Physics I : Oscillations and Waves

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Lecture - 36
Probability

Good morning. In last two lectures we have been discussing a situation where we have electrons coming from a electron gun on to a screen which has 2 slits

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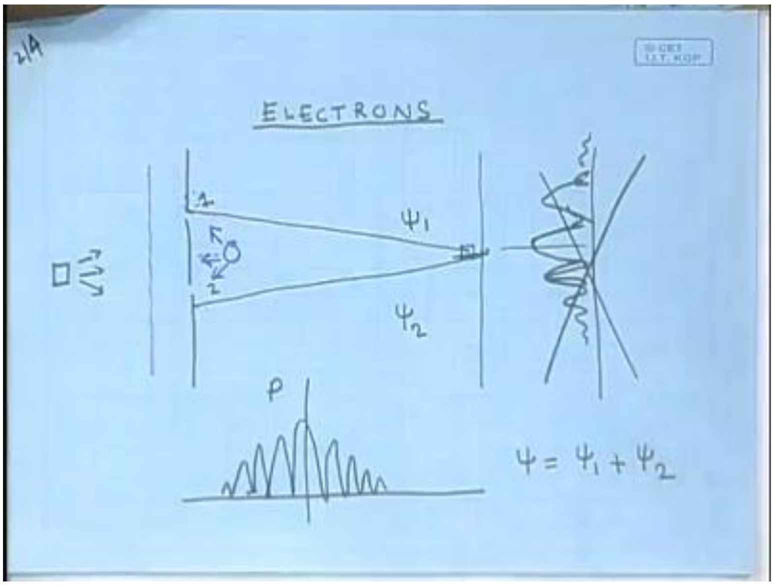

And we look at the pattern of the arrival of the electrons on a distant screen. So, the electrons can arrive at distant screen through these 2 slits, and if you look at the pattern of the electron arrival on this distant screen you will find that that there is something which looks like an interference pattern. So, there will be more electrons arriving over here at the center. And you will find that the number of electrons arriving falls. And then again it rises and falls you will get something which looks like an interference pattern.

And the only way in which you can explain this is if you invoke a wave light property to the electron. So, every electron we found we discussed that if you with every electron you associate a wave. So, when the wave associated with an electron arrives at the slits. Each slit acts like a secondary as a source for secondary waves and if you wish to calculate the resultant at some point on the screen. You have to add up the contributions from this slit and this slit.

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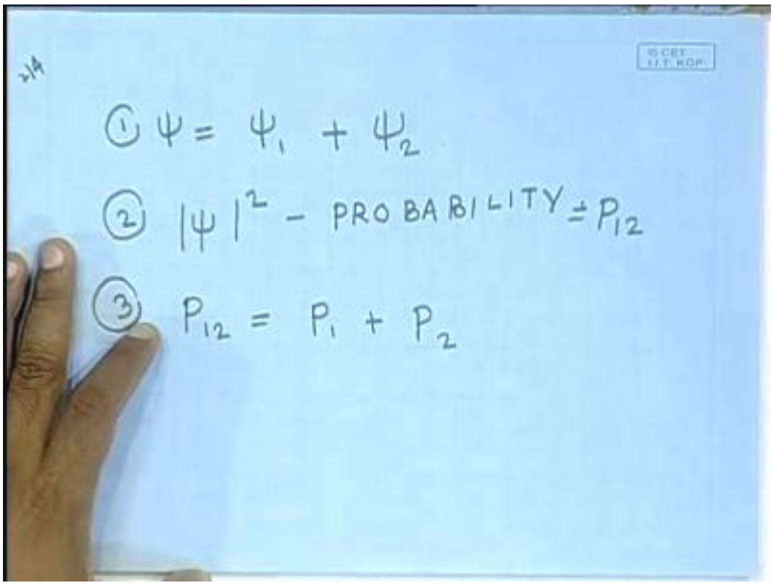

So, the wave psi the value of the wave psi at any point on the screen is psi 1 the contribution from the slit 1 plus psi 2 the contribution from slit 2.

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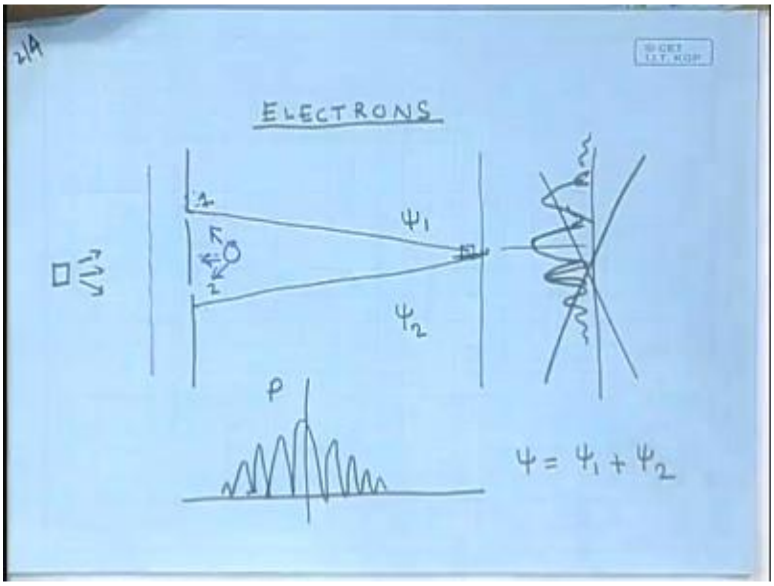

And these two contributions can be in phase out of different points which gives rise to this kind of an interference and the way you interpret this wave.

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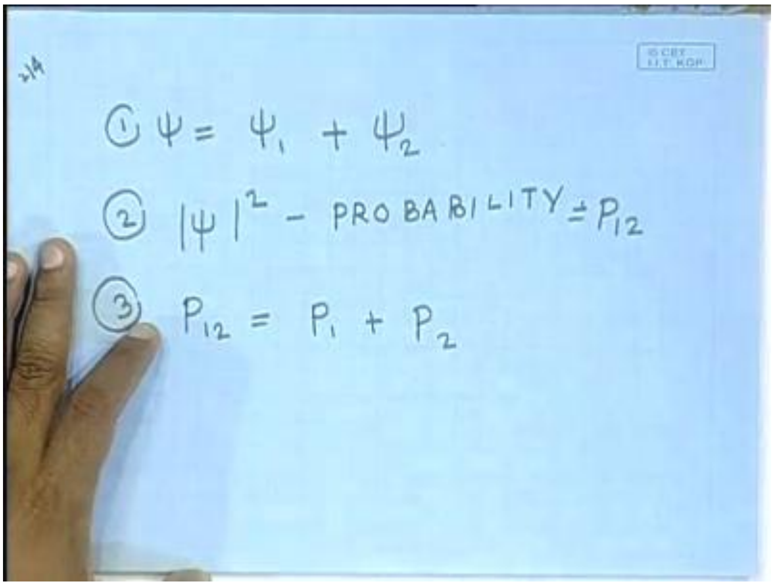

psi is as follows you psi this the probability amplitude and the mod square of this gives the probability of finding the electron at any point on the screen.

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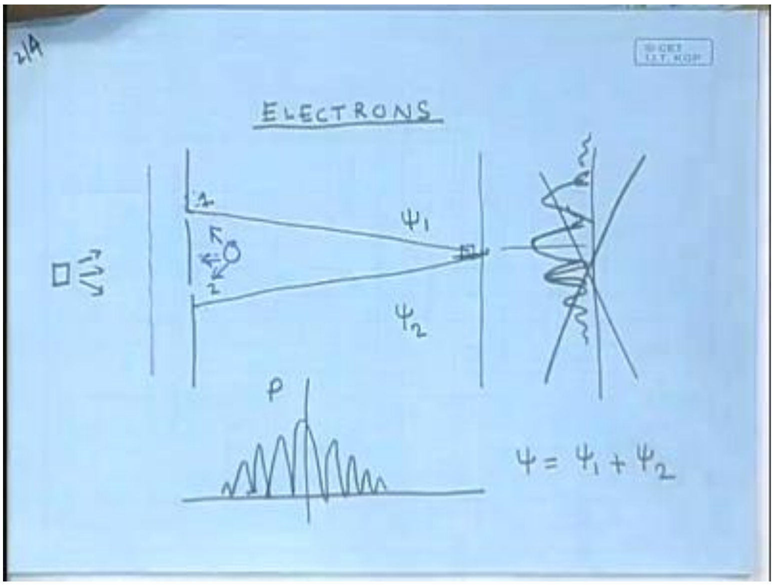

So, you have to evaluate psi at different points on the screen and the mod square of that gives you the probability of finding the electron at any point on the screen. So, what we see is that the moment we adopt this kind of a picture and it is necessary to adopt this kind of a picture, because otherwise there is no way you can explain the interference the probability pattern. That you see which looks like an interference pattern so, the movement you adopt this kind of a wave picture for the electron. You lose the power of predicting the electrons trajectory all you can predict is the probability that the electron arrives at some point on the screen. If you make a measurement you will always find that the electron arrives only at a single point. What you can predict is the probability of the electron arriving at that point on the screen. So, to in today's lecture we shall review revise the concept of probability some of the ideas may be already familiar to you, but some of the ideas may be new. So, we are going to today we are going to discuss probability.

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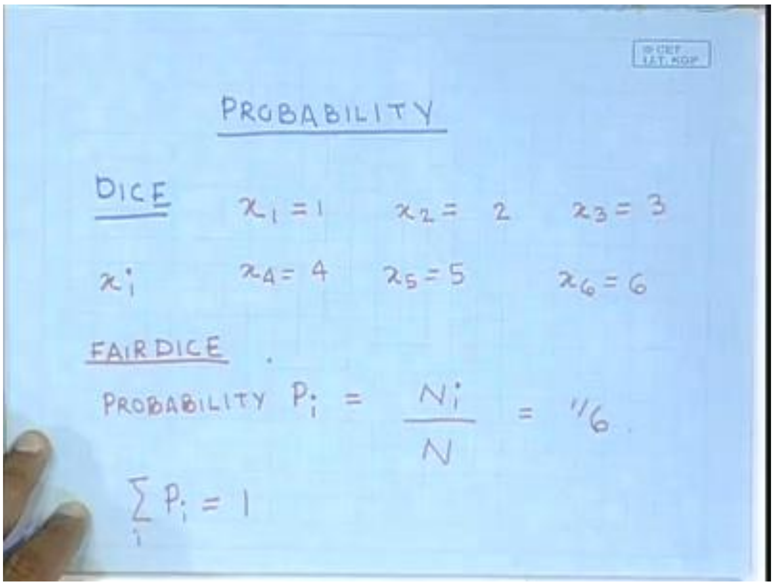

Let us start our discursion by considering a dice. So, we all know that a dice is a cube which has got 6 different sides and each side has a number painted on it. So, the numbers when you throw the dice you cannot predict what the outcome is going to be. The outcome can be any 1 of 6 values. So, let me write down these values. So, the dice outcome of the dice we are going to refer to has x it is a random variable. And it can have values x 1 which is 1 x 2 2 x 3 3 x 4 4 x 5 5 and x 6 6. So, there are 6 possible outcomes which I have enumerated here x 1 x 2 x 3 x 4 x 5 x 6. We shall use xi to refer to them where the index I runs from 1 to 6.

And in this particular case the six possible outcomes are 1 2 3 4 5 and 6. So, the dice can give you any 1 of these values when you throw the dice. Now, if you have a fair dice then the chance of getting any one of these values is exactly the same. And you can define you can calculate you can measure the probability. So, you can ask the question what is the probability? That I will get any 1 of these values so, if you wish to calculate the probability of getting a particular value. So, probability Pi so, this is the probability of getting a particular value 1 2 3 I could take the index I could be either 1 2 3 4 5 6.

The probability of getting a particular value I am the number of times I get that value divided by the total number of times I have thrown the dice. So, this how you can determine the probabilities for associated with each of these 6 outcomes and for a fair dice the probability for any of these outcomes is the exactly the same. So, this is going to be exactly equal to one sixth. So, you can determine the probabilities by repeating the experiment many times. And if you ask the question what is the probability of getting 1 the value 1 then you have of to count the number of times you get the value 1 when you throw the dice. And you have to divide that by the total number of times that you have thrown the dice.

And if it is a fair dice, this probability of getting 1 should be one sixth the probability of getting 2 we also be one sixth for 3 4 5 all of them will be one sixth. The total sum of all probabilities is going to be 1 that you can see from here. The sum of the total all possible wills outcomes have to be the total number of times the dice was thrown. Because the dice will necessarily assume 1 of these values and the total probabilities sum of all the probability of all the outcomes has to be equal to 1. So, we can also draw this probability distribution. So, let me draw it here.

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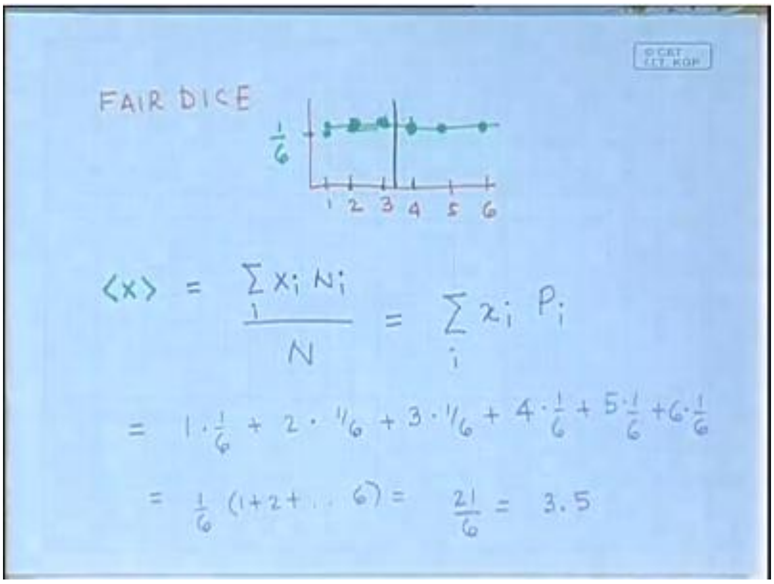

This is for a fair dice. So, these are the different values 1 2 3 4 5 and 6 and each of them has a probability one sixth. So, if this is one sixth each of them has a same value. So, this the value probability of getting 1, probability of getting 2, probability of getting 3. Probability, just straight they are all on a straight line probability of getting 4 5 and 6. So, there all these values are exactly the same one sixth. Now, let us ask the question? If I throw the dice once what is the value of x that I expect what is the outcome that I expect? So, it should be quite clear that if I throw the dice once I would expect to get the mean value I will never get the mean value when I throw the dice.

But I will expect that is the expectation value for example, if I let us ask the question in this class the students have a variety of heights. And if I now, catch 1 student at random and look at his height what is the value that I expect what height I expect him to have what is? So, I would expect him to have summed the average value let that is most that is what I would expect. So, this is what is called the expectation value or the mean value. Say if I throw the dice once I would except the value of outcome to be the mean value it will never be the mean value exactly it will have spread around it.

But that is what I would except that is a typical number which would tell me what value I would expect. Similarly, if in this class if I were to randomly pick 1 student and measure his height. I would except the value to turn to be close to the mean value the mean value tells me what I except if I measure for do the experiment once. If I throw the dice once what do I except if I measure the height once what do I except. So, that is given by the mean value. And the way to calculate the mean value is as follows. So, let we first denote the symbol that denotes the mean value I will use these angular brackets to denote the mean value of the variable x.

And to calculate the mean value I have to count the number of times I receive a particular x. So, xi into Ni divided by the total number of times I have thrown the dice. So, if I get the value 1 3 times. So, I will have to multiply by 3 then if I get the value 2 10 times I have to multiply that by 10. And add up all such possibilities and then divide by the total number of timed I have thrown the dice that is what gives me the mean. Which can I also write as the sum of I xi into the probability of getting that particular value.

Now, in this so, this is how we calculate the mean? And in this particular case the mean value the excepted value let us calculate it. So, there are six possible outcomes so, the first outcome is 1 and the probability of getting this is one sixth the next outcome is 2 and again the probability is one sixth. Then next outcome is 3 again the probability is one sixth plus 4 into one sixth plus 5 into one sixth plus 6 into one sixth. So, this sum is 1 plus 2 plus 3 plus 4 plus 5 plus 6 divided by the whole thing divided by 6. So, it one sixth into the sum of 1 plus 2 all the way up to 6. Now, the sum 1 plus 2 plus 3 plus 4 plus 5 plus 6 we know is 21.

So, this is 21 divided by 6 which is 3.this you can divide this by 3 you will get 7 divide this by 2 you will get by 3 you will get 2. So, it is 7 by 2 which is 3.5. So, the average the excepted value the expectation value or the average value is 3.5 let me show it here. So, the average value expectation value is 3.5. So, if I throw the dice once I would expect to get 3.5 I will never get 3.5. But 3.5 is the representative value it tells me that I will get sum value around 3.5. So, may be 3 may be 4 may be 2. So, the values are spread out around symmetrically around 3.5 and that is the mean value or the expectation value. Now, let me consider next a biased dice.

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What do I mean by a biased dice? In this case what i mean by a biased dice is which has been designed in such way which has been manipulated in such a way. That it has it still has 6 sides, but whenever I throw it I will either get 3 or 4 as the outcome I will never get 1 2 or 5 6. So, we are considering a hypothetical dice which has we manipulated in such a way. So, that when I throw it, it has the numbers 1 2 3 4 5 6 painted on it. But when I throw it, it will either give me a value 3 or the value 4.

