Physics I : Oscillations and Waves
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Lecture - 37
Schrodinger Wave Equation

Good morning. We have been discussing, the behaviour of microscopic particles like electrons. And we considered a situation where we had an electron beam incident on 2 slits and we were looking at the pattern of the arrival of the electrons at a distance screen. And what we found was that the arrival pattern of the electrons on the screen, look like an interference patterns. And based on this to in order to explain this it was necessary to invoke a wave associative wave with every electron. And what happened was when this wave encountered the 2 slits each slit now act like a secondary source. And if you wish to calculate the resultant wave at any point on the screen and it was the superposition of 2 contributions.

So, what we found was that, it was not possible to say that the electron arrives at the screen through either slit 1 or 2 slit 2. And you have to admit the possibility that the electron basically goes through both the slits at the same time, because the wave corresponding to the electron, passes through both the slits. So, the bottom line of all these discussion was that you have to give up the picture where you can think of these microscopic particles. As particles which we are familiar with particles in the familiar world can be thought of when they move from one position to another. You can associate a trajectory with them. You write down the Newton's equations of motion.

And you can solve them and it finally, at the end of the day you have the particles position at every instant of time. As it travels from one point to another, but we found that for microscopic particles like electrons you have to think of it in terms of a wave propagation. And then as a consequence you have all the phenomena like interference diffraction associated with waves also occurring for such particles. I also told you that this wave which you associate with the particle you interpret as being the probability amplitude the modulus square of this wave. So, this wave are the wave function.

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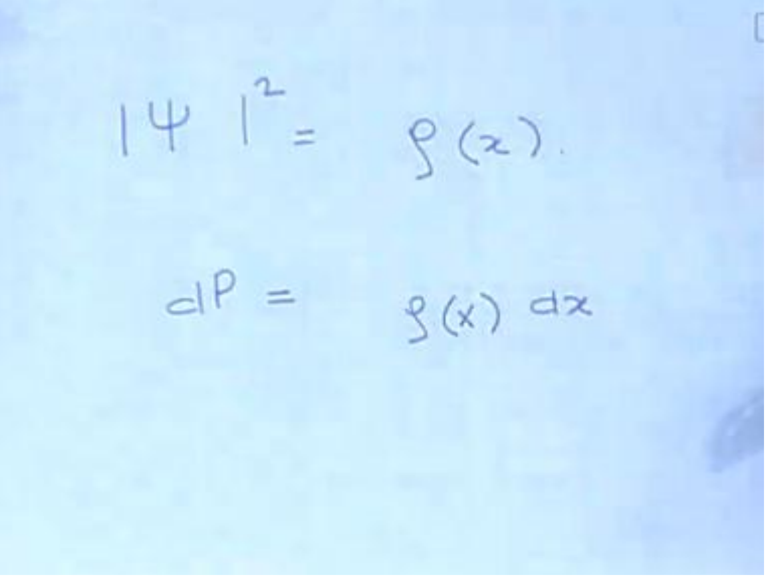

Psi the mod square of this gives the probability density of finding the particle somewhere this is, what I told you in the last class. So, if you ask the question what is the probability of finding the particle, along the interval dx in the interval dx cantered at the point x on the screen this will be given by. The probability density into the interval dx and the probability density can be calculated from the wave psi by taking its modulus and then squaring it. So, this psi is the probability amplitude.

And this is the probability density the modulus square of it gives a probability density, whose interpretation we have discussed in the last class. Now, in today is lecture I am going to tell you the basic rules of quantum mechanics. The previous lectures have all been motivation and interpretation of the wave function and motivation why we need to think of particles as having as in terms of waves. So, today we are going to go into the mechanics of these waves. So, this is what is called quantum mechanics.

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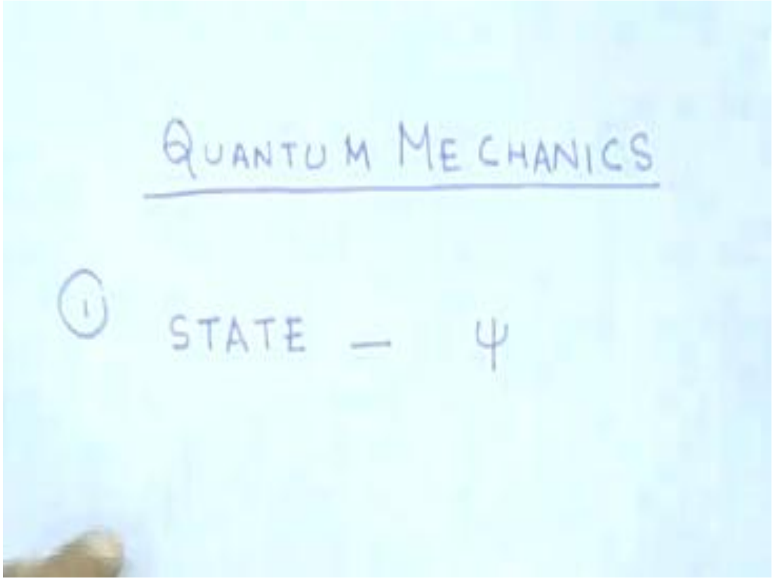

Or wave mechanics and the first postulate of quantum mechanics as I am going to present them to you states that for every possible state of a particle corresponding to every possible state of a particle. There is a different wave functions psi associated with it. So, an electron for example, could be in one of different possible states it could be many possible states and at any time it could be in 1 of them. And or there could be many possible states in which you may find the electron. So, for corresponding to each of these states there is a different wave function psi. So, corresponding to every possible state of the of a microscopic particle there is wave function psi. And this wave function is governed by a wave equation know as the Schrodinger wave equation. So, let me also write down the Schrodinger wave equation.

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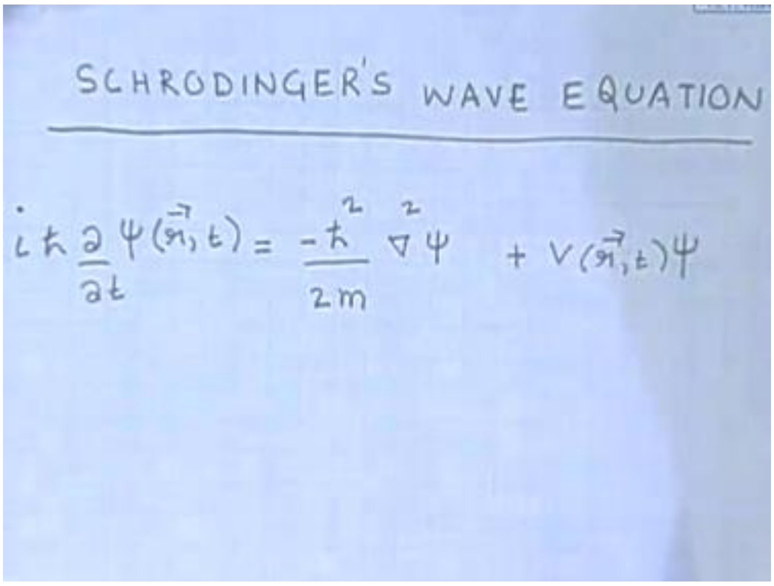

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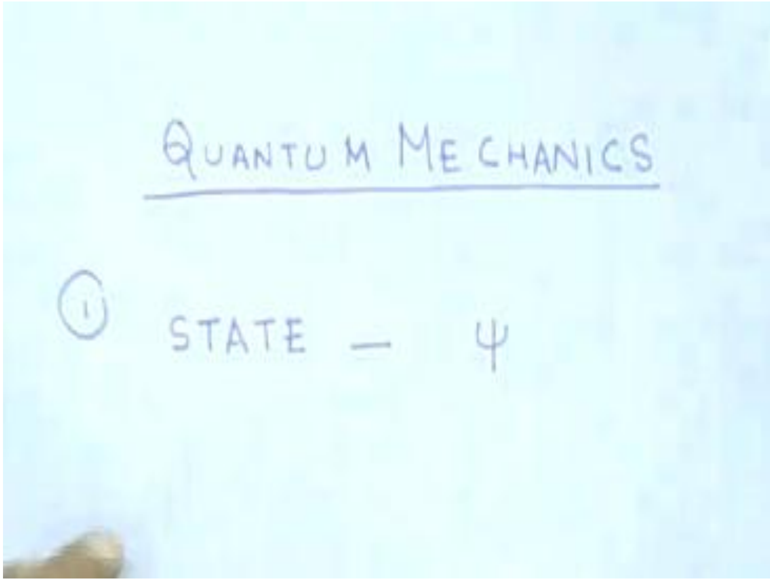

So, the Schrodinger wave equation is the equation is the wave equation governing the evolution of this psi. So, let me remind you once more the first postulate is that corresponding to every state of your system. Say you are dealing with electron corresponding to every possible state of the electron there is a different wave function psi.

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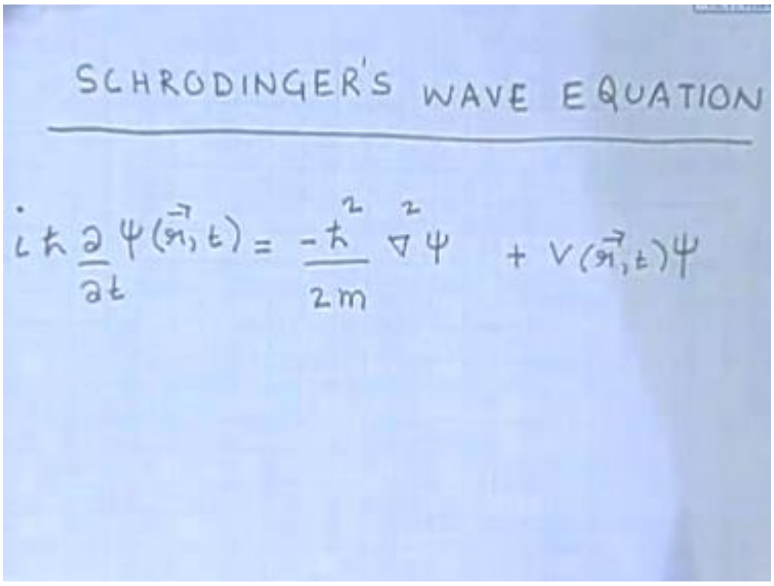

And the evolution of these wave functions is governed by the Schrodinger wave equation, and let me write down the Schrodinger wave equation. So, for a particle moving in which can move in all 3 dimensions. The wave equation is i h cross del by del t psi now psi could be a function of r and t this is equal to minus h cross square by 2 m, where m is the mass of the particle. So, if it is an electron it is the mass of the electron Laplacian minus h Cross Square by 2 m Laplacian of psi plus v which again could be a function of r and t into sigma. So, this is the Schrodinger equation. So, this term over here is the partial derivative with respect to time. This term has spatial derivatives the Laplacian which we have already encountered earlier has spatial derivatives in it.

Now, when you have a particle under the influence of an external force which is the situation quite often for example, if you have an electron the electron could be an external potential. Under the influence of an external electric field static electric field could be represented in terms of a potential. So, if I have a uniform electric field I am sure we all know that there is a corresponding electrostatic potential. So, any motion any force. So, when we do classical mechanics we the force forces can be represented as gradients of potential under some conditions. So, this external force action of the external force on the particle comes in here through the potential v. So, if I have a particle in an external potential.

So, then I have to I have I have to also include this term this term is the potential the gradient of the potential minus the gradient of the potential gives me the force acting on the particle. So, when I thinking of this say a macroscopic particle and I think of it in the usual classical picture. I would write down the equation of motion for a particle moving in an external potential. And for a macroscopic particle I would write down the equation let me just remind you what we are talking about.

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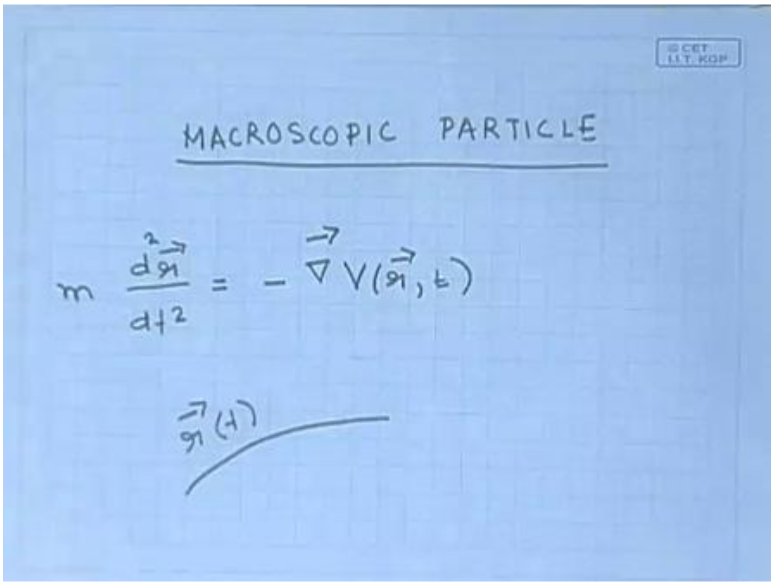

So, for a macroscopic particle say a cricket ball or something like that. So, if I had a microscopic particle under the influence of an external force. Then I would write down an equation which could be that the force if the force could be represented in terms of a potential if it for a potential force. Then usually the force is the minus the gradient of the potential, for potential forces that is how you introduce the potential.

So, then I would write down the equation of motion for the macroscopic particle in terms of the potential m into the acceleration is the force which is minus gradient of the potential. And I would solve this equation of motion which would give me the trajectory of the particle whereby I would know the position of the particle at every instant of time for this under the influence of this external force. Now, if you are dealing with microscopic particles we have to abundant this picture where the particle moves in a trajectory. And we have to describe it in terms of a wave.

