Physics I: Oscillations and Waves
Prof. S. Bharadwaj
Department of Physics & Meteorology
Indian Institute of Technology, Kharagpur

Lecture - 30
Standing Waves (Contd.)

Good morning. We have been discussing standing waves let us take up a problem there is a steel rod.

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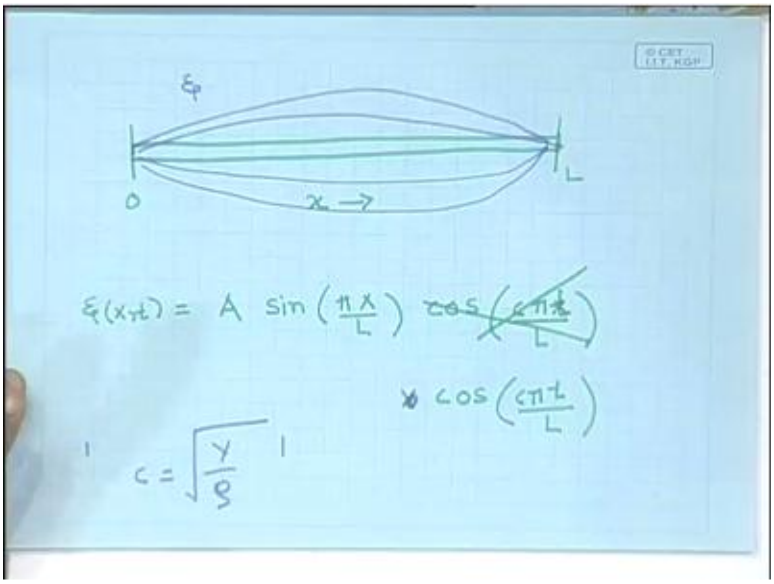

The two ends of the steel rod are fixed so, that it does not experience any deformation and the steel rod is of length L. There is a disturbance longitudinal disturbance xi x t which is the standing wave so, it is of the form A sin pi x by L cos c pi x by L. So, we have a disturbance sorry this should be t c we have a standing wave disturbance introduced in this steel rod of length L cross sectional area A. And the deformation the displacement of any part of the rod, because of this disturbance is as given by the expression over here. This is into product into so, it is a product of one function of x and function of time. And this is the familiar standing wave you can also see that this corresponds to the fundamental mode or the first harmonic.

The disturbance looks like this. It will have a maximum here and this shows you xi the displacement that remember the displacement is not vertical it is a transverse elastic wave standing wave. So, the displacement is in the horizontal direction and it looks like this. It has a maximum at the center and as time evolves this will go down, because of this oscillating cosine c pi t by L C remember is the speed of sound speed of this elastic waves in the steel rod which is square root of Y the Young's modulus by the density of the of the steel.

So, Y is the Young's modules and rho is the density of the medium and the displacement xi looks like this at t equal to 0. And then as time evolves it falls and then at a later the later time it will looks like this. And then it will become 0 and then it will become like this and then like this. And then it will go back and forth those are the displacements of that shows you the displacement pattern in the inside the rod remember the displacement pattern is longitudinal it is not transverse now. And the problem which we would like to address is as follows we have to calculate two quantities first.

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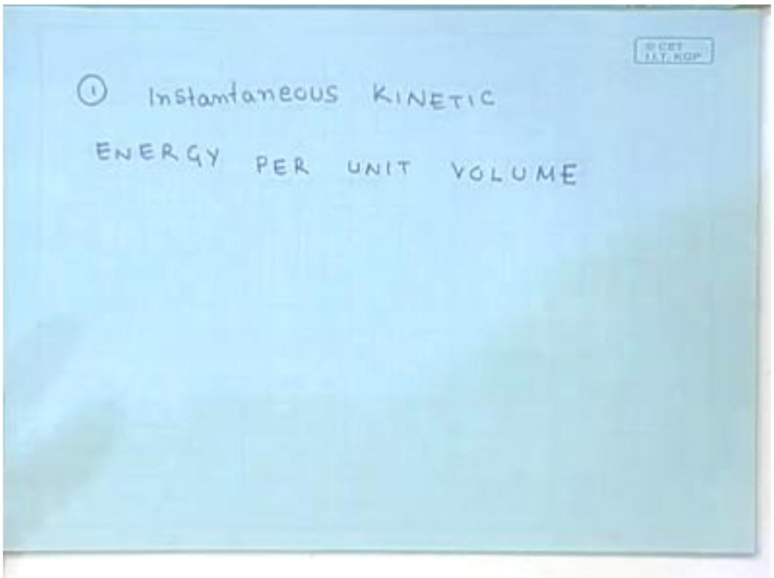

the first quantity which we would like to calculate is the instantaneous kinetic energy per unit volume and we would like to calculate this for any point arbitrary point inside the rod.

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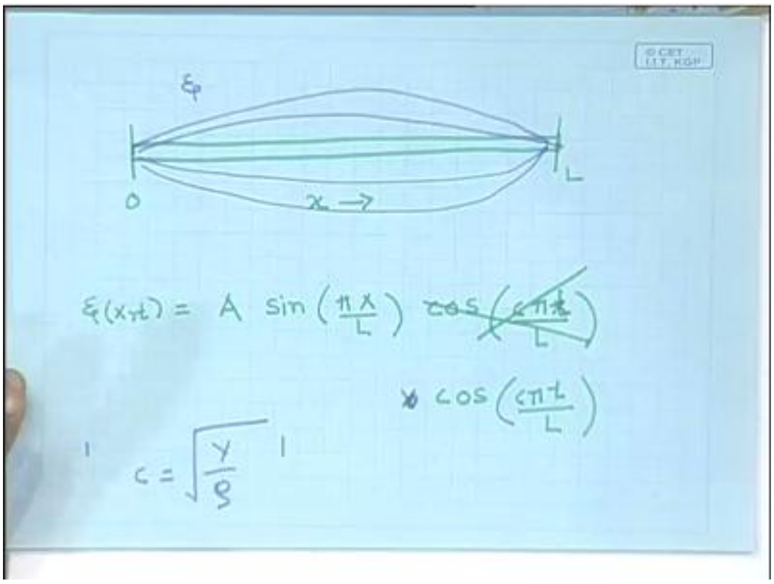

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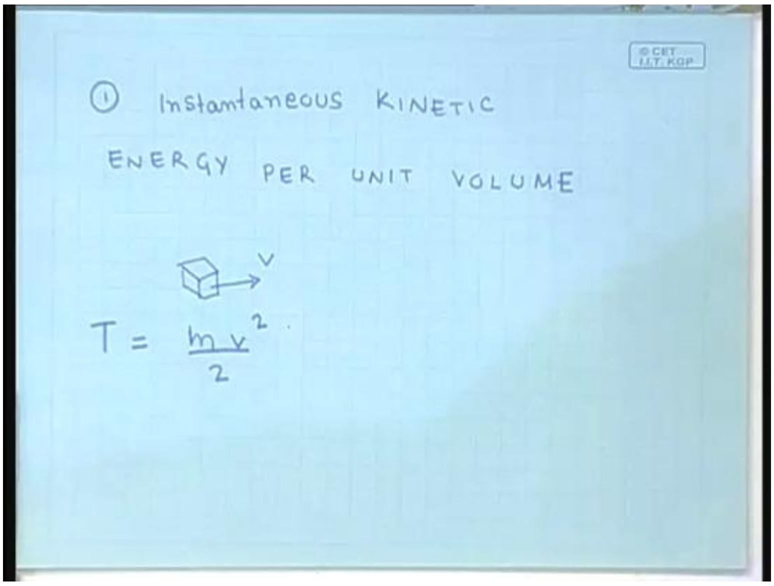

Now, if you have a mass element m. So, there is a mass element m with mass m and if it has a speed v then the kinetic energy. The kinetic energy which we will call T the kinetic which we call capital T is going to be mass into velocity square v square by 2.

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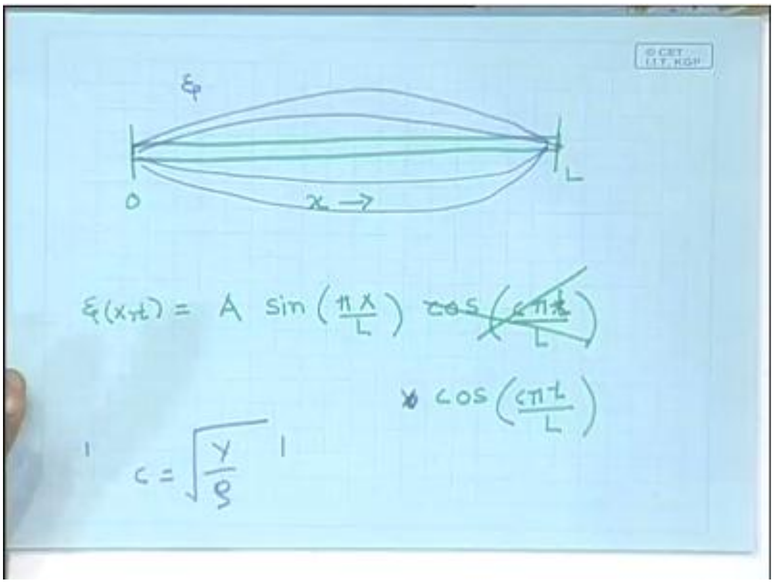

Now, let us take a small volume a small part of the steel rod; so, the element the mass element which we are dealing with is a part of the steel rod.

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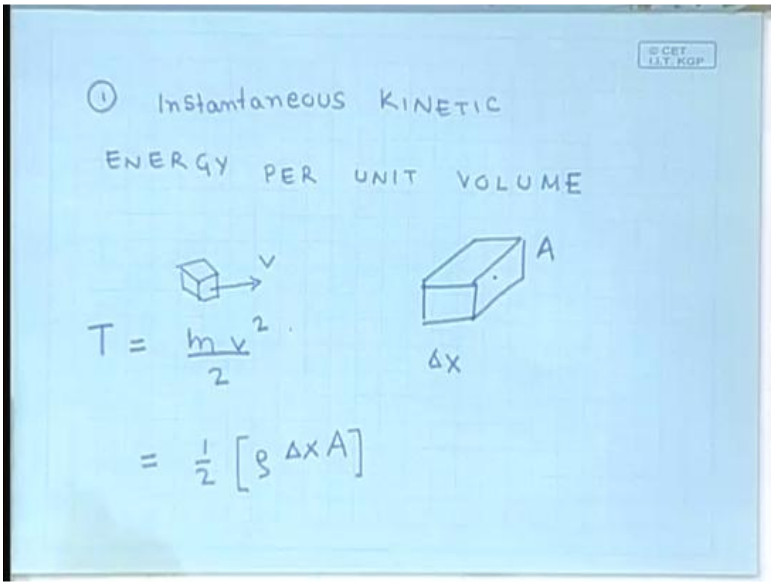

So, this is of actually a part of the steel rod of length delta x and transverse cross sectional area A so, this has an area A. So, the mass of this element of the steel rod is rho into the volume here is delta x into A. So, this is the mass we have half so, half times mass times the velocity squared. Now, the question is what is the velocity of this element of the mass inside.

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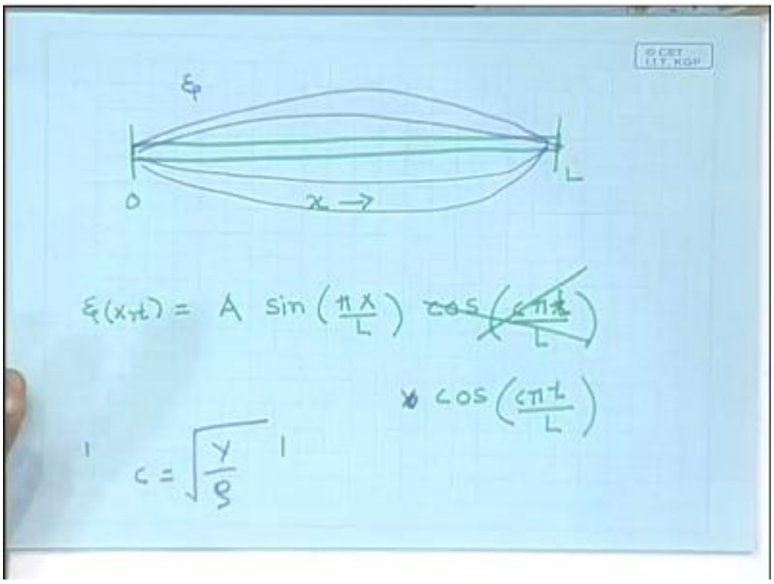

This steel rod now, the displacement of any element of the steel rod is given by xi. So, the time derivative of xi gives the velocity.

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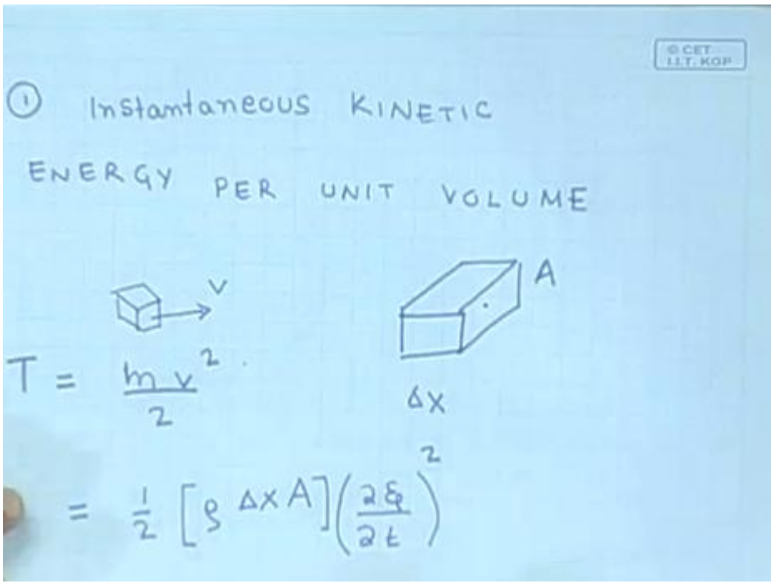

So, it is dell xi dell t square this gives the velocity. So, the total kinetic energy of this volume delta x delta A inside the steel rod so, the kinetic energy per unit volume let me denote rhis by T bar kinetic energy per unit volume this is half.