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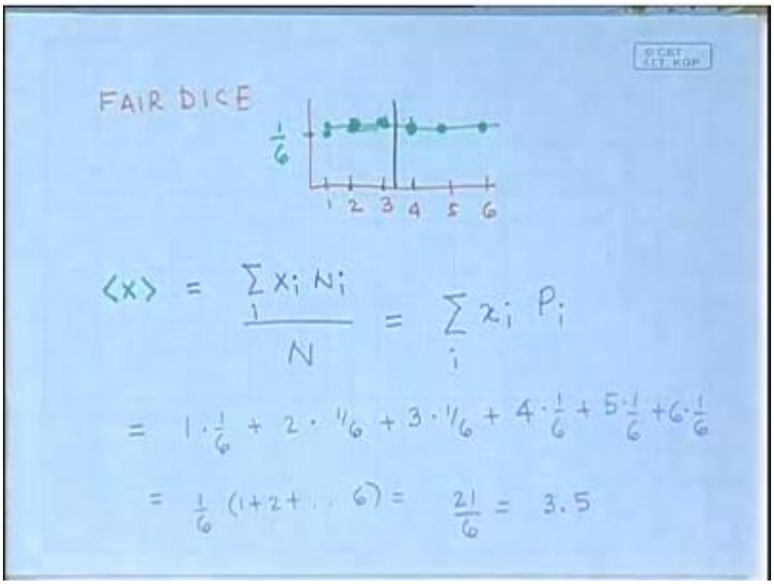

So, again the dice has 6 possible outcomes 1 2 3 4 5 6, but now, the probability of getting 1 the probability getting 2 are both of these are 0. There is a probability of getting 3 and 4 and these probabilities are the equal. Probability of getting 5 and 6 are again 0. So, there is equal probability that you will get either 3 or 4. So, it is if you repeat the exercises if you throw the dice many times and then calculate the probability of getting 3 and the probability of getting 4 you will find that this will be half this will half there is equal probability that you will have either 3 or 4. Probability of getting 1 or 2 5 or 6 is 0.

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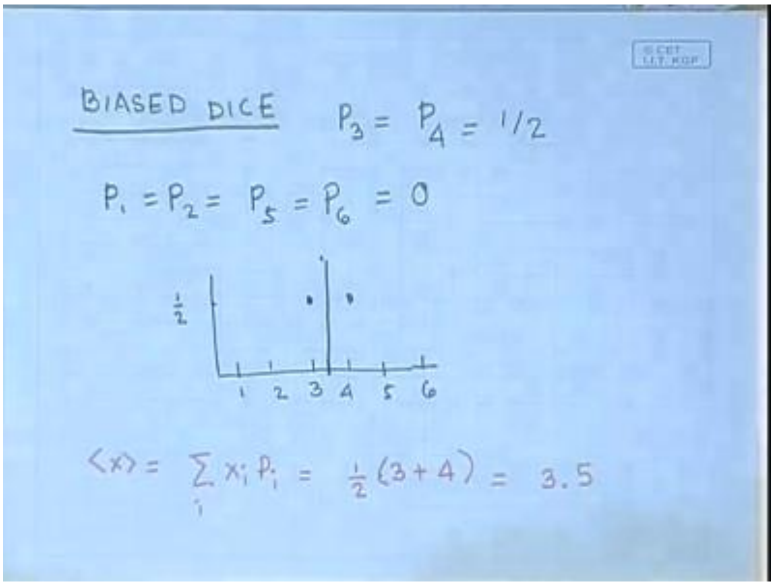

The probability of 3 is equal to the probability of getting 4 and these are both equal to half. Probability of 1 is equal to the probability of getting 2 is equal to the probability of getting 5 is equal to probability of getting 6 and all of these are 0. So, again I can draw the probabilities as a function of the value. So, here I have 1 2 3 4 5 6 and the value is half for 3 and 4 and it is 0 elsewhere. Now, let us calculate the excepted value if I throw the dice once.

So, again we can calculate the mean and the way to calculate the mean I have told you is to take the sum of xi into Pi. So, here there are only 2 possible outcomes 3 and 4. So, the there is probability of getting 3 and the probability is half. There is a probability of getting 4 and the probability is half. So, we have 3 plus 4 divided by 2 and again you find that the expected outcome is 3.5. So, on the average you expect to get 3.5 you never get 3.5 you will get either 3 or 4.

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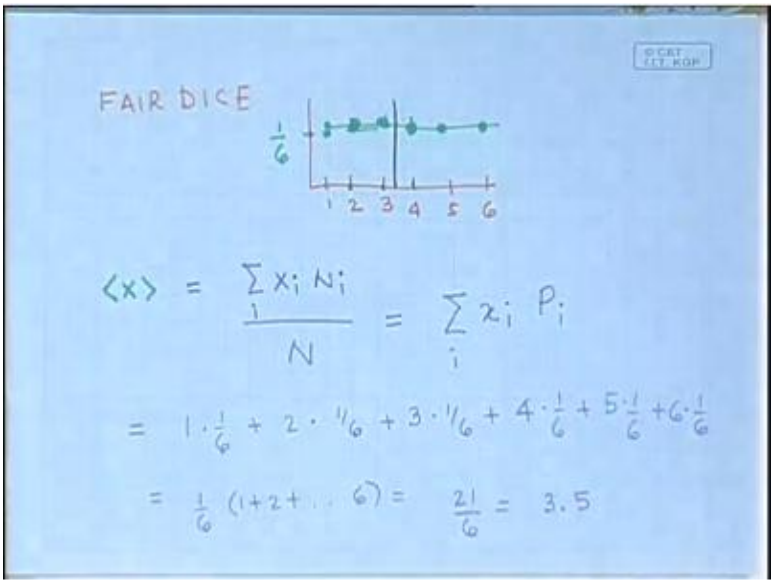

So, what we see is that we have 2 possible we have considered 2 possible situations 1 where we have a fair dice where there is an equal probability of getting any value in the range 1 to 6. The expected value is 3.5 that is the average value.

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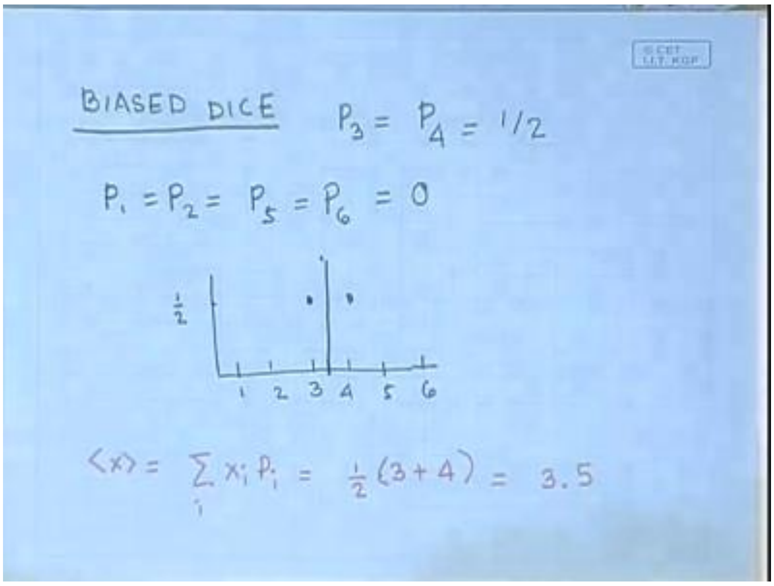

Here we have a biased dice you have equal probability of getting any 2 of these values 3 or 4 the expected values is 3.5. So, for both of these 2 dice the, you expect a value the mean value is 3.5. But you see the probability distributions the probability have a quite different the so, when you throw the dice in this particular case the values.

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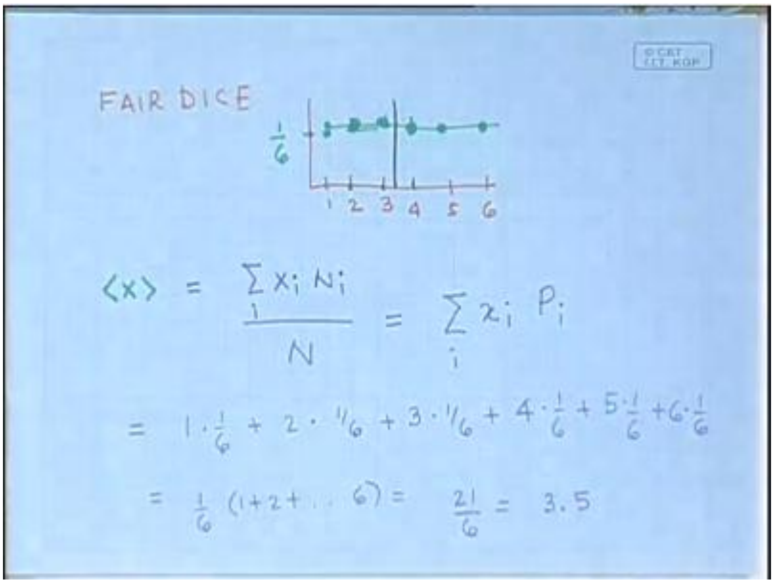

The outcomes are going to have a much larger spread they are going to be spread out quite a bit around the mean it can be 1 2 3 or it could be 4 5 6 which is quite a spread around the mean.

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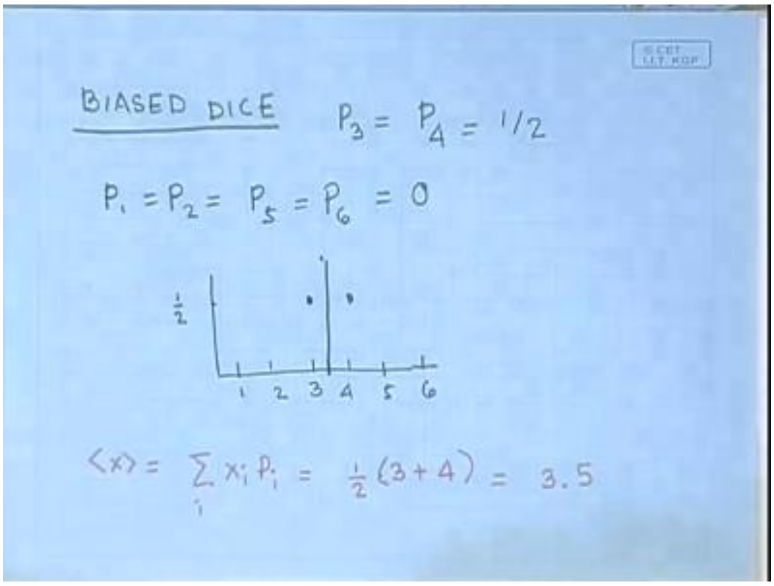

In this particular case for the biased dice they are only 2 possibilities it could either 3 or 4. So, it is called a narrow spread around the mean.

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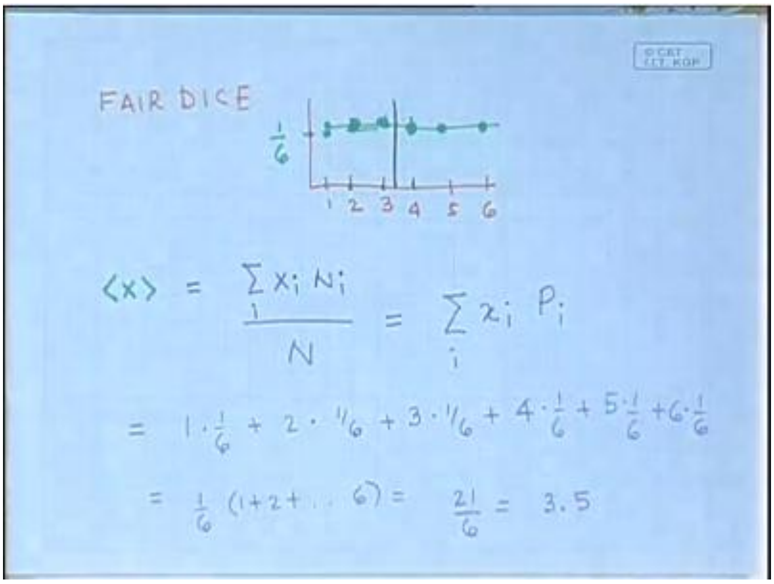

And the question is that if you wish to quantify this spread around, mean it carries information about how wide is the distribution and it is interesting and useful quantify the spread around the mean. It is this which is going to tell us that that the 2 distribution are different. Thus far as the expected values concern both the fair dice and the biased dice are exactly the same. It is only in the distribution around the mean, how the values that you get are going to be distributed around the mean these two are going to be different.

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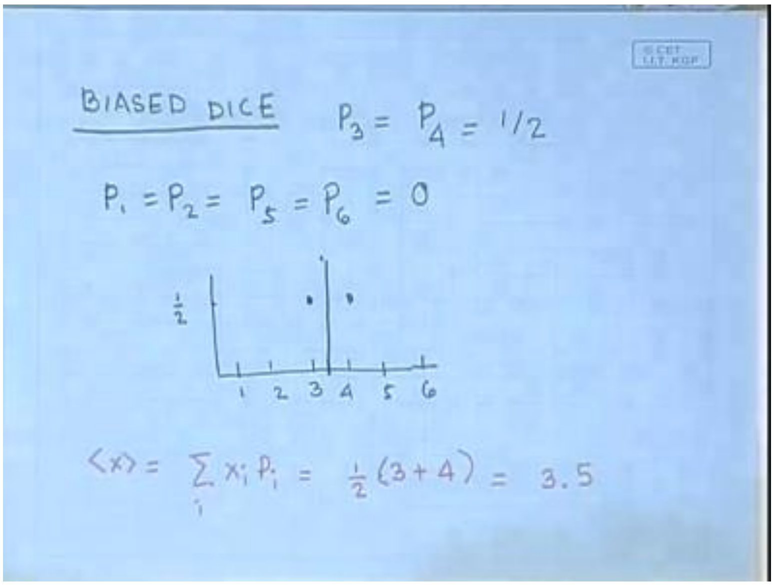

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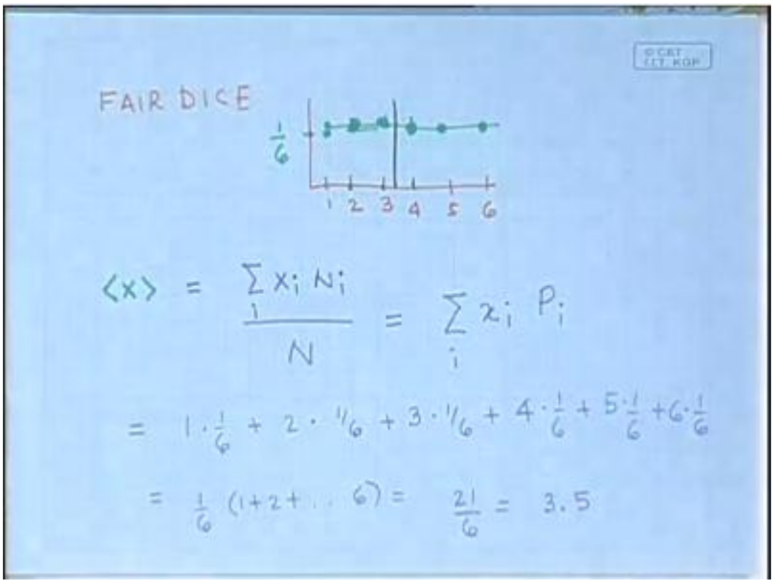

And the fair dice is going to have a much broader distribution the.

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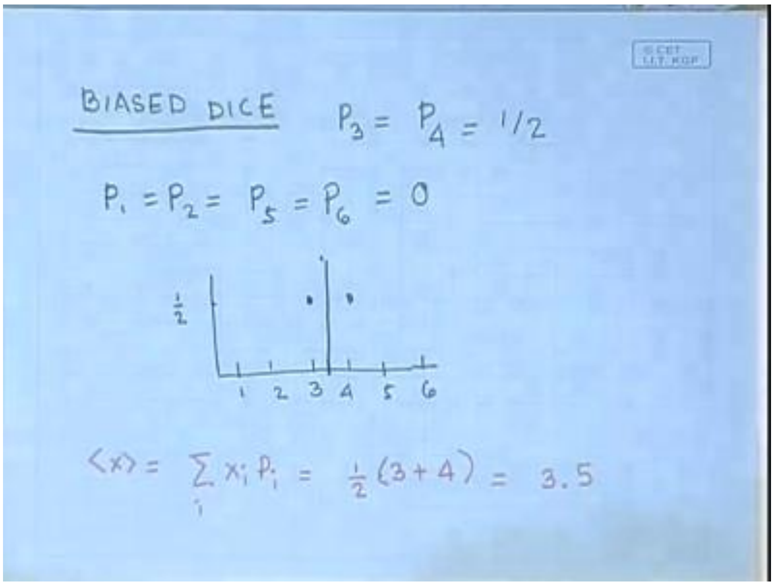

The biased dice is going to have a very narrow distribution in the values around the mean. So, the question is how do we quantify how much is the spread around the mean? I have shown you that it is important to quantify the spread around the mean for, because for the biased dice the spread very small for the fair dice the spread quite large. So, if you can quantify the spread around the mean you can distinguish very easily between a biased dice and a fair dice. For example, in this case, in general it is very important to quantify the spread around the mean. And this is the next thing that we are going to look at. So, let us ask the question how much if I throw the dice once and I will get a value xi.

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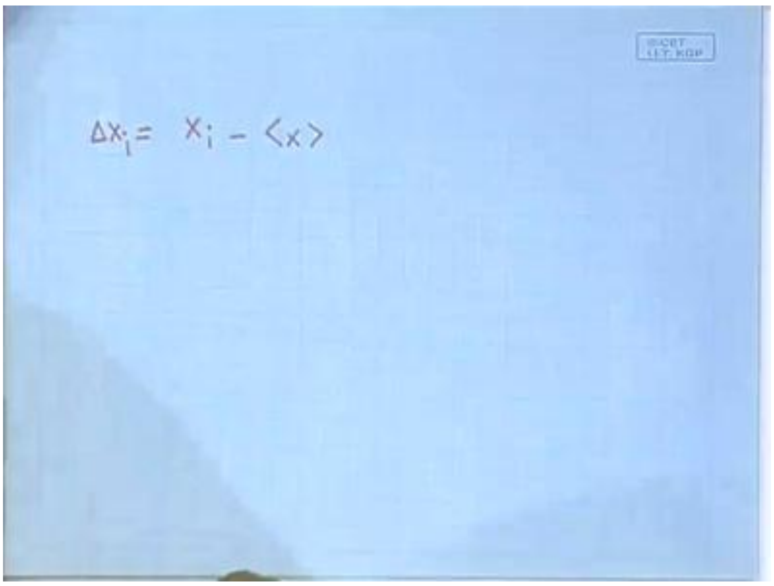

So, if I do the experiment once the outcome of the experiment is xi how much is the deviation from the mean? So, this delta xi that takes the outcome that you get and subtract out the mean value this tells me the deviation from the mean.

