Physics I: Oscillations and Waves

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Lecture - 33
Wave - Particle Duality

Good morning. We have been discussing the Compton effect let we first recapitulate where our discussion end in the last lecture.

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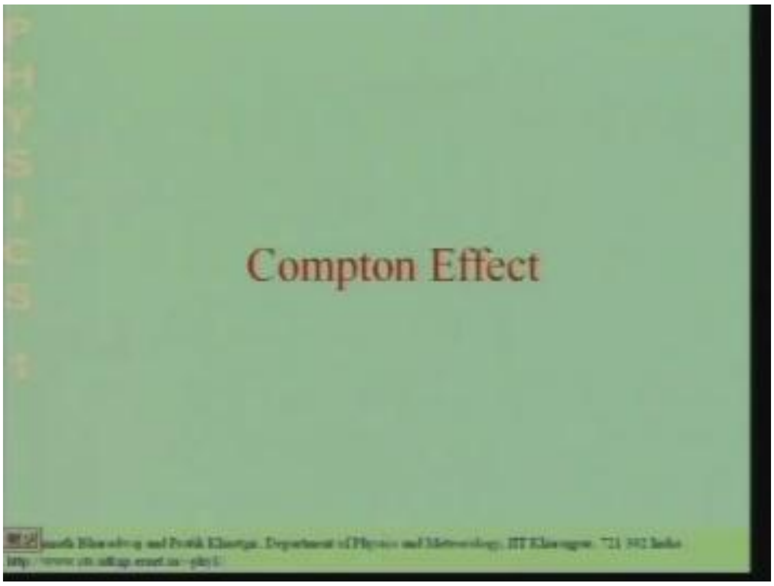

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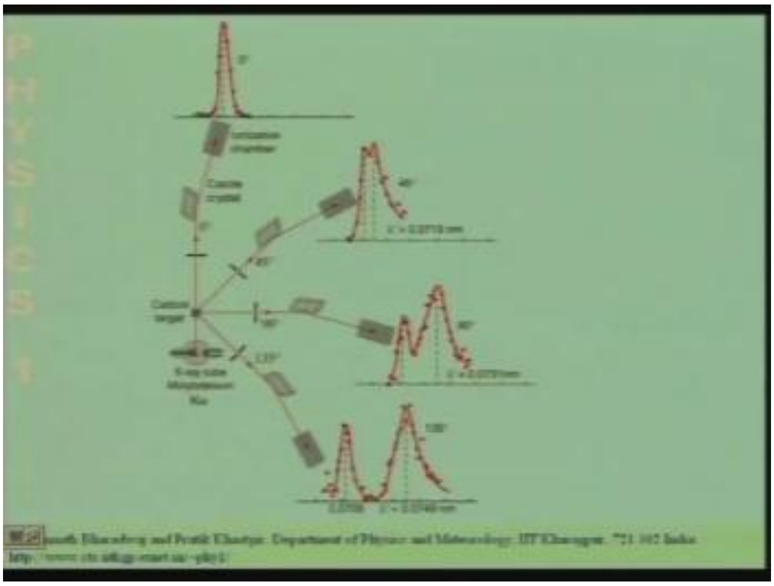

So, we have x ray incident on a carbon target and the carbon scatters the x ray in different directions the x ray is incident in at this from this direction as shown over here. Now, the particular x ray source is such that it produces x ray in a very narrow band around centered around a single frequency. So, it produces a spectral x ray spectral line at centered around a particular frequency or wavelength in a very narrow band around this wavelength. And the quantity shown here is the spectrum as measured in different directions that of the scattered x ray in different directions. Now, if you look in the forward direction the scattered x ray is found only at the same wavelength as which was incident but at other angles you can see that.

The x ray spectrum in addition to the wavelength that was sent in it also exhibits another wavelength which is larger than the wavelength that was sent in. Now, if you try to interpret this in terms of x ray being an electromagnetic wave when the electromagnetic wave is incident on this carbon target the electrons in the carbon. They are set in to oscillation they will oscillate with the same frequency as the incident radiation. And oscillating electron will emit radiation and this radiation 2 will be emitted at the same frequency as the incident radiation. So, we expect to see only the incident frequency or the incident wavelength in these, scattered light in this scattered x ray. But in addition to that there is also this different wavelength a larger wavelength component that is observed question is how to explain this.

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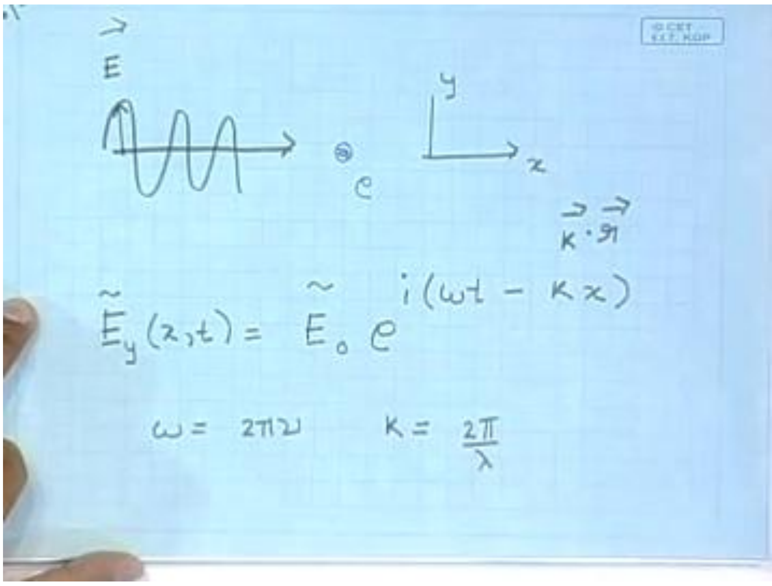

So, we have throughout this lecture being thinking of electromagnetic radiation as like for example, x ray has a wave and every wave can we know that. These waves are oh oscillating electric fields which you can which travels forward in time. And this can be represented mathematically using such a relation.

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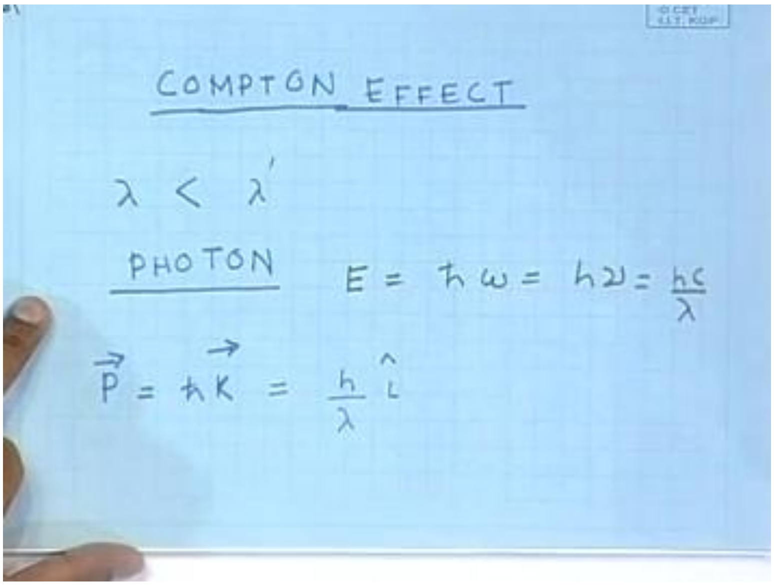

Now, to explain this Compton Effect where you have Compton this extra wavelength?

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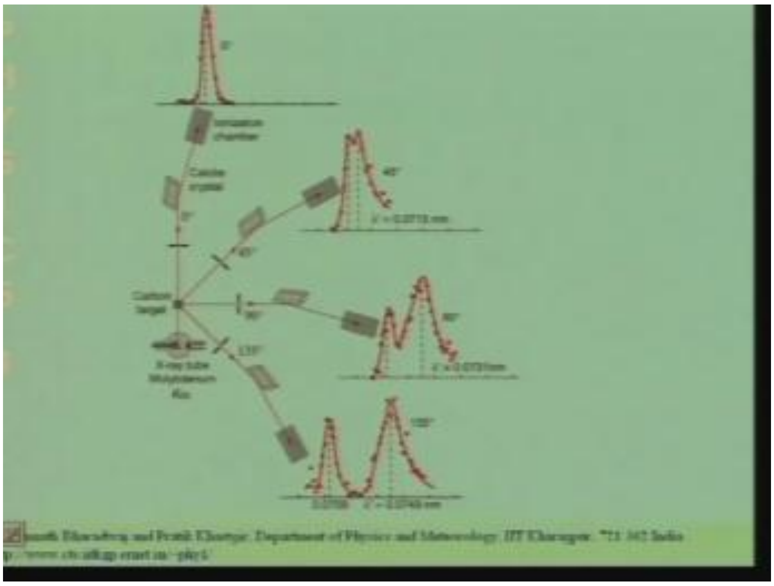

Component that is seen referred to as the Compton Effect and in order to explain this Compton Effect. In order to explain this new wavelength component new wavelength component at a larger wavelength that the one that was sent in it is required to invoke a particle like property to the x ray. So, if you if you postulate that the x ray behaves like a particle called a photon with energy h cross omega where omega is the angular frequency of the wave or h nu where nu is the frequency of the wave which you could also write as hc by lambda the wavelength.

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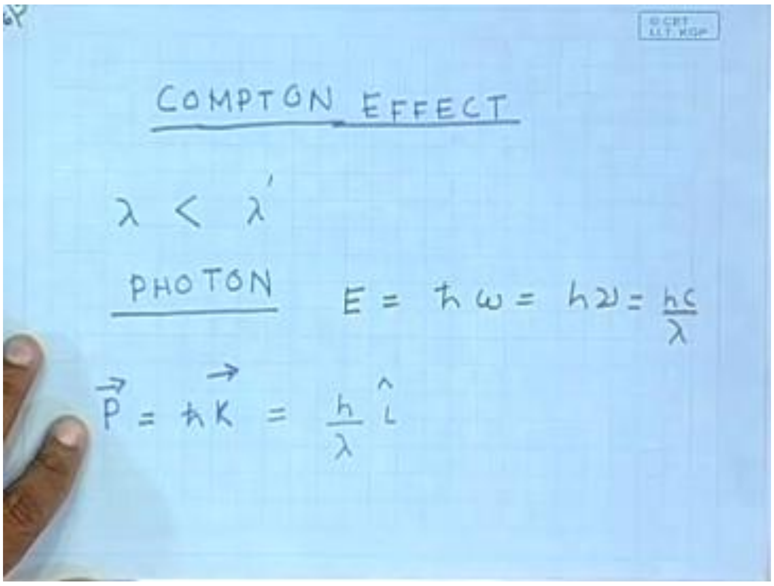

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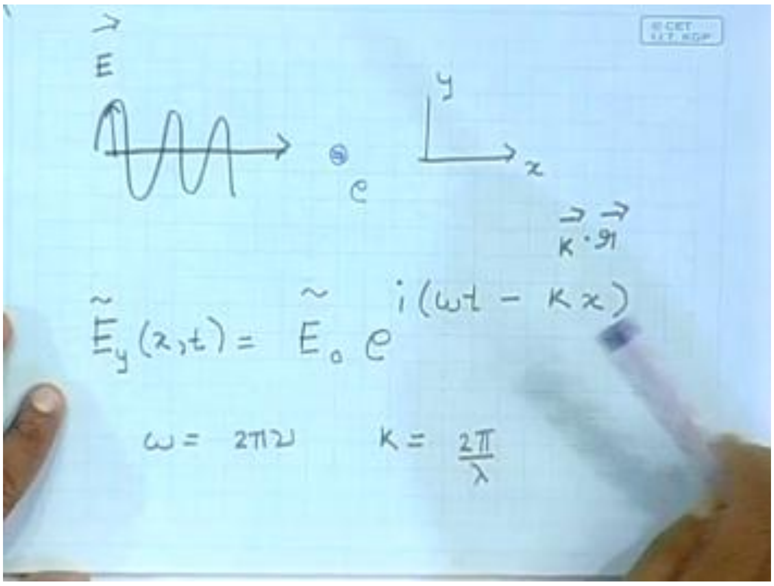

And if you also attribute a momentum P h cross k or h by lambda into the direction of propagation of the wave to. So, along with this wave if instead of this wave you think of the x ray the incident x ray as a particle whose momentum. And energy are related to the angular frequency and the wavelength of the frequency and wavelength of the wave.

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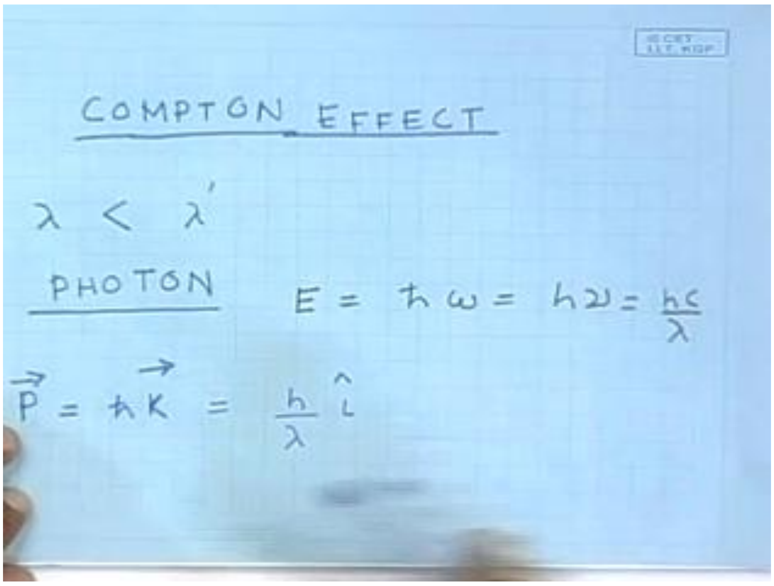

Like this let us see now if this we can explain the Compton Effect.

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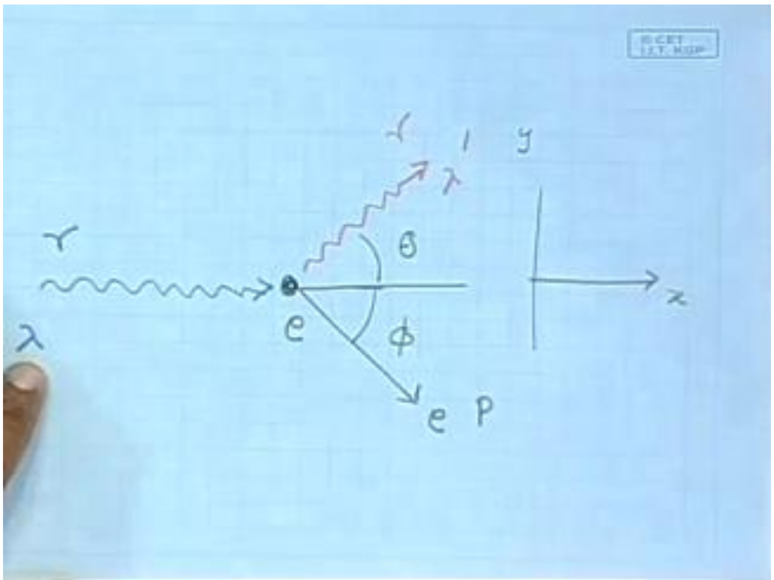

So, these are the problem now is to be thought of as the photon incident on the electron scattered in this direction the photon gets scattered in this direction and we have to apply the laws of conservation of energy and momentum. And then try to relate the wavelength of the scattered photon with the wavelength of the incident photon. The energy of the scattered photon with the energy of the incident photon the energy is inversely related to the wave length. So, if the energy goes down which you expect to happen in such a scattering the wavelength is expected to go up which is what we see. So, which is mainly the main reason why we think that with this kind of a postulate where the x ray is made up of some kind of particles called photons. You can possibly explain of the Compton effect which is what we are now try to calculate. So, we have in the last class we gone into the calculation so, the first thing that we invoked was the conservation of energy.

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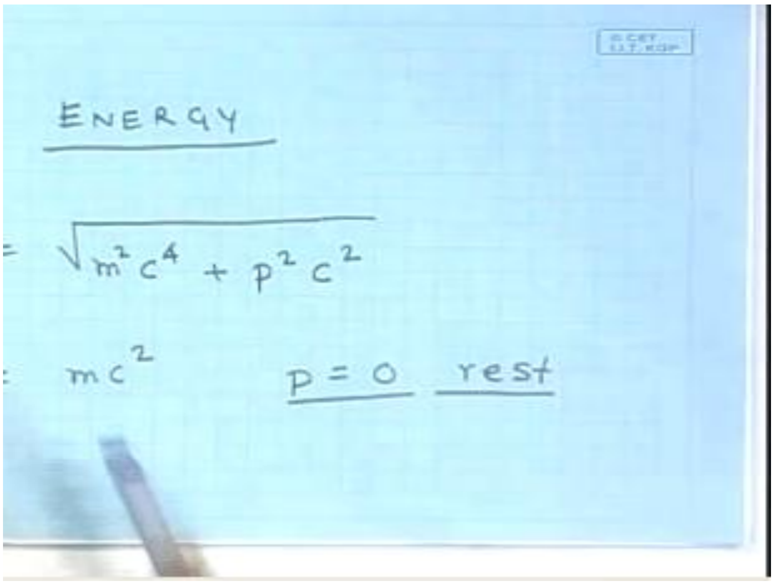

And I had told you that energy of the electron has to be calculated using this formula. This is the correct relative stick formula for the energy of the electron in terms of its momentum.