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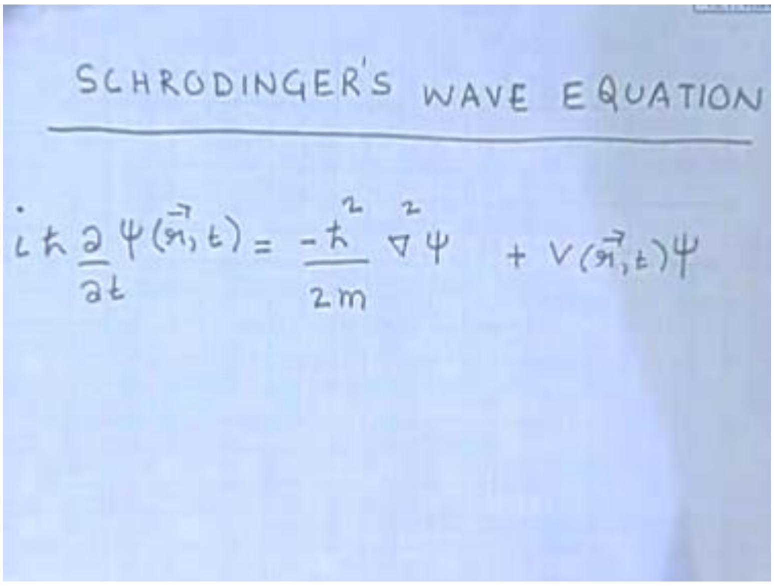

And the evolution of the wave is going to to be governed under the in the presence of the same potential. The evolution of the wave is now going to be governed by this wave equation which tells me how the wave is going to evolve under the influence of that external force. Remember the force is the minus the gradient of this potential. So, I have to now replace the trajectory I cannot I can no longer thing of the particle in terms of a trajectory. I have to think of it in terms of a wave the evolution of the wave. And the mod square of this wave gives the probability amplitude. If I can calculate this wave at different positions the modulus square of that gives me the probability of amplitude this is some which we have already discussed.

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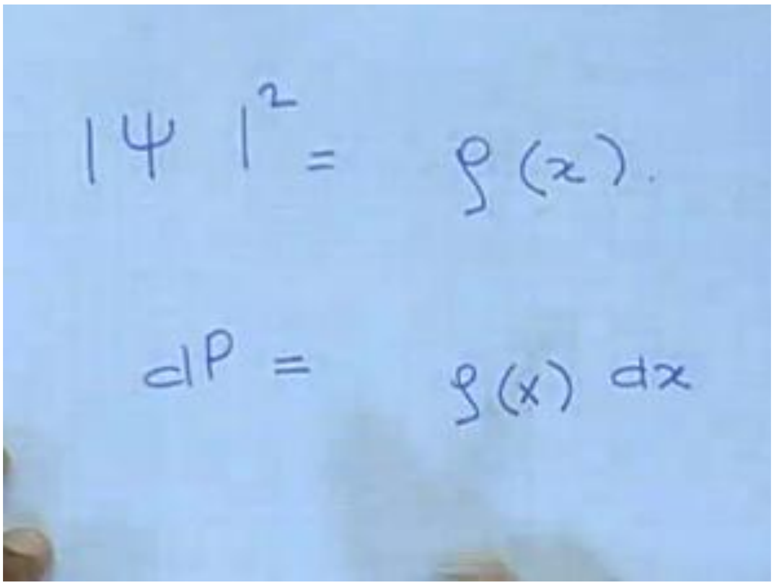

The modulus square of the wave function tells me the probability amplitude. So, once I know the wave function I can tell what is the probability of finding the particles?

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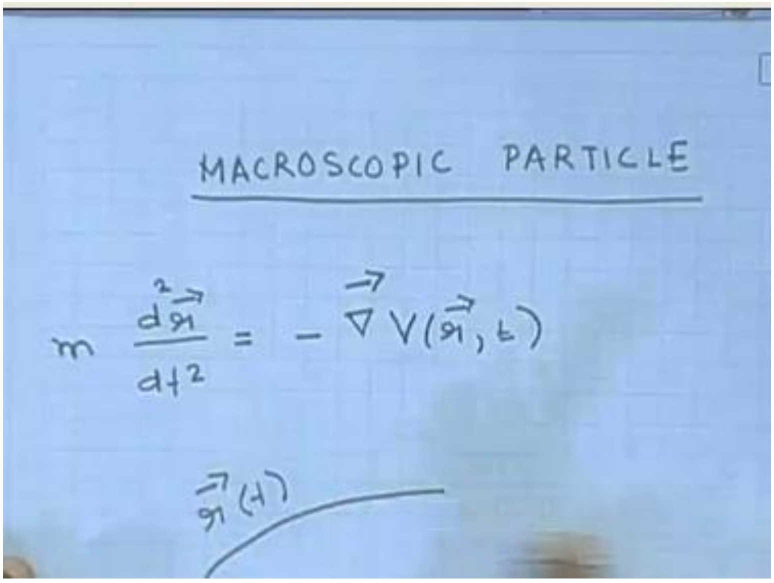

Somewhere, I cannot tell exactly where the particle is like I can do for macroscopic particles, for which I can calculate trajectory this has to be abundant.

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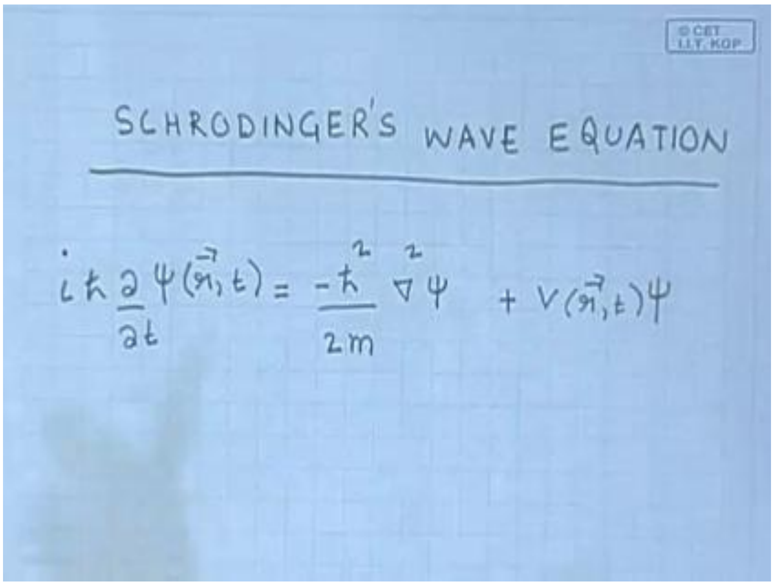

When we are dealing with microscopic particles which we have to think in terms of waves and the evolution of the wave is governed by this wave equation. So, this is a time derivative term which occur in the wave equation this has spatial derivatives and this is the term that arises due to any external influences which we can represent through a potential. So, this is the Schrodinger wave equation.

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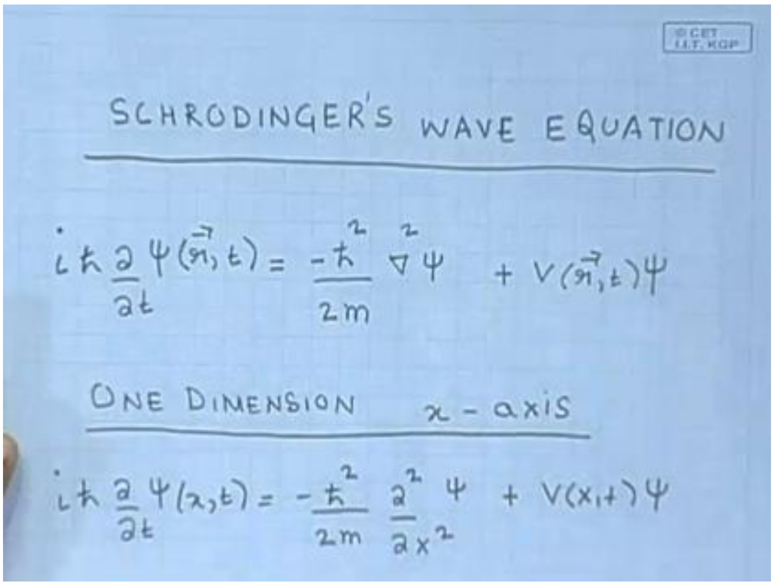

Now, we are going to restrict in this course, we are going to restrict our attention to particles which are free to move only in 1 dimension along the x axis. So, this simplifies the discussion that is all, but from this simplified discussion we can get a clear picture of what would happen if we were to deal with particle in 3 dimensions. It simplifies the mathematic analysis that is why we are restricting our attention to particles which can move only in 1 1 direction as are on the x axis under this simplification. The Laplacian now is just partial derivatives with respect to x that is the only change which occurs psi is a function of x alone and the potential v is also a function of just x. So, the equation now becomes i h cross del by del t psi which is now function of x and t is equal to minus h cross by 2 m partial derivative with respect to x second partial derivative. Second derivative with respect to x plus v, which could now be a function of x and t into psi. We will also make another simplifying assumption we will assume that v.

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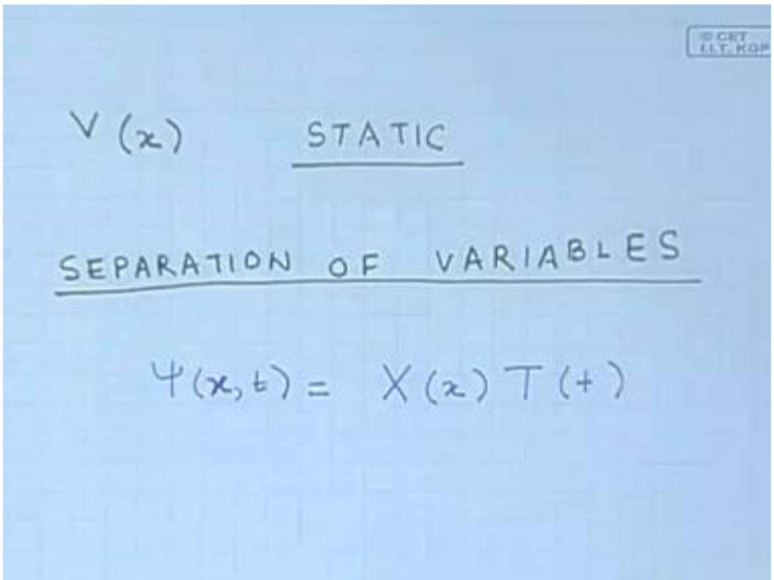

V is time independent. So, v is a function of x alone that is it is static it is a static static potential. So, for example, if I have an a charge particle in an external electric field which is static for example, the uniform electric field we know the potential is proportional to the distance x. We have a static potential, but suppose I have a charge particle let us say an electron bound in an atom and we shine light on this atom. Now, we have discuss that light is essentially an oscillating electric and magnetic field. So, we have a time dependent external force the time dependent external electric field. So, this a situation that is not static the analysis of such situation is more complicated. We are not going to deal with such situations in this course, we are going to deal with situations where the potential is static it is just a function of x and it is not dependent on time. So, under such conditions we will now proceed to look for solution under such conditions.

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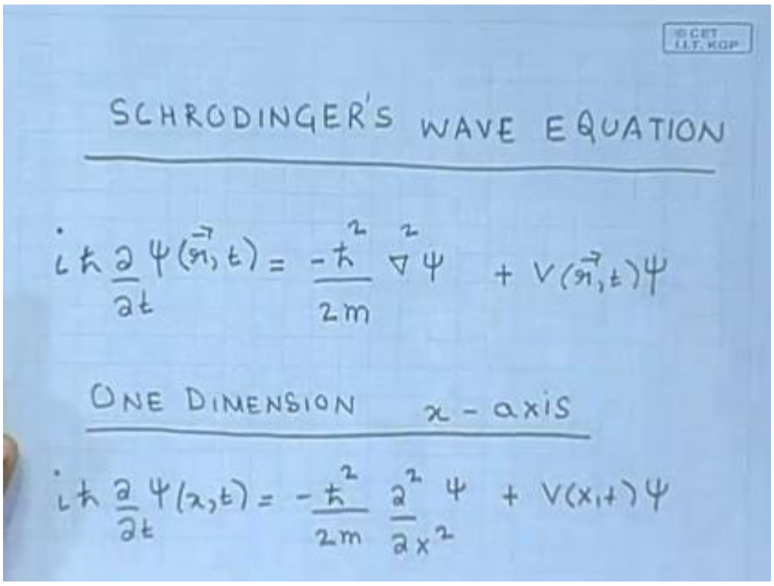

And we are looking solutions to this equations where x is v is now just a function of x not of time.

