is a Hermitian operator rather I did not tell you what is a Hermitian, operator. But the key property of a Hermitian operator which is the property which is important for our purposes is the property that the Eigen values of a Hermitian operator are real. So, a so, we are not going into the definition of a Hermitian operator as far as we are concerned Hermitian operators are operators which have the property that. The Eigen values of such operators are real the definitions are little more complicated and for this the purposes of this lecture we will not go into that.

So, the second point let me repeat it again is that corresponding to every dynamical observable. There is an operator which acts on these wave functions. And these operators corresponding to dynamical observables are Hermitian operators. The crucial property of Hermitian operators is that there Eigen values are always real I also told you the. What the operator corresponding to the position is what the operator corresponding to the momentum of the particle is. And what the operator corresponding to the energy or more precisely the Hamiltonian is and these operators are all guaranteed to have real Eigen values. So, this is what we had discussed in the last lecture. Now, in today’s lecture we are going to first we going to discuss mainly focus on what happens when we make a measurement.

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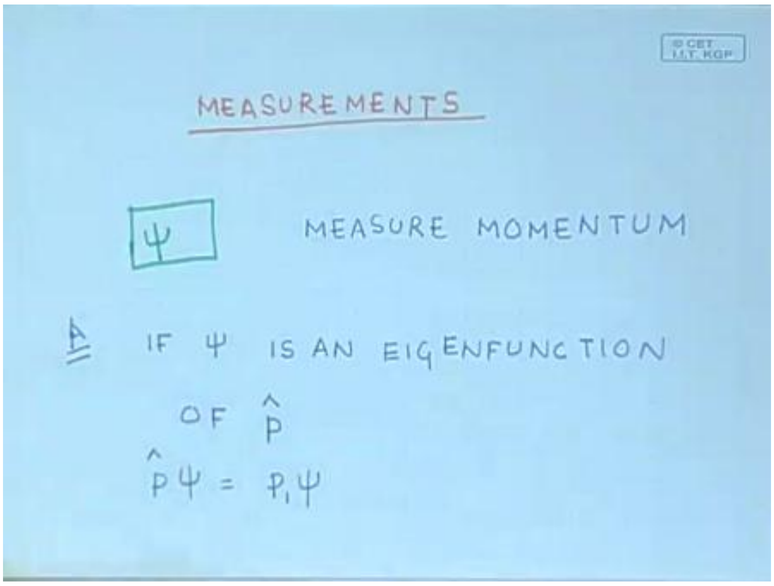

So, the issue that we are going to discuss is what happens when we make measurements before going into this let there is a point which I should make clear?

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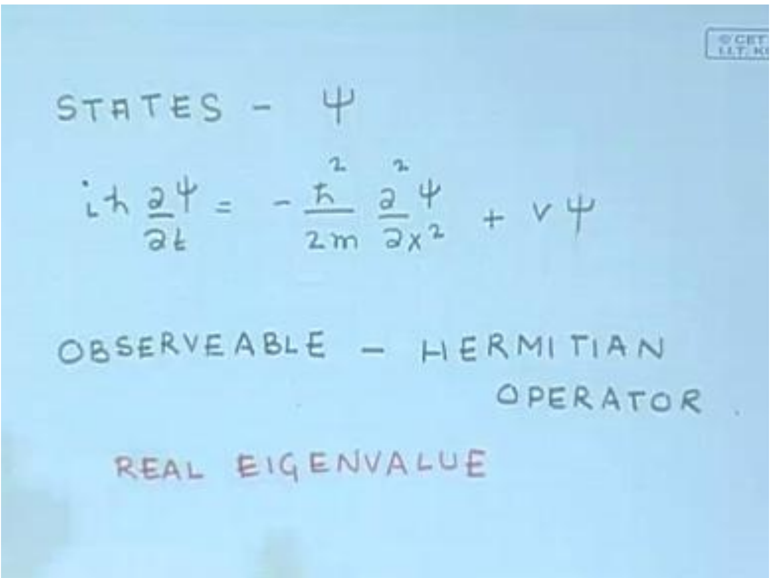

And it has to do with what I mean what do we mean by a dynamical observable I have not written dynamical here what do we mean by a dynamical observable. So, for a particle we have various quantities like it is mass is charge etcetera all of which are attributes of the particle they do not change. So, for an electron it has a fixed mass by mass we always refer to the rest mass of the electron. It has a charge all of which can be measure when the electron is at rest and these do not change for an electron these are fixed numbers. So, when I talk about observables I do not refer to such quantities as the mass or the charge I refer to dynamical quantities which change as the particle moves around and the position the momentum and the energy are examples. So, now, let us get down to the situation that we wish to discuss and the question that we are going to discuss.

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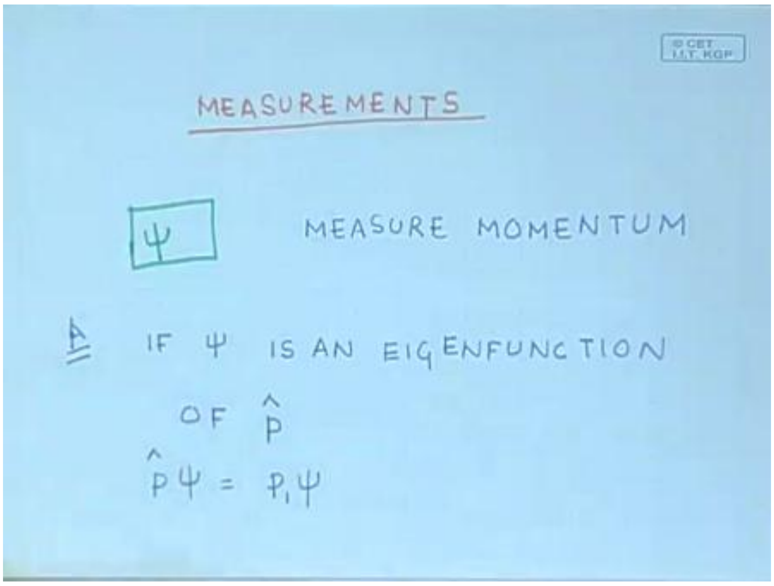

We going to start our discussion with is as follows there is a particle which has a state which is described by a wave function psi. So, there is a particle which has a state which is in a state which is described by a wave function psi. So, what I can say is that the particle has a wave function psi. And let us consider a situation where we measure the momentum of the particle so, we measure we measure the momentum of the particle. And the question is what is going to happen when we measure the momentum of the particle. So, there are 2 possibilities which we will discuss there are 2 possible situations which can arise when I make a measurement of the momentum. The first possibility is that when psi so, let we consider the first possibility which I will refer to as A.

So, the first possibility is if psi is an Eigen function of the operator corresponding to momentum. So, the operator corresponding to momentum is p and if the situation is such that the wave function of the particle is an Eigen function of the momentum operator. So, if this wave function is an Eigen function of the momentum operator let me p psi is equal to p 1 psi. If this wave function is an Eigen function of the operator p then the operator p acting on psi will give me a number which I have denoted by p 1 into psi. That is the property that is what we mean by the statement if psi is an Eigen function of the momentum operator. So, if the wave function of the particle is an Eigen function of the operator corresponding to the quantity that we are measuring which in this case is the momentum. Then the measurement will give a value p 1 where p 1 is the Eigen value so, the measurement.

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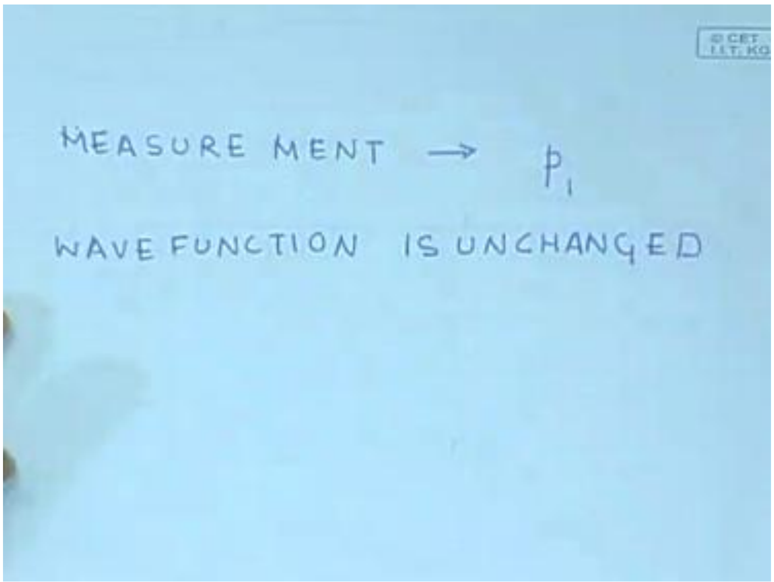

Is going to give the Eigen value p 1 and in this measurement process the wave function is not going to change so, let me write it here wave function is unchanged.

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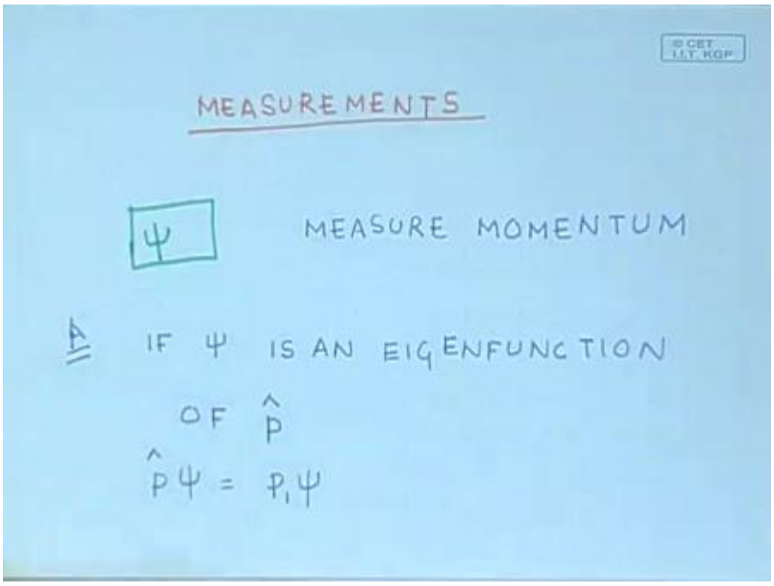

So, if the question let we rewind you again the question we are addressing is that we have a particle which has a wave function psi. And the question is what happens when we measure the momentum of the particle. And we are first discussing a situation where the wave function psi is an Eigen function of the momentum operator which is the operator corresponding to the quantity. That we are measuring if this psi is an Eigen function of this operator with an Eigen value p 1.

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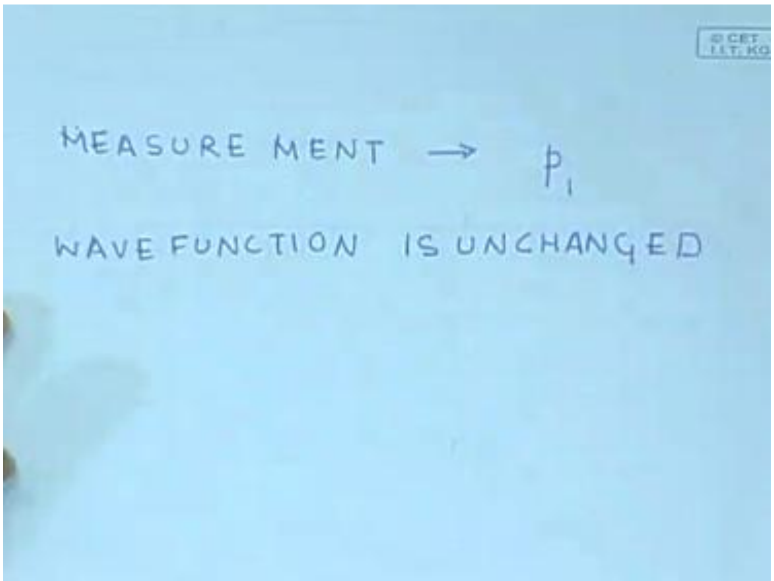

Then the measurement is going yield a value p 1 for the momentum the Eigen value is the value that I am going to get when I measure the momentum. And the wave function is unchanged by the act of measurement.

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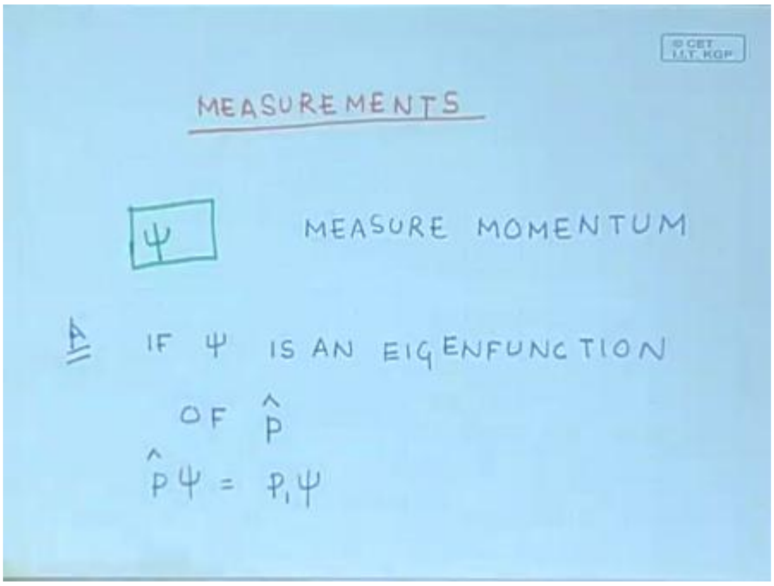

So, if repeat the measurement if repeat and I measure the momentum again.

