Physics I : Oscillations and Waves
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Lecture - 34
Wave - Particle Duality (Contd.)

Good morning. Today we are going to start the lecture by considering a situation where we have a device.

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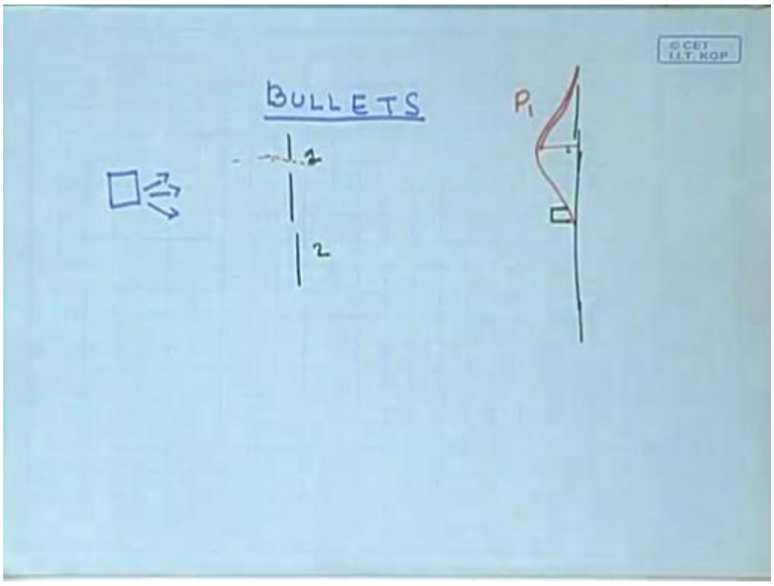

That sends out bullets. So, let me put bullets here, which sends out bullets more or less uniformly in the forward direction. And these bullets are incident on a screen the screen this this is the screen which has two slits we can call them slit 1 and slit 2. So, there is a device here which sends out bullets uniformly randomly in the forward direction and the bullets are incident on a screen. The screen is made up of some kind of a tough material which can absorb the bullets and the bullets are allow to pass through only through the 2 slits, which we have labeled 1 and 2. And the bullets which pass through are finally, incident on a on other screen over here.

So, we have screen here, and on the screen you have some kind of a detector which can be moved up and down which measures the bullets which gives you account whenever a bullet arrives here. Now, the, this device sends out bullets and bullets are always sent out 1 by 1 and the bullets go through the 2 slits and they finally, arrive on the screen over here. We have the detector which can record which gives you a record whenever a bullet comes and strikes it. So, and you can move this around to different places and count the number of bullets which arrive at any given point. So, let us now consider a situation where I first oh shut slit 2 and open only slit 1 in this situation.

So, silt 2 is short and only slit 1 is open, I can now move the detector to different points on the screen. And I can count how many bullets come and hit that particular point in a given time interval. The count is always going to be the bullet count is always going to be an integer we never are going to receive half a bullet. So, we are not considering the possibility where a bullet somehow breaks in 2 and then arrives at 2 different points on the screen. We are not going to consider such a situation. So, the situation which we are considering, the bullet either goes through the slit, and arrives on the screen or it does not go through the slit and it gets embedded in the material which surrounds the slit. So, you are always going to receive either 1 bullet or 0 at the detector.

So, if you place a detector let us say here and for a time period over a time period if you count the number of bullets that arrive here, and if you count the total number of bullets that were emitted. Then you can take the ratio of these 2 and calculate the probability that a bullet will arrive here through slit 1. And this we are going to call P 1 and if you move the detector to different positions on this screen and record the counts at different points. You can by this process determine the probability of the bullet arriving at any particular point on the screen, through slit 1. That is in the situation when only slit 1 is open. And let us draw what kind of what how we expect the probability distribution to look like.

So, when only silt 1 is open we expect most of the bullets to go through and you expect to receive the maximum number of bullets at the point just behind screen 1 the slit 1. There may be some bullets which arrive at go through this slit at an angle and, there will be a spread in the bullets around this point just behind slit 1. So, the probability of receive in a bullet on the screen are the probability of a bullet arriving at any point on the screen through slit 1 is going to have a maxima just behind slit 1. And then the it is going to have a spread and it is going to look something like this and we are going to call this P 1.

So, let me remind you again what P 1 is P 1 is the probability that the bullet arrives at any point on the screen through slit 1. And this is expected to have a maxima somewhere just behind the slit just behind slit 1. And there is a going to be spread, because the bullets may not be exactly in this direction the bullets may be in the other direction also. And because of this there will be a spread in the bullets on the screen and the probability is expected to have something which looks a pattern which looks like this. Now, let us consider the situation where we shut slit 1 and open slit 2 and repeat the same experiment.

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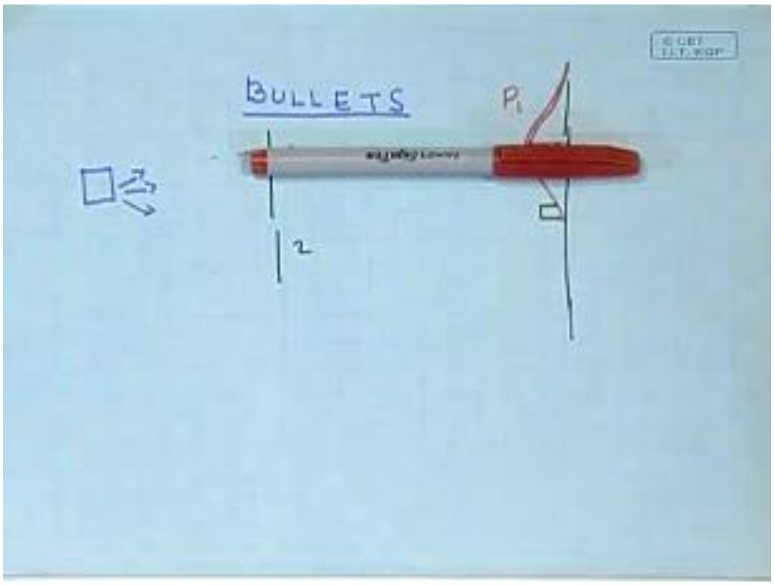

When slit 1 is shut and slit 2 is open, we expect the probability of a bullet arriving on the screen to be maximum, somewhere just behind slit 2 somewhere over here. And again there may be bullets at different angles there may be a spread in the angles in the bullets that arrive over here from this device that emits the bullets.

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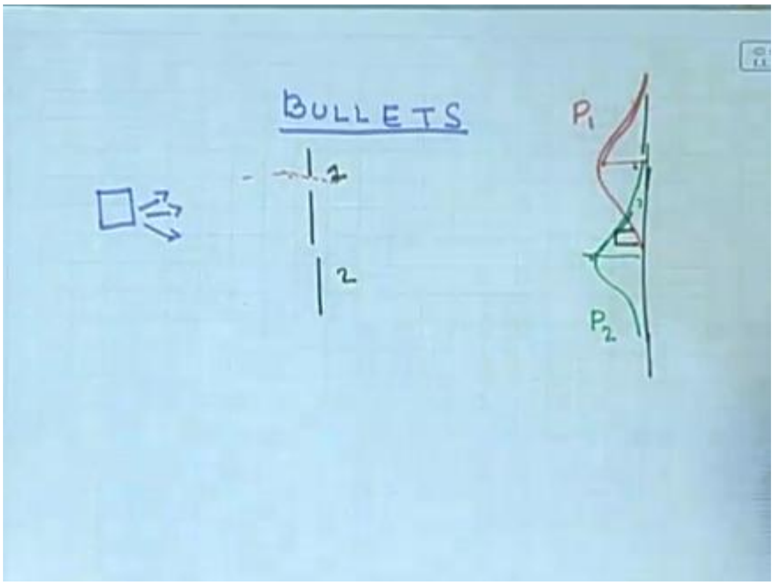

So, there is expected to be spread in the arrival of the bullets on the screen also and there is going to be a spread in the probability around this point. And it is going to look exactly like this, but the whole pattern has shifted so, that the maxima is aligned just behind the slit 2. So, let me draw what we expect when only slit 2 is open we expect something like this when only slit 2 is open. Now, let us ask the question suppose we open both slits and send in the bullets from this device that that is send emitting bullets. And then repeat the whole experiment we move the detector to different points on the screen and the count the number of bullets that arrive.

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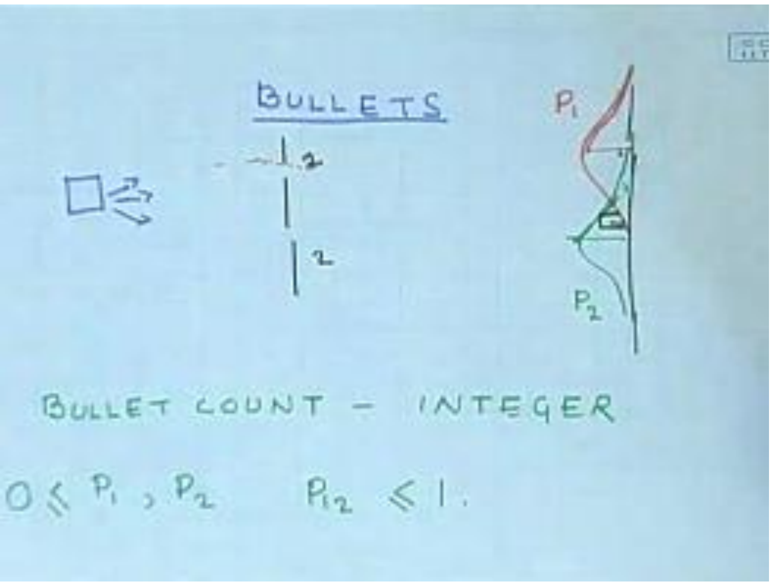

The bullet count is always discreet. So, bullet count is going to be discreet it is going to be an integer 1 2 3. So, if I when I put the detector at any particular position let us say here and record the number of the arrival of bullets. So, what will happen typical is that for some time no bullets will come then suddenly a bullet will come, again after some time another bullet will come and again after some time another bullet will come. So, over a time interval there will be a discreet number of bullets that come may be 4 5 6 it will be some integers. So, we will always get an integer number of bullets that arrive at the detector. Now, I have to divide this by the number of bullets, that were sent out over the same time interval the total number of bullets that were sent out from here over the same time interval.

So, over a time interval I count the number of bullets received over here and I count the and I know also know the total number of bullets that were sent out at the in the same time interval. So, if I divide the 2 I will get the probability that the bullet reaches this point through either slit 1 or slit 2 when both slits are open. Now, and the count is going to be integers. But the probability is not going to be an integer the probability, probability P 1 or P 2 or when I open both of them I am going to call the probability P 1 2. These are both all in the range greater than equal to 0 and less than equal to 1. So, they are going to be positive numbers in the range between 0 and 1 and they are going to be they are not going to be integers they will be some fractions in the range 0 to 1.

Now, the question is how is the probability of a bullet arriving here when I open both the slits related to the probability when the slits the individual slits are open. Now, a bullet at any point on this screen a bullet can arrive at any point on the screen either through slit 1. So, there may be a bullet which goes through this slit and then comes at to this point or a bullet may arrive at this point on the screen through slit 2. So, if I record a bullet over here it could have arrived either through slit 1 or through slit 2. And the total probability of finding a bullet at any point on the screen is the some of the probabilities that it arrived either through slit 1 or through slit 2, right. So, when both silts are open.

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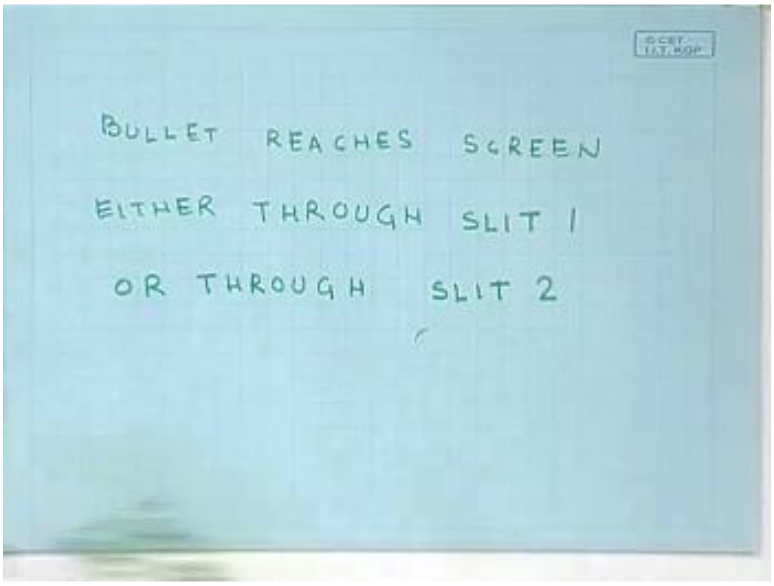

A bullet reaches the screen either through slit 1 or through slit 2 and these are mutually exclusive events if the bullet goes to the screen through slit 1.Then it does not definitely does not go through slit 2. And if the bullet goes to slit the screen through slit 2 same to the same point of the screen through slit 2 we can be sure that it has not gone through slit 1. So, in case we have events which had mutually exclusive like this, and the event, which final event which we are interested in could have occurred by through any of these 2 possibilities. Then we have to add up the probabilities and it will give me the final probability.

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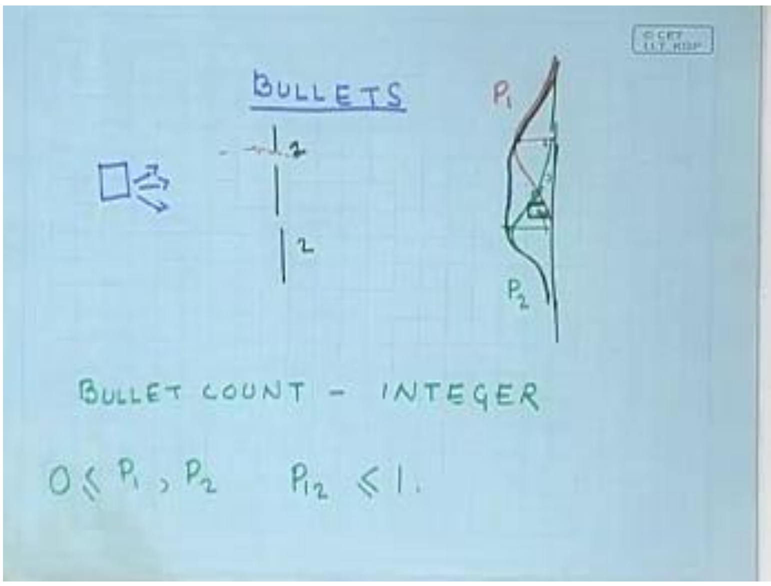

So, I have to add up that the probability of the bullet. So, if I want the probability that the bullet arrives at a particular point any particular point, when both the slits are open. I have I can add up the probability that it arrived through slit 1 to the probability that it arrive by slit 2 and that will give me the total probability. So, it is going to look something like this it is going to be the sum of these two probabilities is going to look something like this. So, at these points P 2 is much larger than P 1 P 1 nearly vanishes over here.

So, it is going to be exactly follow P 2 at same over here it is going to exactly follow P 1 because P 2 is very small the probability of it coming here through P 2. The second slit is quiet small, over here there is a this a significant probability that it comes through this or this. So, I have to add up both the contribution and there it is going to be larger than the individual probabilities. And there will be a dip somewhere in the middle and it is going to be the some of these 2. So, what we can say is that and this we are going to call.