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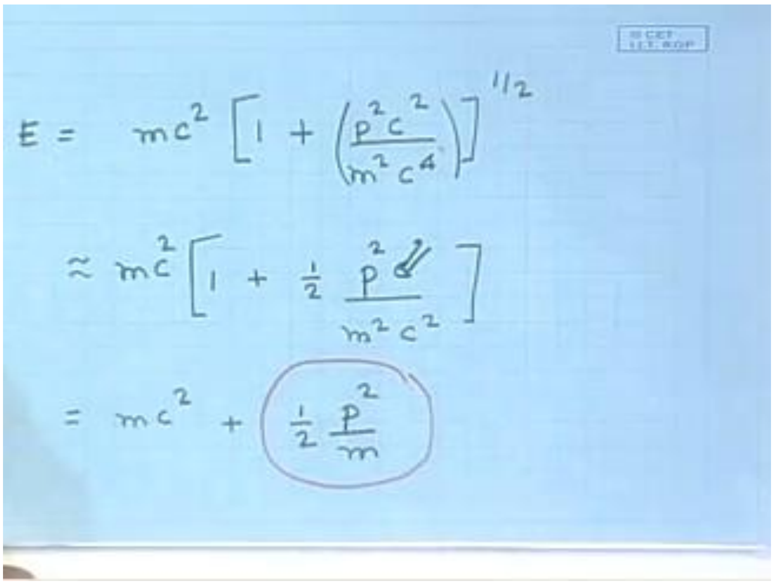

And I had also shown you that this an relation for the energy between the energy and the momentum gives you. The usual Newtonian formula half P square by m with this constant rest mass energy also coming in.

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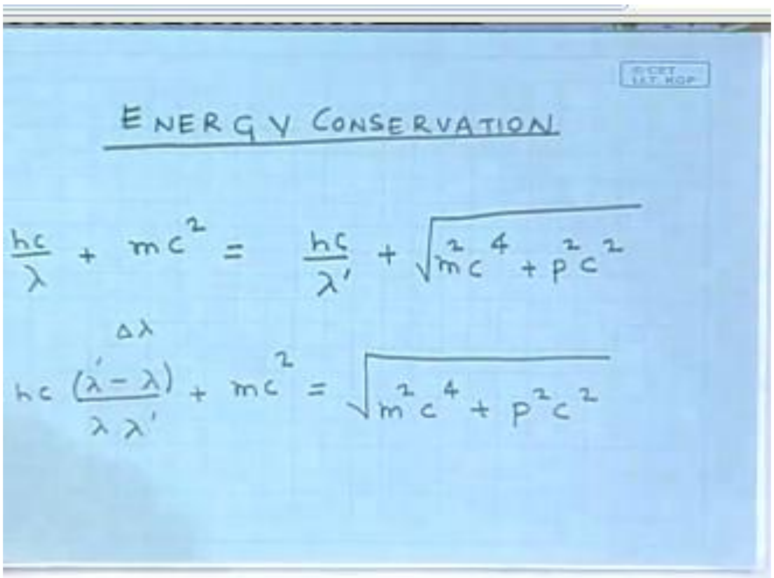

So, applying the conservation of energy the initial energy was hc by lambda for the for the x ray photon. And the energy of the electron which is at rest to start with is mc squared. This is equal to h c lambda prime by lambda prime plus the enhanced energy of the electron because in this scattering the electron, gain some energy. So, this has to increase, because this energy of the electron has gone up to maintain the conservation of energy. This energy of the photon has to go down and by doing some algebraic manipulation on this.

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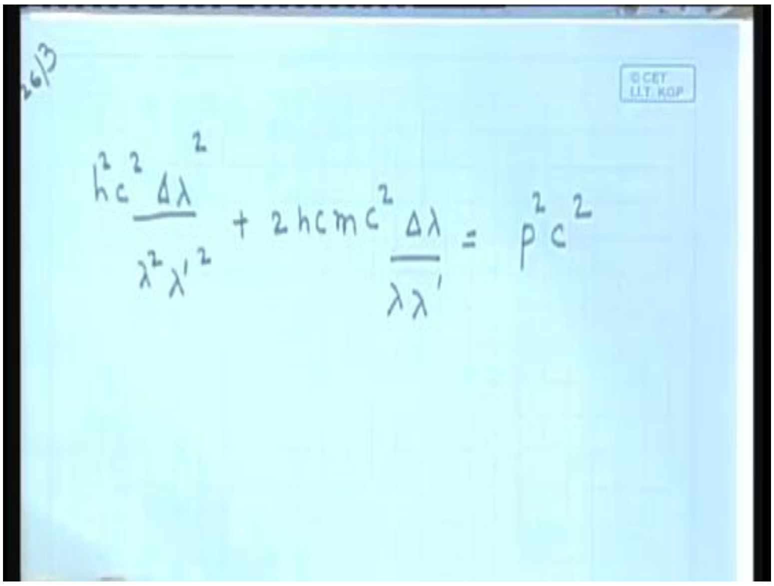

We have arrived at this expression this relation between the momentum of the electron. And the shift in the wavelength of the x ray photon let me now, next apply the conservation of momentum.

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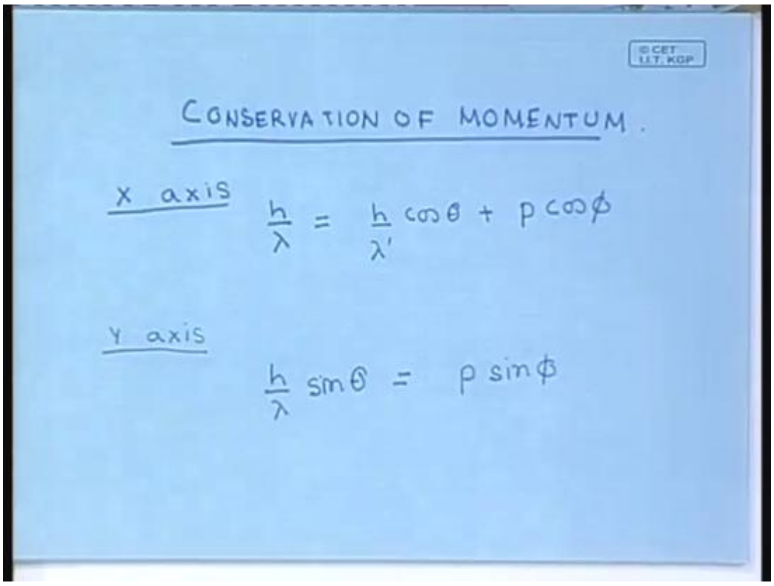

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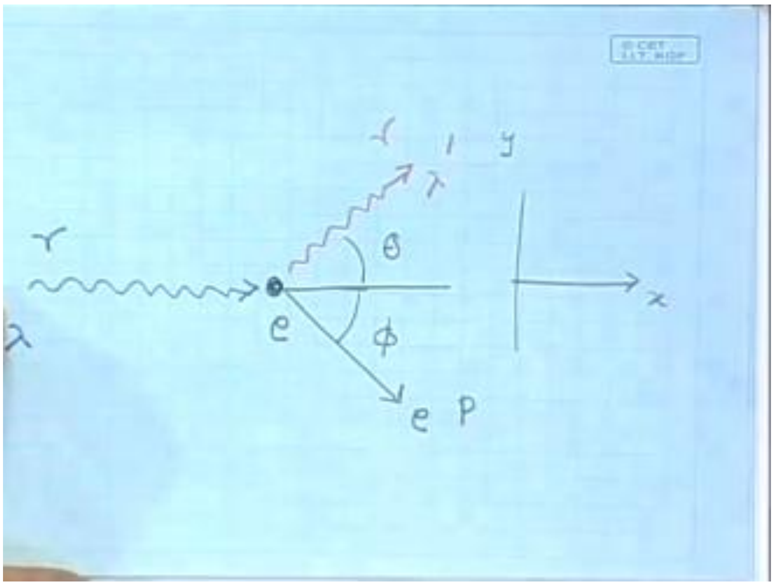

This is the picture showing you the situation let us first considered the x the momentum along the x direction so, the incident photon has a momentum.

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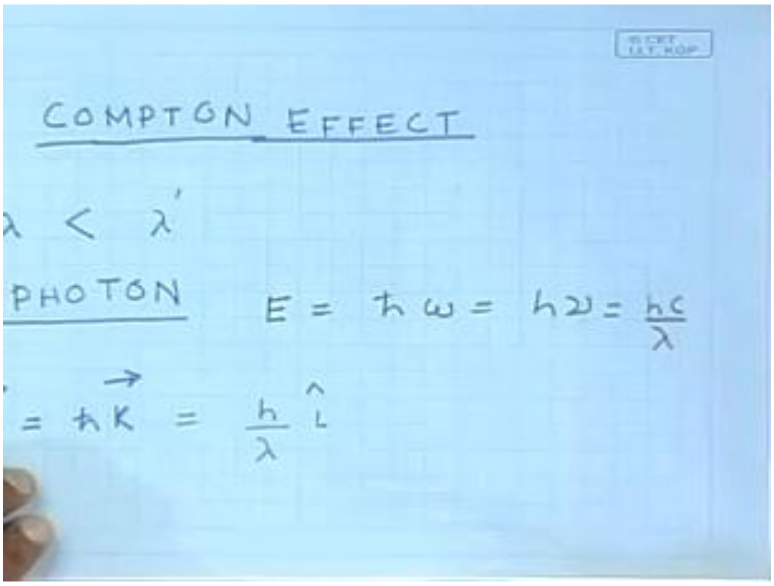

H cross by h by lambda along the x direction the electron has 0 momentum to start with.

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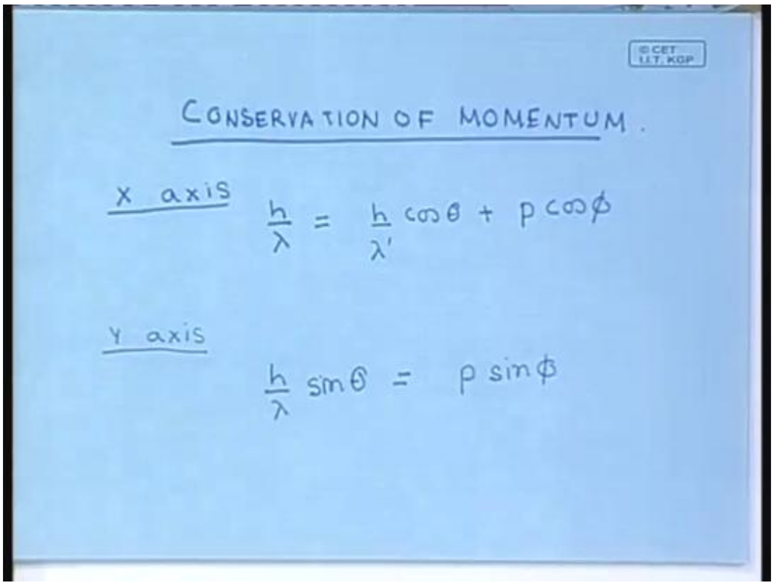

So, along the x axis h by lambda that is the incident momentum of the photon.

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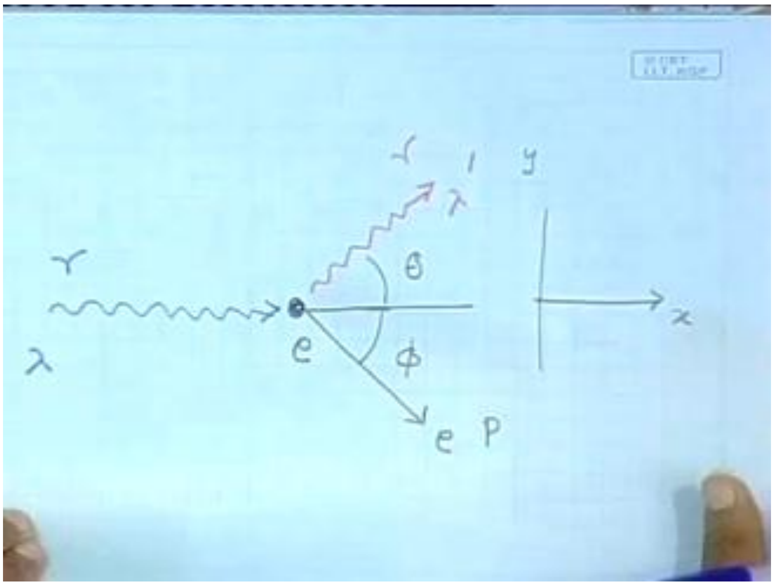

The electron has 0 momentum to start with this is equal to the X component of the scattered photons momentum which is h by lambda prime and we have to take x component which brings in a factor of cos theta plus P. The electrons momentum phi theta is the scattering angle of the photon right that. This angle made by the scattered photon with respect to the direction of the incident photon this is what we call the scattering angle.

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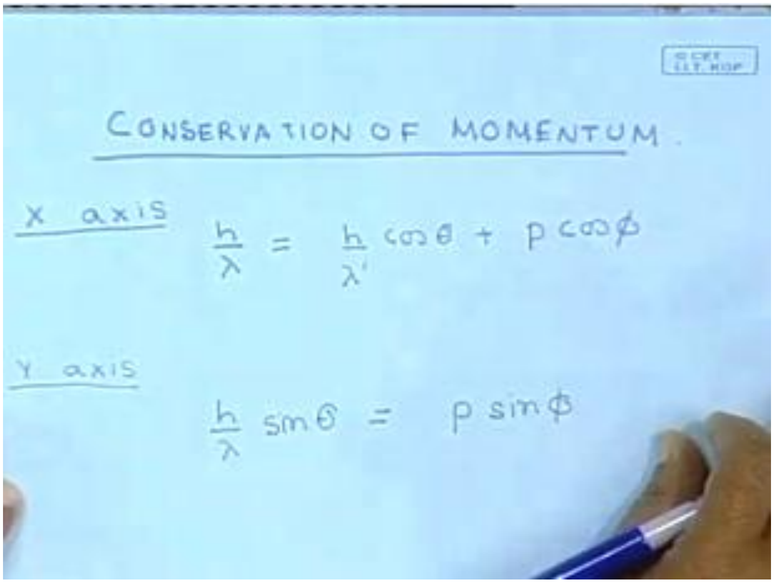

So, we have h by lambda prime cos theta plus P cos phi and along the y axis.

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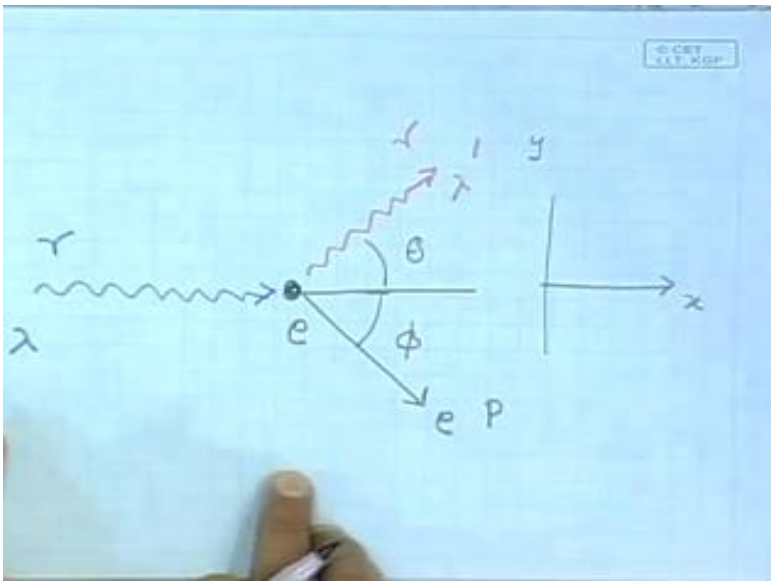

The incident photon has no momentum along the y axis so, the final the electron that comes out. And the photon that comes out there y momentum should exactly balance on another. So, the y component of the photons momentum has involves a sin theta the y component of the electrons momentum involves the sin phi. And I can write down straight away that h by lambda sin theta is equal to P sin phi now, we have to do a little bit of simplification over here. So, let me take this on to the left hand side and square it I will also square this expression if I square this expression what I will get is h let me write on the in the next page. So, I am going to take this on to the left hand side so, that this becomes a minus sign. And then I am going to square it once I take this on the left hand side and square it I will have this square plus this square minus 2 times this into this.

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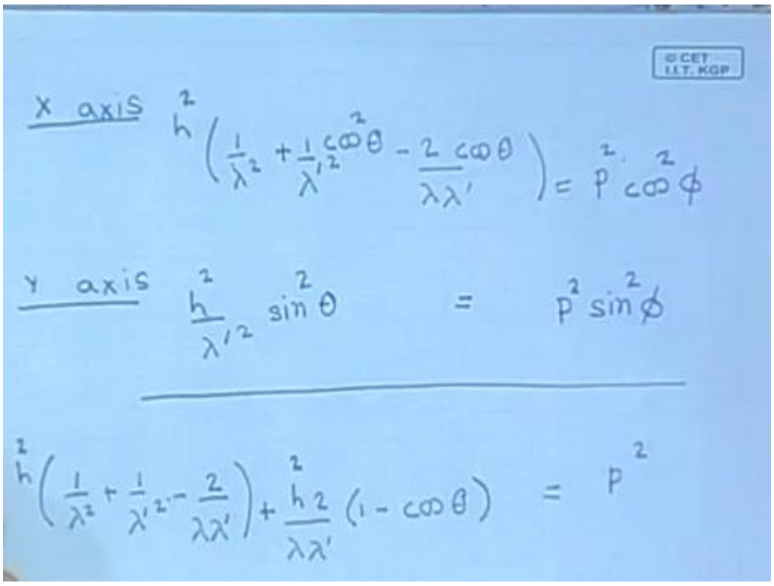

So, let me write it down what I will get is along the x axis I have let me take h square common and I have 1 by lambda squared plus 1 by lambda prime square cos square theta minus 2 by lambda lambda prime cos theta this is equal to P square cos square phi. So, let me rewind you again.