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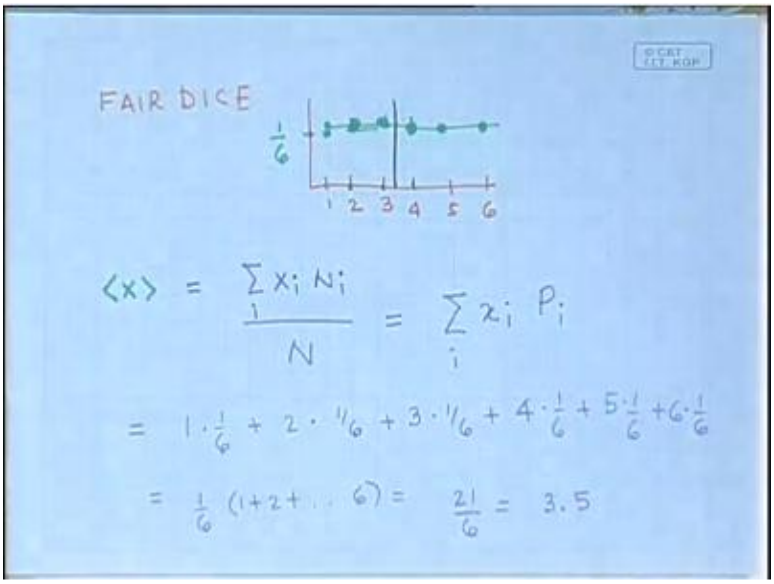

So, if I get an outcome to the deviation from the mean is minus 1.5, because I have to To calculate 2 minus 3.5 that is the expected outcome 2 minus 3.5 is minus 1.5.

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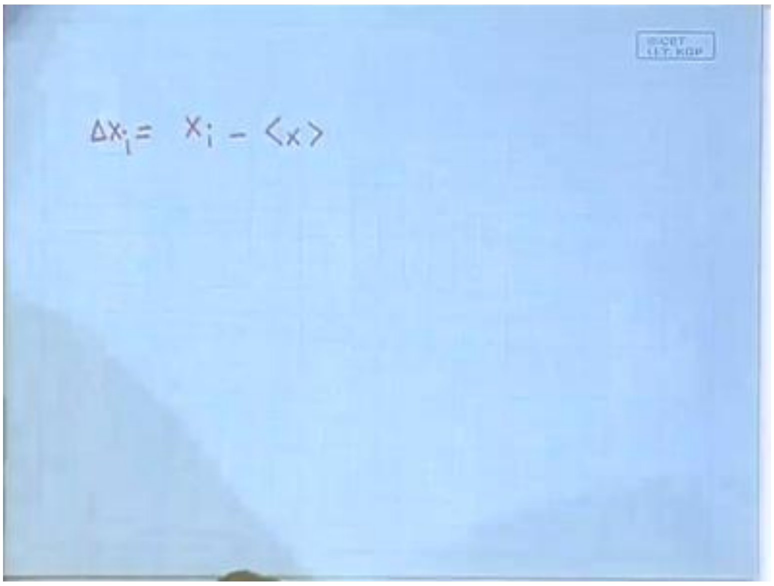

If I get 5 then the deviation is plus 1.5 if I get the 6 the deviation is plus 2.5.

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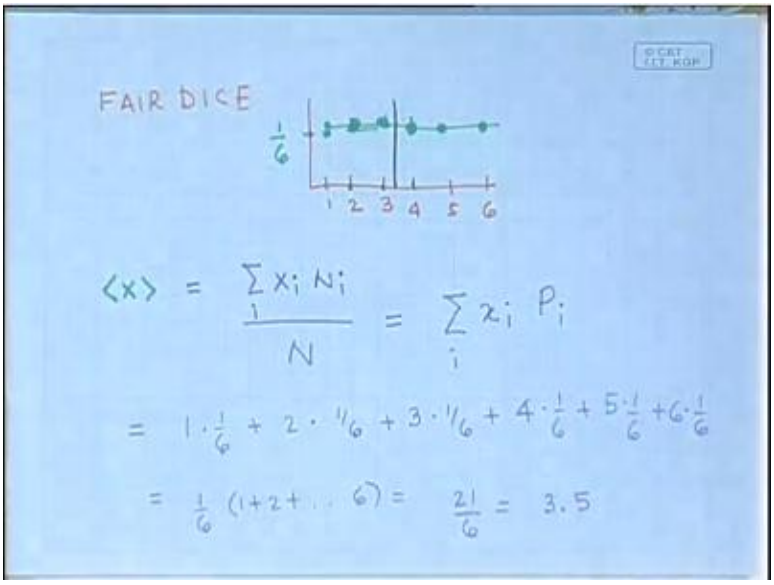

So, there is going to be for each other outcomes that you have whenever you get an outcome you can calculate the deviation.

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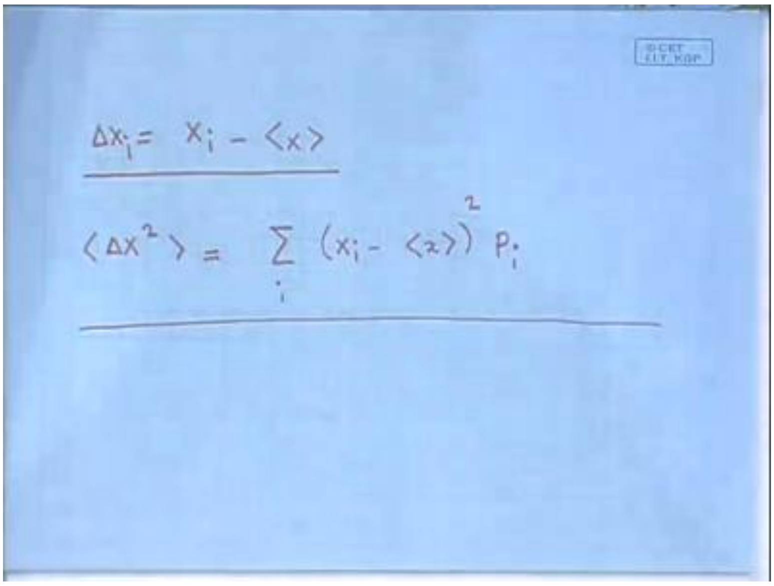

From the mean now, if you ask the question what is average deviation from the mean? This terms out to be 0, because the average outcome is expected to be exactly the same as this. So, if you calculate the average deviation from the mean by definition it is 0. So, you cannot use the average deviation from the mean to quantify the spread in the values. What you have what you can do is you can calculate the square of the average of the square deviation from the mean. So, that is calling the mean square deviation. So, take the deviation from the mean square it and then calculate it is mean.

So, take the deviation of the outcome from the mean square it and then calculate it is mean. So, you can calculate this like this in the following way xi minus the average value that you except square of this in to the probability of that particular outcome Pi. And this quantifies the spread in the values which I will get the expected spread in the values. Let us calculate this for the two possible for these two experiments for these possible dice. So, for let us do it for the biased dice first for the biased dice.

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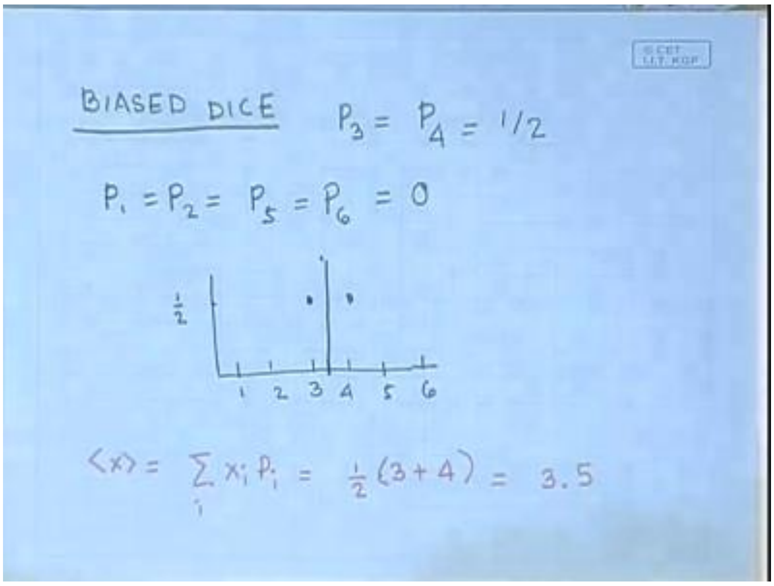

There are only two possible outcomes 3 and 4.

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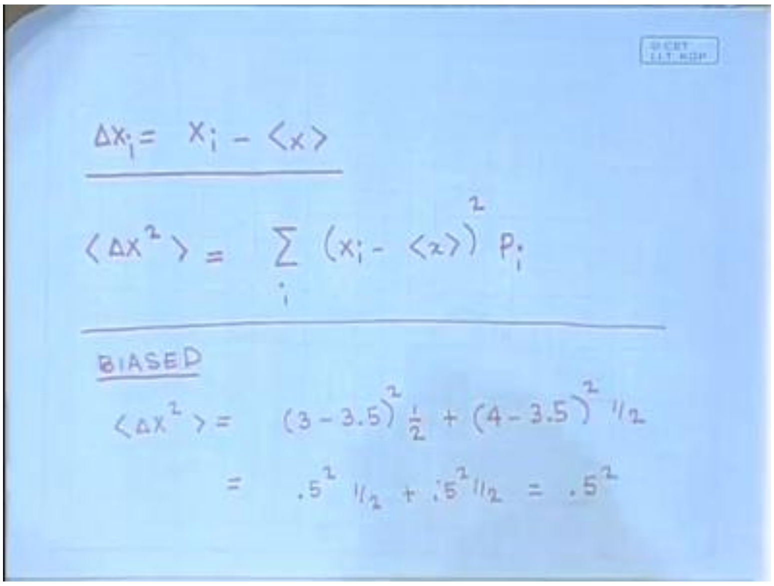

So, for the biased dice there are only 2 possible outcomes 3 and 4. So, if I have 3 then the deviation from the mean the mean values 3.5. So, if I is equal to 3 then let me calculate it delta x square the mean value is equal to 3 minus 3.5 square into the probability of getting 3 which is half plus 4 minus 3.5 the square of this into the probability of getting 4 which is half. So, this we can write as half there are these are the exactly the same this is 1 this is 0.5 square by 2 plus 0.5 square pi by 2 which gives me 0.5 square. So, this is what is refer to as the variance? The variance or the means square deviations for a biased dice the variance.

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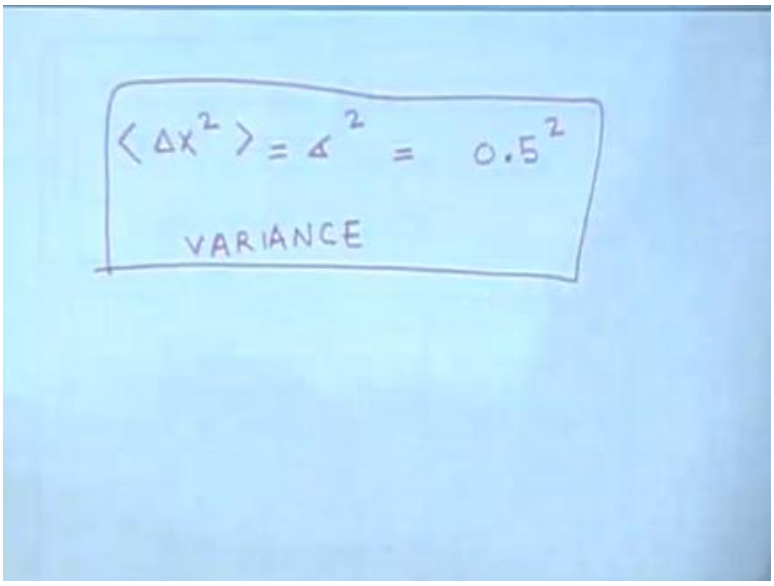

The variance are the mean square deviation this has the value 0.5 squared this is called the variance or the mean square deviation from the mean.

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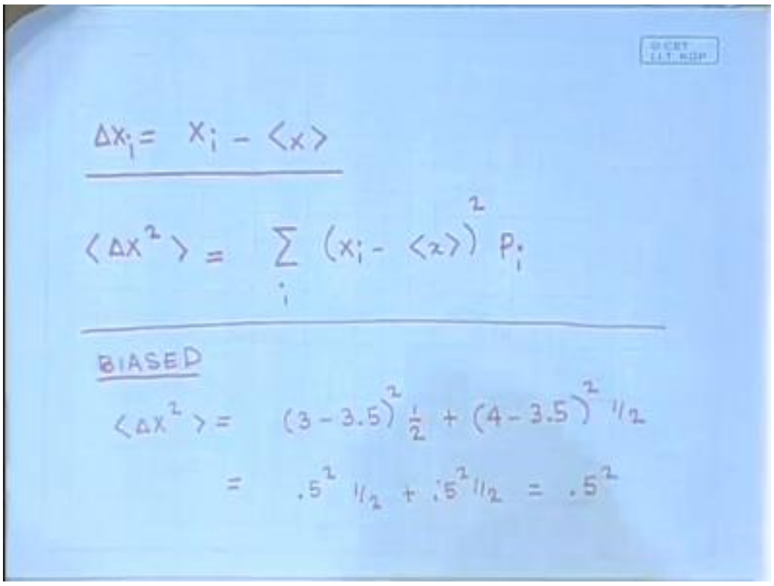

The mean square deviation right the deviation is the difference from the mean and this is the mean of the square of that. So, mean square deviation or the variance.

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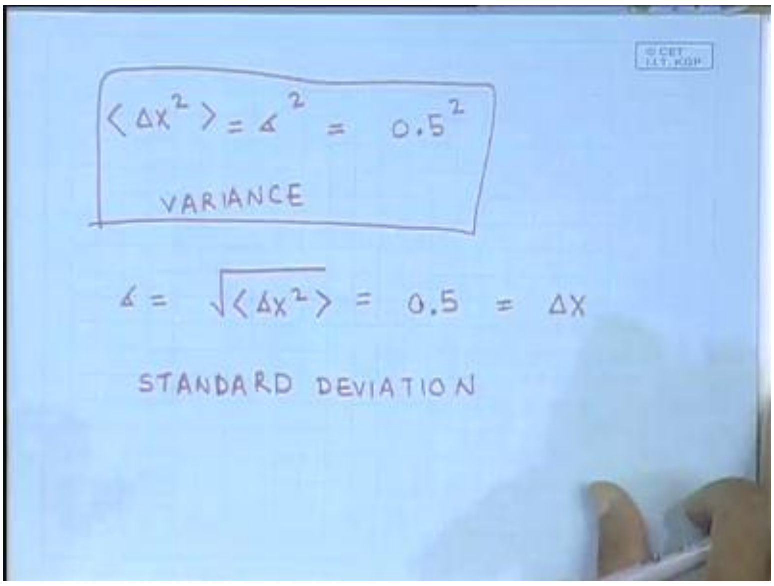

In this case for the biased dice it has the value 0.5 squared. The square root of the variance this quantifies the spread in the values of x this is the uncertainty in x. So, the expected value 3.5 tells me what we can are our prediction of what we value 3.5 is our prediction of what we expect. But we will never get this value when you actually throw the dice. So, there is uncertainty in our prediction and the square root of the variance the standard deviation quantifies the uncertainty and in this particular case the uncertainty is 0.5. So, you expect to get in this particular case there is these are the only 2 possibilities are 3.5 minus 0.5 and 3.5 plus 0.5.

So, that is the uncertainty in x in the value which you expect to get. So, this is called the standard deviation. And it quantifies the uncertainty standard this quantifies we can also use delta x to denote this it quantifies the uncertainty in the outcome. So, it tells you how much of a deviation you expect from the mean value. And in this particular of case for the biased dice it is 0.5. Let us now, calculate the same thing for the unbiased for the fair dice. So, for the fair dice…

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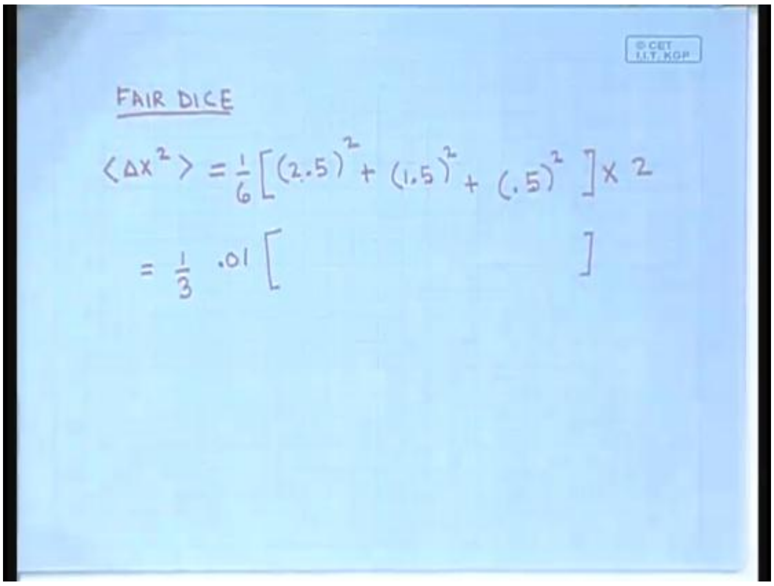

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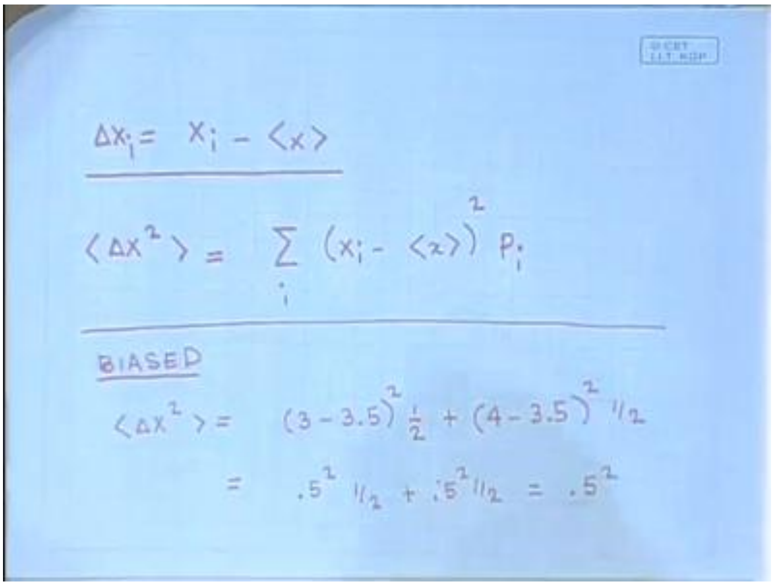

So, you want to now, calculate the variance for the fair dice so, there are 6 possible outcomes. The first outcome is 1 so, 1 minus 3.5 is 2.5 and it has a probability one sixth.