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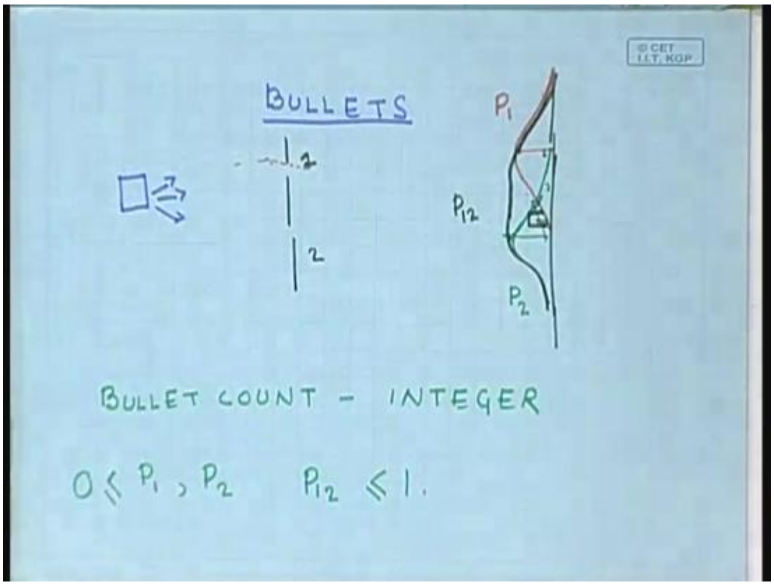

P 1 to the probability of the bullet hitting a particular point in the screen when both the slits are open.

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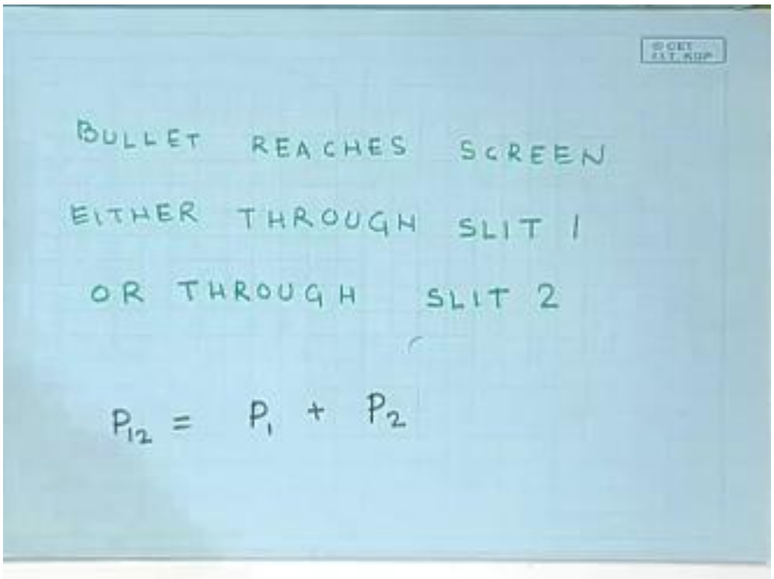

This is going to be P 1 plus P 2. So, let me recapitulate here what we expect.

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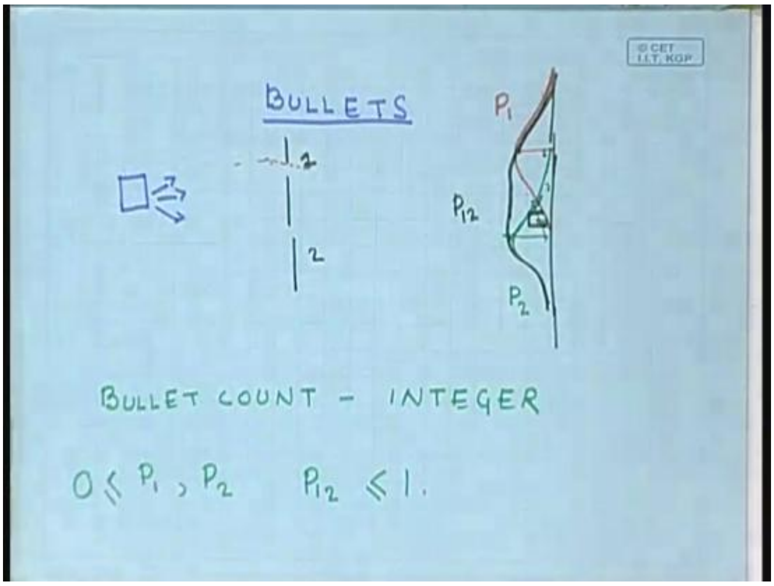

When we do this experiment with bullets, when we do an experiment where we send out bullets more or less uniformly in the forward direction in all direction in the in this all. Over this range of angle in the forward directions and we ask the question, what is the probability that it reaches a particular point what is the probability that the bullet reaches a particular point. We can close 1 slit and calculate the probability that will give me P 1 we can close the second slit and calculate the probability it will gives me P 2. Now, the, if you ask the question what is the probability that a bullet reaches a particular point in the screen when both are open its going to be P 1 plus P 2 that is the probability, when both the slits are open.

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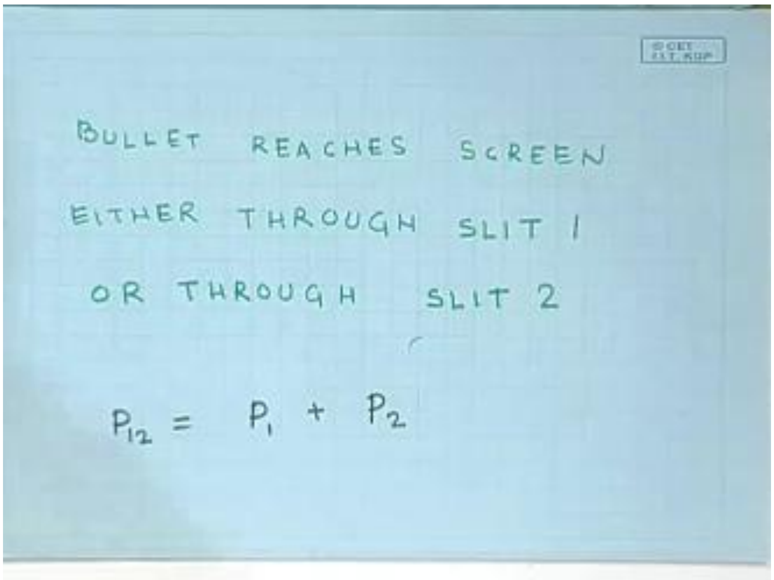

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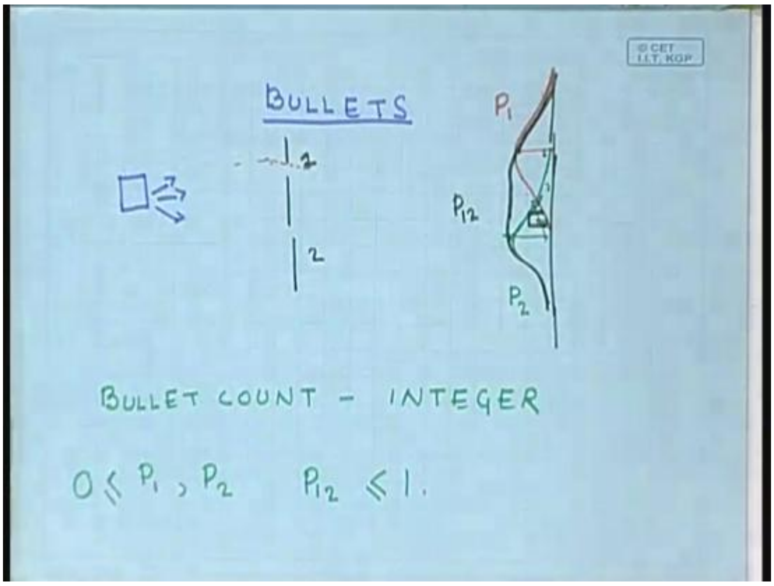

The other very important point which I should again reiterate is that the bullets are going to arrive at the detector on the screen 1 by 1. So, you are always going to get either 1 bullet coming or no bullet coming. So, if you put a detector the detector is going to get the counts the detector and going to occur in units of one you are going to either detect 1 bullet coming or no bullet at all. So, the bullet the arrival of the bullets is an integer event the integer process and the counts are in the bullet count is an integer and they which goes up in units of one every it goes up in units of 1 every time you detect us single bullet. You are not going to detect half a bullet that is the probability could be a fraction. So, you have to do the experiment over a reasonably large period of time and you will find that the ratio of the 2 that is a number of bullets that arrive at the point and the total number that is being sent out is going too slowly.

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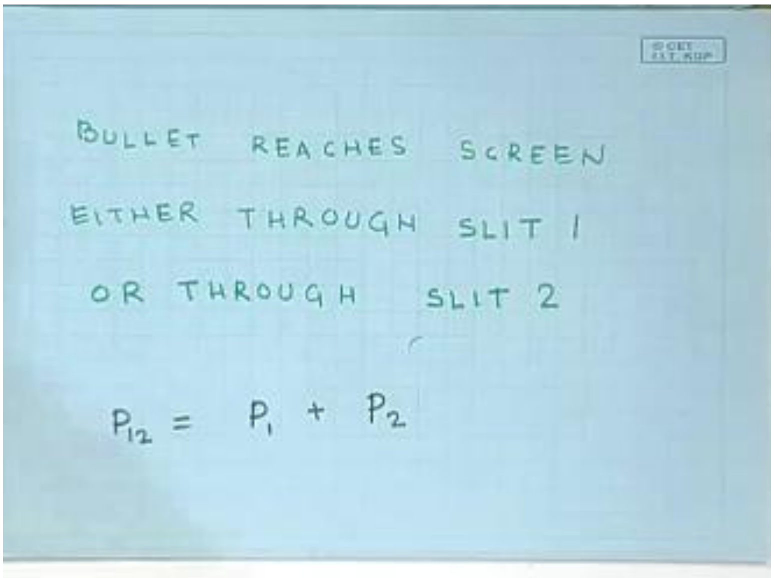

Approach the probability P 1 to that you calculate through this. So, this is as far as bullets are concerned. Now, let us consider another possible hypothetical experiments. So, this was more of a thought experiment than a real experiment. Let us consider another such hypothetical experiment, which we have also been discuss which can easily be done a in reality. So, now instead of a bullet source we have a light source.

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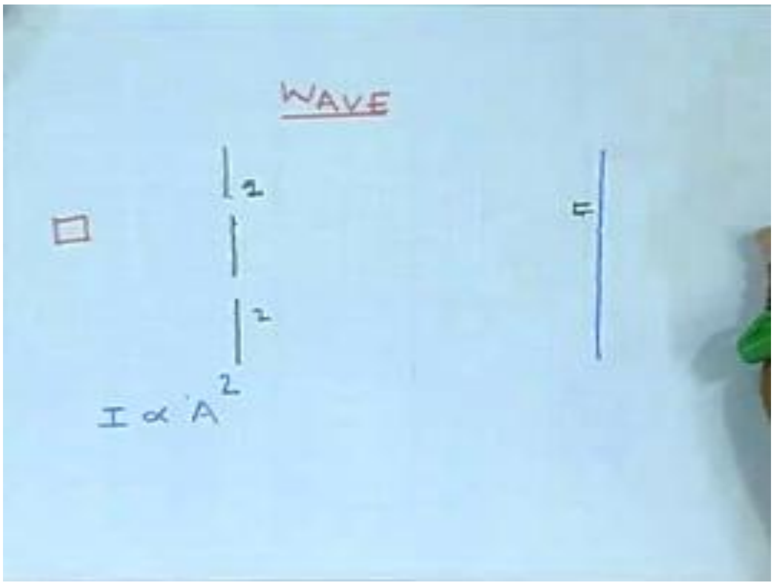

So, there is a source here with a let us say with any wave need not be light. So, there is a source here which emits a wave in the forward direction. This could be a electromagnetic wave as we have been discussed in over a large number of lectures electromagnetic radiation is a wave the electromagnetic wave. So, it could be an electromagnetic wave that is being emitted by this source over here in the forward direction it could be a sound wave or some other kind of wave. And this is incident on 2 slits which are located at some distance away from the source. So, there are 2 slits over here, and then we have a screen located far away over here. Now, the first point unlike the bullet unlike the bullet source, you can vary the intensity of the wave that comes out continuously.

And the intensity of the wave is proportional to the amplitude of the wave that is produced out here. And it is proportional to sorry proportional to the square of the amplitude of the wave that is produced over here. So, you can if you increase the amplitude continuously you can also increase the intensity continuously unlike the bullets which we have to you can just increase in units of one you cannot increase the bullet rate arbitrarily. You have to increase the number of bullets that have been shot out by in in discreet units. Now, let us again repeat the same exercise, we have 2 slits over here on which the wave is incident and let us ask the question suppose I close slit 2 and open only slit 1.

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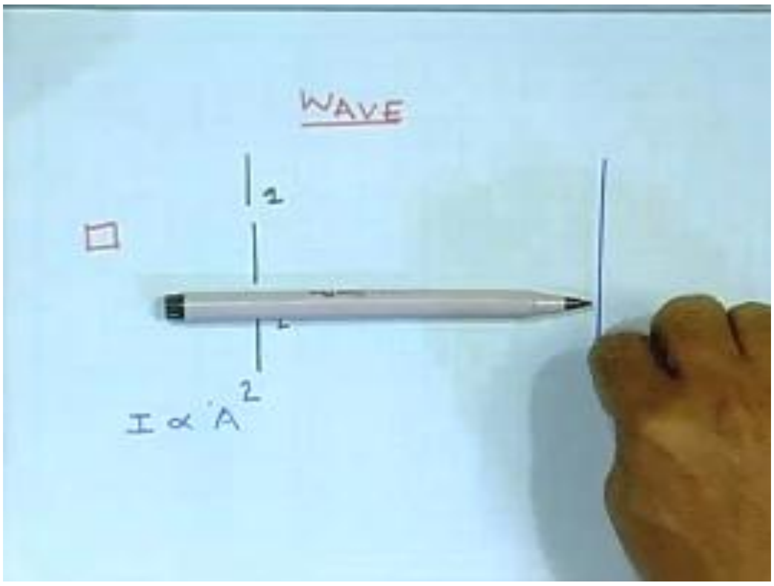

What is the pattern? Intensity pattern that, I expect to see on the screen over here. So, I have a detector again which can measure the intensity I move it to different points on the screen and I measure the intensity of the wave when it arrives here. So, it could be light if it is light electromagnetic wave radiation then I will be measuring the intensity of the light using this detector. Now, when we close slit 2 and open only slit 1 we will get some intensity pattern we have studied this in some detail earlier on, we expect the diffraction pattern of a single slit.

So, we expect the single slit diffraction pattern on the screen over here the intensity pattern over here is going be the diffraction pattern due to a single slit. So, it is going to have a maxima which we know the diffraction pattern of a single slit has a maxima, just behind the slit. And then the intensity falls off there is a spread in the intensity it does not go to 0 immediately at the edges of the slit. It has a just behind the slit if you move away from the point just behind the slit it is going to have a dispersion is going to have a spread. The spread is of the order of lambda by d where lambda is the wavelength of the wave and d is the size of the slit that you are using.

