Physics I: Oscillations and Waves
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Lecture - 32
Compton Effect

At the end of the last class I had given you a problem.

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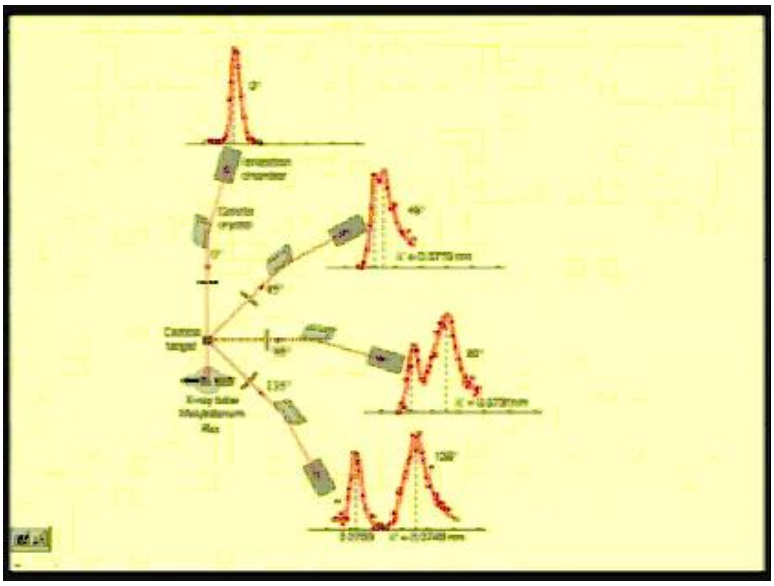

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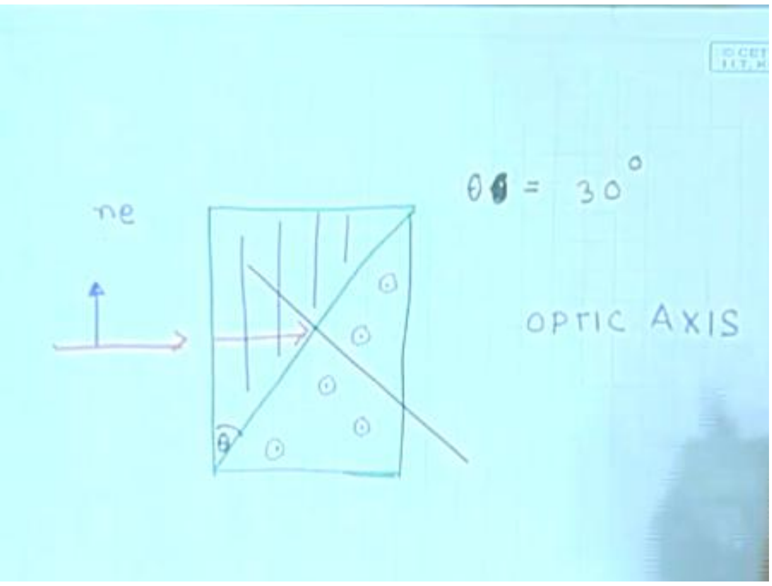

The problem was as follows we had natural light incident on this apparatus. This apparatus is a combination of 2 prisms which are both made up of the same birefringent material calcite the first prism has its optical axis oriented like this. The second prism has its optical axis oriented like this. And this angle in the prism is thirty degrees. So, you could work out what all the other angles are. And the problem was that you have to work out the trajectory of the light through this and through this prism and finally, calculate the angle between the 2 rays that will come out. So, let me out line how to handle this problem?

The incident natural light can be decomposed into 1 polarization 1 which is parallel to the optic axis. So, this is the optic axis this is the optic axis of the first prism. So, we can break up the incidental electric field into 2 components 1 which is parallel to this optic axis this is the extraordinary wave. And when this wave enters this prism when the wave where the electric field is parallel to the optic axis enters the prism. It will experience a refractive index ne. So, for this wave we have ne inside the refractive index ne and the extraordinary refractive index in this prism.

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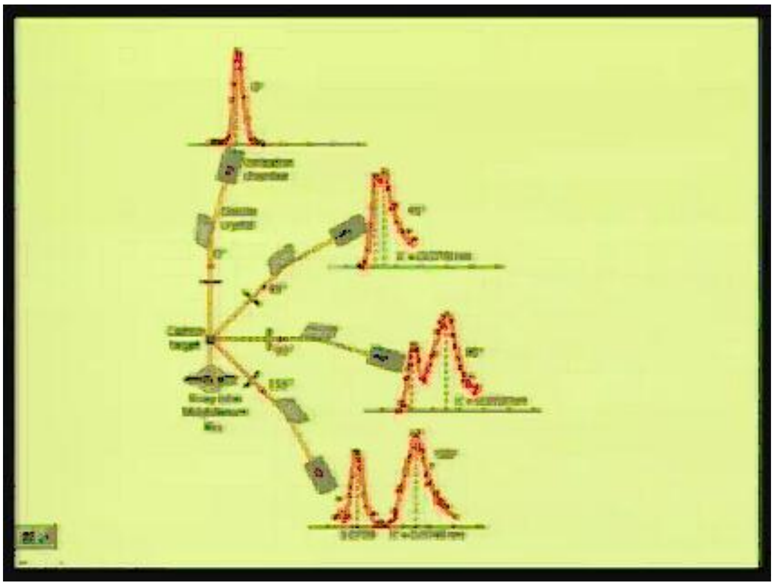

Since it is normal on the surface it will go straight through until it reaches the second prism.

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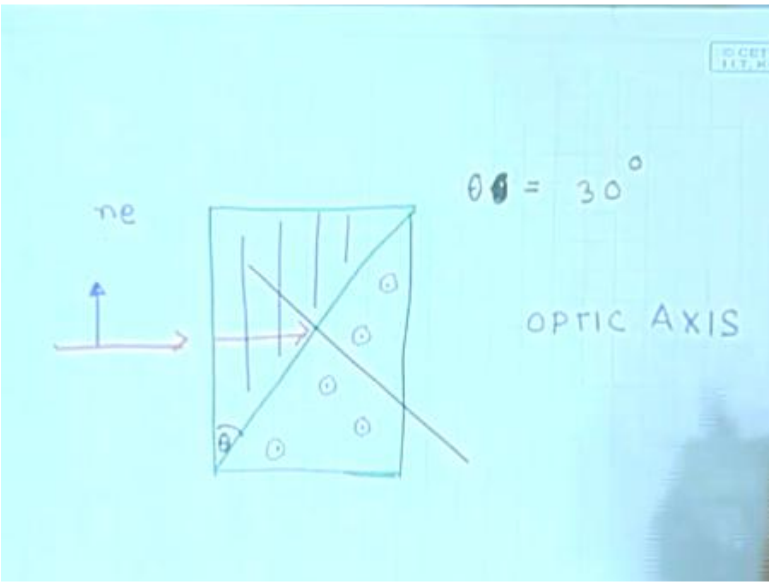

The interface with the second prism, inside the second prism this wave becomes the ordinary wave.

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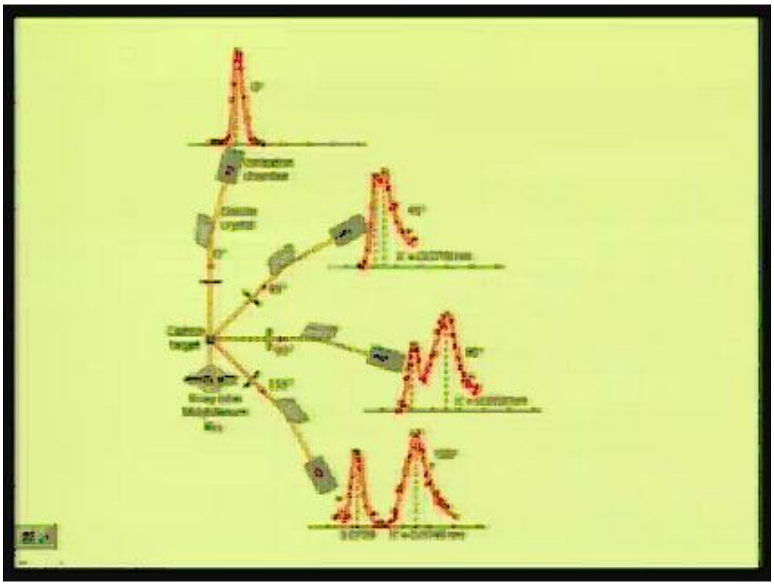

Because this electric field vector is now, perpendicular to the direction of the Optic axis and it will see a refractive index once this same wave enters this prism, the second prism.

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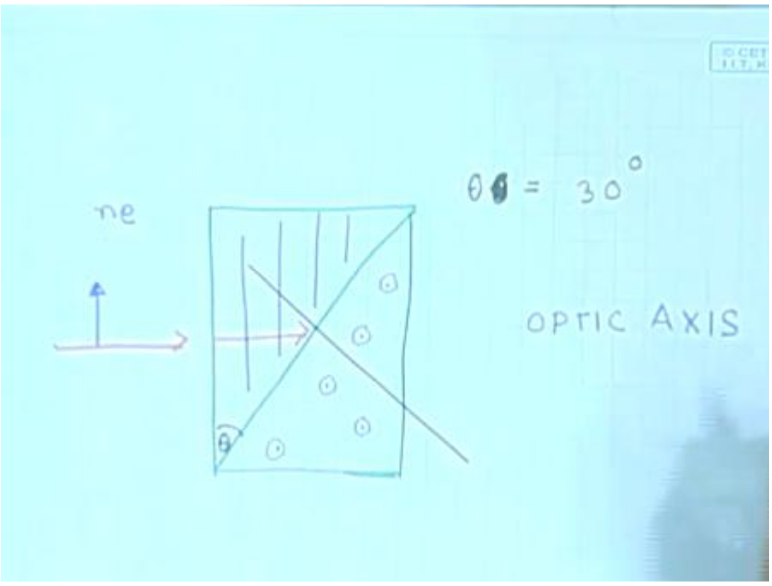

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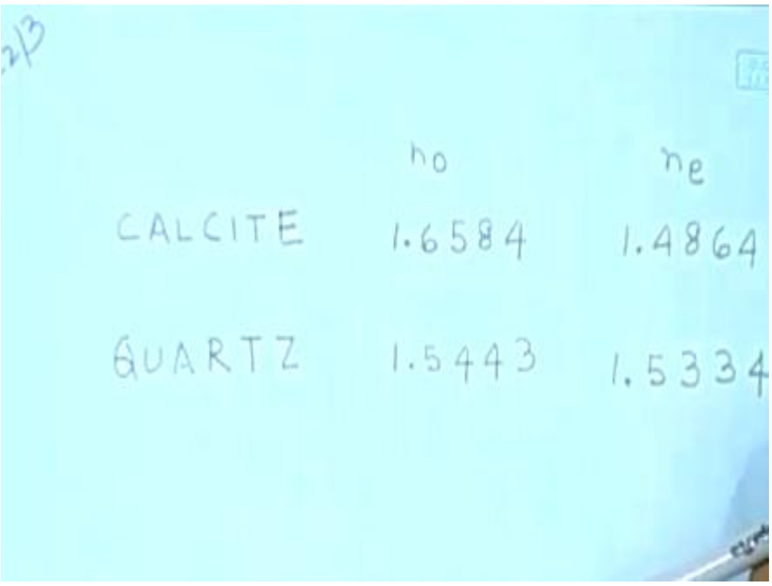

So, ne has a value 1.4 roughly 1.48 and the ordinary refractive index is 1.65.

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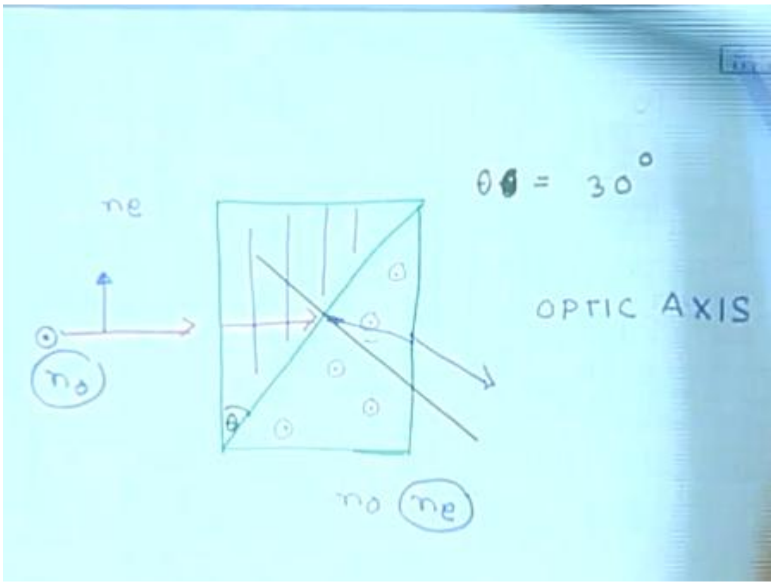

The refractive index which is extraordinary here and ordinary here is going from a rarer to a denser medium. So, it will move closer to the angle of to the normal. So, if I move like this. And then when it encounters this surface its going from a denser to rarer medium. So, I have to draw the normal and it will move away from the normal. So, this is how one wave will propagate and you can just use the good old geometrical optics to follow, determine what this angles all of these angles are. Because we know this angle is 30. Now, let us consider the other wave the other wave is polarized like this.

The incident light can be decomposed into 2 polarizations. And when this wave encounters this prism it is going to be no is going to experience refractive index the ordinary refractive index. And when it encounters this when it enters this is going to experience ne. So, let me draw then with different or if it put circles around them. So, the component of this wave which is polarized perpendicular to the plane of this paper when it enters this prism is going to have experience a refractive index no. And when it enters this prism is going to experience a refractive index ne the extraordinary refractive index and it is going from a denser to a rarer medium.

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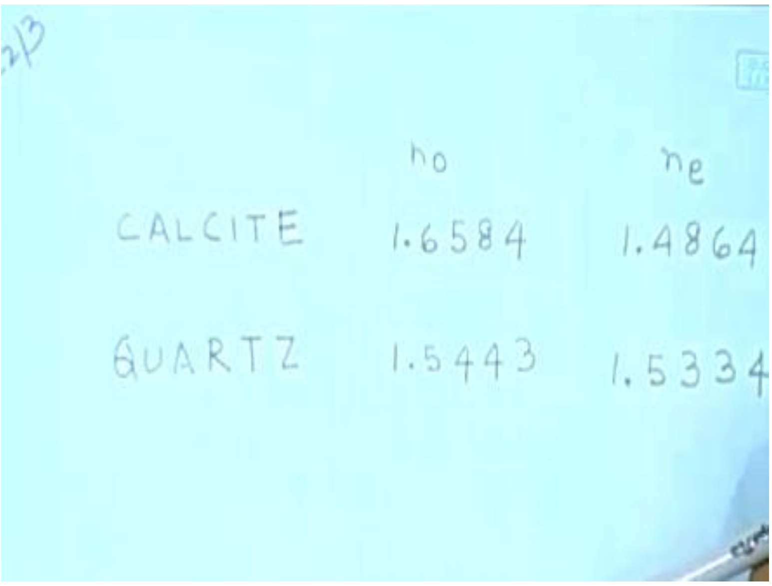

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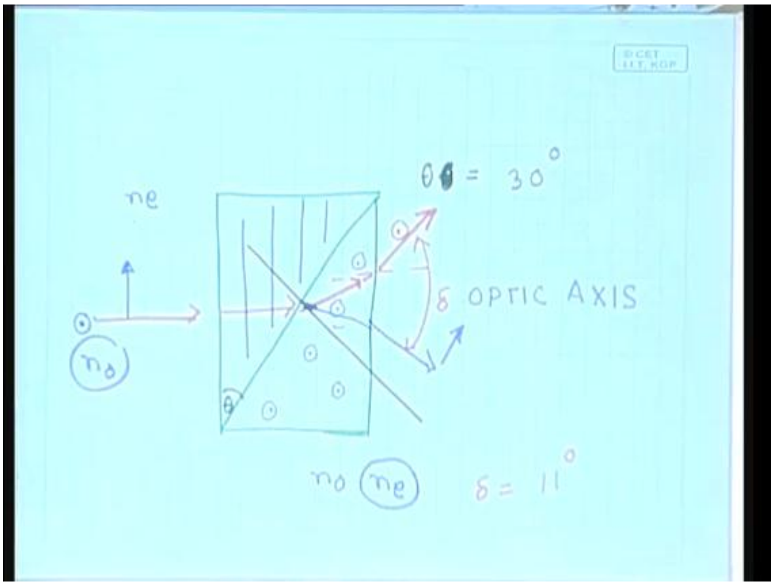

So, it is going to move away from the direction of the normal. So, let me draw it in red. So, it is going to go something like this. And then it is going to move further away from the direction to the normal. So, you can follow this using geometrical optics and then you have to determine this angle which you are which was the problem. So, this angle if you refer to it a as delta and you do the exercise this angle delta comes out to be 11 degrees. So, that brings to an end our discussion of polarization and how we manipulate polarization? So, I have told you that polarization can be generated by various processes reflection, scattering, dichroism and birefringence.