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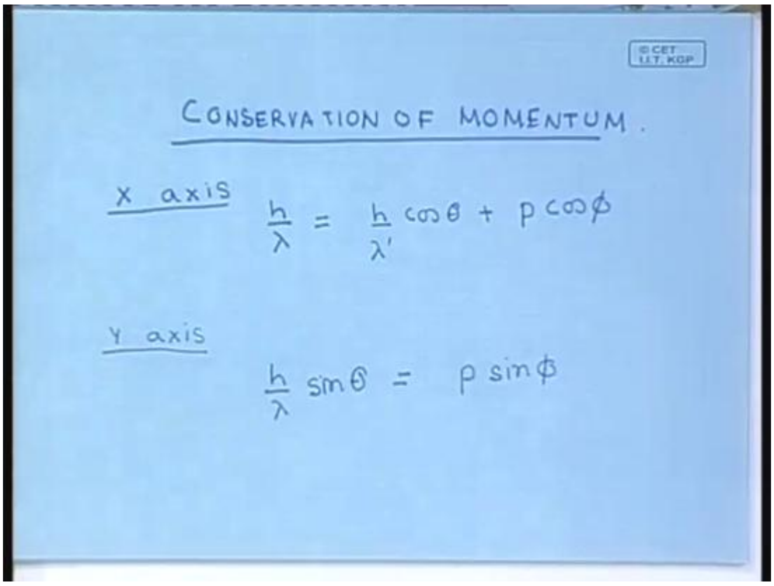

I have taken this on the left hand side and just square this so, I have the square of this square of this and the cross product and along the y axis I have again I can take I have h square by lambda prime square sin square theta is equal to P square sin square phi.

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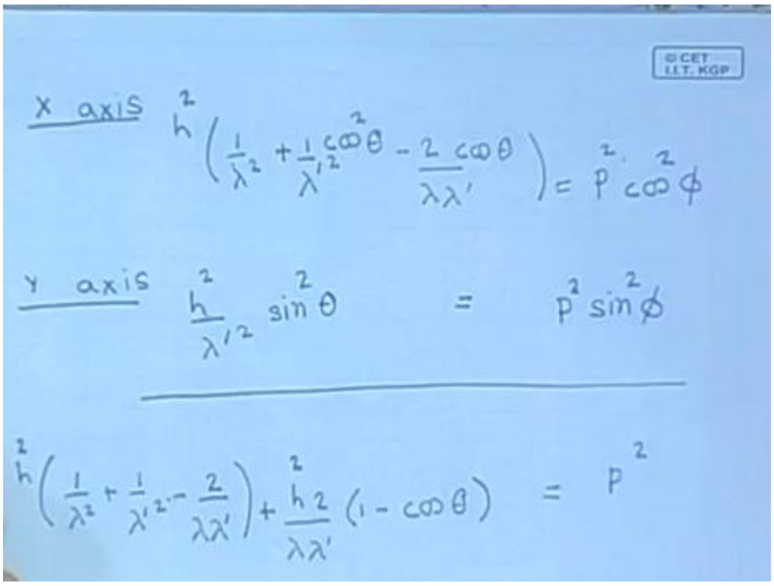

So, now I add both these expressions and this will give me P square and the left hand side is going to be this these 2 terms cos square theta. And sin square theta are going to add up to give 1 so, I have h square 1 by lambda square plus 1 by lambda prime square these are the 2 terms over here. And now, I can subtract let me write it like this I can subtract minus 2 lambda lambda prime I can write it like this.

So, I have written this term the combination of this term this term and this term is what gives me this I have added an extra term over here. So, now I have to I have rather I have subtracted an extra term so, I have to add that. And I have to write this also so, I can write it as plus h square 2 divided by lambda lambda prime 1 minus cos theta. So, 1 minus this minus cos theta term is this and this term exactly cancels out with this. So, this is the some of these 2 terms now, note that this first term the first term over here can be simplified a little bit more.

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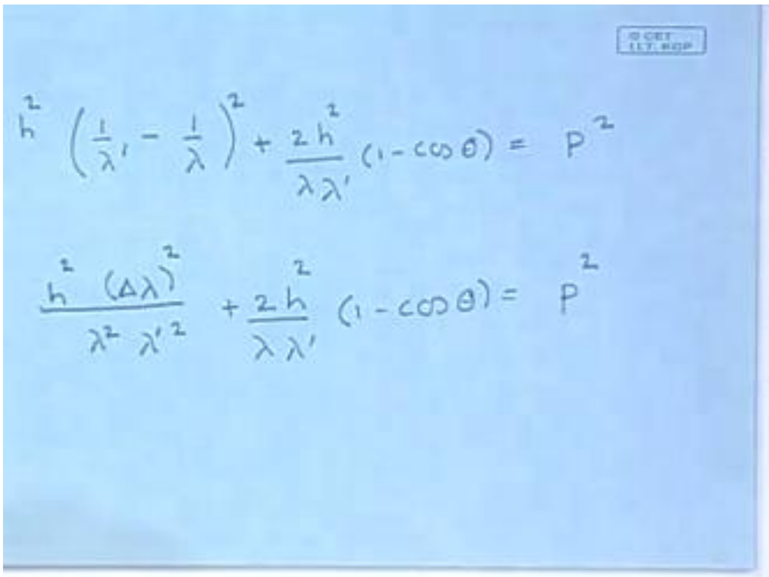

So, what I have is this is a whole square is 1 by lambda prime minus 1 by lambda whole square plus 2 h square.

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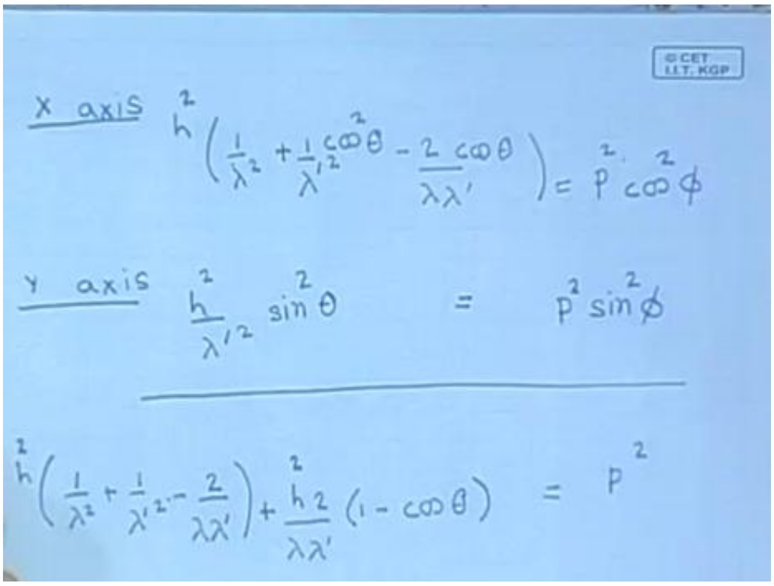

So, this I have written it in this way and this expression you can see can be written as delta lambda divided by lambda into lambda prime square. So, this is this is the same as this because I can multiply this whole thing by a factor of lambda. And this by a factor of lambda prime so, the numerator I have delta lambda then in denominator I have this square. And this square plus 2 h square lambda lambda prime 1 minus cos theta is equal to P square. So, the conservation of energy finally, gives me this the conservation of sorry the conservation of momentum finally, gives me this conservation of energy which we had worked out to the last class finally, gives me this.

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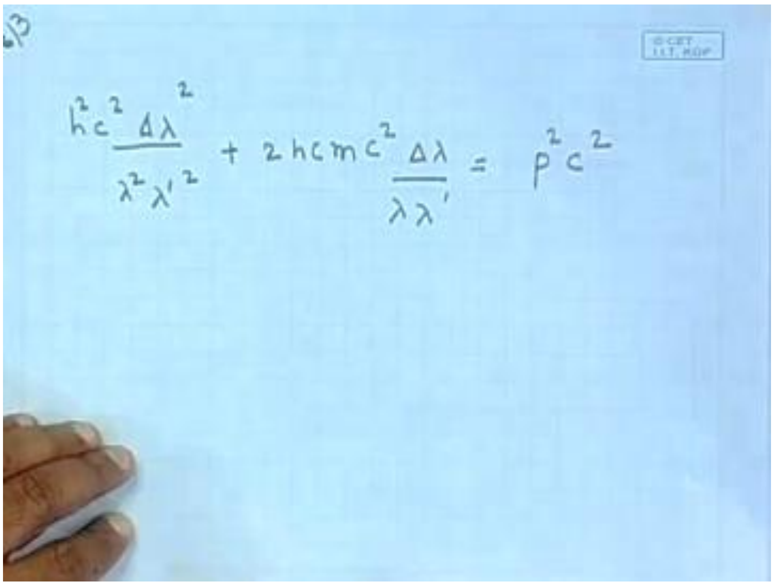

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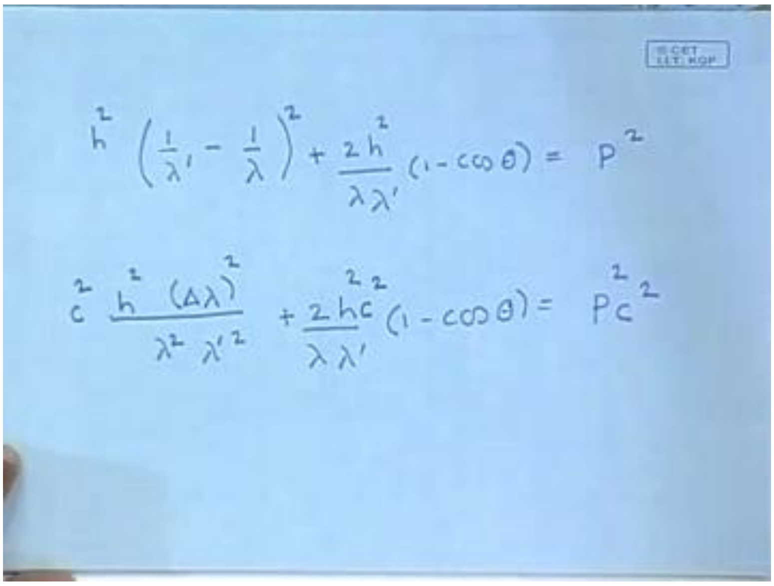

So, I have to multiply this equation by c square so, I want to compare it with the conservation of energy now, we can straight away compare these two expressions this is the conservation of energy.

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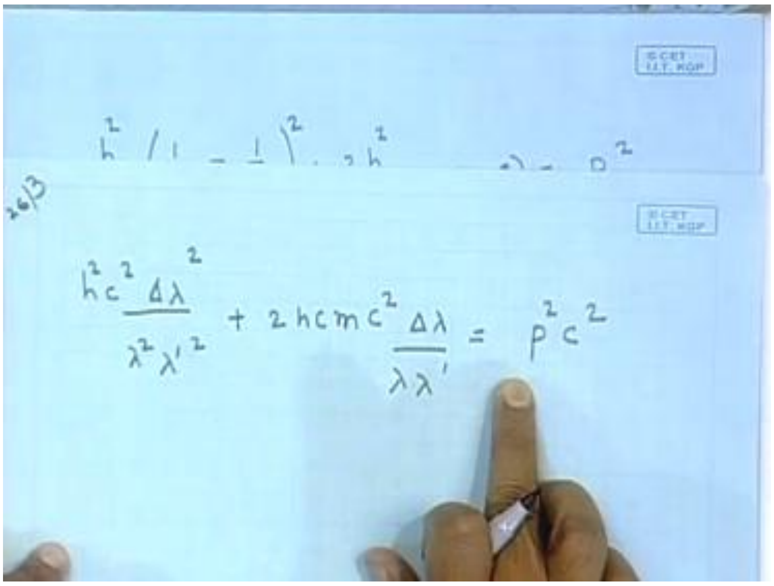

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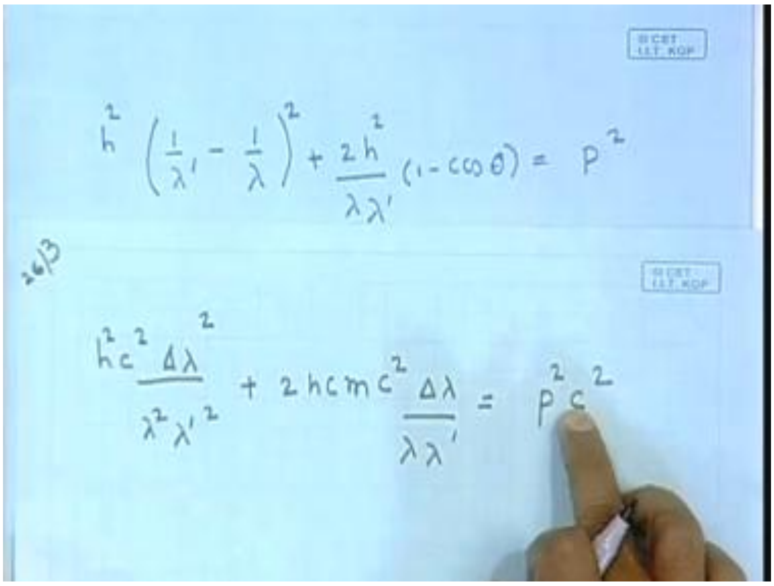

This is the conservation of momentum this term occurs in both the expressions.

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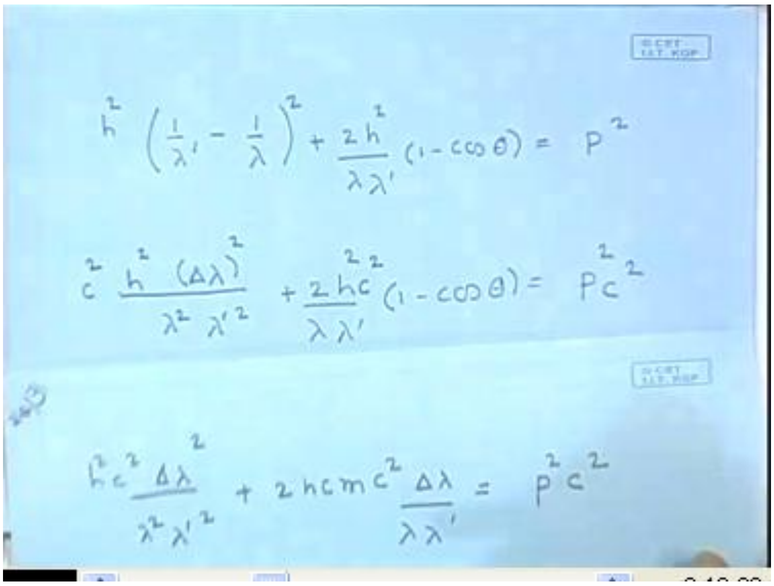

It essentially tells me that this term should be equal to this because the other two terms are there in both the expressions.

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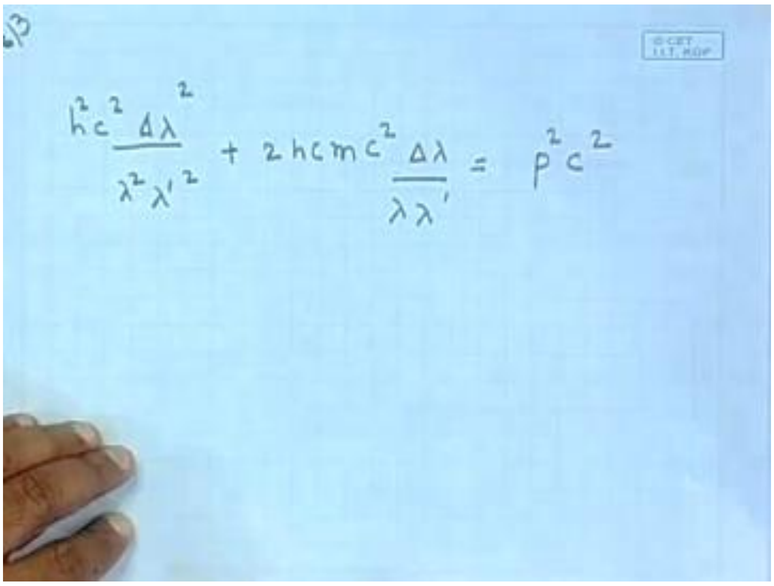

So, finally, what we have when I combine the conservation energy and momentum is that this term should be equal to this

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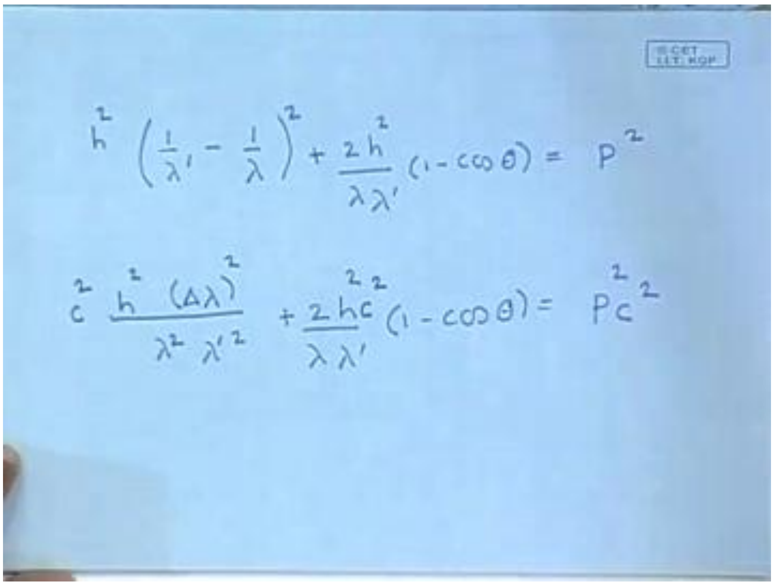

So, let me write it down separately then we can proceed.