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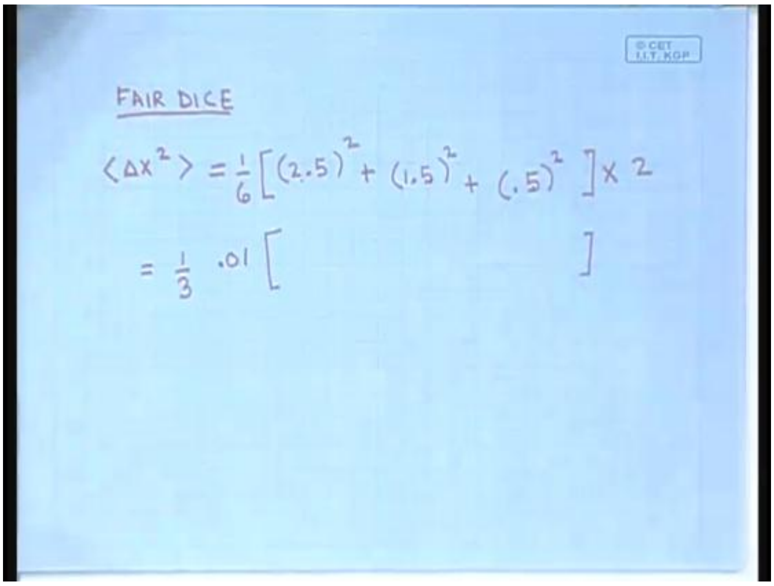

So, this is going to be 2.5 square and there is a factor of one sixth for all of the outcomes possible outcomes plus then you can have an outcome 2.

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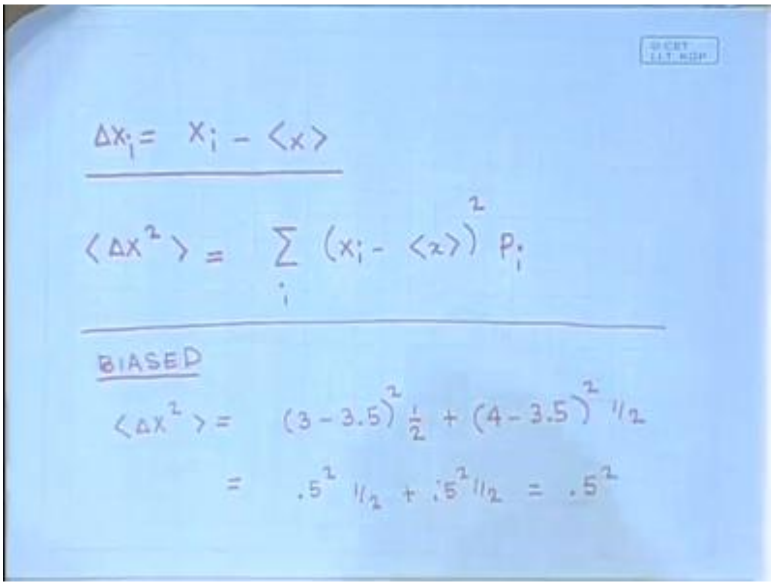

So, you have to subtract out the mean value from 2. The deviation for 2 is 2 minus 3.5 which is 1.5.

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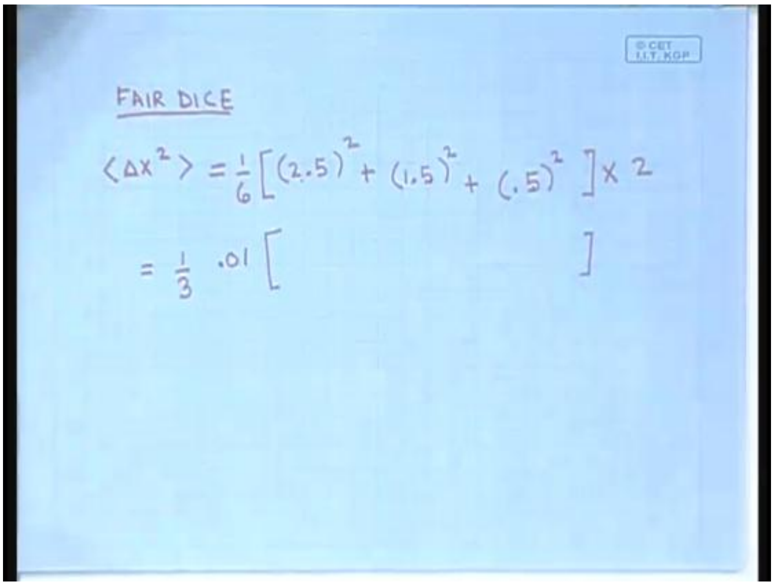

Plus 1.5 square plus 0.5 square, because you can have the outcome 3 for 3 the deviation from the mean is 0.5. And then if you get 4 again you get this for 5 again you get this for 6 again you get this so, it is basically this into 2. So, we have to calculate this is 1 third 2 divided by 6 it gives us factor of 1 third over here 1 by 3 and then 2.5 square. So, let us do 2.5 squares and then you have to calculate 15 square. So, and here you have to

calculate 5 square. So, we can take point 1 square outside. So, this into 0.01 and 25 square. So, let us let me work out 25 square over here 25.

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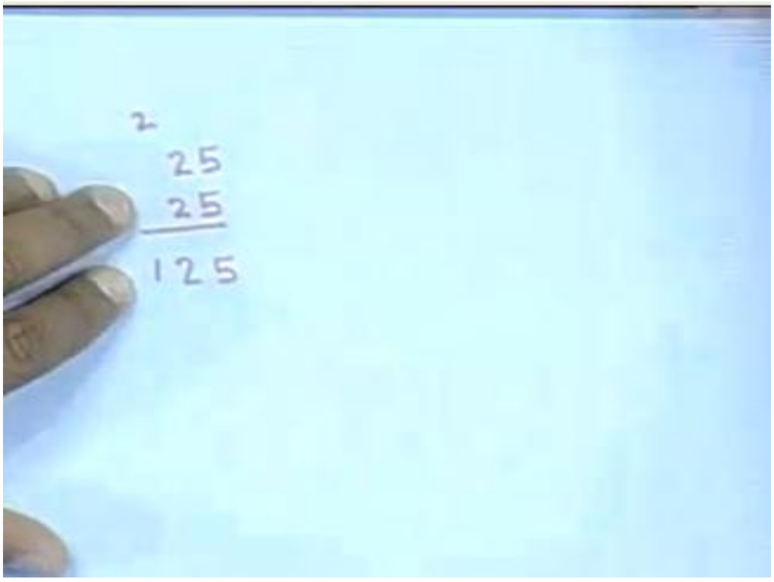

So, this gives me 5 125 and then here we have. So, this gives me 6 25 25 square 15 square.

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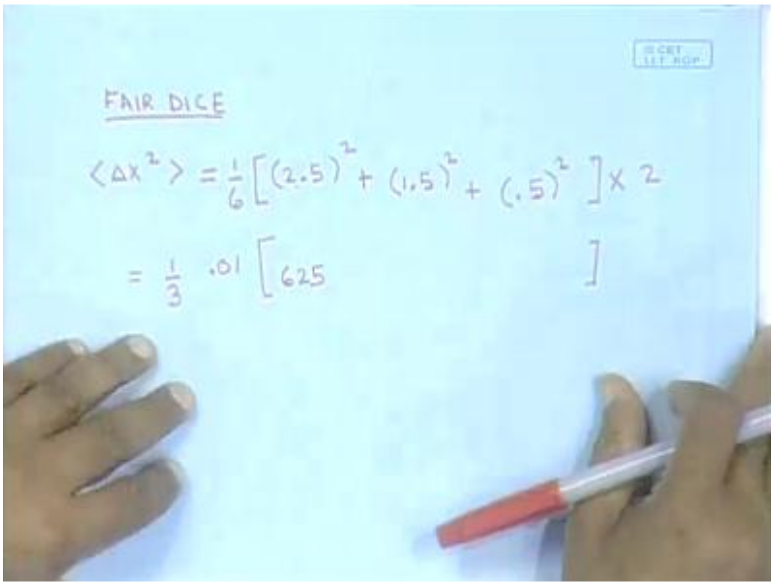

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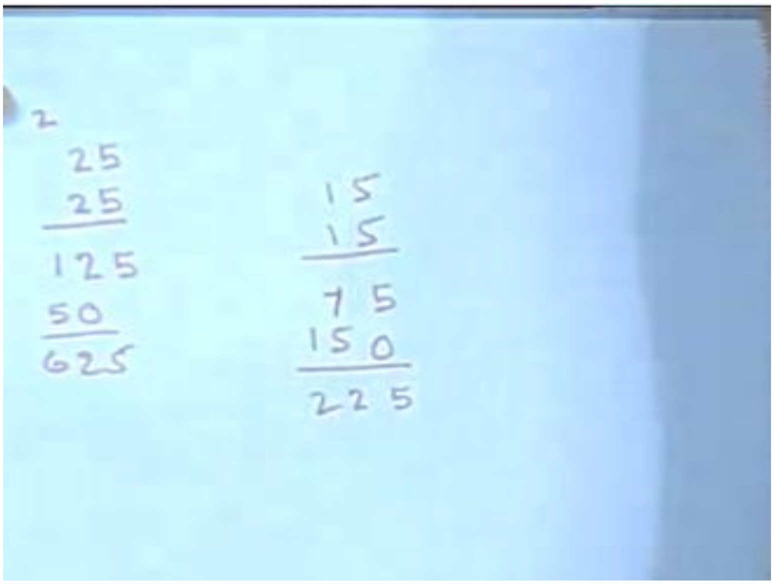

Again I have 5 75 and then I have 150. So, this gives me 5 12 225.

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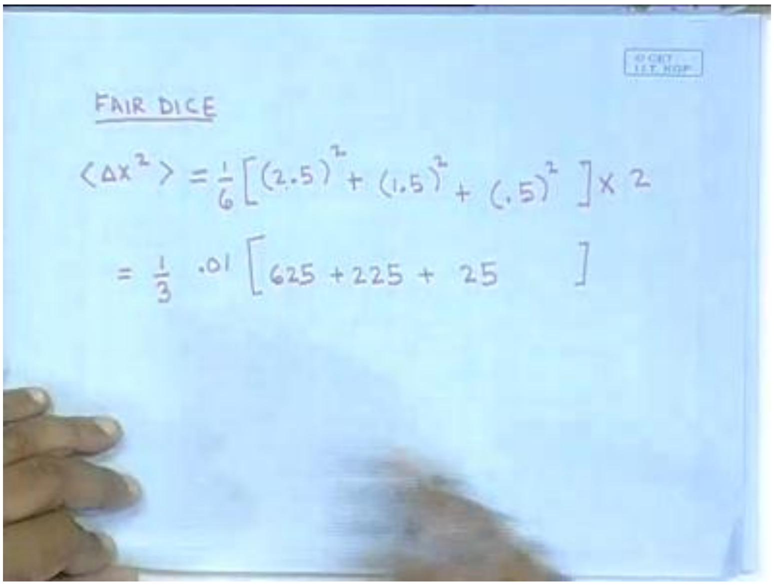

Plus 25 and this whole thing into 0.0 1 by 3 so, let me add up these.

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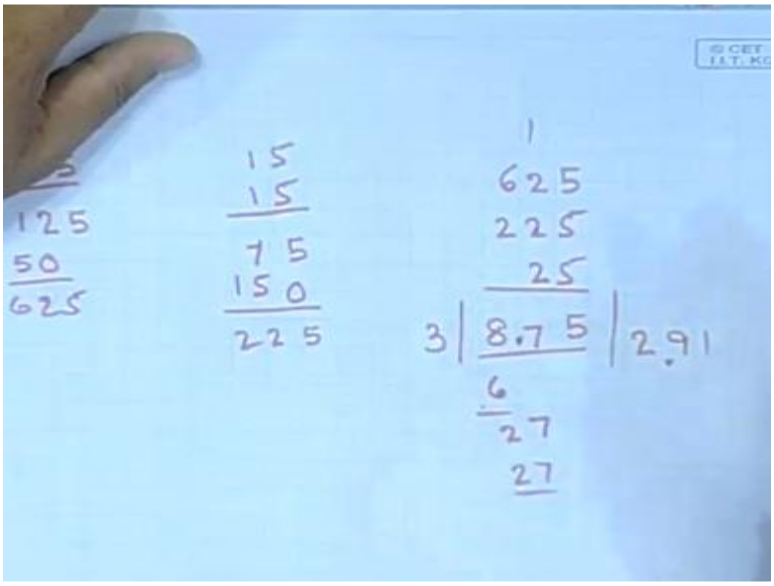

3 numbers 625 225 and here I have 25. So, 15 1 2 4 6 7 and then I have 8 over here 875. So, 875 divided by 3 so, it is 8.75 divided by 3 and if I divide this by 3 I will get a factor of 2 over here 6. So 20 27 2 and then 27 mean 9. So, it is roughly 2.9 and then 27 I have I have 0. So, 29 2.91.

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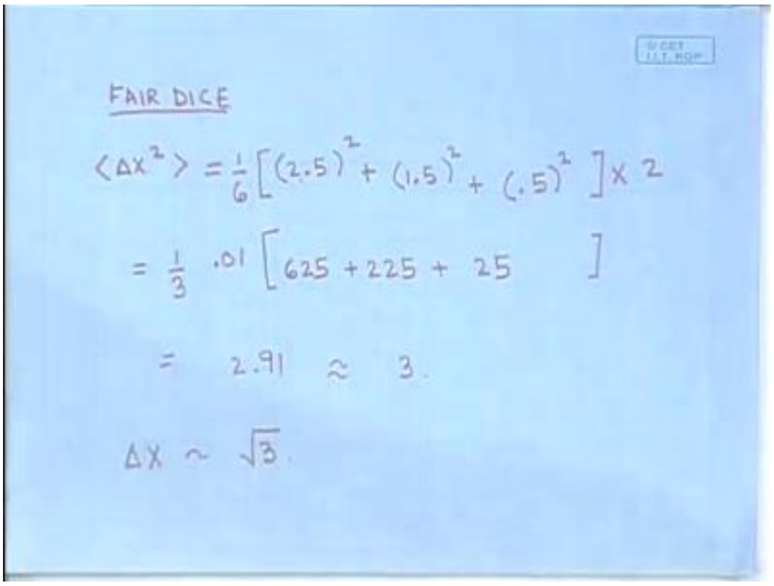

So, this is approximately of the order of 3 the variance. And if I take if take square root of this then it will the square root of uncertainty in x is of the order of square root of 3.

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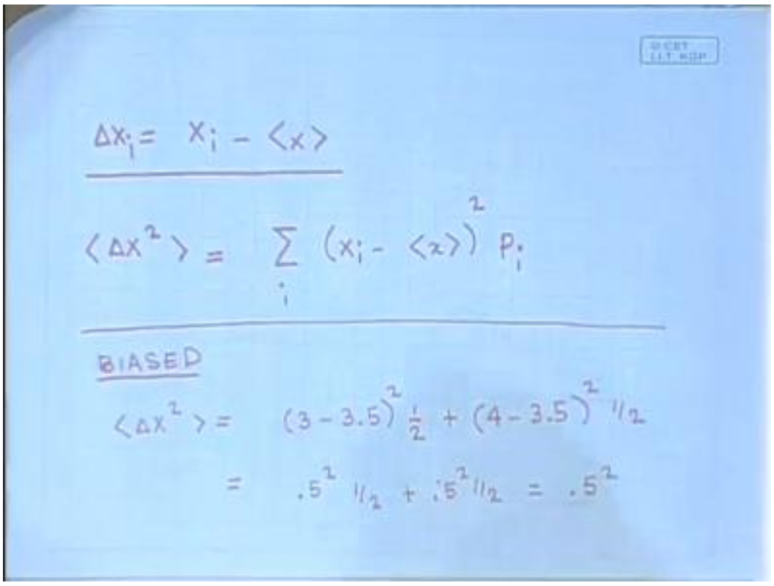

So, you see that the uncertainty in x is much large for the fair dice as compared to the the biased dice the biased dice has an uncertainty which is only 0.5

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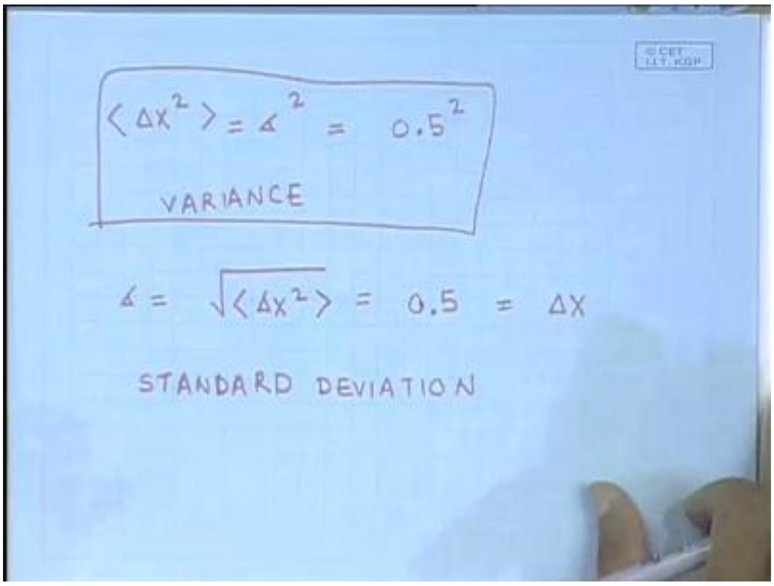

Whereas, here and the variance is 0.5 square where as here is of the order of 3 and the uncertainty the standard deviation is of the order of the square root of 3.