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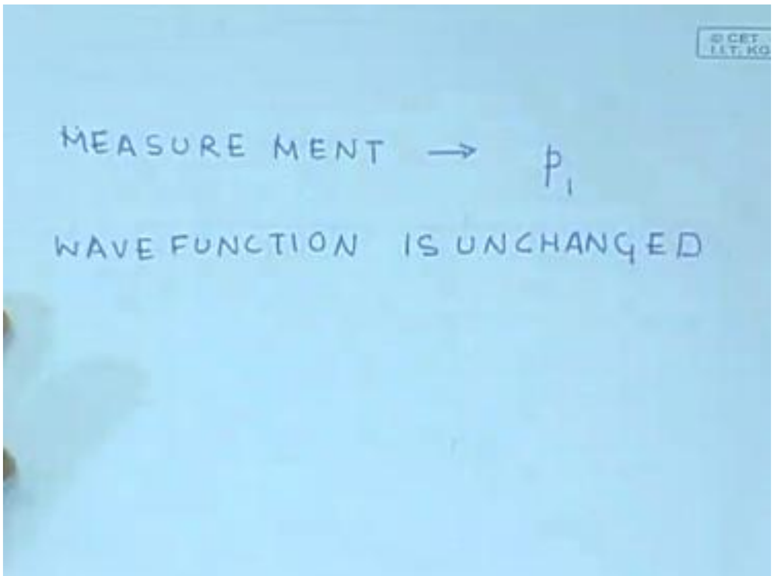

I am again going to get the same value p 1, because the wave is unchanged.

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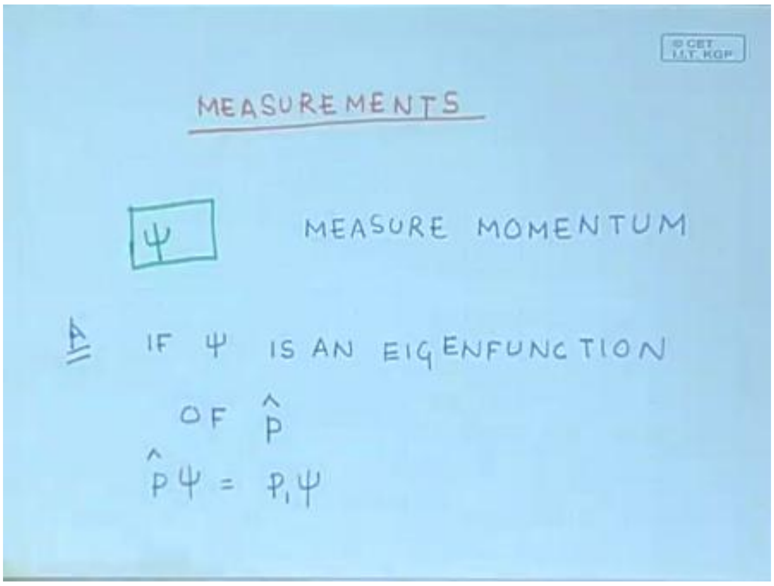

So, this is what happens when I have a particle in a state which is an Eigen function of the operator corresponding to the observable that I am measuring. Now, let we next consider a situation so, the situation is still the same so, I have in I have the same I have a.

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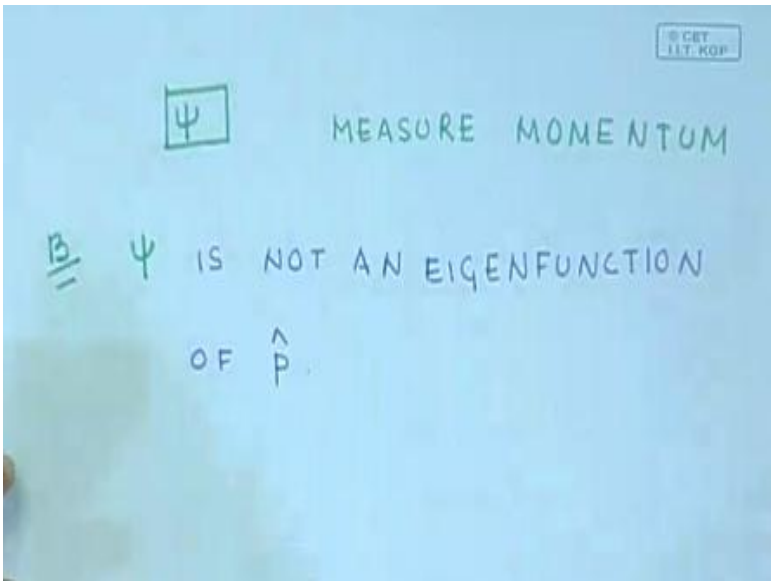

Particle with a wave function psi and I measure the momentum, but in this case B psi the wave function psi is not an Eigen function of P. Before proceeding with this let me just back track to the situation A which I was discussing where the wave function is an Eigen function of the operator p.

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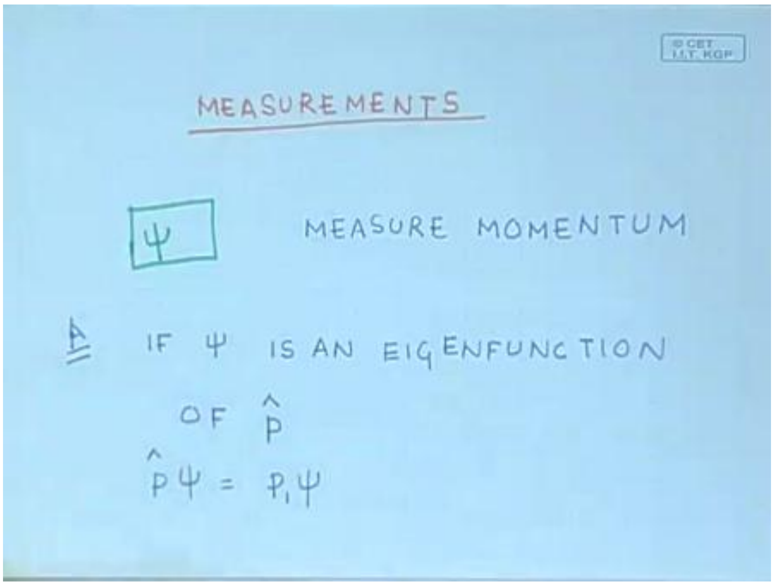

So, let we consider an example of this that is something which I forgot to do. So, let we consider an example.

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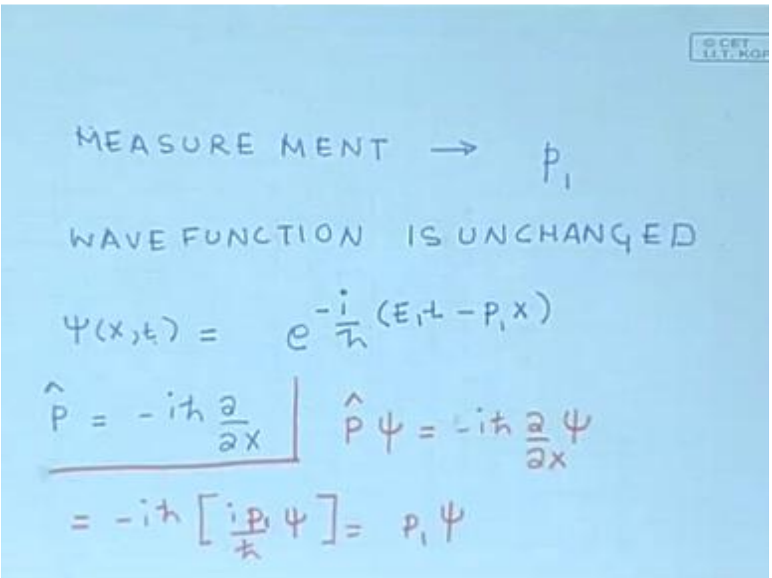

The example which I am going to consider is Psi x t is equal to e to the power minus i by h cross E 1 t minus p 1 x so, this is the wave function that we are considering and the momentum operator. The operator corresponding to the observable momentum I had told you is minus i h cross del by del x. So, let us see what happens when this operator acts on this wave function so, when the momentum operator acts on the wave function psi given over here. What we have to do is we have to multiply this wave function by minus i h cross and then differentiate it with respect to x. Now, when we differentiate this wave function with respect to x we get the same function multiplied by minus multiplied by i p by h cross.

So, we get the same function multiplied when I differentiate we get the same function multiplied by this is equal to minus i h cross. And the derivative of this function is i p by h cross into psi right that you can check easily if you differentiate. This with respect to x this is e to the power i by h cross into p x the minus here and minus here give plus. So, when i differentiated I will get e to the power of i p by h cross into psi and this p 1 into psi because there is a p 1 here. And this gives me p 1 minus i into i is 1 h cross cancels out this is gives me p 1 into psi. So, it is clear that this wave function is an Eigen function of the momentum operator. And it has an Eigen value p 1 the constant that appears over here so, for this wave function if I make a measurement of the momentum.

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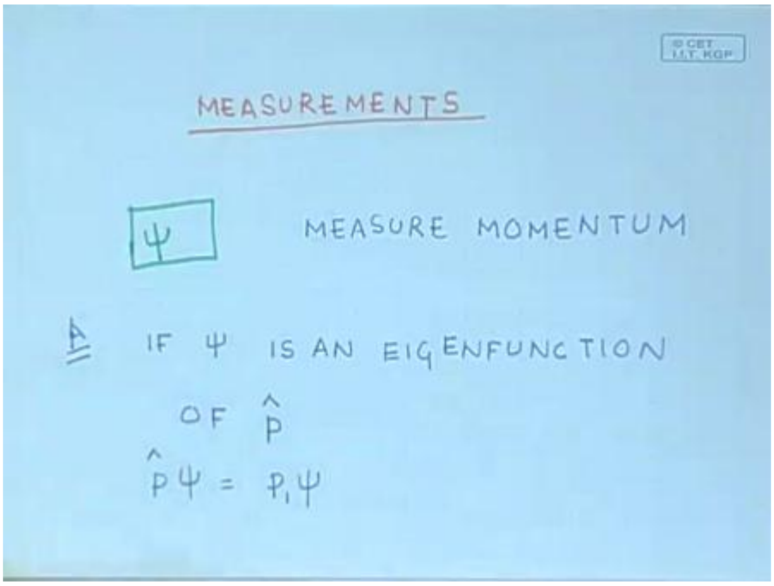

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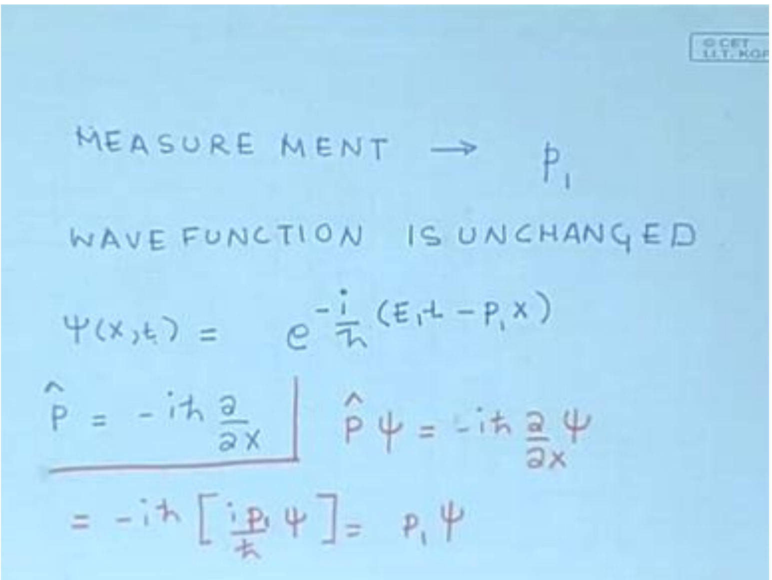

This wave function is an Eigen function if I make a measurement of momentum I will get the value p 1 and if i in this process of measurement the wave function is unchanged. So, if I repeat the experiment I will again get the value p 1 if I repeated again I will still get the value p 1 so, in this process the wave function is unchanged.

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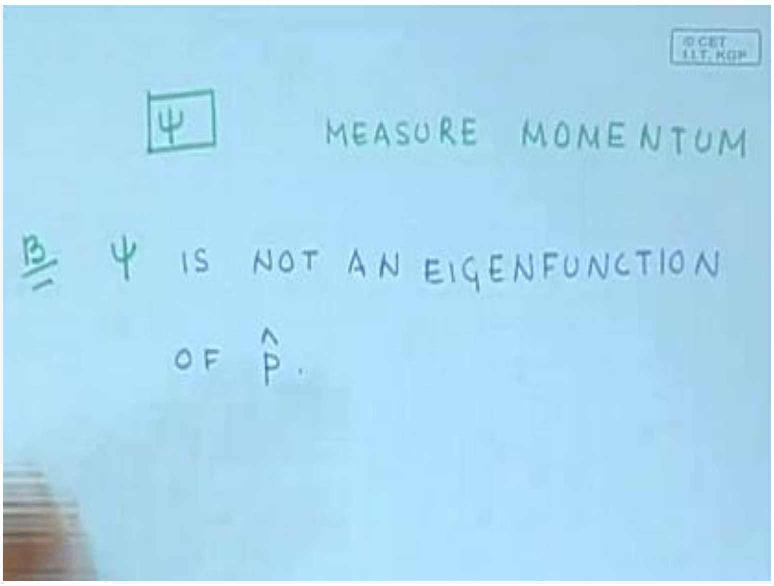

Now, let us go back to the situation where the wave function psi is not an Eigen function of the momentum operator question is what will happen now. So, let me now give you an example of this and the example that I will consider is as follows we have.