And I have told you how birefringent materials can be used to make half wave plates quarter wave plates. And here is an example how birefringence can be used to generate polarization, because you see the two different rays that come out are going to have different polarizations. This ray is going to be polarized in this direction in the plane of the paper. And the ray that comes out here is going to be polarized perpendicular to the plane of the paper. So, you have produced 2 polarized rays coming out with mutually orthogonal polarization. Now, we shall take up different topic. So, let me introduce the topic by considering a particular situation. And the situation that we are going to consider is as follows there is a there is an electron.

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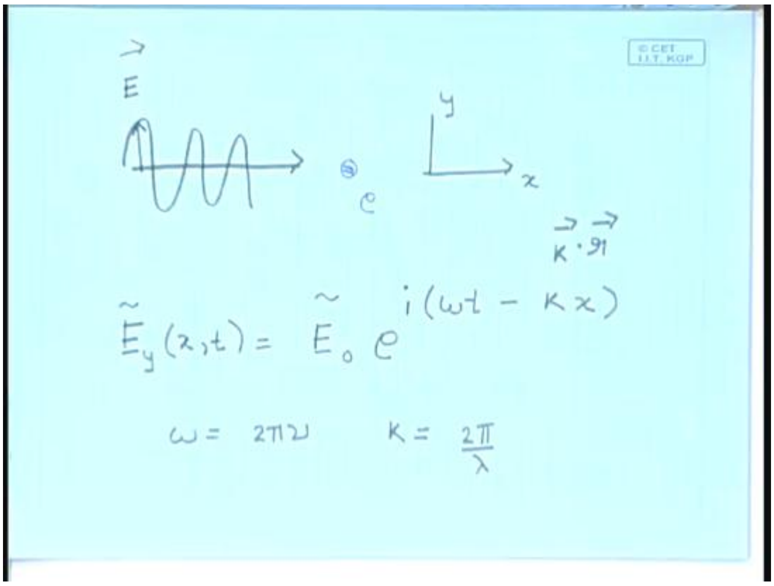

On which there is an electromagnetic wave some radiation there is radiation incident on this electron. So, this is the direction in which radiation is incident on this electron. It could be visual or radio or x ray we know that all of these are electromagnetic waves, all of these are radiation and we have we know that we can treat them all as electromagnetic waves. So, our discussion is quite general you could have, you have a… we are discussing a situation where there is an electron and you have some kind of electro the radiation incident on it could be light could be x ray could be radio waves could be radio. And we know that this radiation electromagnetic radiation is an electromagnetic wave. And so, what we have here is an oscillating electric field which at any given instant looks like this. So, when we speak of radiation incident on an electron what we have?

If you put apply all that you have learnt in this course till date you will think of an electromagnetic wave a travelling pattern of electric field. And if it is a sinusoidal plane wave electric field is oscillating up and down and the whole thing is moving forward that is incident on this electron let us chose axis to simplify the discussion. So, this I am going to call the x axis this the y axis and the z axis is perpendicular to the plane of the paper coming out words. And the electric field the wave the radiation is incident in the x direction. So, the electric field of this radiation could be anywhere in the y z plane we are going to deal with only 1 of the two possibilities where it is aligned with the y axis.

So, there will be another component which is alignment with the z axis I am not going to talk about that. So, for our discussion here we will think of the only 1 component of the electric field which is oscillating up and down the y axis. And we can we have learnt that we can describe this electromagnetic wave using an equation which looks like this. Ey in the complex notation this is a function of x and time, because it is a plane wave travelling in along the x axis will be some amplitude E zero tilde e to the power I omega t minus k x. And let me remind you what these parameters that appear are so, you have omega and omega is the angular frequency which is related to the frequency of the wave. And this determines how fast the wave the electric field goes up and down in time.

And you have the wave number k which is related to omega through the dispersion relation. And that determines the spatial reputation the wave length of this wave how fast it repeats if it moves along if I move along the x axis. And it is the wave number and in general if the wave is travelling in some arbitrary direction I will have k dot r. So, for a wave in some arbitrary direction this will be the wave vector k dot r. And the amplitude of the wave is the complex amplitude which may have a phase compartinate is given by this number E not tilde which is their outside. So, this determines the amplitude of the wave. And if you wish to calculate the energy the intensity the radiation intensity that is the energy per unit second per unit area.

Then that is determined by the amplitude. So, it you want to increase the intensity of the radiation the energy carried by the radiation you have to increase the amplitude it is proportional to the amplitude squared the modulus of the amplitude squared we have seen this. And it does not depend on omega and k. So, if you want increase the intensity of the incident radiation the energy content you have to increase the amplitude. So, there are 3 things the amplitude the angular frequency and the wave vector or wave number.

And the wave number are the wave vector let us talk in terms of the wave number in this particular case is related to the wave length as follows 2 pi by lambda. And the energy of this wave is proportional E naught square.

Now, let us analyze again what happen when this wave is incident on this electron? The electron now, experiences an external acceleration and external force, because of the oscillating electric field. And we have also discussed this situation what happens to a particle a free particle when it is under an oscillating external force remember that. When I have a spring mass system which is being driven by an external force if the external force has a frequency which is much higher than the resonant frequency of the spring mass system then it is going to make the mass oscillate at exactly the same frequency as external force.

But with pi phase difference and in this situation where the frequency of the external force is much higher than the natural frequency of the particle you can ignore the spring. So, usually the electron that we have talking about is going to be bound. So, you can think of it as being a spring mass system electron bound to some positive charge by a spring. And if the external the frequency of this wave is much higher than the resonant frequency of that spring mass system I can ignore the spring. And I can say that the electron is going to oscillate with exactly the same frequency as this with pi phase difference.

In the phase difference is going to be different if we are close to the resonance. And if you are if the external frequency is much smaller than the resonant frequency the phase difference is going to be zero them going to be approximately the same phase, but the key point is that the electron is going to oscillate at the same frequency as the wave which is making it oscillate. So, under the influence of this electromagnetic wave of this oscillating electric field the electron is also going to oscillate up and down at the same frequency and the same angular frequency of the incident wave.

Now, once an electron oscillates starts to oscillate like this we have again discussed what this is going to cause an oscillating electron is going to emit radiation. So, if an electron oscillates at an angular frequency omega it is also going to it is you can think of it as at dipole oscillation like this. And you for away you will see the dipole radiation pattern and the radiation is going be at exactly the same frequency at which the electron is oscillating which also is the same frequency at which you are sending in the radiation? So, finally, what we what happens is that if you have an incident radiation on the electron the radiation the radiation which with we think of as an electromagnetic wave is going to cause the electron to oscillate up and down which in turn is going to reradiate and this radiation is going to go out it different directions.

Because you can think of this is a dipole and far away. So, the radiation is going to have a dipole pattern the radiation pattern of a dipole. And it is going to be at the same frequency as incident radiation. So, if you put radiation in along the x axis on an electron it is going to send out radiation in all direction. This is what is called scattering. So, you have sample which has got electrons in it all materials have got electron you send radiation on it. In 1 direction the radiation gets scattered out in all directions, because the radiation causes the electron to oscillate and that again causes radiation. And the radiation that you expect to come out is going to be at exactly the same frequency as the radiation that you sent it. So, this is what we expect on the basis of the electromagnetic theory where you think of the incident radiation as an electromagnetic wave. Now, there was an experiment performed by Compton a scientist called Compton

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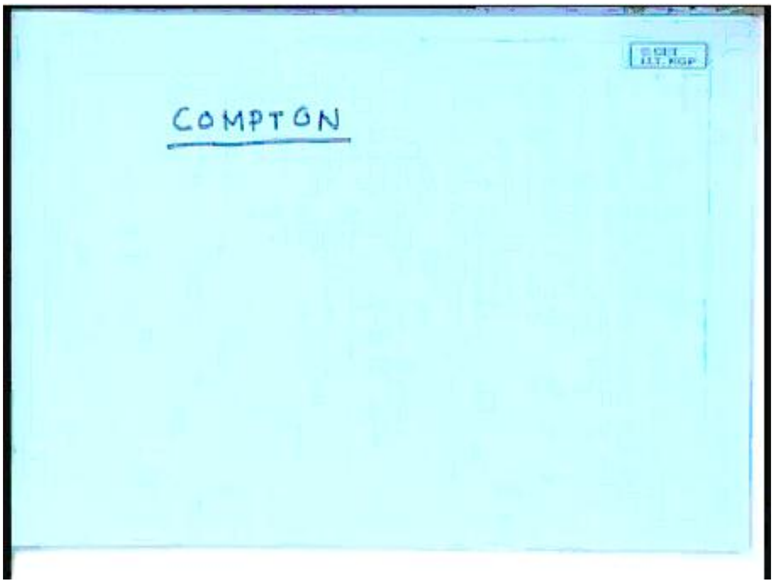

In the early part of the 20th centuries Compton conducted an experiment. Let me outline what this experiment is over here. So, this shows you schematically what the experiment which a Compton conducted.

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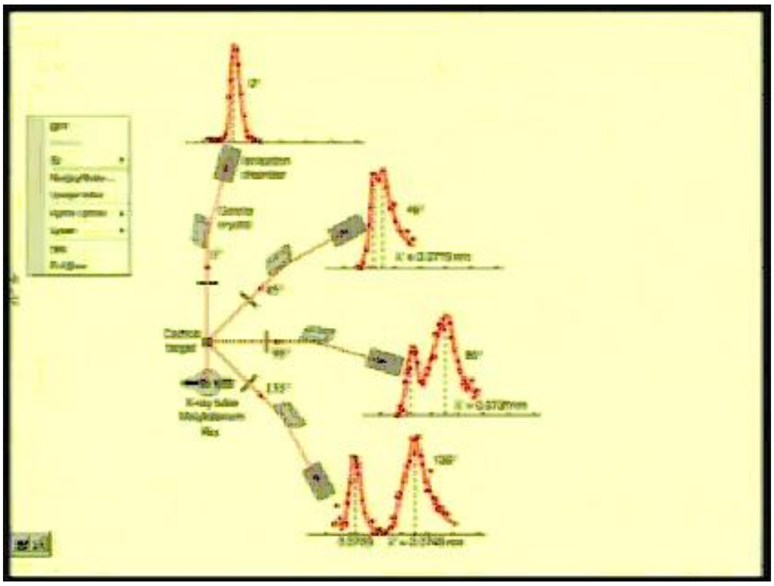

So, you have here you have a source of radiation. And in this particular case the radiation is in the x rays x ray band. So, it has a very small wavelength of the order of 1 Armstrong. And here this is a source of x ray radiation and this is a very particular type of source we have discussed how x ray could be generated. So, you could have for example, a copper target on which you bombard electrons. Once these electrons encounter the copper target send them with very high energy when they encountered the copper target they will slow down inside the copper target.

And this deacceleration of the electrons inside the copper target will give rise to radiation. Finally, they will come to rest in the copper target and this gives rise to radiation if the electron the sufficiently energetic. They will be x ray emitted in with all possible frequencies or all possible wavelengths so, over all, rather all large bands of frequencies and wave length. Now, here we would like to check whether this prediction is correct or wrong.

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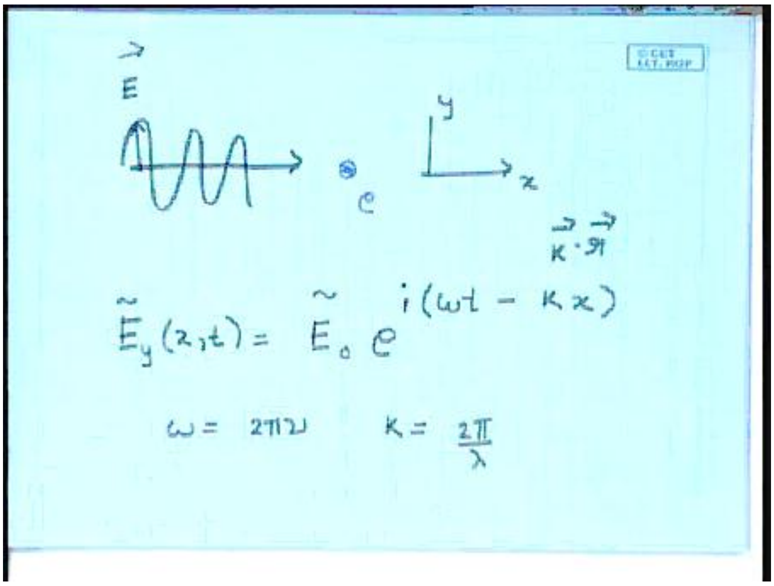

So, the question that we are interested in checking is if there is any shift do. You get wave length radiation that comes out of exactly the same frequency. Or is there some how some shift in the frequency if they if we if that would so, then this whole theory would be in trouble.

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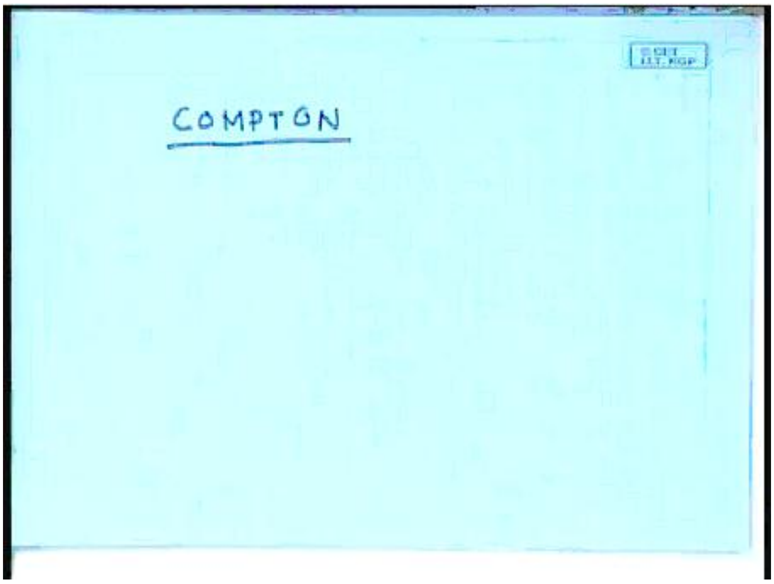

So, this is the different kind of x ray source.

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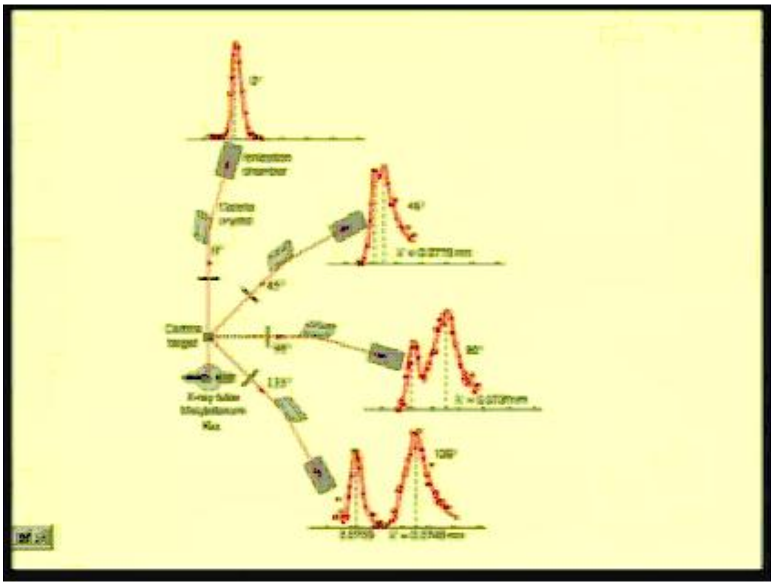

It uses molybdenum and it uses a k band k shell transition of molybdenum. So, you have atomic transitions we have also discuss this and they can produce radiation. If an electron goes from a more energetic atomic shell to a lower energetic atomic shell the difference in energy comes out as radiation. If you have an outer shell transition the radiation the energy of the radiation the radiation typically corresponds to visible or ultraviolet. For example, hydrogen has the each alpha line which is in the ultraviolet sodium lines sodium has spectral lines which are in the visible we have discussed the sodium lines when we were discussing the Michelson interferometer. So, the outer shell transitions give rise to optical or ultraviolet radiation, but there are inner shell transitions where the energy differences are quite large and if you have such transitions you can get x rays. X rays with the very specific frequency x ray radiation with the very specific frequency.