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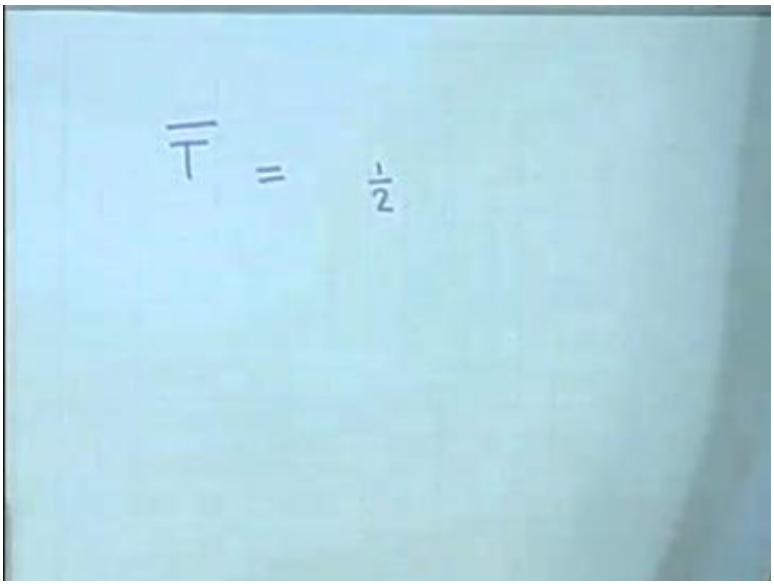

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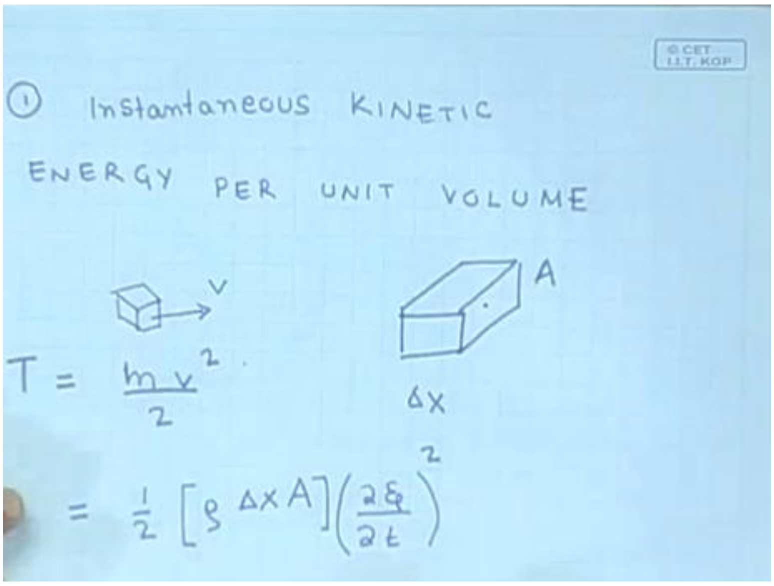

So, I have to divide out the volume form here. If I divide the volume out then this delta x into A is gone.

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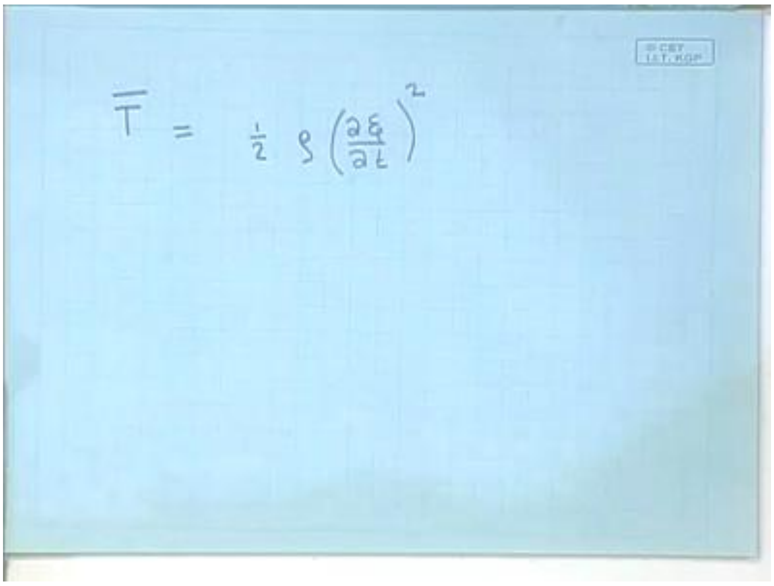

And I have that fact that this is half rho dell xi dell t squared. This is the kinetic energy per instantaneous kinetic energy per unit volume inside this.

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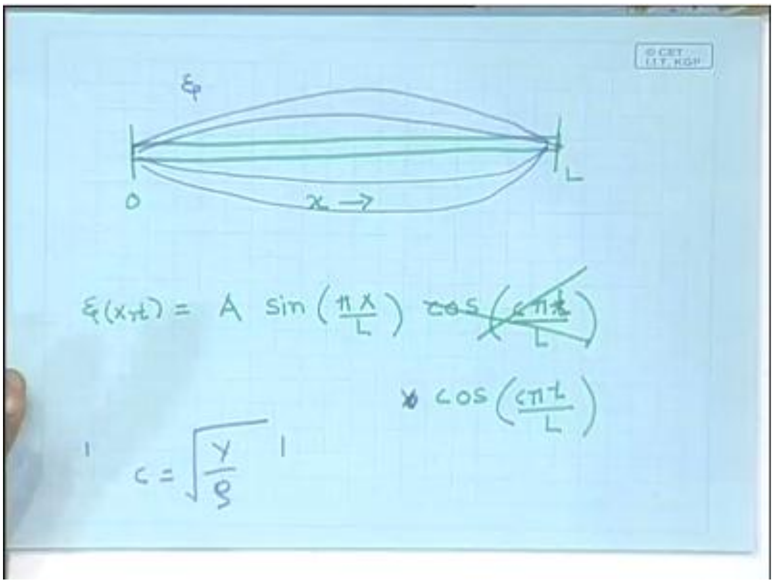

Steel rod and this is the displacement pattern has a function of time inside this rod. So, we have to take the time derivative of this if you take the time derivative of this. You get an extra you get a factor overall you pick out a factor of c pi by L if you differentiate cos with time you get a factor of c pi L outside and then this becomes minus sin. So, when I square it I will have sin square over here.

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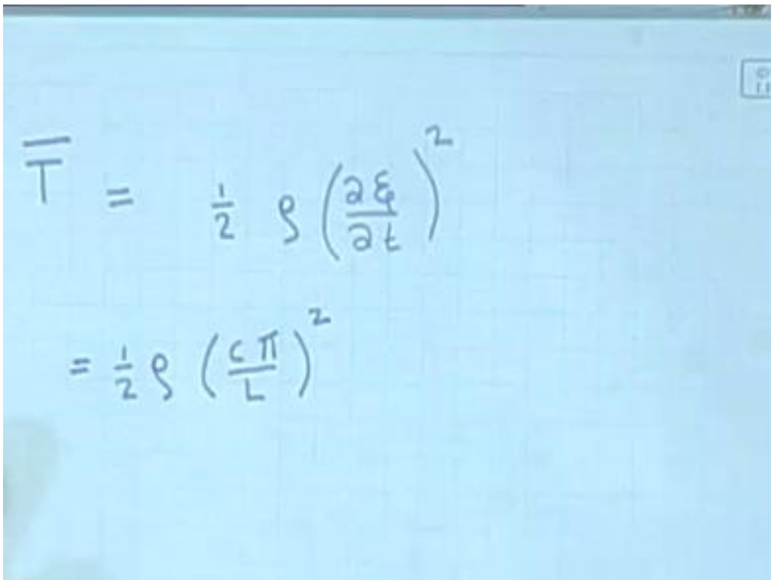

So what I have is that this is equal to half rho and the derivative gives me a factor of c pi by L squared and then I have A square sin square of this term into the sin square of this term.

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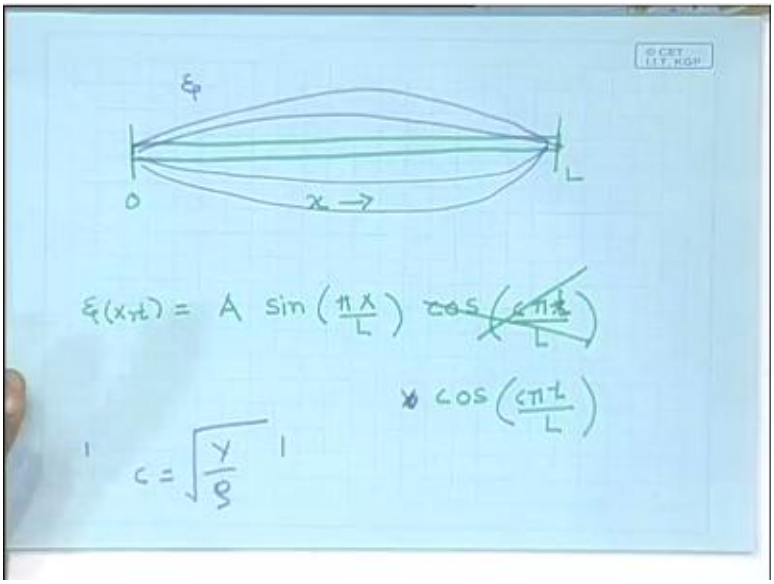

Because I have differentiated it this cos becomes sin.

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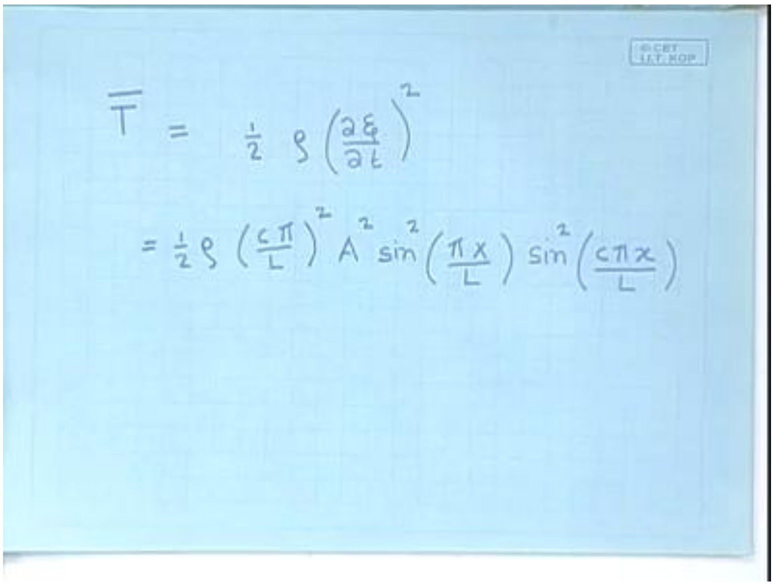

So, what we have is A square sin square pi x by L into sine square c pi x by L this gives the kinetic energy per unit volume. Let us know at any instant of time the instantaneous kinetic energy per unit volume let me now calculate the instantaneous potential energy per unit volume Let so, we again to calculate the potential energy per unit volume. The kinetic energy calculation is quite simple. Now, the question is how to we calculate the potential energy per unit volume? The potential energy calculation is a little not so, simple, but it is not very difficult either.

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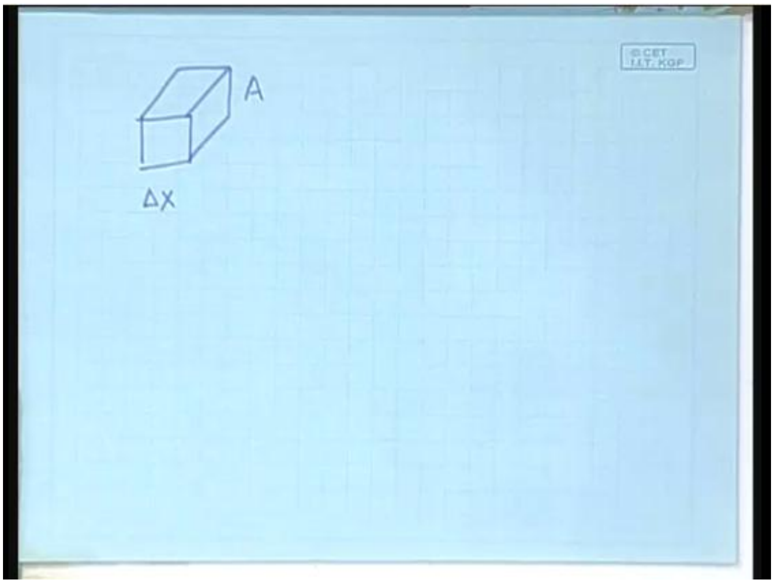

So, let us take a small element of the steel rod of length delta x and cross sectional area A I am not I can put that here also and cross sectional area A the same element. And we have seen in earlier lectures that this can be thought of as a spring. So, any element any mass element inside this steel rod can be thought of as a spring.