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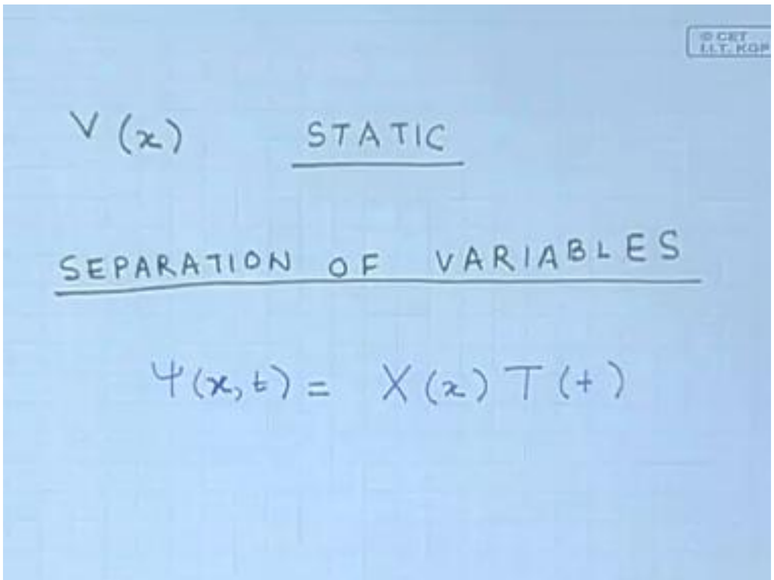

So, we going to use the method of separation of variables which is familiar. Remember we have introduce this method when we were discussing standing waves. And what you do is we have to we can we will take a trail solution of the form where psi x t is a function of is product of 2 functions capital X which is a function of the position alone and capital T which is a function of time alone. So, we will take a trial solution like this this recollect that this is the method of separation of variables. So, we are going to put a trial solution like this into our wave equation.

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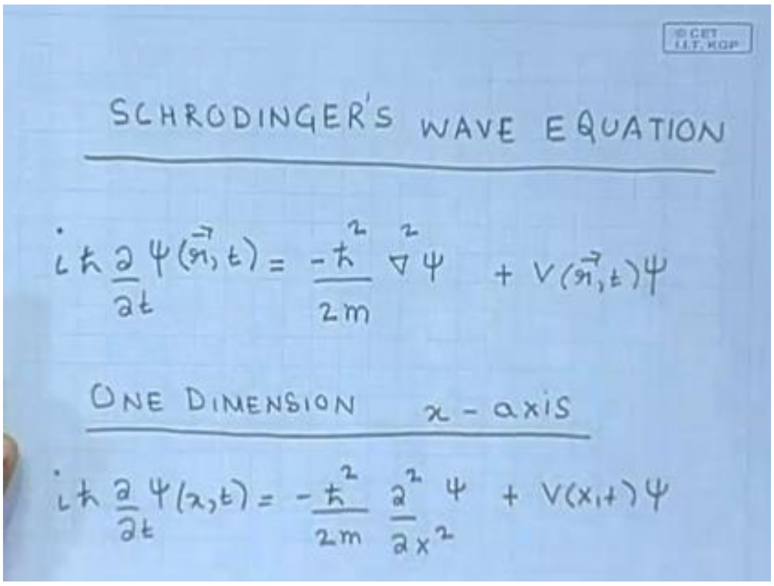

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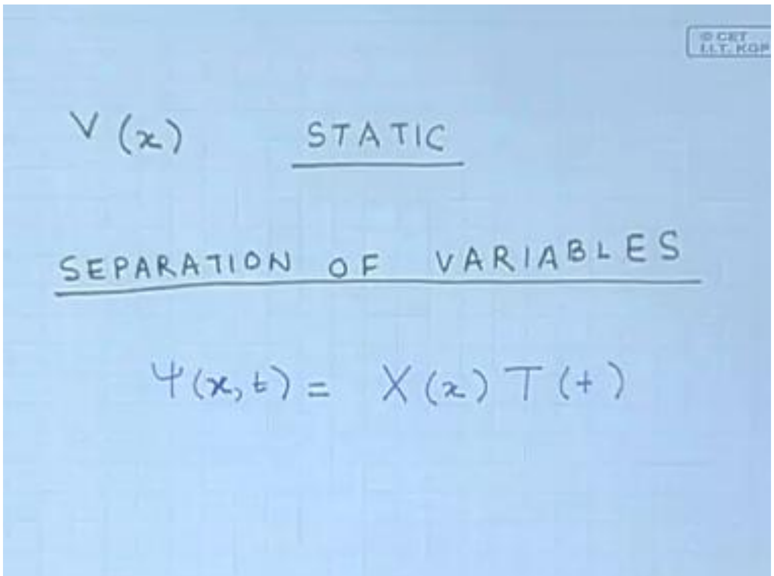

Now the partial derivative of this psi with a partial derivative with respect to time is going to act only on this function and it is going to be now a total derivative.

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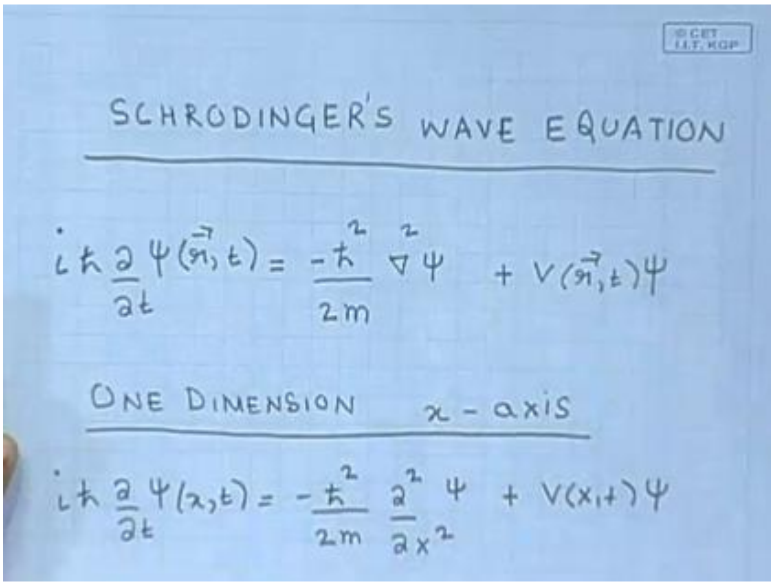

So, the equation the Schrodinger equation in one dimension now reads as follows. So, I have to replace this with x into t and the x will go out I will have the time the function of time left over here.

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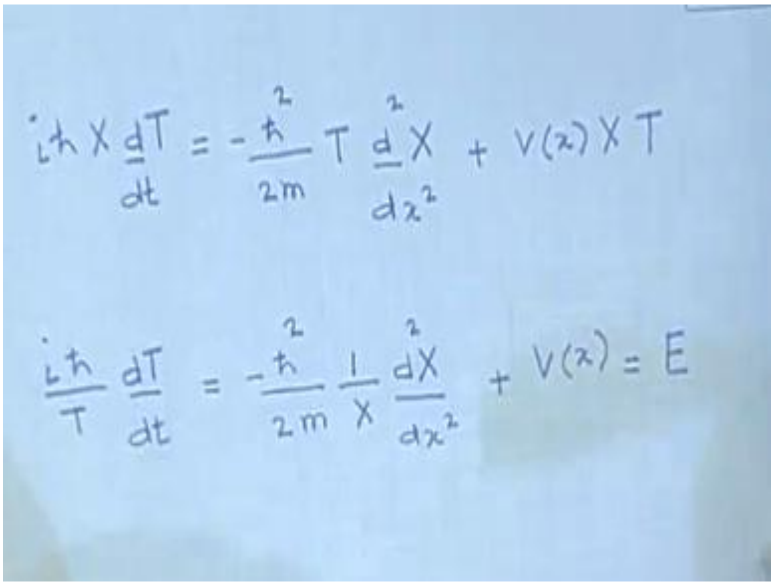

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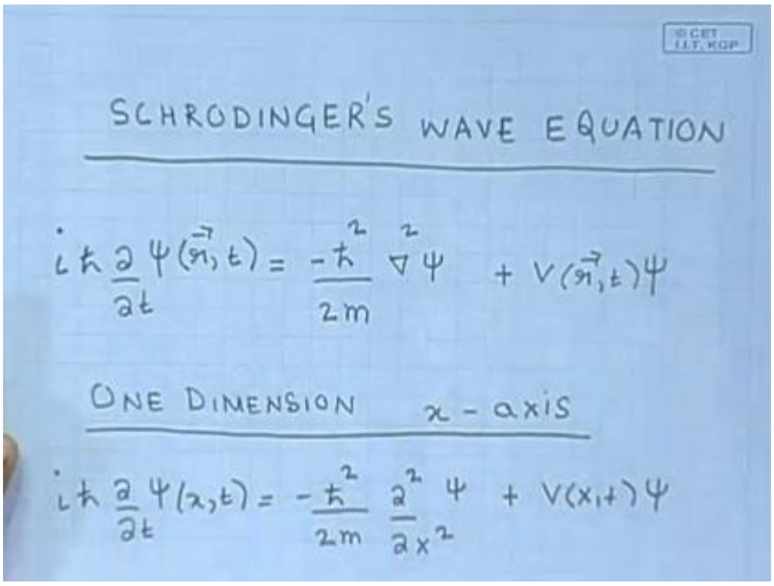

So, what I have is I h cross x d by dt of capital T that is the first term again I replace the this psi as a product of capital x and capital T. Capital T will now go outside because this is the derivative with respect to x. And what it gives me is…

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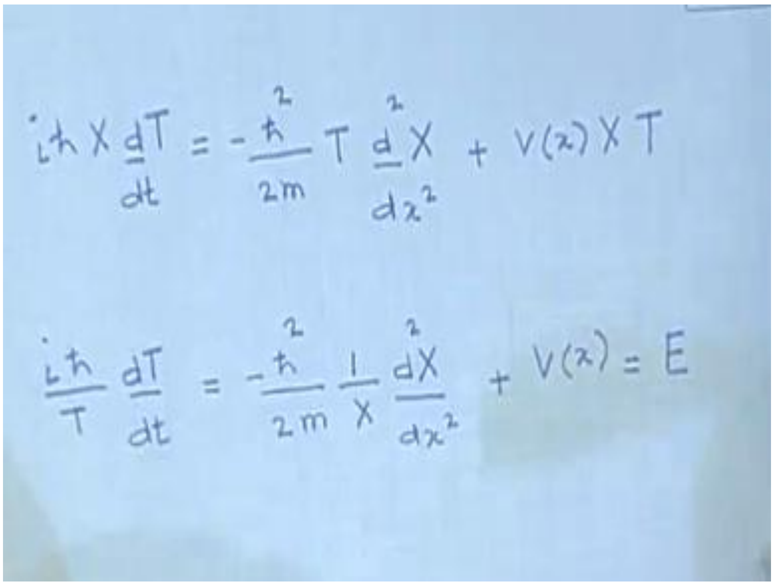

This equal to minus h cross square by 2 m capital T now comes out d square dx square of capital X and I will do the same replacement is here where is going to be no change.

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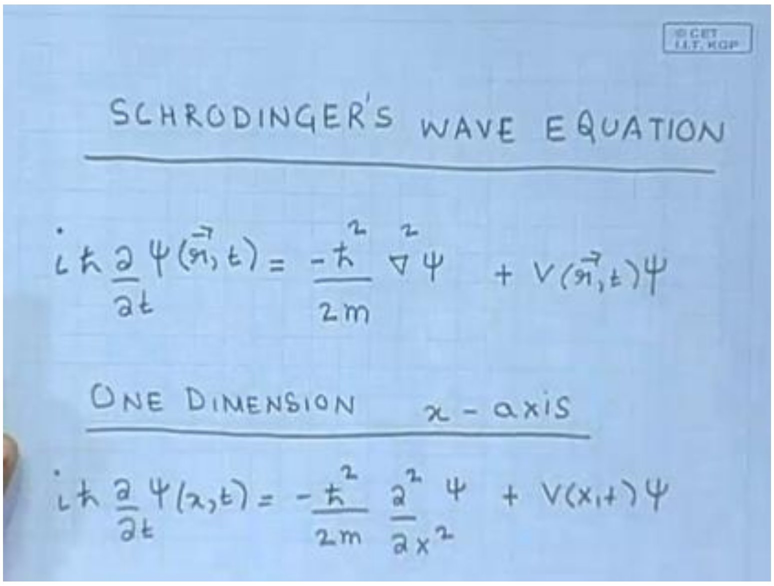

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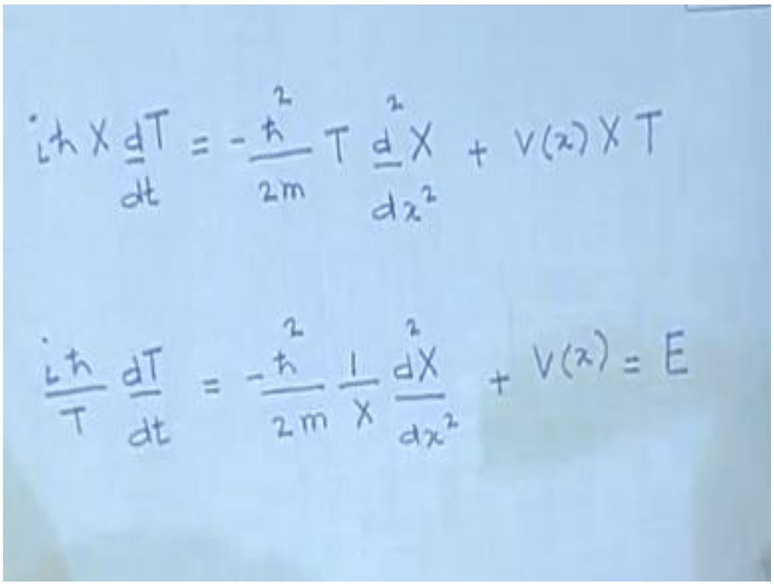

V which is just a function of x now into capital X into capital T. So, this is the Schrodinger equation with the separation of variables put in. Now, what we do if you remember we have to divide this equation complete equation by psi which is X into T. So, when I divide this equation with X into T. If I divide this term with X into T X cancels out and what I have is i h cross I am dividing it by X and T. So, 1 by T remains the d by dt of capital T if i divide this by X into T capital X into capital T what I have is minus h cross square by 2 m. I am dividing by capital X and capital T. So, capital T will cancel out I will have a capital X left. This is and this term will give me this v which is a function of x alone I am dividing by capital X and capital T.