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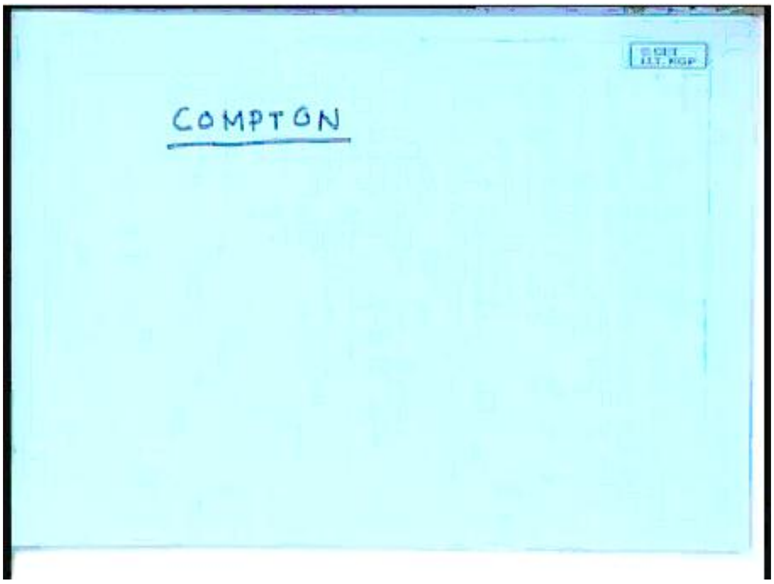

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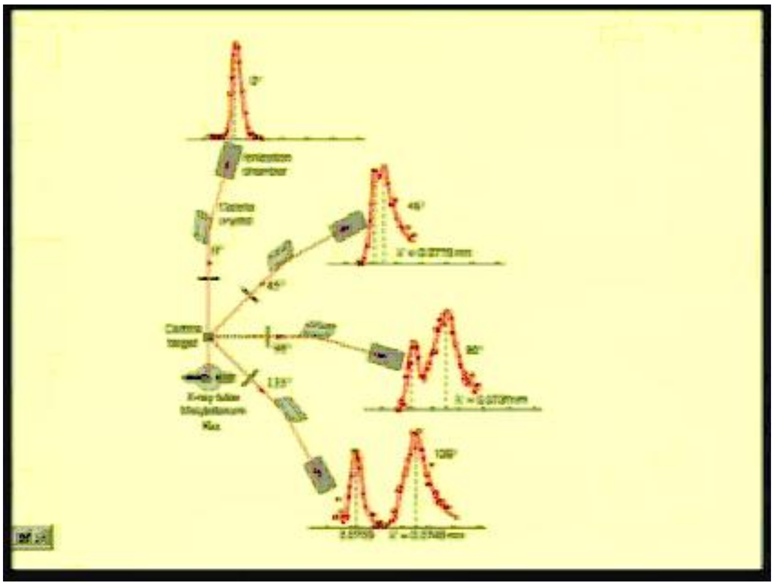

So, here there is a x ray source which uses the K alpha transition so, it is a k shell transition of molybdenum to produce x ray with the very particular frequency very narrow range of frequency spectral line. So, it is an x ray line. So, specter line in x ray. So, if you do not have any thing over here and you measure the spectrum of this x ray then you will get you will find that the x ray is being emitted at 1 at a single frequency. So, it peaks at a single frequency.

And you will have this frequency fall of which again you can as we have discussed you can assembly model using a Lorentzian or may be Lorentzian and a Gaussian. The key point is that this source emits x ray radiation at a particular wave length. Now, the again this brings. So, this again brings another very interesting issue how do you measure, how do you measure the spectrum of x ray? So, how do you construct a x ray spectrometer in the visible we know we can use a prism for example, the prism has different refractive indices for different wave lengths. And if you send light through a prism us different wave lengths come out at different angles.

This will not work for x ray you cannot use x ray glass to diffract x rays. So, we it will not work. Another possibility is diffraction if you send light of different wave lengths through a diffract meter the higher order maximal's corresponding to different wave lengths will occurred different angles. Now, the question is how will you construct diffract diffraction grating which works at x rays? X rays have a wave length of the order of 1 Armstrong. So, you require a diffraction grating which has spacing of the order of an Armstrong.

Now, we have seen that x ray is diffracted by crystals. So, you can use crystals to as a diffraction grating for x rays. So, you can use the crystal to diffract the x ray the bract criteria for a maxima which we have a again discussed earlier is that 2 d sign theta should be equal to m lambda where theta now, is the incident angle the gracing on the gracing angle not the incident angle. So, 2 d sign theta is equal to m lambda. So, the different wave lengths will have maximas at different gracing angles and you can use this to measure the spectrum of x ray.

So, here there is a calcite crystal and what the calcite crystal will do is if you send in x ray of different wave lengths the different wave lengths will have maximas in different directions. Now, you take a detector and move it around and you can measure at which angle each maxima occurs and then you can determine what the wave length is, because you know that the maxima has to satisfy the criteria that 2 d sign theta is equal to m lambda d is known. So, you can determine lambda so, here a calcite crystal has been used as these as the diffraction grating and this shows you the spectrum of the light which comes out from here.

Now, what is done is you put a carbon sample carbon over here carbon target and the x ray is now, incident on this carbon target. So, inside carbon you this carbon target you have these atoms and the atoms have a nuclei and they have electrons. So, these electrons are going to start oscillating up and down under the influence of the electromagnetic radiation which comes through and if you think of it is a wave. You expect the radiation that comes out in different directions to all have the same frequency as the incident wave. So, if you look at the spectrum in different directions you expect to see only one wave length which is the wave length that was sent in which is this.

Now, what Compton found was that when it this experiment that in addition to the wave length that was sent in there is another wave length component radiation at another wave length component which is produced, because of the scattering. So, in addition to this now, there are there is another spectral line which you can see here at a slightly larger wave length. And the difference in the wave lengths this is the so, you see there are 2 lines here there are 2 lines here and there are 2 lines here. The first line is the one that was sent in. So, this has the wave length that had that was sent in.

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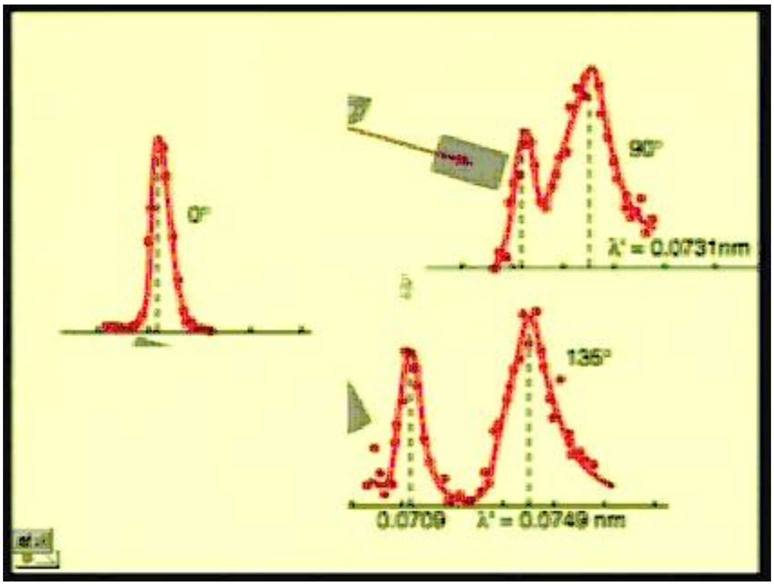

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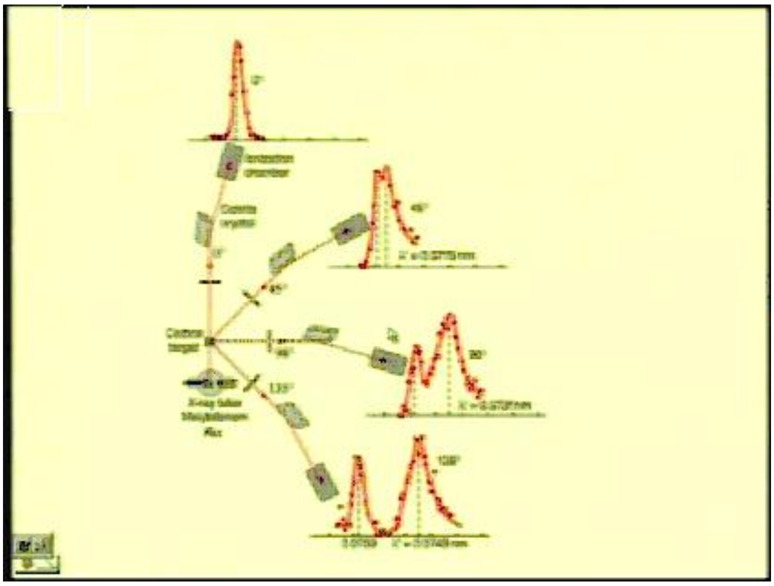

Now, you have another wave length which is being produced which has a large slightly larger value than the 1 which was sent in. This is what Compton found. So, there is Compton found that there is a shift in the wave length, because of scattering. So, there is this comp1nt which you expect to see from the electromagnetic wave theory of electromagnetic radiation. But you in addition to that you also have another comp1nt which has a slightly different wave length which is larger than the 1 that was sent in which is also being produced and the difference increases as the angle is increased which you can see here. So, the question is how do you explain this discovery which was made by Compton? And it is called the Compton Effect.

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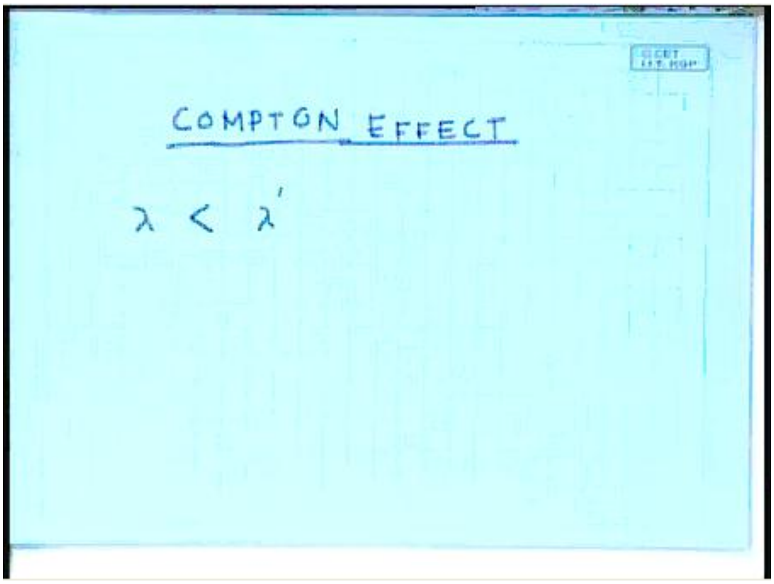

So, let me summarize what the Compton Effect is as follows you have radiation of wave length lambda incident on an electron or on a collection of electrons on a target. And the radiation that comes out you finds has a different wave length lambda prime which is larger than the incident wave length. This is the Compton effect and the question is how can you explain this? Now, it is quite clear that to you cannot explain this.

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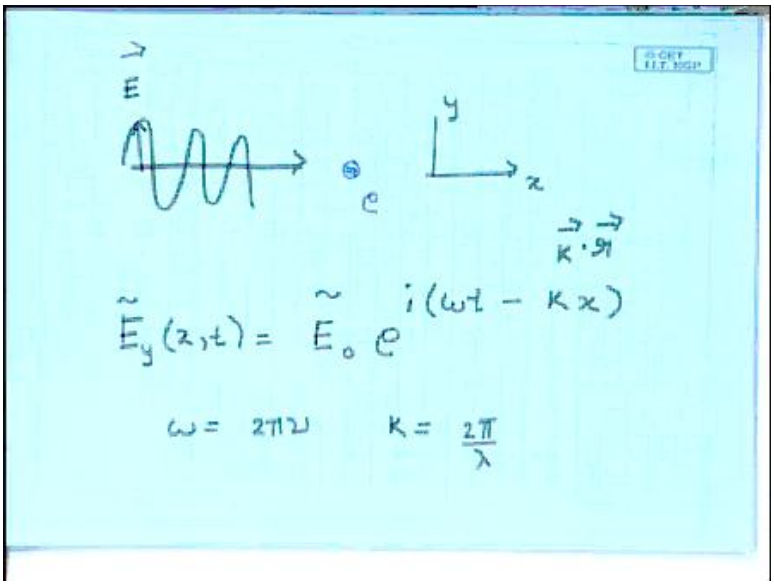

Using the wave picture of electromagnetic radiation the wave picture predicts that the wave length or the frequency of the scattered light x ray should be the same as the one which you sent in. Because the electron also oscillates with the same frequency so, the question is how can you explain.

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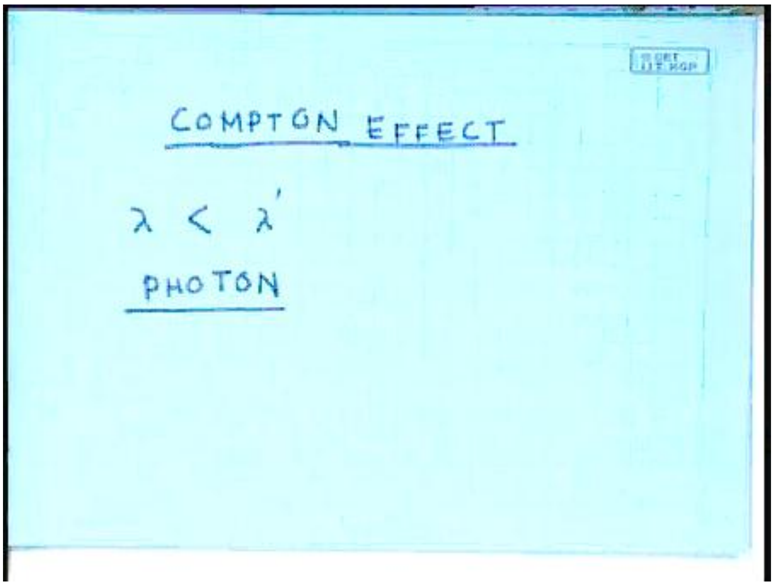

This now, the wave you can explain the Compton Effect is as follows. So, you have to in order to explain the Compton effect you have to postulate we have to change the way you think of the radiation of the x ray that is coming in. You have to change the way you have a adapt to new picture a new way of thinking about this x ray that comes in. So, till now, we have been thinking of all kinds of electromagnetic radiation as being electro waves. We now, have to give this up and we have to adopt a picture where we think of the electromagnetic radiation as a particle a particle called the photon. The concept of a photon was already in existence when the Compton Effect was discovered it had been proposed by Einstein to explain the photo electric effect. I shall not going to the details of the photo electric effect in this course I presume that you are already aware of it.

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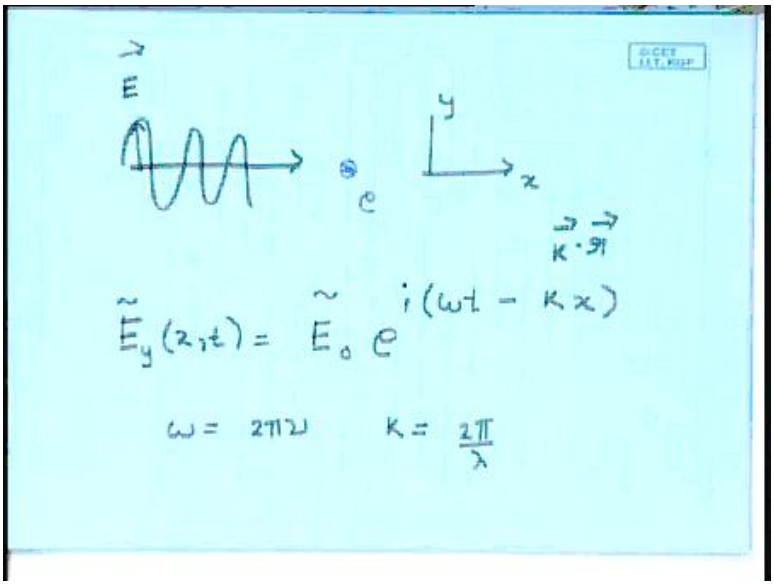

The let me just outline the basic idea. You have a metal plate on which you are shining light. The metal plate we know the metal has free electrons. And when we shine light on this we expect the light to be able to knock out some of the free electrons which are inside the metal from the metal surface. So, there was be some metals some electrons which will come out of this metal surface and if you put and electrode if you give a charge if you give a bias to this metal surface and say negative. If you give a negative bias to this metals are metal which metal sheet on which light in incident and you put a positive electrode elsewhere when you can measure the current that can be produced, because of these electrons which are coming out from the metal surface. So, the key point is that we expect the radiation to be able to transfer some of its energies to the electrons inside the metal.