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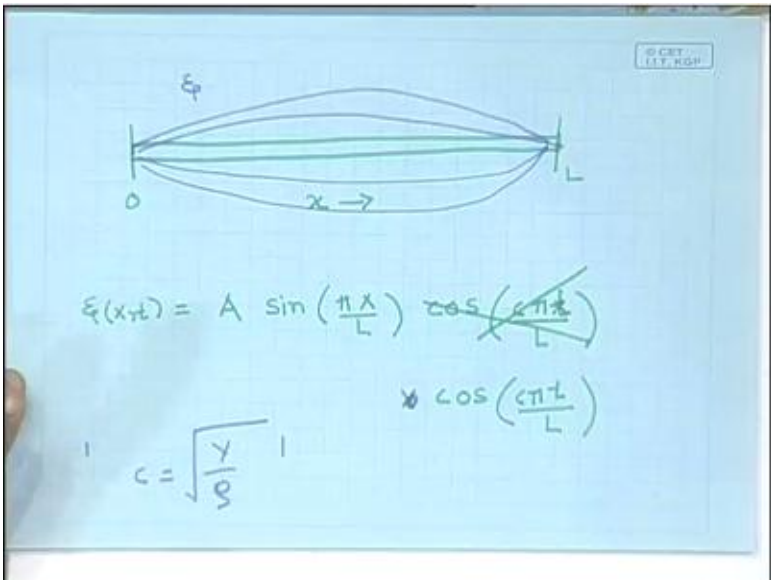

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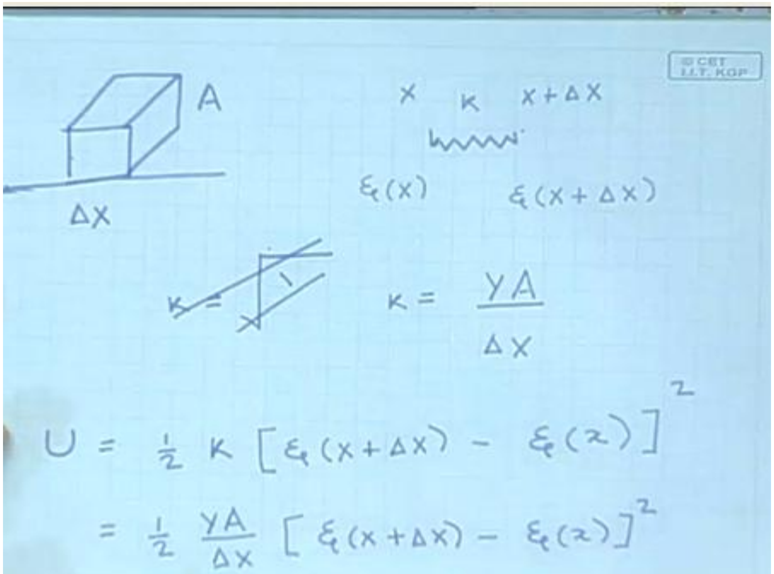

So, I have told you that we can think of this as a spring and the. So, we can think of this as a spring with a spring constant k which is Y sorry there is no square root. So, let me cut this out k is equal to the Young’s modulus into the area divided by the length of this element which is delta x. So, we are looking at deformations of this mass element in along the x axis only you should bear thus bear this in mind. So, we are looking at deformation of this mass element along the x axis only along this direction. And for this purpose you can think of this mass element as a spring constant k where k is Y into the cross sectional area divided by the length delta x we have discuss this quite a few lectures ago. Now, the question is what is the potential energy of this mass element? If it undergoes a deformation we know that the potential energy is half.

So, the potential energy I am going to denote by U, U is half into the spring constant k into the deformation of the spring into the change in the length of the spring. Now, this spring actually corresponds to this mass element this is the so, this spring this end of the spring is at x this end of the spring is at x plus delta x. The displacement of this end is xi x I am not showing the t explicitly, but the t is there and the displacement of this end is xi x plus delta x. Now, the deformation the change in the length of the spring is the difference between these 2 displacements right if I move both ends of the spring by the exactly the same amount there will be no deformation of the spring there will be no change in the length of the spring.

So, the change in the length of the spring is the difference between the displacement of this end and the displacement of this end it is a difference these 2. So, the potential energy of the spring is half k into the change in the length of the spring which we are all familiar with this concept. So, and the change in the length of the spring is xi delta x plus delta x minus xi x the so, this is the potential energy of the spring. Now, we know that k the spring constant k can be written as Y the Young modulus into the cross sectional area divided by delta x. So, this becomes half YA divided by delta x into xi x plus delta x minus xi x to square of this. Now, if I divide this by delta x squared this let we do it here.

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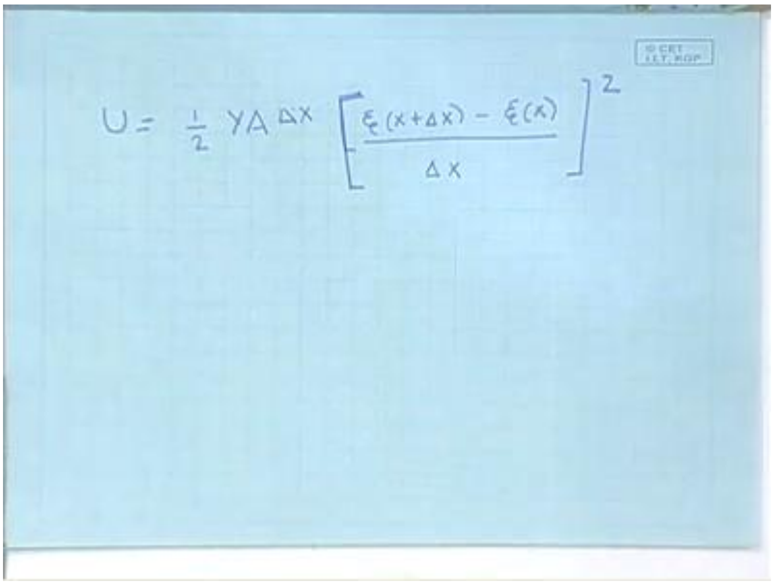

We can write this as U is equal to half YA into delta x into delta x over here xi x plus delta x minus xi x square. So, we have been calculating the potential energy.

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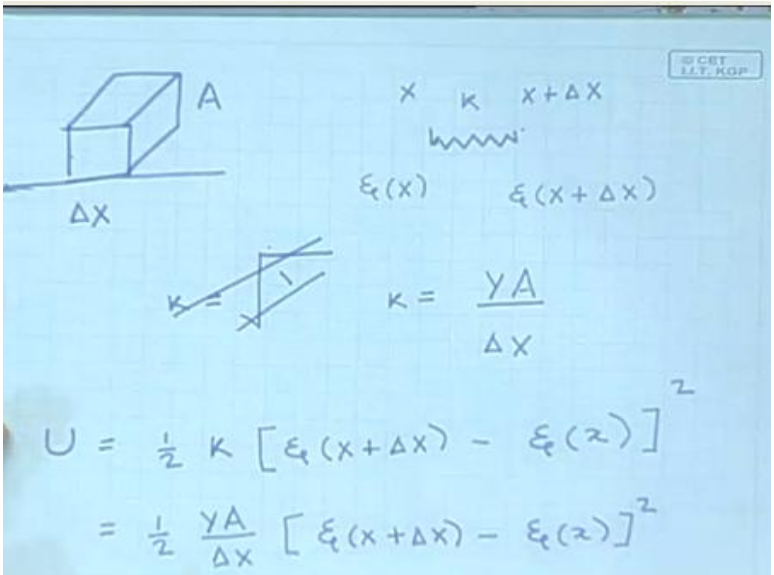

The potential energy is half times the spring constant into the difference of the displacement of the 2 ends of the spring. And we so, which is given over here and then we took we divide this whole. We write this in the form of this difference divided by delta x whole square and then we have put in a extra delta x over here.

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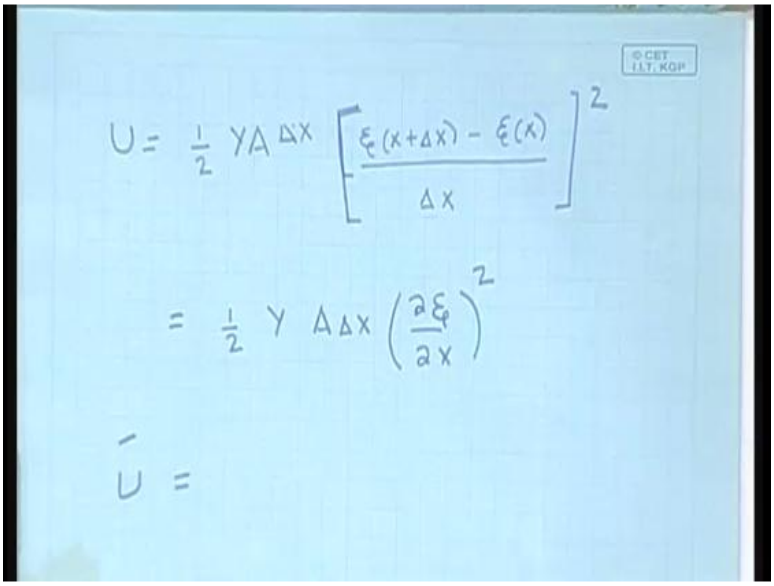

And this is the derivative of xi with respect to x. So, the potential energy is half into YA delta into dell xi dell x the square of this. So, the potential energy per unit volume I divide out the volume the volume is A into delta x the volume of this mass element is A into delta x.

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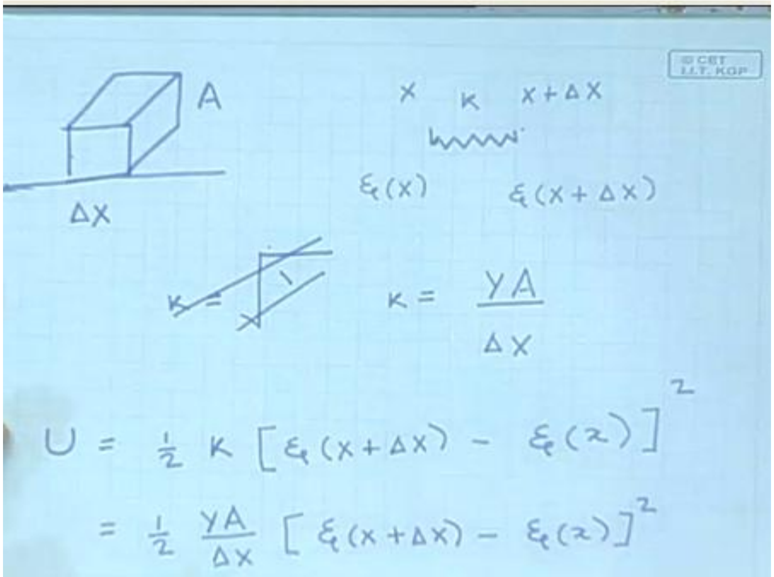

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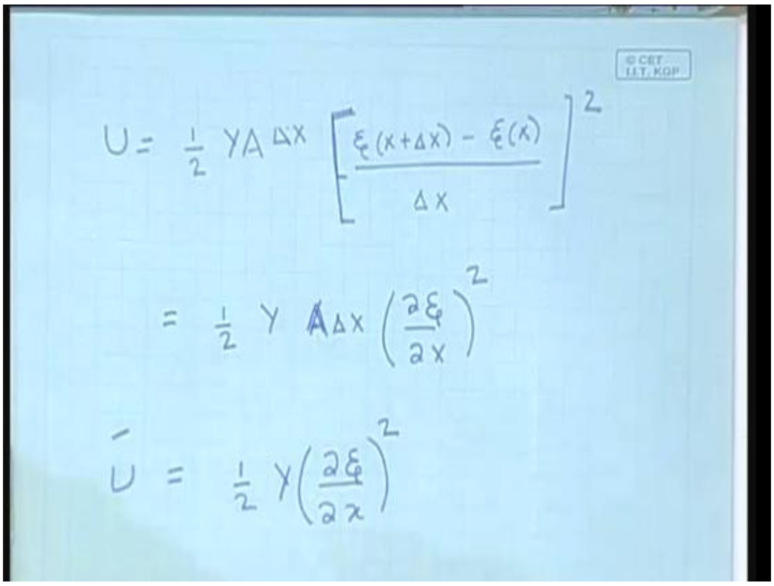

So, I divided out the volume and what we get is the potential energy per unit volume is half Y dell xi dell x squared. So, we have calculated the potential the kinetic energy per the kinetic energy per unit volume which is half rho dell xi dell t whole square.

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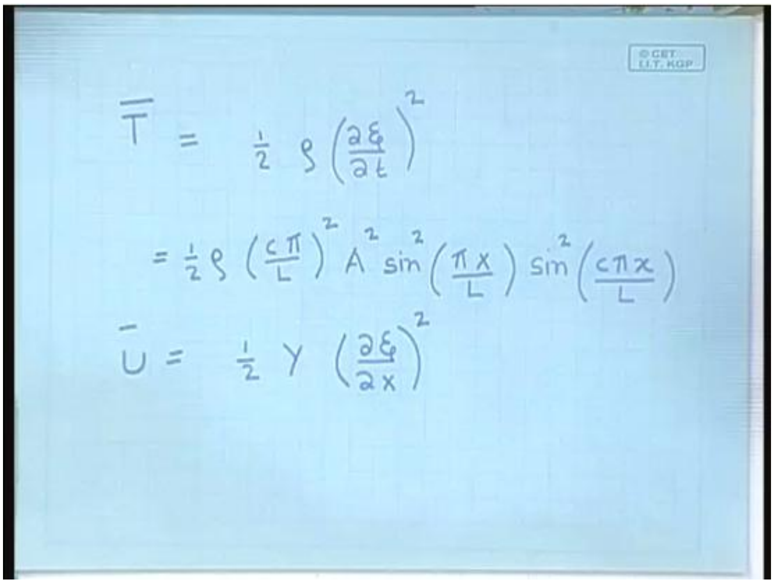

We have also calculated the potential energy per unit volume which is half Y dell xi dell x square. And for the particular displacement pattern for the particular deformation the standing wave that we are dealing with.

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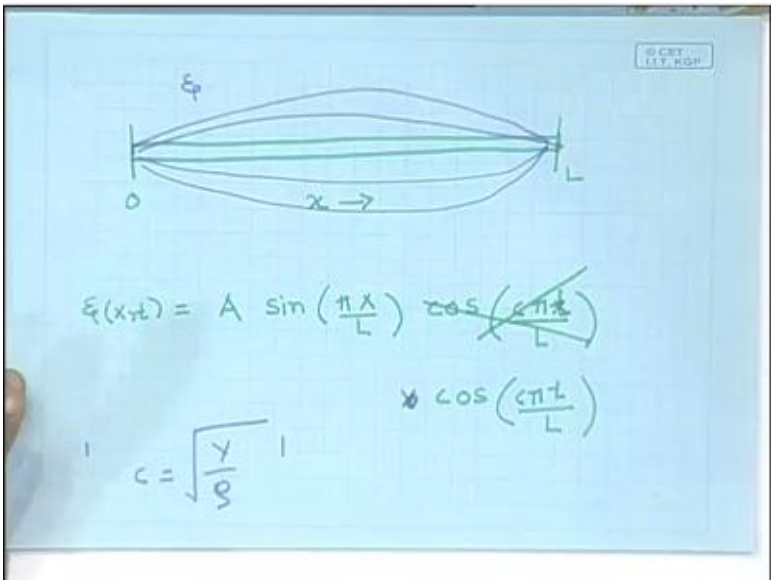

This xi is given over here. So, when I differentiate this with respect to x I will pick up an extra factor of pi L outside this will become cosine and I have to square it. So, what we have is.