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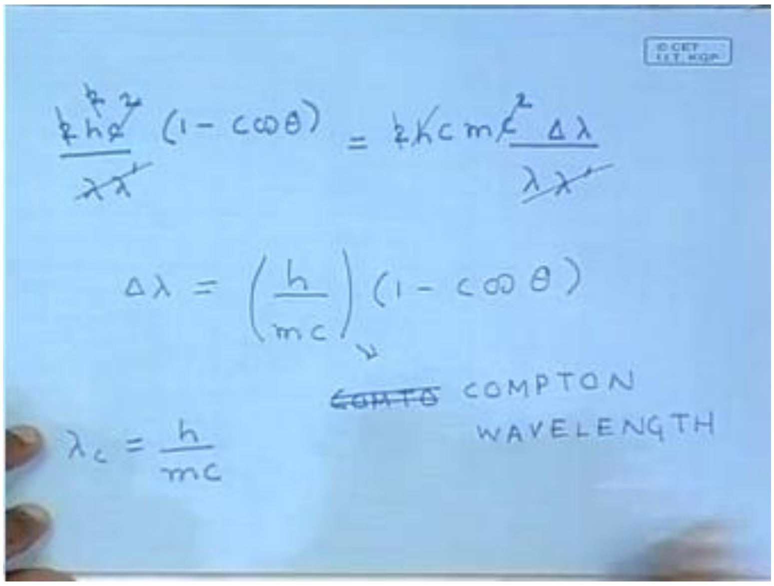

So, what it tells is the 2 h square c square 1 minus cos theta this is the conservation of momentum this should equal to the term that occurs in the conservation of energy which is 2 hcmc square.

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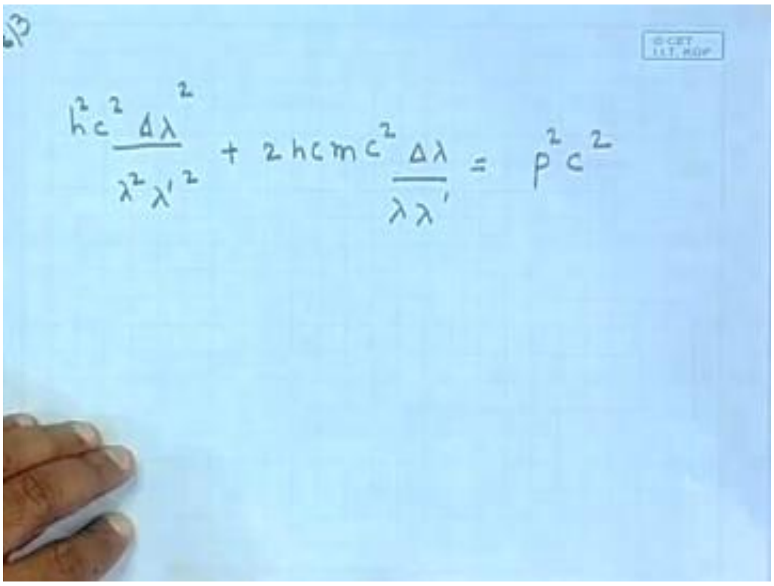

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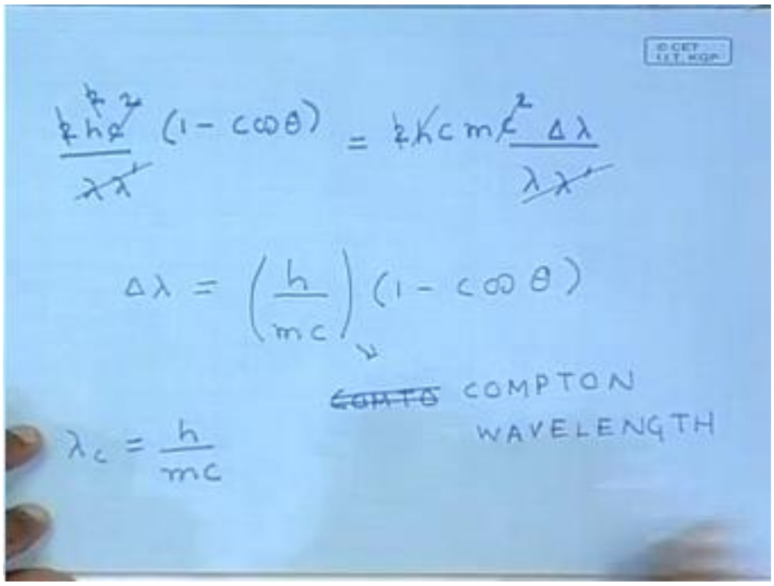

2 h c m c square into delta lambda lambda by lambda prime and this factor of 2 is gone h 1 of the h over here is gone. So, this h square is gone this c square cancels out this c square this lambda lambda prime cancels out over here. And what we are left with is delta lambda is equal to h m divided by mc into 1 minus cos theta or. This number h by mc is called the Compton wavelength this is called the Compton wavelength Compton sorry lambda c which is equal to h by mc this ratio.

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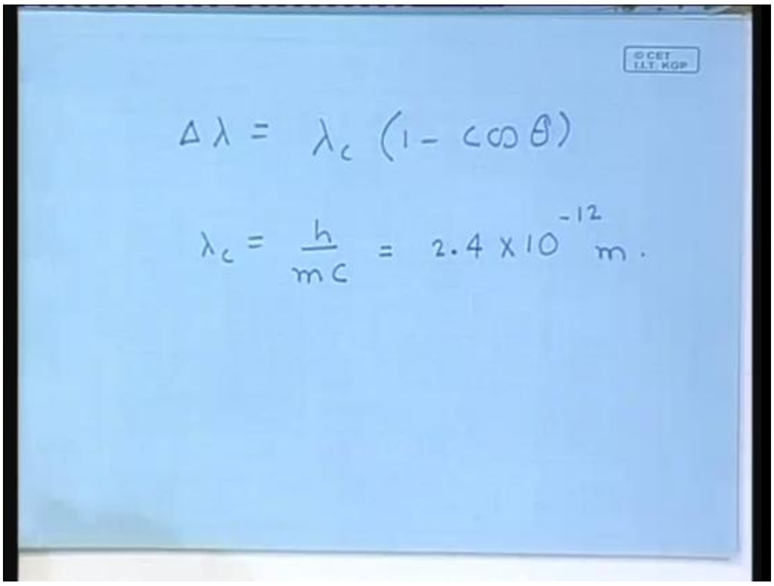

Or in brief what you could write is that delta lambda the change in the wavelength is the Compton wavelength into 1 minus cos theta where the Compton wavelength h by mc. So, what we have achieved is that we have been able to predict the expected shift in the wave length if the x ray can be thought of in terms of particles called photons. And we find that the expected shift in the wavelength is the Compton wavelength associated with the scattering particle into 1 minus cos theta let us go back to the situation that we are discussing.

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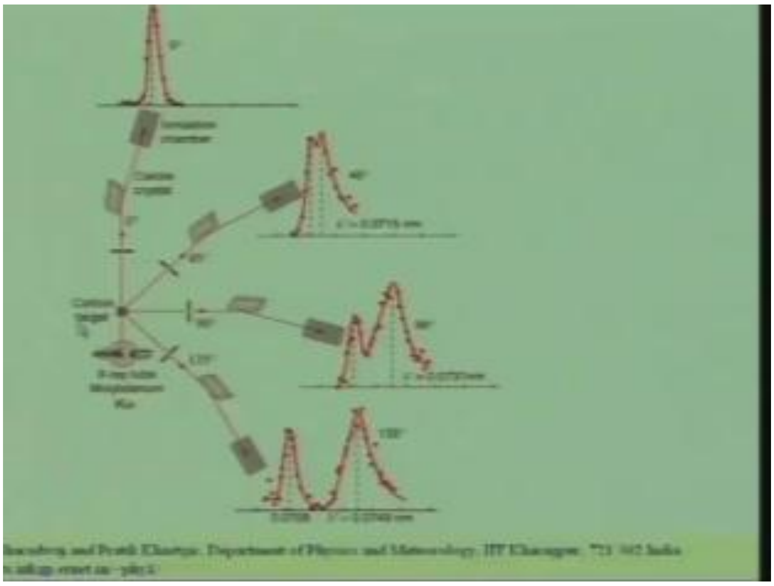

So, here we have x ray incident on a carbon target and in the carbon target we know there are these carbon nuclei. And which has electrons around it so, when the x ray is incident on the carbon target. It is the nuclei as well as the electrons both are going to scattered the x ray both are charge particles both are going to scatter the x ray. And the shift in the wavelength that is going to be produced is the Compton wave length.

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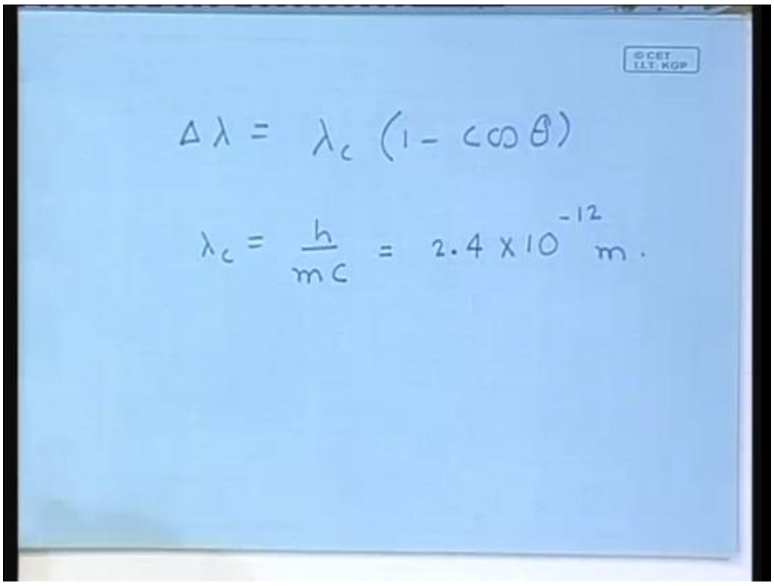

Corresponding to each particle into 1 minus cos theta which tells you the angular dependence. Now, let us you see the Compton wavelength for any particles so, for electron the Compton wavelength is going to be the Planck's constant divided by c. And also divided by electron mass for the nucleus is going to be h c divided by the nuclear mass. Now, the nuclear mass is we know are much larger than the electron masses the electrons are the light lightest particles in the carbon sample lightest charge particles. So, the electrons are one which are going to be most effective in producing the, is carbon to produce a largest shift in the wavelength. They are going to be most effective in this process so, let us estimate the Compton wavelength for the electron now the electron.

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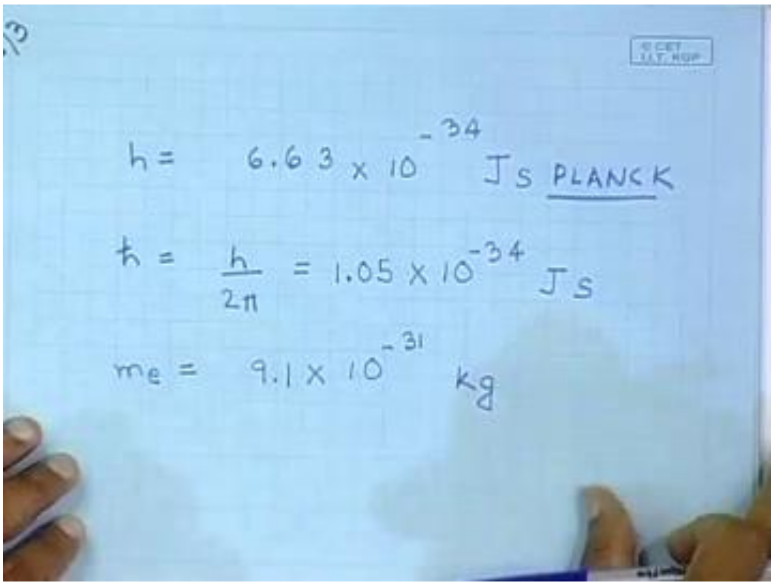

So, the Planck constant so, this is the Planck constant each and this is the Planck constantan h cross plank constant. So, we the values given here the electron mass the values given here 10 to the power minus 30 1 kg 9.1 into 10 to the power of minus 31 kg.

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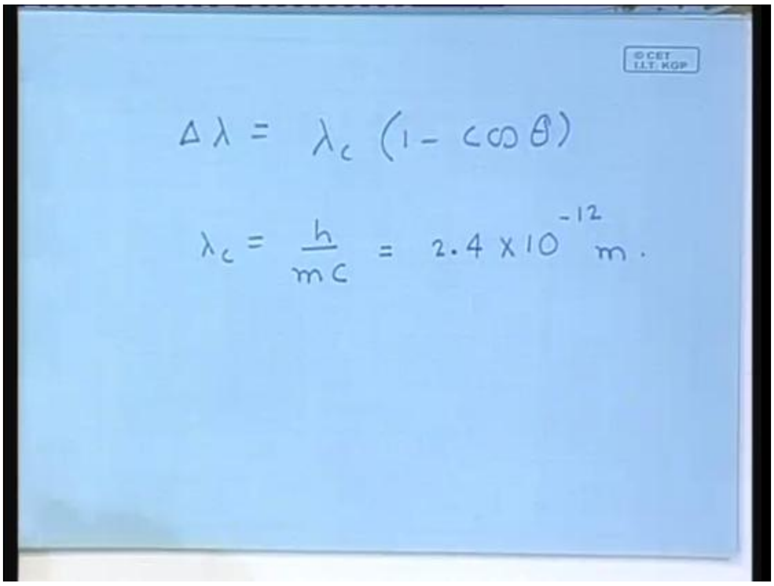

And the speed of light is also known. So, let us put in the values and so, this if you put in the values and calculate this; this comes out to be approximately 2.4 into 10 to the power minus 12 meters for the electron and for the proton neutron.

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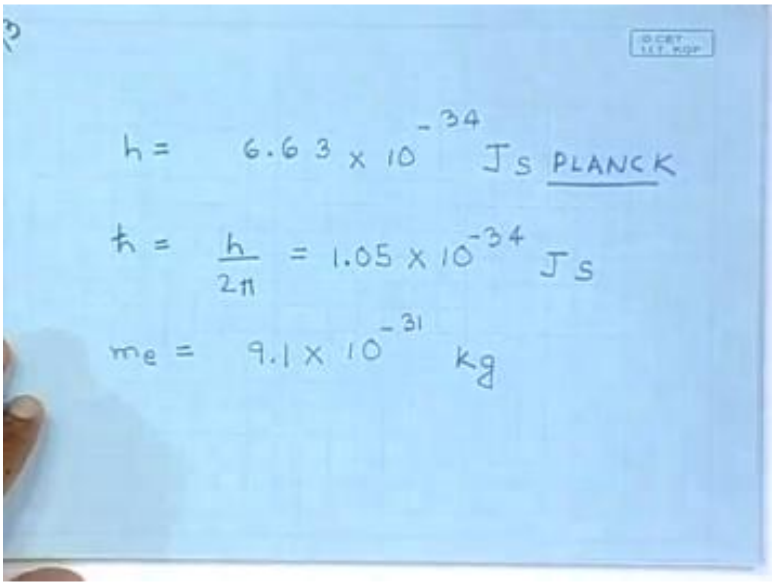

And the nuclei it is going to be much smaller so, we see that the, that there is if you assume that the x ray. The incident x ray comes in the form of particles called photos the incident x ray if you think of it in terms of particles called the photons. And then calculate the energy transfer in this particle particles scattering and then calculate the shift in the wavelength of the photon.

Then you get this prediction and you find that the there is a going to be a shift in the wavelength which is of the order of 10 to the power minus 12 meters a little more than that. And it is going to have a 1 minus cos theta dependence so, let us first so, we have estimated the magnitude of the shift. Now, let us look at the angular dependence of the shift what this predicts is that. The shift is going to be minimum when theta is 0 when theta is 0 cos theta is 1. So, this is going to cancel out. So, there should be no shift in the forward direction which is what you see.