So, that it gets the energy and it can come out from the metal surface. Now, you send in light say in the visual and you find that there are no electrons coming out. So, now you think why what could be the reason and you thinking you are thinking of the light as a wave you want to increase the energy the electrons the not getting sufficient energy. So, what you do is you increase the amplitude which increases the intensity of the radiation. So, you increase the amplitude and see if is now, able to knock out some of the electrons which transfer sufficient energy to make them leave the metal surface and come out. But; however, large you make the amplitude it is found that if you are working with low frequency radiation which has got frequency below a certain threshold however large you make the intensity you cannot make the electrons come out.

Whereas, if you work with high frequency radiation there is the frequency threshold if you go slightly if you just work at that frequency or slightly higher than that; however, low the intensity be you can knock out the electrons from the metal surface. So, you see the energy you increase the energy you increase the intensity, but if the frequency is low you are not able to knock out the electrons. You decrease the energy decrease the intensity decrease the amplitude essentially and increase the frequency and you find that low you are able to knock out some of the electrons. How do you explain this? An Einstein provided an explanation he postulated that the energy in the electromagnetic radiation does not come uniformly it comes in the form of packets which he called photons.

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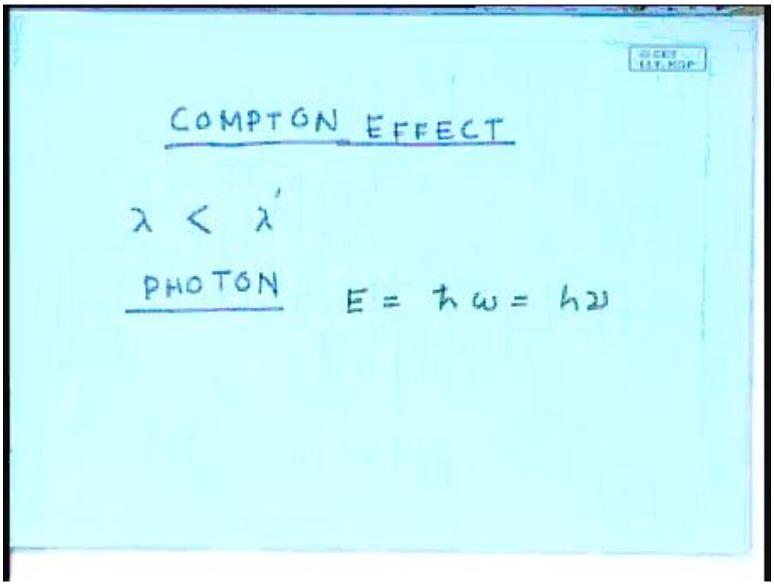

And each photon each packet has an energy h cross omega or h nu. This h and h cross are nu constants which appear here.

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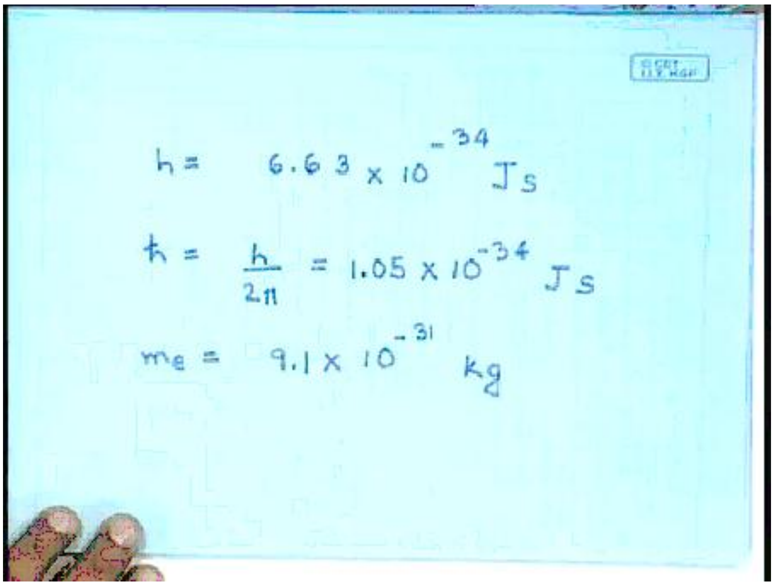

So, let me tell you what they are h and h cross these are called planks constants. So, you could refer to either this are this as the plank constant h has a value 6.63 into 10 to the power minus 34 Joule second. So, it is energy into time if you multiplied with frequency you get energy H cross is h divided by pi and its values given here. So, what you have to postulate?

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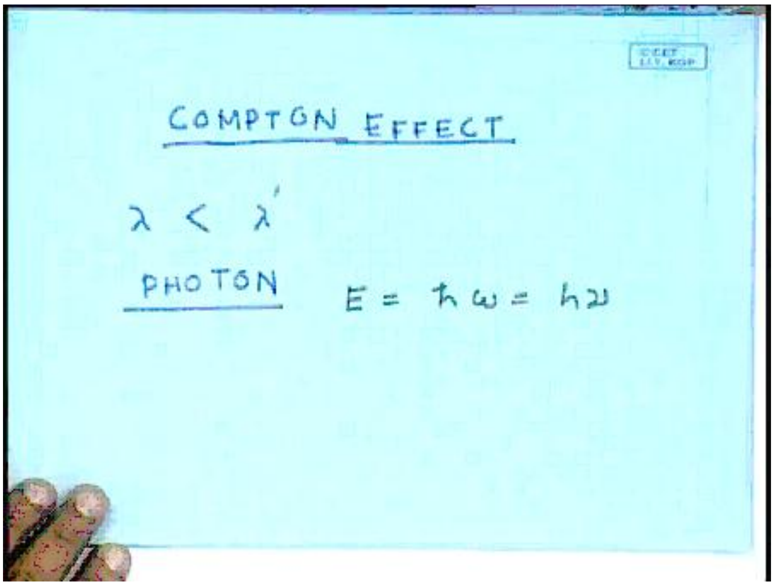

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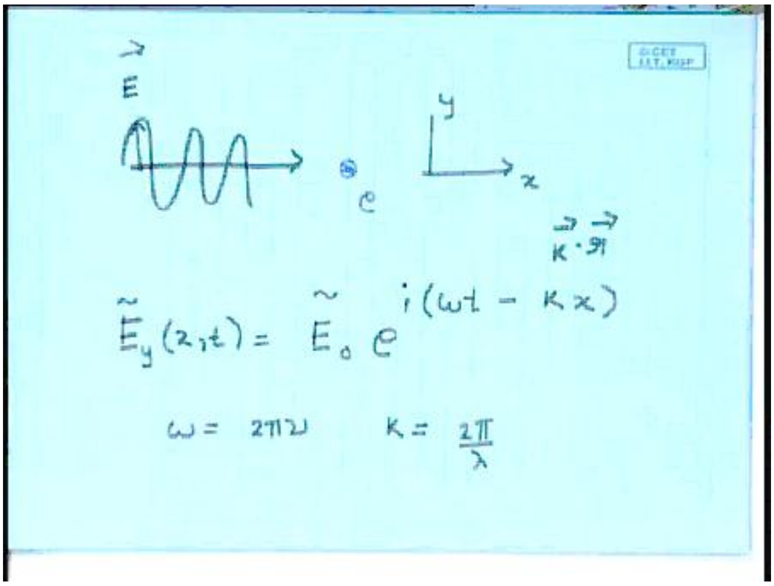

To explain the photo electric effect is that the energy of the electromagnetic radiation does not arrive continuously, but it arrives in packets called photons

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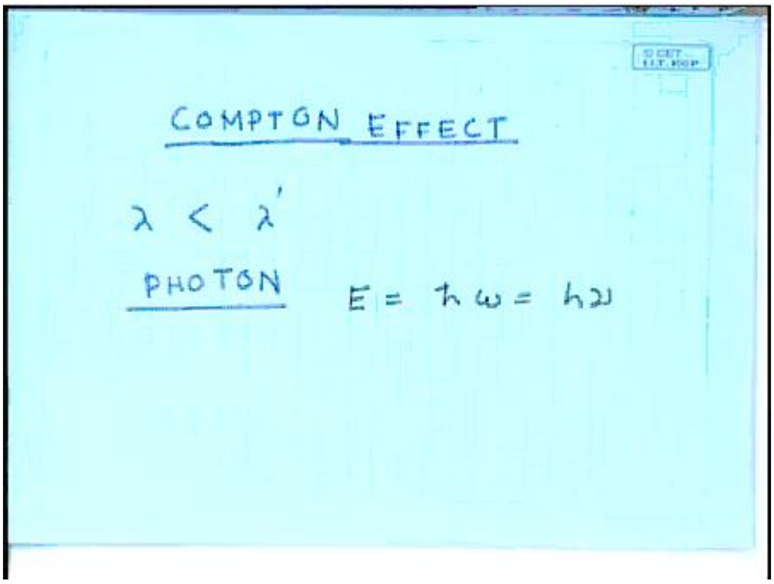

And each photon have an energy which is proportional to the frequency. Now, with the same postulate you can explain the Compton Effect.

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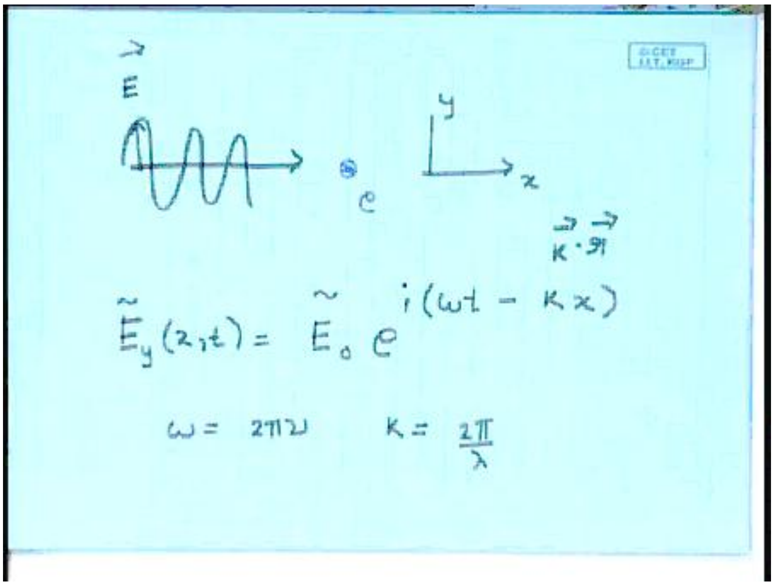

So, when you have this electromagnetic this radiation incidents so, we are dealing with the situation where you have radiation incident on this.

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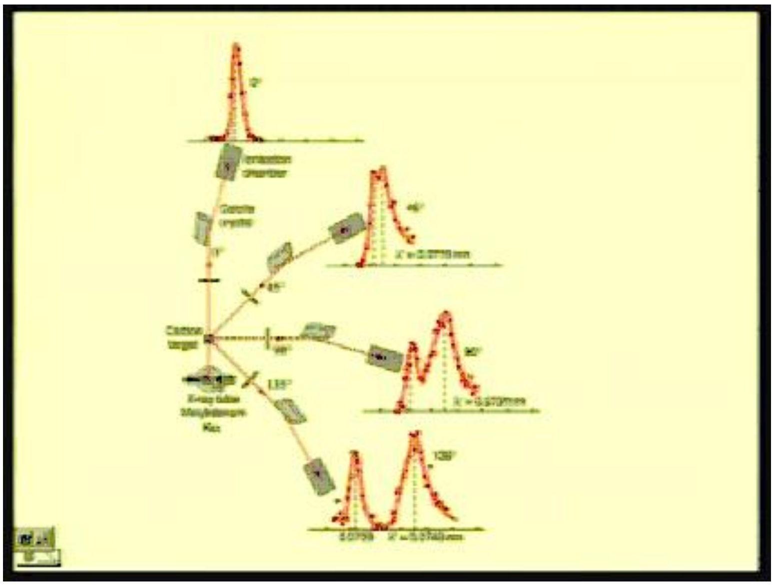

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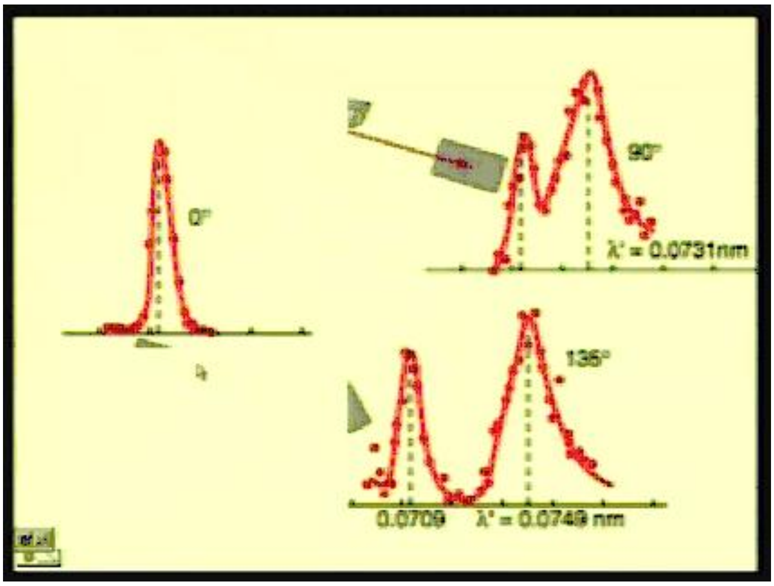

Target.

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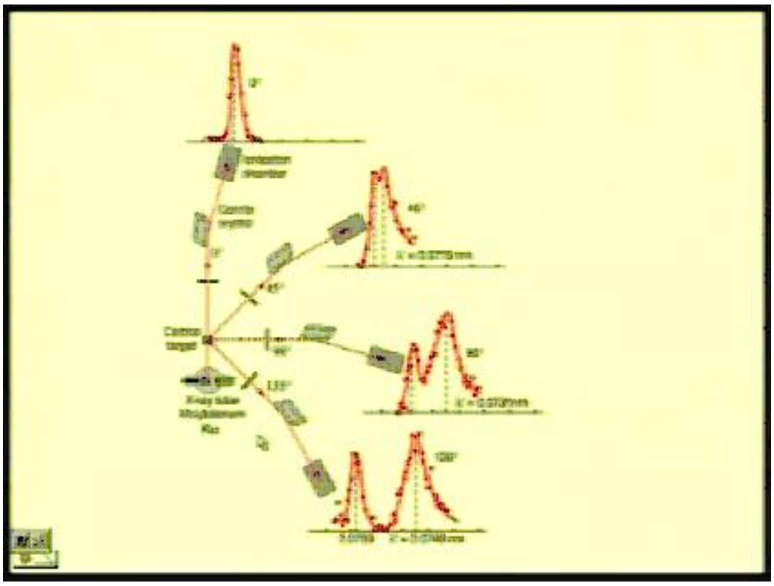

When you have this radiation incident on this target which has got electrons in it you have to think of the incident radiation.

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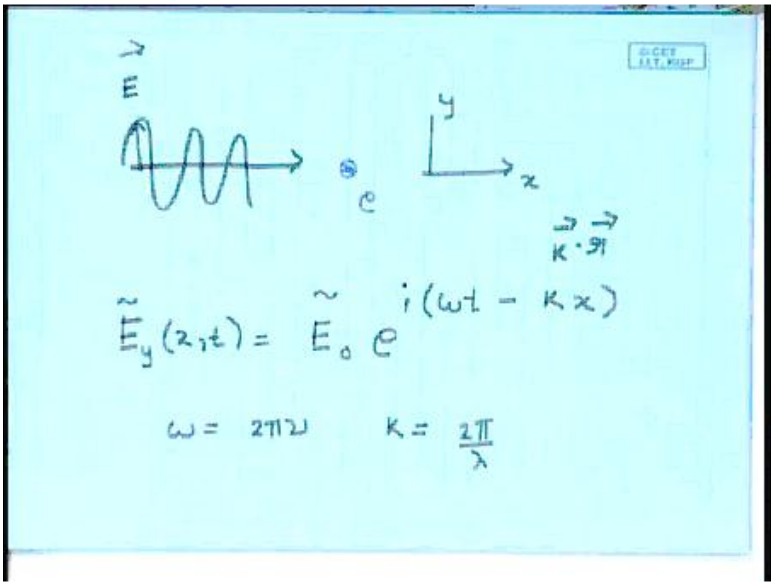

In terms of particles called photons.