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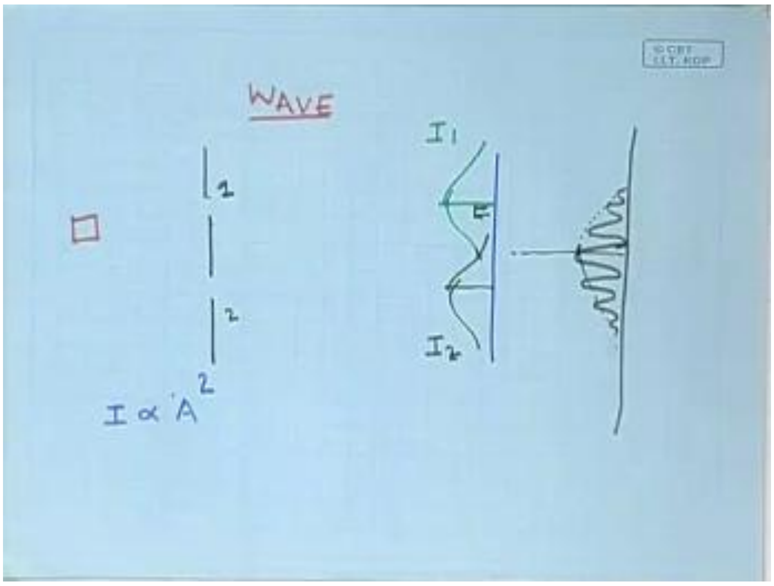

So, there is going to be a dispersion the spread in the intensity around the point which is just behind the slit. So, when I have opened only slit 1 and if I measure the intensity pattern on the screen, we are going to get something which looks like this. It is going to have the spread and there will be some secondary maxims which are going to be much fainter I am not drawing them, right. We have discussed that it is going to be sinc square function. The sinc square function we know the secondary maximas the other higher order maxims have very low intensity. So, I am not drawing them so, this is just the primary maxima it will have a spread around the point which is just behind the screen. So, it is going to look like this. Now, we are going to call this I 1 the intensity when only slit 1 is open. Now, suppose I close slit 1 and open slit 2 and repeat the same exercise I am going so, if I measure the intensity on the screen over here.

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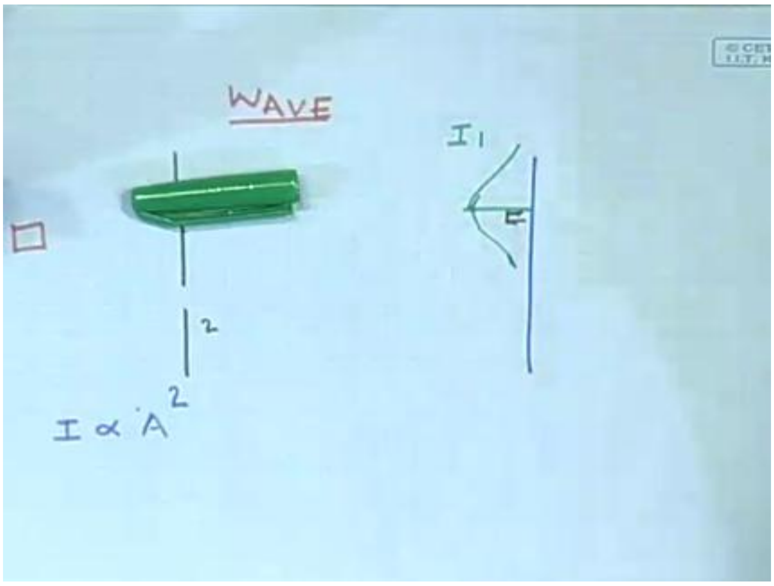

The question is what do we expect the intensity pattern to look like, the intensity pattern is essentially going to be exactly the same as when slit one is open, but whole thing is going to be shifted down. So, that the center where the intensity is maximum is just behind slit 2. Now, so, when slit 2 is open and slit 1 is closed, the intensity pattern is going to look something like, this it is going to be just it is going to be peaked somewhere here and it is going to look exactly the same.

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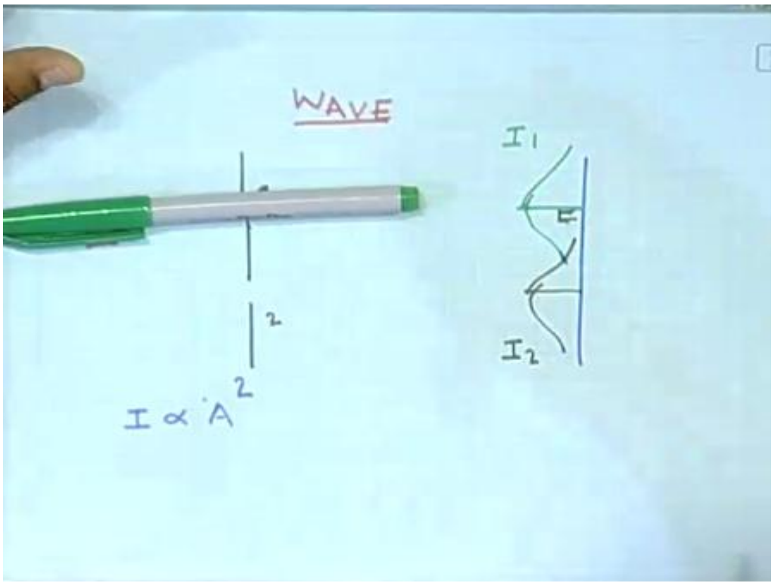

So, it is going to be something like this, and this is going to be I 2. Now, we also know that when we have 2 slits like this and we have a single wave incident on this produces 2 secondary waves. And if I open both the slits the intensity pattern over here, is not going to be the some of the individual intensities it is going to be the this is the Young's double slit experiment is going to produce an interference pattern. And the interference pattern is going to have a period which is lambda by this d the distance between the 2 slits. So, it is going to vary much faster than the individual diffraction pattern and it is going to be modulated by individual diffraction pattern.

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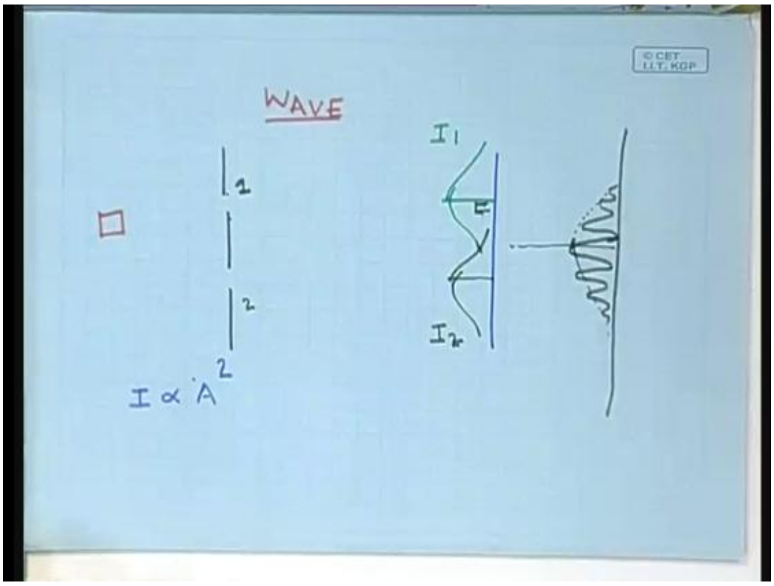

So, it is going to look finally, it is going to look like this, the maxima is going to occur and the point in the middle of the 2 slits and is going to be modulated by this. So, I can also draw that this is what the intensity is going to look like. And the intensity pattern is going to look something like this, and we have analyze this in great detail. So, the resulted intensity let me write it down over here.

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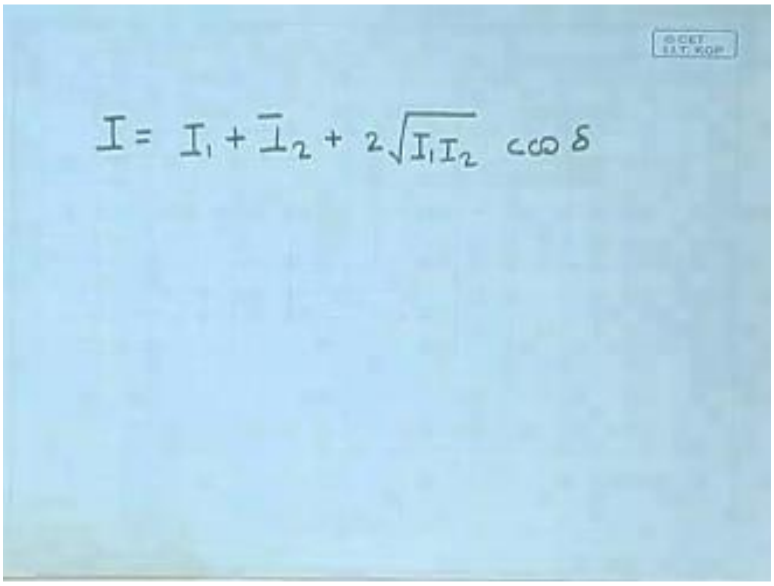

The resultant intensity I is I 1 plus I 2 where I 1 is the intensity of when only slit 1 is open I 2 is the intensity when only slit 2 is open plus 2 root I 1 I 2 cos delta. Where so, this is a formula which we have derived we have spent quite a bit of time discussing this.

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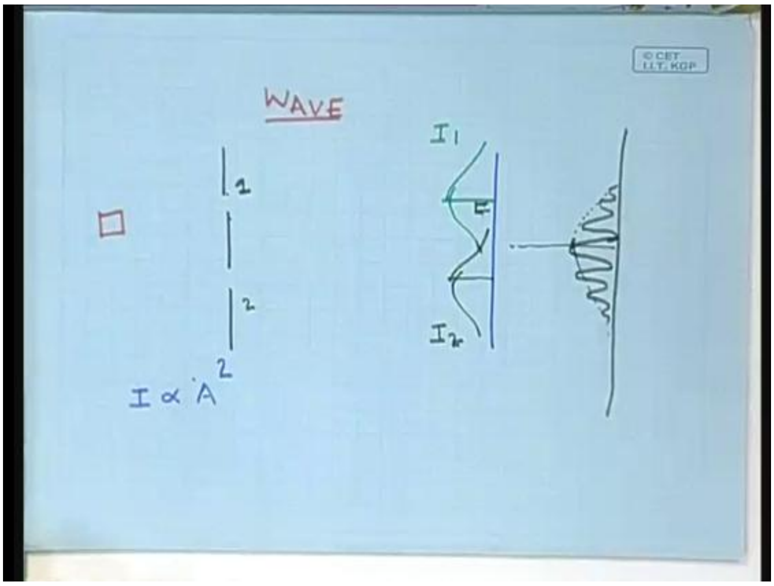

Let me just remind you what this delta is. Delta is the phase difference between the two waves that arrive at the screen from the 2 slits. So, delta is the phase difference if I am calculating the intensity at any point particular point in the screen, the contribution from slit 1 and the contribution from slit 2 are going to be at 2 difference phases and delta is that phase. So, let we write this down quantitatively.

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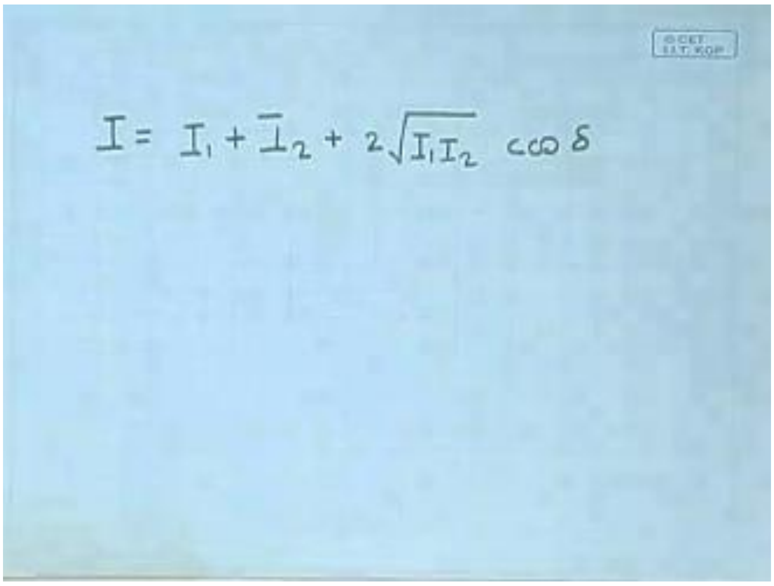

So, this is and this term it is this term with cos delta with a phase difference that gives rise to the interference pattern. You see the big difference over here is that the intensity.

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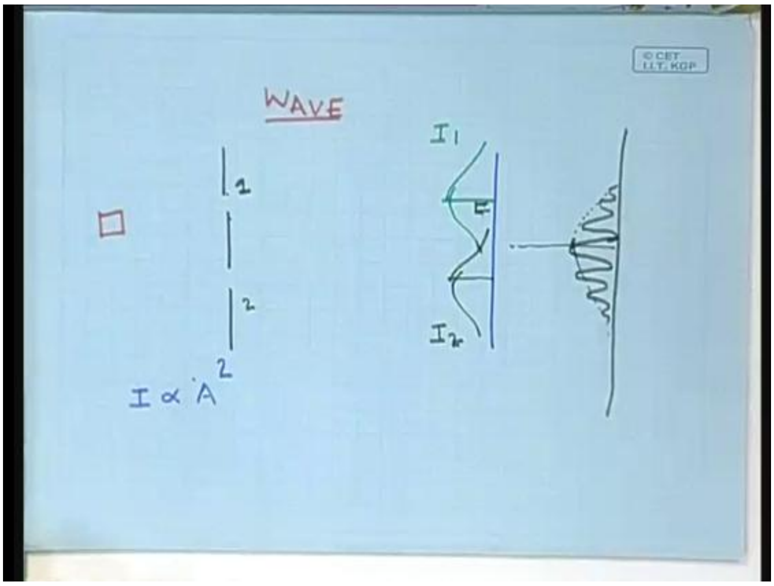

When I open both the screens at certain points, the intensity becomes more than, what I had if I able to add up I 1 and I 2. But at certain points it becomes less than what I will what I would get if I were to add up I 1 and I 2 and it is this cosine term over here which gives rise to this pattern.

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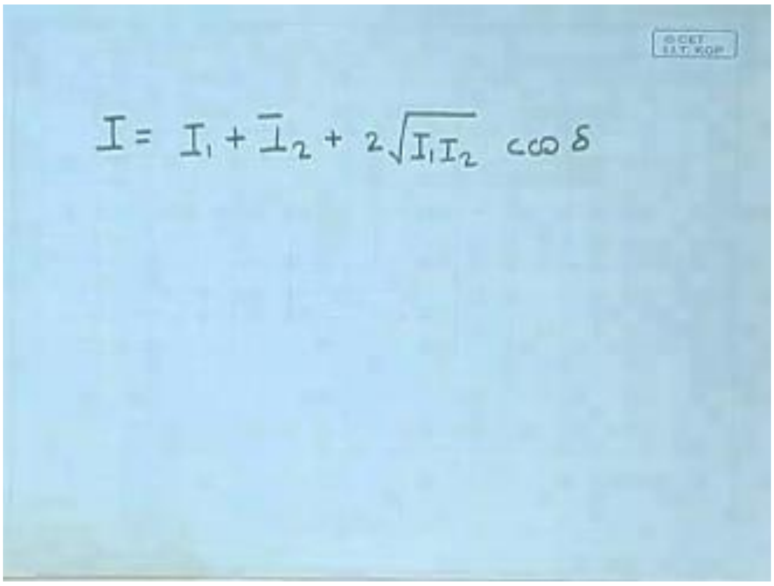

So, at the points where the intensity falls below the sum the 2 waves I which arrive from the 2 slits are going to be out of phase.