Now, notice that the left hand side is a function of time alone the right hand side is a function of x alone. I am free to change time without changing x I am free to change x without changing a time the fact that this equality must hold. Basically tell us that these 2 terms must be equal to a constant and I am writing that constant as E. So, we have used the method of separation of variables and it tells us that these terms should separately be equal to a constant we have to now solve these equations. Let us first look at the time part of this equation. So, the time part means that we have to look at this term which is equal to a constant E. What this gives us [Vocalized Noise] is this differential equation.

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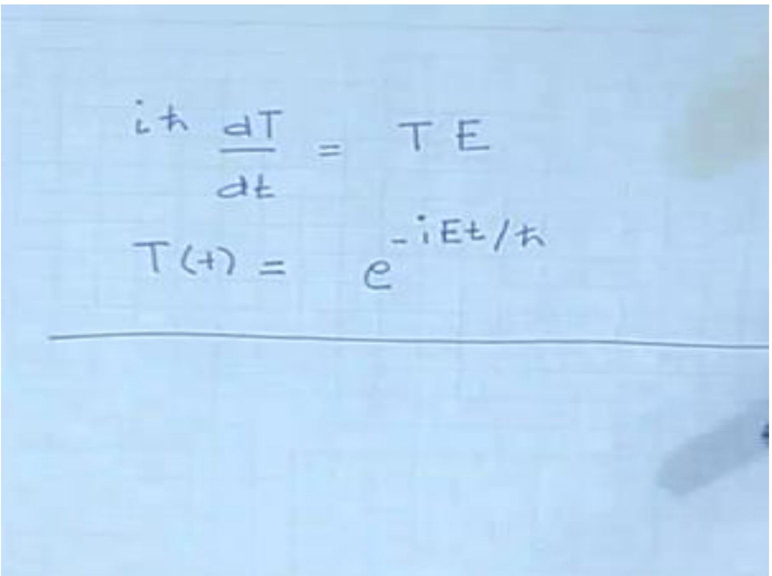

i h cross this equal to T times E where E is a constant this capital T is a function of time. Now, the solution to this is easy to write down it the solution is T as a function of time. Capital T is the function time is equal to there could be a constant which I am not writing down and overall constant into e to the power minus i E t by h cross. And you can easily check that if you differentiate this once with respect to time you will pull out a factor of minus i E by h cross minus i into plus i will give me 1. You have a one by h cross will cancel out this h cross here and i will get E. So, what this if i differentiate this i will get exactly the right and side. And there could be a overall constant which I have not written down this is the time part of the solution. Now, let me write down the the the spatial part of the solution. So, when dealing with the spatial part of the solution it is convenient to introduce a constant another constant which is define like this.

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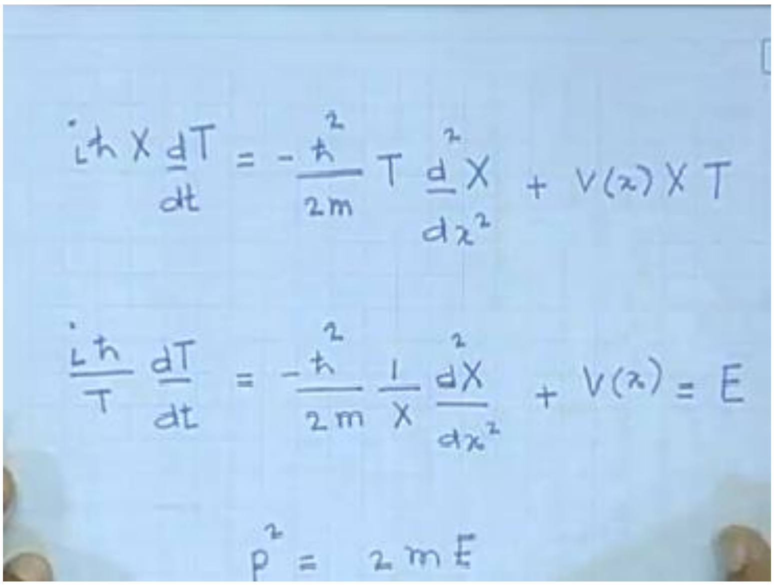

So, we are going to introduce another constant P square which is 2 m into E. So, we are going into introduce another constant P square which is 2 m into E. We are now looking at the spatial part ho before that. we are now going to focus our attention on a particular situation where we have a free particle.

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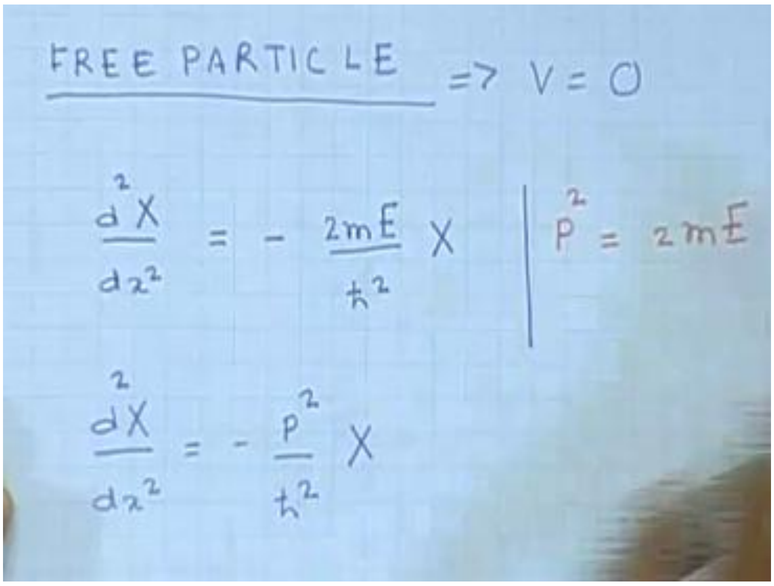

Let me first make this point. So, the first situation that we are going to consider is where we are dealing with the free particle. So, for a free particle the potential v…

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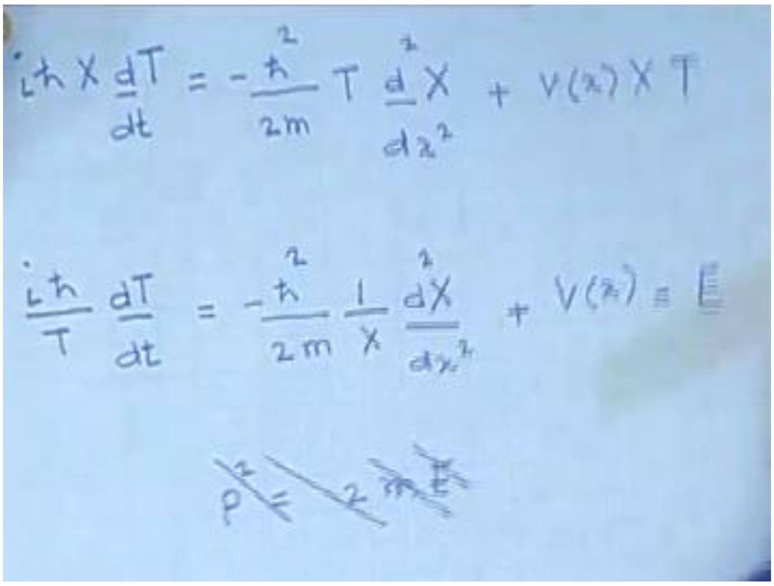

Let me put this elsewhere this need not come here. So, we are going to deal with the free particle and see what kind of solution. This Schrodinger equation gives us remember a free particle refers to a particle on which there is no external force. So, this kind of a particle has no potential is as no potential acting on it. So, this potential v is going to be set to 0 if i set the potential v to zero the spatial part of this equation now becomes.

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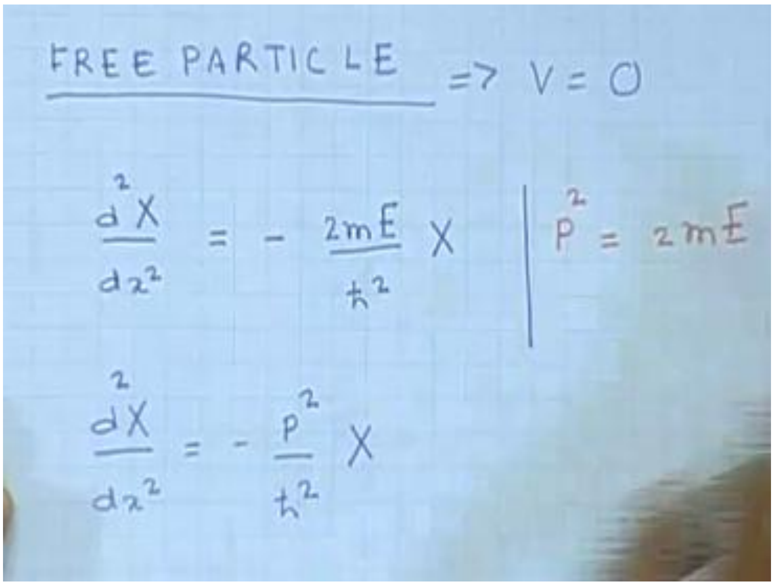

D square capital X dx square is equal to minus 2 m E by h cross square into X. Now, it is convenient to introduce a new variable let me put it here. So, it is convenient to define a new variable. So, define this numerator to m E as a new variable P square equal to 2 m into E. So, with this new in terms of this new variable P the equation governing capital X. Now, becomes d square X dx square is equal to minus P square by h cross square into X, and this has to 2 solutions basically this we know that this is the simple harmonic oscillator equation. And the solution to this is quite straight forward to write down.

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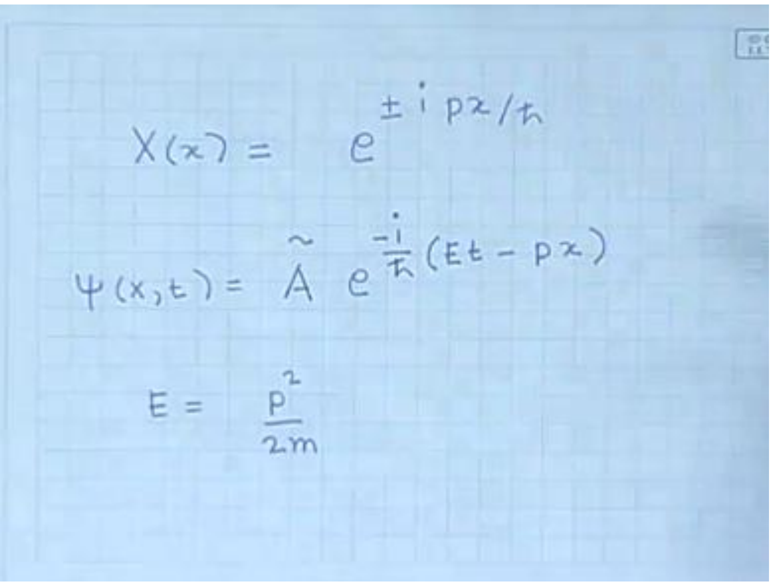

And the solution is X as a function of x is a constant again I could have a constant which I am not writing explicitly into e to the power plus minus i into P into x by h cross. If I differentiate this twice I will pick up minus P square by h cross square which is the equation that I have over here.

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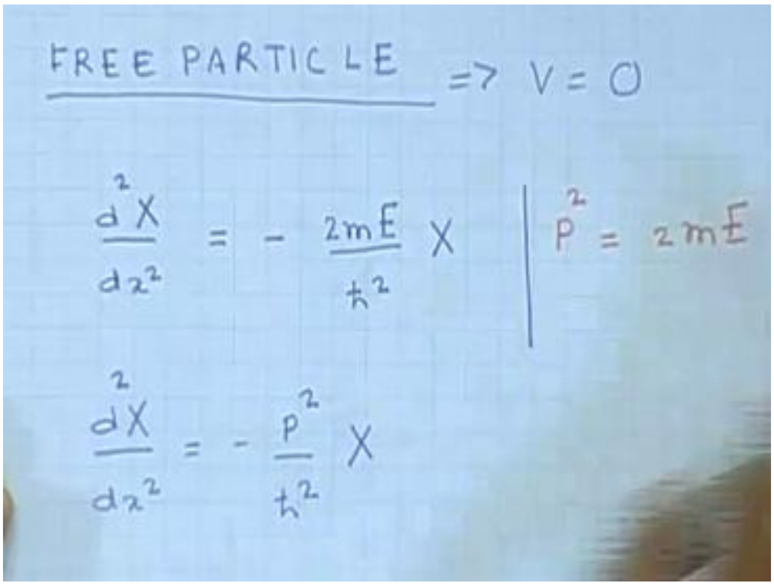

So, it is very easy to check that this is just the simple harmonic oscillator equation and it is quite straight forward.