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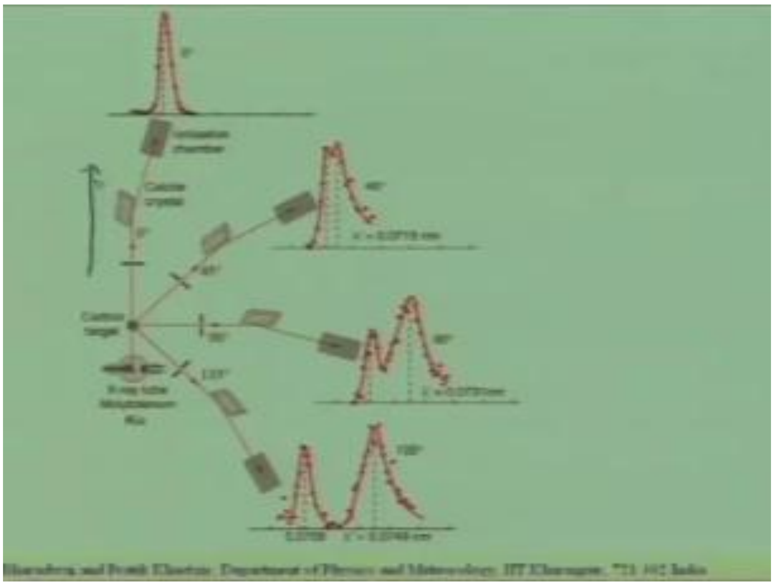

The photon is not going to transfer any energy to the electron if it gets scattered in the forward direction which is also what you see over there and then as you look at larger and larger scattering angles.

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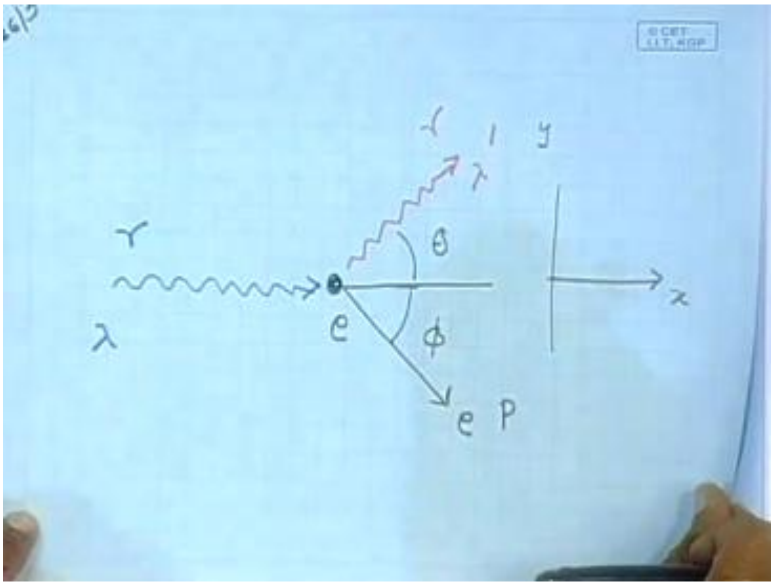

You expect the shift in the wavelength to increase right and the transfer of energy is going to be maximum to the electron is going to be maximum. If the photon is scattered straight back then the electron is going to be important momentum in this direction. And that is going to be the maximum situation where the maximum energy is transferred to the electron.

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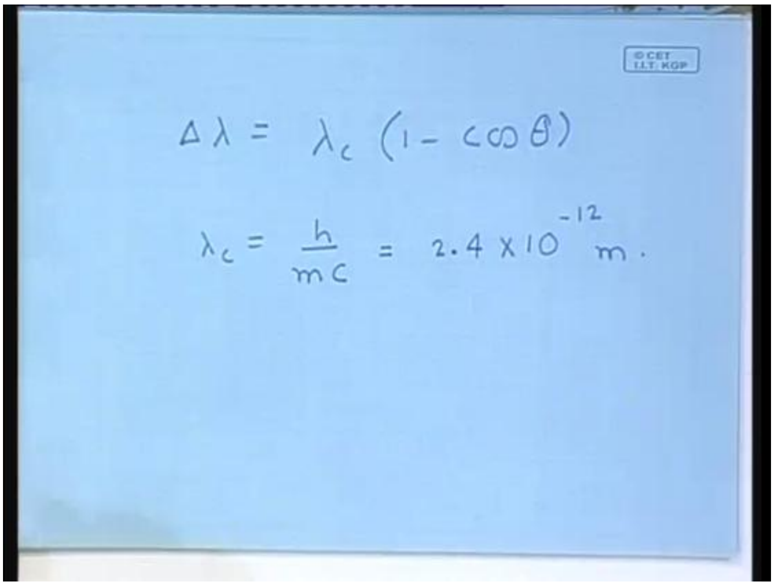

So, cos theta pi is the, is minus 1 where this factor becomes 2 or the shift in the wavelength is twice the Compton wavelength. This is the situation when the maximum energy is transferred to the electron which is also what you see.

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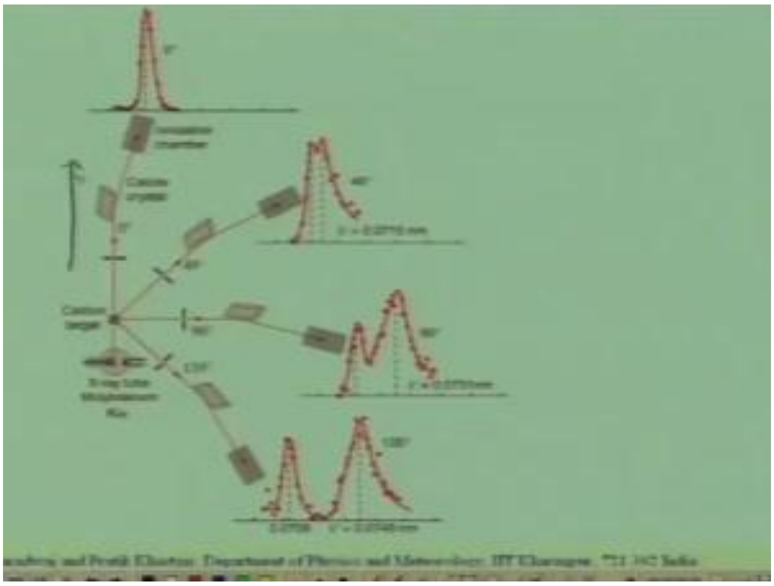

So, the pattern that you see here is very is quite close what is very similar to what you expect from this qualitative discussion.

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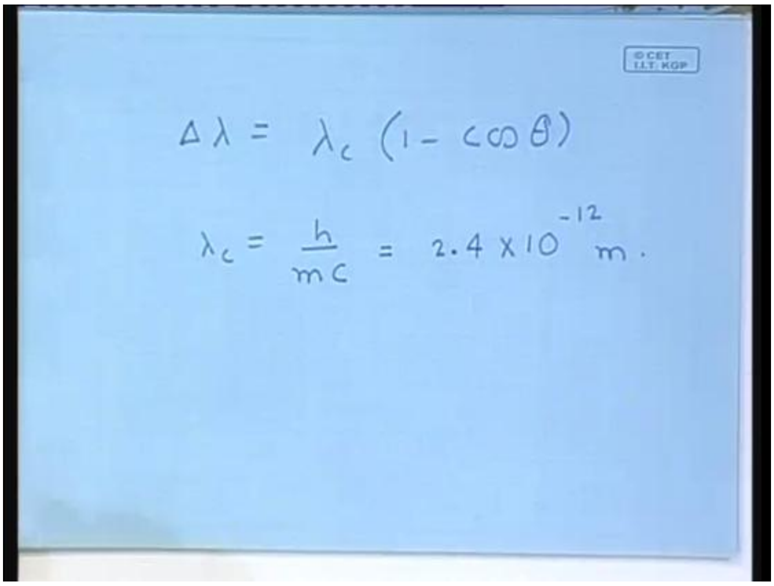

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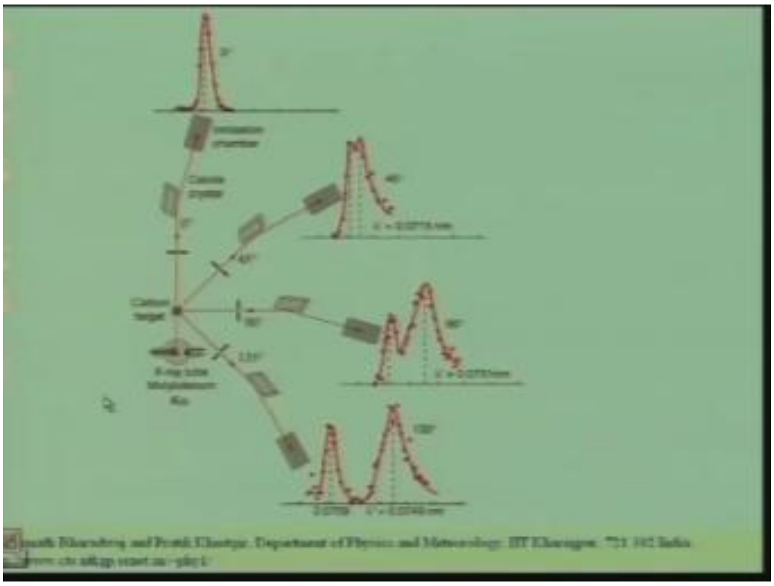

The shift in a wavelength you can see here increases at as with theta and this shows you at 45 degrees 90 degrees 135 degrees. And notice that at 135 degrees the shift that you see here is maximum.

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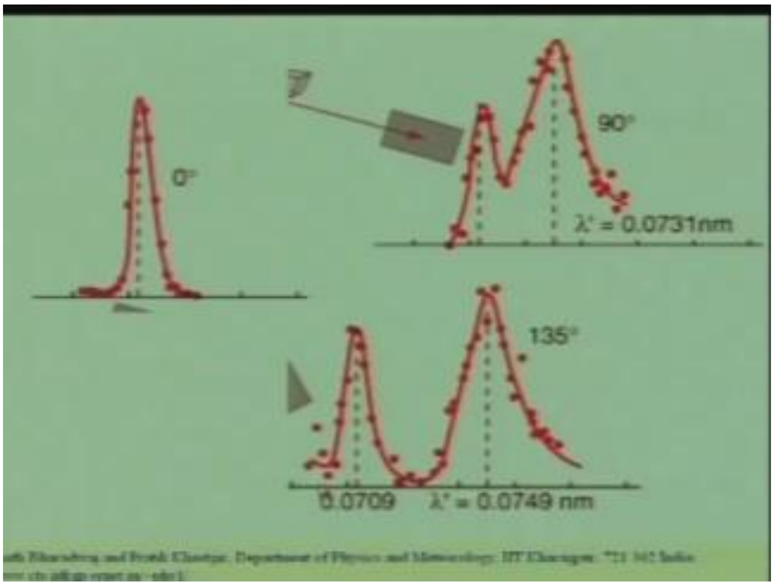

This shows you the same spectra blown up so, this is the spectrum at 0 angle theta equal to 0 that is in the direction of the incident or in x ray. So, the forward direction so, there is only 1 spectral lines seen over here this shows you the spectrum at minty degrees this shows you. The spectrum at 135 degrees now, the incident x ray the x ray is incident at an angle at a wavelength which you can read from here it is 0.0709 nanometers 0.07.

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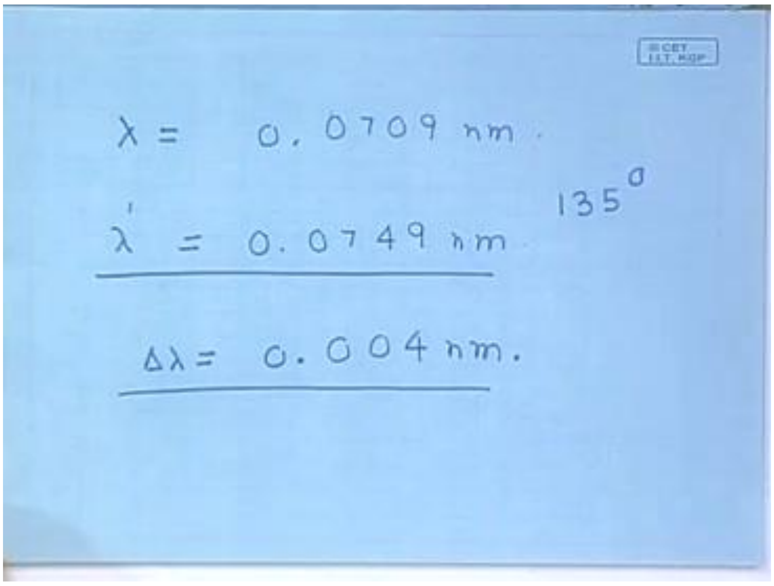

So, in this particular situation the x ray is incident and the x ray which is scattered the the Compton effect the larger wavelength.

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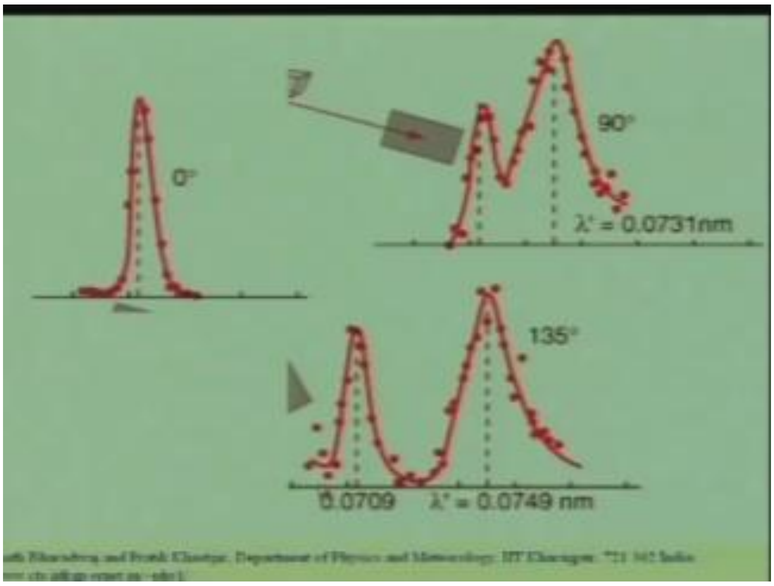

This is at you can again read this from here it is 0.0749 nanometers and this is at 135 degrees scattering angle. So, when the scattering angle is 135 degrees the delta lambda the shift in the wavelength is 0.004 nanometers you have to check you should check.

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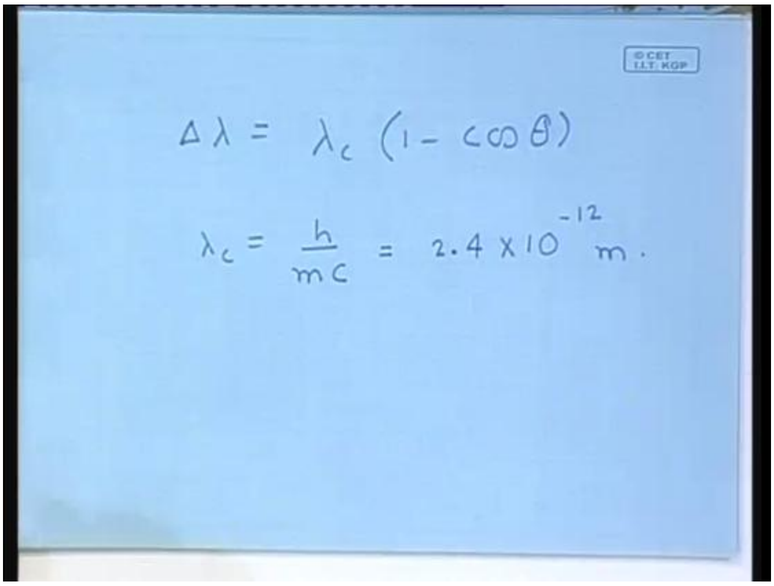

That this actually is consistent with the predictions of the calculation that we have done so; you should check that if you put in 135 degrees over here for theta. And put in the value of the Compton wavelength 2.4 10 to the power minus 12 meters.

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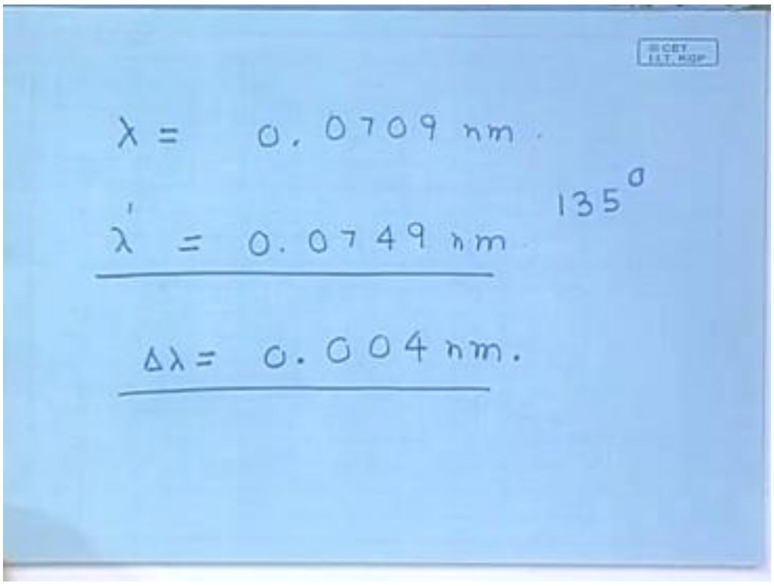

You can explain you can explain this shift in the wavelength which is observed you get exactly 0.004 nanometers. So, what we see at the the photon picture the Compton effect if you wish you explain the Compton effect you have to assume that. The, that the x ray let that the electromagnetic radiation behaves like a particle. This particle is called the photon so; you have to assume that the x ray behaves like a particle, call the photon. And the photon energy is h cross omega it has a momentum h cross k where k is the wave vector of the wave which you would have thought of which you would have attributed to the x ray. If you had thought of it as a wave and omega is the angular frequency that you would have attributed to the x ray if you had thought of it as a wave.