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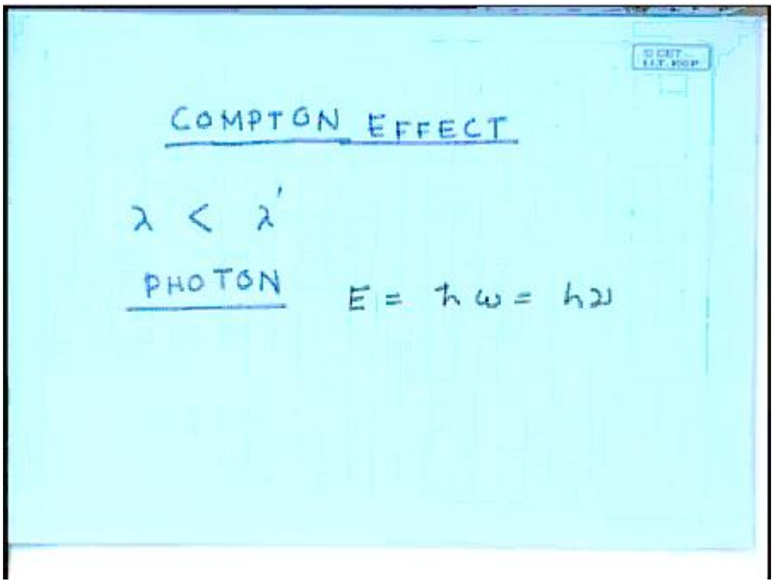

So, we are going to postulate that you can think of the incident radiation in terms of particles called photons of this each particle is a photon.

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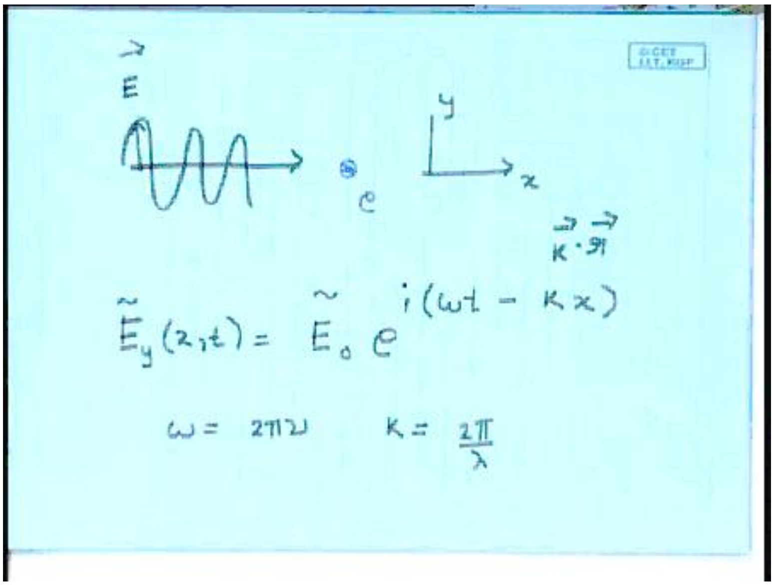

We are going to think of as a particle in terms of particles. So, each one of these particles is called a photon. And there are many photons in this incident radiation each such particle.

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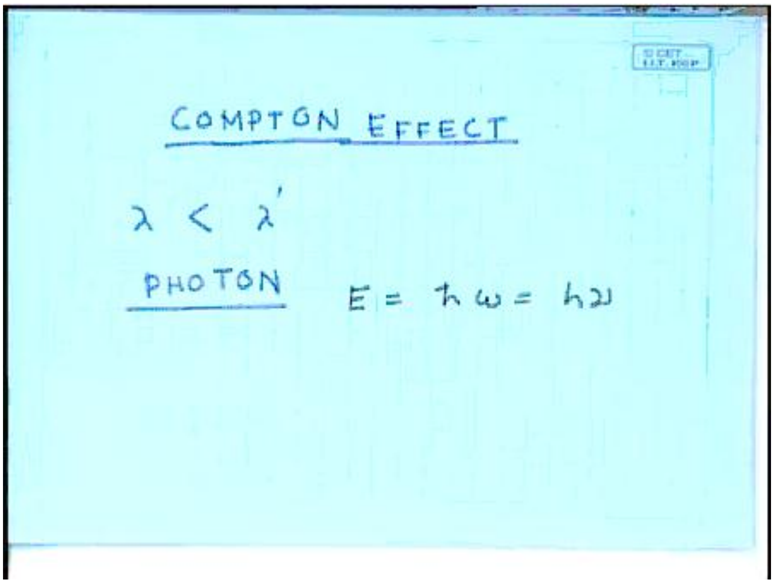

Has energy h cross omega where omega is the angular.

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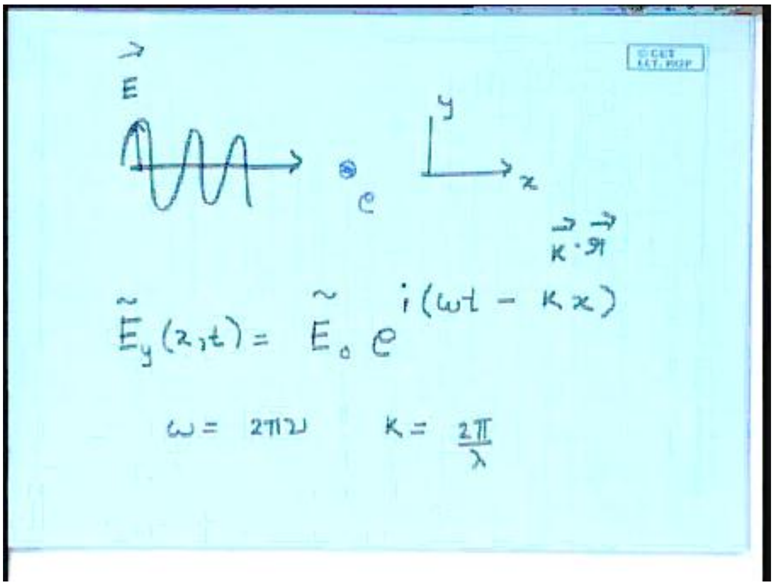

Frequency that you would have a scribed to this if you had thought of it as a wave. So, if I had modeled this incident radiation as a wave I would have written down some angular frequency some wave number. So, we are going to postulate that instead of thinking of this as a wave describe by this plane wave sinusoidal plain wave we are going to think of it.

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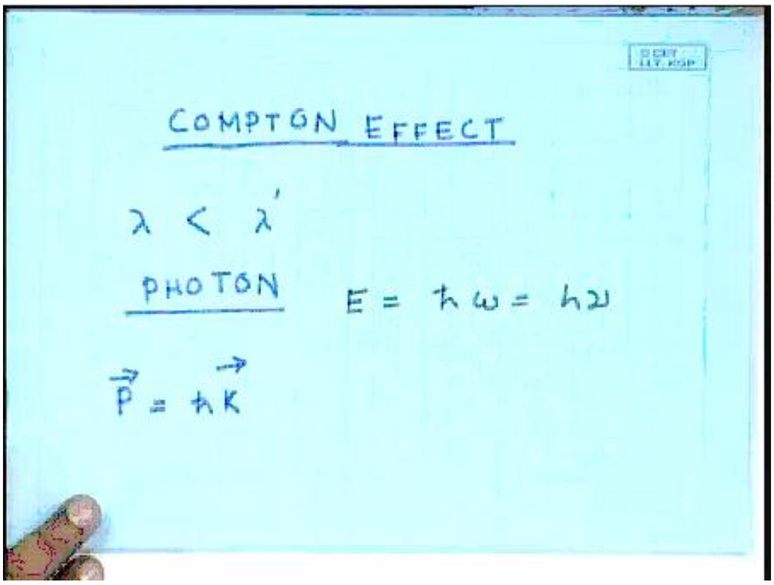

As particles called photons each photon has energy h cross omega or h nu. An each photon has momentum p which is h cross into the wave vector k which in this case…

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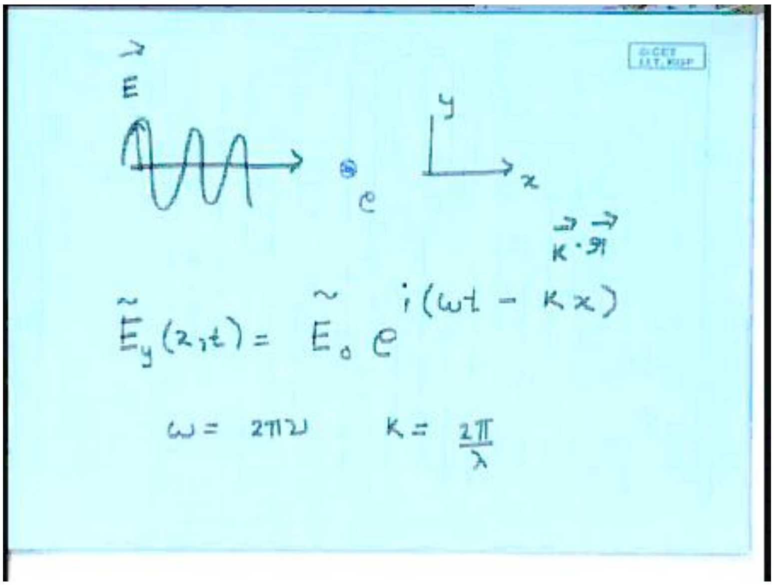

In this case the wave vector is along the x axis and it has a value 2 pi by lambda.

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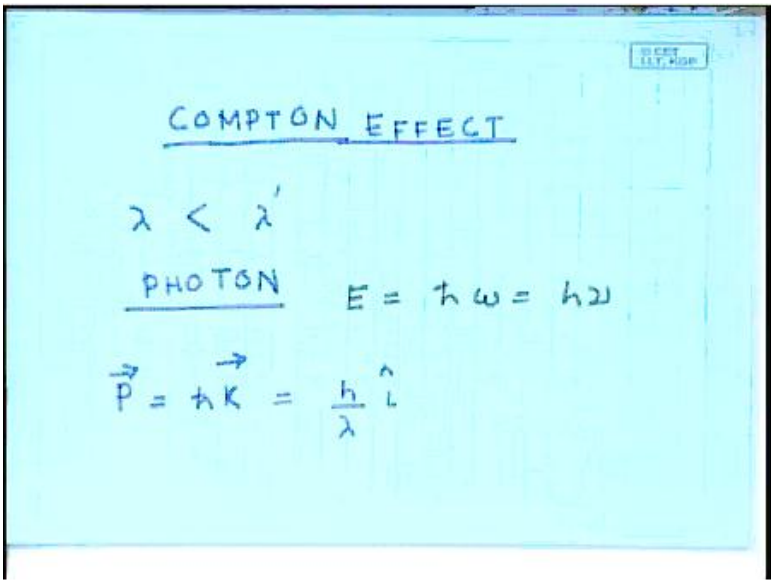

So, we can also write this as h by lambda into the unit vector along the x axis i. So, let me recapitulate what we are doing we are making a big jump. Until now, we have being thinking of radiation.

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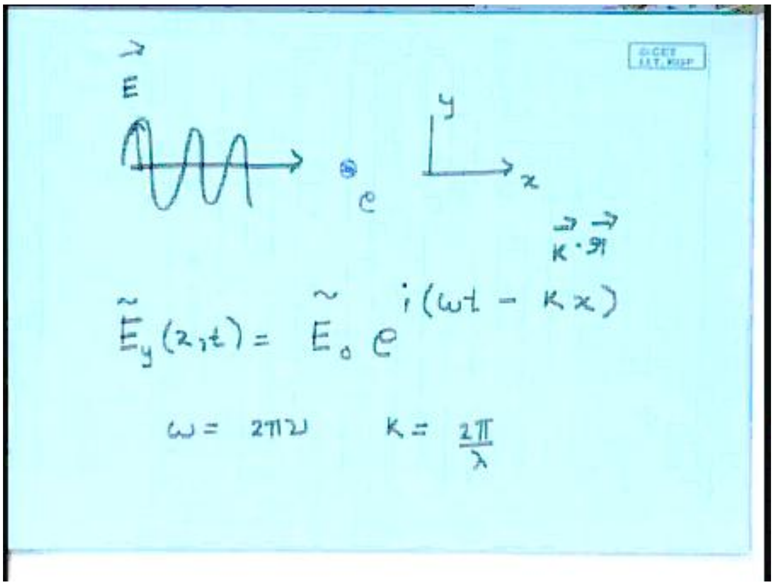

As electromagnetic waves so, whenever I spoke of x ray optical radiation, radio, microwave, infrared what we… the picture. That we had was that there is an oscillating electric and magnetic field pattern which propagates. So, when I say that there is a x ray incident over here the way to think of it is that there is an oscillating electric field and magnetic field incident in this direction it is a wave. But what we found is that with this picture you cannot explain the Compton Effect where there is a shift in the wave length of this scattered light. So, now we are postulating another picture another way in which we should think about the incident radiation. In this different way of thinking of the incident radiation should think of it as be made up of particles these particles are called photons. These particles have energy E is equal to h cross into omega. So, if when I think of it is a wave if I had written a wave equation like this. Now, when I think of it is a particle I should say the particle.

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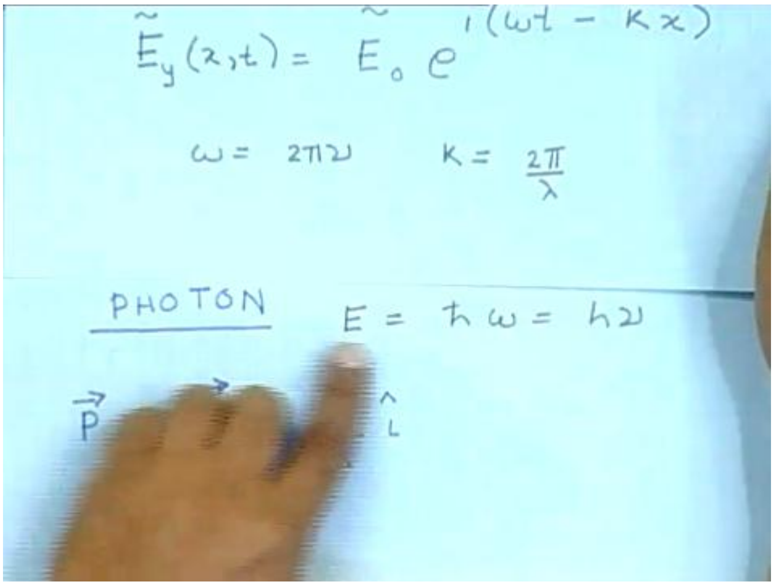

Photon has energy.

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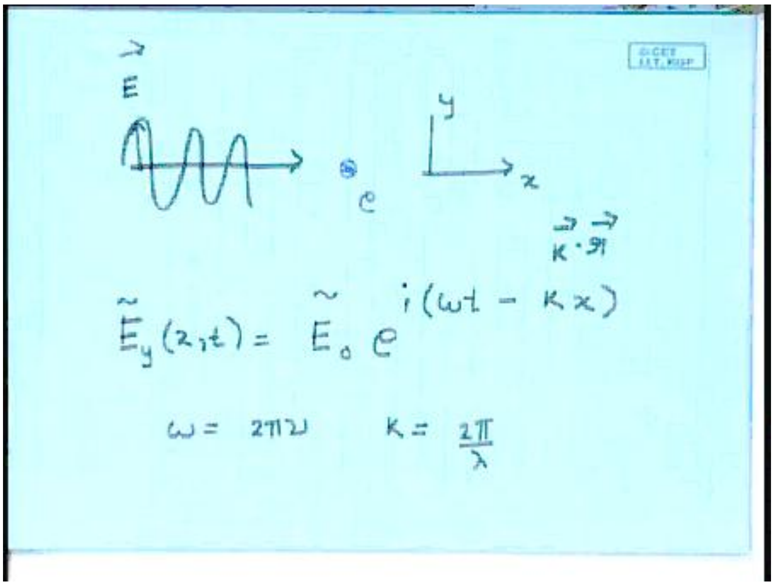

H cross into the same omega which I would have written if I thought of it as a wave. And the particle has a momentum h cross into k where k is a same wave vector which I would have written to describe the radiation.