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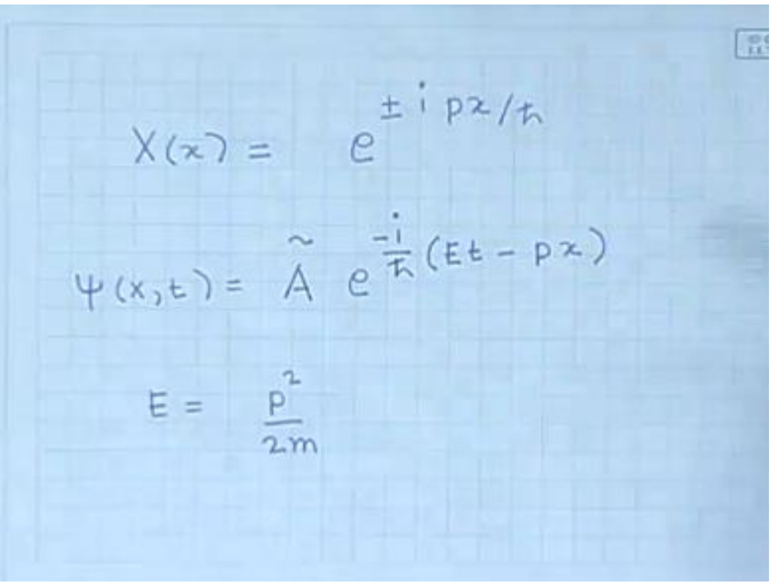

To realize that this is the solution there could be either plus here plus P or minus P because we have to take the square root when we are dealing with that equation. So, this could plus or minus we can choose 1 of them and go ahead.

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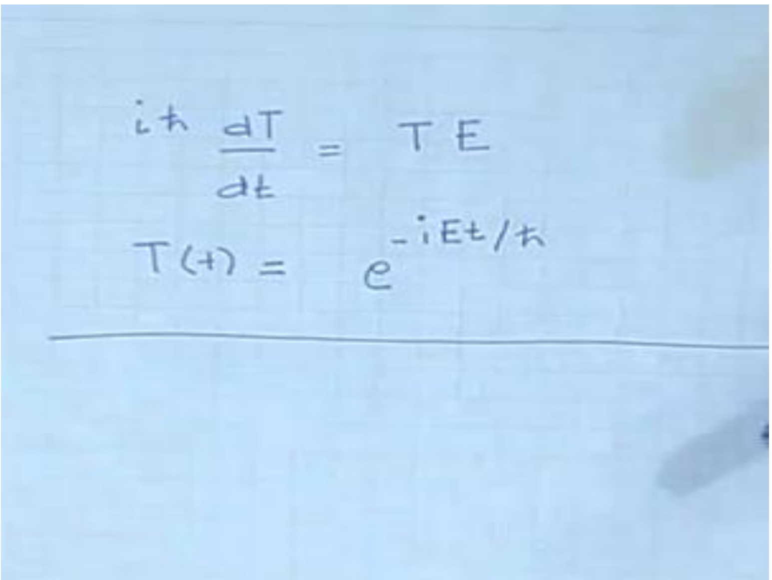

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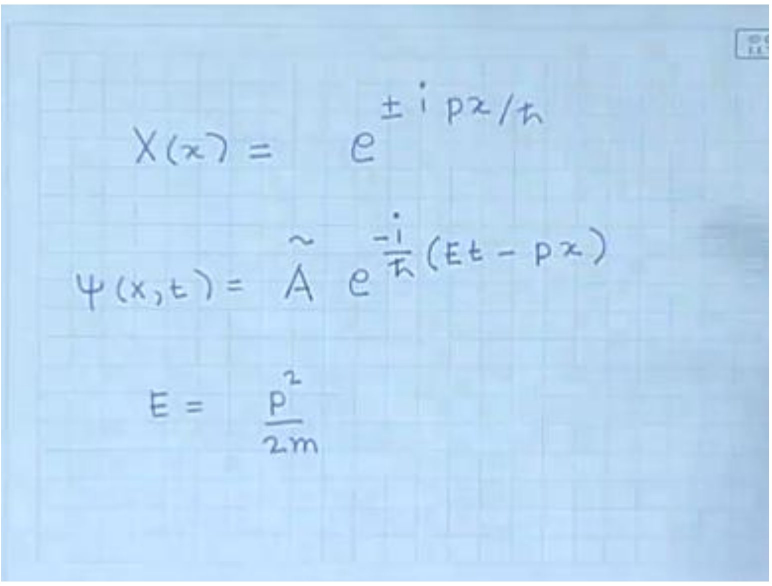

So, combining this with the temporal solution over here what we find is that the resultant is Psi x t can be written as some constant. I am now putting in the constant remember both the this and the temporal part had constant in front and I can take the product of those 2 and get a another constant E to the power minus i by h cross. So, I am writing the temporal part first.

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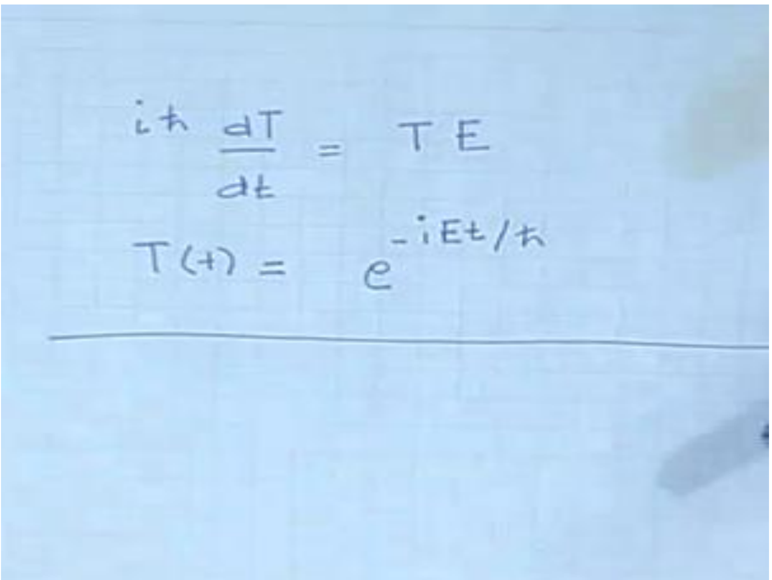

The part is minus i E by h cross into t.

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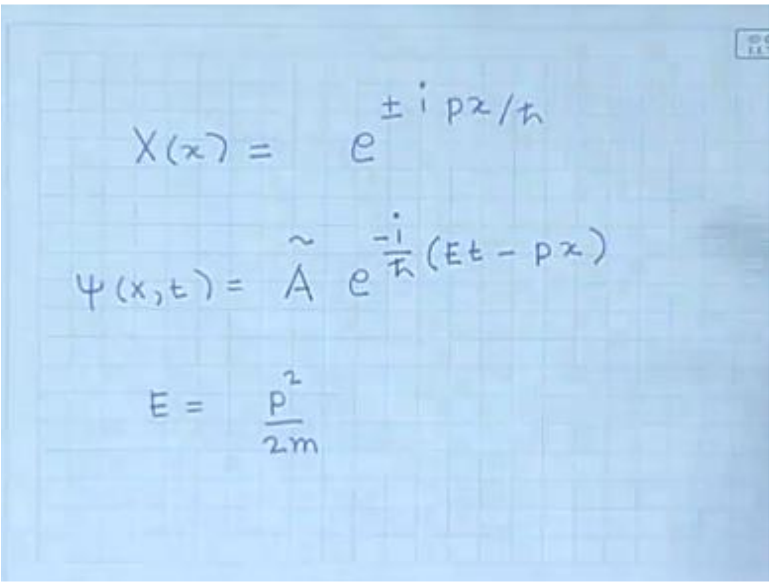

So, which I have written as minus i by h cross E into t minus P into x which is the spatial part. So, the spatial part has a solution which is given over here plus minus i P x by h cross I have taken only the plus the solution with the plus sign which is the 1 I have taken here. You could also have a solution with the minus sign which will introduce a plus sign over here. So, bear in mind that there are 2 possibilities I have just taken 1 of them over here. So, this is solution of the of of the Schrodinger equation for a free particle where there could be an overall constant which is still on determined. There is also a an arbitrary constant E which appears which I have not told what it signifies and there is another constant P which is related to E which is not independent, but related to E as follows.

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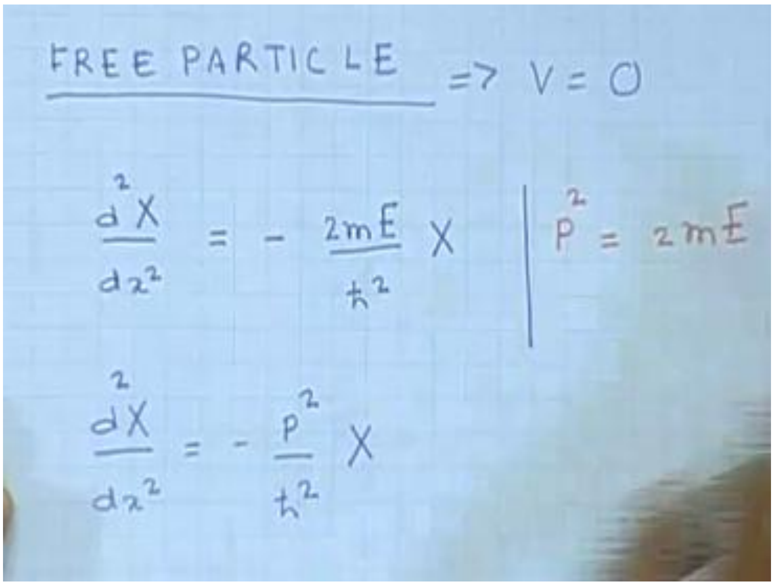

P square is 2 m E.

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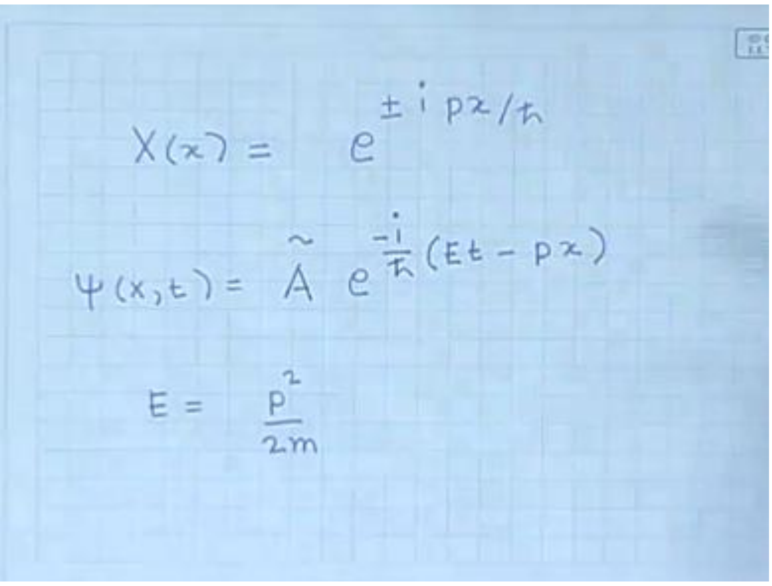

So, there is basically 1 constant either you can think of it has either P or E which I have to tell and E. If I tell you P then E can be E is fixed over if I fix then P is fixed and there is an overall normalization constant over here. And the E and P are related as follows. E is equal to P square by 2 m. So, this is a solution of the Schrodinger wave equation and solution has an overall constant over here. And it has 2 constants over here which i are not independent which are related like this. So, I can fix any 1 of them and I can get the other constant if I fix P I will E from that and that is it. So, there are this is a solution of the Schrodinger equation we have still not discussed the significant of this constant E. Which appears when I integrate this equation the Schrodinger wave equation or the constant P which appears, when we have when we integrate the Schrodinger differential equation.

So, there are till now unknown constants whose significance I have not discussed. Now, if you remember de Broglie hypothesis then you can straight away say what the significance of this E and P are. But we shall come back to the significance of this E and P in later on as we go long in this lecture or possibly in the next lecture. Another point which I should make here let us ask the question, what is the angular frequency of this wave? Now, you can straight away read the angular frequency of the wave if you think of this as e to the power of minus i omega the so, omega you see is e by h cross. If you ask what is the wave number of this wave for this wave the wave number if you write this as e to the power of i k x. Then you can straight away read the fact that P by h cross is the wave number.

So, this solution of the Schrodinger equation has arbitrary constants which come about. There is only one of them that is independent in the exponent there is only one of them which is independent. And it is this constant that also determines the angular frequency of the wave. And the wave number of the wave. So, this is a plane pure sinusoidal wave solution for the Schrodinger wave equation. If I had chosen the minus sign I would have this is the wave propagating to the right, if I chose the minus sign I would have got the wave propagating to the left. Now, the super position of two such solutions is also a solution let me also write that down. So, I can have two different you see this constant e is arbitrary or in other words the constant P is arbitrary.

Suppose I chose two different values for E and P if I so, I will refer to them as E 1 P 1 and E 2 P 2. Then i E 1 P 1 the set of values E 1 and P 1 will give me particular solution thus of the Schrodinger equation which as I have told you right in the start corresponds to a particular state of the particle. If I chose a different set of values E 2 and P 2 I will get a different solution which corresponds to a different state of the particle and I can now superpose these two solutions that also is guaranteed to be a solution of the Schrodinger equation. Because it is an linear equation. So, the superposition of two solutions is also a solution and I can write down this super position solutions. So, let we write it down here.