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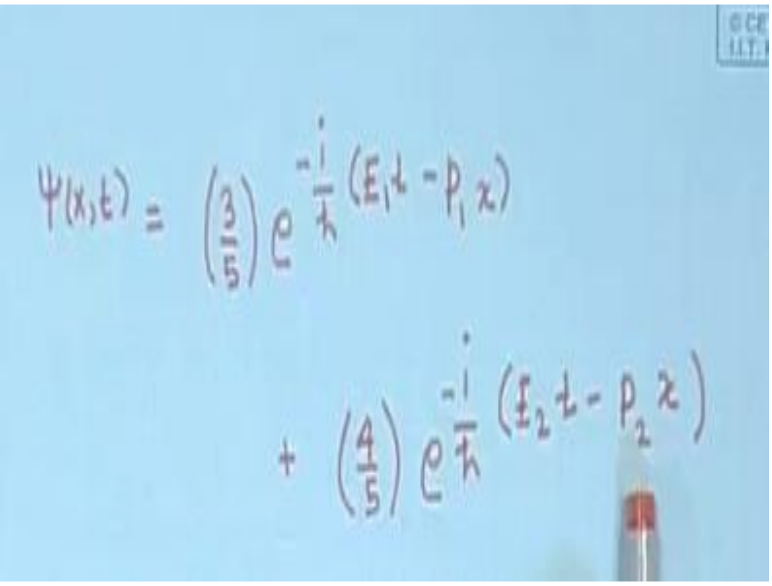

A wave function psi x t which is equal to 3 fifth e to the power minus i by h cross E 1 t minus p 1 x plus 4 fifth e to the power minus i by h cross E 2 t minus p 2 x. So, note that this wave function is a super position of 2 wave functions which looks like this with different constants the constants being e 1 and e 2 p 1 and p 2.

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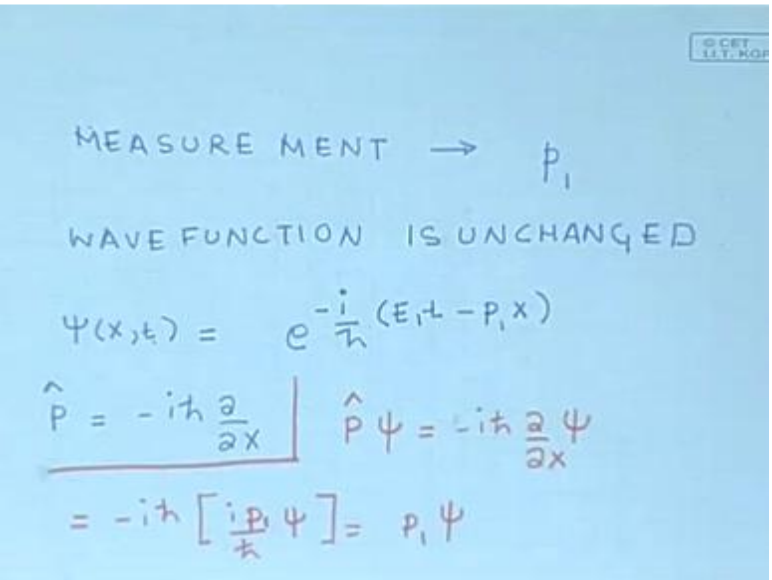

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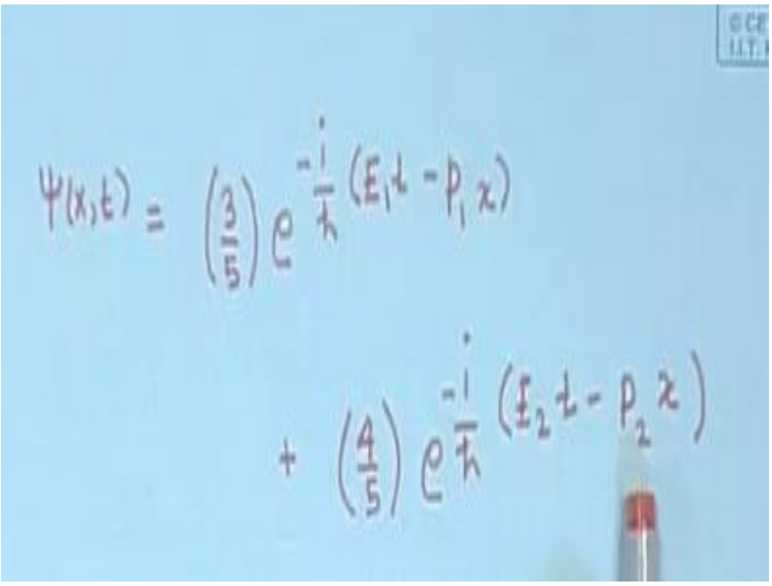

So, this itself is an Eigen function of the momentum operator with Eigen value p 1 this itself is an Eigen function of the momentum operator with Eigen value p 2. The question is now what happens when I make a measurement of the momentum for this wave function which is a combination of 2 Eigen function. This wave function itself is not an Eigen function of the momentum operator remember the wave function that. We are dealing with now is a superposition of 2 different Eigen functions of the momentum operator. And this is not an Eigen function of the momentum operator the questions is what happens when I make a measurement of the momentum. Now, what happens is as follows this wave function is a superposition of 2 Eigen functions each of which this one has an Eigen value p 1 this 1 has an Eigen value p 2.

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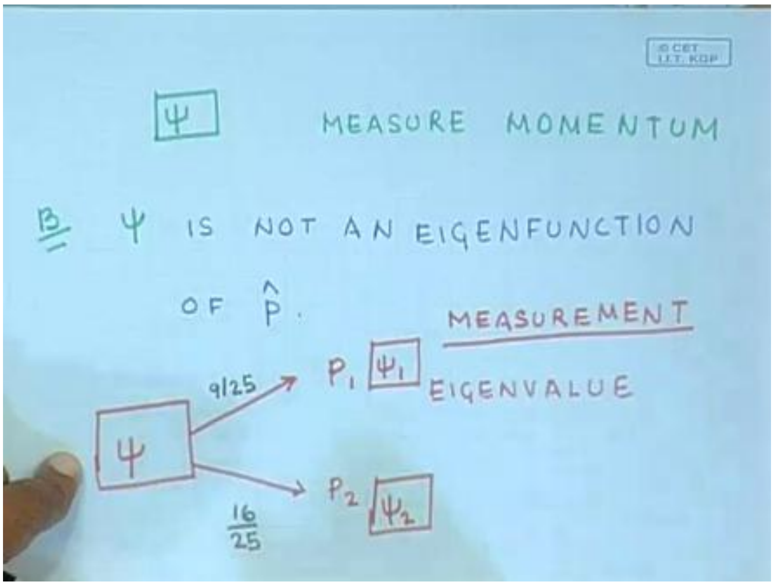

So, what happens when I make a measurement is that when I make a measurement of the momentum for the wave function psi I can get one of 2 possible outcomes the p 1 or p 2. So, when I make a measurement of momentum for a wave function which is not an Eigen function of the momentum operator then the value that I get is one of the Eigen value.

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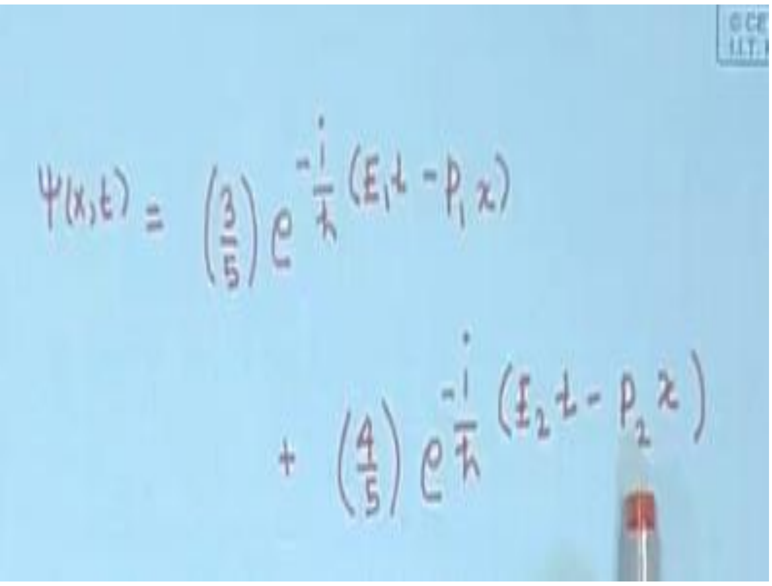

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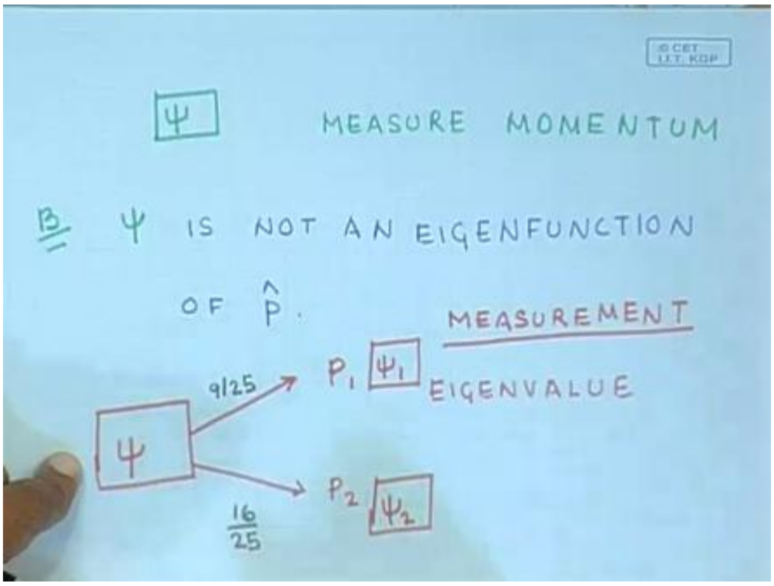

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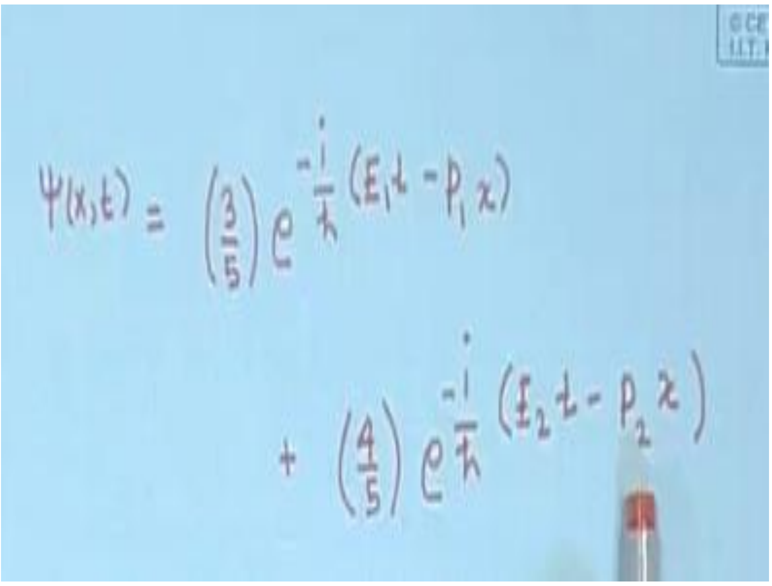

So, what would what I have to do is I have to express this wave function interns of Eigen function of the momentum operator which is what I have done here so, the wave function psi which is not an Eigen function of the momentum operator is a super position of 2 different Eigen functions. So, whenever I make a measurement of the momentum I will get either this value or this value.

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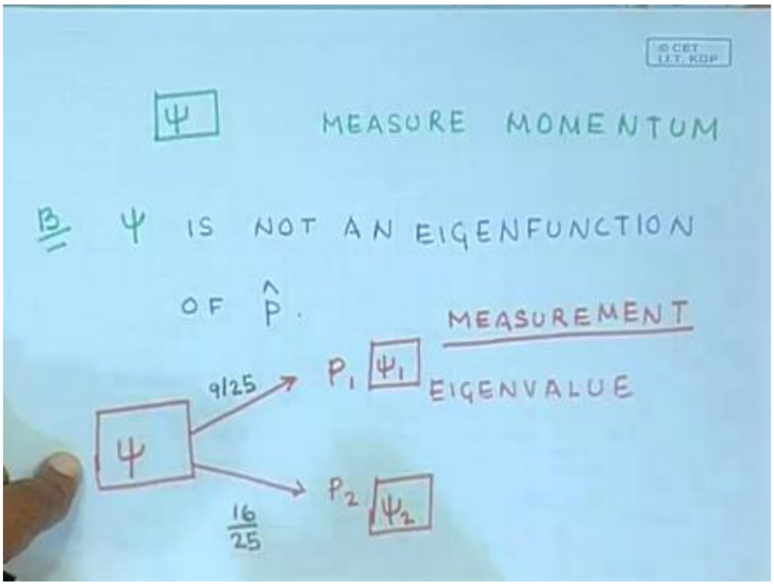

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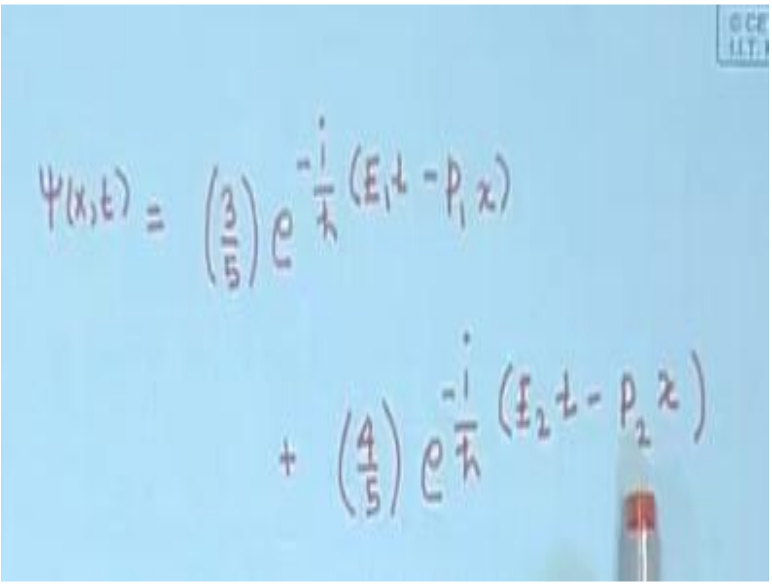

I will get either p 1 or p 2, because this wave function psi is a superposition of 2 Eigen functions with Eigen values p 1 and p 2 we could have considered. The situation where psi is a superposition of 3 Eigen functions P 1 with value p 1 1 with value p 2 1 with value p 3 in such a situation if I were to measure the momentum I would get one of these 3 momentums when I make a measurement of the momentum one of these 3 values. So, whenever I make a measurement I will get one of the Eigen values of the operator so, the act of measurement.