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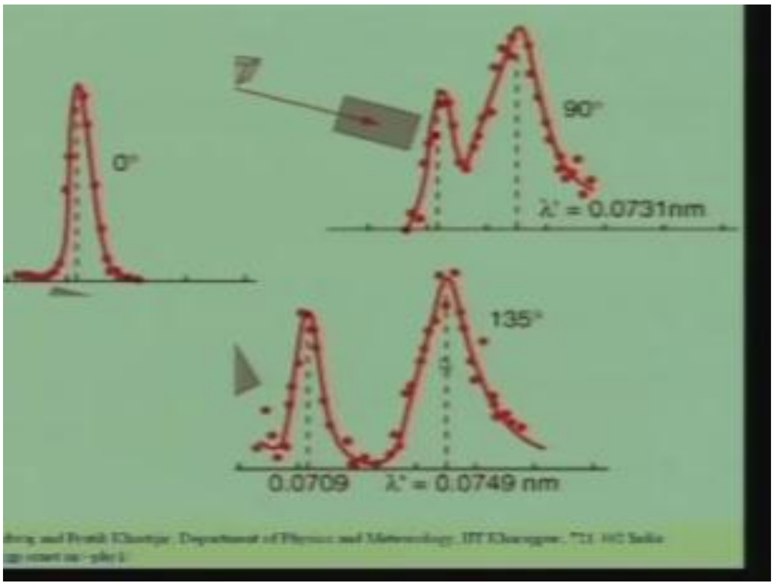

So, in order to explain this, these shifted wavelength lines the lines with larger wavelength you have to invoke the assumption that x ray behaves like a, that particle. And this true not only for x ray it is true for the whole of electromagnetic radiation. So, the photo electric effect for example, which we have I have told you briefly yesterday requires us to think of the radiation in ultraviolet visible or the blue end of the visible in terms of particles call the photon. And the same think is required here so, you have to now invoke the particle picture for light for the whole of the electromagnetic spectrum. But this does not mean that you have to abandon the wave theory of light if you abandon the wave theory of light you cannot explain the phenomena of diffraction.

And interference which have been discussing extensively the, a large part of this course and which there are many experiments which you might have some of which you might have possibly done where you have demonstration clear demonstration of diffraction. And interference using light using electromagnetic radiation light radio waves all of them exhibit interference. And diffraction x ray exhibits diffraction and you cannot explain this without invoking the wave property of x rays. But you cannot explain the Compton Effect without invoking the particle a part the particle without assuming that. The x ray also behaves like particle call the photon so, you what we see here is that light shows a dual behavior. It behaves like both a particle and a wave and this is what refer to as the wave particle duality.

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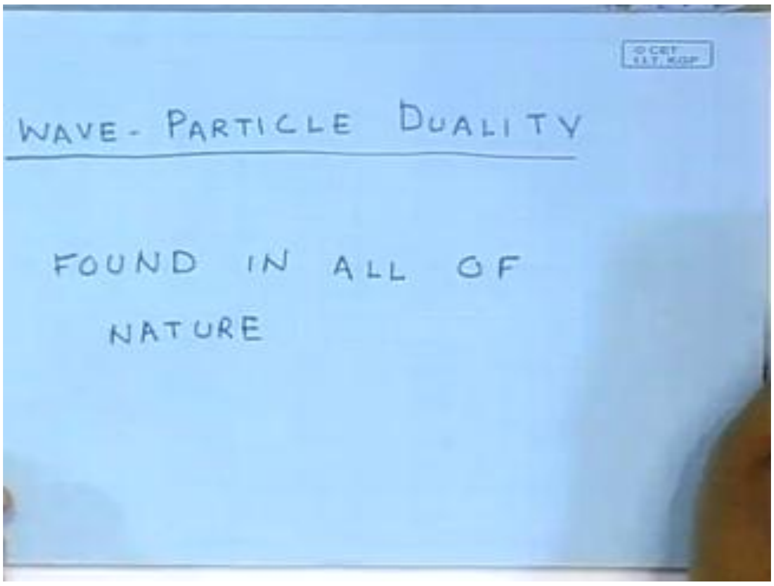

So, light behaves like both particles and waves weather it behaves like a particle or a wave depends on what you do with the light depends on the context depends on the situation. And you would have to invoke either the wave picture or the particle picture to describe the light. It is not that light is a wave and not a particle or light is a particle or not a wave light is both it behaves like both a particle. And the wave which is what we refer to as the wave particle duality it has it exhibits both. The properties of both sometimes it exhibits a property of a wave sometimes it exhibits the properties of a particle. Now, this dual behaviour is seen not only for light it actually extends to everything in nature let me repeat this dual behaviour that. We have which the Compton which we have brought in to our discussion through the Compton Effect. This dual behaviour of light that it behaves like a particle and a wave this is not restricted to light alone such dual behaviour is found in all of nature.

So, this is found in all of nature let we write it down so, that you get the point this extends to all of nature. So, this is what we are going to discuss in the rest of a today is lecture. And the, this is going to be what we going to discuss in the remainder of these all of the lectures that are going to follow. So, the rest of this course is going to be completely devoted to discussing this wave particle duality. So, all of nature exhibits this dual behaviour where it behaves sometimes like a waves sometimes like a particle depend depending on the situation. And this wave particle duality is what is going to what is going to be the centre of our discussion in the lectures that I going to follow it was. So, the Compton effect I have told you he was discussed was discovered in are in may have told you was discovered in the 1920's and 1924.

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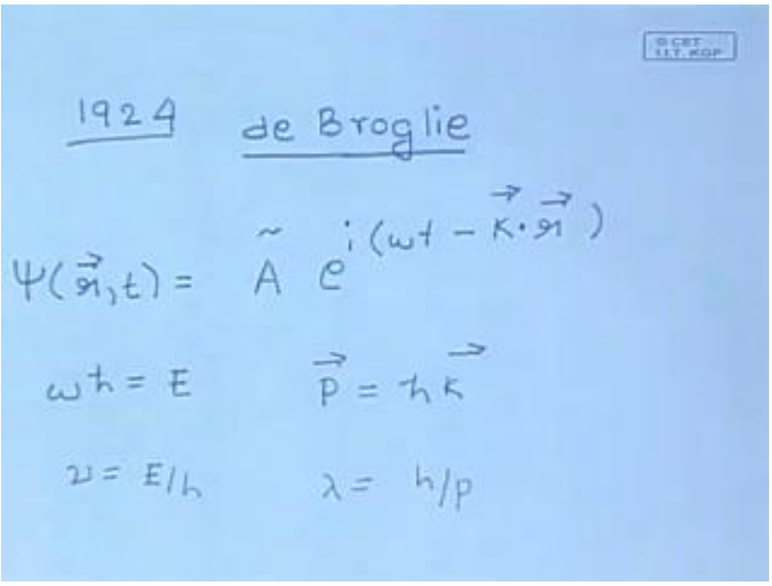

The French scientist De Broglie so, in 1924 the French scientist De Broglie he hypothesized that associated with every particle. We have a wave what we have seen until now is that associated with every wave for example, electromagnetic radiation. It is sometimes as said it to think of it as a particle called a photon now, what De Broglie hypothesized was exactly. The other way around what he hypothesized was that for every particle for example, for the electron for this pen over here for any particle for that matter. There is an associated wave so, for every particle there is an associated waves so, let we write down a wave. We are all hopefully by this time familiar with how to write down waves equations for a, for waves so, let we write down a wave.

So, this is the wave this represents a wave so, what De Broglie hypothesized was that associated with every particle. There is a wave and this is what represents a wave and the angular frequency of the wave omega is related to the energy of the particle through this h cross omega is equal to the energy of the particle. And the wave vector k is related to the momentum of the particle just like we did for the photon or you could write this in terms the the frequency and the wave length. So, the frequency is related to the energy the frequency of the wave that is associate with the particle is related to the energy of the particle through this. It is E by h and the momentum and the wavelengths so, the wavelength of this wave is related to the wavelength of the particle through this relation h by p.

And this wavelength is called the De Broglie wavelength this wavelength of this particular wave is called the De Broglie wavelength. So, let me again repeat what De Broglie's De Broglie's hypothesis was his hypothesis was that associated with every particle. There is a wave and the angular frequency or the frequency of the wave is related to the particles energy by a relation like this. The wavelength of the wave is related to the particles momentum like this and I have written down here an expression for a wave. And since we have been working in terms of angular frequency and wave vector I have also write down these in terms of the the momentum. And the energy of the particle and these are very similar to the relations we had for the photon now, the first question. So, let us ask a few let us take up a few questions, but before we take up a few question regarding this there is a point which I should make. So, the point is a very important point when we talk of a particle.

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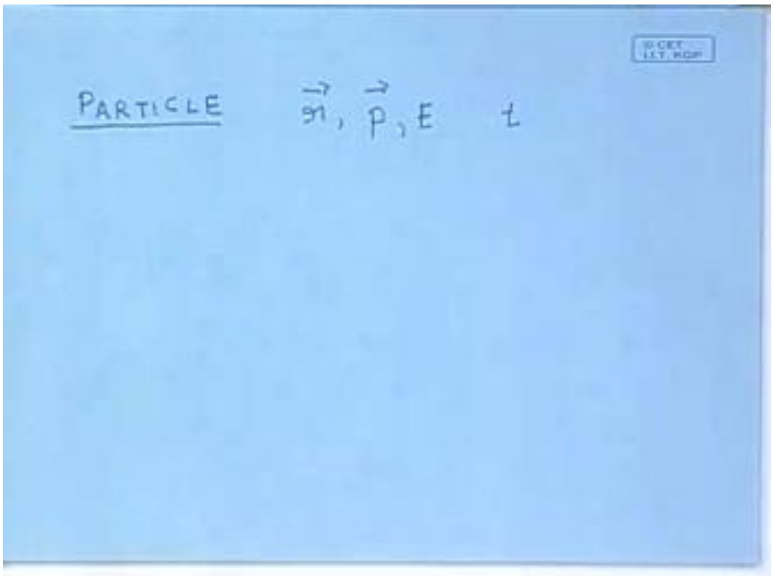

When every we talk of a particle we the position of the particle its momentum it is energy all of these are very well defined quantities at a any given time for example if we discuss this pen. And think of it as a particle which would be appropriate if you were not interested in the orientation of the pen and such things or you cloud take the the cover of the pen and think of it as particle. Then its position at any given time its momentum its energy all of them are very well defined. And when you write down the Newton's equations of motion for a particle Newton's laws it assumes that the momentum the position everything is well-defined. And you write down second order differential equation you solve them. And it will give the trajectory for any later time that that these are things which are familiar.

So, for a particle its momentum its position we know where it is you know what speed it is moving with at any instant of time. And the, you expect a particle the, we know that the particle is going to be only at 1 point. So, this particular pen cover is located at this point at present at t let us say this is t equal to 0 so, its present it is exactly over here. Now, if I leave it its position is going change and it is going to go somewhere else. So, there is no ambiguity as to the position of the particle at any given instant time it has a well-defined position it has a well-defined momentum. And you can follow it at its position and momentum at all instants of time there is no ambiguity about it. Now, you look at let us look at the wave associated with the particle.

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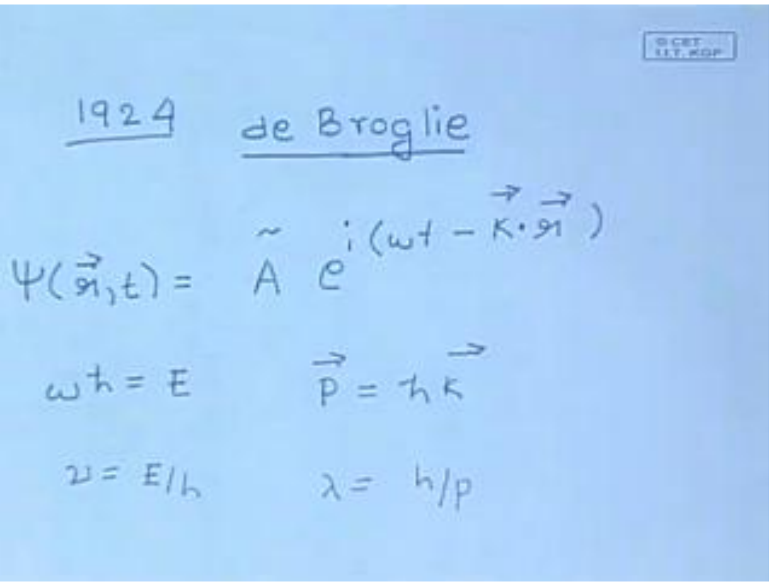

So, associated with the particle we have a wave let us ask the question where is this, wave defined. Now, this question the answer you see this let us look at this wave psi is a function of r and t. And if you give me any value of r I will have a value of psi and this is a sinusoidal wave.

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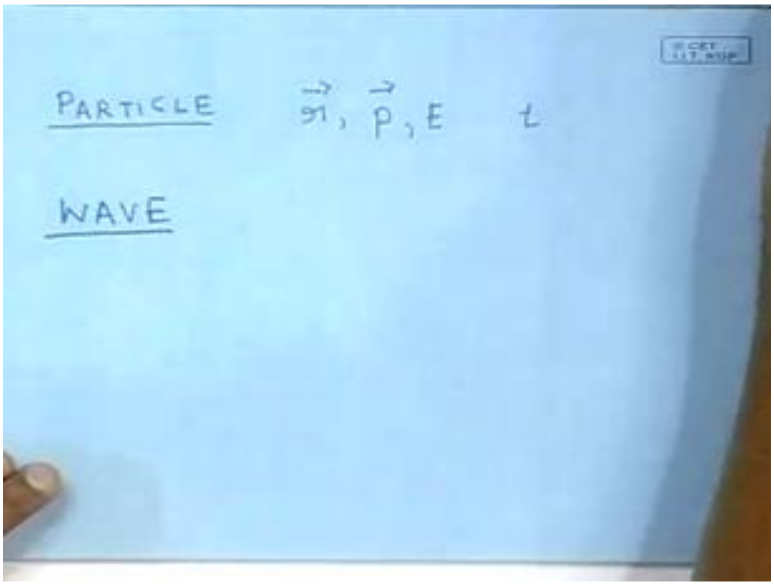

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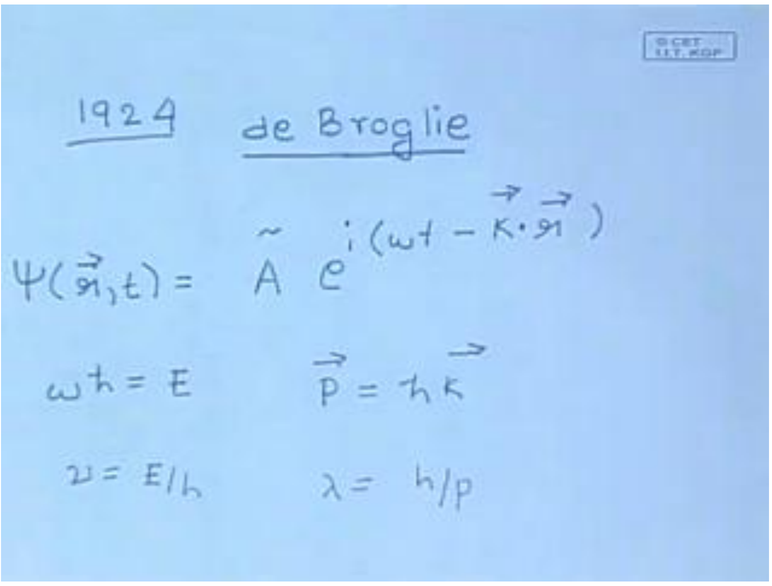

So, it is defined if you draw it let us say the wave let us for an instant assume that the wave is along the x axis.

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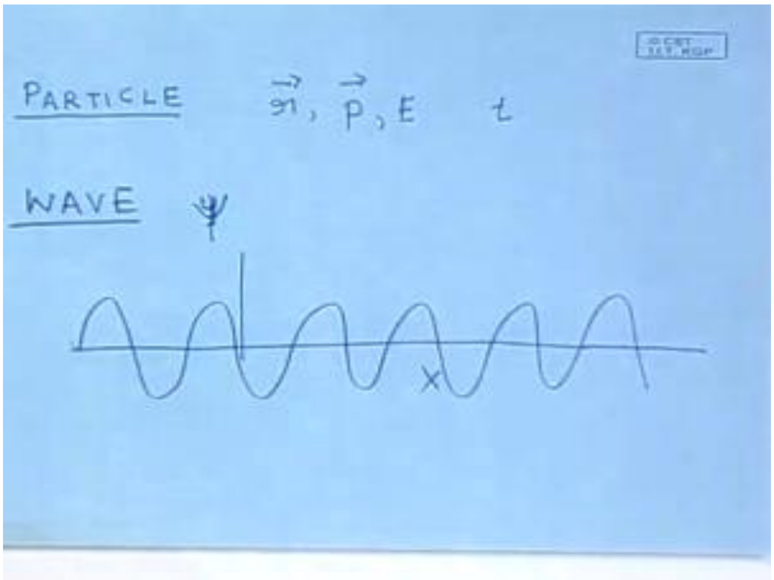

So, if I draw the real part of this wave as a function of x this is psi the real part of psi it will look something like this. So, if you ask the question where is the wave? Wave is distributed all along the x axis. It is defined over the whole of space so, the wave is not defined at any one unique point.