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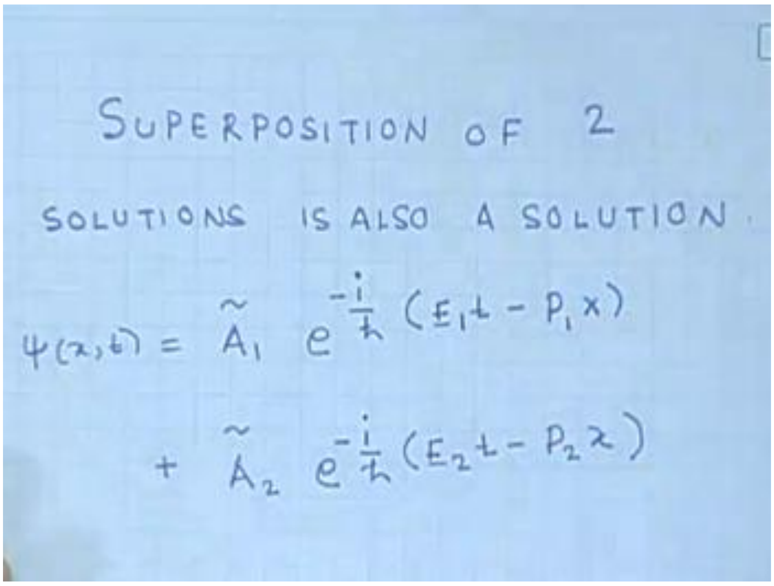

So, this psi is a super position of two different solutions. This solution has the constants in for the in this solution the constants have value P 1 and E 1 the overall amplitude is A 1 the amplitude remember could be complex. This solution the constant has values P 2 and E 2 and there is an overall amplitude A 2 which is again different from this. This corresponds to a particular state of the particle in this case the electron if you are dealing with an electrons wave function this corresponds to a different state from this for the particle. The superposition of these 2 is again at a different solutions. So, it corresponds to a third state of the particle. And if I change these constants and I will get different each of them will give me a different state of the electron. I can generalize this further. So, I can write down a solution.

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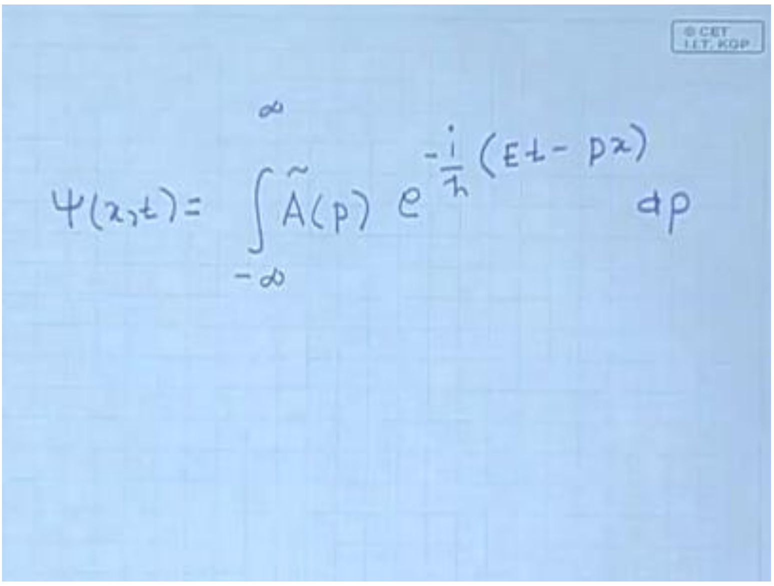

That looks like this. So, here the constant P which appears in a in our solutions can take any value in the range minus infinity to infinity. I am superposing all such values with different amplitudes.

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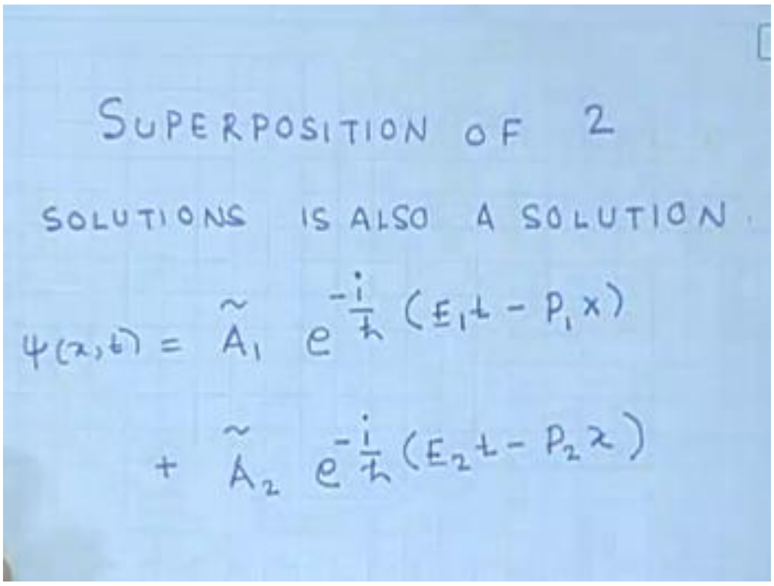

Here I have only 2 such values I am superposing them with different amplitudes so, corresponding to P 1. There is a E 1 and I have a particular amplitude for that wave corresponding to P 2 there is an E 2 which is the function of P 2 and for that. For this particular wave I have a different amplitude and the superposition of this also gives me a solution.

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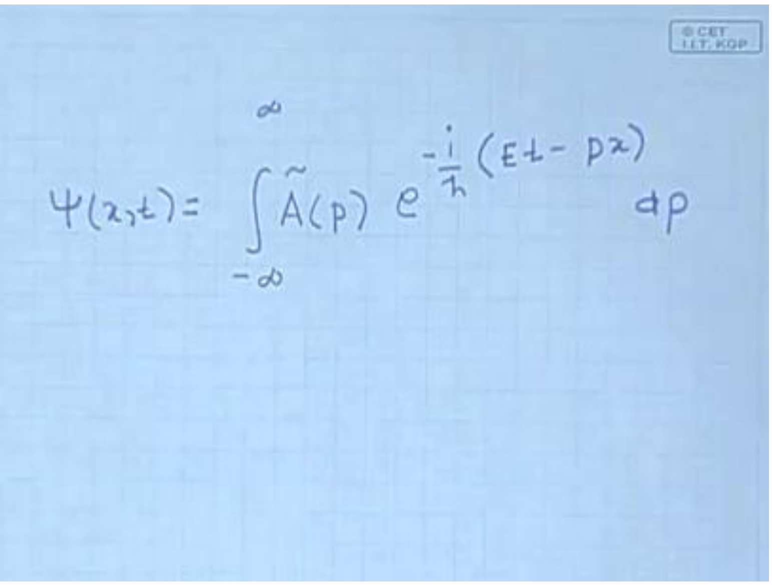

Now, what I am doing is I am superposing infinite some of infinitely many momenta. So, I represented as an integral. So, P will change and when P changes the particular wave corresponding wave is going to be added with the different amplitude. So, this amplitude also changes with P and I am superposing all of these solutions to produce another solutions psi of x of psi which is a function of x and t. So, different set of amplitudes will give me different psi wave functions and 1 sign no psi I can calculate the probability amplitude of finding the particle somewhere. So, if I know if I ask the question what happens if I make a measurement of the position of the particle if I measure the particles position I know how to what I can I can predict with the wave function what I can predict with the wave function is as follows.

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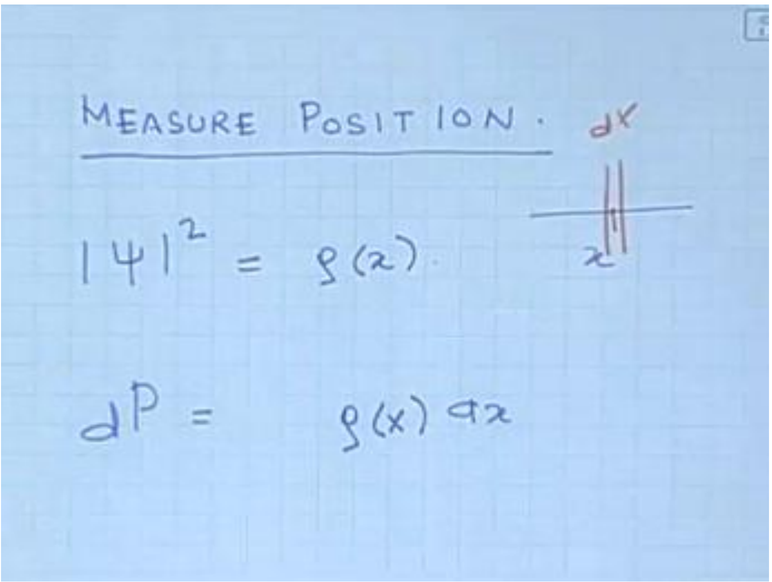

If I know for a particular solution psi the mod square of this gives the probability density rho x and the probability of finding the particle and an interval dx. So, if this is the region space I am dealing with this 1 dimensional if I ask the question what is the probability of finding the particle in an interval dx around this point x that is given. That probability can be calculated if I know the wave function. And it is it has the value the rho x. So, rho x into the interval dx gives me the probability of finding the particle in this small interval around the point x.

So, we know we can make some kind of a prediction of what we expect. If I make the measurement of the position of the particle what we can do with the wave function is that, we can predict the probability of finding it at some at any position. So, once we know the wave function we can tell, what is the probability of finding the particle at different positions? But for a particle free to move in 1 dimension position is not the only physical observable attribute that you can measure physical observation that you can do. You can measure various other things for example, you could measure the particles momentum or you could measure the particles energy.

For a particle which can move in 3 dimensions all 3 dimensions you can also talk about the particles angular momentum. The question is in quantum mechanics where we represent the state of the particle as waves. How do we deal with other such observable quantities? Physical observable quantities how do we deal with a situation where I measure the momentum of the particle? So, I have represented the state of the particle by a wave the question is, what will happen when I make a measurement of the momentum? So, here we have to introduce another postulates. So, I have already told you that the state of the particle is represented.

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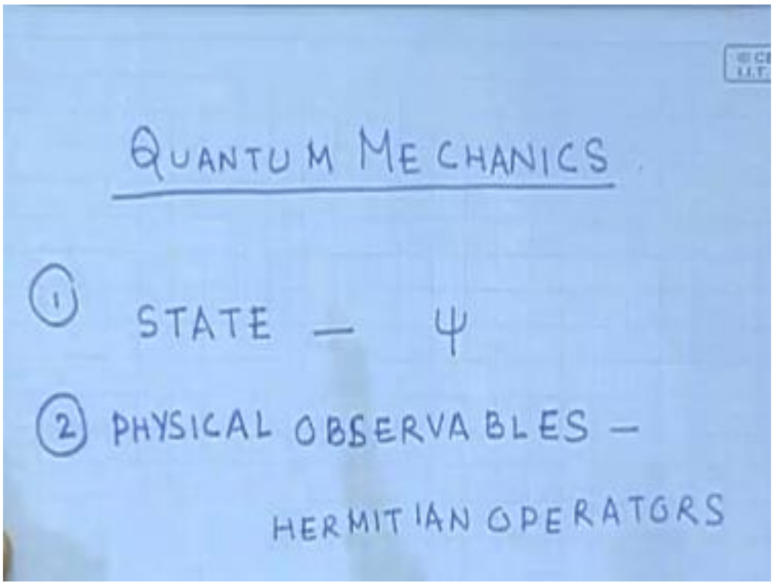

By a wave function psi, the second thing is physical quantities physical observable quantities are represented by Hermitian operators. So, corresponding to every physical observable quantity like position momentum energy etcetera for a particle there is an operator there is a Hermitian operator. So, we have now introduced to new words I have to explain what they mean the first thing is the Hermitian and the second thing is operator or you may say the other way around I have introduced this Hermitian and operator. We going to take a operators first and then I am going to tell that what you mean by a Hermitian operator.

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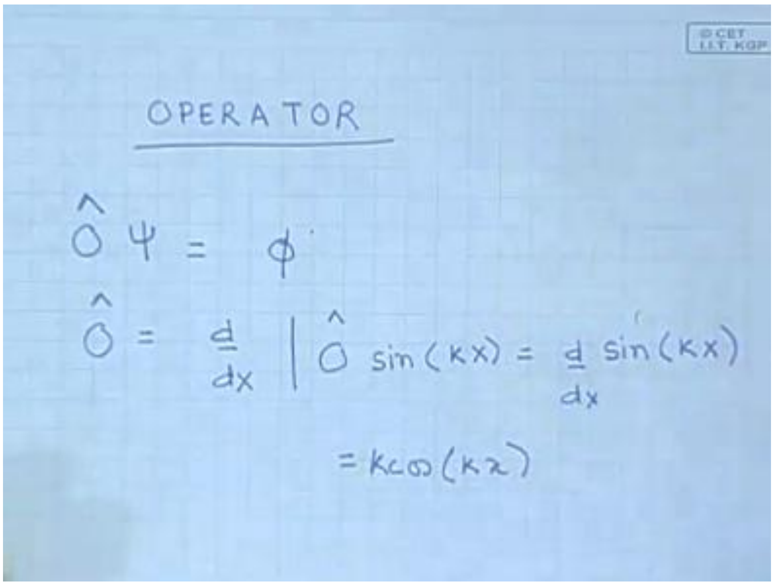

So, an operator an operator which I will denote with a any symbol with a cap on top. So, this is the operator o with the cap the cap fact that there is a cap on top or a hat on top tells going to tell us at this is an operator not a number. And operator is something that acts on a function to give me another function. So, let me give you the examples an example of an operator is d by dx. So, the operator acting on the function let us say sig k x is equal to d by dx of sin k x which is equal to Cos k into Cos kx. So, what we see is that this operator acts on a function to give me an another function. In this case the function is sin k x the function that I get when the operator o acts on this function or operate at d by dx acts on this function is k into Cos kx. So, this is what is an operator let me give you another example of an operator.