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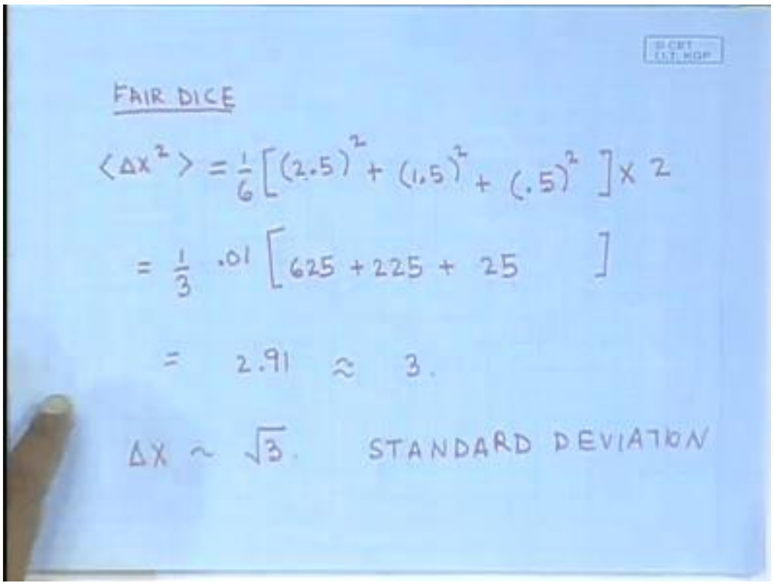

So, you have a much large spread in the values which of the outcome for the fair dice then you would have for the biased dice. So, what we have seen here is that the expectation value the mean value gives you tells you the outcome that you expect when you if you throw the dice once. But you will never get this value by enlarge what you will get is a spread of values around this value and the variance the standard deviation tells you the uncertainty.

That is there when you actually do the experiment you will get some other value not the expected value. And the uncertainty the deviation from the mean is what is quantified through the standard deviation or the uncertainty? Now, our discursion until now has been with a situation where you have only a discrete possibility as outcome. Now, let us deal with the situation where we have a continuous variable instead of district.

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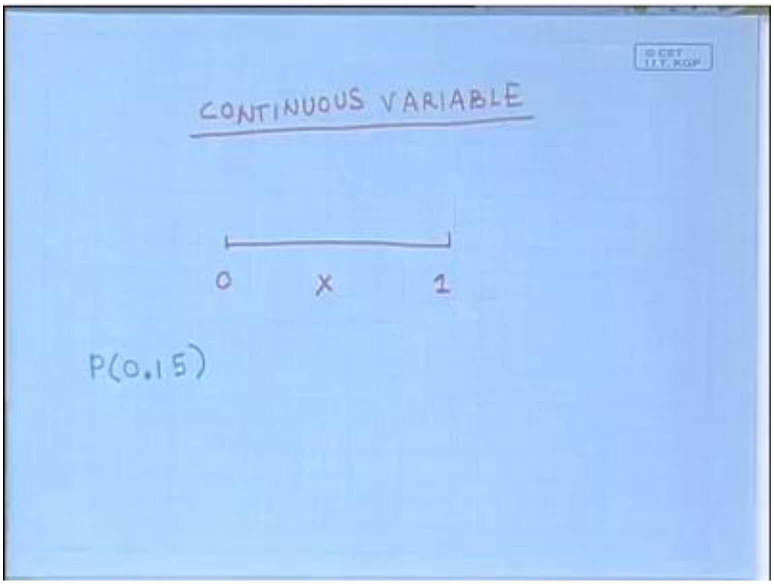

So, let us consider a situation where there is a particle the position the x position of the particle is somewhere in the range 0 to 1. I do not know where it is, but there is a particle which whose x position is somewhere in the range 0 to 1. Now, let us ask the question what is the probability? I go and look for the particle and i find it somewhere. And I know beforehand that the particle has equal probability of being somewhere in the range 0 to 1. So, the probability of it being anywhere in this range between 0 to 1 is equal. Now, I ask the question what is the probability that the particle is at the position 0.1? Let us say it is at the position 0.15 what is the probability that the particle is at the position at the point 0.15? Now, we have just a few minutes ago we have discussed how to calculate probabilities? So, what we have to do is we have to determine.

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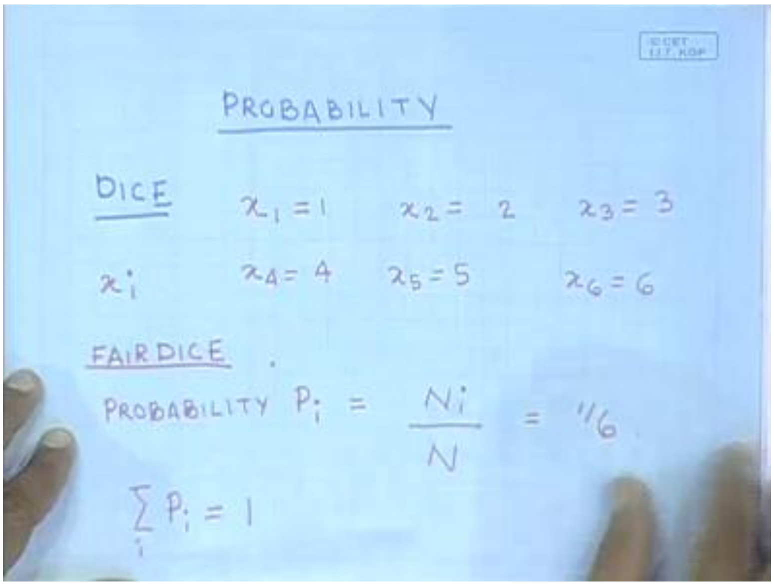

The total number of total possible outcomes the total number of successful outcomes divide that by the total number of possible outcomes.

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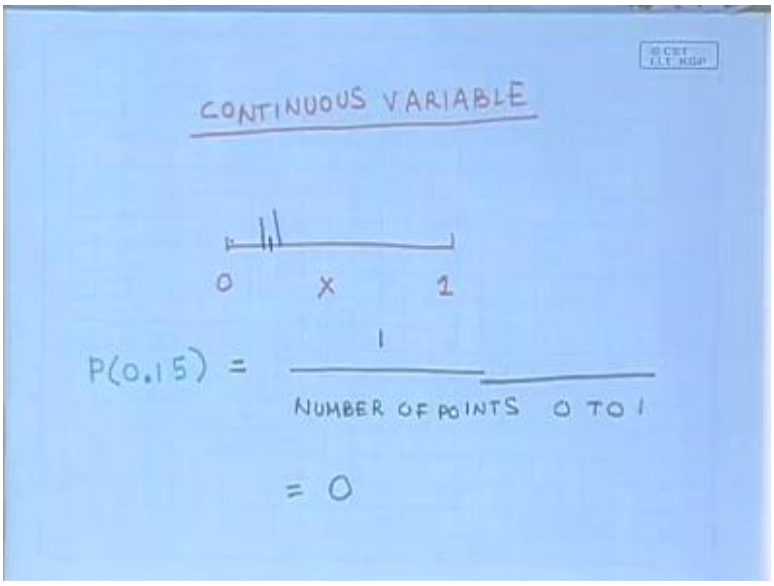

So, here the particle has equal probability of being anywhere in the range 0 to 1. And the question that we are interested in is what is the probability that it is a, at the point 0.15 at this particular position? Now, this you see is so, the particle can be anywhere between here and here. So, what we have to do is we have to count the number of successful events. So, the particle possible ways in which you can find the point here which is only 1 and divide this by the number of points in the range 0 to 1. So, this is only 1 point in amongst all the points that are there in the range 0 to 1 particle can be at any of these points with the equal probability. So, to find the probability of finding the particle at this point I have to divide 1 that is this particular point by the total number of points in the range 0 to 1. Now, we know that there are infinitely many points in the range 0 to 1.

So, this probability actually turns out to be 0, because there are infinitely many points in the range 0 to 1. So, essentially what the point here is that the question this kind of a question cannot be answered. So, if you ask the question what is the probability of finding the particle exactly at 1 point in the range 0 to 1 the probability will always turn out to be 0? The question the right question to ask is as follows. So, at the right question a question a correct question that could be ask here is as follows what is the probability? If I ask the question what is the probability of finding the particle in a small interval around this point? Then the question can give a correct then I can get the answer. So, for example, let us ask the probability.

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So, let us ask the probability..

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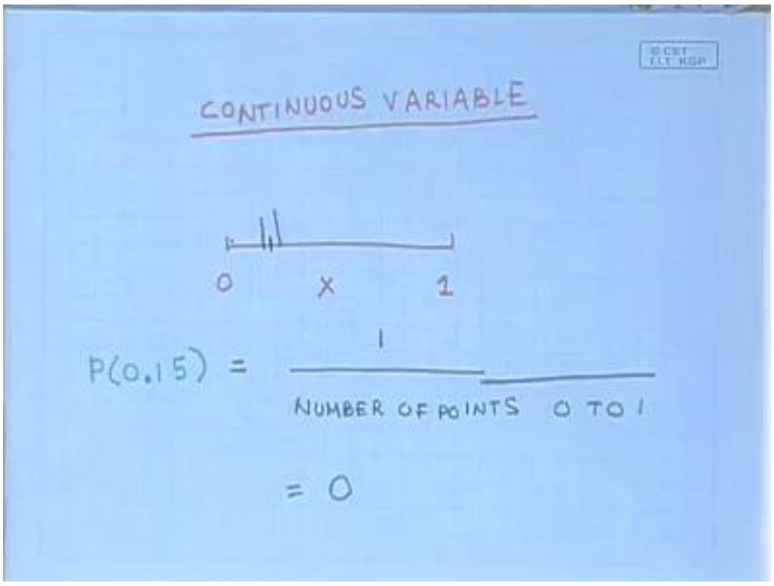

The particle is at in an interval around this point. So, let us choose the intervals.

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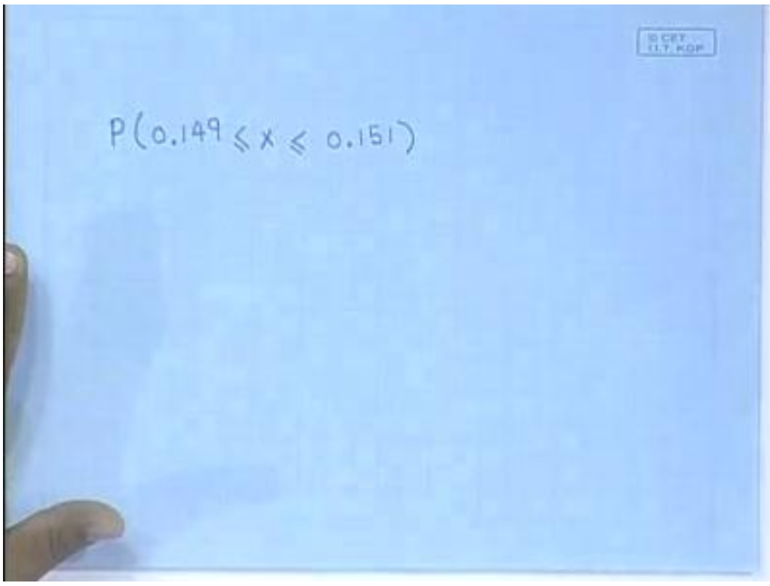

Is in the range 0.149 x is what is the probability that x the position of the particle is more than this and less than 0.151. So, this is a question whose answer you could get.

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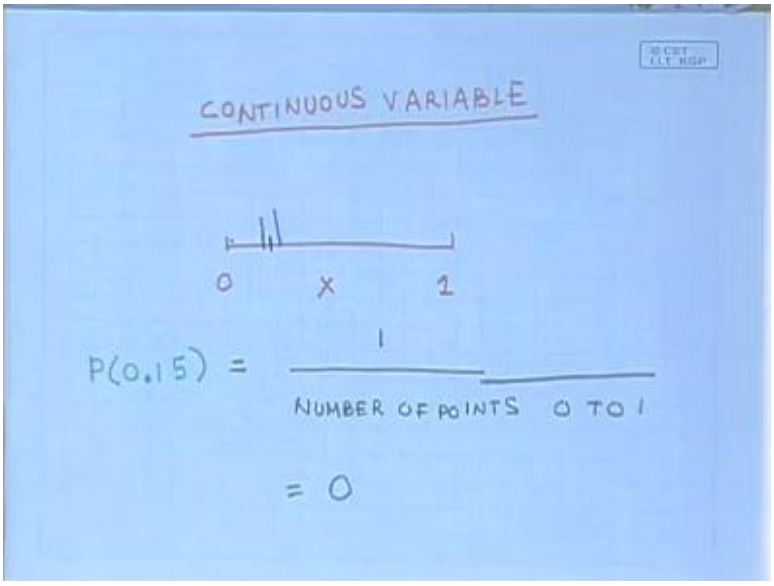

So, we have asked the question that we have asked is what is the probability that the particle there is found in the range 0.149? Which is, very close to the point 0.15.

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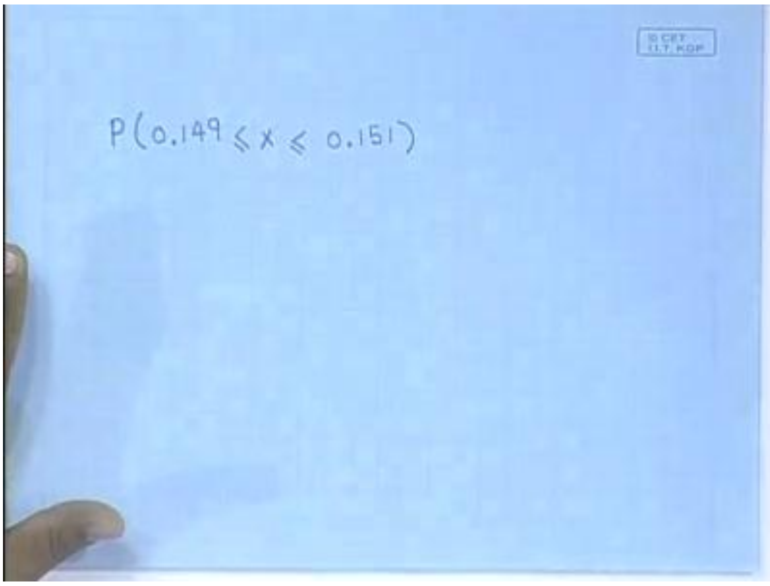

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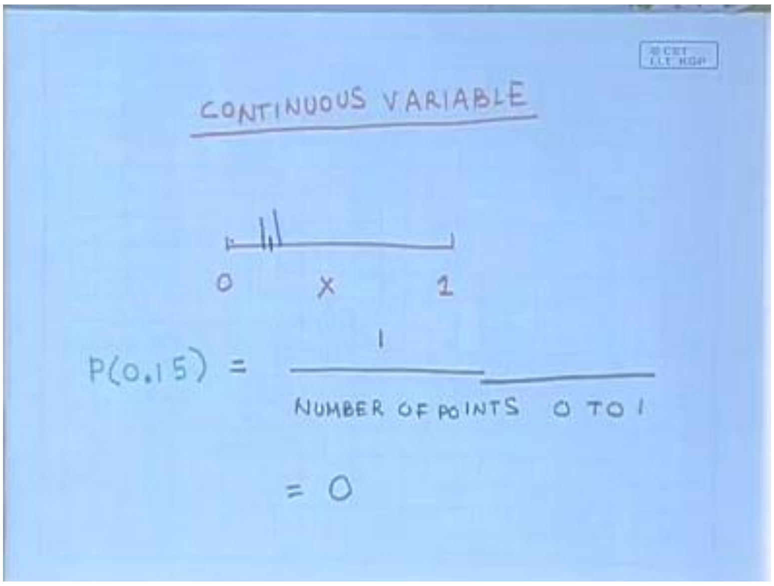

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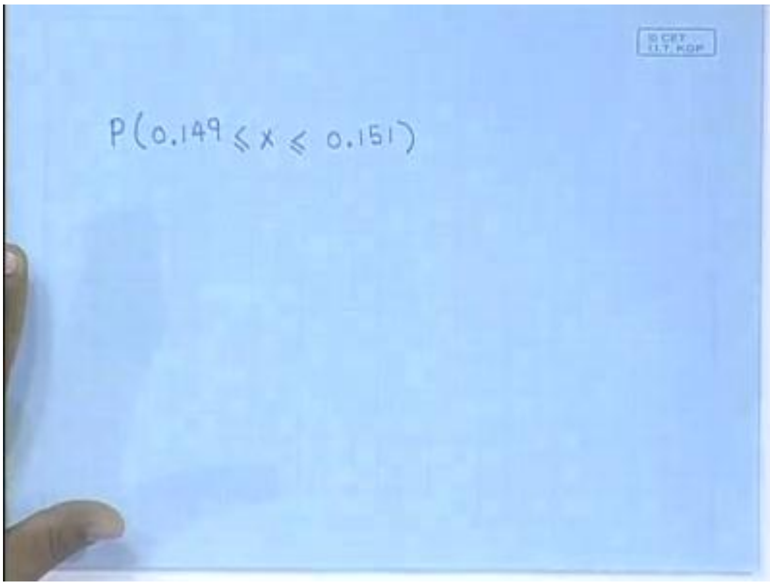

So, what is the probability we ask the question, we are asking is what is the probability that the particle is somewhere in this range?

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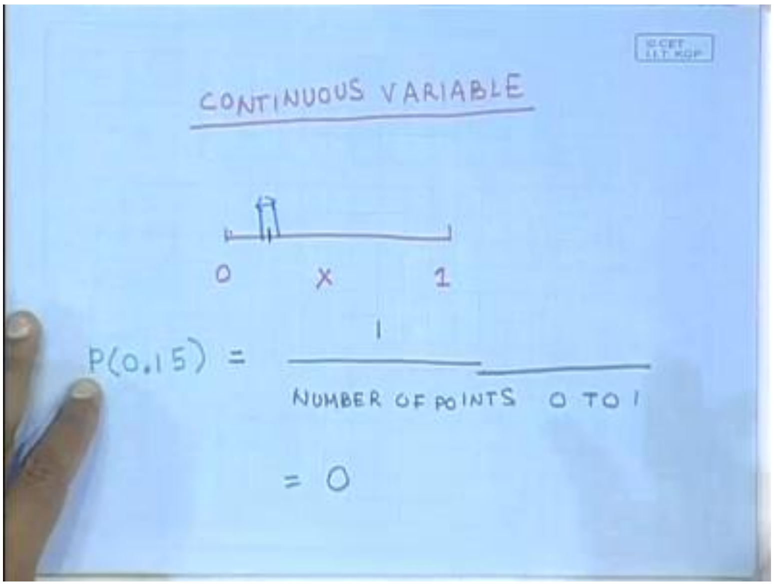

Now, this question can be answered, because you see the particles has equal probability of being anywhere in this whole range 0 to 1. Now, the probability of finding the particle in this particular interval is the ratio of the length of this interval to the total interval.