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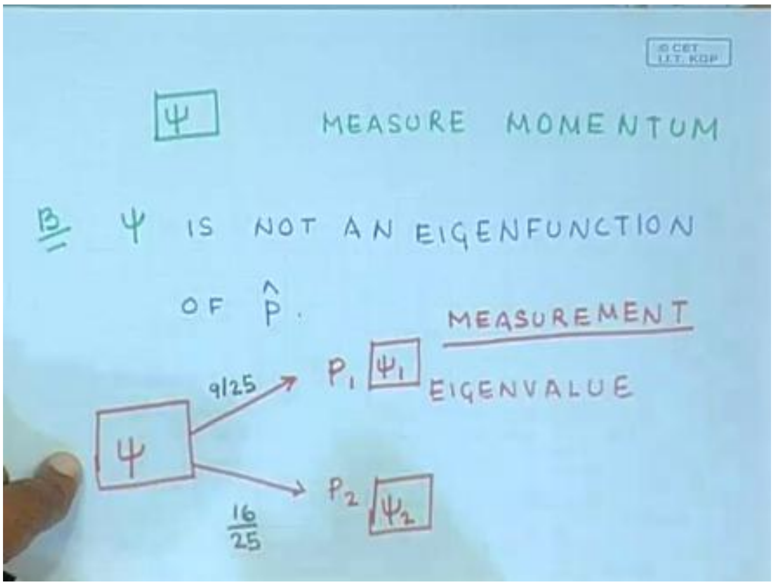

The act of measurement is going to give me an Eigen value of the momentum operator and the psi. So, the Eigen values the possible Eigen values here are p 1 and p 2 and the psi the wave function itself is going to change if I get p 1 as my momentum. Then the wave function after the measurement is going to be psi 1 if I get the momentum p 2 as my result. Then the wave function after the measurement is going to be psi 2 the probability that I get p 1 is the square of this.

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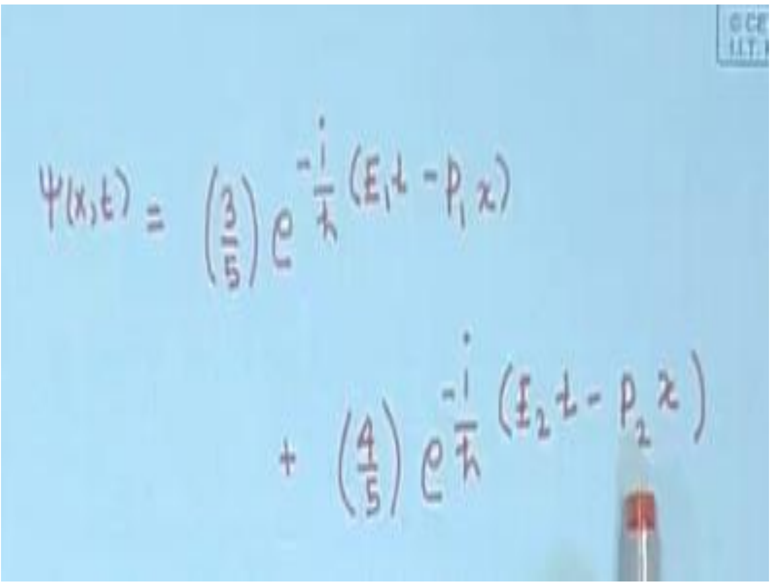

Coefficient over here so, in this case the probability of getting p 1 is 3 by 5 square which is 9 by 25 so, the probability of getting the value.

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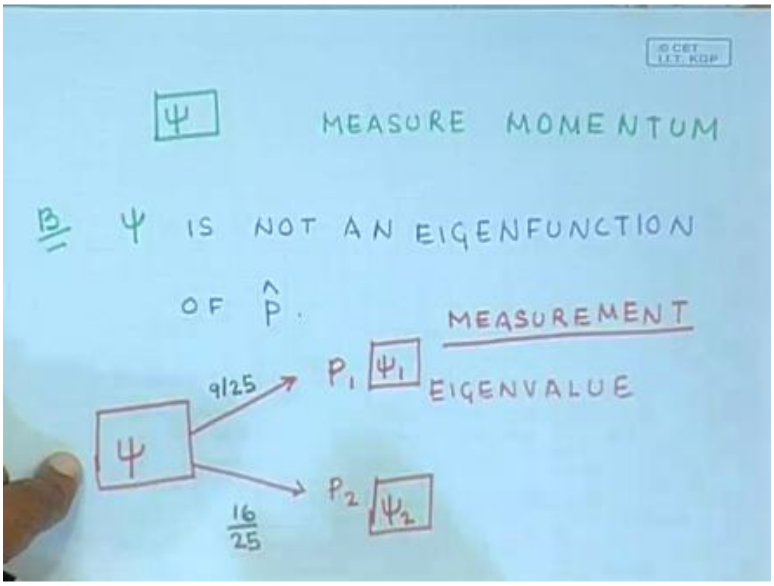

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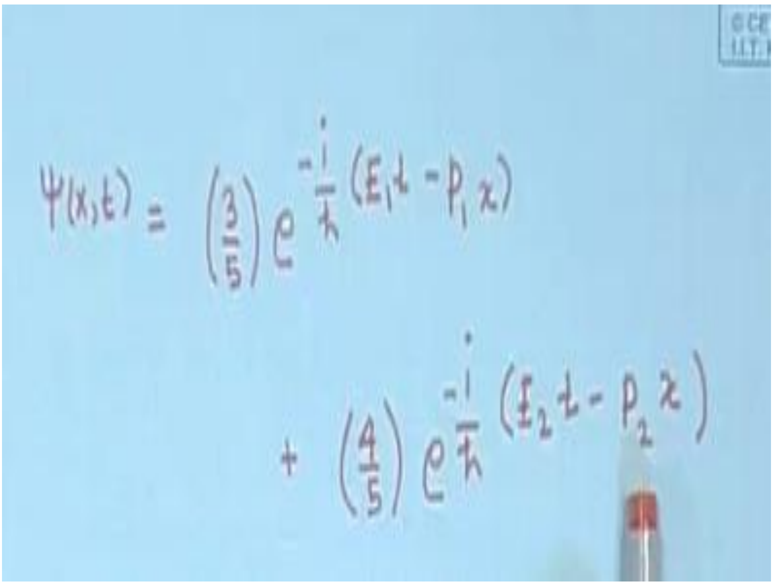

P 1 is 9 by 25 the probability of getting the value p 2 is the square of this coefficient which is 16 by 25?

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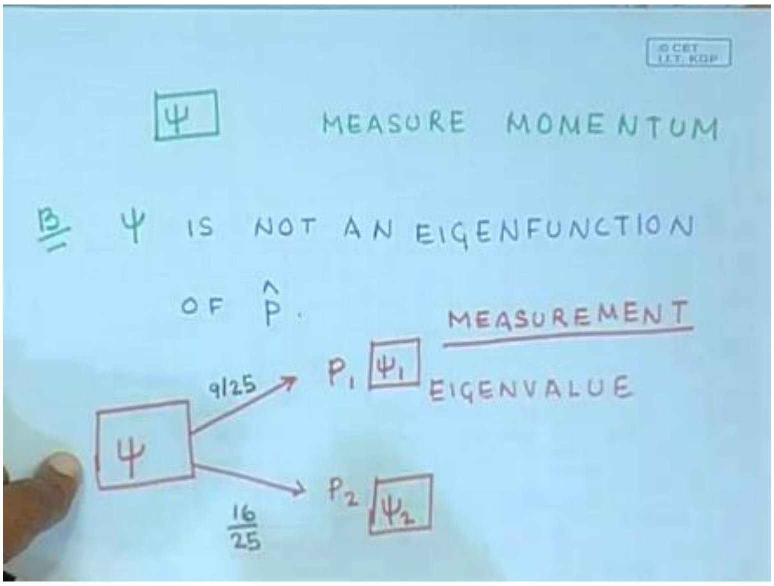

So, let me summarize these here again.

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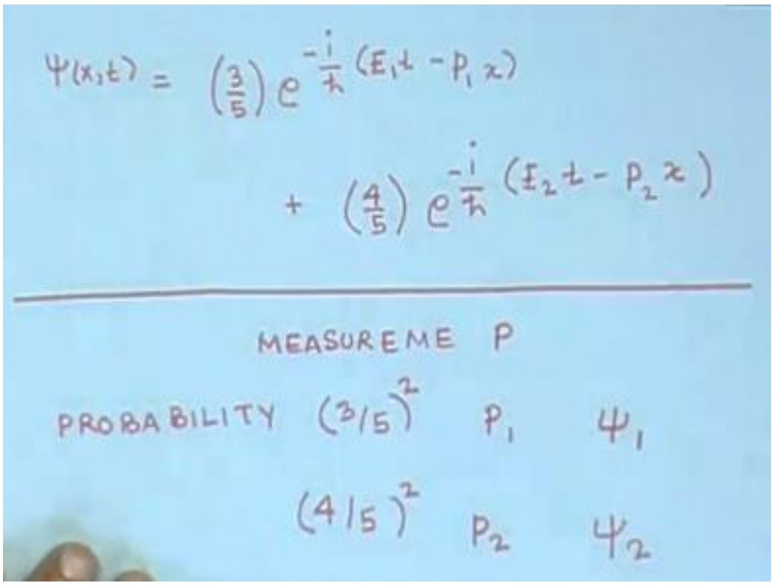

So, there is a probability so, when I do a measurement of the momentum there is a probability 3 by 5 square which is nine by 25 that I get the value p 1. And if I get this value p 1 then the wave function becomes psi 1 after the measurement. There is a probability 4 by 5 square that I will get the value p 2 if I get the value p 2. Then the wave function gets changed it is becomes psi 2. So, depending on whether I get p 1 or p 2 after the measurement the wave function will becomes psi 1 or psi 2 where psi 1 is this function. And psi 2 is this function.

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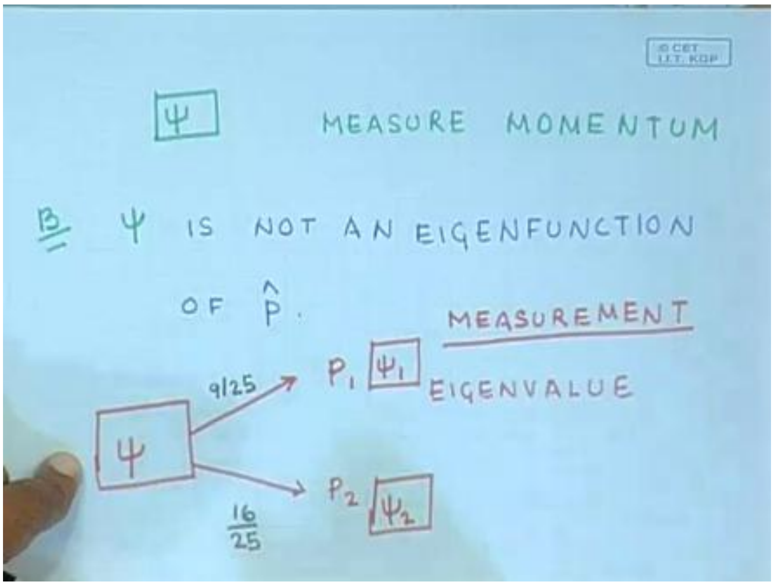

So, let me recapitulate the point and trying to make the point I am trying to make is that if I have a particle in a wave function which is not an Eigen function of the operator corresponding to the dynamical observable I am measuring. Then the measurement will give me one of the Eigen values of that operator. So, the momentum operator for example, can have different Eigen values the act of measurement is going to give me one of the Eigen values. And I can calculate the probability of getting any particular Eigen value so, what I have to do is I have to expand this wave function.