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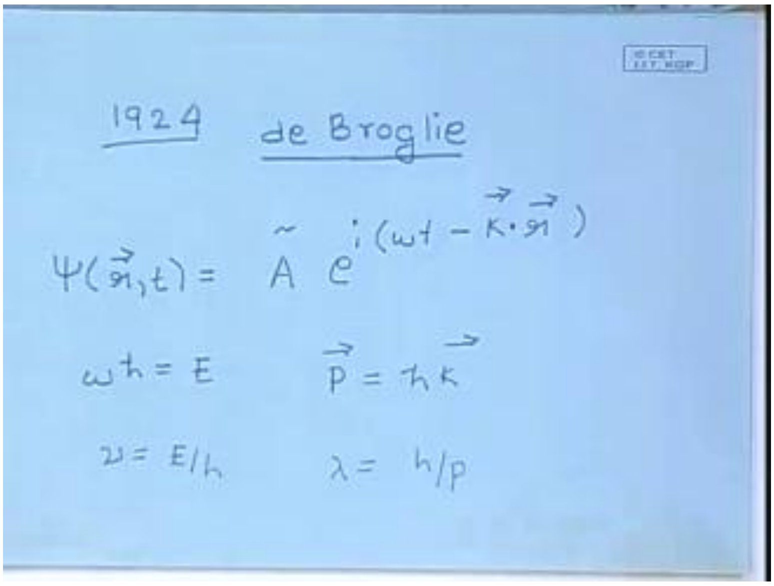

It is defined all over space the whole of space you could a value for the wave at any point you could ask what is the value at any point. So, this particular wave is defined all over space it has a well-defined angular frequency. And momentum and wave vector so, it and you can relate these to the particles energy and momentum.

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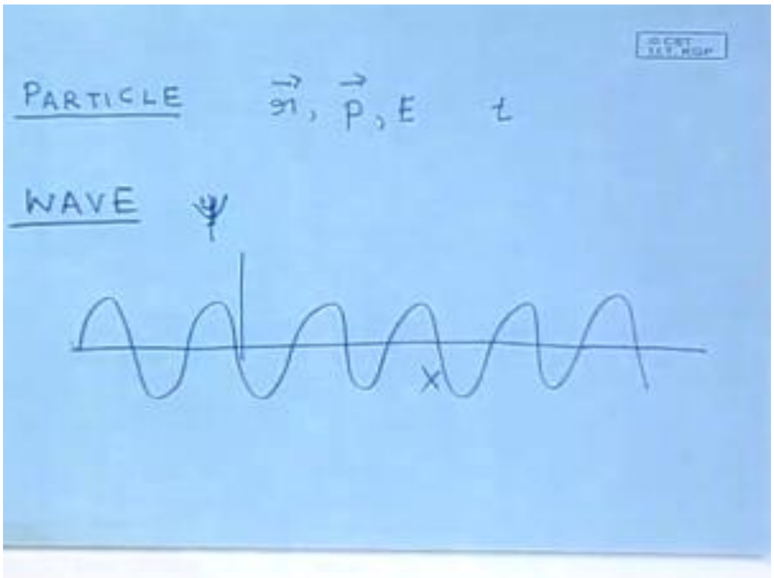

So, the wave precisely represents the particles momentum and energy, but this particular wave has no information about the particles position. So, you see the there is a certain amount of ambiguity which is introduced.

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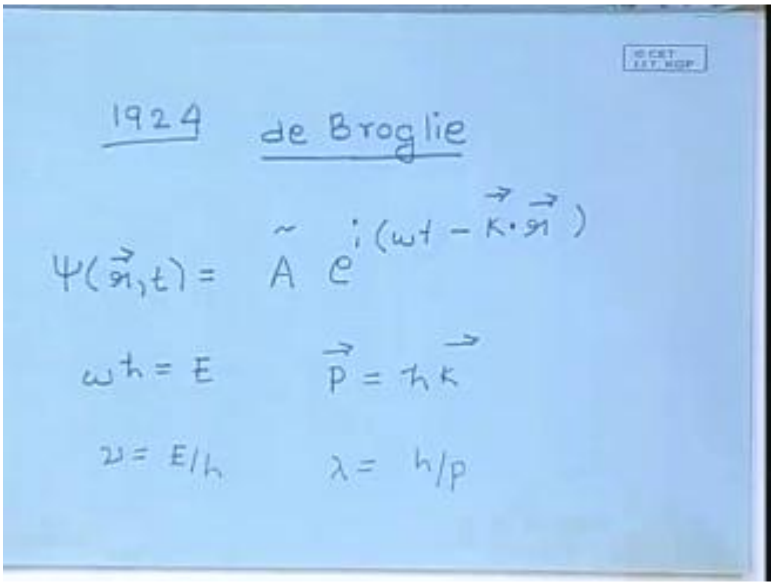

The moment you think associate when you think of the wave when you the moment to think of the particle as a wave instead of a particle. This wave contains only precise information only about the particle energy and momentum it has no information about the particles position, because the wave is defined everywhere it has no position information this is an important point which you should note.

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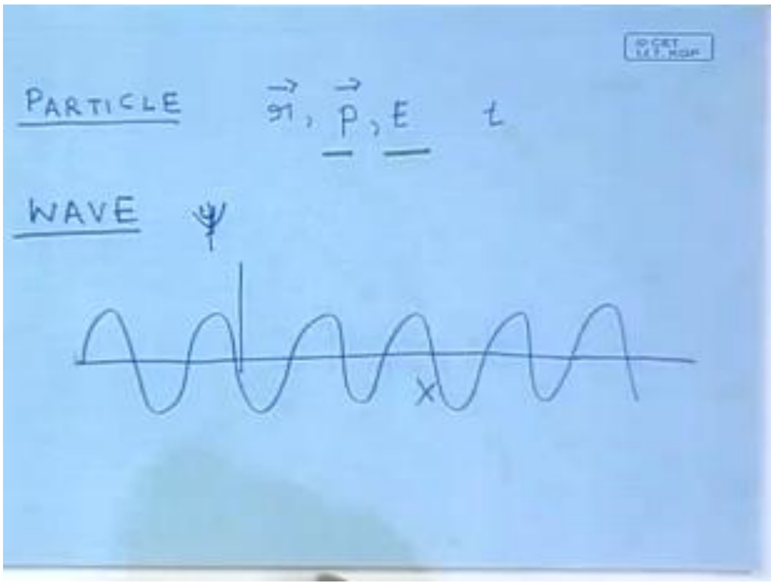

And we shall discuss this point more detail when we take up the interpretation of the wave that is associated with the particle.

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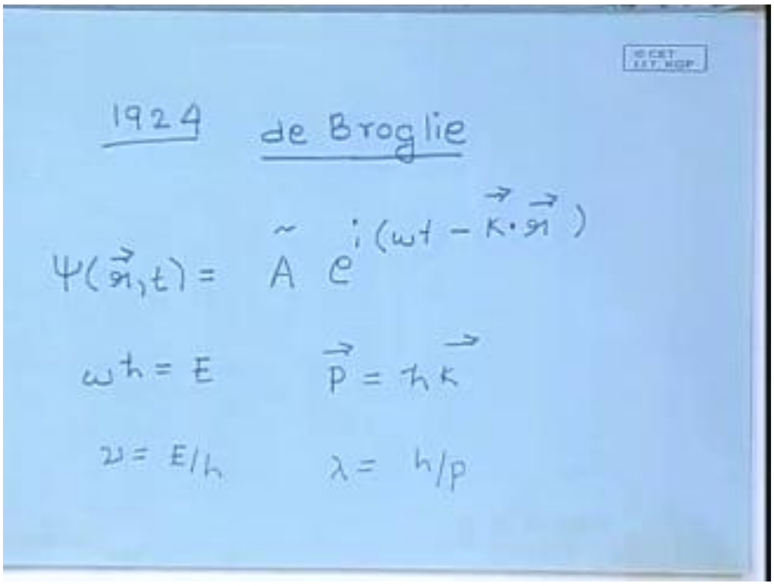

Next let us calculate, ask the question we were supposed to. Now take up a few questions so, the next question that we are going to take up is what is the dispersion relation for this wave. What do we mean by the dispersion relation we have already discussed this, a dispersion relation is a relation between the angular frequency and the wave vector. So, we know that the energy and the momentum for such a particle are have a relation so, let me write down the relation between the momentum.

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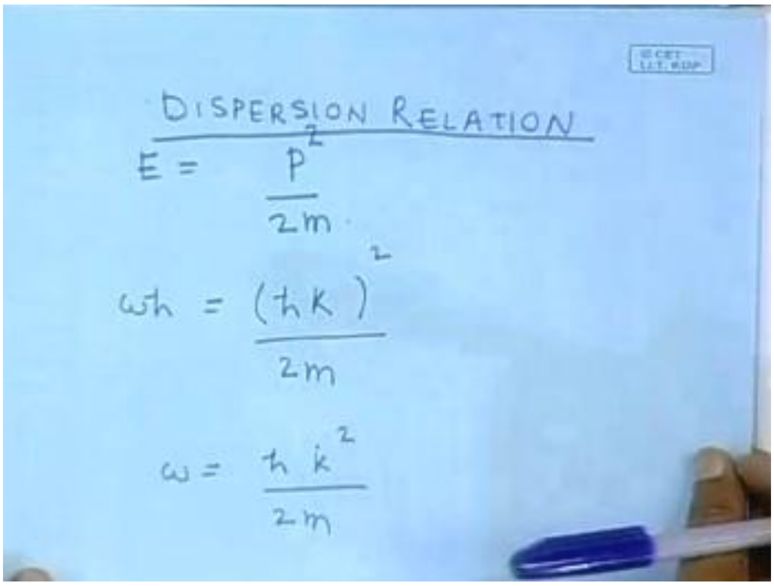

And the energy if the particle is non relativistic if it is moving slowly we now that the energy E is P square by 2 m and the energy is related to the angular frequency of the wave as omega h plus the momentum is related to the wave vector as h cross k. So, this is the, what it tells us the relation between the energy and the momentum. And it gives us the dispersion relation that this is equal to h cross k square by 2 m. This is the dispersion relation. I leave it to you to calculate the phase velocity you know we, how to calculate. The phase velocity you also have been thought how to calculate the group velocity. So, I leave these two things to you. You should please calculate the phase velocity.

And the group velocity for this wave now, I have what De Broglie hypothesized was that associated with every particle there is a wave. Now, the question is that we when we think of a particle when we see a particle we do not think of it as wave. There is no need for us to think of it as a wave because at De Broglie's time there was direct evidence that there is a wave associate with a particle. So, the question is then on what basis was this hypothesis made was this hypothesis ever verified or is a still a hypothesis. Now, at the time when De Broglie made this hypothesis it was purely on theoretical and philosophical grounds.

But it was it has later on been verify then it it is a very important development in our understanding of the whole of nature. So, let us go it into a go into it into a little more detail to get a feeling for what the wave if there is a wave associated with a particle we should make an estimate of what is the expected wavelength. So, let us start with an electron let us take a situation where there is an electron and you might have heard of electron guns. These are there in tv's in oscilloscopes the electron gun is the device that accelerates an electron. And then the electron comes out with a particular speed so, the way the electron is accelerated there is negatively charged.

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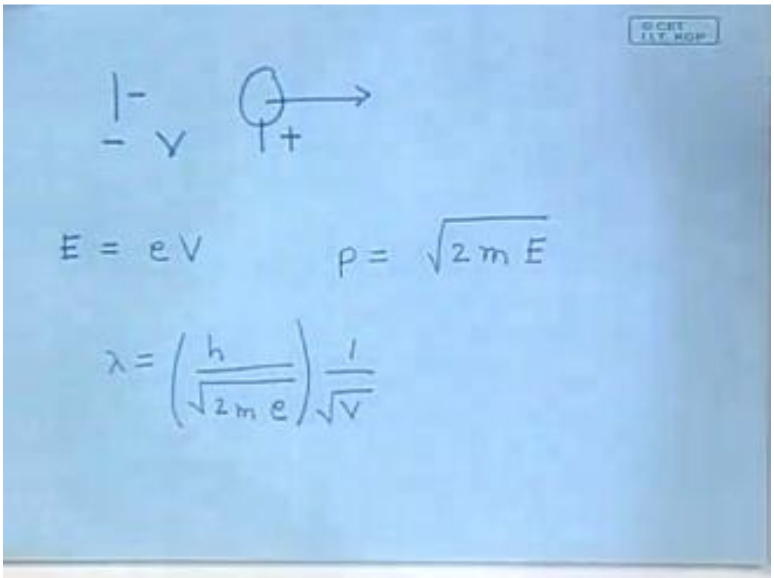

Metal plate over here a cathode and which is heated and if you give a negative voltage to it then the electrons will be emitted and then here you can put a positively charged. So, this has given given a positive charge and this is has a negative charge it is heated. So, it will emit electrons and electrons will be attracted and then some of them will come out through. And they would have through a potential difference v which will cause it to accelerate. The question that we are interested in is suppose there is an electron gun where an electron is made to go through a potential difference v. What is the wavelength of the electrons that come out? Let us calculate this. So, the energy of electron when it comes out which goes through a potential difference v is the charge of electron into the potential difference.

And the momentum of the electron is 2 m into E the square root of this we are assuming that the electron is non relativistic. So, E is P square by 2 m or the momentum is P is the square root of 2 m into the energy of the electron. So, we can now calculate the wavelength of the electron this will be h by the momentum so, h divided by 2 m into the charge of the particle in to the potential. So, I will write it like this where v is the voltage and what we have to now do is we have to calculate this number. So, the Planck constant we know is 3.3 4 4 to the power sorry 6 .6 3 10 to the power of minus 34.

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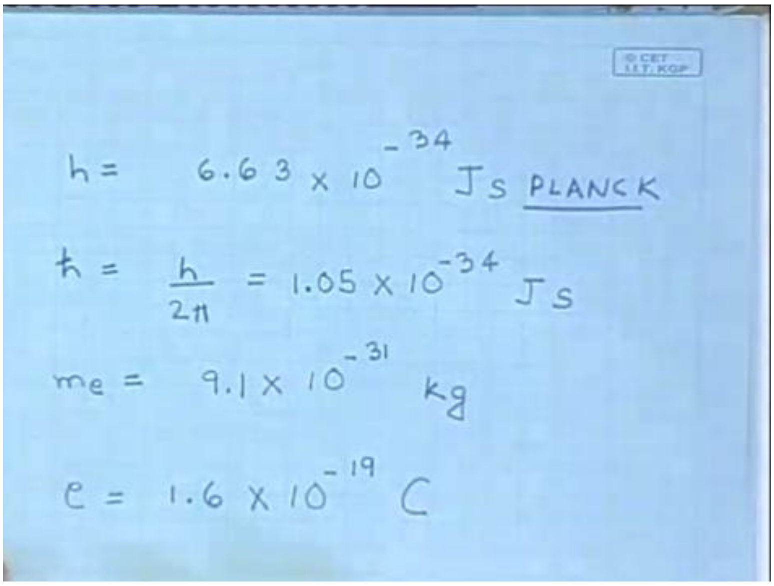

The mass of the electron is 9.1 10 to the power minus 31 kg’s and the electron charge is 1.6 into 10 to the power minus 19 Coulomb’s.

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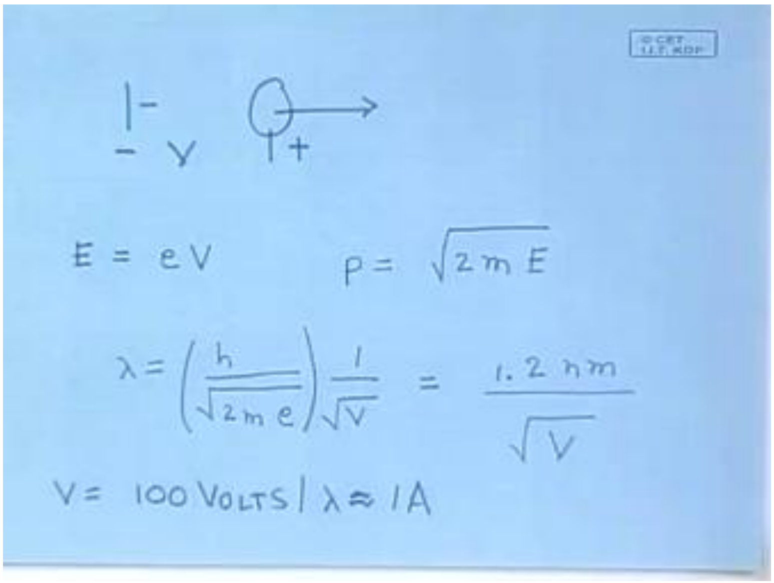

So, if use these values in this expression what you will get is that this is equal to 1.2 nanometers divided by the square root of the potential difference. So, all that you have to do is you have to put in these numbers in volts and if it is a 1 volt potential difference the the wavelength of electron. That comes out is going to be 1.2 nanometers and if the potential difference is hundred volts a 100 volt potential difference is not something which is very difficult to generate. So, if the potential difference is hundred volts then the wavelength is 1 Armstrong where it is point nanometer 1.2 Armstrong. But approximately of the order of 1 Armstrong let me make it approximately equal to 1 Armstrong. So, what we see is that if we send an electron through a 100 volt potential difference. Then you get expect to get a wave if De Broglie hypothesis is correct you expect to get a wave of wavelength 1 Armstrong now, if you compare this electromagnetic radiation 1 Armstrong 0f the order of 1 Armstrong is what you have where you have x rays.