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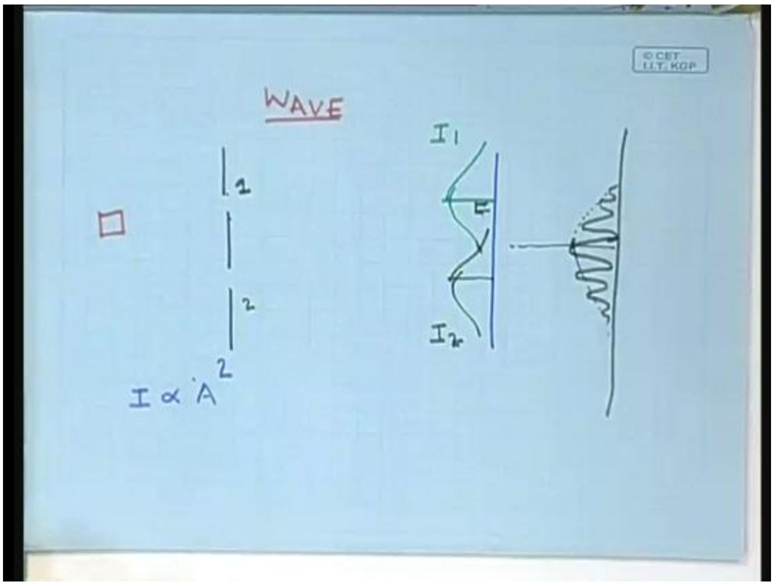

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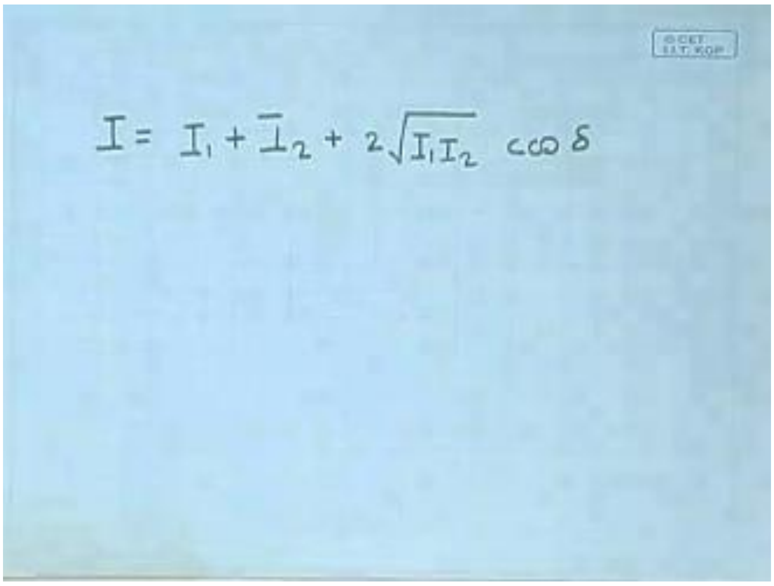

So, remember that the intensity it is the amplitude of the wave which is crucial.

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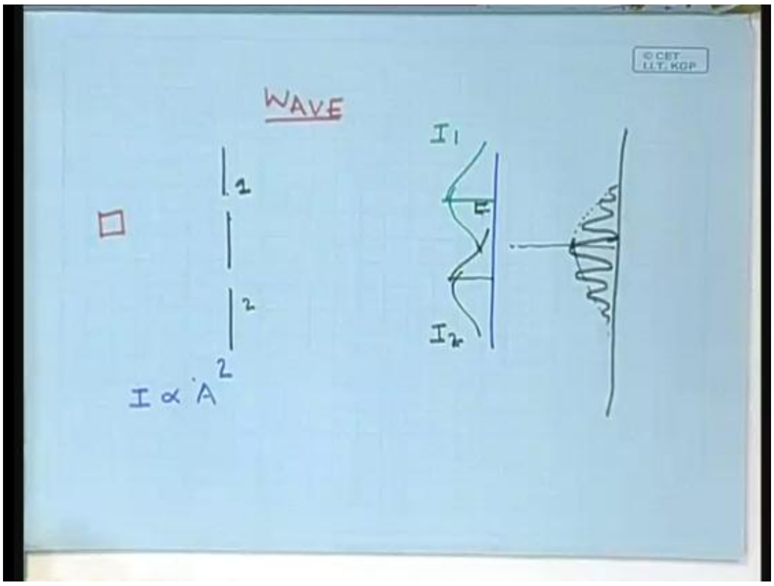

So, the wave which is arriving at the screen if I represent it as A the amplitude of that wave as A.

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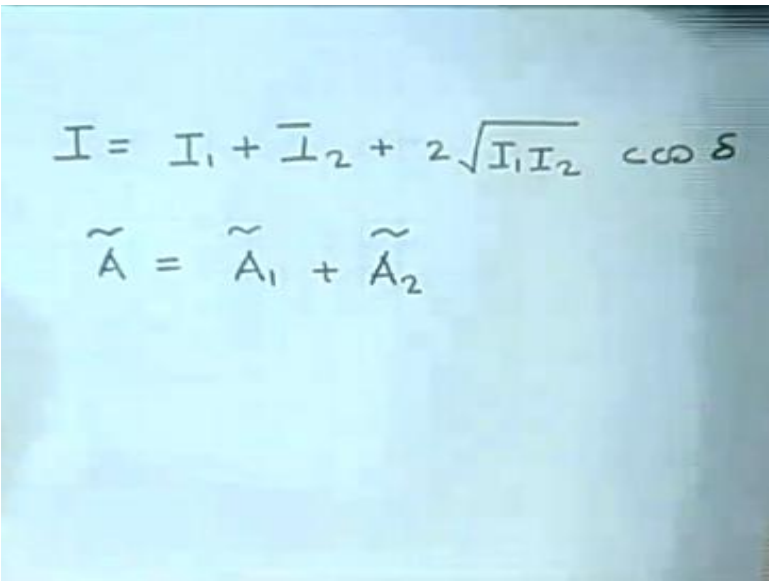

In the complex notation then it is going to be A 1 plus A 2.

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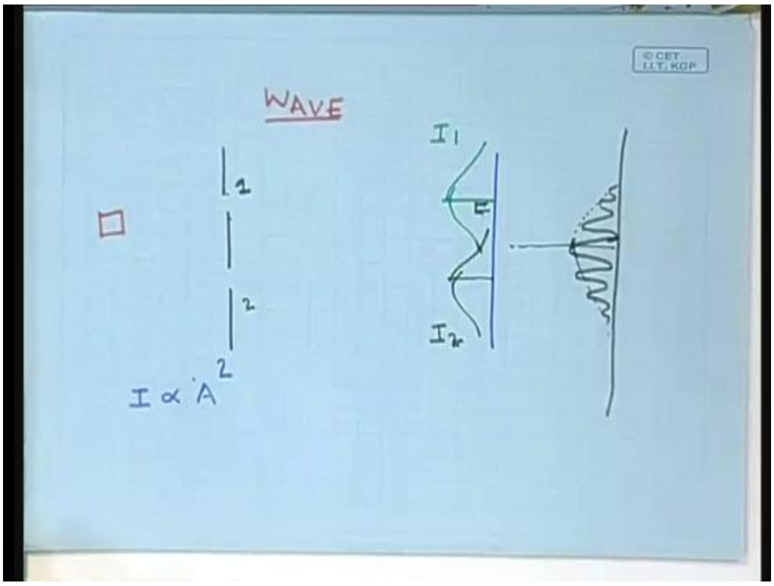

So, A is the amplitude of the wave arriving at any particular point on the screen. A 1 is the part that arrives from slit 1 A 2 is the part that arrives from slit 2. If I close this only there would be a only A 1 if I were close to this there would be only A 2, when I open both of them there is going to be A 1 plus A 2 the waves.

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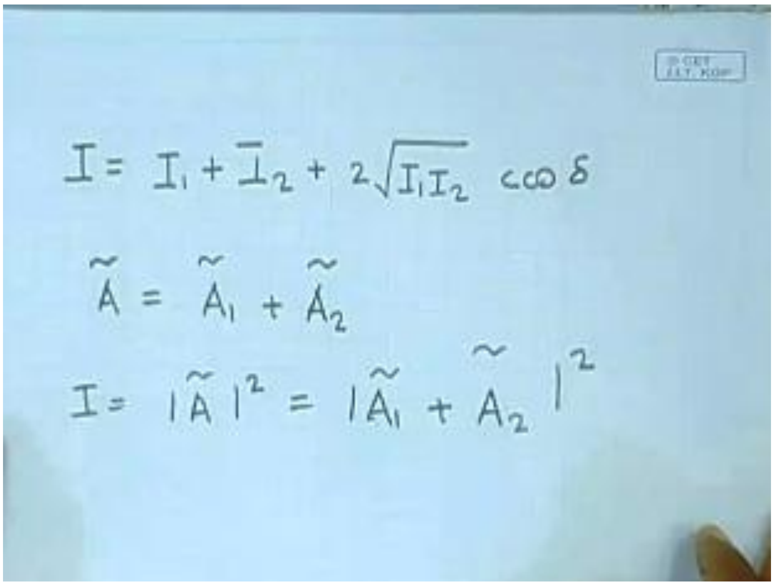

Arriving to the 2 slits are going to add up and I am going to get the resulted amplitude A. Now, the intensity is the square of this the modulus of this and this is A 1 plus A 2 the modulus of this squared. So, this is what gives rise to this expression where delta is the phase difference between the 2 waves that arrive at that point. So, delta is the phase difference between A 1 and A 2, and it is this phase difference between the 2 waves that arrive through the 2 slits that causes the interference. That may cause the cancelation of the waves the reduction in the intensity at some points on the screen.

So, this again is something which is quite familiar to you, the difference with the situation where we have bullets is quite clear quiet obvious. And here the intensity can vary gradually there is its unlike the bullets it does not have to increase in integer values. The intensity can be made to vary gradually by changing the amplitude of the wave. So, we have discussed the same very similar situation when I use bullets and when I have for example, electromagnetic waves. Now, let us ask the question what would happen if I had exactly the same thing.

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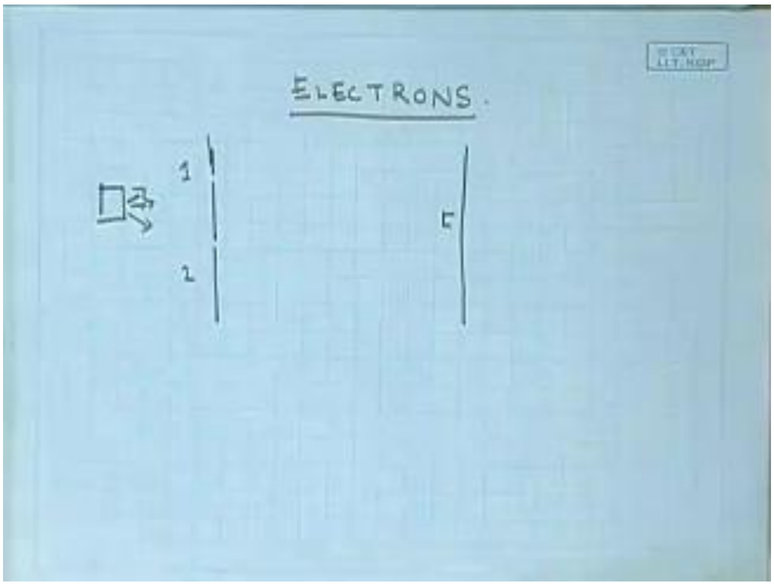

I had a source and I have 2 slits which I call slit 1 and slit 2. So, this source now sends out electrons. So, there is a source which sends out electrons in the forward direction and these electrons are incident on 2 slits. which I call slit 1 and slit 2. The electrons which hit the region outside the slits they are absorbed over here. The electrons which fall on the slit allowed to pass through and then far away on a screen fall on a screen far away. And then we have a detector which can be move up and down the screen the detector can detect individual electrons. So, whenever there is an electron which comes and hits the detectors. So, suppose I place the detector here if an electron comes and hits the detector it will give me account for example, whenever an electron hits a TV screen it gives some light is emitted.

Let so, we can have detectors which can count number of electrons that have incident on it an. So, electrons are emitted here 1 by 1 integer. So, these are integer events and the electrons will be incident on the screen. So, if I put a detector here and keep it on for a certain time it is going to tell me the number of counts, the number will tell me the electron counts. So, the number of electrons that arrived here in that particular interval of time, if I know the total number of bullets that were emitted I can then calculate the probability of finding the bullet at any point on the screen when these slits are open. Now, let us again deal with this into steps in steps.

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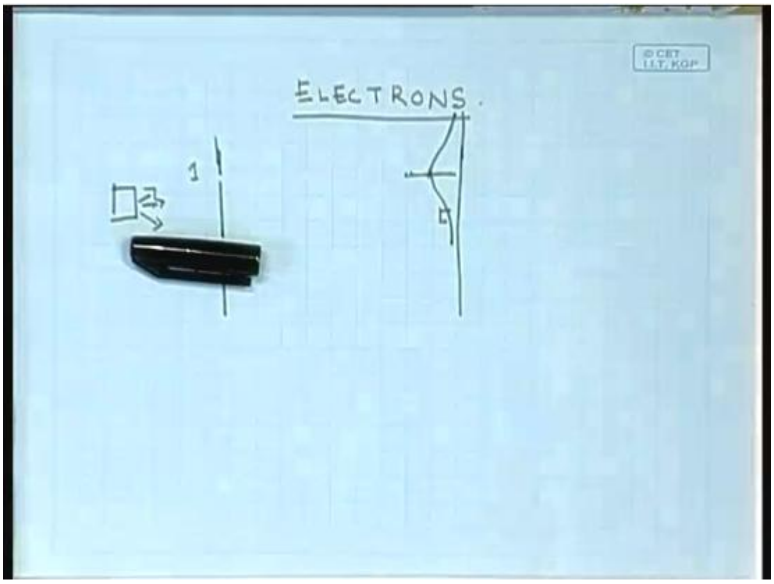

So, the first situation that we are going to consider we close slit 2 open slit 1 and we record the probability of the electron arriving at different points on the screen. Now, if you do this experiment we will find that the electrons mainly arrive on the screen at the point just behind the slit. But then there will be a spread around it will not be restricted to just this point. And if you calculate determine the probability of finding the electron at different points on the screen by moving the detector to different positions counting the number of electrons, that arrive in a given time interval you will find that the probability looks something like this. So, it will have a spread around this value and it will look something like this. So, that is the probability of an electron coming to the particular point on the screen when slit 2 is closed and only slit 1 is open.

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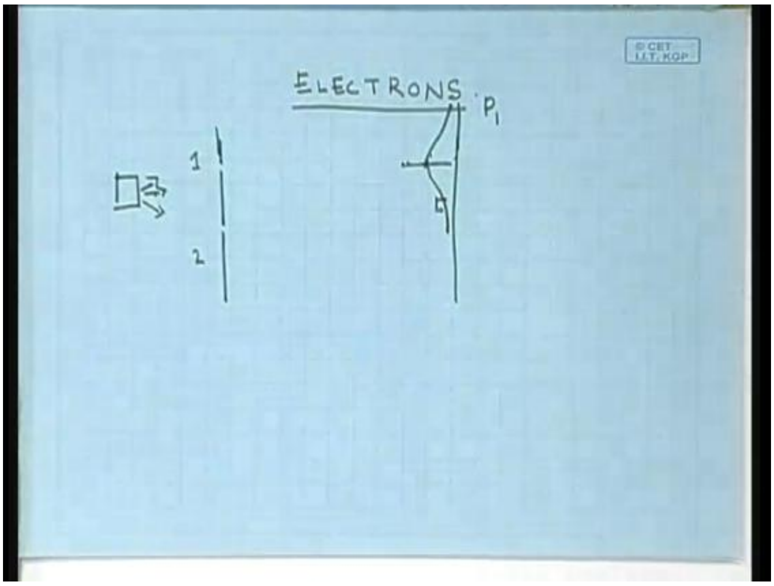

We are going to call this P 1.