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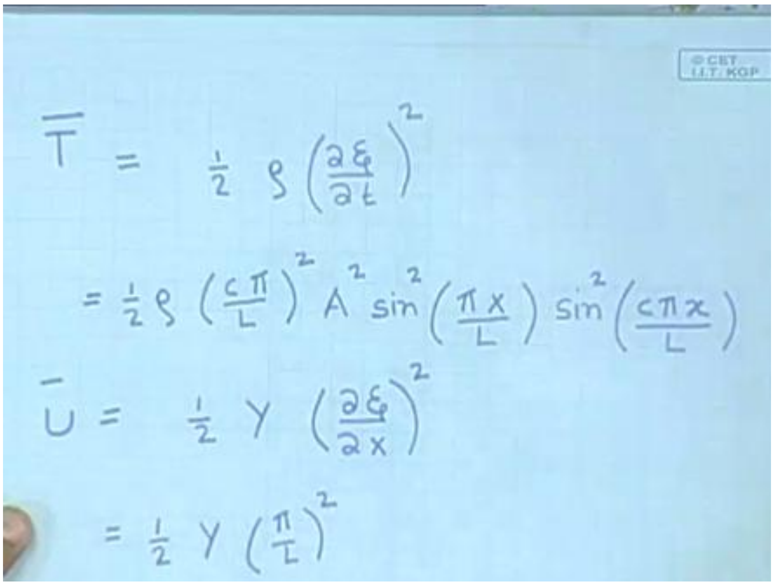

This will be equal to half Y into pi by L square this pi by L square comes when.

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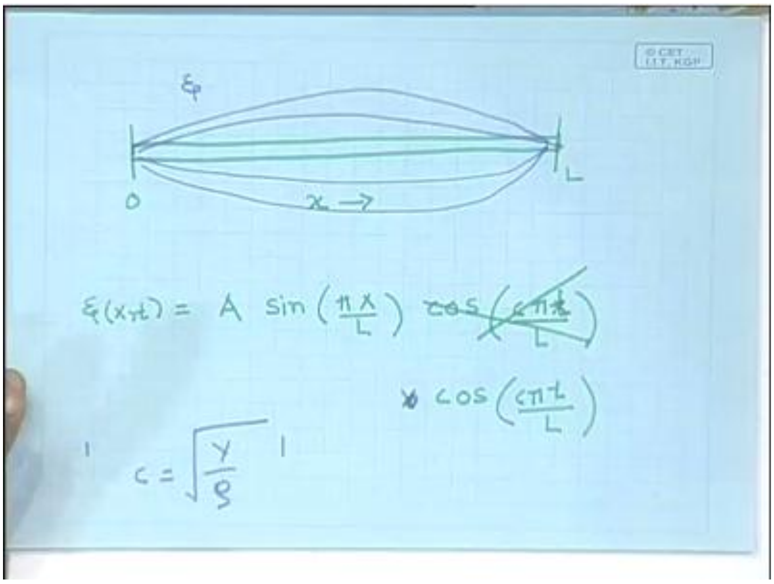

Differentiate this sin and then square it. So, I will have a factor of pi by L square.

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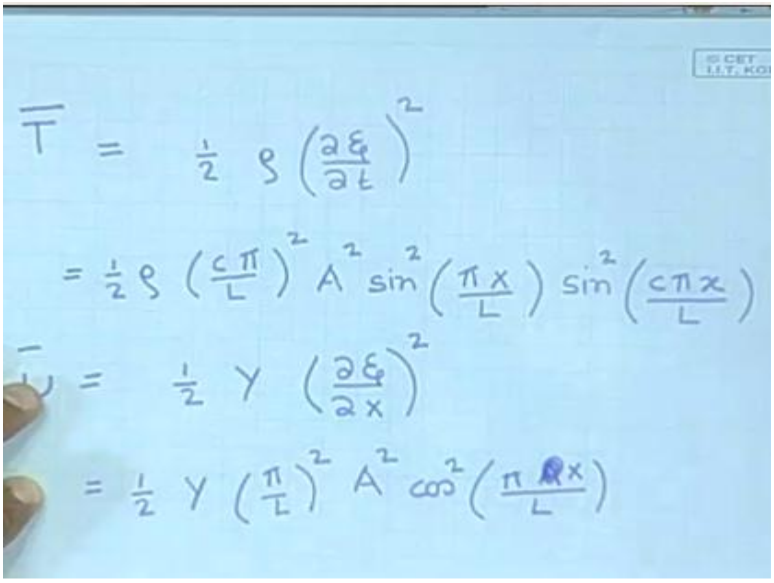

And then I have A square cos square pi A pi x sorry pi x by L into cos square into cos square c pi t by L.

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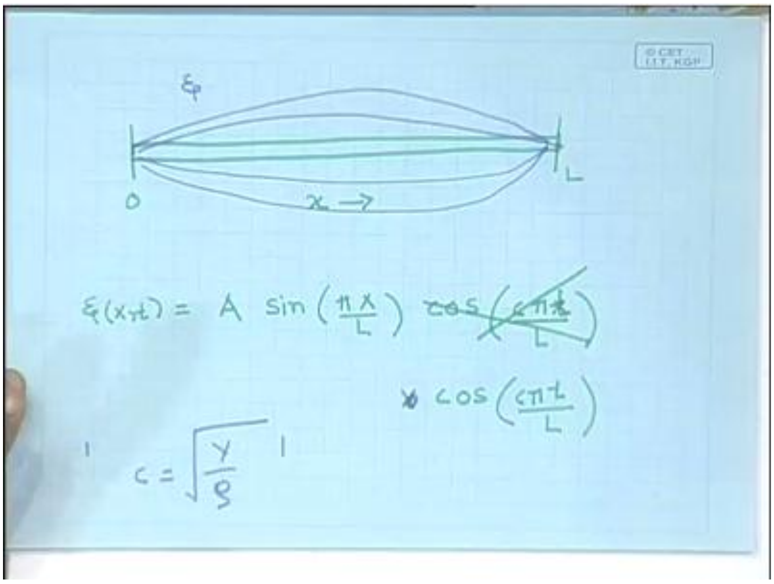

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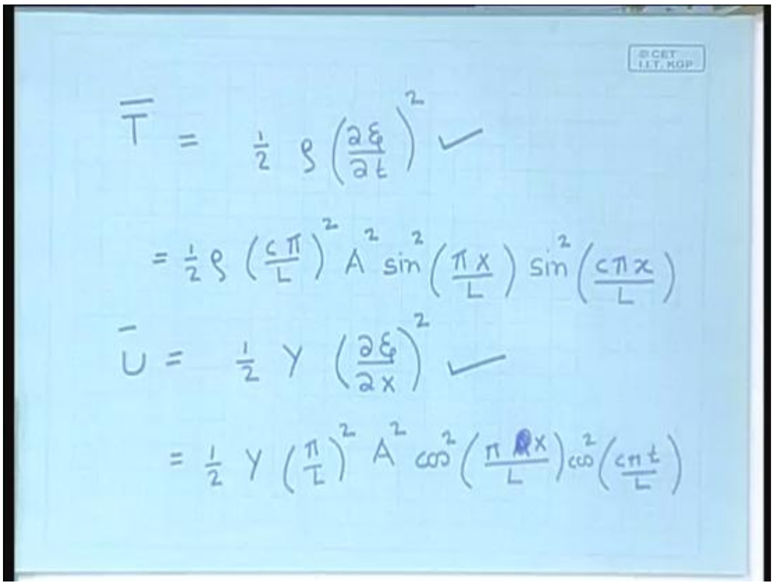

So, into cos square c pi t divided by L so, we have what we have done till now, is we have worked out the instantaneous kinetic energy and potential energy inside this steel rod. And these expressions are valid whether you have a standing wave or you have a traveling wave or if you have some static deformation of a any kind of deformation of a of an elastic rod. If you have the deformation in one direction then these expressions are valid. They are also valid if you have deformation in all 3 directions, but we are not really interested in that. So, these expressions are very general expressions this and this they are valid in a large variety of situations even if I have a static deformation for example.

Or if I have a travelling wave and for the standing wave which we are considering the instantaneous kinetic energy per unit volume is given over here. The instantaneous potential energy per unit volume is given over here. Now, let us spend a few minutes in analyzing the significance of what we have just calculated the first thing that we calculate is as follows. Let us see how the kind kinetic and potential energy evolve with time. So, let us plot at t equal to 0 let us plot the evolution of the kinetic and potential energy. So, we will consider the kinetic energy at t the let us plot these as a function of x at t equal to 0.

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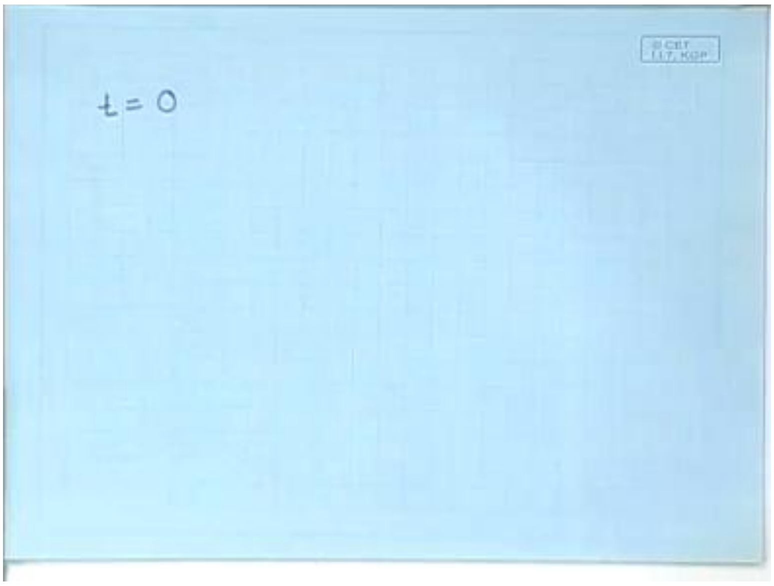

So, at t equal to 0.

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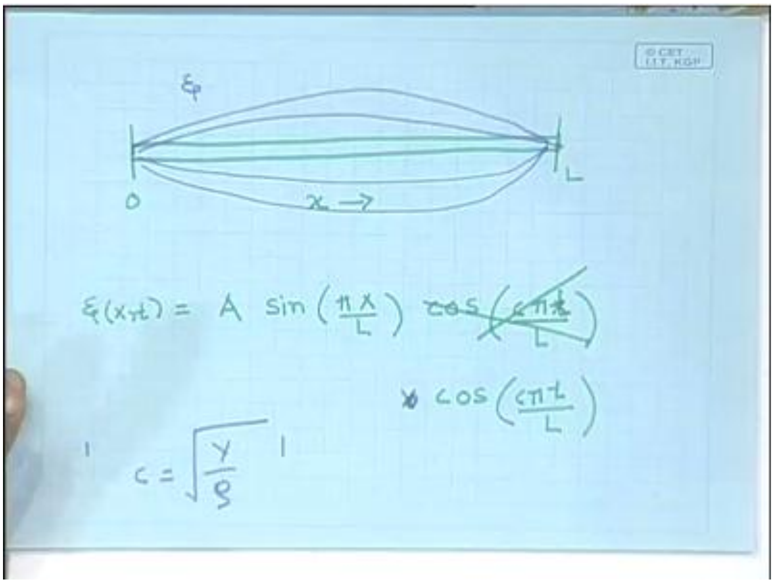

At t equal to 0 what does the displacement, the disturbance look like? Let us first plot that at t equal to 0 the disturbance looks like this where is the at any instant of time the disturbance is given by this. So, we set t equal to 0 at t equal to 0 the cosine term is 1. So, the displacement is A sin pi x by L let us set A also equal to 1.

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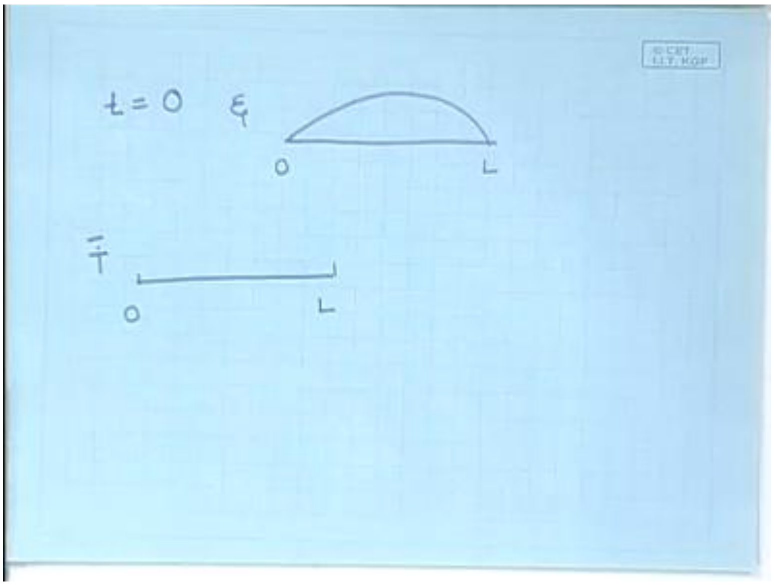

And the displacement xi as a function of x at t equal to 0 looks like this we have already plotted this and discussed it, but it let me do it again it looks like this. So, the deformation is maximum at the center it is always 0 at the 2 ends and at t equal to 0. The deformation is the maximum and it looks like this the deformation. So, it is maximum at the center and it falls off at the 2 ends. Now, let us plot the kinetic energy per unit volume as a function of x at t equal to 0.

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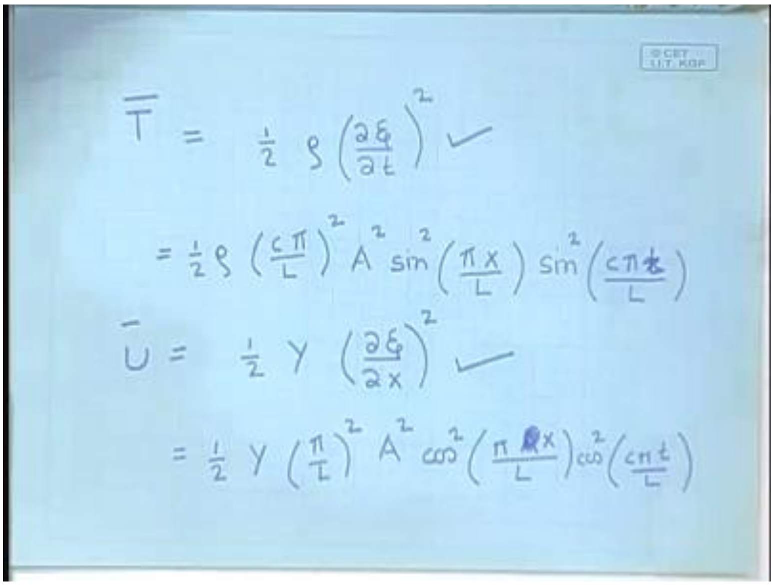

So, at t equal to 0 we have sin square c pi t by L sorry this should have been t not x the kinetic energy term has a t in it not an x it is a time derivative. So, the kinetic energy per unit volume this should have been t not x please note the change there was a when we differentiate this with respect to time.