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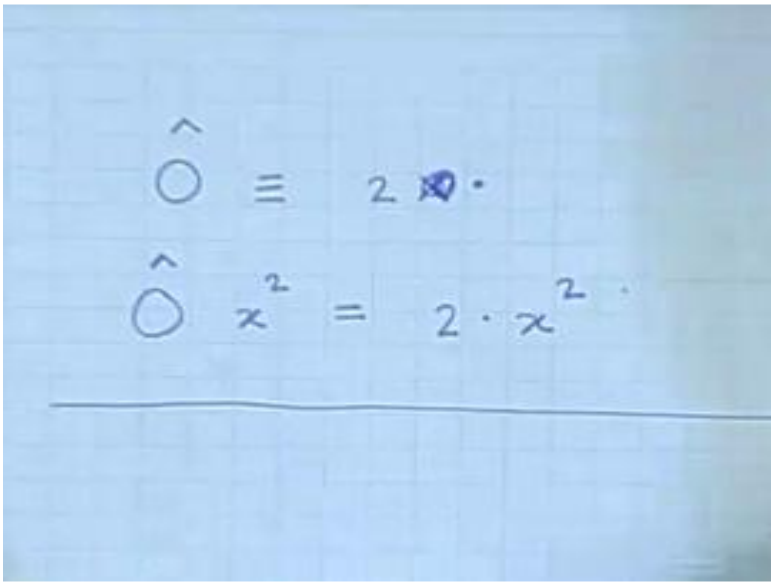

We have an operator o I am using the same symbol let us say who which is multiplication by 2. So, o acting on let us say x square when where o is defined as multiplication by 2. So, 2 times this is 2 into x square that is is the the operator here multiplication by the number 2. So, when this operator acts on the function x square it gives me two into x square. So, this is what we mean by an operator an operator acts on a function to give me different function.

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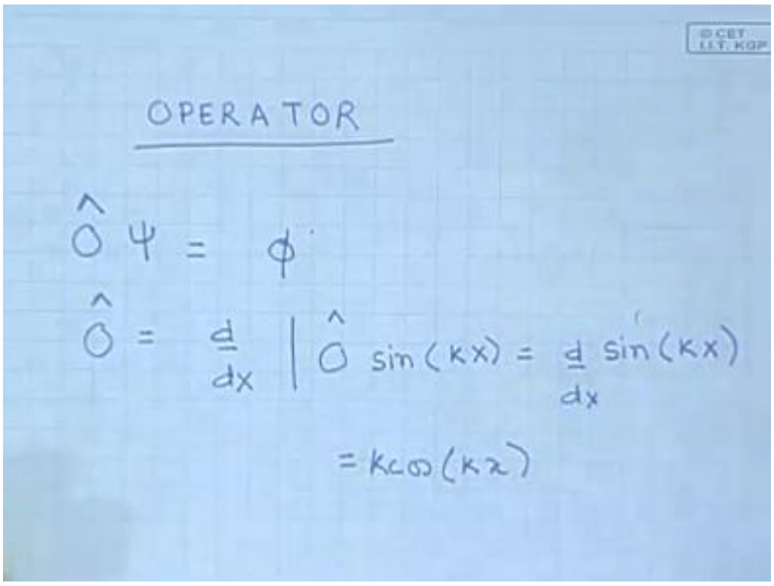

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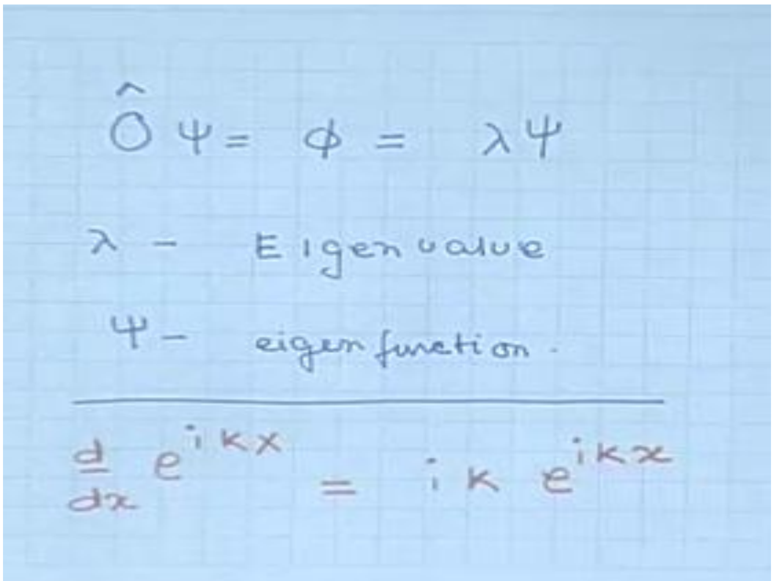

Now there are situations where an operator acts on a function to give me a different function, but this different function which is the resultant of the operator acting on psi is the same old function psi multiplied by a number. If this is true then this number is said to be an Eigen value of the operator o. And this functions psi is said to be an Eigen function. So, example an example of a of an Eigen function and an Eigen value is given below I am going to give you 1 now. So, let us consider the operator d by dx and this operator is going to act e to the power i k x this is going to give us i k into e to the power i k x. So, what we see is that the function e to the power of i k x is Eigen function of this operator d by dx. And it has a Eigen value i into k let me give you another example the next example is when the same operator d by dx acts.

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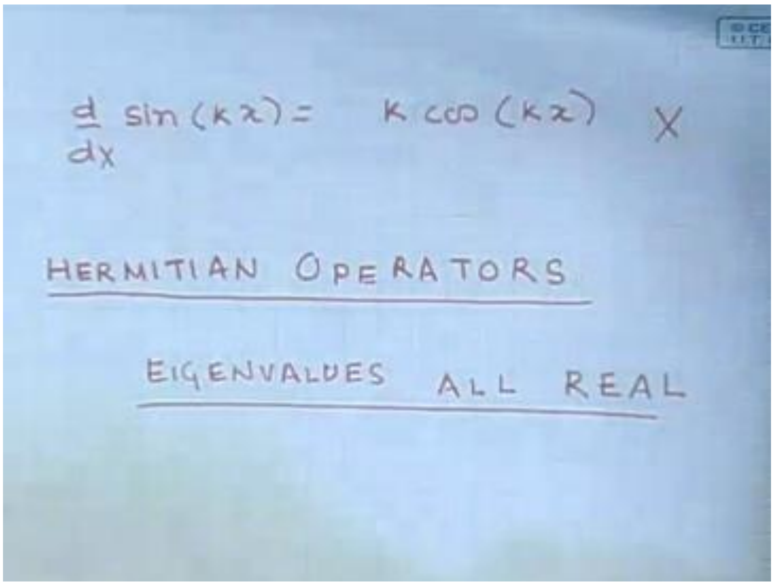

On sin k x it gives me k into Cos k x we see that this is not an Eigen function sin k x is not an Eigen function of d by dx right. Because when d by d the operator d by dx acts on sin k x it gives me a different function which is not the same function there is way, i can write this as the same function multiplied by a number. So, this is not an Eigen function and it has no Eigen value.

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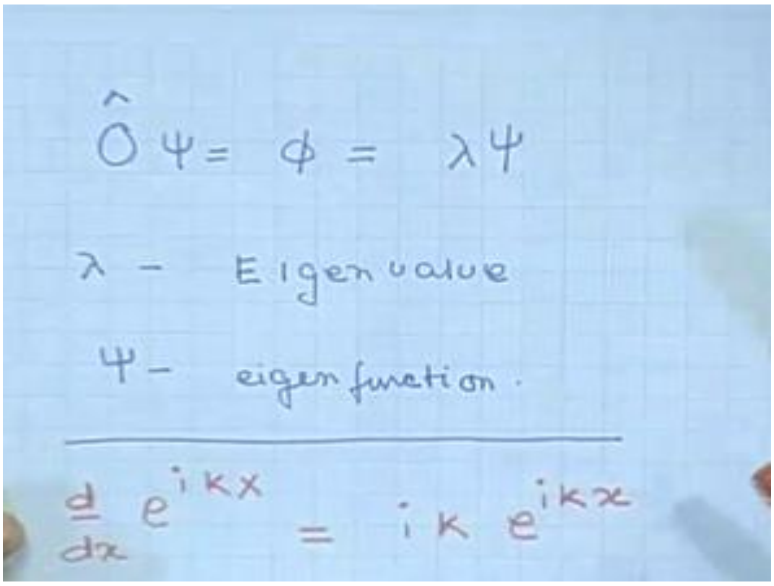

Therefore for this operator similarly this operator this Eigen this function is an Eigen function of this operator. And it has an Eigen value i into k now I told you that the that what we that in quantum mechanics corresponding to every physical observable.

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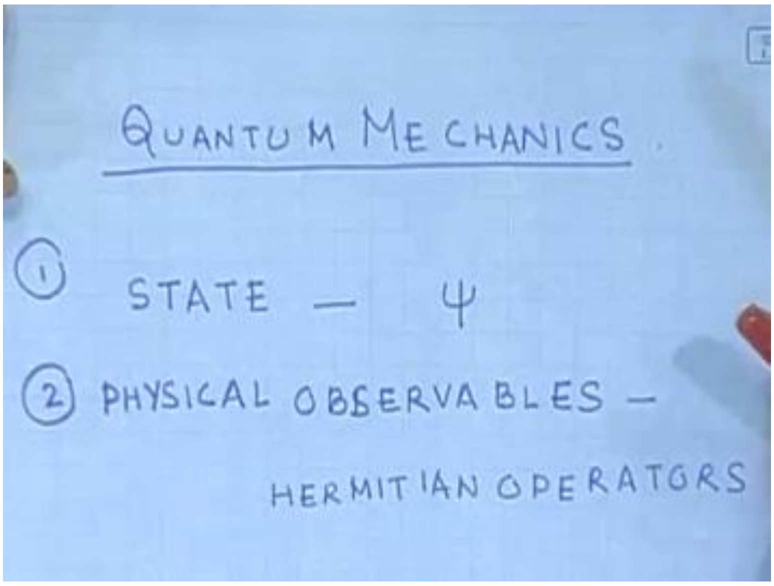

There is a Hermitian operator, what do we mean by a Hermitian operator? I have told you what we mean by an operator I have also told you what we mean by the Eigen value and the Eigen function of an operator. Now, the Hermitian operator is is a little complicated there is definition which I shall not go into the as far as we are concerned and we are we are mainly interested in one property of Hermitian operators. And we I am going to just tell you about this property. So, Hermitian operators are operators all of whose Eigen values are real so.

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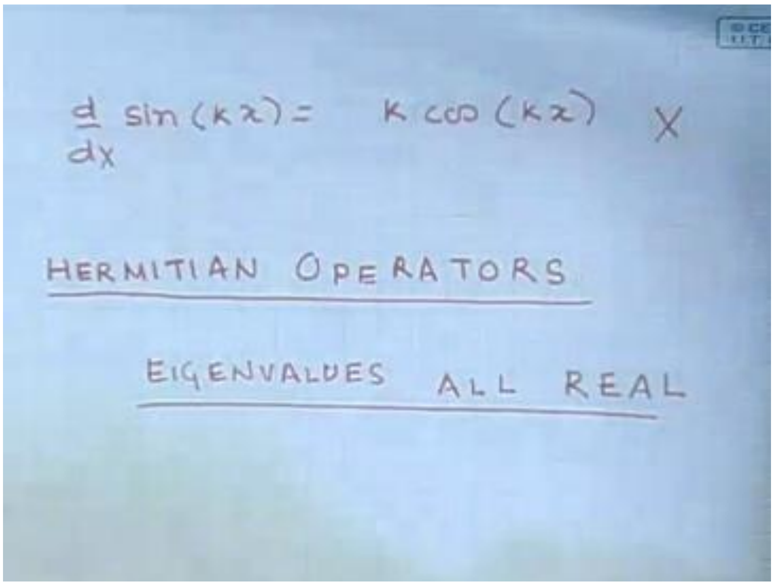

Hermitian operators are type of operators have this special feature that its Eigen values are all real. So, operators can have typically more than one can have many Eigen values.

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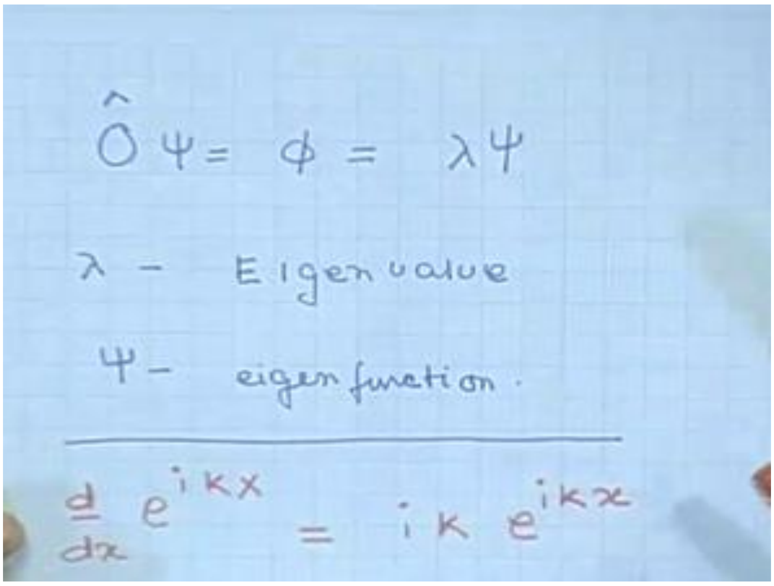

For examples the operator d by dx which I have introduce as a example d by dx has many Eigen values the function e to the power of i k x is an Eigen function for any value of k this is an Eigen function. So, for each value of k I will have a different Eigen value i into k. So, if k is 1 there will be a particular Eigen value i if k is 2 the Eigen value is 2 i, if k is 3 the Eigen value is 3 i. Any value of k is an Eigen is an Eigen value is a different Eigen value of this operator.