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And if you want to verify if the electron weather the electron that comes out actually has wave properties or not you have to look for some properties some phenomena which is exclusive to waves.

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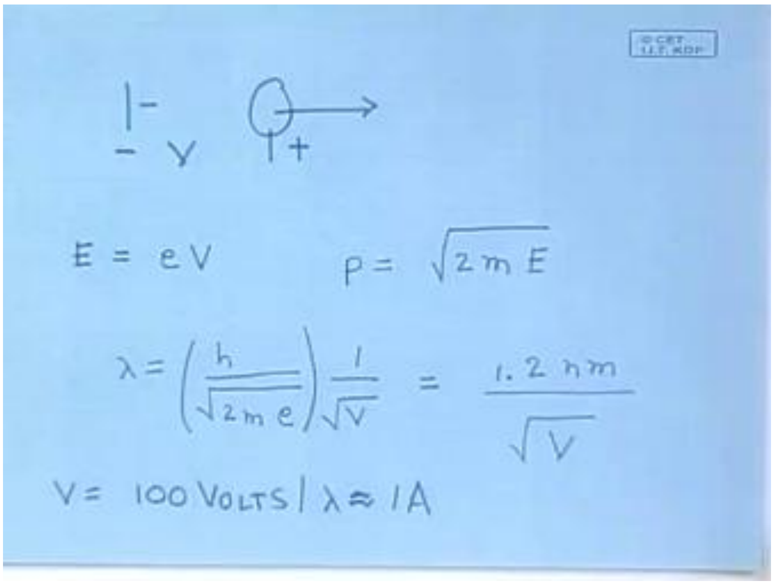

And we have discussed quite extensively that 2 thing which you cannot explain if you do not have waves two things which you see only for waves are interference and diffraction. So, if by some means you could observe diffraction with this electron beam that comes out from the electron gun. So, you have an electron gun over here which produces a beam of electrons that come out if you could somehow observe interference or diffraction with this wave. If this electron beam that comes out you would able to verify that this is actually a wave and the wavelength that you expect is around one Armstrong which you know is the wavelength of x ray.

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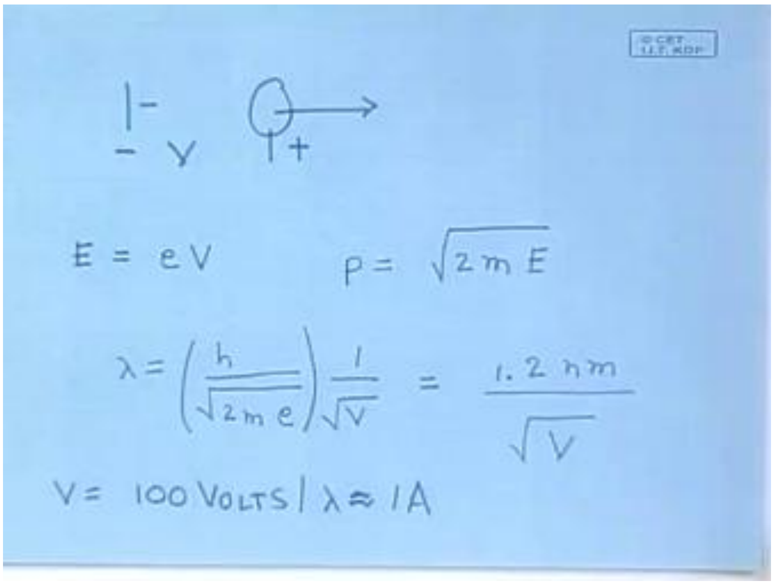

So, this is not an electromagnetic wave, but it is wavelength is comparable to that of x ray now, x ray we know exhibits diffraction when it is incident on a crystal.

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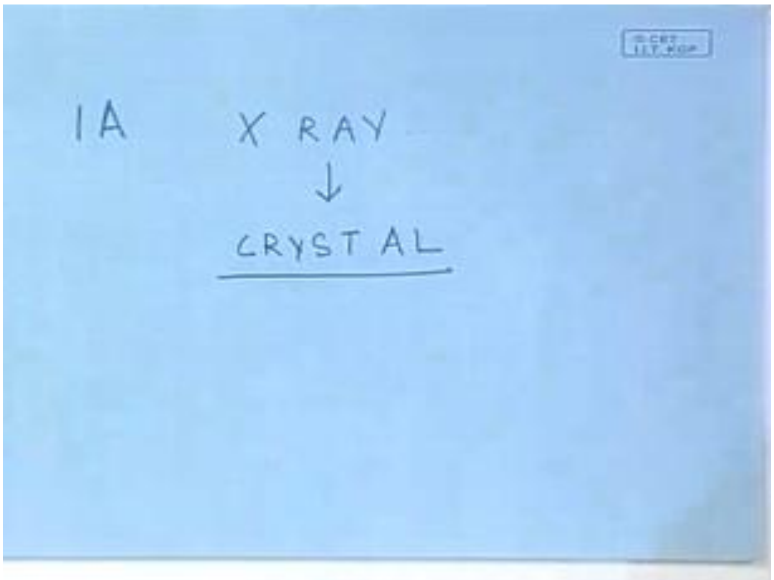

When x ray is incident on crystal which is what we have used which also been used in the Compton effect to study the spectrum of the x ray. When x ray incident on a crystal it exhibits a diffraction pattern and the maximas occur whenever 2 d sin theta is equal to m lambda. So, two scientists Davisson and Germer in 1912 73 years after De Broglie made his hypothesis. They used large crystal of nickel on which they put an electron beam and they found that there were in the electrons that come out they found a pattern of maximas just as you would if we have used x rays. And this was clear evidence that electrons do have the wave associated with them otherwise there is no way electron can exhibit diffraction.

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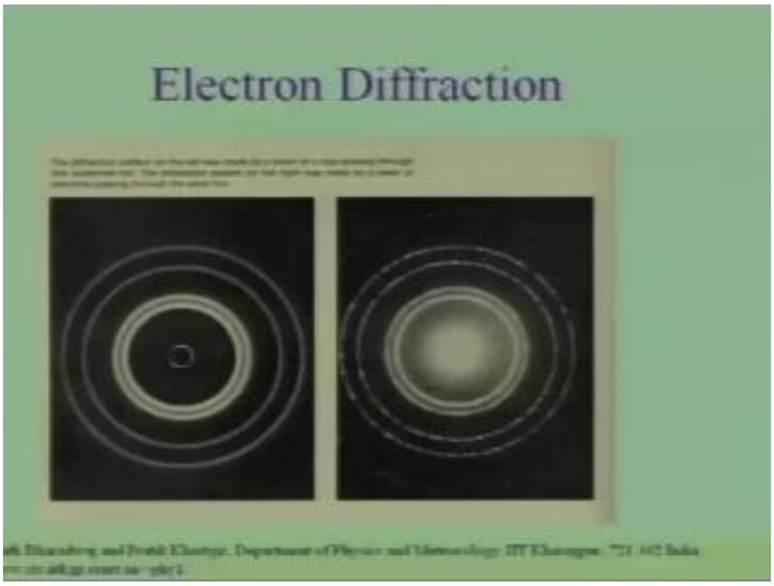

Let me show you a picture of electron diffraction. So, this picture is very similar to the kind of thing that was done by the Davisson and Germer. Here an aluminum foil has been used and this shows you the diffraction pattern that you obtain when x ray is incident on the aluminum foil. So, when x rays is incident on aluminum foil it produces a diffraction pattern the aluminum crystals produce a diffraction pattern. The same aluminum foil when there is beam of electron incident it also produces an a diffraction pattern which is what is shown over here.

So, the fact that you can have a diffraction pattern when you electrons are incident on a crystal. Is clear evidence that the electrons also behave like a wave and this is the demonstration of the fact that you need to attribute wave properties to the electrons. So, in addition to thinking of electrons as particle it is also necessary to think of electron as waves and it is not only electrons for all particles there is a wave associated with it. Now, before we are going to discuss this extensively in in in the coming lectures, but before finishing today’s lecture I should a there are few points which I should make. The first point is as follows.

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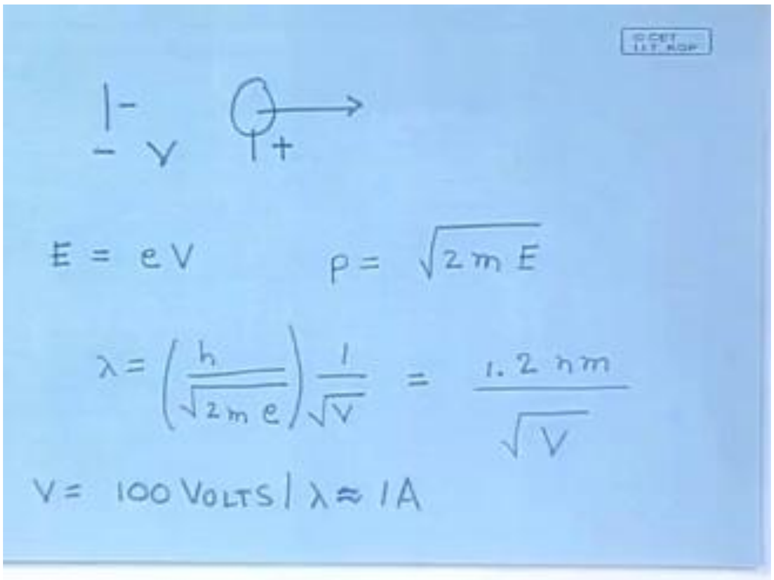

When we calculated the wavelength of the electron in the electron gun we assumed that the electron is non relativistic and the momentum and the energy are related as follows. Now, I have told you in quiet detail that this is valid only when the particle is non relativistic if the electron is relativistic.

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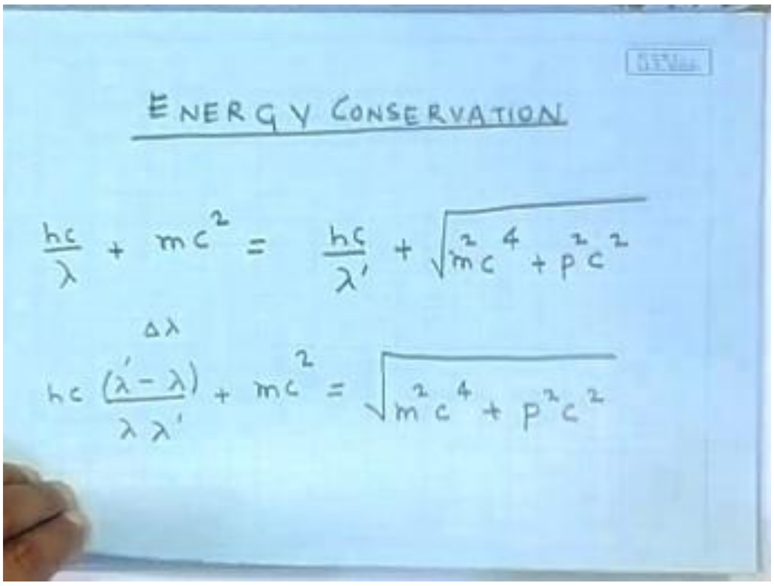

If the electron moves very fast it is necessary sorry to use this Relation between the energy and the momentum.

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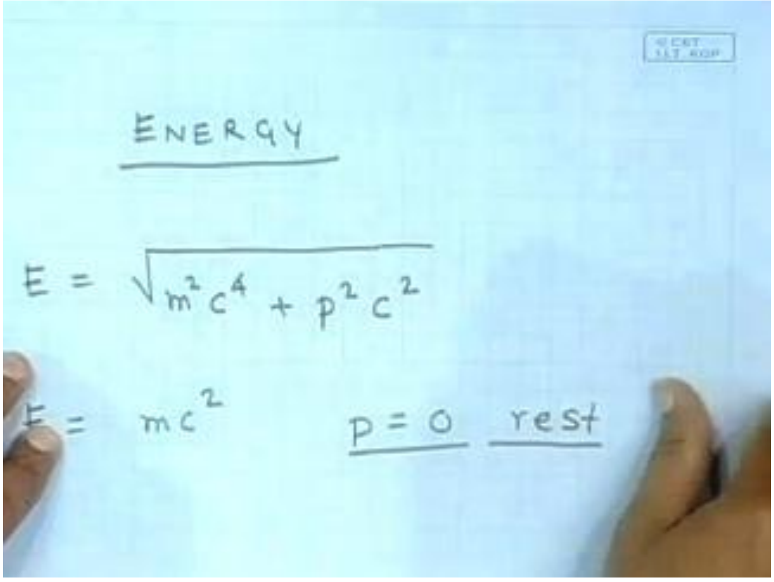

Let us discuss this quite extensively in the last lecture.

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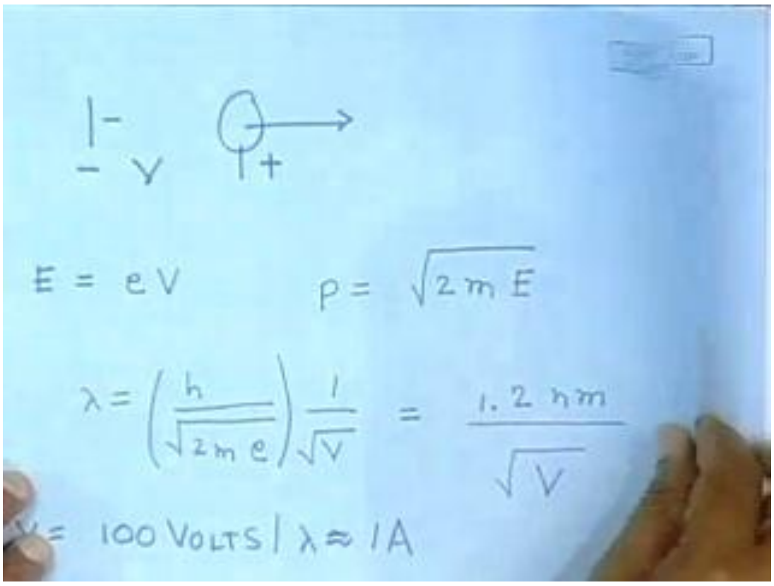

So, it turns out that if the potential difference is of order of a few 10000 volts. So, few 10000 volts then it is necessary to take into a account the full relativistic formula. If you are working at voltages below that tens of thousands then you can safely use this formula to calculate the wavelength. But once you cross that then you have to use the full relativistic formula. The second point which I should make is as follows. So, what we have what the De Broglie hypothesis is that De Broglie's hypothesis is that associated with every particles there is a wave.

Now, you this is; obviously, is counter intuitive to all of us, because whenever we think of massive particles we do not think of it of the wave associated with it. For example when I walk or when I see somebody walking when I see a dog walking on the street I do not think of it of the wave associate with it. The question is why it should I think of it has a wave which is their distributed or should I think of it as a particle being at a very well defined position? Why do not the wave attributes the wave aspects of such a big thing? Why do them not manifest themselves why does it not manifest itself the wave aspect?

Well, there are it is actually very complicated issue why in the macroscopic world we do not see this everywhere we do not see this wave the wave aspect, but we could make some preliminary estimates over here. As to what the wavelength of the wave is expect to be a that would gave a some idea some idea some very crude idea of why we do not see the wave nature all the time for very macro big macroscopic objects. So, if for example, if I have a man walking.

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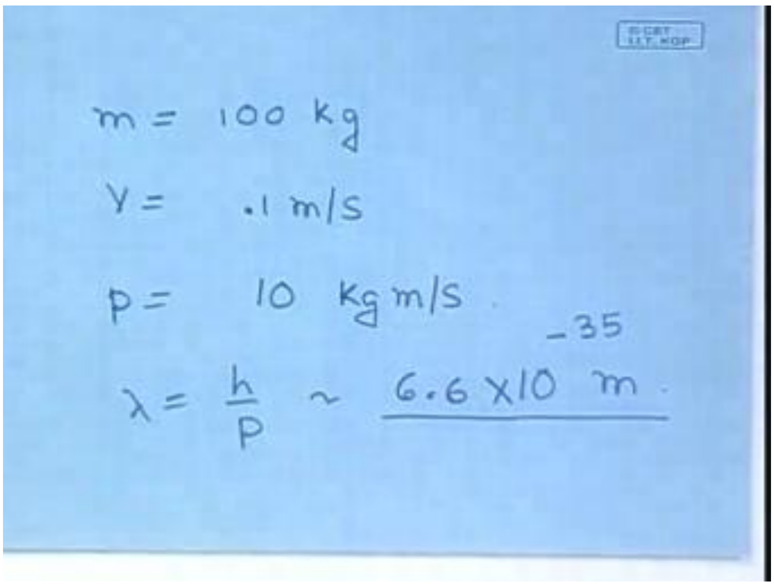

So, let me take the weight the mass of the ma person to be around 100 kilos will be typically less, but for simplicity I am taking it to be 100 kilos. Let me take the speed to be 0.1 meter per second. So, it is his momentum is going to be 10 kg meter per second. So, the wavelength is h by P and this is going to be of order of 6.6 into 10 to the power minus 35 meters which you see is a very very small number the wavelength turns out be extremely small right. So, I think that is a sufficient discussion for today let me stop here. We shall resume on this topic in the in the next lecture.