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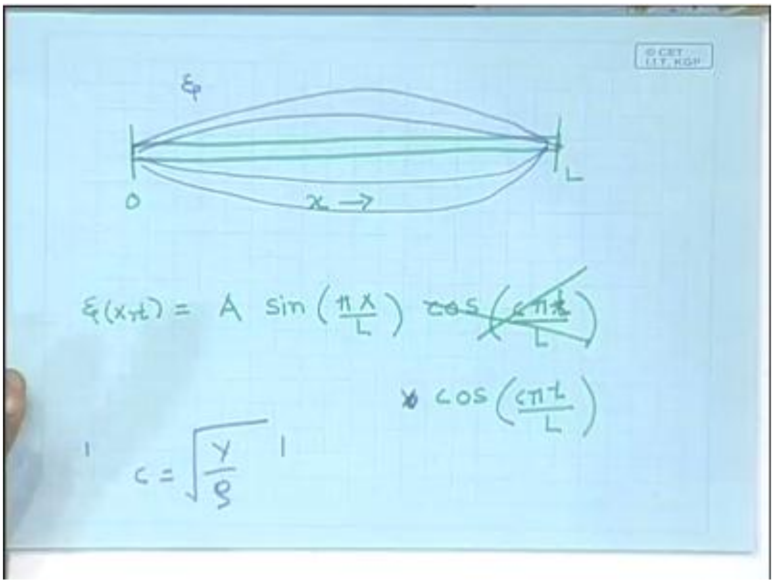

You get sin square c pi t by L.

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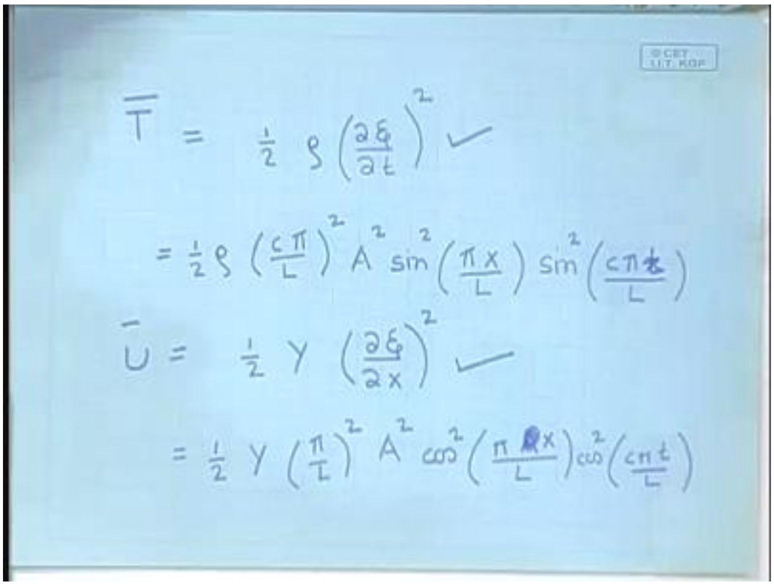

It should be this is the term right correct thing. So, at t equal to 0 this term is 0 and what you see is that the kinetic energy per unit volume is 0 throughout.

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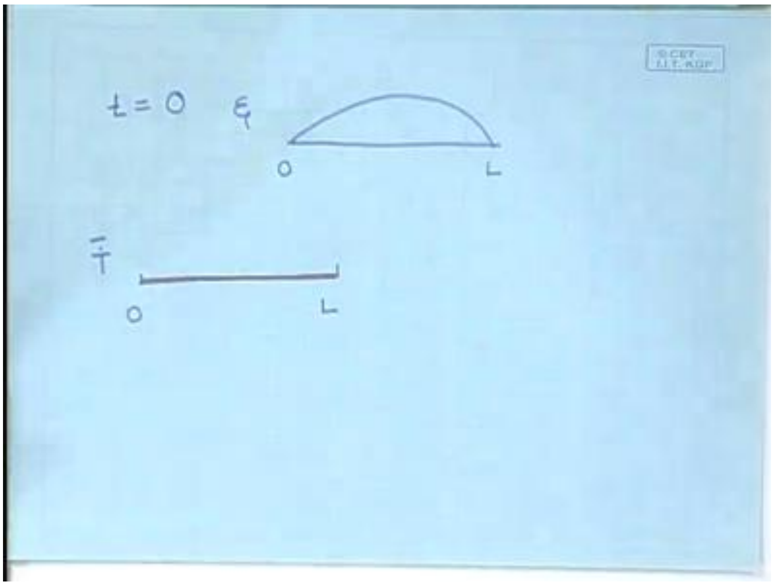

So, the kinetic energy per unit volume at t equal to 0 looks like this it is 0 everywhere. So, it is just here now, let us draw the potential energy per unit volume.

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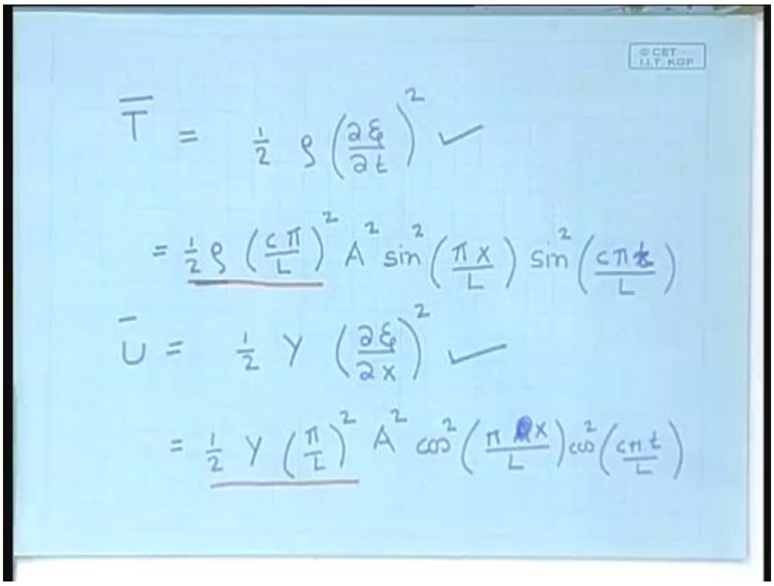

So, the potential energy this term is going to be 1 at t equal to 0 and the potential energy per unit volume is given by these constant factors. Now, there is a point which I should make here that the constant factors here and the constant factors here are exactly the same. This is a point which I should have made earlier that we make it now, the constant factors given over here which appear in front of a kinetic energy per unit volume. And the constant factors which appear in front of potential energy per unit volume for this standing wave are exactly the same. That is you will realize that they are exactly the same which you substitute c with the Square root of the Young's modulus divided by the density.

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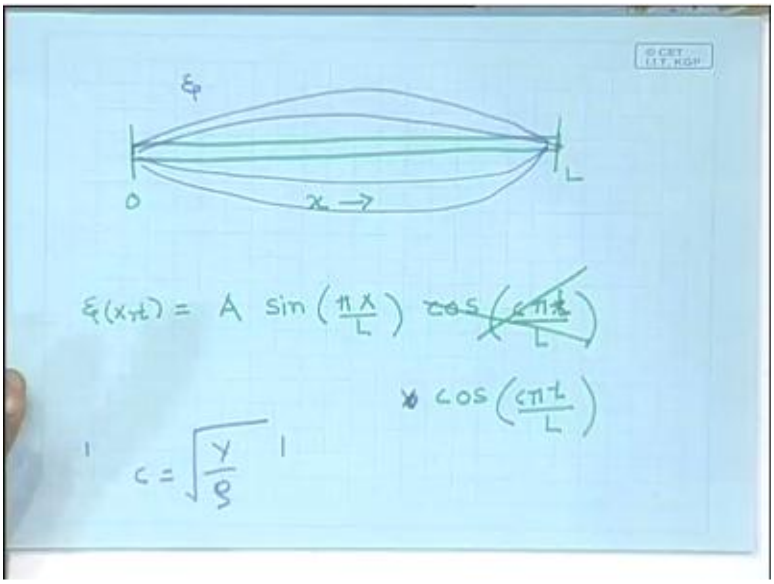

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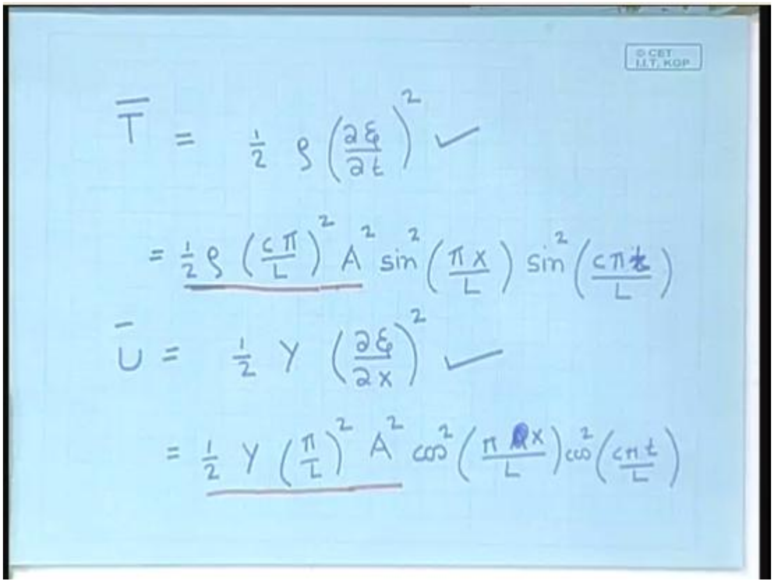

If you substitute this the rho here will cancel out you will be what you will have is Y into pi by L squared so; this term and this term are exactly the same. So, we are not going to bother about these two terms, these two terms are also exactly the same. So, we are not going to bother about this either what we are going to be interested in is in the behavior of this and this, because these overall factors are exactly the same for the kinetic and the potential energy per unit volume. So, let me now, draw the potential energy per unit volume at t equal to 0 this term is 1. So, what we have to draw is cos square pi x by L and this has the maximum value at x equal to 0. It also has a maximum value at x equal to 1 it has a minimum in the half way exactly half way between that L by 2.

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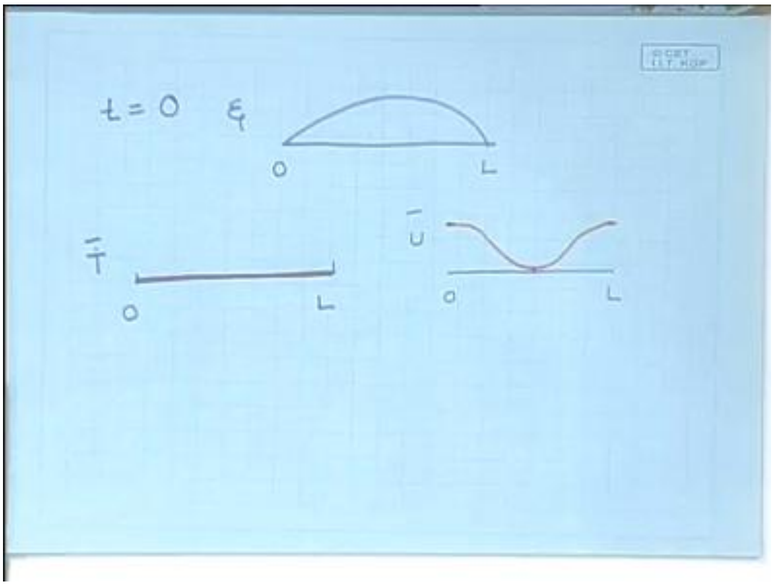

So, the potential energy per unit volume at t equal to 0. Let me we are going to plot that so, this is the x axis it goes from 0 to L. This is the potential energy per unit volume and the potential energy per unit volume is maximum at the 2 ends it is minimum at the center. And the function is cos square pi x by L so, it will look like this. So, this is the potential energy per unit volume at t equal to 0. Now, let us see what happens the time evolves. So, let we take another instant of time the instant of time which we are going to consider is the instant where the kinetic energy becomes maximum and the potential energy becomes minimum.

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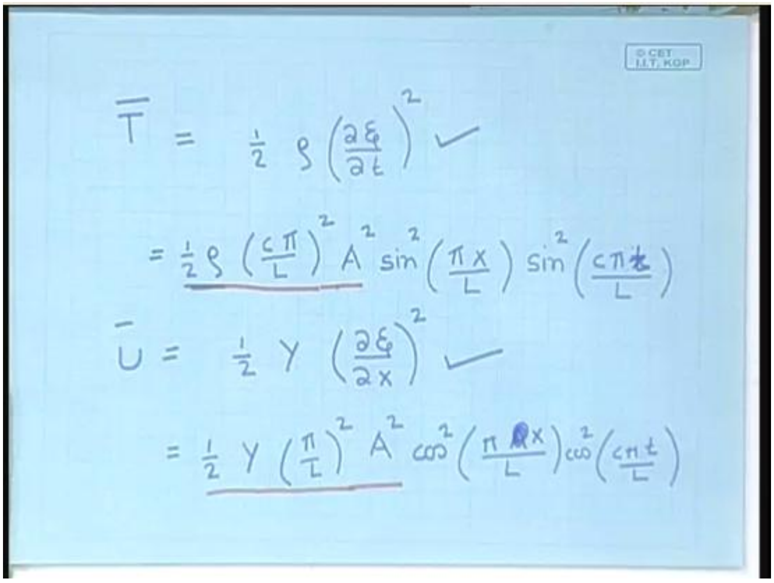

So, we should be look at a time instant where the factor the argument of this sin square term is pi by 2. And the argument over here is pi by 2 when t is equal to L by 2 c. So, we are looking at the time instant where t is equal to L divided by 2 c.