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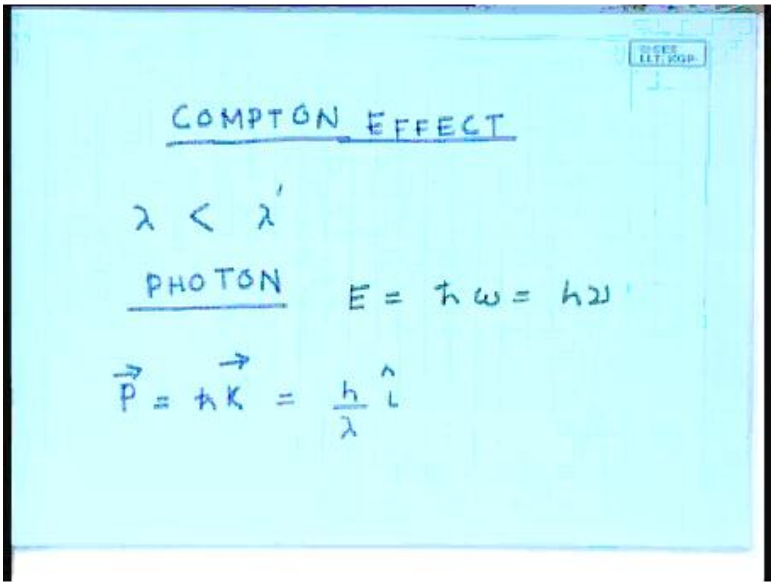

If I had thought of it as a waves so, instead of thinking of the radiation as a wave we are now, going to try and see if I think of it in terms of photon as particles which have discreet energy h cross omega or h nu. And momentums h by lambda h cross k or h by lambda. So, instead of thinking at of the x ray as a wave if we think of it us a particle can you explain the Compton Effect. At this picture was already present by when the Compton Effect was this covered it was already been proposed by Einstein in order to explain the photo electric effect. Einstein could give a given explanation of the photo electric effect using this photon picture where the energy arrives in packets. So, we are now going to adopt this picture where the radiation is actually not a wave or instead of thinking rather let us not say it is not a wave. Let us say that instead of thinking of the radiation in terms of a wave.

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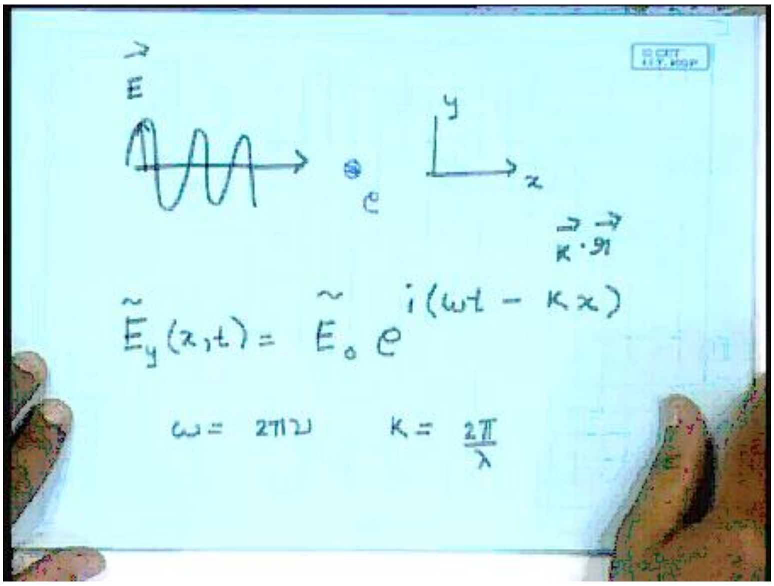

We are going to now think of it in terms of particles called photons and their photons have this energy and this momentum. So, the same situation which we have been discussing I am going to now draw it again so, we have a photon now.

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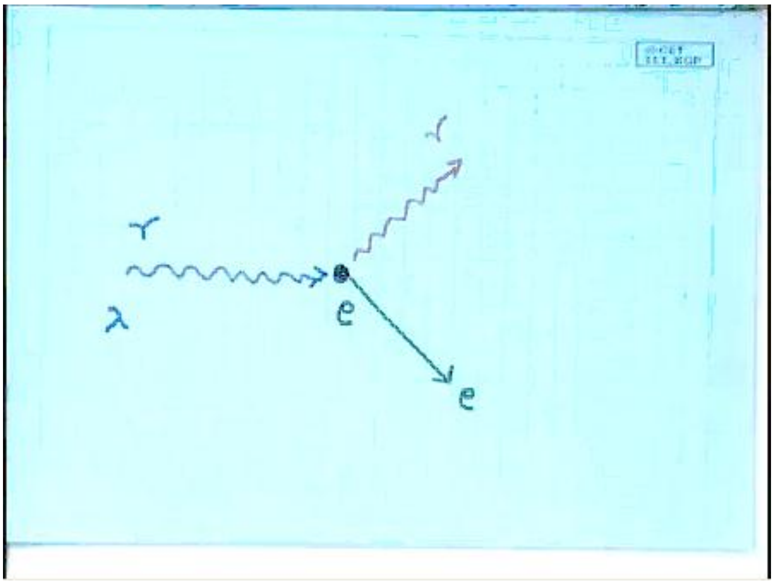

We have a photon propagating along the x axis so, this is my photon which I am going to denote by a gamma. This photon has wave with lambda it is incident on an electron. So, this is my electron which is at rest. So, this is the particle now, so, 1 particle is incident on another. So, the photon I am going to represent by gamma and the electron by e. So, there is one particle over here which is incident on another. Now, this is something which we have learnt in school. We have two particles what happens when 2 particles come and one comes and hits the other this is what is called scattering. So, now, the 2 particles will emerge could emerge in different directions. So, let us considered the situation where the photon comes out like this. So, this is the gamma and where the electron comes out like this. So, the same situation which we had been discussing here.

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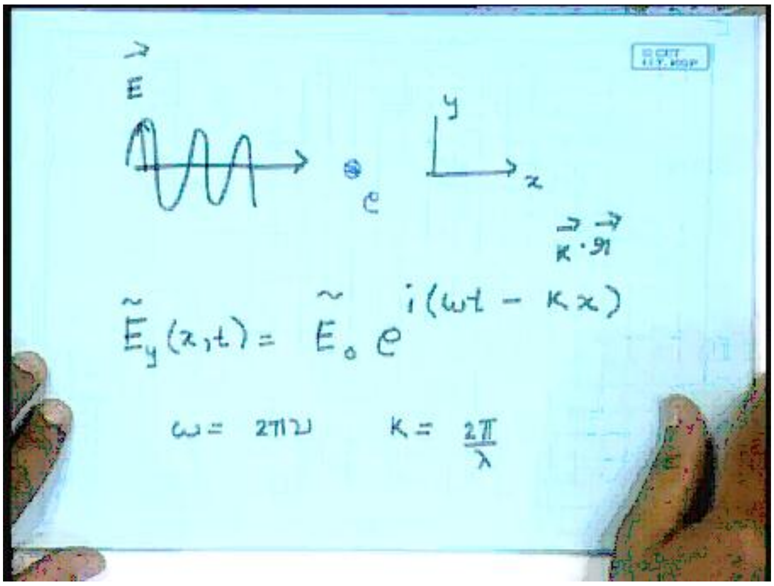

In terms of waves of a wave incident on an electron. And that is the x ray is incident on the electron we have a thinking of in terms of a wave incident on the electron. You now thinking of as a particle called the photon.

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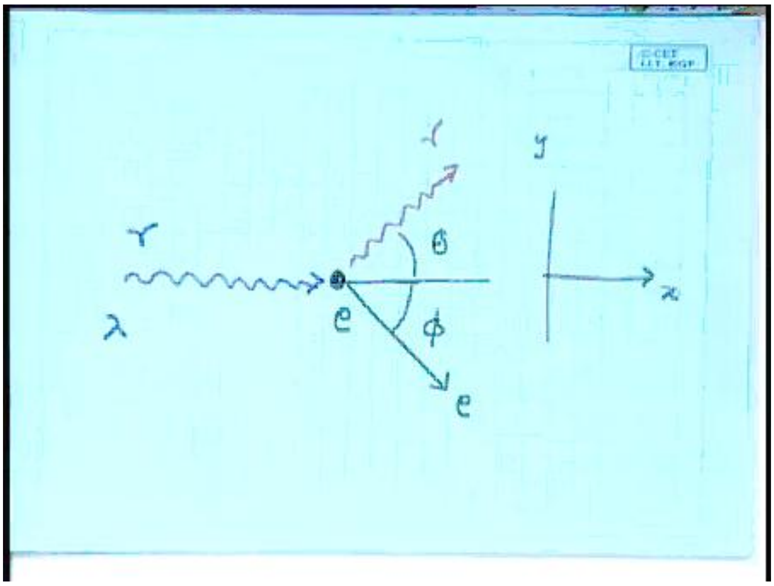

Scattering of the electron as a consequence of this scattering the electron moves in this direction the photon moves in this directions. Let us put in angles so, this is the angle theta this angle we are going to call phi. And remember that this angle theta is the scattering angle, because this is the direction in which the photon was initially the x ray is incident or the direction in which the photon was initially travelling the x axis the scatter photon makes an angle theta with the incident direction. So, this is the scattering angle theta and we will call this the y axis. We are not going to be interested in the z axis which is perpendicular to the plane of the paper.

Now, notice that when the photon scatters of the electron and sets the electron into motion over here it imparts some momentum energy to the electron. Now, momentum an energy we know are both conserved this is an elastics scatterings. So, momentums an energy we know are both conserved. So, if the photon imparts some energy to the electron it must be at the expense of its own energy and momentum. So, the photon that comes out over here if it has imparted energy to the electron it must be having a lower energy and a lower momentum.

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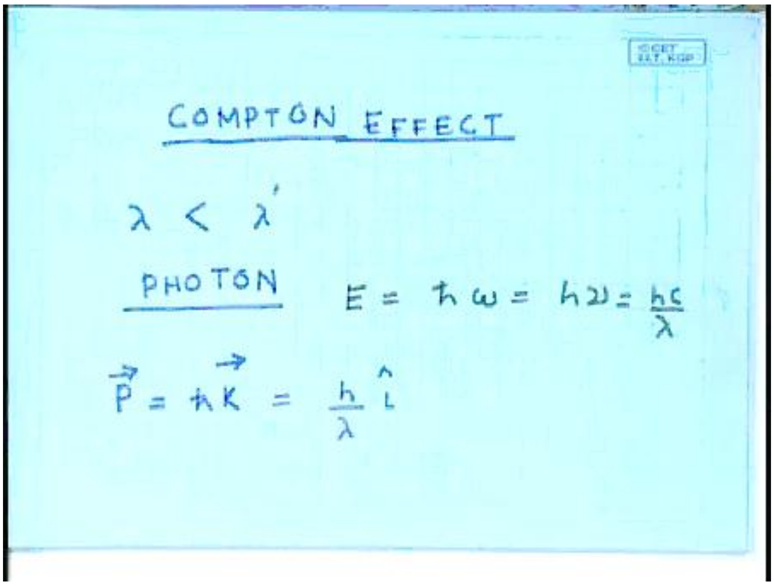

Now, the energy of this photon is related to the wave length we could write this in terms of the wave length as h c by lambda. So, energy is inversely related to the wave length. So, if the energy goes down the wave length has to go up. So, what you could say is.

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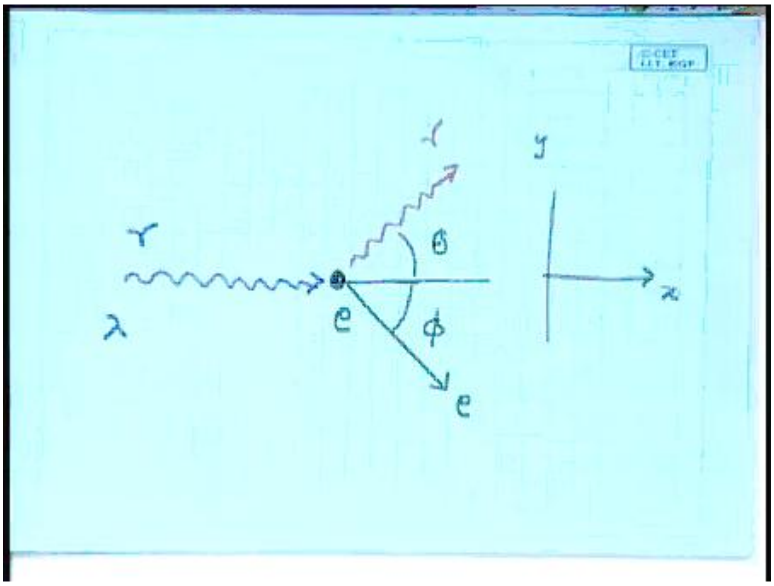

That if the photon comes out at an angle different angle when it imparts some of it is energy to the electron. Then its wave length must increase, because the energy of the photon has to go.

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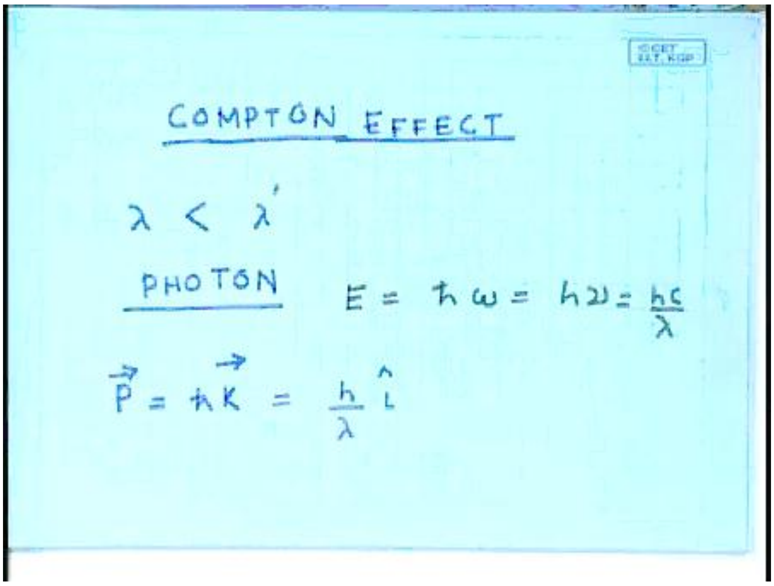

Down total energy is conserved it has imparted some of its energy to the electron. So, the wave length of the photon has to increase.

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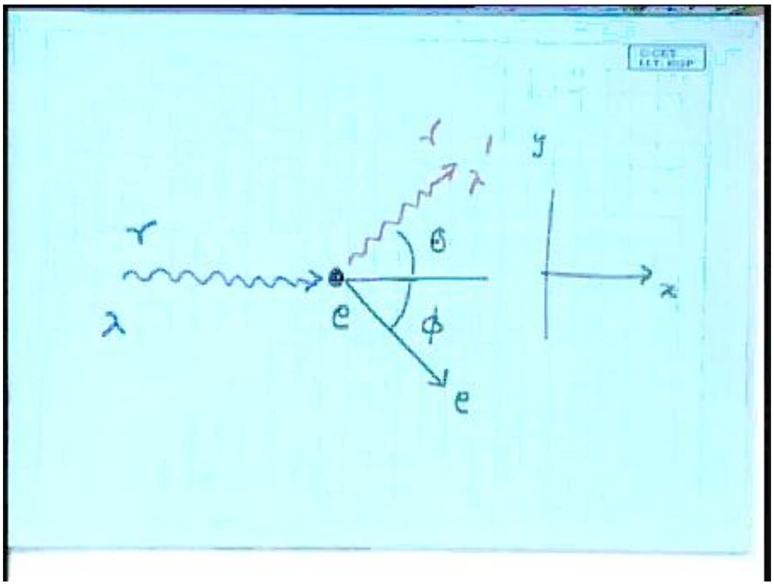

So this photon is going to come out at a larger wave length lambda prime. So, we see that with this photon picture it is we could possibly get an explanation for the Compton Effect. You could possibly it seems possible it seems possible here that you could explain the increase in the wave length due to scattering. We have to now, do the calculation and check whether it really matches with the observed angular dependence of the change in the wave length.

So, we have to now, calculate and see whether you can really predict how much the wave length is going to change? And its angular dependence correctly with this picture photon picture. So, let us do that it is a very simple exercise which I am sure all of you are a familiar with, because you must have done exercises where 2 particles scatter of each other and in your make an a such situations in a in your mechanics course. So, let us first write down the conservation of energy. What does the conservation of energy tell us? So, we are going to first apply the conservation of energy so, the energy of the incident photon.