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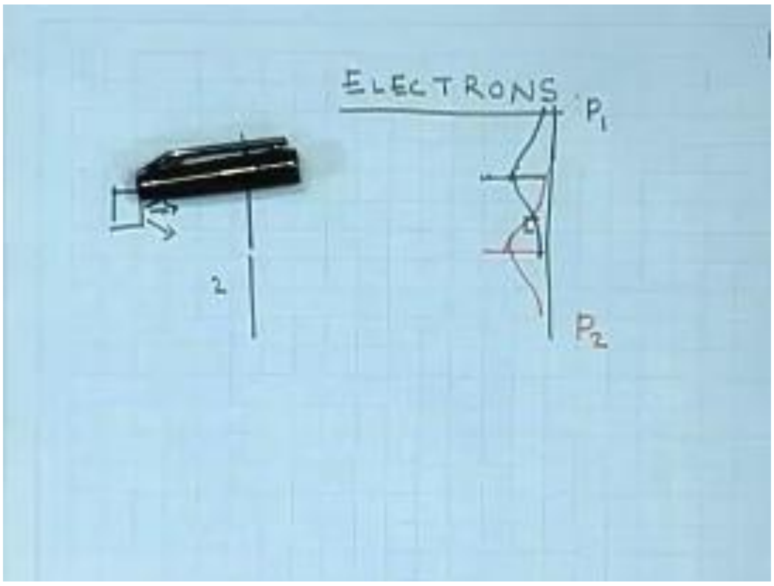

Now, if you repeat the same thing, but close slit 1 instead of slit 2 then the whole pattern is going to shift. So, that the maxima now occurs behind slit 2. So, it is going to maxim is now going to shift to this position. But the curve is going to look exactly the same, and it looks something like this and we are going to call this P 2. So, let me remind you again we are sending out electrons from this electron source electrons are sent out as discrete particles. Electrons can go through the slits or they can get absorbed in the region outside the slits the electrons that goes through the slits are now then incident on a screen far away. And we have a detector which we can move to different points on the screen the detector counts number of electrons that arrive there. So, if the so, if the detector is kept at any point for a particular amount of time and you measure the number of electrons. So, you measure the electron count.

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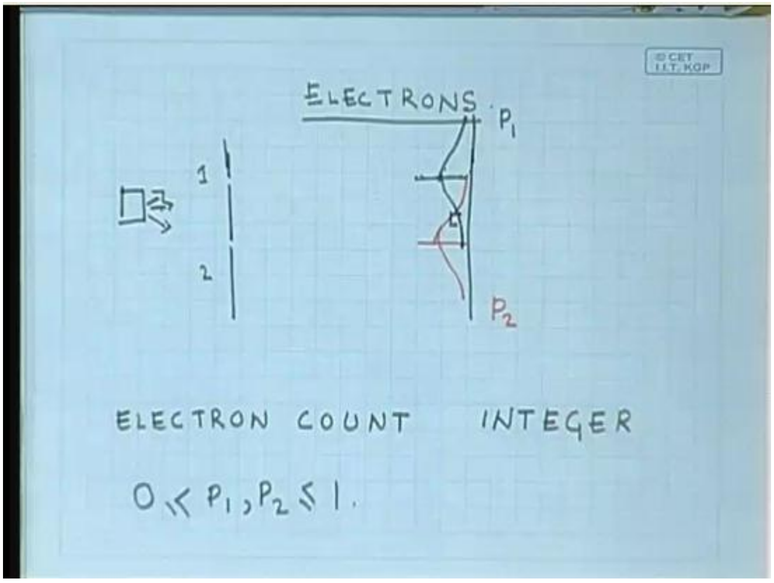

This will always be an integer you will not be you will not get half an electron coming somewhere, you will always record either an 1 electron or no electrons at all. So, if you put a detector here and keep on sending out electrons from this for some time you may not get anything. And suddenly you may get a click, and then again after some time you get another click, and again after some time you get other click. So, you have 3 electron count is 3 at that point you divide this by the total number of electrons that was sent out.

And if slit 2 is closed the ratio 3 divided by the total number of bullets that were sent out in the same time, will give you probability of the bullet arriving at that point in the screen. The point here is that the electron count always going to be integer the probability P 1 or the probability P 2 or probability as such any probabilities always in the range 0 to 1. The integer the count is always going to be an integer, the probability is going to be typically a fraction is a range 0 to 1, but the electron count is going to be an integer. Now, the question is what will happen when both the slits are opened simultaneously? And then we repeat the experiment.

So, let me repeat again we now have this device which sends out electrons in the forward direction, and the electrons are incident on 2 slits the electrons which are incident on the slit, will pass through the electrons which encounter the abstraction are absorbed. The electrons are pass through we record their arrival on the screen over here. So, we put a detector here and we can count how many electrons arrive at this point. We put a detector here and we can count the number of electrons at arrive over here etcetera. So, if you believe if so, if lying the bullets if you believe that the electrons. So, if you believe that the electrons reach screen.

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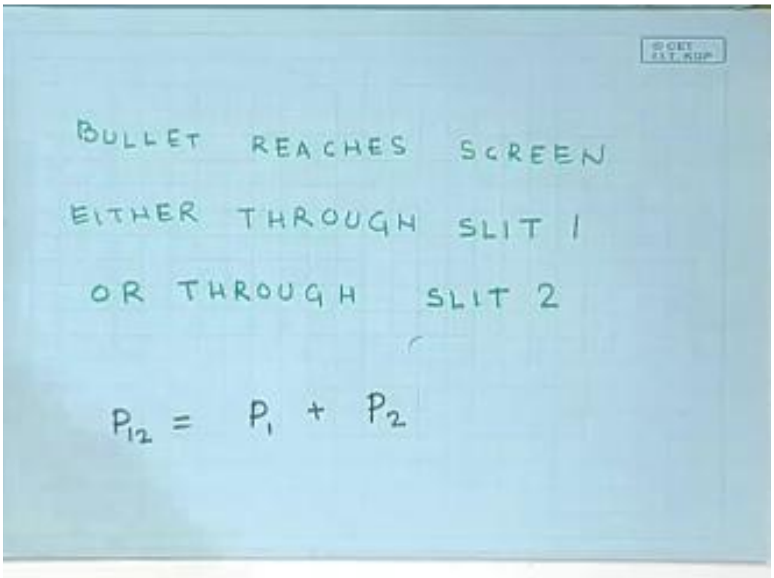

The screen either through slit 1 or through slit 2.

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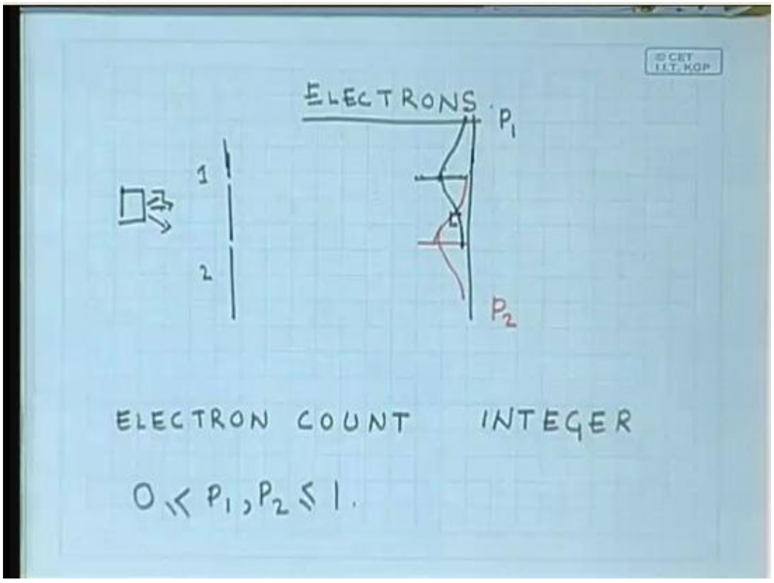

So, if you believe that if I measure an electron detect an electron here it should have come either through this or through this. Then I can say that the total probability of the bullet arriving here is the probability of the bullet arriving through slit 1 plus the probability that it arrive through slit 2. So, if this assumption is correct what we expect is that the total probability, when I open both the slits the probability of the bullet arriving here should, let me plot it here should looks something like this very similar to what we get expect if they be if we did the experiment with bullets, right. So, the probability over here is going to be somewhere above the individual probabilities over here it is going to be more or less equal to the individual probabilities and this is what you expect. Now, if you actually do the experiment where you send the electrons from this side on to slits like this, what you get does not look like the sum of P 1 plus P 2 it does not look like this.

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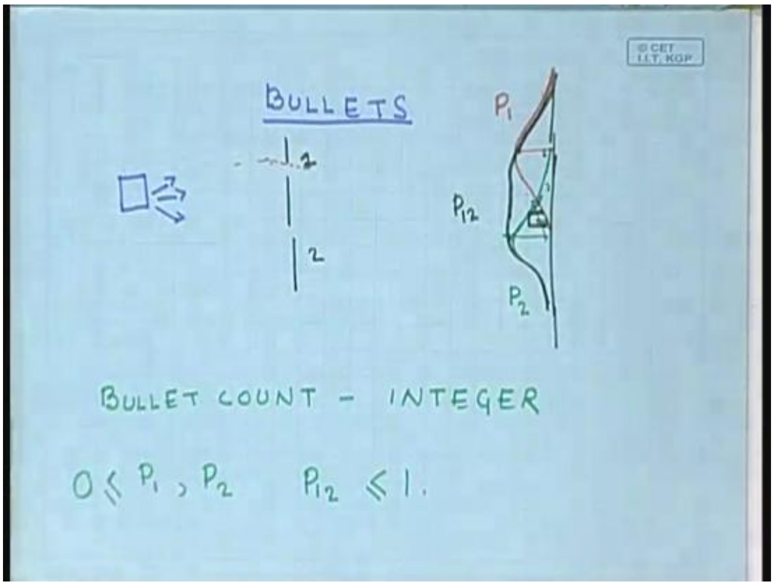

The sum of P 1 plus P 2 it looks like this.

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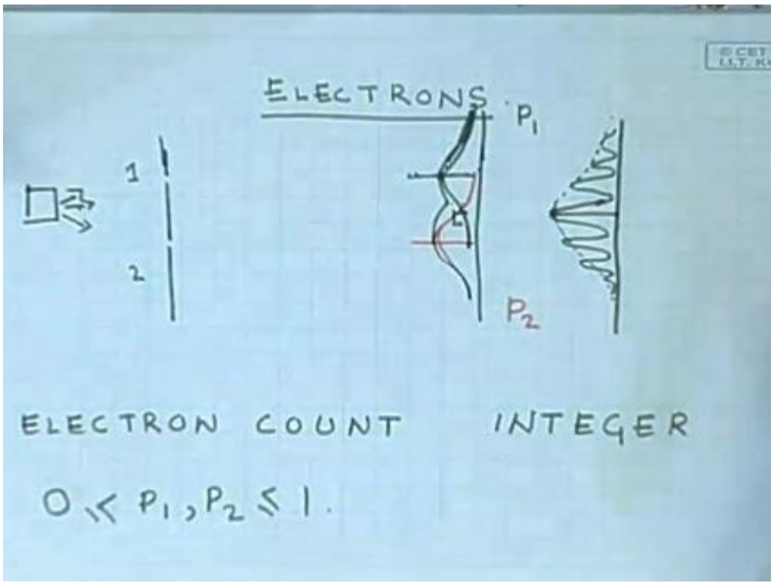

And this is what the probability of finding a bullet, at any point on the screen when both slits are open looks like. So, there are points notice where the probability actually goes below the probability when only 1 slit was open.

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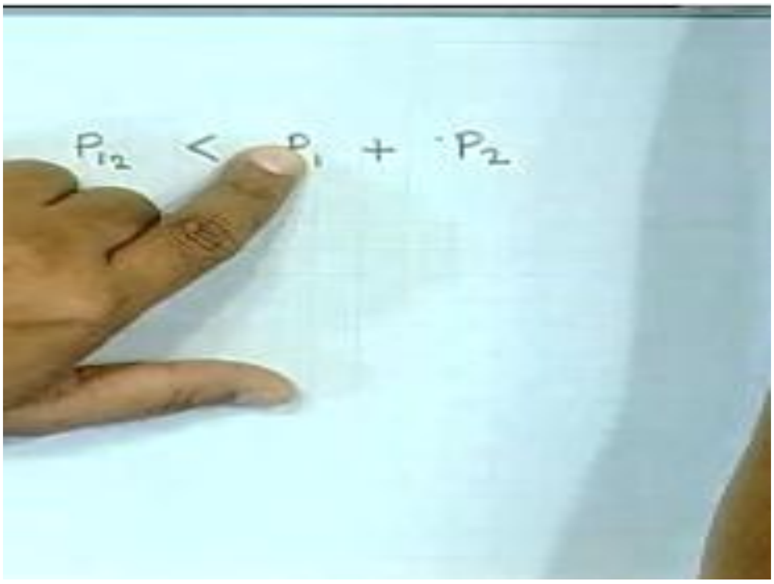

There are points where the probability of a bullet reaching the screen when both slits are open is less than the probability, when only slit 1 is open and the some of this with the probability when only slit 2 is open.

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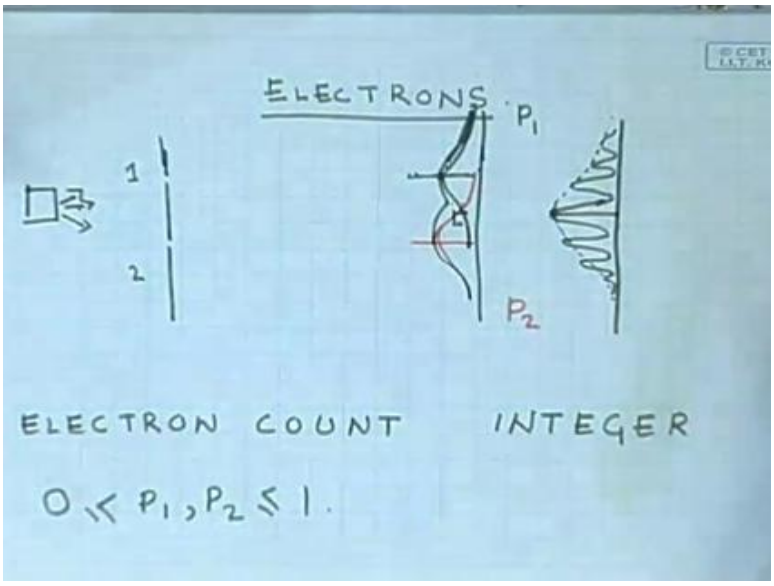

And then there are points where the probability P 1 2 when both slits are open is more than the sum of only P 1 and P 2.

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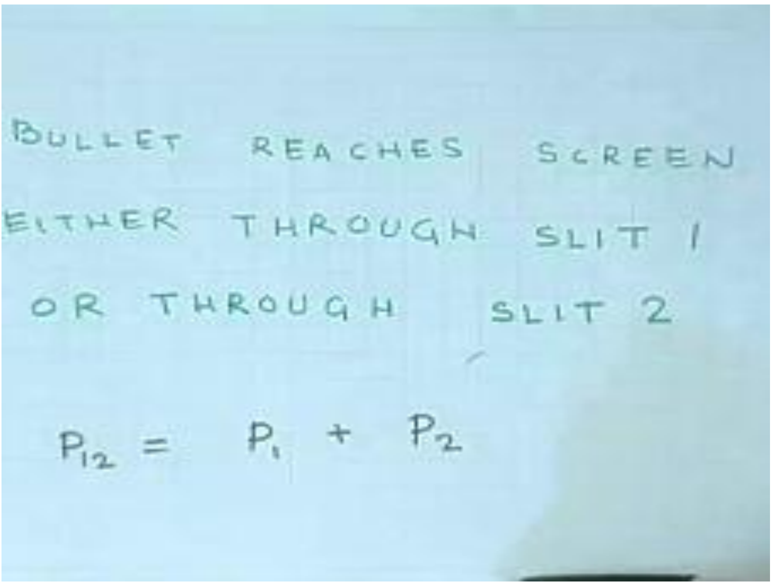

So, you hypothesis does not work in the case of electron you cannot really say that the electron went either through slit 1 or slit 2 and you cannot add the probabilities of the bullet going through slit 1 with the probability of the bullet going slit 2 and get the probability, when both slits are open.