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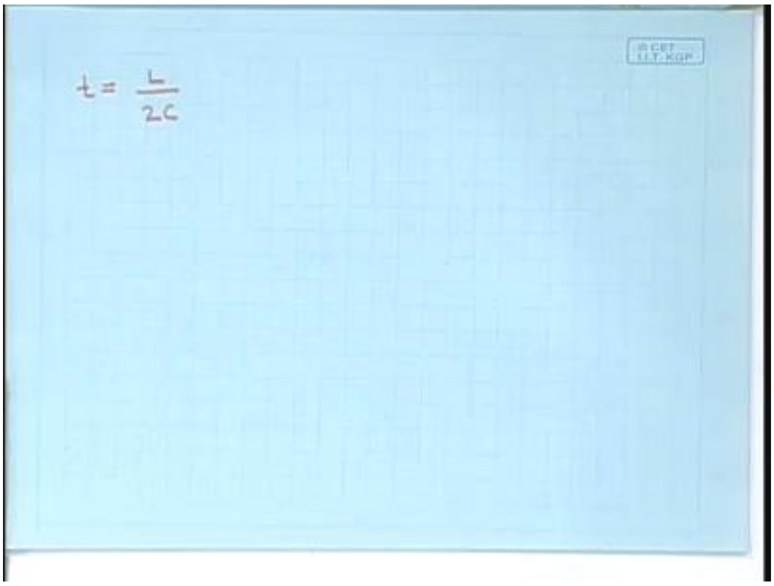

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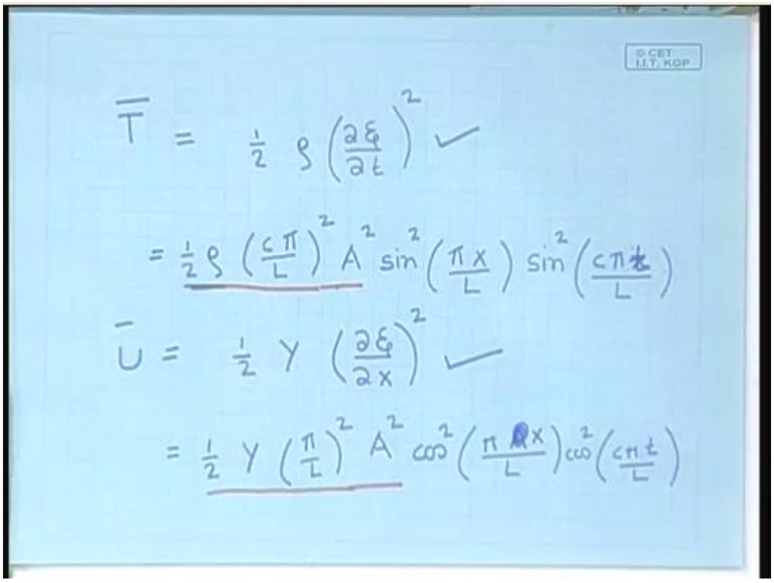

If you put L by 2 c over here this becomes pi by 2 sin square of pi by 2 is 1 the potential energy also has argument pi by 2 cos square of pi by 2 is 0. So, the potential energy per unit volume now becomes 0.

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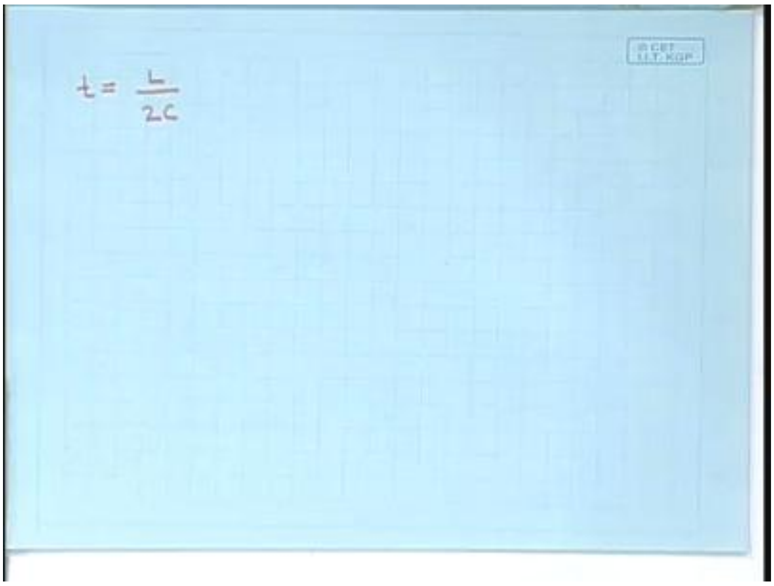

And first let me draw the vibration the displacement pattern in the string t equal to L by 2 c t equal to L by 2 c this becomes pi by 2.

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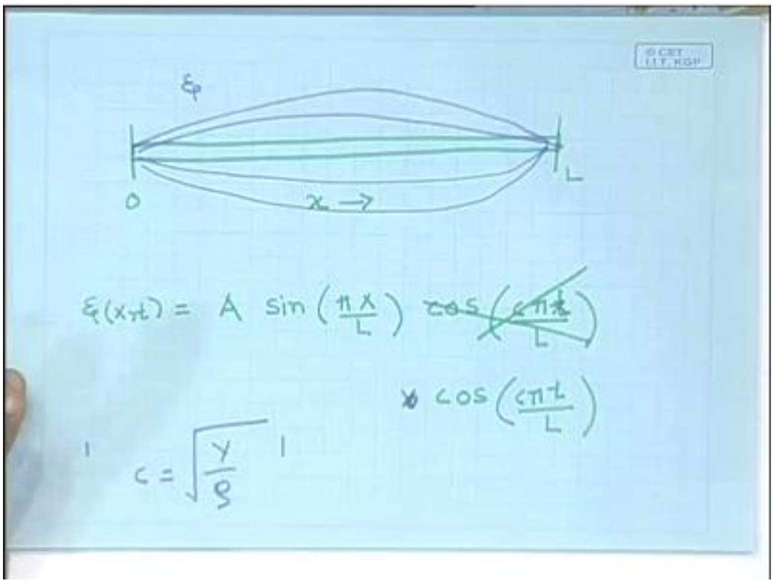

So, the displacement is exactly 0 everywhere.

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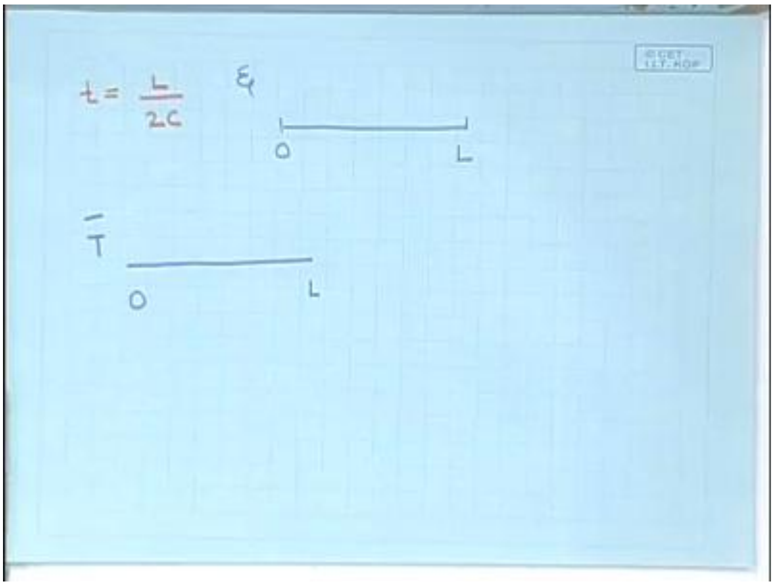

So, the displacement xi is 0 everywhere and the kinetic energy per unit volume at that instant.

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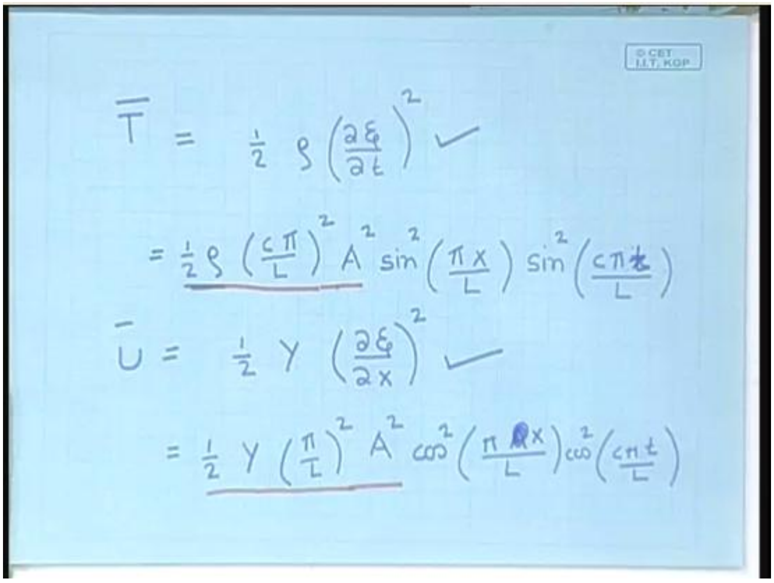

This term is 1. So, what we have now, is sin square pi x by L sin square pi x by L has a maxima when x is equal to L by 2 the argument becomes pi by 2 is has minima when x is equal to 0 or x is equal to L.

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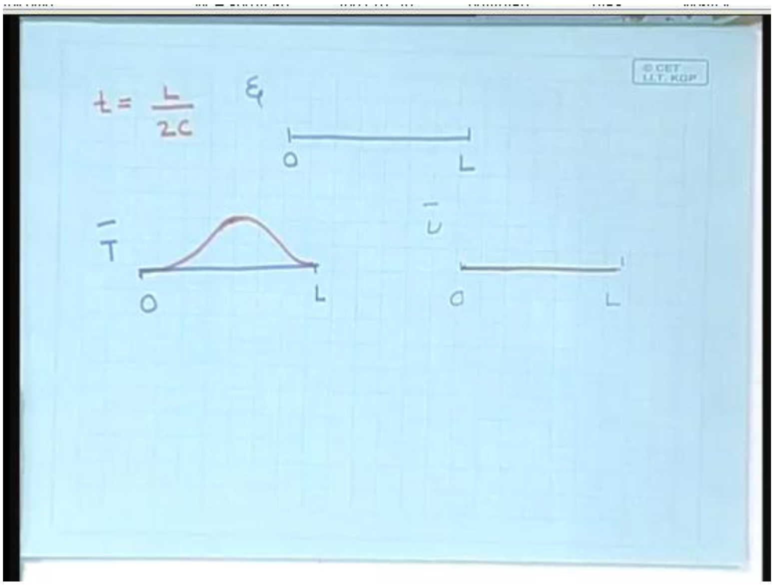

So, this curve is going to look like this. It is going to start from 0 here and 0 here and it is going to have a maximum in the middle. So, it is going have a maximum over here. So, it is going to look like this and the potential energy per unit volume is now 0 everywhere it is exactly 0 everywhere. So, what is this tell us? Let us just now, we are in a situation to analyze what it tells us. So, what it tells us is as follows? Let we can summarize what we learnt from this whole exercise.

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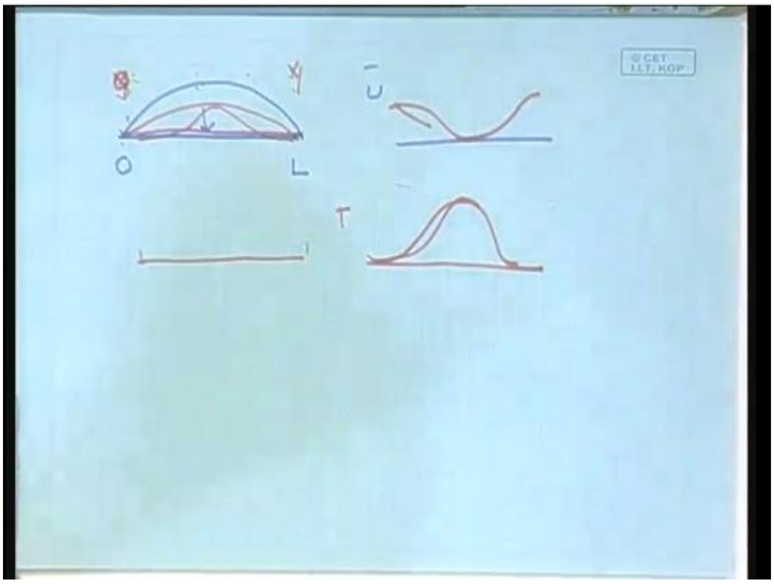

What we learnt from this whole exercise is as follows that the deformation in this steel rod is going to be initially like this is the deformation pattern inside the steel rod. The longitudinal deformation pattern is initially going to be like this. And then the amplitude of this is going to go down and then it is going to become like this. Now, the when the deformation is maximum the kinetic energy is 0 we expect that, because when it reaches the maximum the velocity becomes 0 when the displacement is maximum the velocity become 0. So, the kinetic energy per unit volume becomes 0 so, in this situation at this position when the deformation is at its maximum. The kinetic energy is 0 the potential energy is maximum and the potential energy is concentrated at the 2 nodes. So, it looks like this the potential energy is concentrated at the 2 nodes.

\So, the potential energy is concentrated at the 2 around the 2 nodes and it falls off and it will 0 at the center. Now, as time evolves the deformation goes down. So, at a later time the deformation will become will have gone down the potential energy also would have gone down the kinetic energy would have picked up. So, the total energy has now flown towards from has been converted from potential to kinetic energy and sum of the energy flows in from the 2 nodes and it is now distributed along the entire rod. And then finally, the deformation will become 0 everywhere at this instant all the energy has gone into the form of kinetic energy and all the energy is now concentrated at the center it will look like this.

And then again energy will flow out from the center towards the 2 nodes at the ends and it will go into the potential form. So, what you say is that the as you when you have the standing wave the energy flows in out in out like this. And it is get converted from the potential energy where the energy is mainly concentrated at the nodes to the kinetic energy where the energy is mainly in the antinodes and then it flows back nodes and to the antinodes. So, this tells us how the energy flows and in how the energy is distributed and how it moves around in the in a in a rod like this when we have a standing wave you could also calculate the time average kinetic and potential energy.