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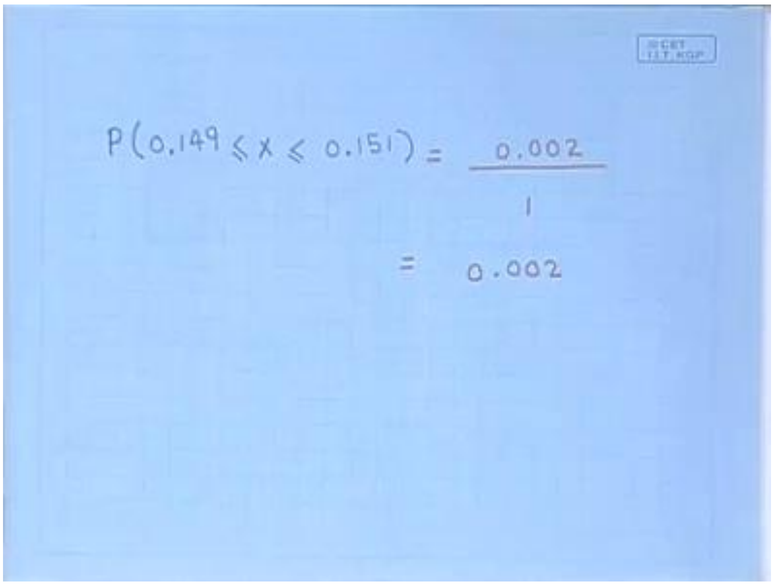

So, this probability will be the length of this interval the length of this interval is 0.002 divided by the of the total interval which is 1. So, this probability is 0.002 so, this the question which has a well-defined answer.

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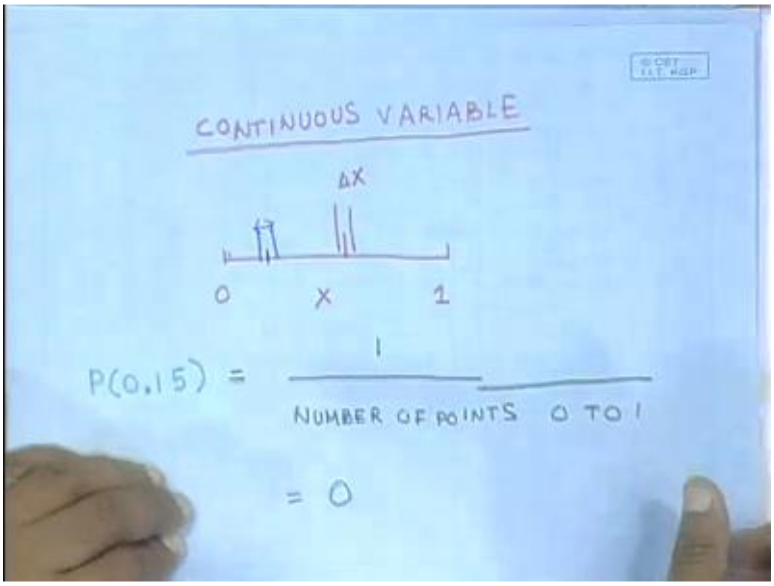

Now, let us ask the same question what is the probability of finding the particle in a in an interval delta x around the point x. So, in this particular case the probability of finding the particle in this interval delta x around a point x.

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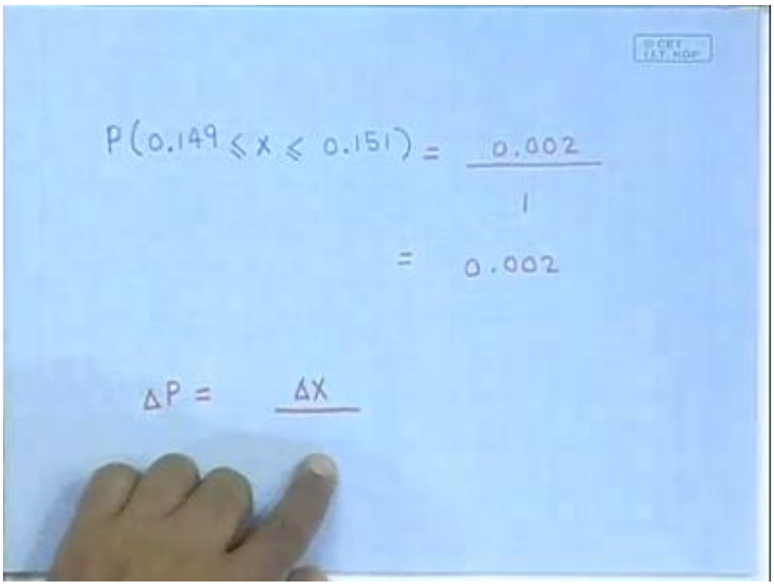

This we will denote by delta P, is going to be just equal to delta x divided by the total interval.

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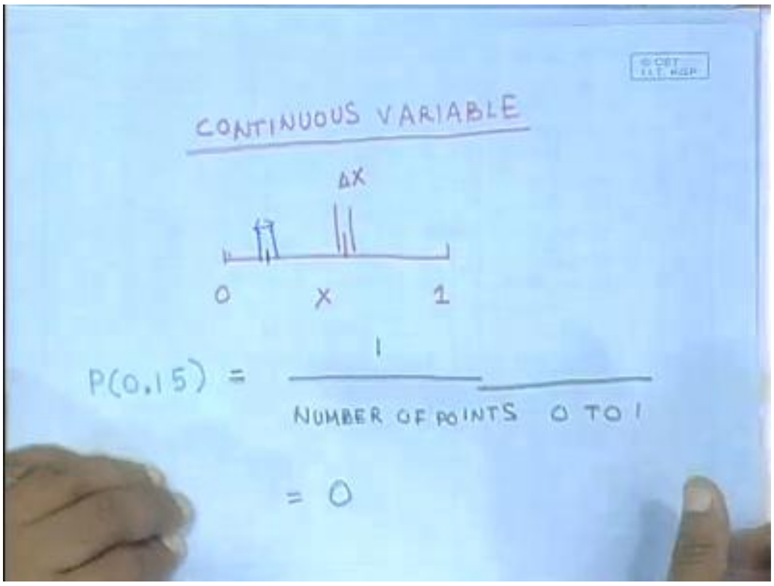

Total range is 0 to 1.

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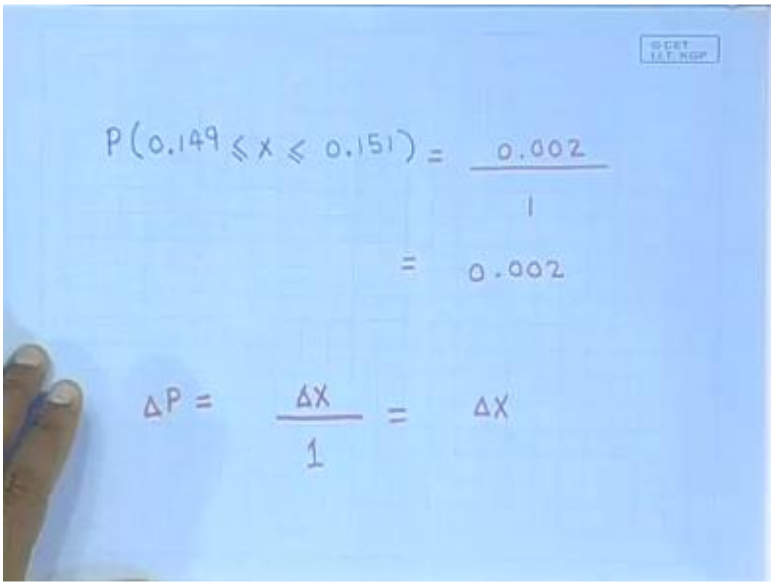

It is just equal to delta x divided by 1 it is just equal to delta x.

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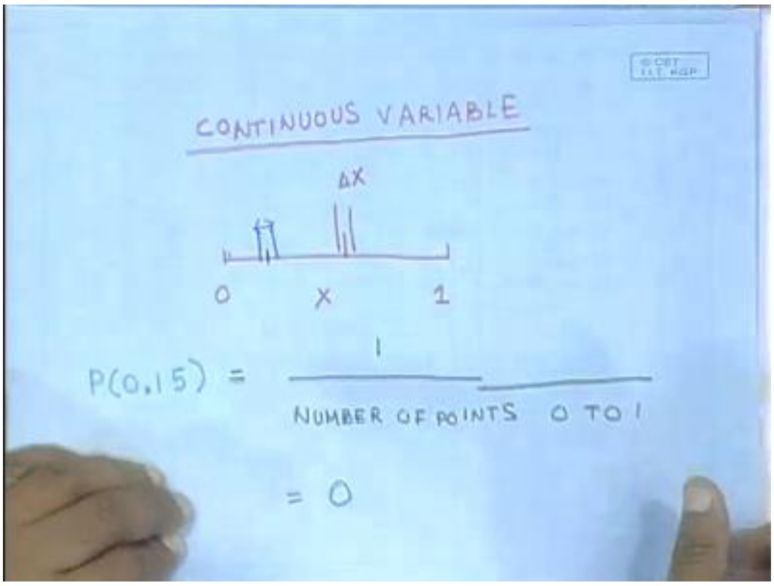

Now, here we have be considering a very specific situation where the particle is known to be in the range interval 0 to 1. And where the particle has equal probability the probability of finding the particle at sum value is the same in throughout this interval. Now, in general in general so, in a more general situation you could the question could be like this.

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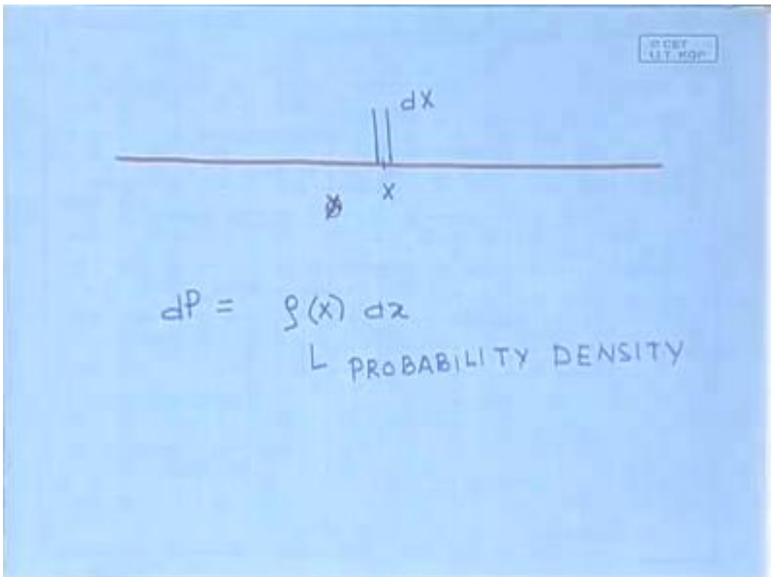

That the particle or the random variable x is sum where in along the real line and you could ask the question what is the probability of finding the particle in the interval dx around the value x? And in general this will be given like this. So, this can be written as rho x into dx where dx is the interval. So, the probability of in general when a particle position is random it is a random variable. So, x is as a random variable which can have any value any real value. Then the probability of finding the of getting the particular value of in the interval dx around the value x is the probability is dP of finding the of getting this particular value will be sum function call the probability density into the interval dx.

So, in the previous situation this probability density was 1, because the particle we knew was in the range 0 to 1 and the probability was equal everywhere. But the in general the probability density is sum function such that that function into the interval dx tells me the probability of finding the particle in that interval. So, the so, what we have see now is that when you have a continuous variable you have to deal with a probability density you cannot deal with the probability of finding the particle at some point. The question that you can ask is what is the probability of finding the particle in sum interval around a point? And this is given in terms of the probability density rho into the interval x. So, the probability of finding the particle in this interval is the probability density into the interval dx. Now, the probability we know is positive.

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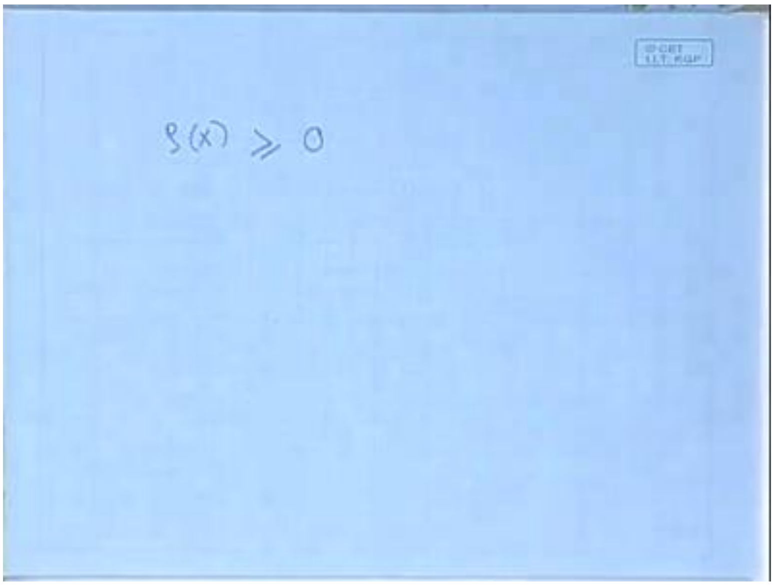

So, the probability density rho x is also necessarily positive that is the first condition.

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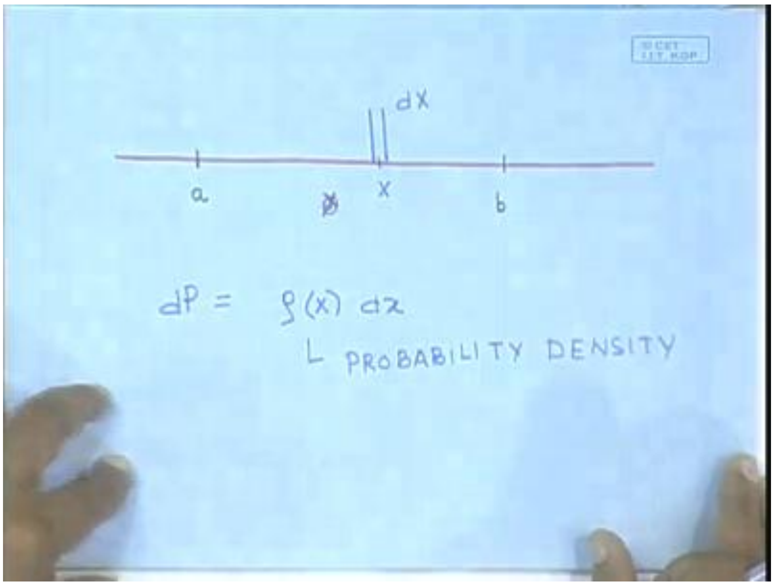

The second point we are discussing the properties of the probability density. So, the first property which I have told you is that the probability density should be positive. The second useful property is as follows if I ask the question what is the probability that the particle is in the interval a to b? What is the probability of getting, finding the particle in the interval a to b or more general I have a random variable x. What is the probability that if I make a measurement e if I go and that the outcome is of this random variable in one experiment is going to be in the range a to b. This can be calculated by integrating this from a to b.

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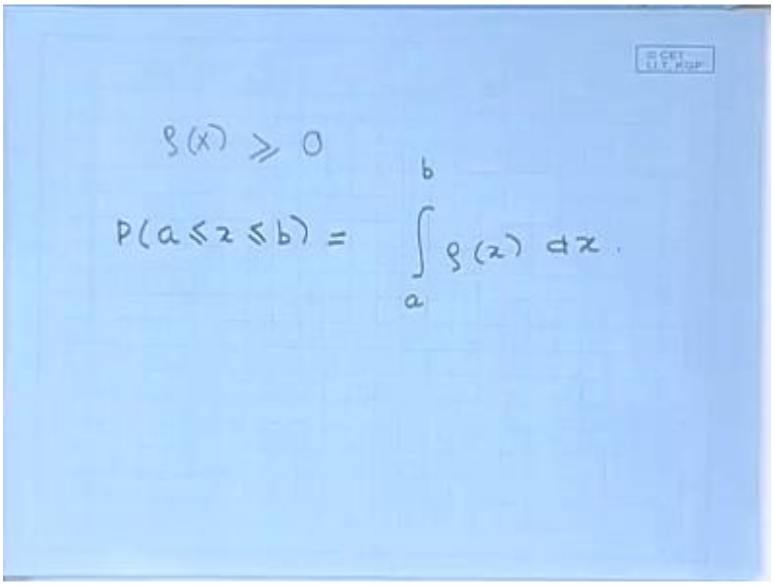

So, the probability of getting a value of x in the range a to be is a to b is the integral of the probability density between the limits a to b.

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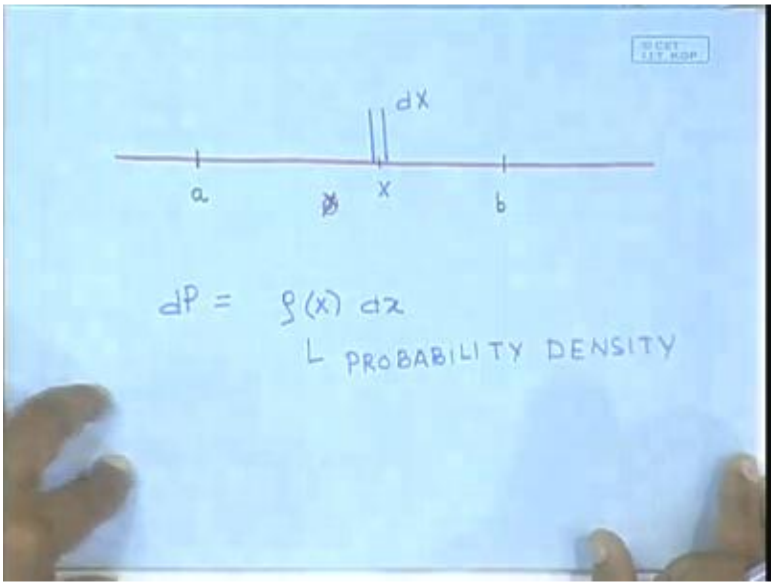

And if I ask the question what is the probability that the particle is somewhere has sum value that you get sum value of x in the range minus infinity to infinity?