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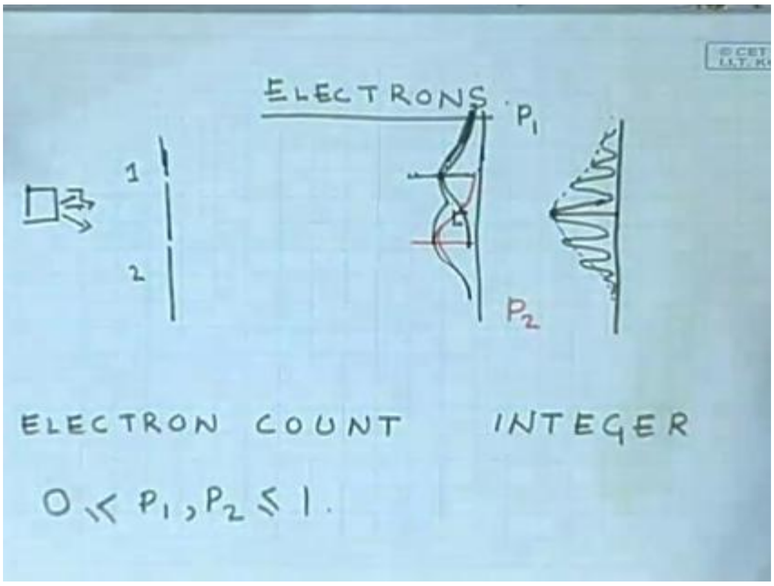

You have to change the picture of how the electron behaves when it encounters 2 slits like this, and the crucial point is that there are points where the probability.

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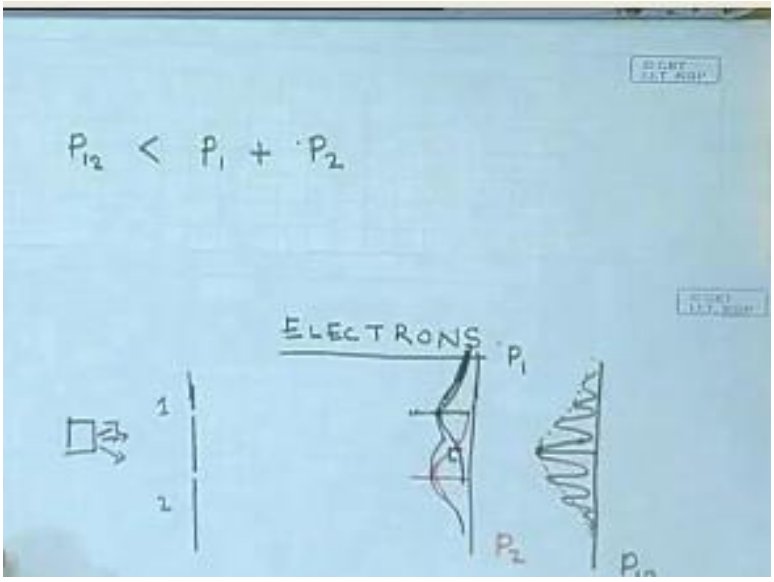

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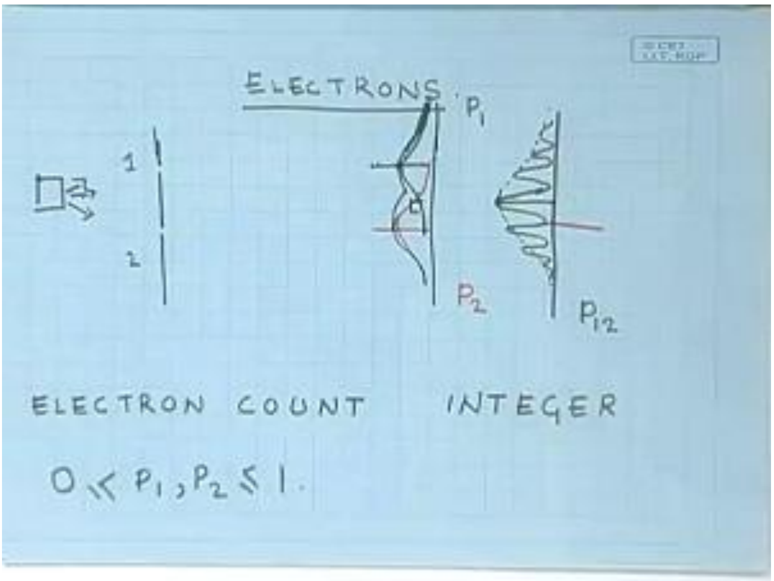

Notice there are points for example, this point over here where the probability of the electron of the electron arriving here, you see is smaller than the probabilities when the sum of the probabilities when the individual slits were open. So, there could be a situation where if I go and sit at a point I open only slit 1, I get I wait for I put a detector for some time. And I get 2 counts same point I close slit 1 and open slit 2 and I put my detector and I get 1 count the there is a probability that the bullet can come here through slit 1 or through slit 2. Now, when I open both of them when I sit quietly my detector there some time a find that get count at all.

So, when only slit 1 is open there is a probability that the bullet can reach here, when only slit 2 is open there is a probability that the bullet can reach here. But when I open both of them together I find that there is no probability then the bullet can reach here. There are see there are situations like this, which occur which when I open both the slits and I send electrons in from this side. The question is how do we explain this? Now, this is you see you must have realized by now that this is very similar to what happens, when we have a wave incident on the slit on 2 slits when we have a wave incident on 2 slits there are points. So, when I have a wave not electrons when I have a wave over here.

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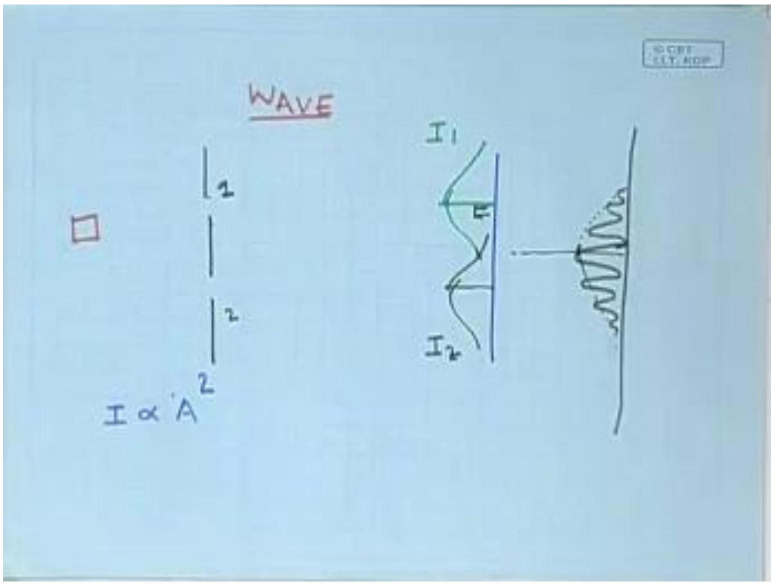

There are points where the 2 waves that come from the 2 slits are out of phase and it they exactly cancel out and the intensity goes to 0. Whereas, if I had only this open there would be a finite intensity at that point similarly if only this were open there would also be a finite intensity at that point. But when I open both the 2 waves which are exactly out of phase cancel out and the intensity goes to 0.

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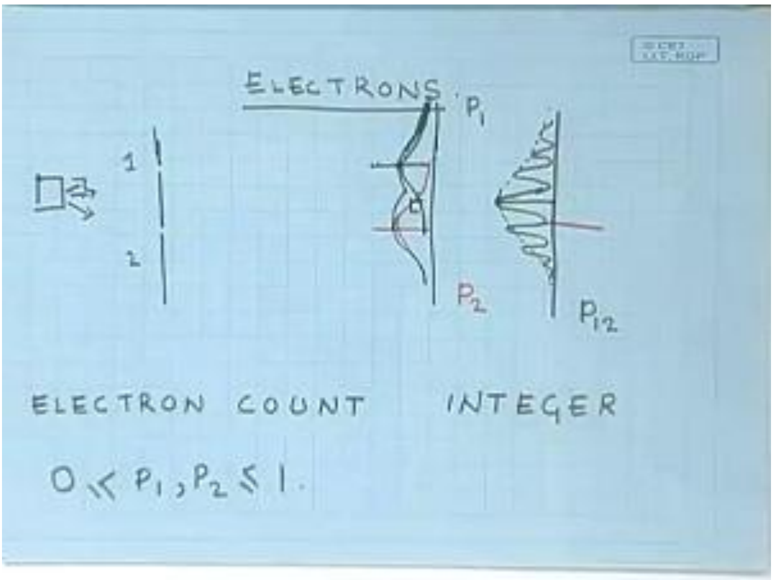

So, the situation that you see for electrons is very similar to that, so, we have to there must be something going on which is very close to what happens in waves, where you have a cancelation of 2 waves interference. Now, let us just ask the question under what condition does do 2 waves exactly cancel out at a point. Now, 2 waves we know that if the 2 waves which we are superposing at that point are pi out of phase it is only then that they cancel out. Now, if 2 waves are pi out of phase and if one of them is positive, then the other 1 has to be negative for the cancelation to occur. Now, here for the electron we are measuring the electron count we are dealing with probabilities, probabilities we know are restricted in the range 0 and 1.

So, there is no way there can be such a cancelation for the probabilities, the probability is restricted in the range 0 and 1. So, when I open this I have the probability over here, when I open this I have the probability at the same point when I open both. How do we explain the fact that when I open both the probability actually goes down instead of going up. This you cannot explain this with probabilities alone because the probabilities we know have to necessarily be positive in the range 0 to 1. So, you cannot explain this fact that the probability goes down by using just probabilities. So, it is necessary to introduce another quantity. So, the way we can explain this is by introducing a wave associated with every particle, which is what we had been discussing yesterday the de Broglie hypothesis.

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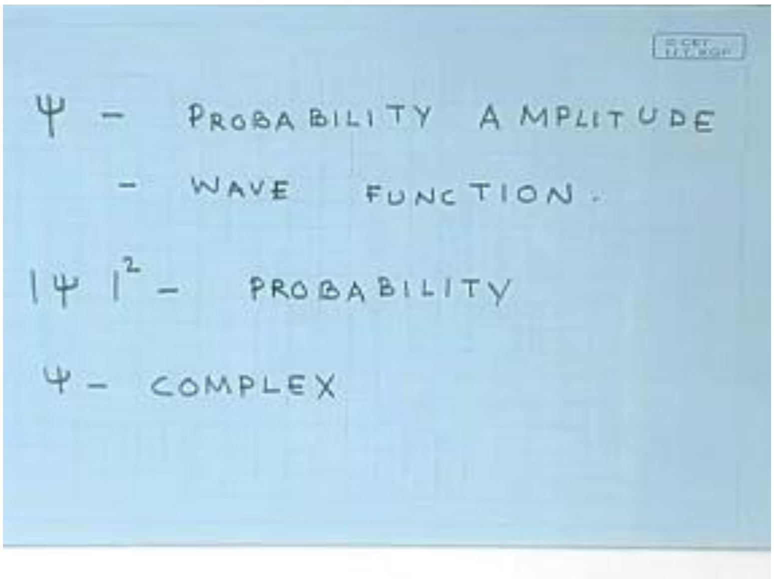

So, if you associate a wave Psi with every particle, this wave is called the probability amplitude or it also refer to as the wave function and probability amplitude is defined in such a way. So, that the square of this the modulus square of this gives the probability, psi itself is necessarily complex, but it is modulus square is positive guaranteed to be in the range 0 more greater than 0. And we can always choose an overall constant to make it in the range is 0 to 1. So, this we will associate with the, if we associate this positive number with the probability then you have an explanation for this phenomena. Let you see over here.

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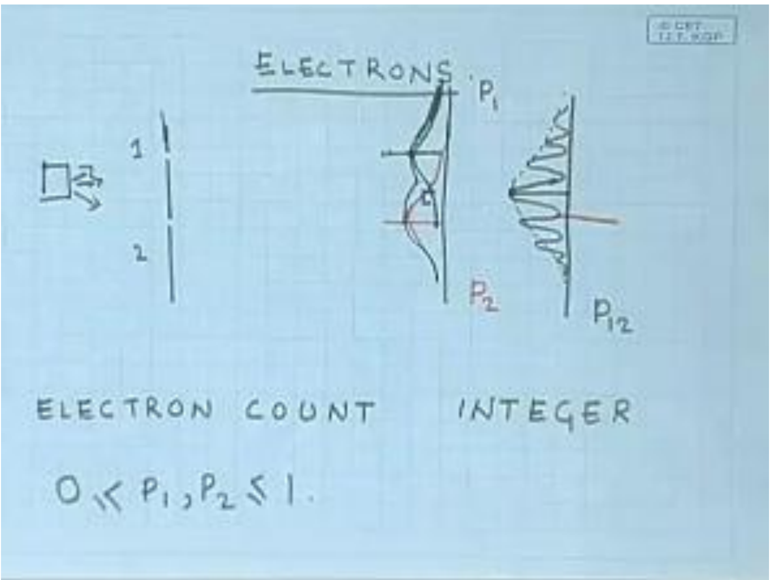

So, let we go through this. So, let we go through the through this. So, consider an electron which is sent out from this device which sends out electrons consider an electron. Now, associated do not let us not instead of think of this electron as a particle we will think of it as a wave psi. Now, when this wave when a wave is incident on 2 slits we have already discussed this, when the wave is incident on 2 slits the part of the wave which is on the slits will pass through and this will act as secondary sources. So, if you wish to calculate the value of psi on any point on this screen. So, we want to calculate the value of this wave associated with the electron on any point on the screen. So, let us choose a point on the screen we want to calculate psi here, the psi at this point is going to be the superposition of a contribution from this slit. Which we will call psi 1 and a contribution from this slit which we will call psi 2.

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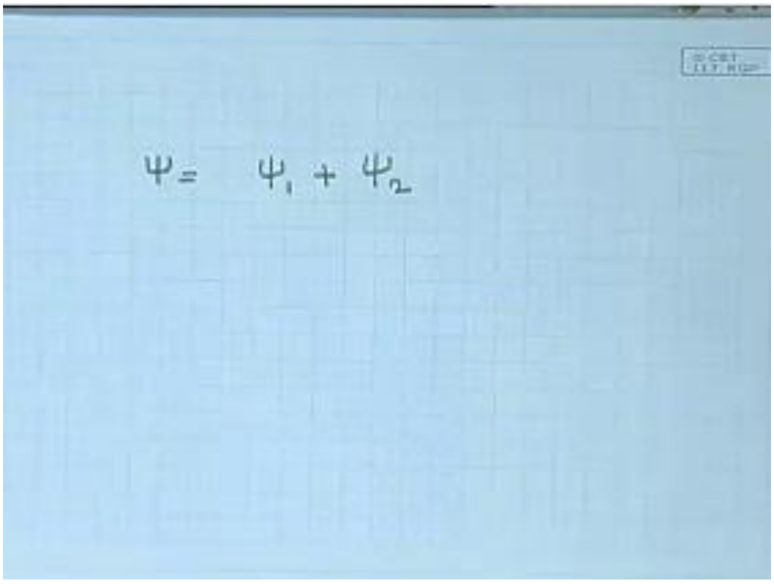

So, on the screen at any point Psi is going to be psi 1 plus psi 2 let me remind you, that with the electron we have associated a wave. So, we are no longer thinking of the electron as a particle moving going to the slit, and going through we are associating a wave just like we had discussed in yesterday’s lecture. So, today also we are doing the same thing we are continuing on the same topic, with the particle we are associating a wave. And we are not we abandoning for the time being the particle picture we are thinking of as the wave propagating it. And we are interested in the value of the wave on the screen over here.