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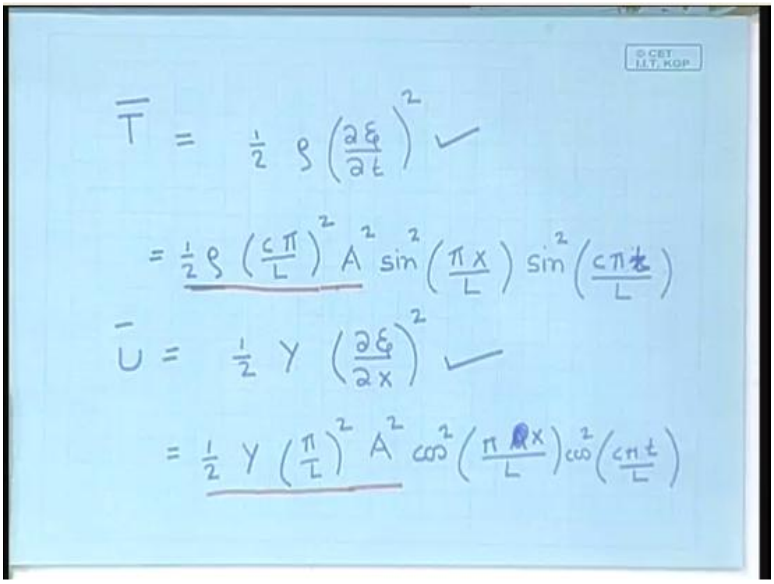

So, the time average kinetic and potential energy are very easy to calculate the time average of the kinetic energy per unit volume just average over time. So, will get a factor of half over here if you take a time average of this you will get a factor of half over here. So, the time average kinetic energy is another factor of half so, it is essentially given by a sin square pattern. This is given by a cos square pattern the time average potential energy so, the time average kinetic energy.

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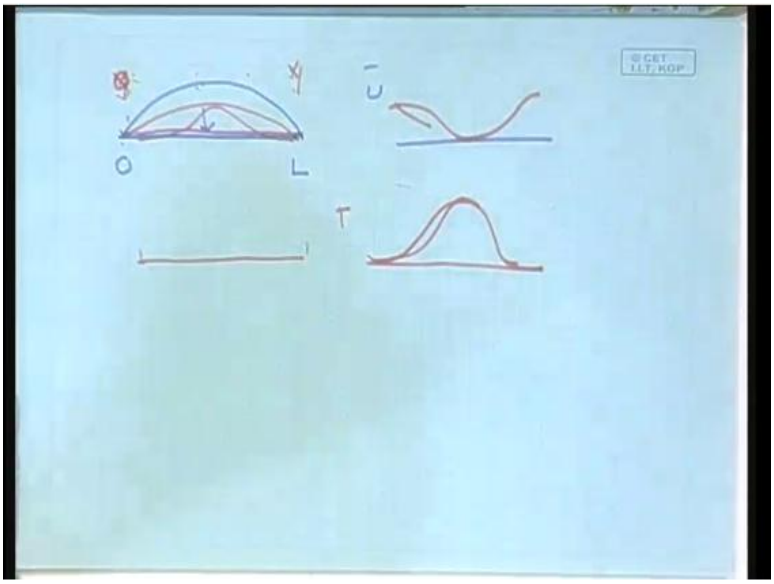

Also looks like this, but it is amplitude is lower the potential energy also looks like this it is amplitude is lower.

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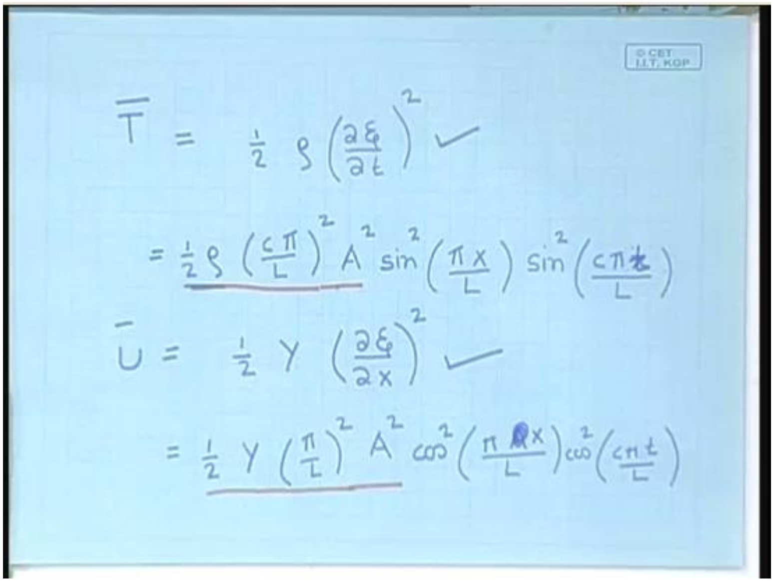

And the time average of these 2 things is a constant. So, that brings us to our to the end our discussion of standing waves. And the next topic that we are going take up is polarization discuss is polarization.

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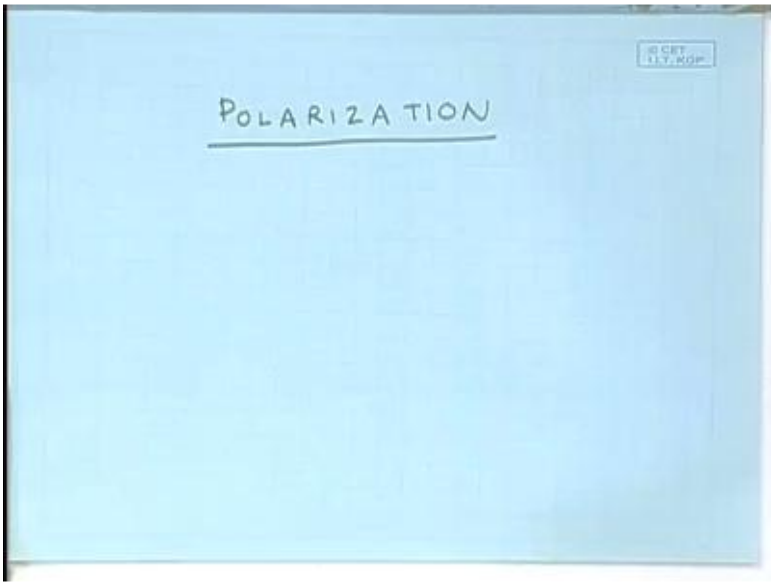

I have already told you quite a few lectures ago that light is an electromagnetic wave and if I have a wave travelling along the z axis. The disturbance in the electric and magnetic field these are transverse waves it is a transverse wave. So, the disturbance in the electric and magnetic field are going to be perpendicular to the direction of the wave. So, if the light is travelling along the z axis the electric field can be anywhere in the x y plane and same with the magnetic field. And they will be both in phase and they will be mutually perpendicular to each other. I have a also told you that the electric field vector could be oscillating up and down in a straight line such light is set to be linearly polarized. The electric field vector could be going around in a circle such light is set to be the circularly polarized or it could be going around in an ellipse.

And such light is set to be elliptically polarized elliptically polarized light is the most general polarization state of light. We have discussed all of this quite some time ago I have also told you that the natural radiation that we receive the natural light is unpolarized. What we mean by unpolarized light is that the electric field direction changes randomly in the plain perpendicular to the direction in which the way is propagating. So, for natural light if it is propagating along the z direction the electric field vector just goes around randomly in the x y plane. It has no fixed pattern it has no predictable pattern it jumps around randomly in the x y plane. So, this is natural light which is largely unpolarized. Now, there are certain optical devices called polarizer’s.

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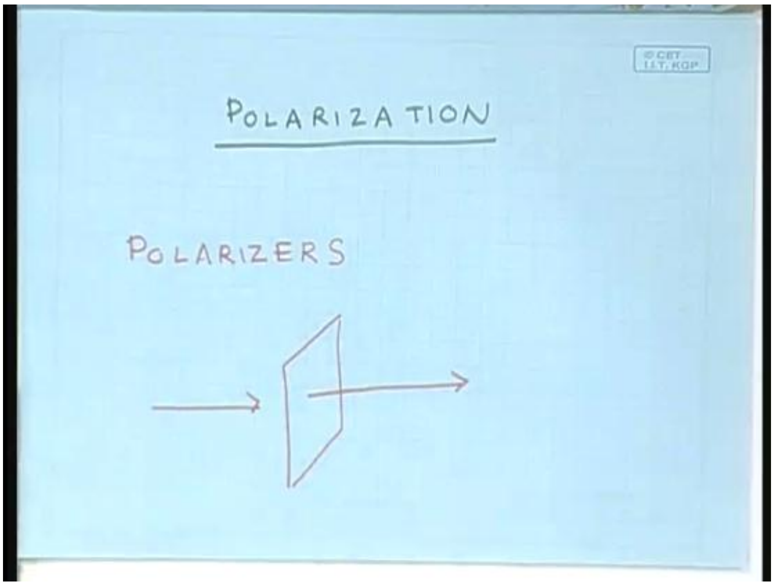

So, there are optical devices called polarizer's. So, let we draw a polarizer over here schematically this is a polarizer and typically we would send in radiation perpendicular to the plane of the polarizer and the radiation would come out here. And the property of the polarizer is that if you send in natural light the radiation that comes out is going to have some kind of polarization. And it could be fully polarized or it could be partially polarized it depends on the polarizer. And you could have linear polarizer's which produce linearly polarized light that comes out you could have circularly circular polarizer which produce a circular polarized light that comes out or it could have elliptical polarizer's. So, let us discuss linear polarizer's.

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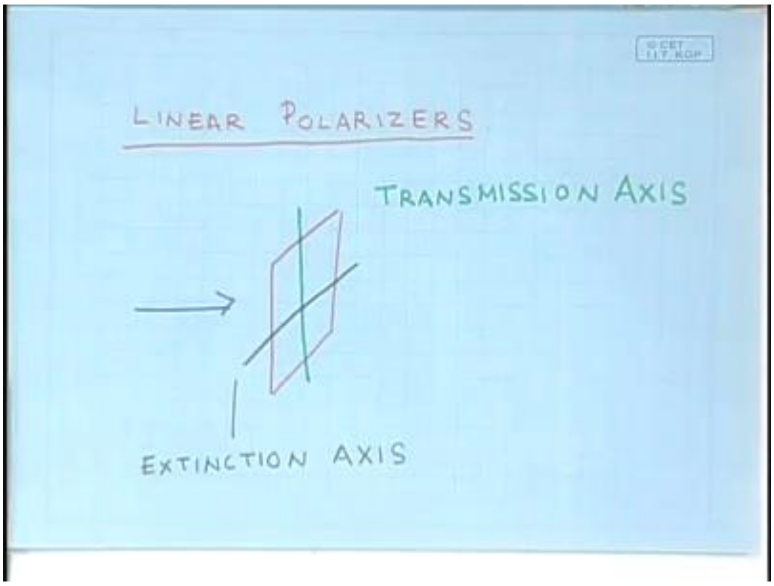

So, let me tell you again what is a linear polarizer a linear polarizer is a device where if you send in this is the linear polarizer i am sending in light. The light that comes out is going I am sending a natural light. The light that comes out is going to be linearly polarized. So, if this is a linear polarizer and we are sending in radiation like this it has 2 axes which are mutually perpendicular so, there is one axis over here called the transmission axis.

And the linear polarizer has the property that it will allow the electric field to pass through only the electric filed comp1nt parallel to the transmission axis to pass through. There is another axis called the extinction axis and the electric field vector if it is the compound of electric field vector parallel to the extinction axis is not allowed to pass through. So, let us first consider the situation where we have natural light incident on a linear polarizer.

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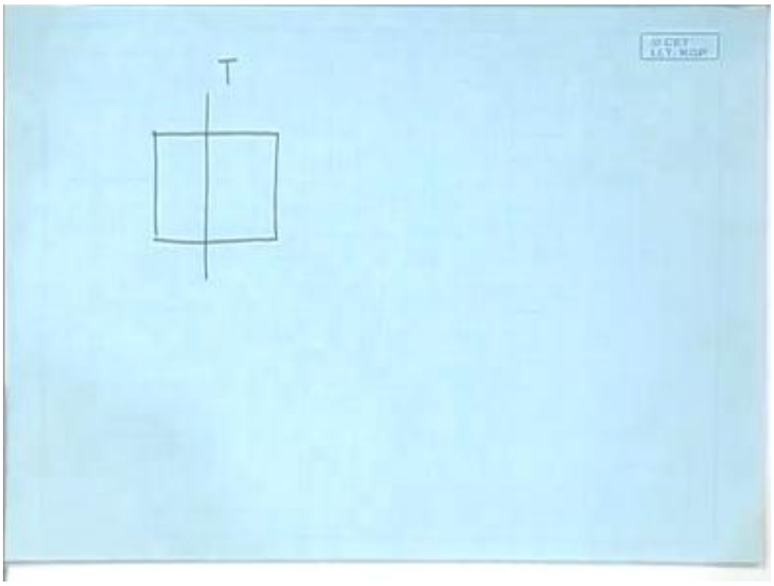

So, natural light if this is the polarizer and this is the transmission axis natural light the intensity of the natural light which is incident on this.

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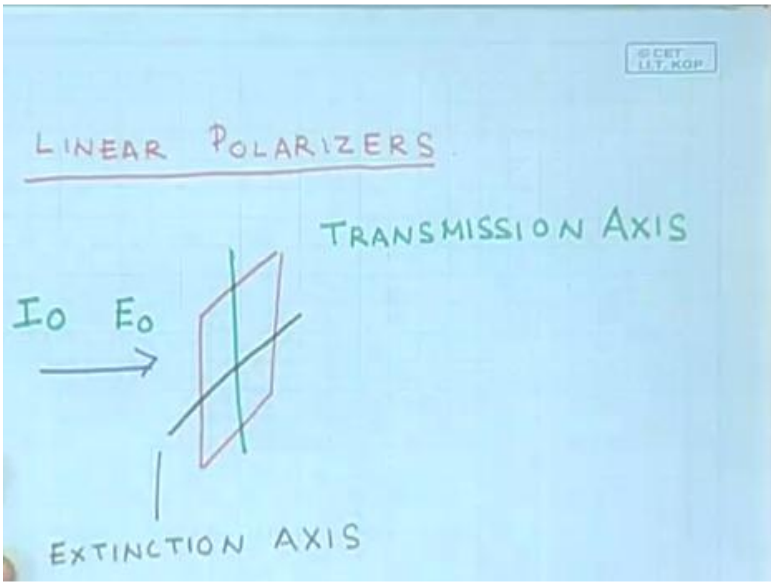

Let us say is I naught we know that the intensity of the natural light is if it is I naught. Then the amplitude of the electric field should be E naught we can call this E naught. And I naught is proportional to E naught squared we will be using I naught exactly equal to I naught E naught squared.