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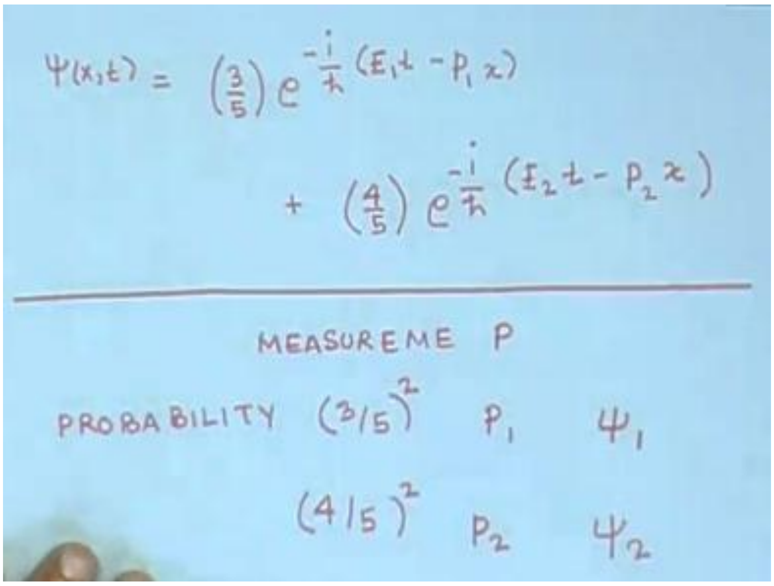

In terms of Eigen functions of the momentum operator which is what I have done for this particular wave function psi it is possible in general to always expand any arbitrary wave function in terms of Eigen functions of these observable quantities. And now when I make a measurement I will get one of the possible Eigen 1 of the Eigen values as my result. The probability of getting particular e Eigen value is the coefficient in the expansion in this case the probability of getting p 1 is the square of this. The probability of getting p 2 is the square of this note that I have chosen these coefficients. So, that the some of these 2 probabilities is 1 in case the sum is not 1 I can I have to multiply the wave function with an overall number which ensures to finally, ensure that. The sum of all the probabilities is equal to 1 that is called normalizing the wave function. So, act of measurement returns a value which is an Eigen value of the operator corresponding to the observable. That I am measuring and the act of measurement also changes the wave function.

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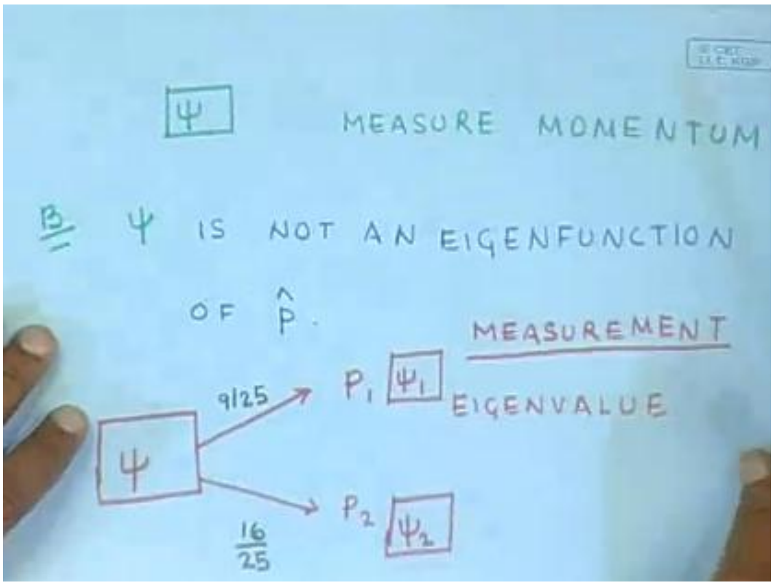

So, if I start with the wave function psi and I make a measurement of the momentum if I get a value p 1. Then after the measurement I can say that the wave function as changed and it has become psi 1. That is the Eigen function corresponding to p 1 if instead I had got the value p 2 I could say the after the measurement the wave function is in the as. Now, changed to psi 2 where psi 2 is the Eigen value Eigen function corresponding to this Eigen value.

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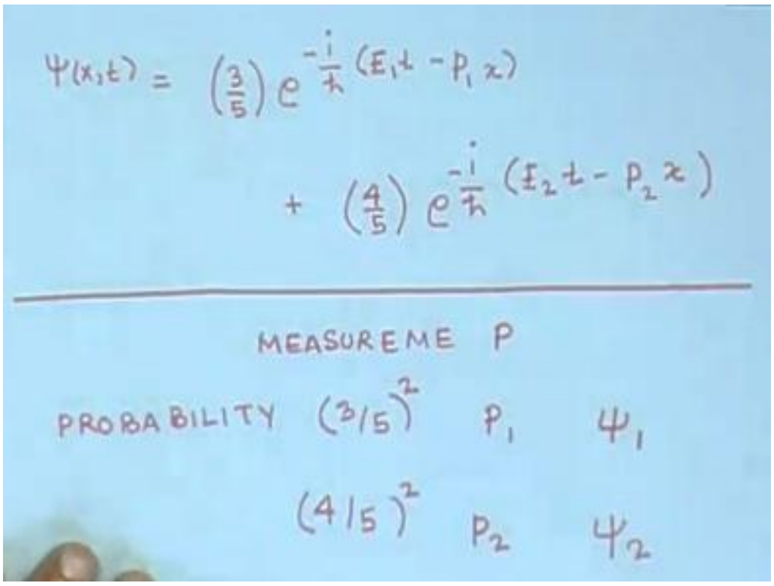

So, the point here very important point is the act of measurement changes the wave function. It is only when the wave function is an Eigen function of the operator that you are measuring the variable that measuring that. The wave function does not change in any general situation if I make a measurement it will change the wave function. And the wave function will go over to one of the Eigen functions of the operator corresponding to the observable that I am measuring. And I have given you an example with this particular wave function and where we make a measurement of the momentum I can have 2 possible outcomes p 1 p 2. If I get the outcome p 1 I know that after the measurement the wave function is this if I get p 2.

Then I know after the measurement the wave function is this the, normalize the co efficient outside as to be chosen to normalize the wave function. So, I have told you what happens in both the situations 1 where the wave function is an Eigen function of the operator corresponding to the observable that we are measuring. The second where the wave function is not one of the Eigen functions of the operator corresponding to the observable. That we are measuring with this background we can now go back. And interpret the wave function that we had derived in yesterday's lecture, in yesterday's lecture we had solved a Schrodinger equation. And the solution that we obtained for a free particle where there is no potential we use the method of separation of variables to obtain a solution and the solution that we obtained was psi.

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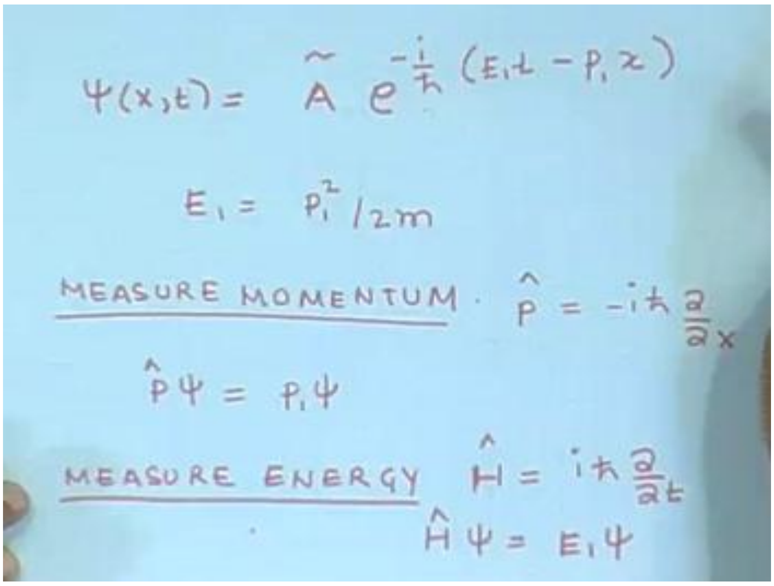

A, some normalization constant e to the power minus I by h cross E 1 t minus p 1 x this is the solution that we had obtained in the last class. This is was the solution of the Schrodinger equation and in this solution A tilde was an overall constant out side in addition to this. There was this constant p 1 and this constant E 1 both of which were related as E 1 is equal to p 1 square by 2 m. And I could arbitrarily chose either E 1 or p 1 E 1 had to be positive p 1 could be negative. So, if I fix E 1 and p 1 was fixed if i fix p 1 then e 1 was fixed one of them is can be chosen arbitrarily. And we had not gone into the physical significance of these constants. Now, let us ask the question what happens if I measure the momentum of the particle if it is described by this wave function. So, there is a particle which is described by this wave function which we saw was a solution to the Schrodinger equation free particle described by this wave function.

Let us ask the question what happens if we measure the momentum. Now, the operator corresponding to momentum is minus i h cross del by del x so, when this operator acts on psi it gives me. So, what happens if when you differentiate this exponential with x you pick up minus I by h cross into p 1 i into h cross I into p 1 by h cross i into minus i gives you 1. The 1 by h cross into h cross gives you 1 so, what you have is this is equal to p 1 psi so, what you can say is that if a particle is described by this wave function. And if you make a measurement of momentum since this wave function is an Eigen function of the momentum operator. It will give you the value P 1 it has an Eigen value p 1 so, whenever you measure momentum you will find that you will get a value p 1.

So, the momentum measurement of momentum will always give you a value p 1 and this measurement is not going to disturb the wave, function. Similarly, if you measure energy the operator the Hamiltonian operator whose Eigen values are the energy? So, strictly speaking there is a Hamiltonian operator which is which gives you the energy of the particle the Hamiltonian is the energy of the particle. So, the Hamiltonian operator is i h cross del by del t. And this wave function is an Eigen function of the Hamiltonian operator the operator corresponding to energy h psi. So, when you take the time derivative of this you will get minus i by h cross E 1 minus i into I will give you 1 is the 1 by h cross into h cross will you give once what you get is that this is equal to E 1 into psi.

So, when if you make a measurement of the energy of the particle you will also get a value you will get a value E 1. And this is not going to disturb the wave function so, if you repeat the experiment again. And measure the energy you will get the again the same wave function multiplied by E 1. So, it is so, whenever you make a measurement of the energy it has a well-defined value E 1 if you make a measurement of the momentum it has a well-defined value p 1. So, this wave function corresponds to a particle with energy E 1 and momentum p 1. Now, the energy of the momentum are related like this which is what you expect now what happens when you measure the particles position.

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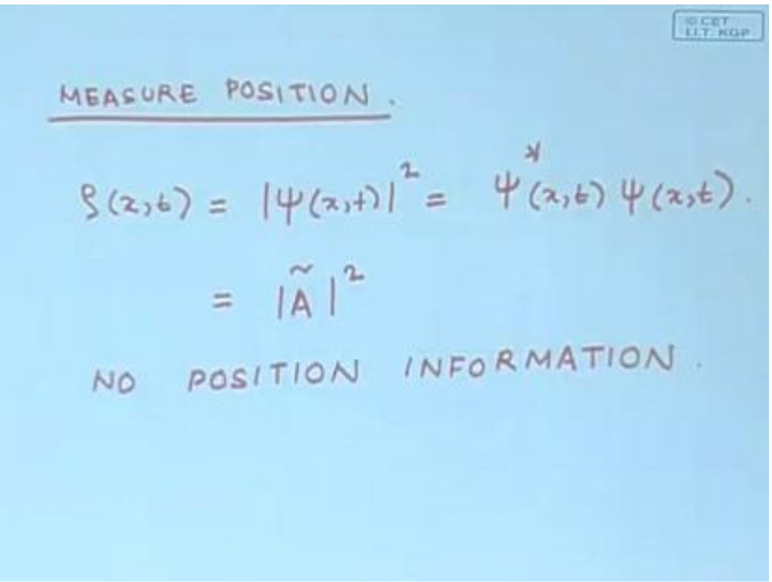

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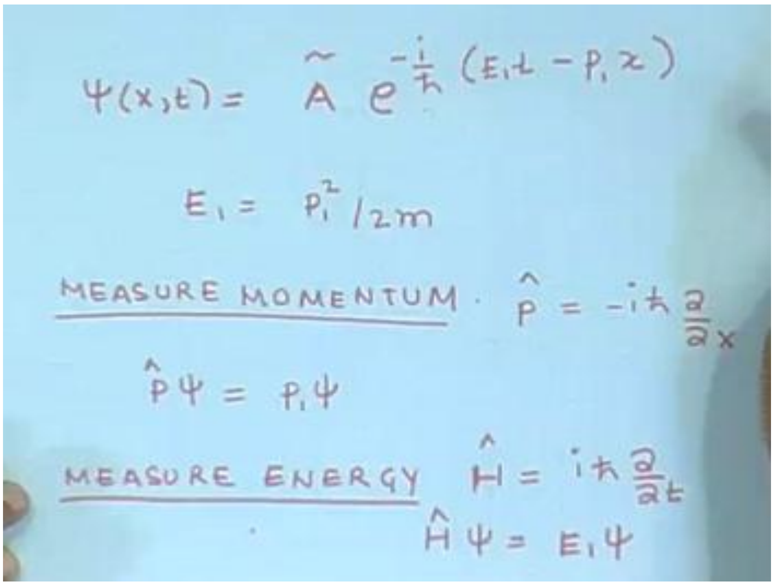

Now, when the position operator acts on this it basically multiplies this with x so, it is clear that this function this wave function is not an Eigen function of the position operator. We have discussed what will happen when you make a measurement of the position. The way you interpret what you can say about, what you can predict for a measurement of the position is that the probability density of finding the particle at some position is what you can predict.