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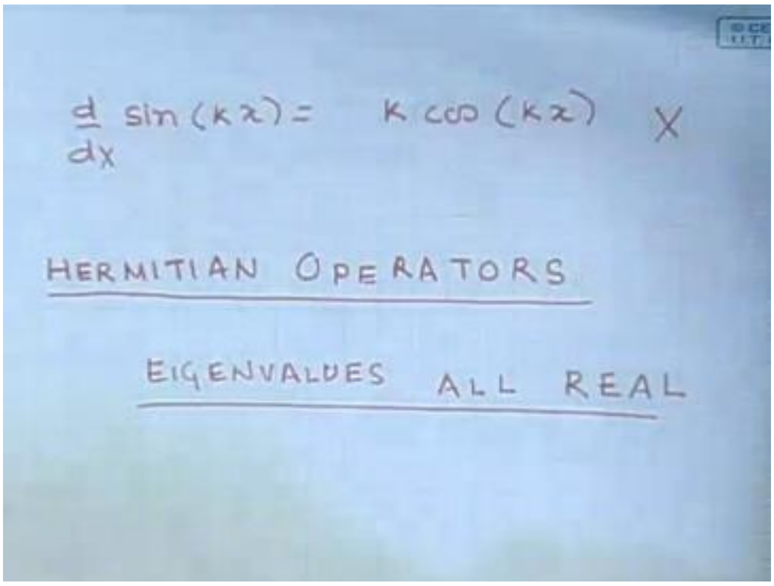

So, operators typically have many Eigen values many and correspondingly many Eigen functions. Now, Hermitian operators are a special class of operators all of whose Eigen values are real.

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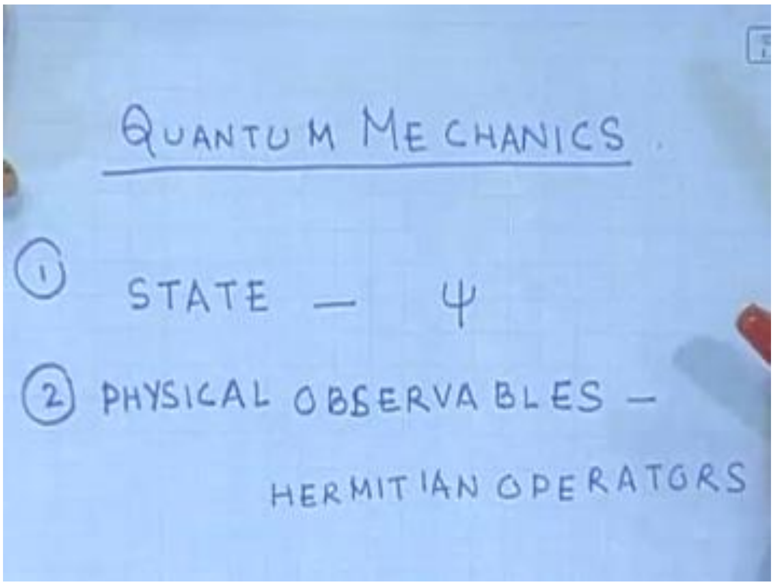

And it is the postulate in of a quantum mechanics that corresponding to every physical observables, there is a Hermitian operator associated with it. So, let me give me an example of a Hermitian operator the operator. So, let me give you an example.

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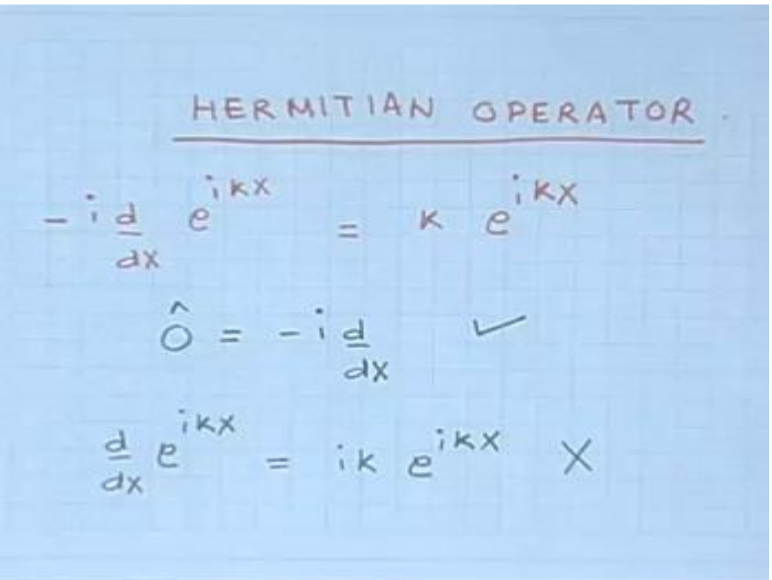

. So, let us consider the operator i d by dx and let us look at the function e to the power let us i k x let us consider instead minus i d by dx. So, let us look at the function e to the power i k x. And when this operator acts on this it give us what does it give us. So, if I differentiate this with x I will pick up a sign of factor of i into k and i will retain the original function i into minus i gives 1. So, I have a k. So, this gives me k into e to the power i k x. So, what we see is that this operator o defined as minus i d by dx this is Hermitian because its Eigen values are all real. Then you can check that it has no imaginary Eigen values Eigen values are all real you could I could also give you an example of an operator let us look at this operator d by dx.

So, d by dx acting on i k x gives me i k into e to the power of i k x now notice that this e to the power i k x is an Eigen function of this operator and it has an imaginary Eigen value such an operator is not Hermitian. So, I have given you an example of Hermitian operator I have also give you an given you an example of an operator that is not Hermitian. And in quantum mechanics corresponding to every physical observable there is a Hermitian operator let me give you an example. Examples so, for a particle moving in one dimension we have…

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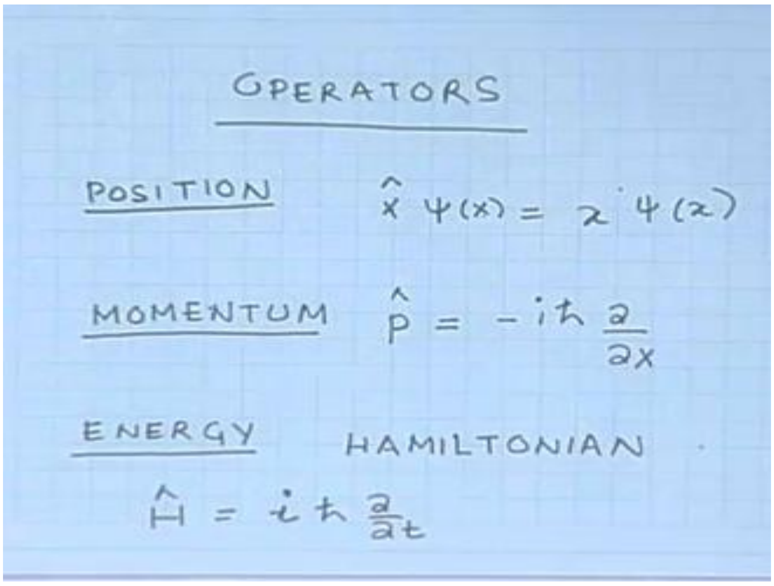

What are the physical observable set you can think of the position the corresponding to the position. We have an position operator x which is defined as follows when x acts on a function psi x it gives me the variable x into psi x. So, the function psi x is not an Eigen function of this operator x because x acting on psi x the operator x acting an psi x is the variable x into psi x. If you change x this is under constant if you change x the value of this variable also changes. So, this is not an Eigen function of this operator. And corresponding to position we have this operator x which is defined like this corresponding to momentum. We have the momentum operator P which is minus i h cross del by del x. So, this is the momentum operator and the other quantity which you can measure for a particle observable quantity for a particle free to move in 1 dimension is the energy.

Now, the operator corresponding to energy or more rigorously strictly speaking there is an operator corresponding to the Hamiltonian which as far as we are concerned in this course is the energy. And we shall denote this by h and this is this is i h cross del by del t i h cross del by del t. So, what we see that we have to postulate 2 things 2 postulates that we have encountered of quantum mechanics. The first postulate the first assumption is that associated with every state of the particle there is different wave function. And these wave functions are governed by the Schrodinger equation I have told you what the Schrodinger equation is associated with every physical variable which you can measure every physical observable quantity of the attribute of the particle that you can measure.

For example, for a particle in 3 dimension in moving only in a single dimension you have its position.

Momentum and energy corresponding to the position there is a position of operator corresponding to momentum operator corresponding to energy or strictly speaking the Hamiltonian there is Hamiltonian operator. Which is given over here i h cross del by del t the momentum operator is minus i h cross del by del x. So, corresponding to every physical quantity there is an operator. In the next lecture I shall tell you how to what happens when we make the measurement what is the physical interpretation I mean what is the what role do these operators play that I shall discuss in the next class. There is a point which I should make before I close today's lecture in today's lecture we worked out.

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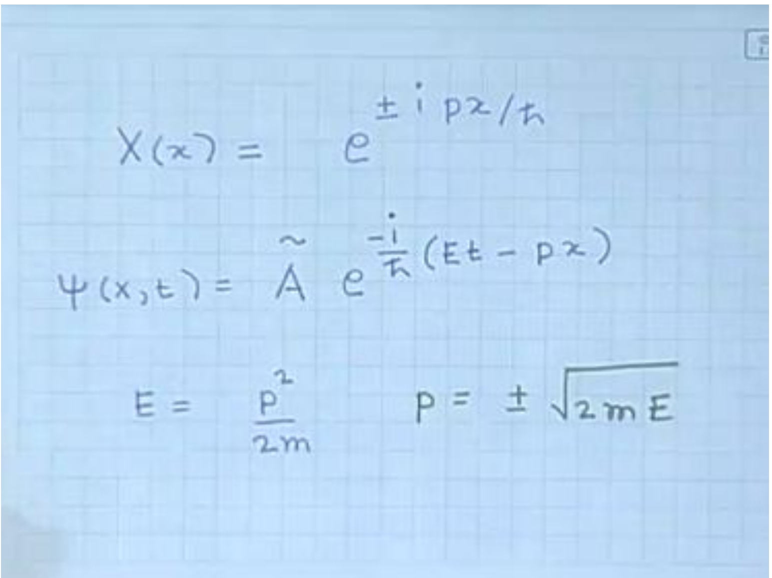

In the solution corresponding of the Schrodinger equation corresponding to a free particle and in this solution I told you that there is this constant E which comes up when I solve this equation there is this constant E which comes up. And related to E there is a constant P which is related to E like this. So, P is essentially the square root of 2 m into E with the plus plus or minus sign possible right. Now, the point which I forgot to tell you is that the E the constant E has to necessarily be positive it cannot be negative and the reason for this is as follows.

The constant E is related to the constant P over here. So, P is equal to plus minus 2 m E and till now our in our discussion E and P are both are arbitrary constants which are related to one another. And they essentially fix the dispersion relation for the wave. Now, E has to be real and positive because if E is negative. The constant P becomes imaginary if I have an imaginary constant over here and I multiplied with i then the x dependence now becomes of the form e to the power some real constant into x let me make this point clear. Why does E have to be necessarily positive?

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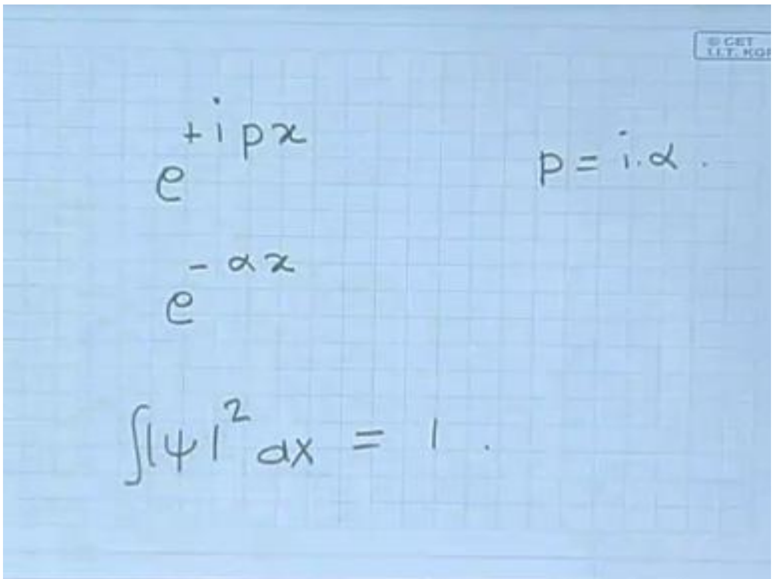

If E is negative then P becomes imaginary and if I have E to the power minus i plus i P x that the x dependence if this P is imaginary. Let us say P is i alpha then this becomes minus alpha x because i and i gives minus 1 and the range of x is from minus infinity to plus infinity. Now, notice if the x dependence of the wave function is of this form. It blows up when x goes to minus infinity the wave function should not blow up because the wave function gives the probability and the modulus square of the wave function integrated should be 1.

So, if you want this to be satisfied the wave function should not blow up which basically tells us that P cannot be imaginary, and if P cannot be imaginary E has to be real. So, in this arbitrary solution in this solution that we have worked out E could be arbitrary an arbitrary constant as long as it is real. That is the point which I have forgotten to mention. So, let me bring today is lecture to an end over here we shall resume over discussion of how to interpret these operators how to manipulate with them in the next lecture.