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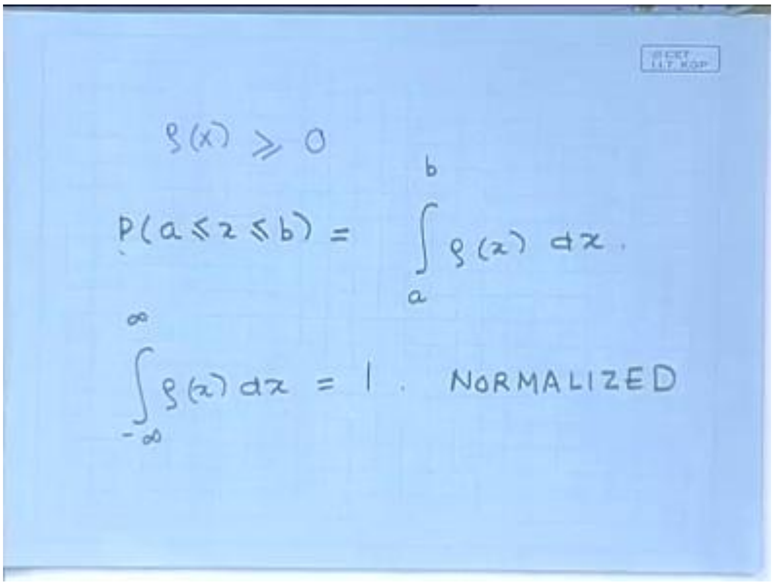

The total probability of finding the particles somewhere in the range minus infinity to plus infinity has to be 1. So, these are some of the useful properties of the probability density. So, whenever we deal with continuous variables like the position of a particle you have to introduce a probability density the probability itself is not of finding the particle at some at a position is not well defined it gives a value 0. But you have a probability density which tells you the probability of finding the particle in a particular interval and the probability density satisfies these properties. It has to be normalized this is what we mean by that it being normalized the total probability of finding the particle has to be 1. And this is how you can calculate the probability of finding the particle in some interval a to b and the probability density has to be necessarily positive. Let us take an example. So, the example which I am going to discuss is as follows.

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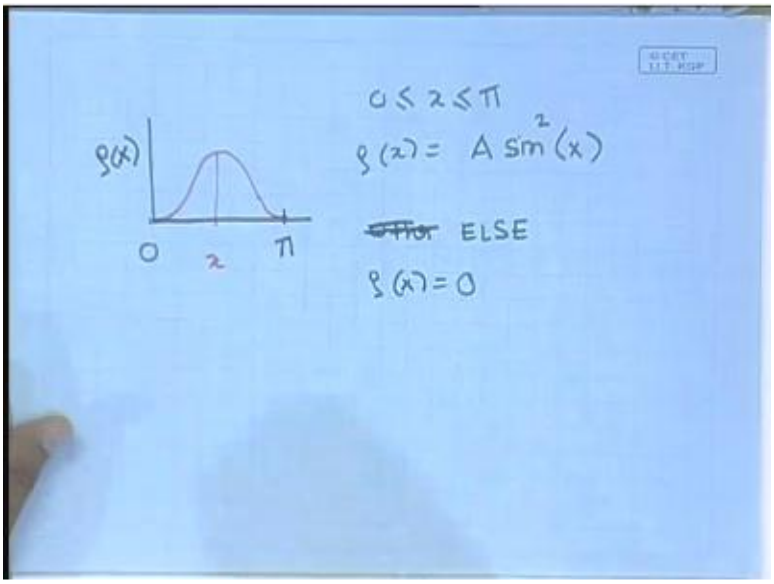

We are going to consider a particle which is restricted in the range 0 to pi so, the quantity I am plotting over here is the probability density. And the probability density rho x is A sin square x for x in this range 0 to pi and it is 0 otherwise. So, in this range it is sin square x in to a. So, let me draw it sin square x has maxima at x equal to pi by 2 and then it will it falls to 0 at x equal to 0 and pi and it will look something like this. So, this is the probability density now, the there is a unknown coefficient A over here we have to first determine this.

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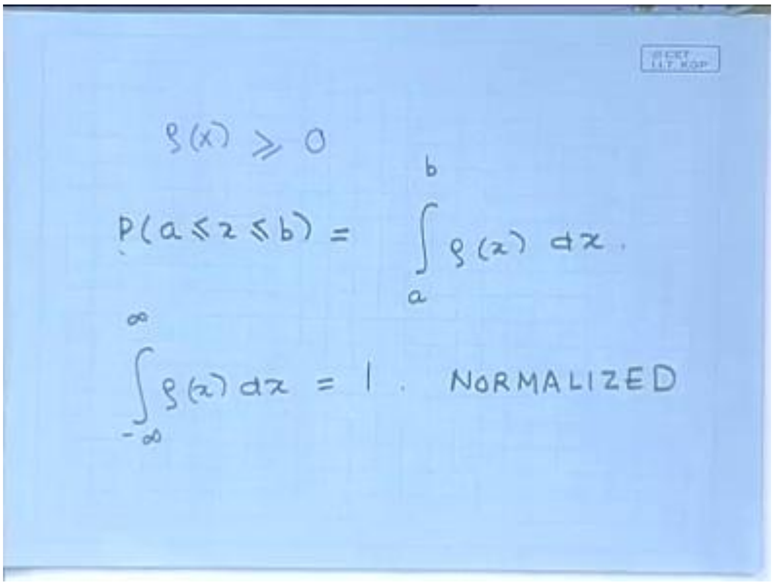

So, the probability density has to satisfy the property that the integral from minus infinity to plus infinity has to be equal to 1.

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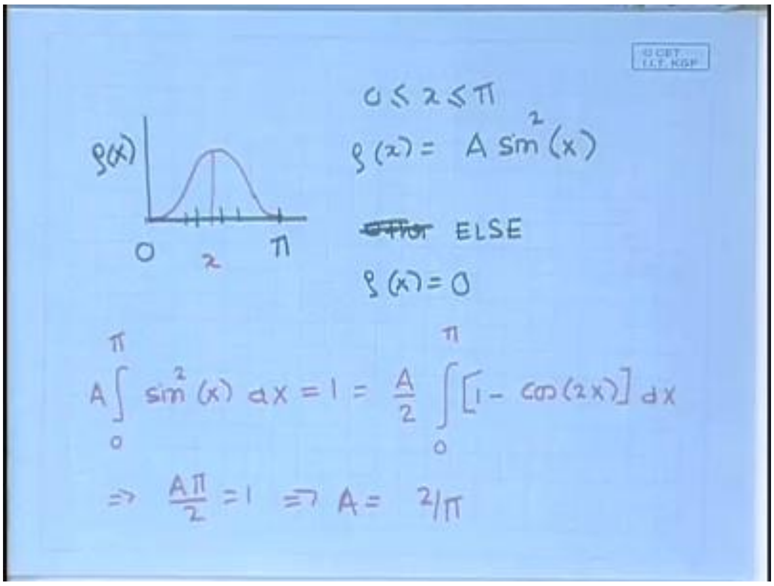

In this particular case the probability density is 0 outside the range 0 to pi. So, it has to satisfy the condition that 0 to pi A sin square x dx this should be equal to 1 now sin square x we know can be written as half into 1 minus cos 2 x. So, 1 minus cos 2 x divided by 2 is sin square x. So, this integral has to be equal to 1 now this integral of cos 2 x dx from 0 to pi is 0, because you are integrating over a whole period. So, what you have this tells you that pi when you integrate 1 from 0 to pi you get pi. So, A pi by 2 is equal to 1 or A is equal to 2 by pi. So, you have normalized the probability density and determine the value of this amplitude A A is 2 by pi. So, this gives us the normalized probability density now we could ask. So, whenever now you go and make a measurement of the particles position you will get a value which is decided by this probability density.

So, there is large probability you will get a value somewhere over here there is a very low probability you will get a value somewhere over here. The probability is 0 that the value will be at the ends right. So, this into the interval dx tells you the probability of getting that particular value a value in that interval. So, very time you make a measurement to will get a different value the question is now next question that we will take up is what is the expected value if you repeat the in for say you go and measure the position once. What is the value that you expect to get and as we have discussed this is the mean value. So, the expected value is the mean which can be calculated as follows.

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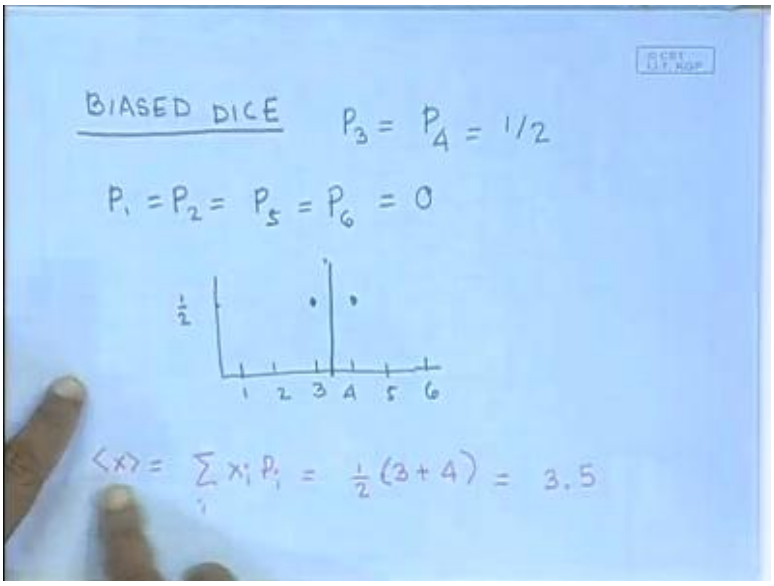

So, this gives the mean, the value that you expect to get if you do the experiment for 1. If the outcome of one measurement of the particles position what you expect to get is this. That is the mean value and this how you calculated. So, in this particular case you have the probability density you do not have probabilities you have the probability density defined like this.

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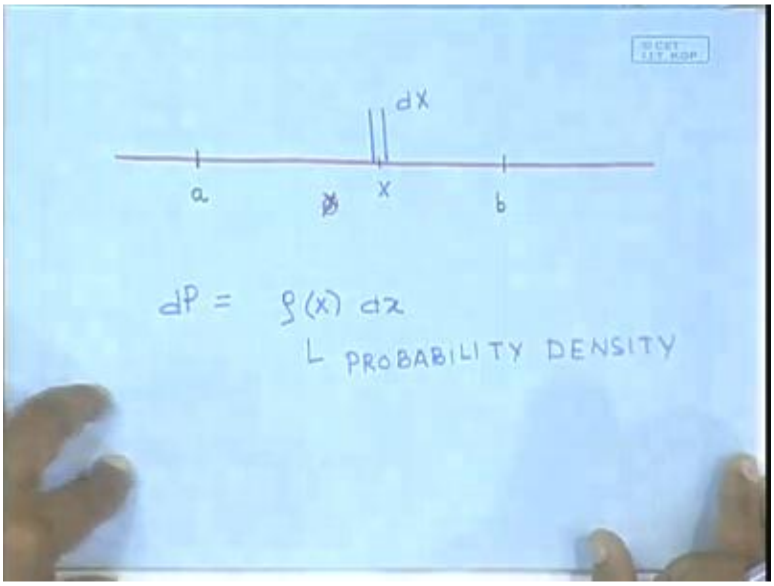

So, it should be very clear that the quantity the way you calculate the mean is as follows.

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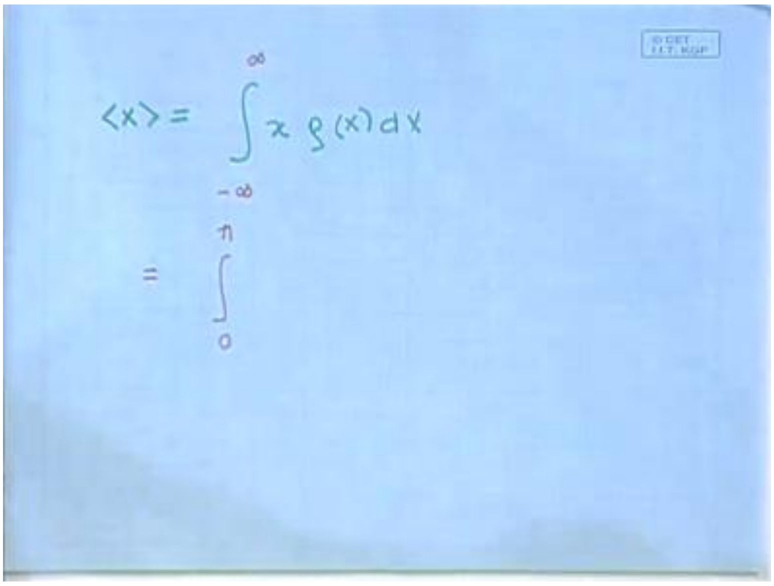

You this you have to replace the sum by an integral and x rho x d x this is essentially the probability of getting a value in an interval dx around the value x. So, you have to add up add this up for all possible values of x. So, this in the range minus infinity to infinity will give me the expected value. And in this particular case I have to calculate the integral 0 to pi the probability density we have already determined the normalized probability density it is.

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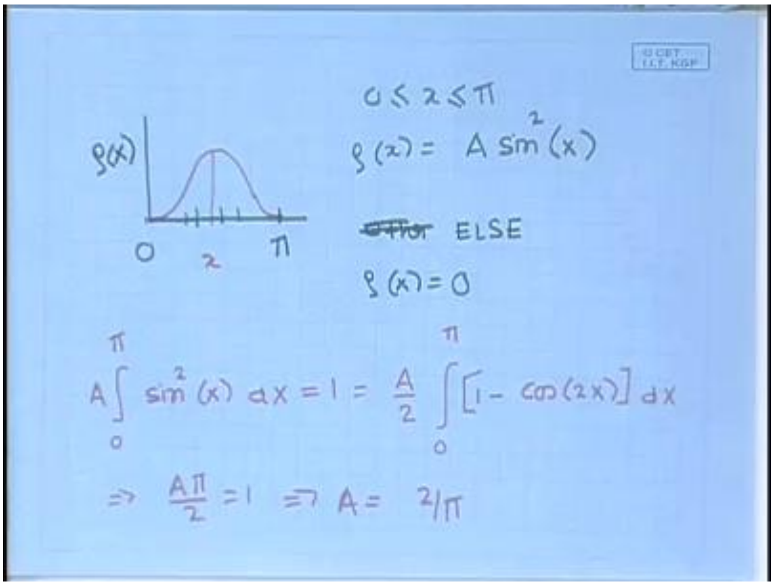

A sin square x. So, A has a value 2 by pi.

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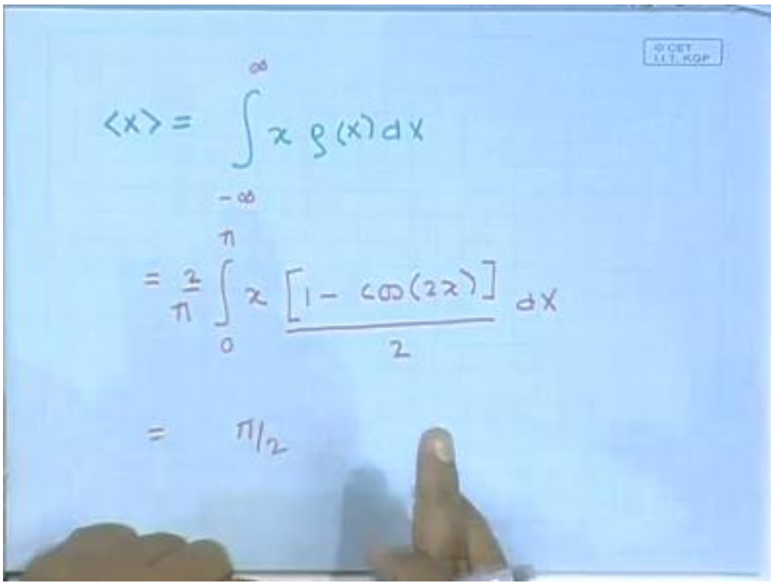

So, there will be a factor of 2 by pi here and we have to now calculate x into sin square x sin square x remember is 1 minus cos 2 x divided by 2 this is sin square x dx. So, we have to evaluate this integral and if you do this integral it will give you a value of pi by 2. So, the expected value the expectation value is right.

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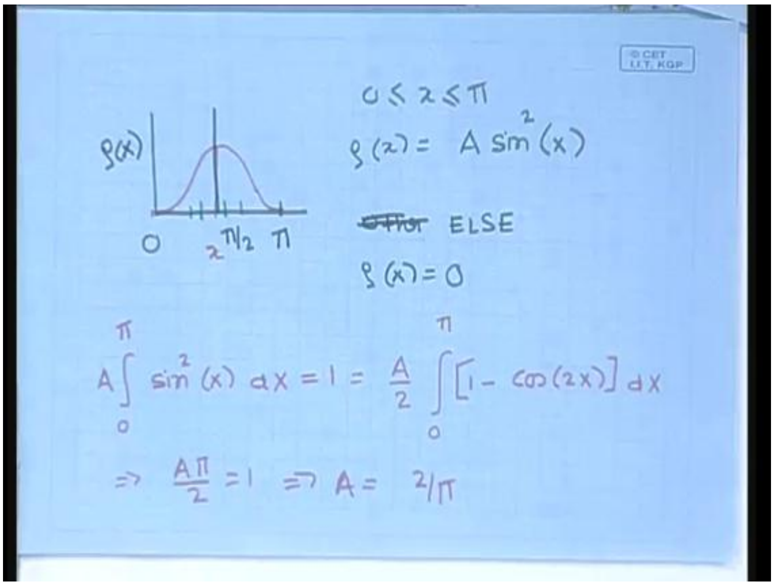

At the center in this particular case it is the value pi by 2. So, if you do the experiment this is the outcome that you expect if you go and measure the position of the particle this is the outcome that you expect. But when you actually go and do it you will never get you will not get this outcome in general you will get a spread of values. So, if you do it once you may get it here and if you repeat it you may get it here here here. So, the next question is how do we quantify the spread? And I have told you that the way to quantify the spread is through the standard deviation or. So, you first calculate the deviation from the mean square it and calculate it is expectation value. So, in terms of the probability density the way to calculate it, calculate this.