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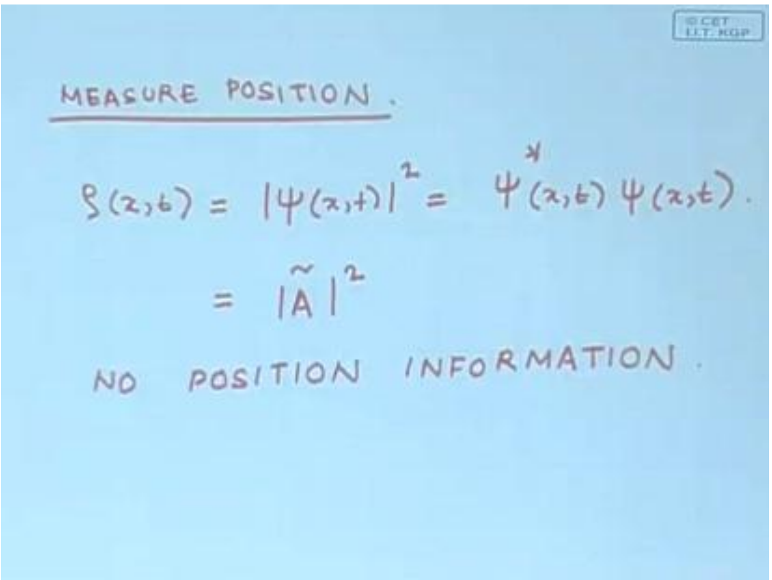

So, rho x t is going to give the probability density of finding the particle at some point. And this is equal to the mod of psi x t square or you can write it as psi star x t into psi x t now, let us look at the wave function.

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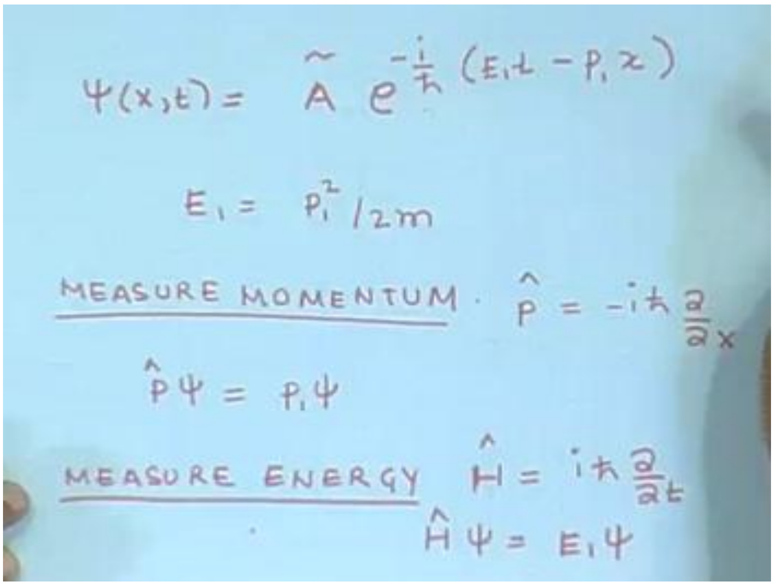

That we have we have this wave function so, when I take it is complex conjugate it will not have this minus sign. And if I multiply this function with this complex conjugate this into its complex conjugate is going give us 1.

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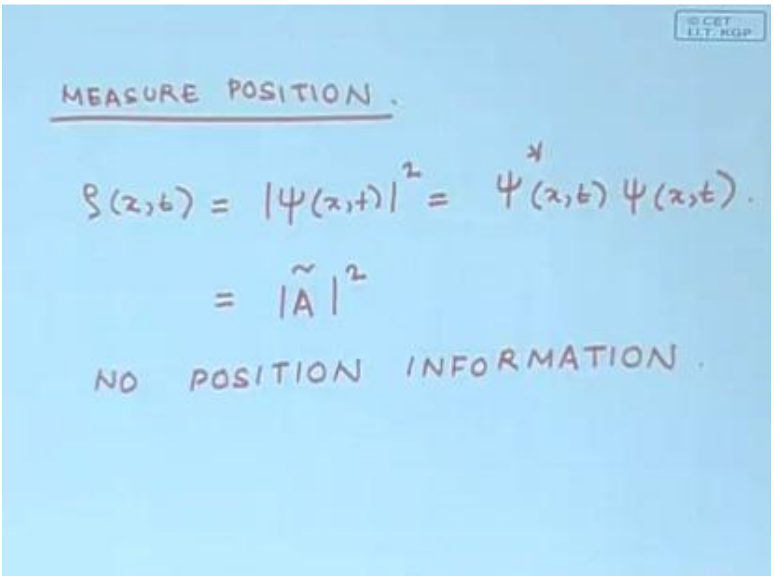

So, what am going to get is that the probability density of finding the particle somewhere the probability density is a constant and the constant is the modulus of A square. So, it is equally probable that we find the particle anywhere the probability density of finding the particle at any position the same it does not depend on x. And it is the same everywhere so, this wave function has a precise refers to a particle which has got a precise momentum p 1 it has precise energy E 1. But, it has you cannot make any prediction as to the particles position. It is equally probable that the particle is there anywhere in the whole of space. So, this wave function has no position information it has no information at all about the particle's position. So, this is the interpretation of the wave function which we had derived in yesterday's class.

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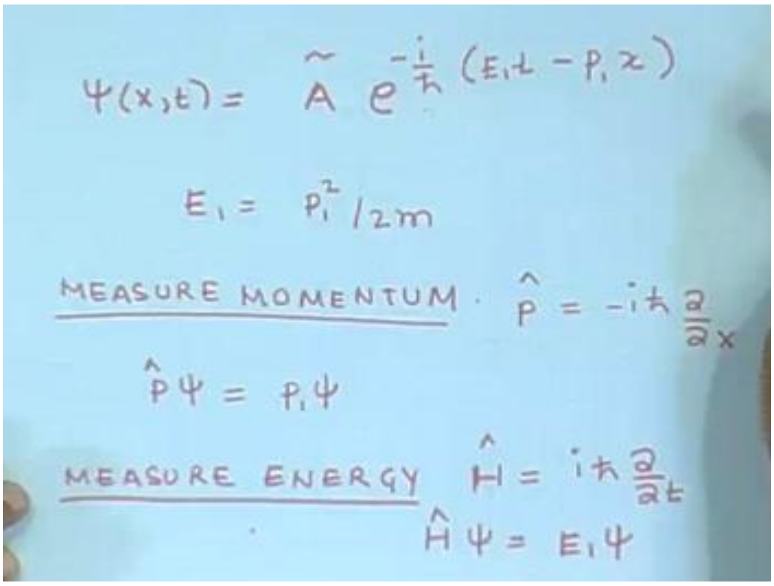

It has very precise information about the particles momentum you can interpret this constant as being the particles momentum you can interpret. This constant as being the particles energy, but it has no information at all about the particles position momentum. And energy are very absolutely well defined, but the uncertainty in the momentum in the position is infinite it has no information about the particles position. So, the first point is that this wave function is consistence with a de Broglie with the de Broglie hypothesis the de Broglie principle. And so, the Schrodinger equation is a Schrodinger wave equation essentially incorporates the de Broglie hypothesis. It incorporate the prediction of de Broglie regarding the relation between the wave number and the energy the momentum the angular frequency and the energy. So, it is consistent with that we also find that this wave predicted by the Schrodinger wave equation has very precise energy and momentum, but it has no position information.

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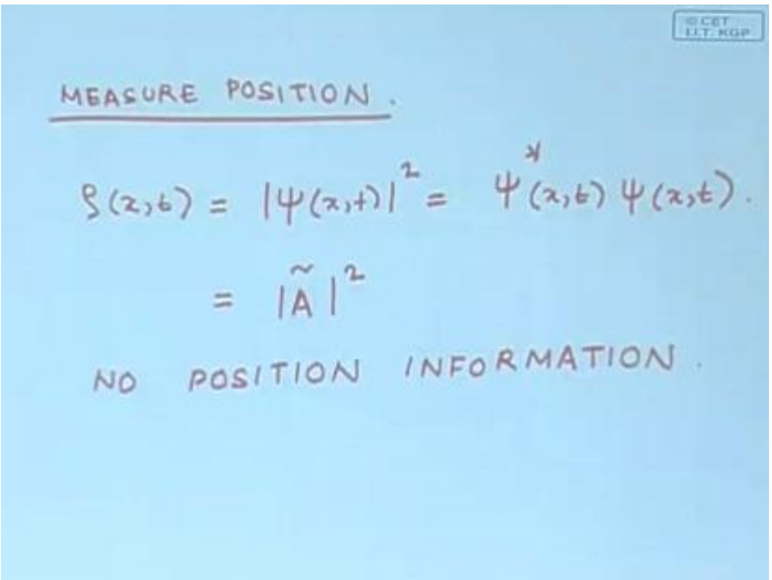

So, this is quite consistent with the Heisenberg uncertainty principle which we have which I had told you about it tells you that. There is a fundamental restriction to the accuracy with which you can determine the position and momentum of the particle of a particle simultaneously determine the position and momentum of a particle and the uncertainty.

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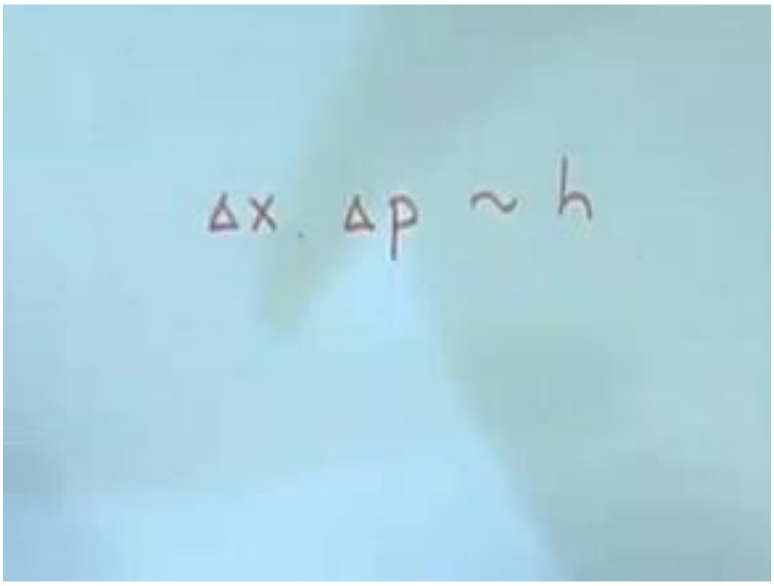

In the position the uncertainty in the momentum the product of these will be of the order of h at least of the order of h may be more could be more than this, but it cannot be very less than this. So, the product the on the uncertainty in the position and momentum is going to be of the order of h in this particular case the wave function that we have seen. There is no uncertainty in the momentum this uncertainty in the momentum is 0 as a consequence. The uncertainty in the position has to become infinite, because the product has to be of the order of h cannot be less than this. So, is this is 0 this has to become infinite so, this uncertainty principle is an, is. So, you see the wave the wave nature is basically I mean if you want to have you if you want the particle to have a wave to have to a to be describe by a wave. It is absolutely essential that it is a consequence basically that if you want to have a wave picture there is going to be an un uncertainty in the position. And in the product momentum right let we illustrate this with one more example consider a situation where we have a.

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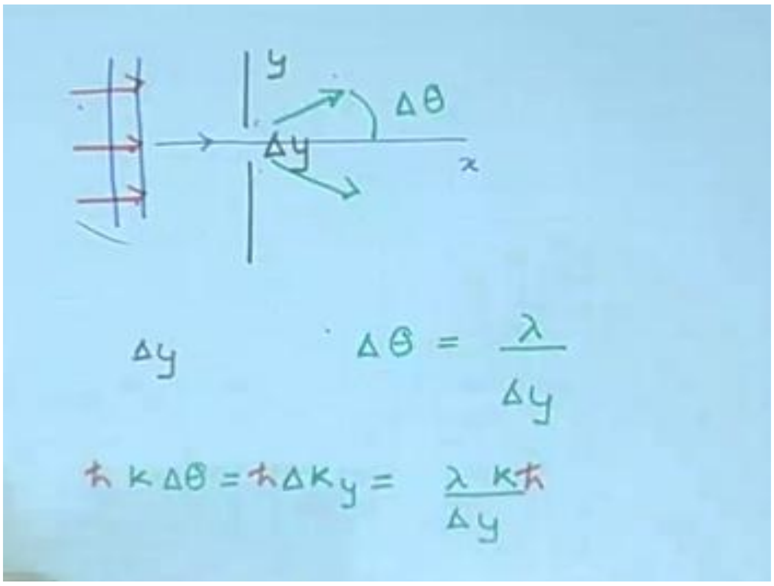

Particle which is incident like this in this direction so, we are supposed to in quantum mechanics we know that we should thing of this as a wave. So, we have a wave propagating like this and the wavefronts now, we know what the wavefronts look like. The wavefront are going to be like this so, what we have is a particle we will call this the x direction what we have here is a particle incident along the x axis. And we are considering all 3 we are considering all 3 directions know.

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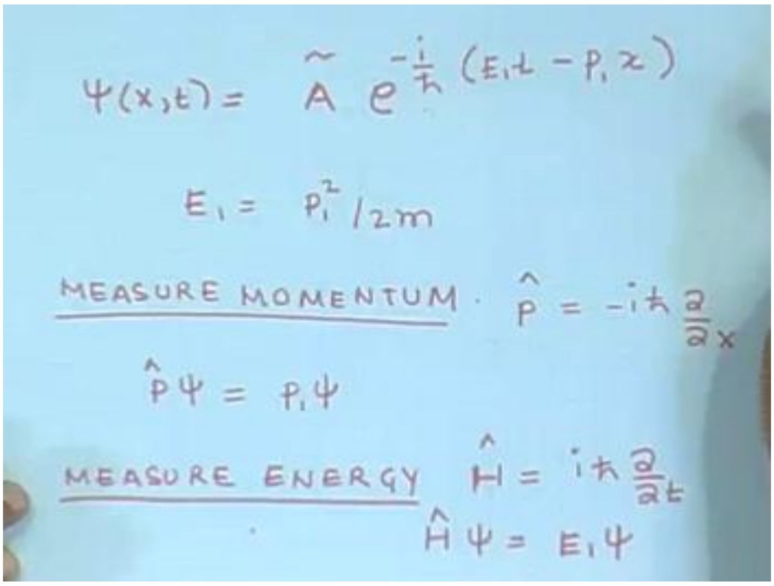

So, we have a particle incident along the x axis the particle is straight by a wave function like this it is now a function of x y and z all 3, but it depends on only on x. So, we have a particle incident along the x axis it has a well-defined energy. It has a well-defined momentum but, we have a absolutely no information about the particles position.