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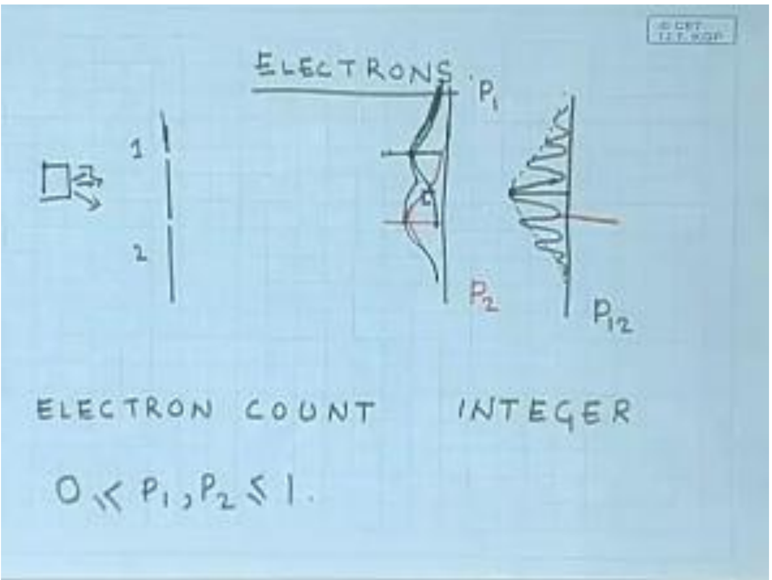

To calculate the value of the wave on the screen over here, you have to the wave is incident on 2 slits, which will act like secondary sources. We have to add up the contribution from this secondary source and this secondary source which is what I have written over here.

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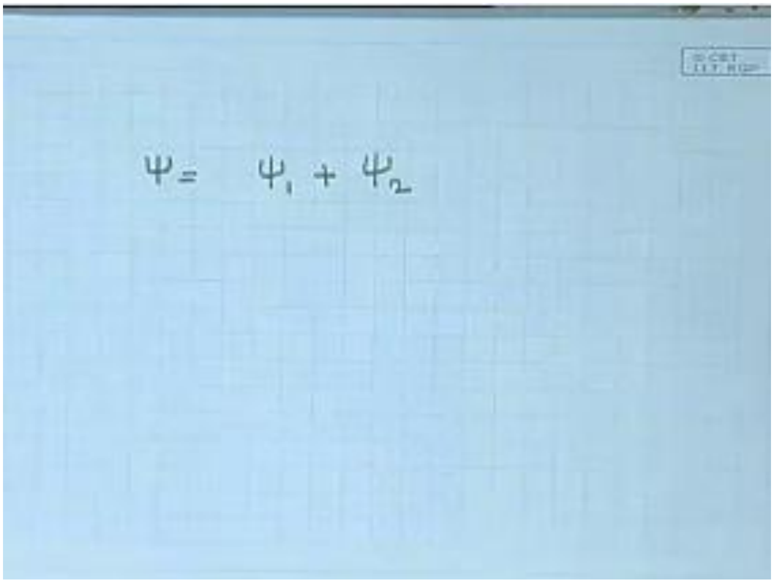

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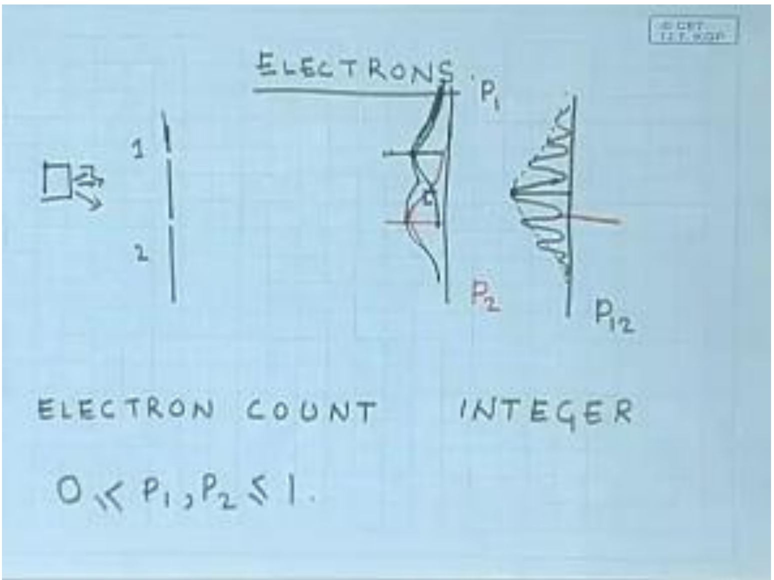

Now, if you ask the question what's the probability of finding the that electron on any point on the screen.

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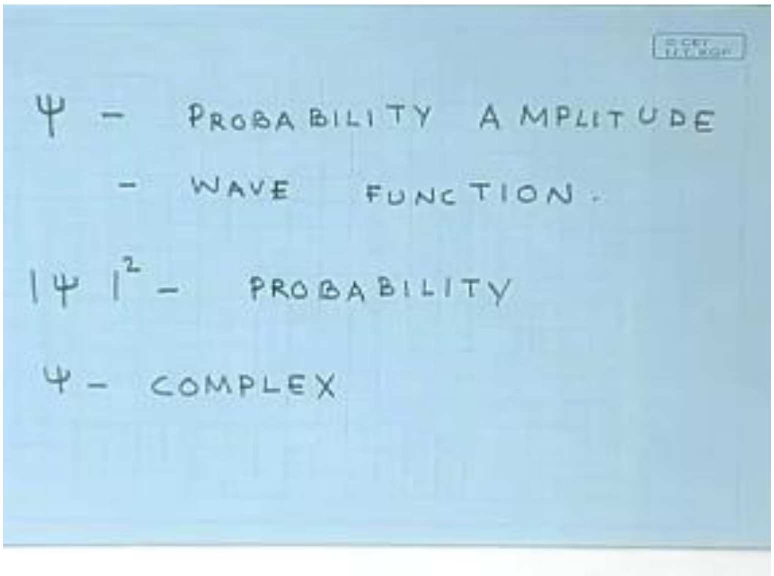

This psi is called the probability amplitude, it is a complex number it is a complex function defined at all points on the screen the square of which gives the probability the modulus square of this is what gives the probabilities.

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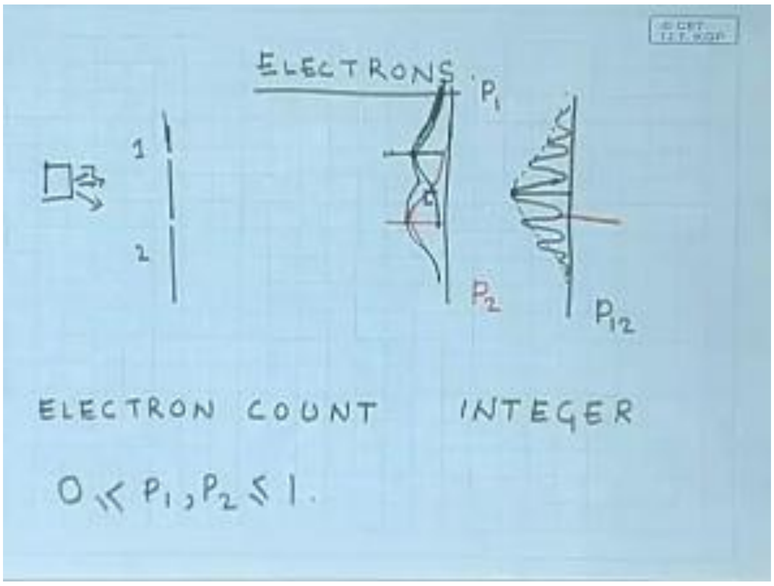

So, if I wish to calculate the probability of finding the electron on any point at any point on the screen say at point on the screen this point on the screen. I should take the value of psi at this point.

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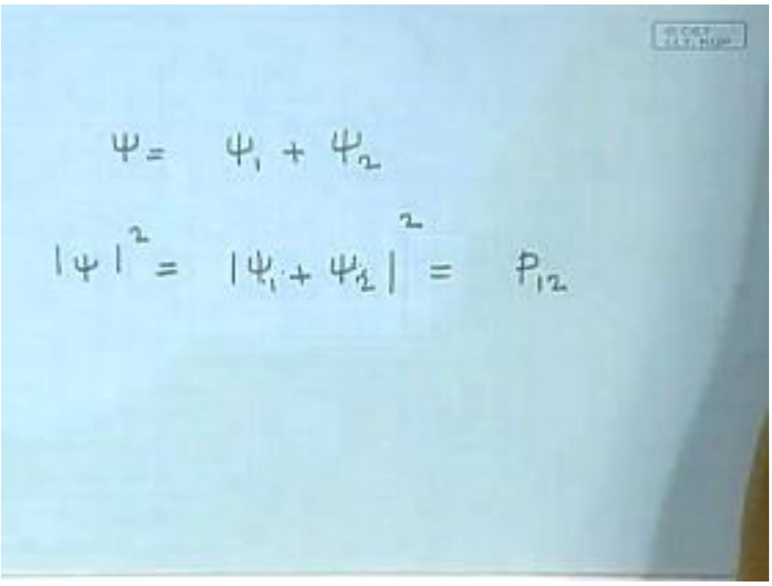

And square it this will give me the probability of finding the electron at that point of the screen. So, I should evaluate the value of psi at that point and this will have 2 contributions. You see psi is a sum of 2 different parts 2 different contributions, and there could be a phase difference between these 2 contributions and as a consequence of this. If there is these 2 contributions from the 2 different slits are exactly out of phase.

They are going to cancel out and the resultant is going to be less than the individual size psi 1 or psi 2 and you will have a cancelation you will have a dip in the probability. So, this is going to give me the probability of finding the particle on the at any point on the screen, when both the slits are open.

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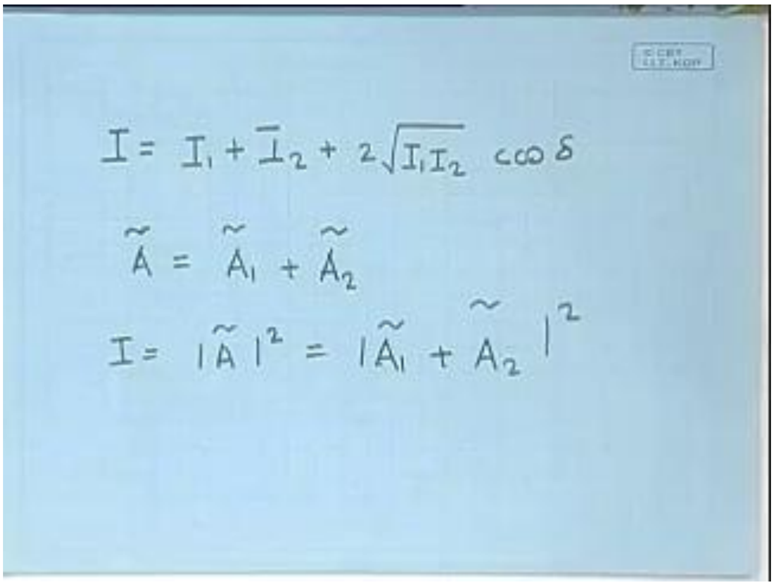

Now, you must have realize by now that this psi which we have introduced the probability amplitude is very similar to the amplitude for the wave which we have been dealing with earlier, it is exactly analogous to the has been introduced in a fashion which is exactly it is. So, that it is exactly analogous to the amplitude of the wave, which we had encountered for this amplitude of the electromagnetic wave. You should think of it has being exactly analogous as for as this purpose is concerned. So, the quantity that you measure is a intensity for the electromagnetic wave here the quantity that you measure is the probability. And so, the intensity and the probability are again related in the same way as how the amplitude of the wave and the amplitude of the this probability amplitude are related.

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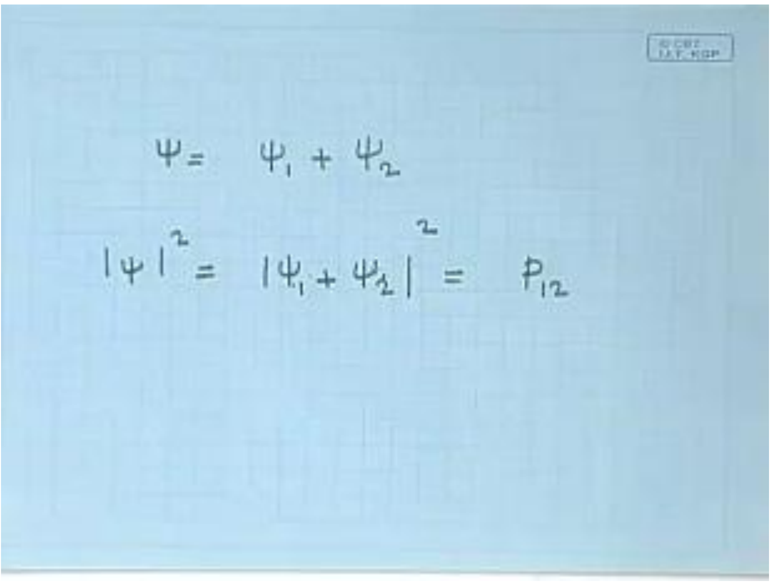

So, when you want to calculate the probability of finding the particle at any point on the screen of finding the electron at any point on the screen.

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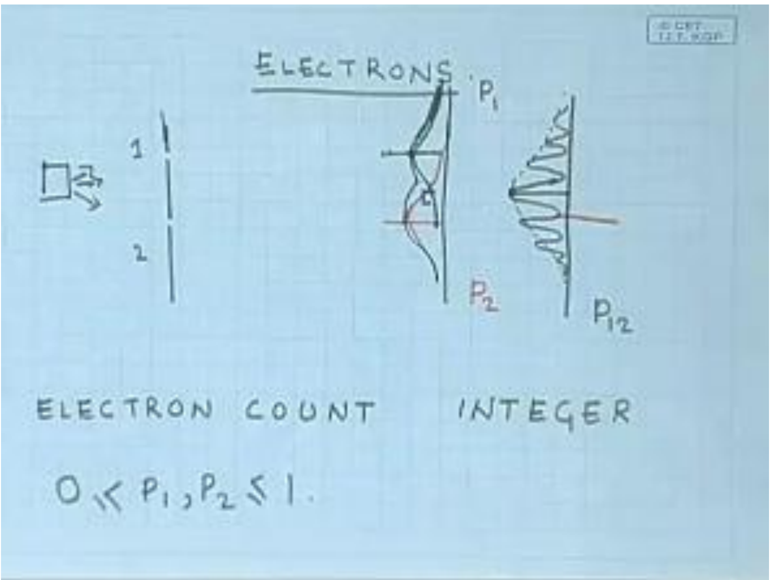

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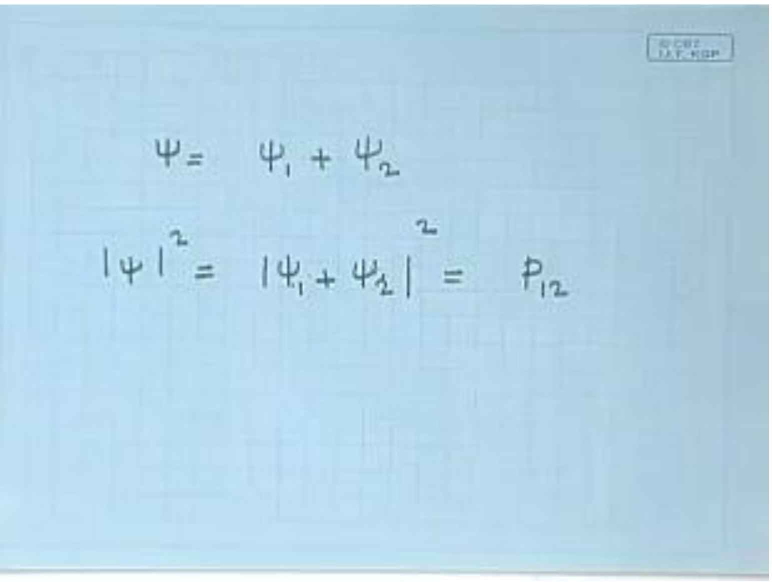

It will be the square of this when both slits are open it is going to be the square of this.