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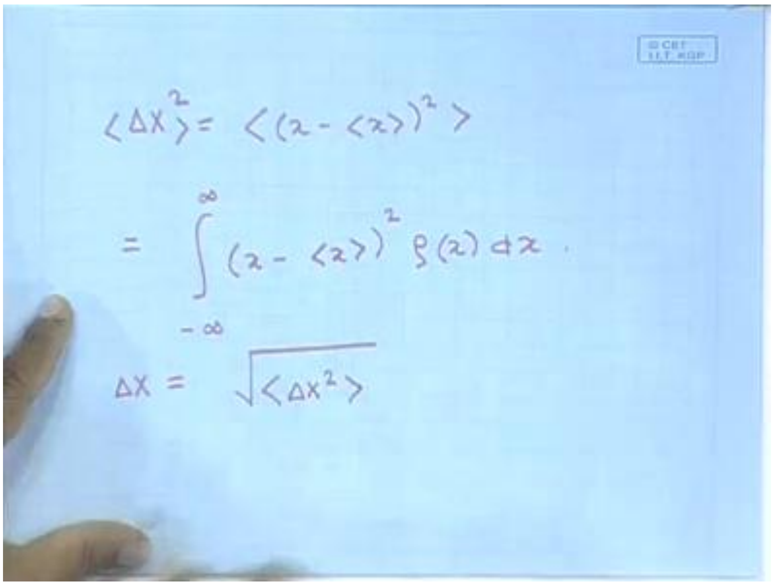

So, the quantity you have to calculate is the expectation value the of the deviation delta x square which is x minus the mean value square of this and then take the mean this gives the variance. So, in this particular case what you have to calculate is x minus the mean value which in this case is pi by 2 square rho x dx. So, you have to calculate this in the minus plus infinity. So, in this particular case you have to do the integral from 0 to pi x minus the mean value which is pi by 2 rho x is known and you have to do the integral from 0 to pi this will give me the variance.

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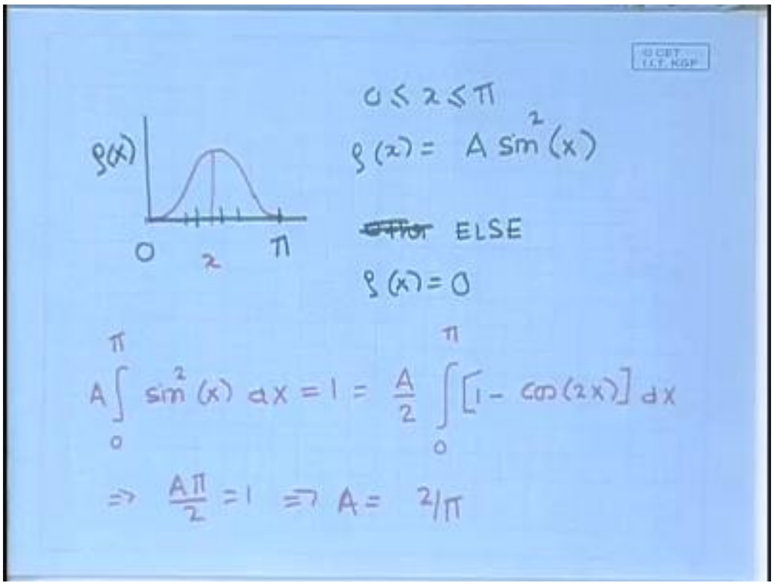

The square root of this will give me the uncertainty in x this will tell me the spread in the values the larger the uncertainty the more is the spread in the values of the outcome. So, if repeat the experiment I will get some values which are not going to match with mean by enlarge they will be spread out around the mean and this.

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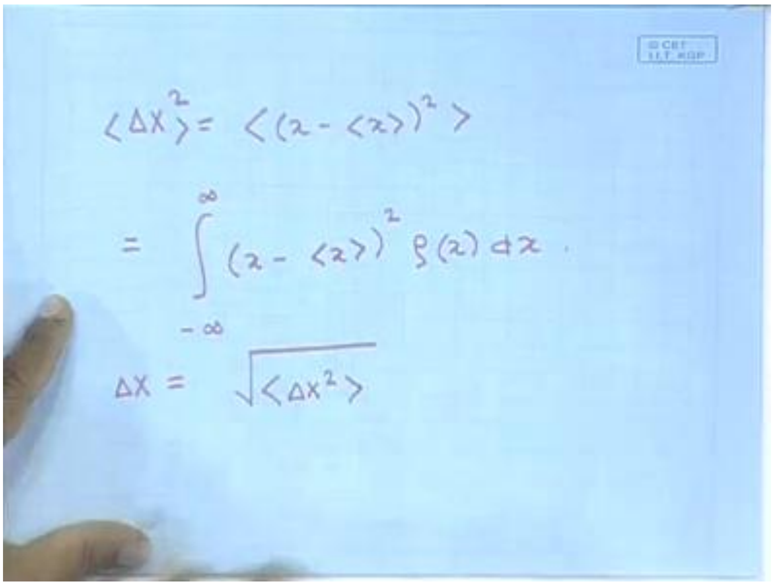

The uncertainty the standard deviation quantifies that spread and here I have told you I have shown how to calculate the standard deviation in terms of the probability density. Now, next we have to relate all of this to quantum mechanics now in quantum mechanics in when you are dealing with electrons.

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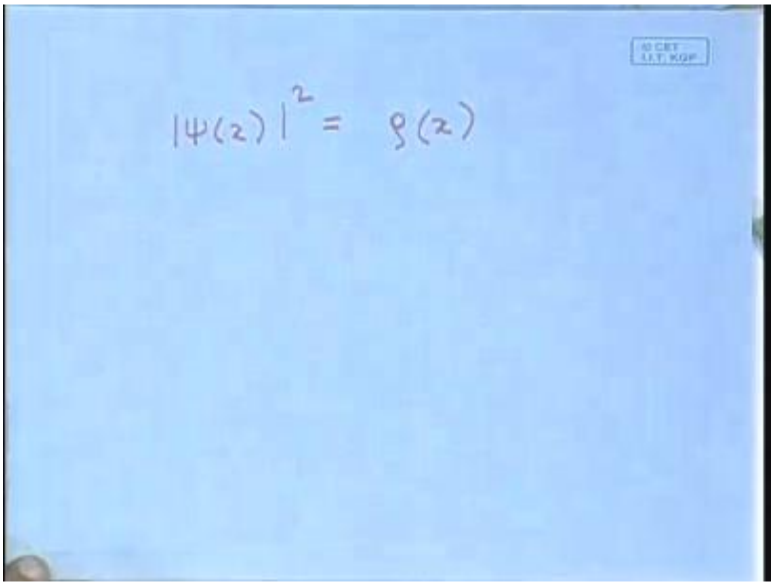

We have introduced the probability amplitude psi x. So, the modulus square of the probability amplitude is basically the probability density. So, this if you know the probability amplitude the modulus square of it gives us the probability density. So, once you know the probability amplitude you can calculate the probability density once you know the probability density. You can calculate where you expect to find the particle you can calculate the uncertainty that is if you do the experiment what is the expected spread around the mean value? You can calculate all of these. So, what we have what I have told you now is that how to what I have told you essentially how do how to interpret the probability amplitude or the wave function?

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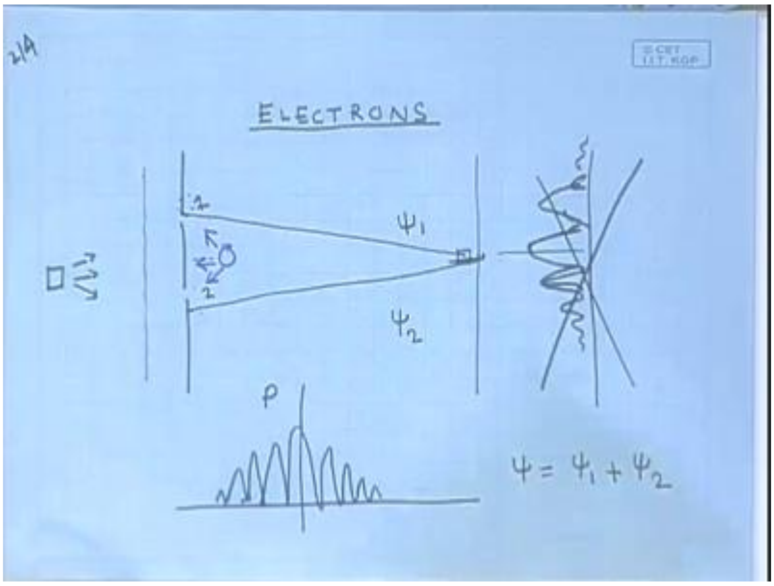

So, in the situation where we have an electron incident on 2 slits you can calculate the probability amplitude psi at any point on the screen by superposing the contribution from the 2 slits.

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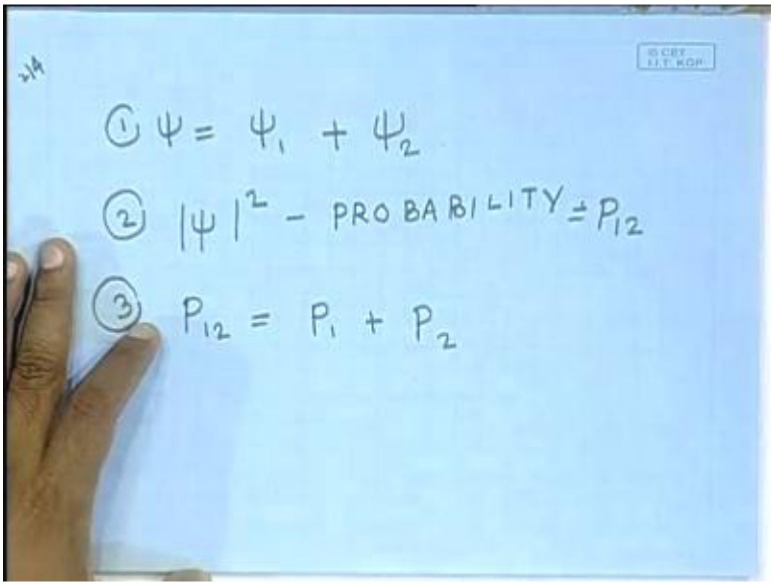

So, you can superpose the contribution from the 2 slits to calculate the psi at any point on the screen.

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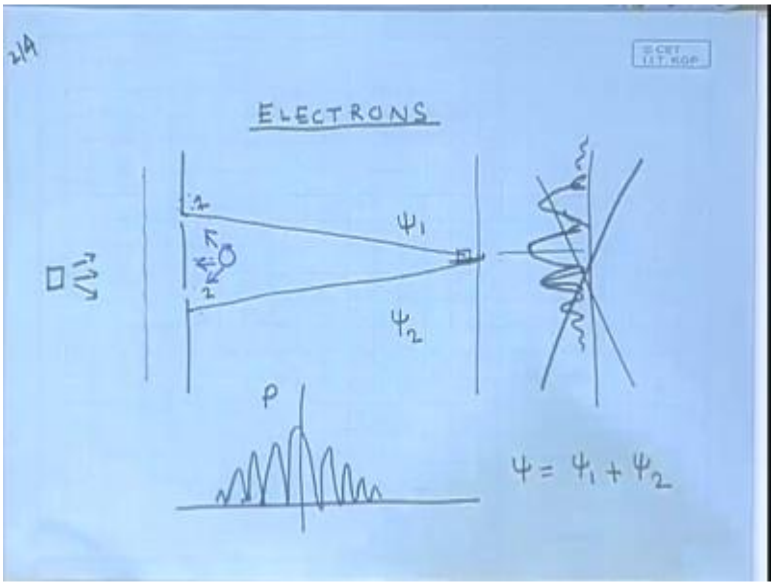

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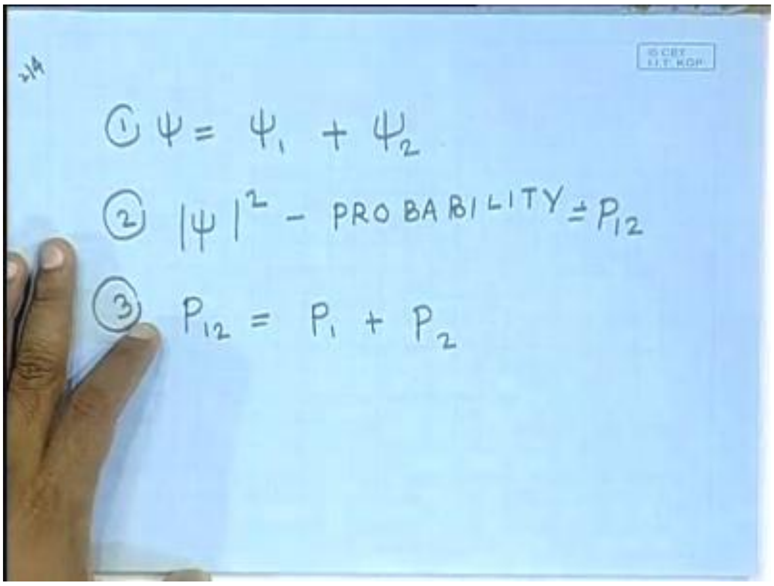

The mod square of this psi gives the probability density of finding the particle at that position.

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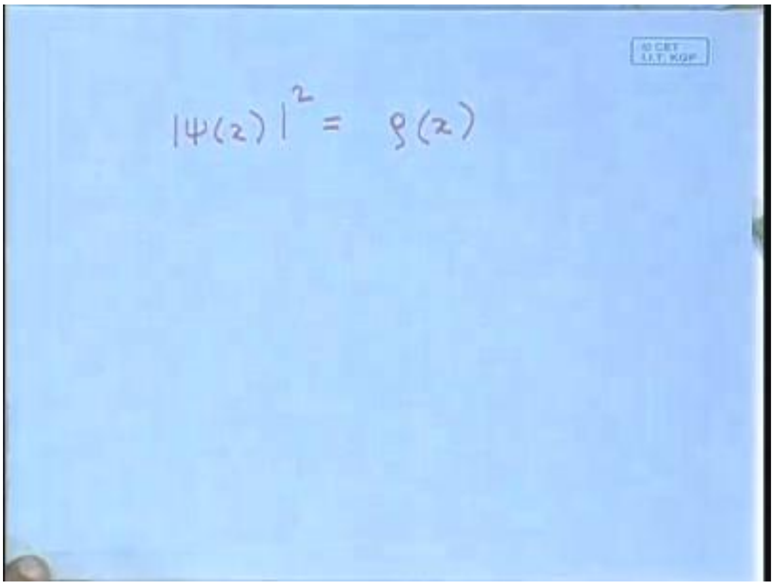

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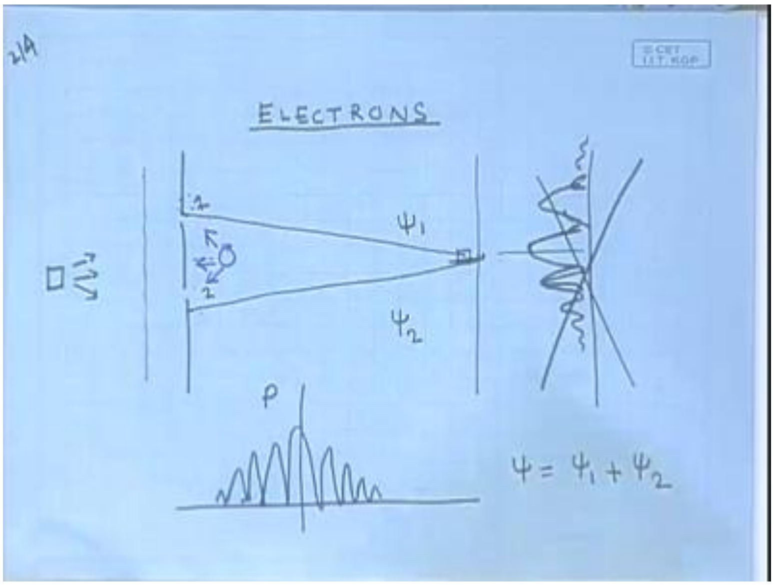

So, the modes square of psi at any point on the screen will give the probability density of finding the particle there. And you know how to interpret that probability density the probability density into the interval dx gives the probability of finding the particle there.

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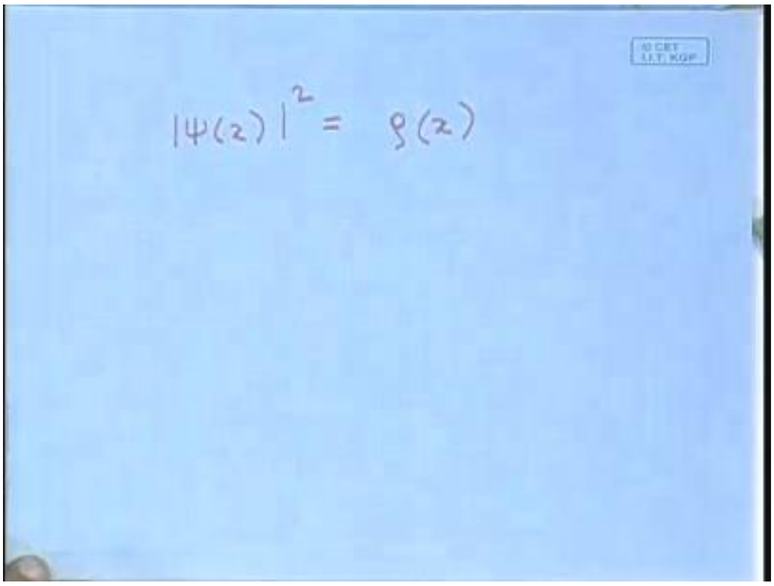

So, now we know how to interpret the probability amplitude or the wave function in the next lecture I am going to tell you how to calculate the probability amplitude or the wave function in some simple situations? So, let us stop here and wait till the next lecture where I will show you how to calculate this in a few simple situations.