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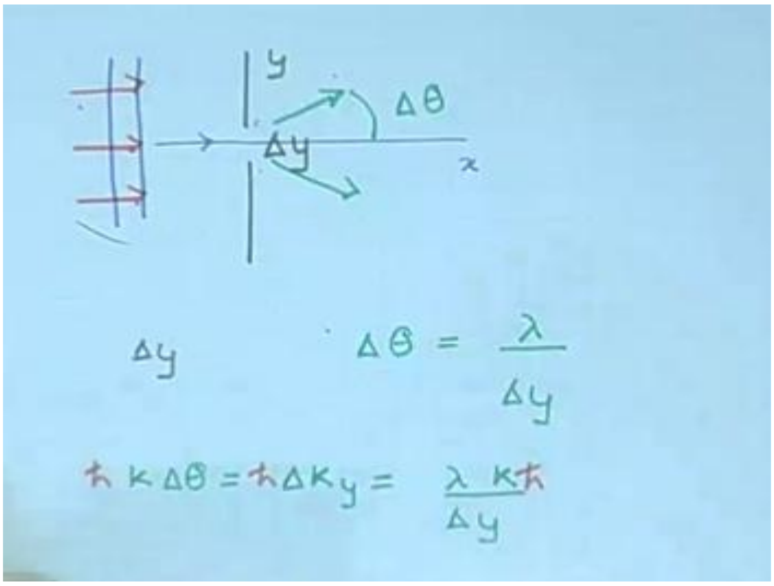

It is equally likely the probability of finding the particle is all the same everywhere so, this particle is propagating along the x axis. Now, what we do is we introduce a slit over here a slit of width delta y so, before the particle is incident on the slit. The wavefronts are now parallel to the y this we will call the y axis this is the x direction the wavefronts are parallel to the y axis. And probability of finding the particle anywhere is the same because it has no why dependence. So, the mod square of psi is constant actually it is a constant everywhere now, what happens when the particle is incident on the slit when the particle emerges. We know that the uncertainty in y is of the order of delta y, because the particle has been blocked here. And here so, we know that when the particle emerges the uncertainty in y has been reduced and it has now of the order of delta y. So, initially there is no uncertainty in the momentum we know that p x has a value the x component of the momentum has a value which is what we have here.

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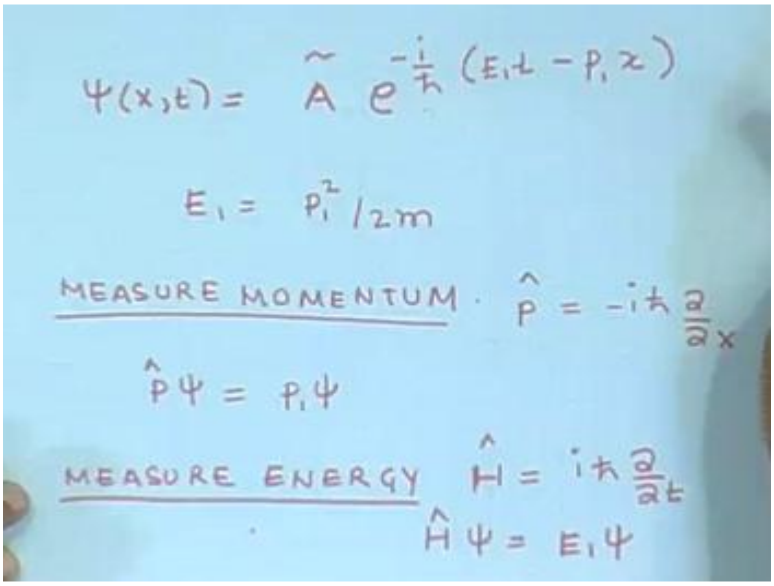

The Eigen value here is the value.

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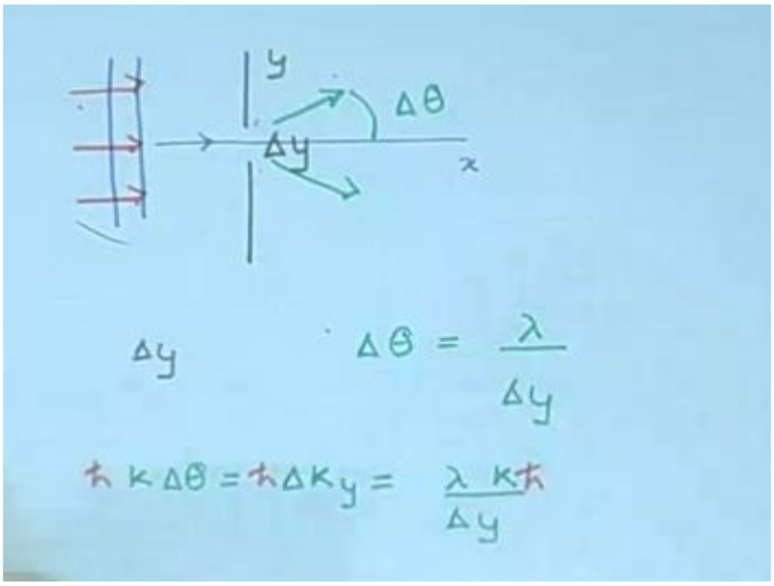

The y and z component have value 0 so, there is no uncertainty in the momentum, but there is an uncertainty in the position it could be anywhere. Now, when this emerges from the slit when it is sent through a slit there is now reduced uncertainty in the y direction in the y position. And the uncertainty in the y position is delta y, but as a consequence of sending this through a slit we know that when we send a wave through a slit. There is going to diffraction and the wave that emerges is going to be spread out over a range of values. And we know that it is going to be spread out over a value delta theta where delta theta is of the order of lambda by the width slit width which delta y.

This is something we have learnt in diffraction so; the wave which was initially propagating along the x axis when it comes out is going to have a spread in direction. The spread in direction delta theta is of the order of lambda by the slit width delta y now if I multiply this by the wave number of this wave. So, k into delta theta is a spread in the wave number in the y direction this is delta k y. And this is equal to lambda k by delta y let me also multiply this whole equation so, this with h cross. And what it tells me is that the spread in the momentum of the particle in the y direction in this direction is this is the spread in the momentum h into delta k y so, the spread in the momentum.

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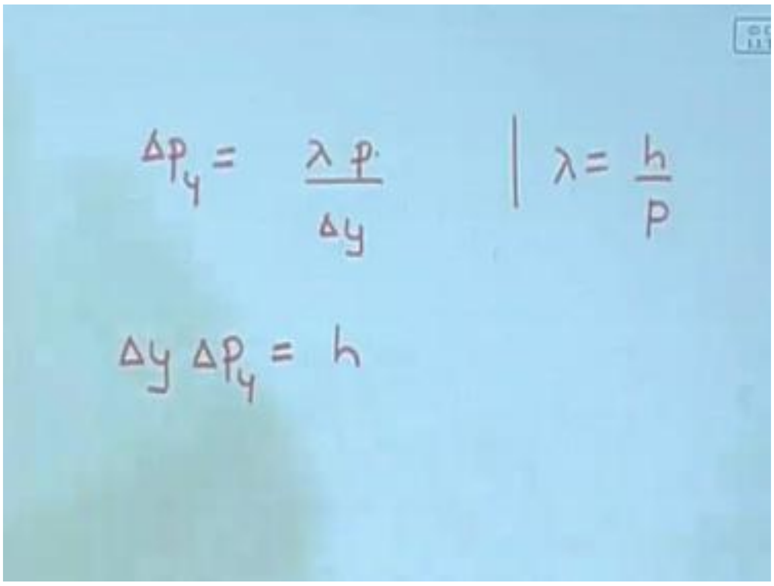

Delta Py is equal to lambda.

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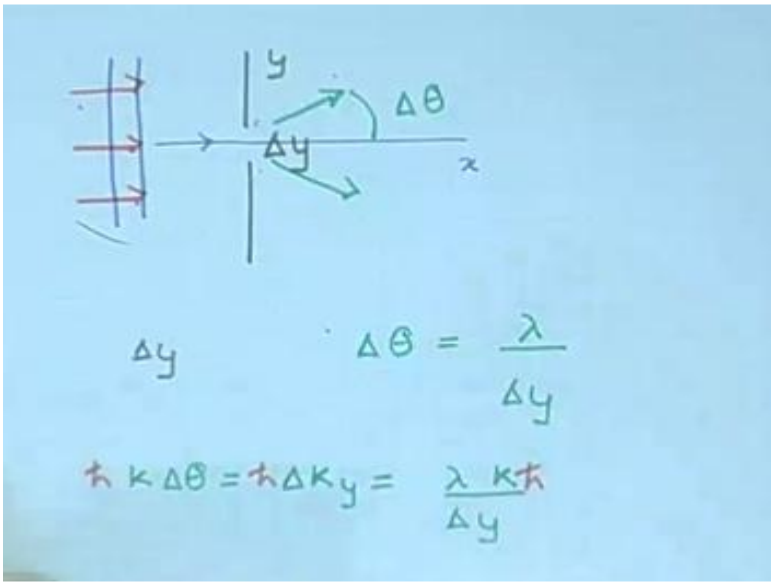

Into the initial momentum k into h cross p divided by delta y;

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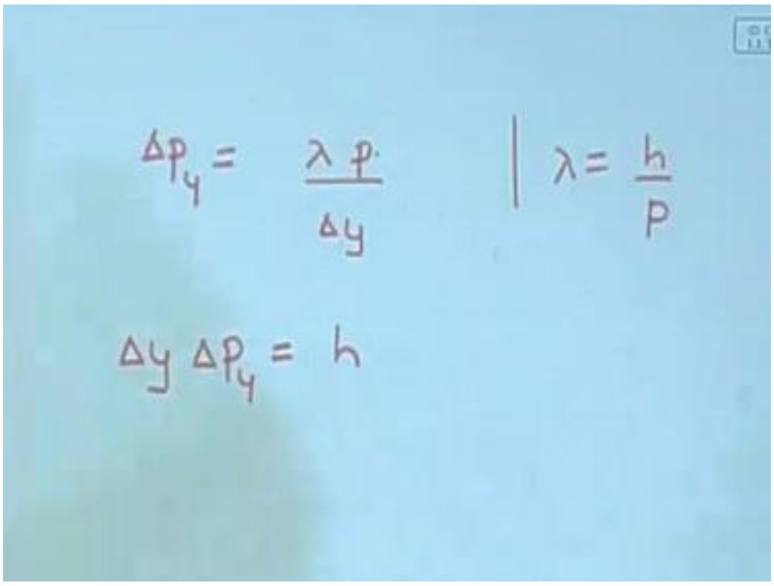

Now, the momentum of the particle and the wavelength the de Broglie wavelength we know are related as follows the de Broglie wavelength is h by p. So, what this tells us is that the uncertainty in the position and the uncertainty in the y component of the momentum is equal to h is of the order of the h. right.

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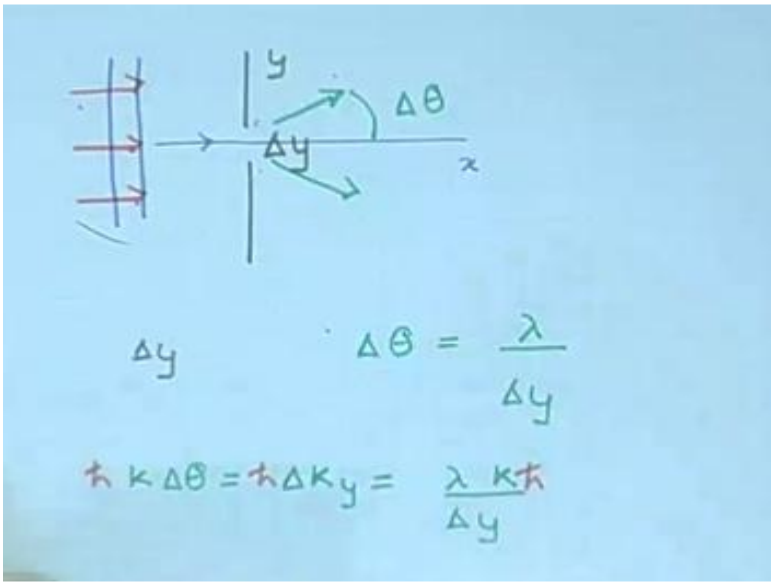

So, what have we seen here we see that if this particle if you represent a particle as a wave initially this wave has no uncertainty in the momentums the momentum is precisely known. But there is absolute total uncertainty about the position because we have no position information. Now, what we have done is we have send the wave through a slit. What the slit is does is it localizes the particle the moment you localize the particle you introduce an uncertainty in the wave vector through, because of the diffraction. And this uncertainty in the wave vector gets converted to an uncertainty in the y component of the momentum.