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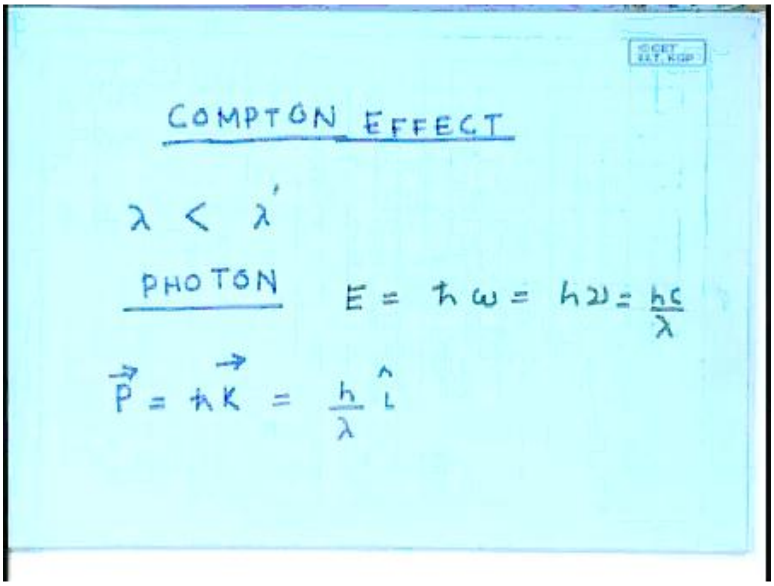

Is h c by lambda so, let me write that down. So, we are doing the conservation of energy.

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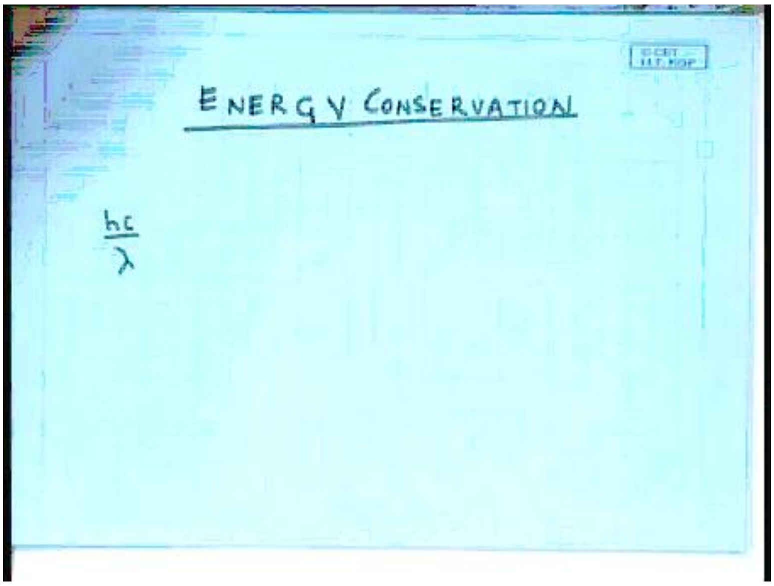

And the incident photon has energy h c by lambda.

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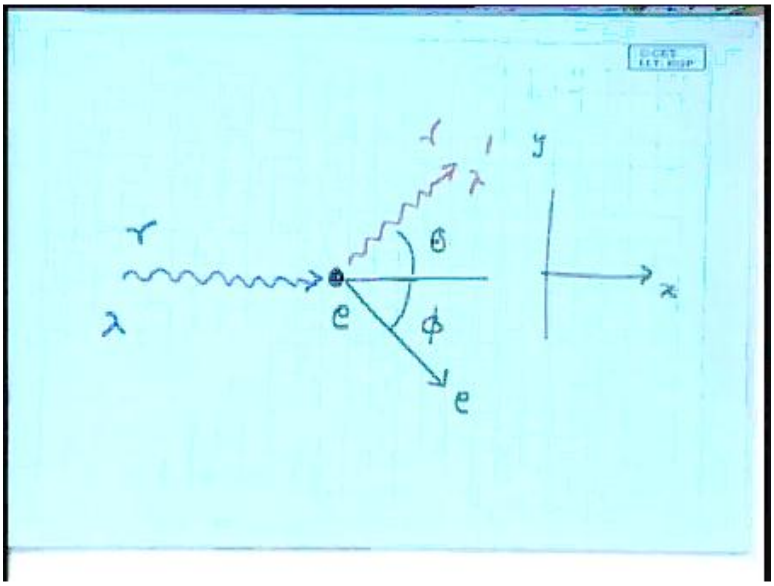

This as energy h c by lambda now, what is the energy of the electron to start with? So, this is an interesting question and Einstein theory of a relativity tells us that a particle has energy even when it as at rest. And the way the formula to calculate the total energy of a particle which is moving let me write that down for you.

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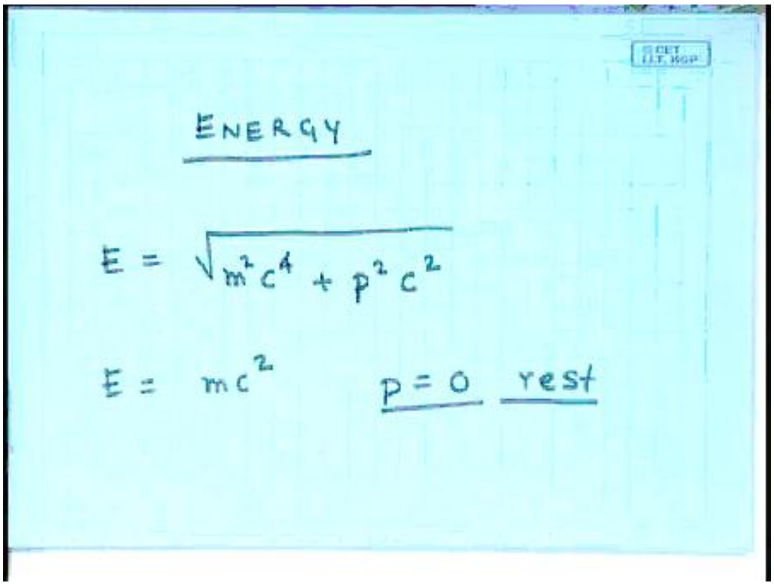

So, the way to calculate the energy of an A particle which is moving in Einstein's theory of relativity is that you should m square c four plus p square c square. So, this is the relation between the energy and the momentum of a particle according to Einstein's theory of relativity. And we have to use this formula in order to get the correct results. So, we will straight away use this and Einstein's theory of relativity was known by the time with the Compton effect was discovered. So, let us use the this formula of a relating the energy to the momentum before just applying this formula. Let me discuss this very briefly the first thing that this tells us is that if the particle is at rest it will have an energy E is equal to m c square if the particle is at rest.

If it has no momentum which means it is at rest this is called the rest mass energy. So, corresponding to the mass of the particle there is an energy associated with that which is there the particle is at rest and that energy is m c square. And if the particle has a momentum which is non zero then the energy and momentum are related like this. Now, let us also see that it produces reasonable results when the particle is moving slowly we know in Newtonian mechanics that we have a formula for energy we should be able to recover that when the particle is moving slowly. So, the same expression can be written in this way.

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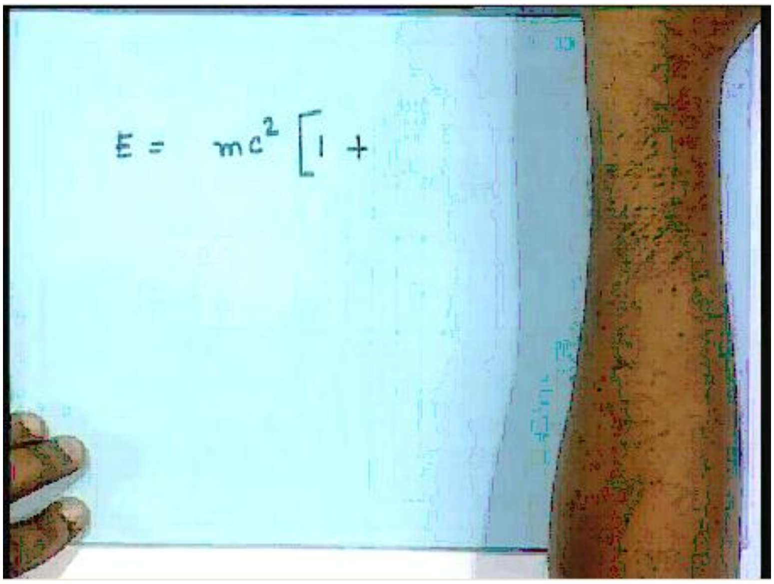

. So, if you going to do is we are going to take this term common outside.

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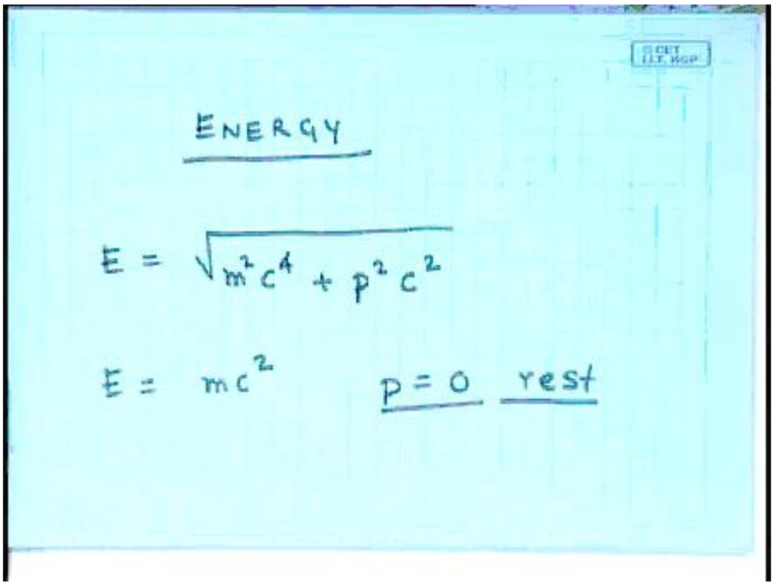

So, I have to take the square root of this term the square root of this term is m c square. So, I take the term m c square outside and then I have to divide

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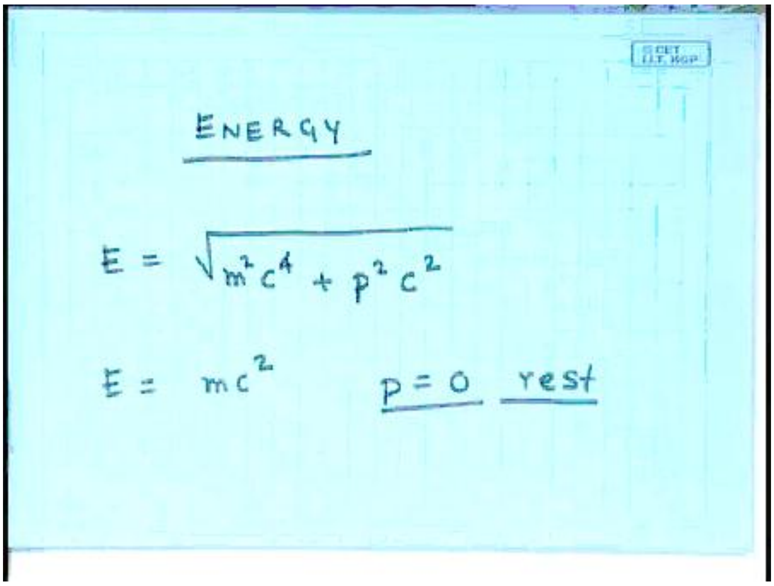

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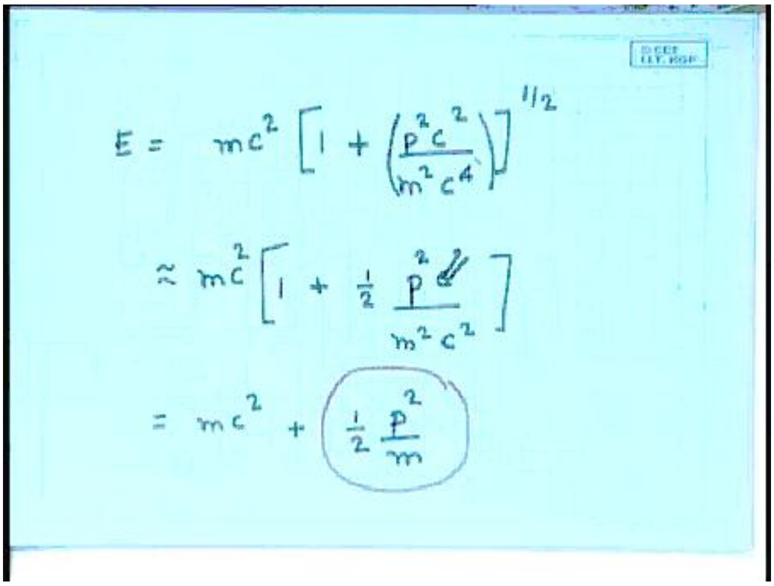

This by this so, I have T square c square divided by m square c four to the power half. So, this is the same expression for the energy given by Einstein in his theory of relativity. Now, this is the rest mass the square of the rest mass energy and this is the contribution that the extra thing that arises when the particle moves. Now, let us assume that the particle moves slowly. So, that the contribution from the momentum is small under this assumption this number is going to be very small if you assume that this number is very small. You can do a Taylor expansion and ignore all the higher order term. So, what that gives you is m c square 1 plus half p square c this c square will cancel out with this and what I will have is p square by m square c square. Which is equal to m c square 1 m will cancel out with them outside the factor of c square will cancel out with this plus half p square by the mass. Now, this term over here is what in Newtonian mechanics? We call the energy of a moving particle now notice that this formula which Einstein gives which is the correct full formula relating the energy to the momentum in the situation where the particle is moving slowly.

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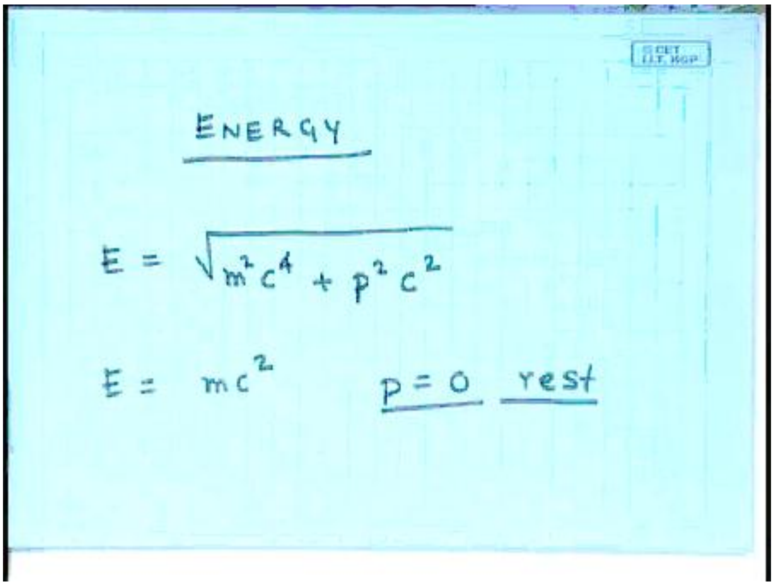

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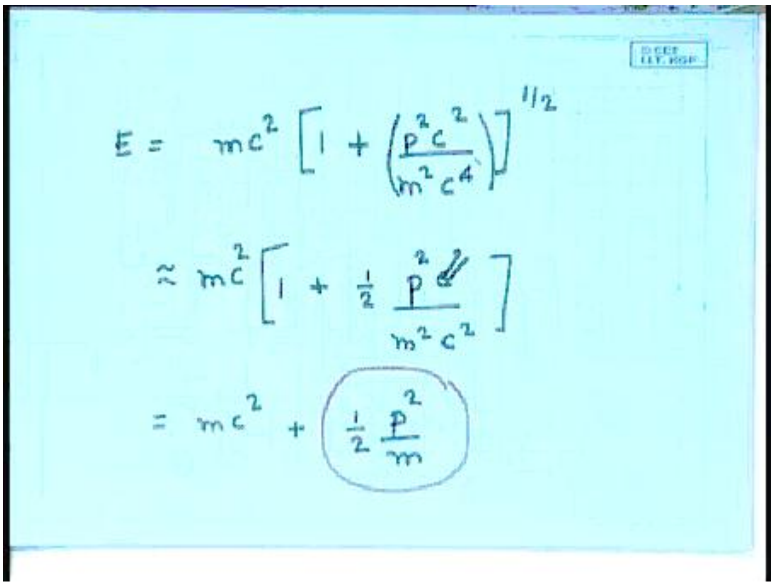

Gives you the only Newtonian result that the energy is half P squared by 2 m P square by 2 m plus a constant m c square which is the rest mass energy. Now, in Newtonian mechanics we can always add a constant to the energy does not make any difference.