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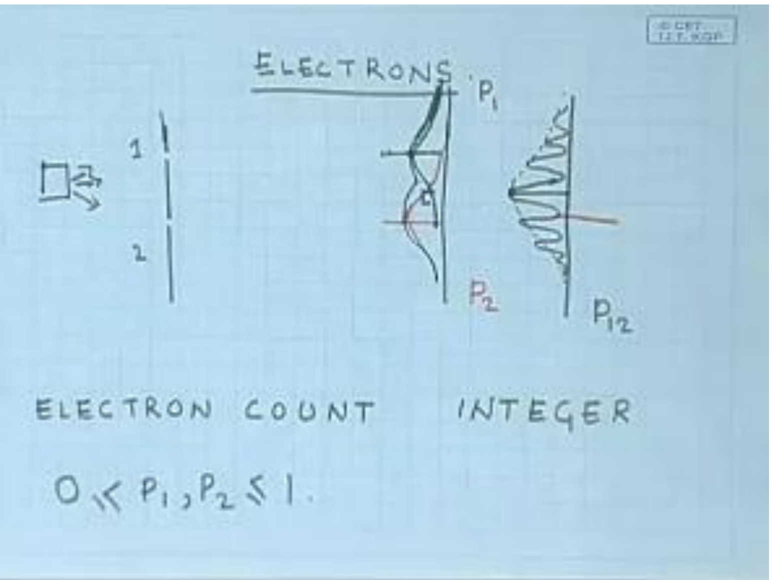

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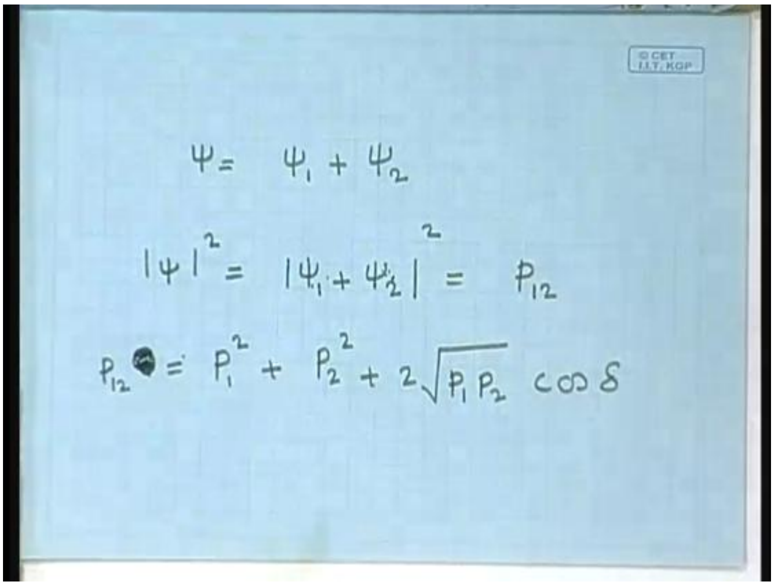

And you can know very easily guess that this P 1 2 is going to be equal to equal to P 1 squared plus P 2 squared plus 2 root P 1 P 2 cos delta where delta is the phase difference in these between these 2 waves these 2 probability amplitude waves that come from these 2 slits.

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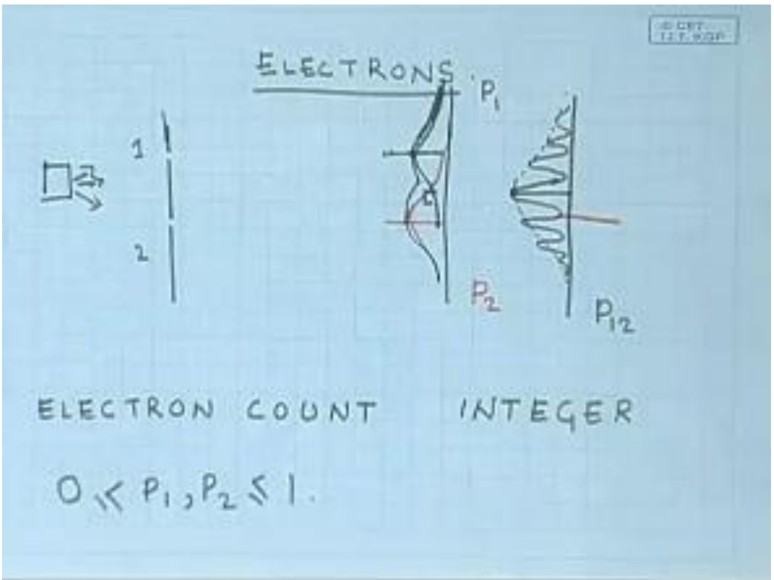

So, with this if you make… So, that is what I have shown you here, is that if you make this assumption that associated with the particle there is a wave Psi which I am referring to as the probability amplitude.

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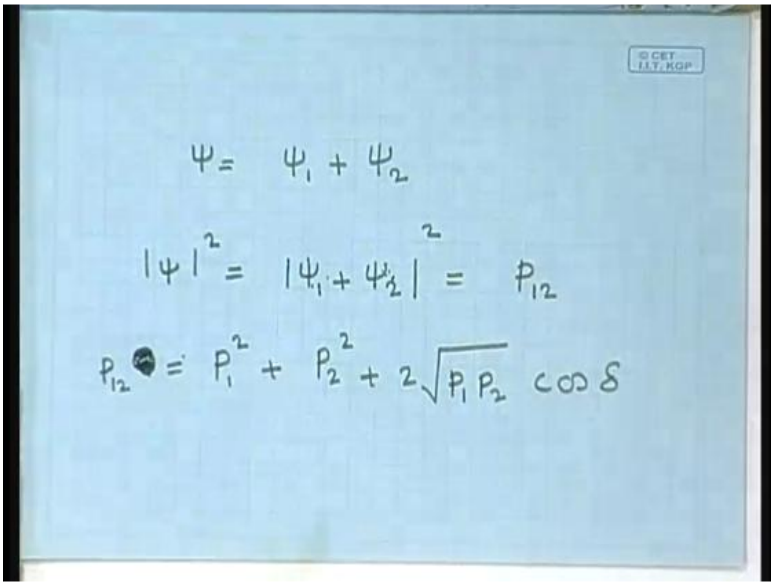

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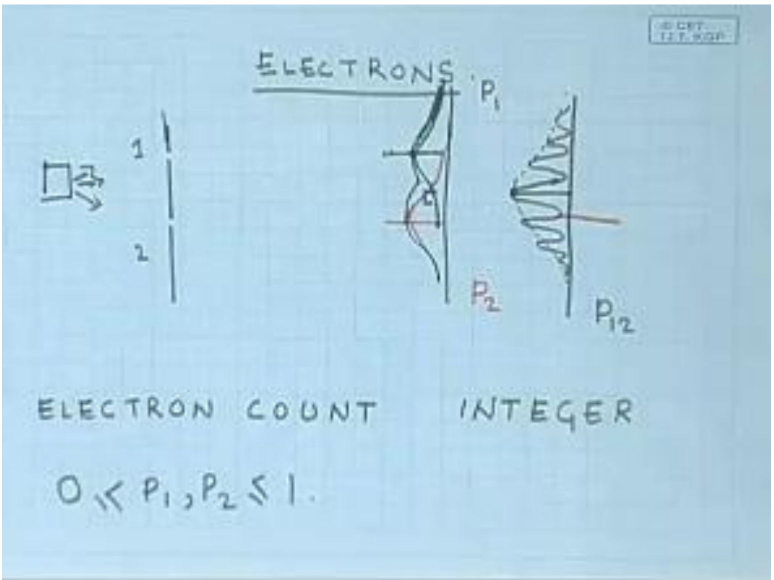

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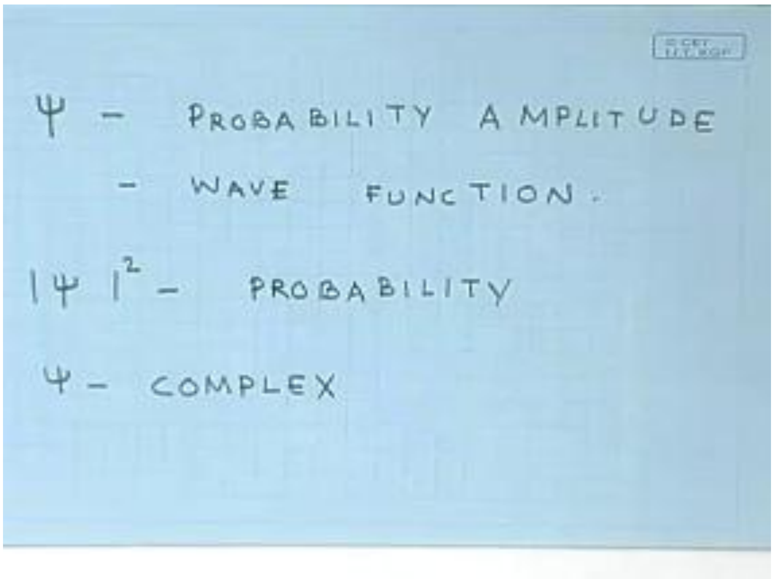

Why probability amplitude we know that the square of the amplitude gives the intensity which is a positive number which you measure. So, the probability amplitude has been defined in such a way. So, that just like you calculate intensity you calculate the intensity for this take the modulus take the square. And it gives you the probability of finding the particle which is supposed to be positive. And it is supposed to be in the range 0 to 1 which you can adjust easily by just multiplying in by a real number and this psi is complex.

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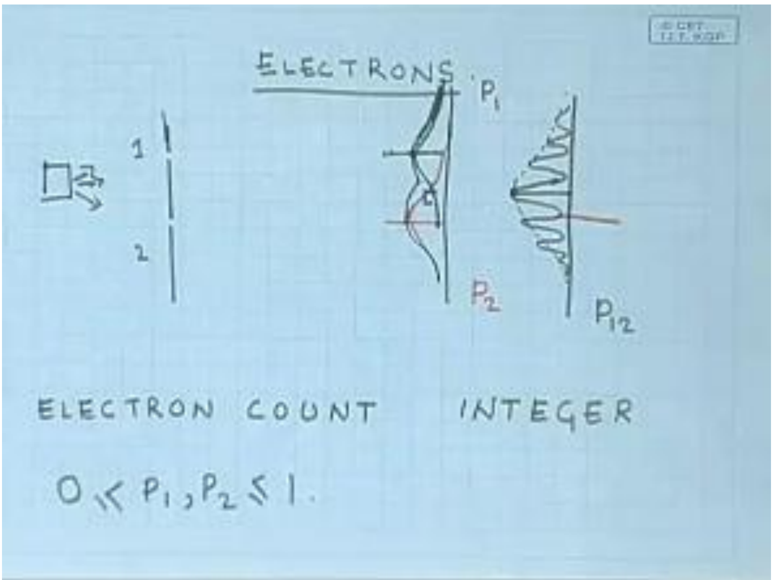

So, this observed probability pattern that you get when you do when you send electrons on to 2 slits can be explained. If you make the hypothesis that associated with the electron there is a wave.

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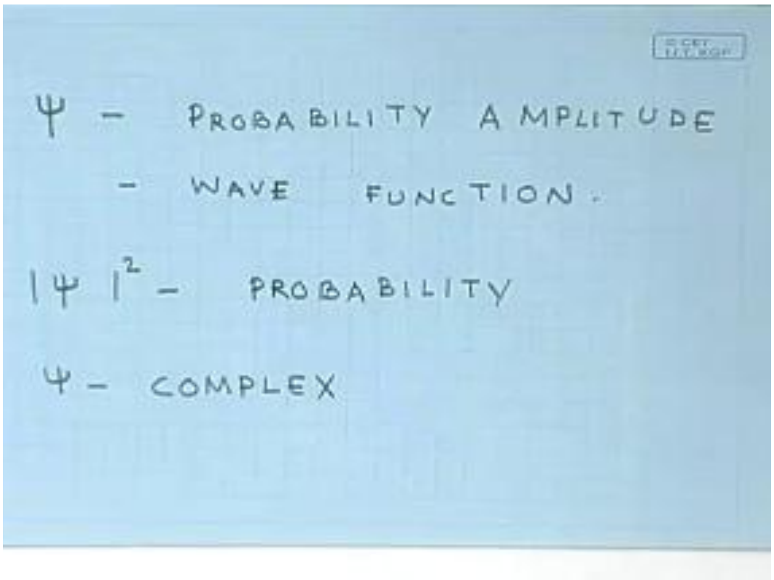

The wave is the probability amplitude wave or also called a wave function the square of that gives the probability, if you make this assumption that with every electron there is such a wave associated then you can explain the kind of probability pattern that you will get on the screen over here.

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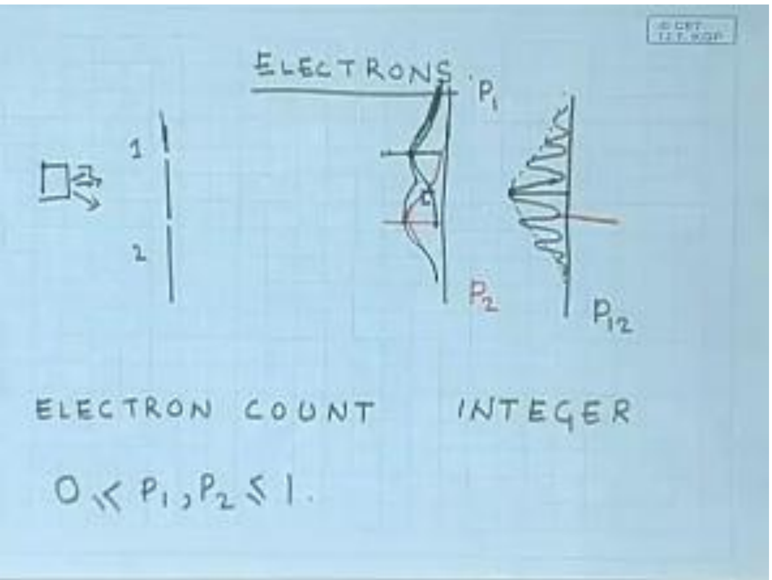

Now, remember that the probability is going to be a number between 0 and 1. So, with the electron I have a wave the wave is going to be defined everywhere it is going to give me a probability everywhere. But the moment I measure moment I make a measurement I am going to get the electron only at one position.

So, although the wave is defined everywhere the wave associated with the electron is defined everywhere it passes through two slits simultaneously. It comes here there is an interference we are dealing with the single electron there is a wave associated with the single electron. When that wave comes to the 2 slits and passes through both of them these 2 waves come on to the screen they produce an interference pattern the interference pattern the square of that gives the probability of finding the electrons somewhere on the screen.

Then you actually put a detector and measure where the electron is you will find that you have one electron at a particular position. Now, if you repeat this experiment many times you will get a count you will get counts at different points. And then you can convert these counts to probabilities and you will find that the probability follows this pattern. So, let me end today’s lecture over here we have really moved on to something quiet new. So, I am going to repeat it again in the next lecture and in forth coming lectures. So, let me end today’s lecture over here and we shall take it up again in the forth coming lectures.