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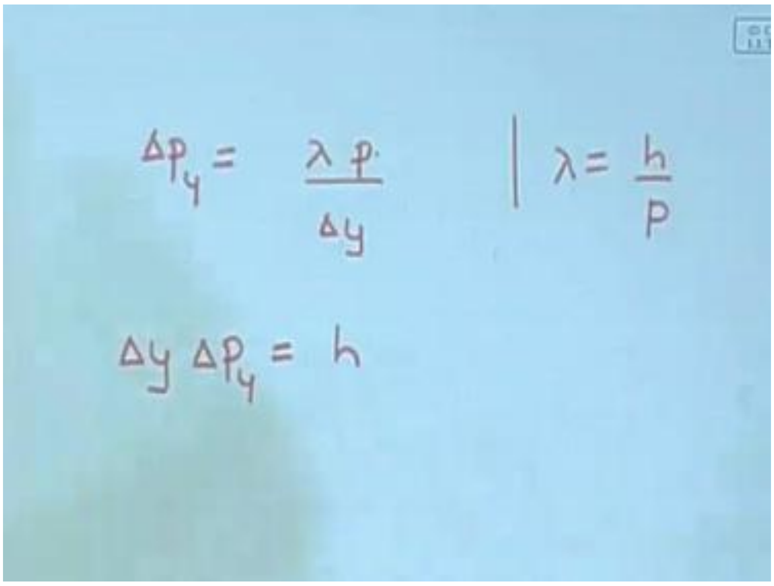

So, you have reduced the uncertainty in the y component of the y position of the particle, but, at the expense of introducing an uncertainty in the y component of the momentum. And the product of these two has to be of the order of h that is what we see so, this is a consequence of the diffraction. So, what we find is that this uncertainty the fact that the product of the uncertainty is cannot be below a certain number is a direct consequence of the fact that you have a wave you are representing the particle by a wave. And if you want the wave picture to be consistent this kind of an uncertainty relation has to be valid right. So, we see that the uncertainty, the uncertainty relation and the wave nature are at both are both have to be if you have the wave picture. Then there should be uncertainty relation you cannot determine the position and momentum of the particle both to arbitrary level of accuracy simultaneously right. So, let us know go back to the point which were discussing the point being what happens when we make a measurement.

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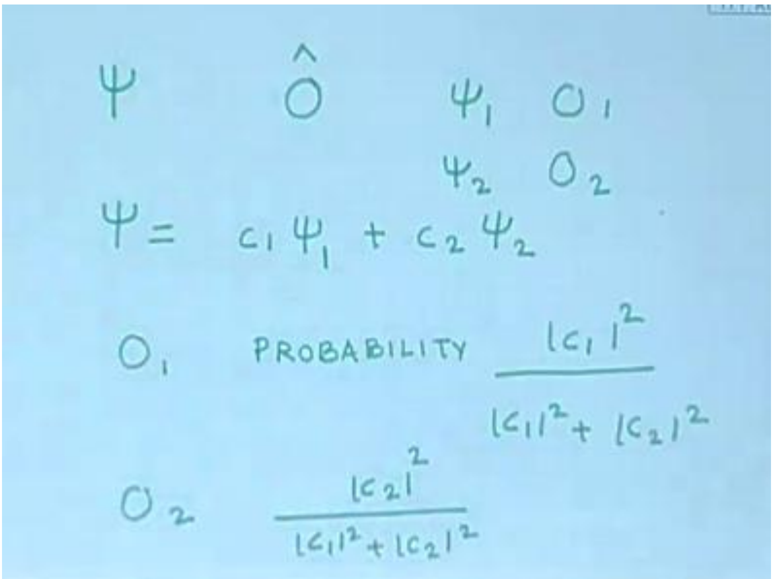

So, if I let me know consider a general situation if I have some A particle in a state where it has a wave function psi. And I have an observable o which is represented by an operator o. So, I am making a measurement on this wave function I am measuring a quantity corresponding to which there is an observable. There is an operator o then if I make this measurement the outcome of this measurement is going to be one of the Eigen values of this operator. And if I can represent this psi as a superposition of different Eigen functions of this operator which I can always do it is a theorem that can always represent any arbitrary wave function in terms of Eigen functions of this physical observable. Then if I can represent it as say c 1 psi 1 plus c 2 psi 2 psi 1 and psi 2 are Eigen functions of this operator psi 1 has an Eigen value o 1. So, psi 1 is an Eigen function of this operator with Eigen value o 1. Psi 2 is an Eigen function of this operator with Eigen value o 2. So, we are considering a situation where there is a particle in a state psi which can be represented in terms of these 2 Eigen functions of the operator o.

In general there may be 3 4 5 you may require many more Eigen functions of this operated to represent this wave function, but you can always represent it as a superposition of Eigen functions of this operator the general situation. So, if in this particular case I can represent psi as a superposition of 2 Eigen functions 1 with Eigen value o 1 another with Eigen value o 2 when I make a measurement of this observable I will get either o 1. And the probability of getting o 1 is c 1 mod square by c 1 mod square plus c 2 mod square and the probability of getting o 2 is similarly c 2 mod square divided by c 1 mode square plus c 2 mod square that is the general situation. Let me know consider a different question again to do with measurement. And the question is as follows the situation is as follows suppose I have a particle in a state psi.

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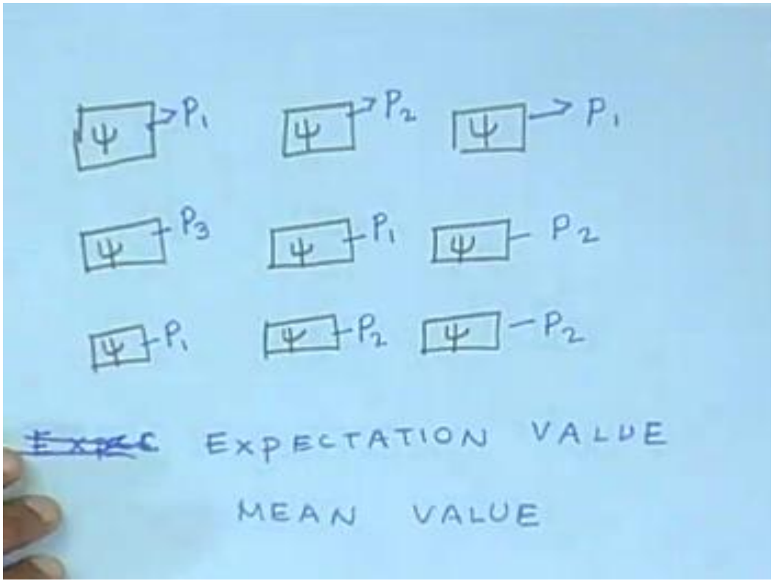

And I have many replicas of this particle. So, I have many replicas of the particle I will draw few of them. And we the question the situation that we are going to consider is that I have this particle many replicas of this particle all in exactly the same state psi and what I do is I measure the momentum of the particle. And this state psi is not an Eigen function of the momentum operator. So, when I make a measurement of the momentum let us say that I get a value p 1 here. If I make a measurement of the momentum I will get I could get a different value p 2 I will get the say the first value p 1 here I could get a different value p 3 here I get p 1 here I get p 2 p 1 p 2 p 2. So, the situation that I am considering is as follows I have particle which is in a certain state psi I have many replicas of this particle all of them are in exactly the same state psi.

Now, I go and measure the momentum of the particle is exactly identical and I have many replicas of it, but it is not an Eigen the state psi is not an Eigen function of the momentum. So, I cannot predict exactly what outcome I am going to get what I can predict is this probability of different outcomes the outcome is going to be an Eigen value of the momentum operator. So, if I do the experiment I can predict the what the probability of getting a particular Eigen value is and suppose I do it then I will get one of the Eigen value. So, here I get p 1 a possible Eigen value of the momentum operator here I get p 2 which again is an out possible out Eigen value of the momentum operator here I get p 1 again here I get p 3 etcetera.

Now, the question is what is the expectation value or the mean value of the momentum or the mean value? So, if I do the experiment if I have many many replicas I can predict what the mean value I can calculate what the mean value is going to be if I get p 1 10 times if I get p 2 3 times if I get p 3 4 times. I will multiplied 4 into p 1 plus whatever 10 10 into 4 p 1 plus 4 into p 2 etcetera and divide by the total number that will tell me the mean value. The question is how can I predict what the mean value of the momentum should be from the wave function? So, the way to calculate the mean value or the expectation value the value that expect to get if I do the experiment only once the way I calculate that.

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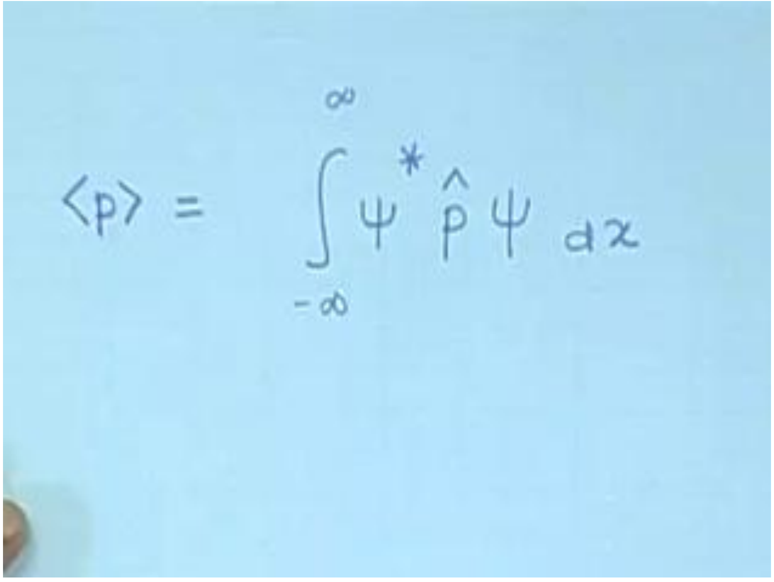

This is what we call this the expectation value or the mean value is through this integral minus infinity to infinity psi star. The momentum operator p into psi dx this integral gives me the makes a prediction for what the mean value is going to be.

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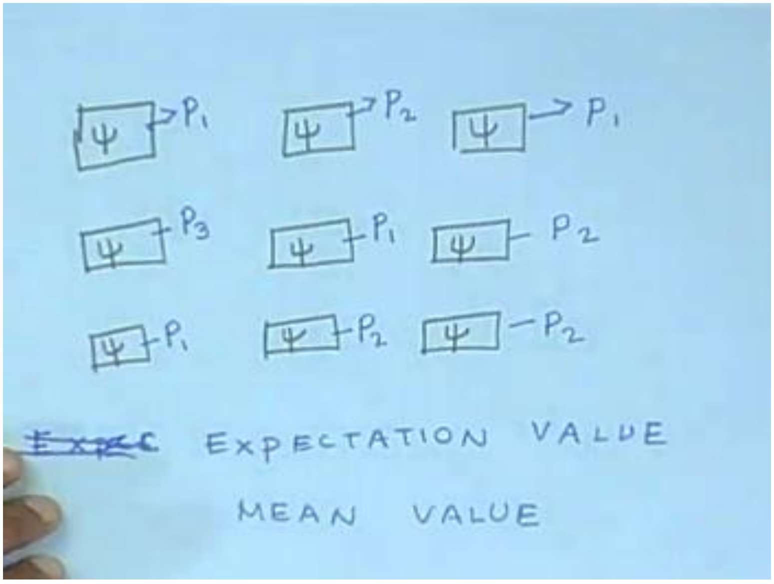

I can now do the experiment have many replicas of this particle in the state psi. And then measure the value of p that I get in all of these experiments. And then calculate the mean I am going to get p 1 some number of times p 2 some number of times p 3 some number of times. So, I am going to get all the possible Eigen values some number of times and then I calculate the mean.

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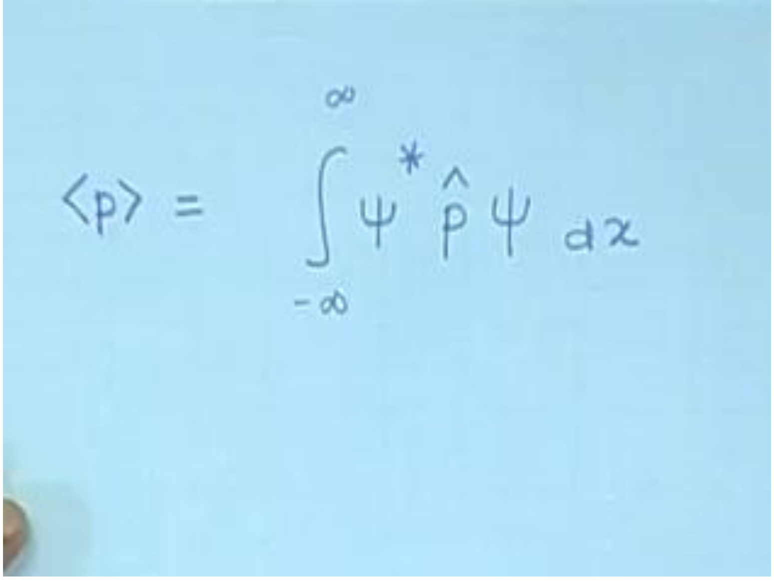

I can predict what the mean should be from the wave function by doing this integral. So, the integral that I have to do is as follows. I have the wave function psi which the particle that is the state in which the particle is I act with the momentum operator remember the momentum operator is minus i h cross del by del psi i multiplied by psi star. So, this is a function of x I have to integrate this from minus infinity to infinity. This integral is going to give me the the mean position or the expected position of the particle. Let me stop today's lecture over here and take up the question of how to calculate the uncertainty in the momentum and related issues in the next lecture.