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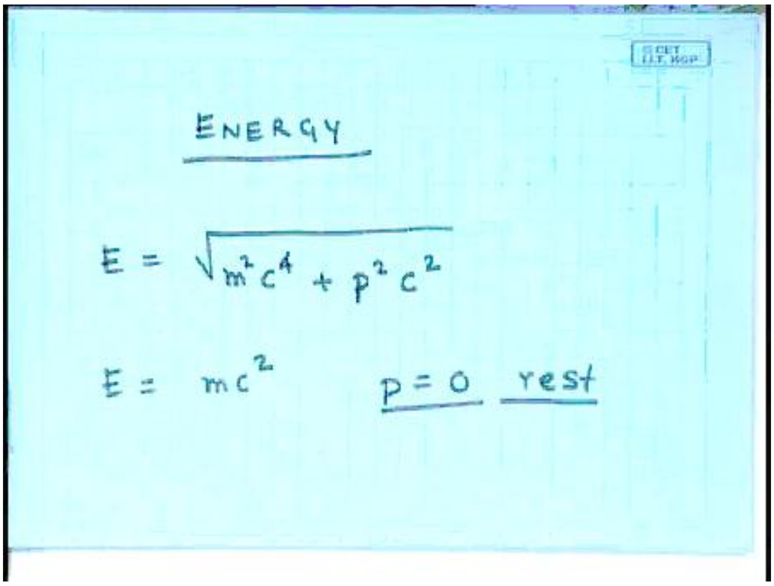

So, we see that this does give the usual expected results when the particle move slowly and we will be applying this relation between the energy and momentum to the electron over here.

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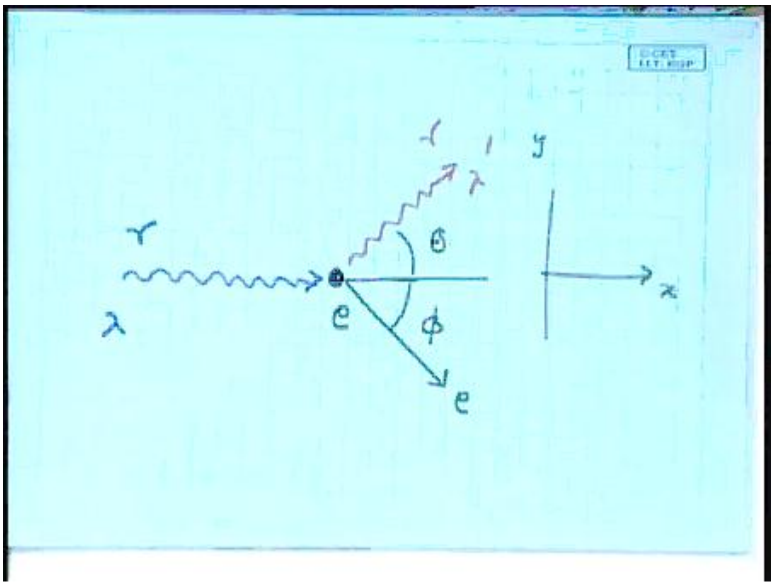

So, coming back to the situation that we have discussing we have this photon whose energy we have already written down. And now we have to look at the initial energy of the electron the initial energy of the electron we seen electron is initially interest.

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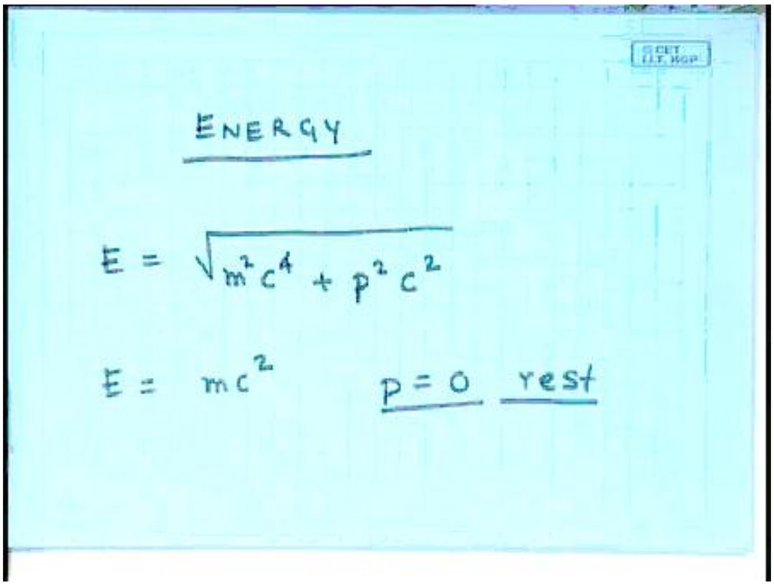

So, the initial energy of the electron is going to be m c square so.

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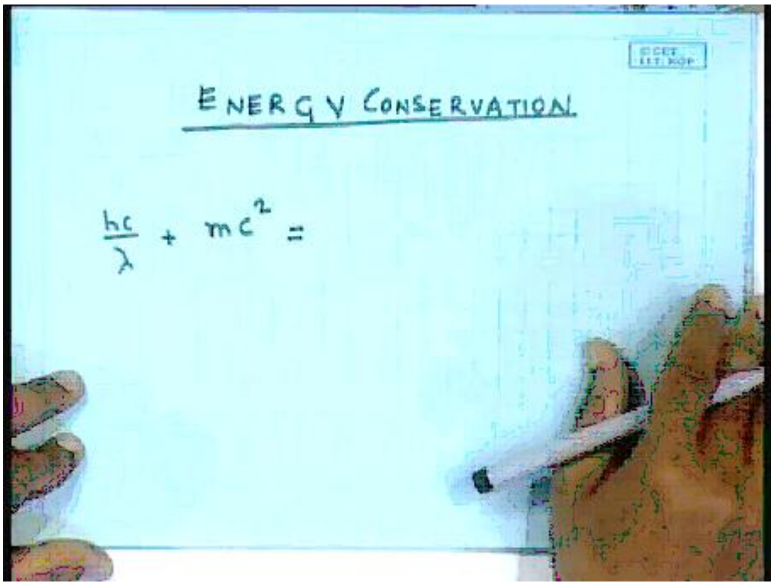

This is the initial energy of the electron a plus m c square. So, this is the initial energy of the photon this is the initial energy of the electron this should be equal to the final energy of the photon M c square.

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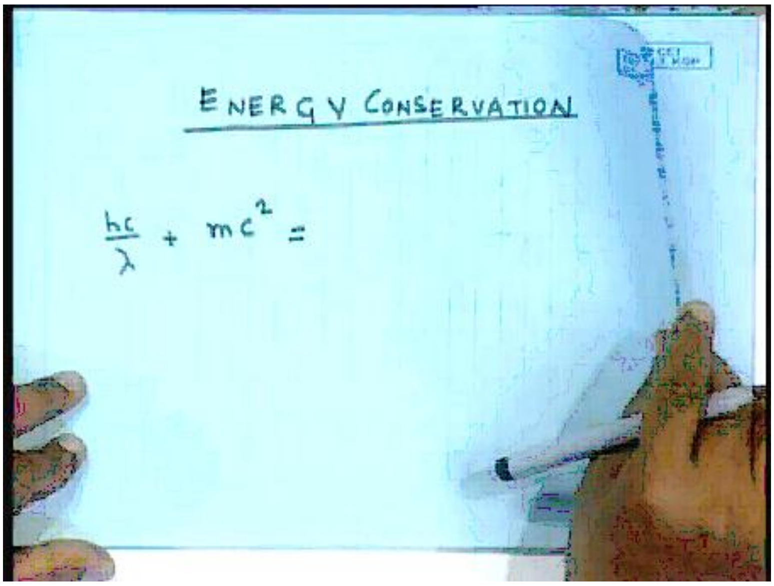

So, this is the initial energy of the photon this is the initial energy of the electron this should be equal to the final energy of the photon.

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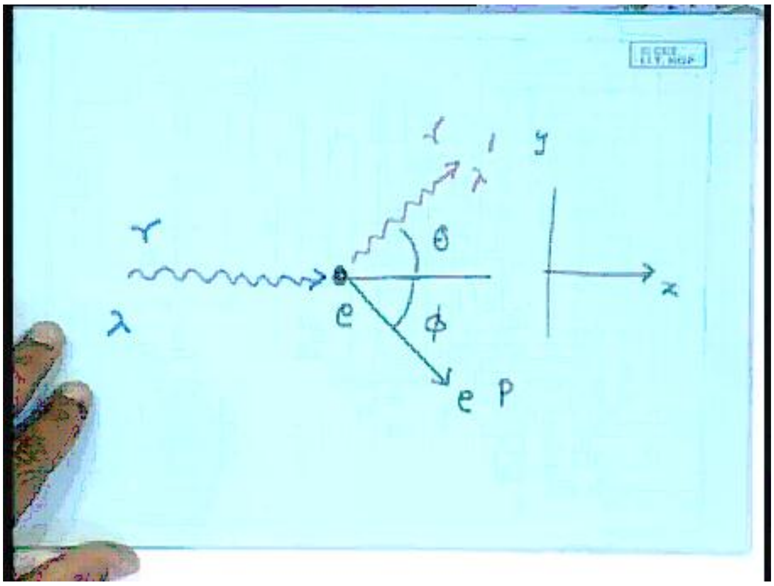

So, the electron which comes out is going to have a momentum because it is going to be set in motion as a result of the scattering and we can write the final energy of the photon as hc by lambda prime well lambda prime is larger than lambda.

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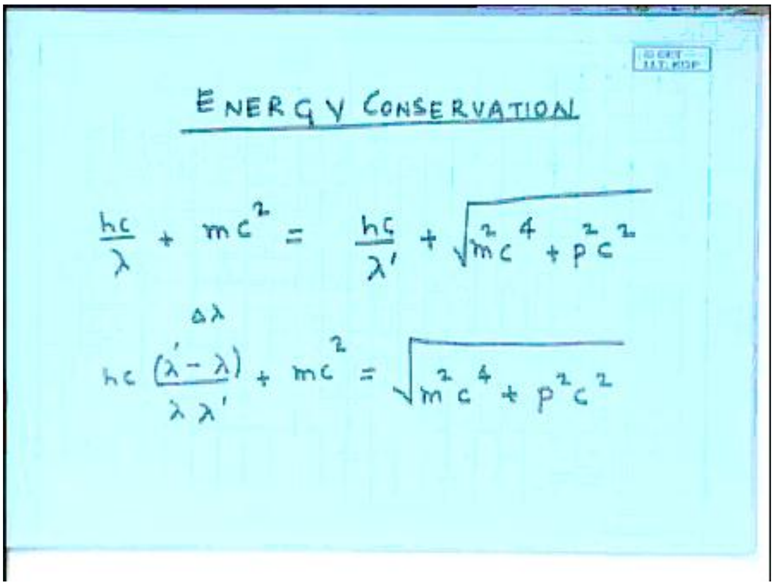

Because the photon is going to lose energy and some of it is going to be transferred to the electron. So, this plus the electrons energy which is now going to be m square c to the power of four plus p square c square. Now, we can simplify this a little bit. So, let me take this on to the left hand side. So, if I take this on to the left hand side what is going to give us is this h c lambda prime minus lambda divided by lambda lambda prime. I have take it this term on the left hand side plus m c square is equal to the square root of m square c 4 plus p square c square. Now, notice that lambda prime minus lambda is the change in the wave length the quantity that we are interested in and we are going to call this delta lambda.

So, this is what we are going to call delta lambda the shift in the wave length. So, now we can square this the left hand side and square the right hand side and write down what this gives us. So, this is going to give us. So, we are squaring this and then squaring this also. So, when I square this there will be one term which is the square of this which is going to cancel out exactly with this. So, this term is g1 I have another term which is the square of this which I have to retain and they will be a term which is the twice this into this which I have to retain and I am going to have this term over here. So, what let me write it down straight away what I am going to get is h.

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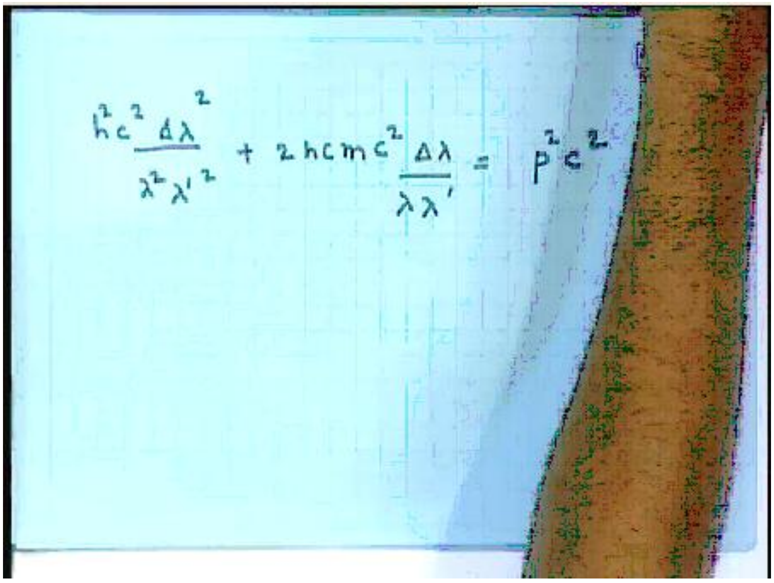

Square c square delta lambda square divided by lambda square lambda prime square plus 2 h c m c square delta lambda by lambda lambda prime is equal to p square c square. So, all that we have done is that we have.

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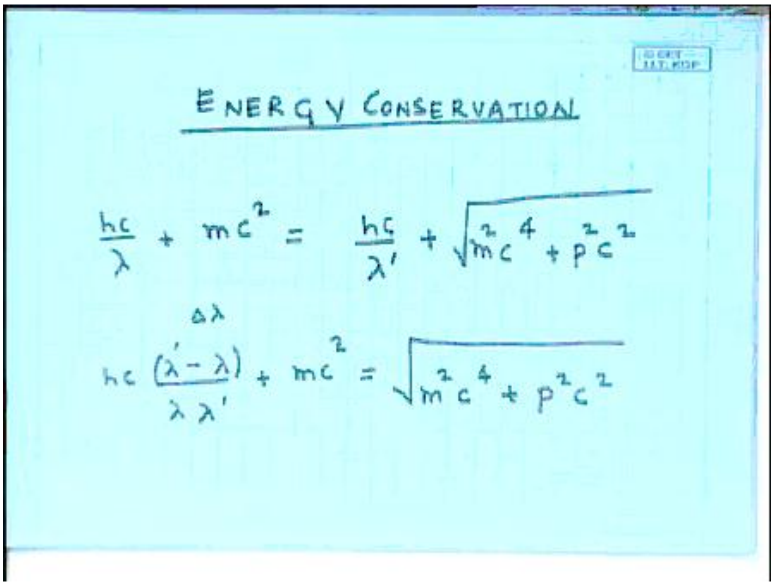

Squared this and canceled out the terms which are there common both in the left hand side and the right hand side.

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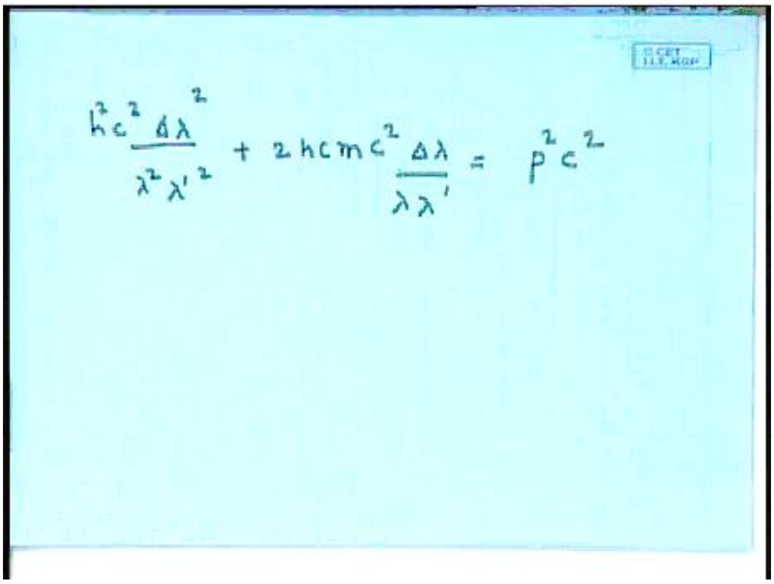

So, this is the relation that we get from the conservation of energy let me stop here for today and in tomorrow’s class I am going to take up what we get from the conservation of momentum. And then we shall see whether this can explain the observed wave length shift in comp in the Compton Effect.