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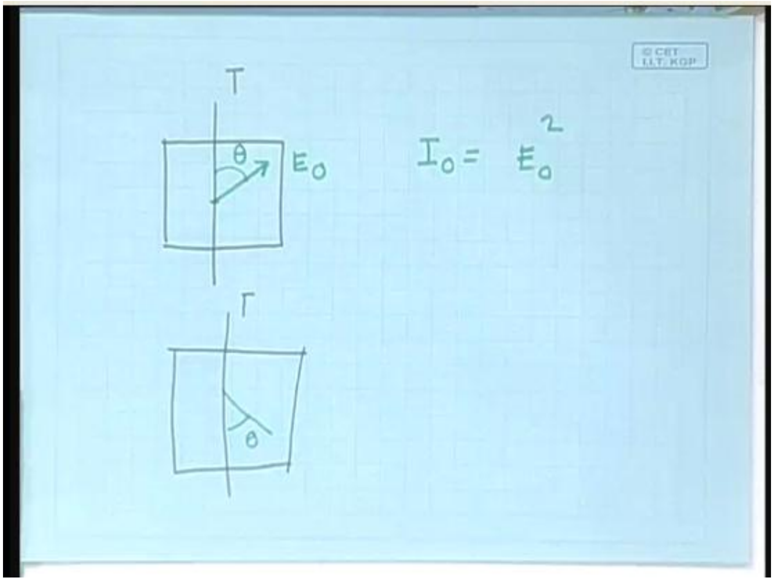

So, the electric field of the incident natural light is going to be of amplitude E naught such that I naught is E naught square there will be other constant. So, over here half epsilon naught etcetera. We are not interested in those constants over here if they are important we will write it down, but in most of our discursion they are not going to be important. So, we will not them explicitly we are going to assume that E I naught the intensity is proposal to the amplitude of the electric field squared.

So, we have natural light incident on the polarizer and the direction of the electric field with respect to the transmission axis is given by an angle theta. And this direction is going to vary randomly with time so, at one instant of time it is going to look like this. At the next instant of time the for the same polarizer the same natural light the direction of the electric field vector would have changed and it may it will be at some other angle. So, at some later instant of time it will be at a different angle theta over here and it is going to oscillate around it is going to randomly change around over time.

And the time scale over which this change occurs is quite large of the order of the coherence time of the light. So, it is going to change around quite rapidly and the quantity that we measure is the average intensity averaged over. So, we are assuming that for natural light the electric filed vector changes around quite rapidly on a scale time scale which is much faster than the time scale over which we measure the intensity. So, when you measure the intensity of the light what you are going to measure is the average over different orientations of the electric field.

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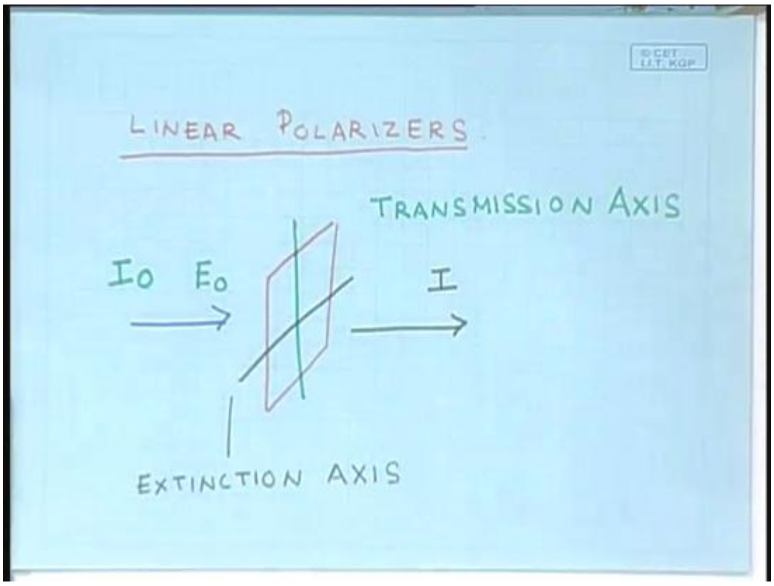

Now, the radiation that comes out, we are interested... So, the first question that we are interested in is what is the intensity of the radiation that comes out this will have some intensity I. And the question that we are interested in is what is the intensity of the radiation that comes out?

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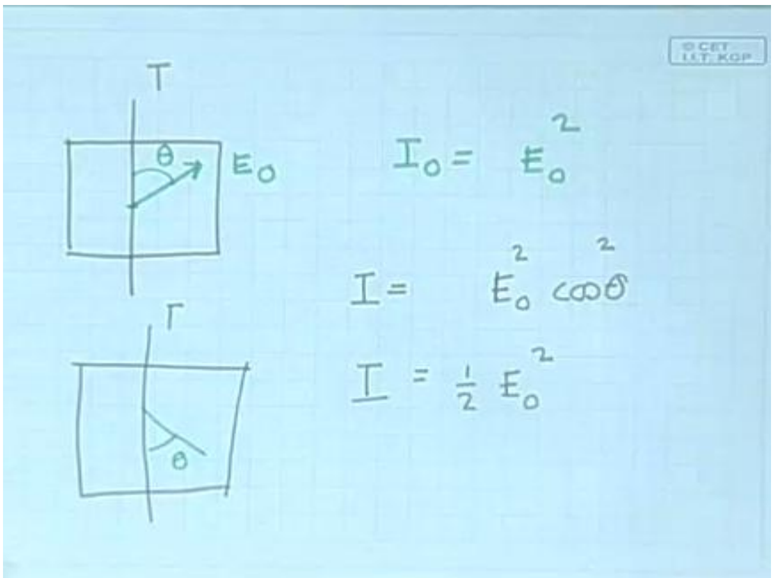

Now the amplitude of the vector that is allowed to pass through remember that only the component parallel to the transmission axis is allowed to pass through. So, the amplitude of the vector that is allowed to pass through is E naught cos theta. So, the intensity of the light that come that is allowed to pass through is I is equal to the square of this into the square of this. And this cos square theta is not fixed it varies randomly over time varies rapidly randomly over time. So, what you really measure is the average and the average of cos square theta is half. So, the intensity that comes out is half E naught squared. So, let me remind you again the situation that we have considered.

We have considered the situation where there is naturally polarized naturally natural light which is unpolarized incident on a polarizer whose transmission axis is along the direction shown over here. Indicated by this unpolarized light has an electric field of the amplitude E naught whose direction varies randomly very rapidly. The polarizer only allows the comp1nt parallel of the electric field parallel to the transmission axis to come through so, this comp1nt is E naught cos theta. So, the intensity of the light that comes through is E naught square cos square theta and since this direction theta varies randomly what you measure is the average. So, what you measure is the average of this average of cos square theta is half. So, the intensity falls to half the value

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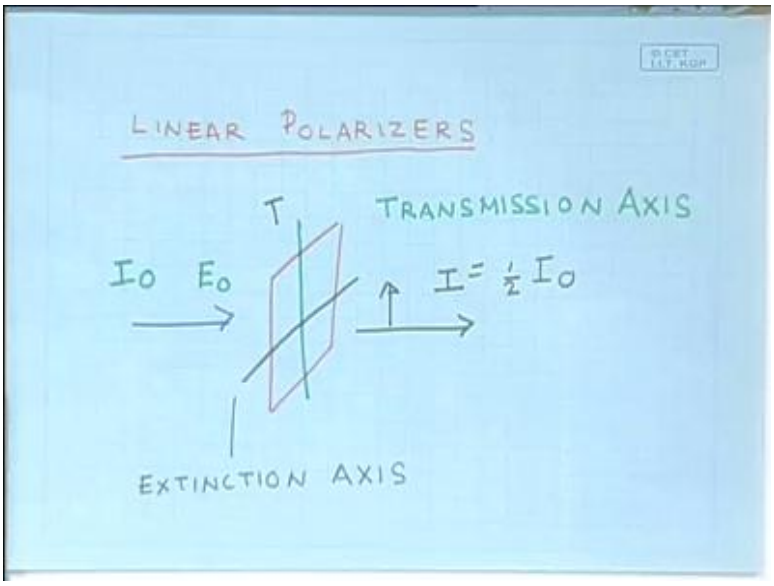

So, this intensity of the light which comes out from the polarizer is half I naught. And it has the property that the electric field is parallel to the transmission axis. So, it is linearly polarized that is why it is called a linear polarizer. Let us now, consider a slightly different situation more the situation is as follows we have 2 polarizer's.

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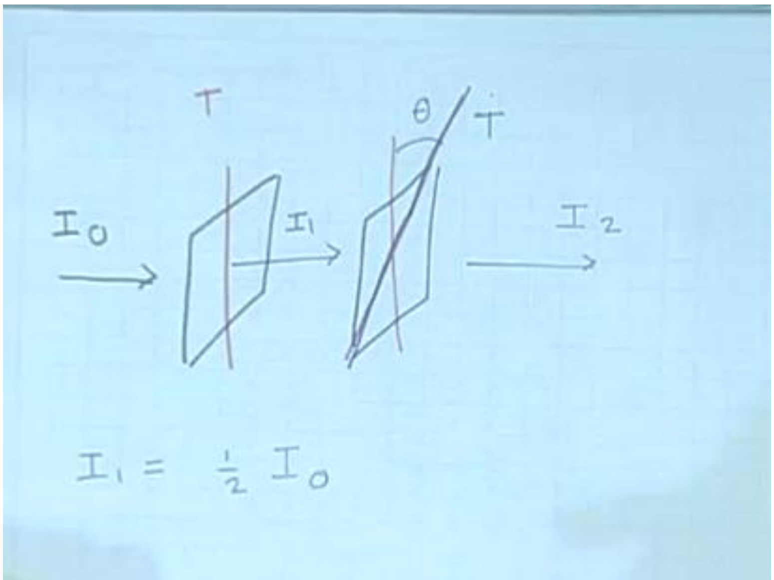

And we have natural light of intensity I naught incidents on the first 1 whose transmission axis is like this and then we have another polarizer which is refer to as the analyzer whose transmission axis is at an angle theta. So, the transmission axis of the second 1 of the second polarizer which we also refer to as the analyzer and polarizer are exactly the same thing just that we are they are being used for different purposes. So, this is another polarizer whose transmission axis is at an angle theta with respect to the first 1. So, the question is what happens to the light that comes out after the analyzer?

So, the intensity of the light that comes out here is I 1 and the intensity of the light that comes out here we are going to call I 2. And we have already seen that I 1 is going to be half I naught. Now let us analyze let us discuss what happens at the second at the analyzer? This we refer into this as the analyzer it is also a polarizer, but which we are using it for a different purpose we are using it to analyze the polarization content of the light that comes over here. And the transmission axis is at an angle theta to the transmission axis of this.

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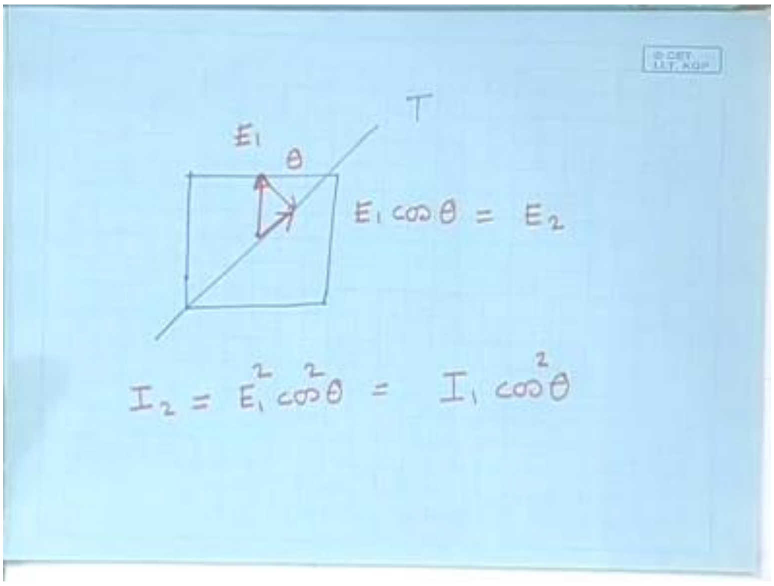

So, let us draw a picture which shows the second, which shows the analyzer whose transmission axis is like this along this direction and the incident radiation has the electric field oriented like this. Now, the analyzer will only allow the comp1nt of the electric field parallel to the transmission axis to pass through. So, if this is E 1 the if the incident light has amplitude the electric field is amplitude E 1 the amplitude of the electric field that passes thorough is going to be E 1 if this angle is theta is going to be E 1 cos theta. So, if I calculate this is E 2 the amplitude of the electric field that comes out. So, if I calculate the intensity of the radiation that comes out this is going to be E 2 square which is E 1 square cos square theta which I can write as I 1 cos square theta.

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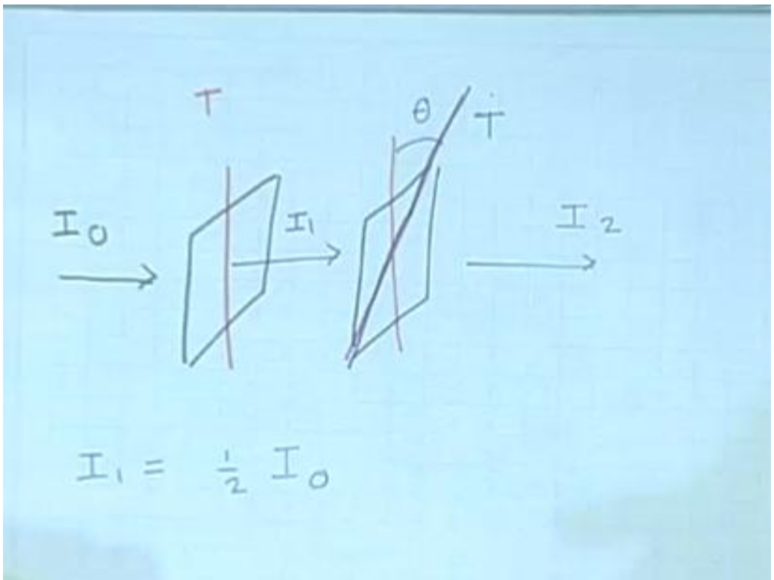

So, what we see is that if I have 2 polarizer’s at an angle theta the intensity of the light goes down by a factor of cos square theta where theta is the angle between the transmission axis of the 2 polarizer’s or another way of thinking of it is.

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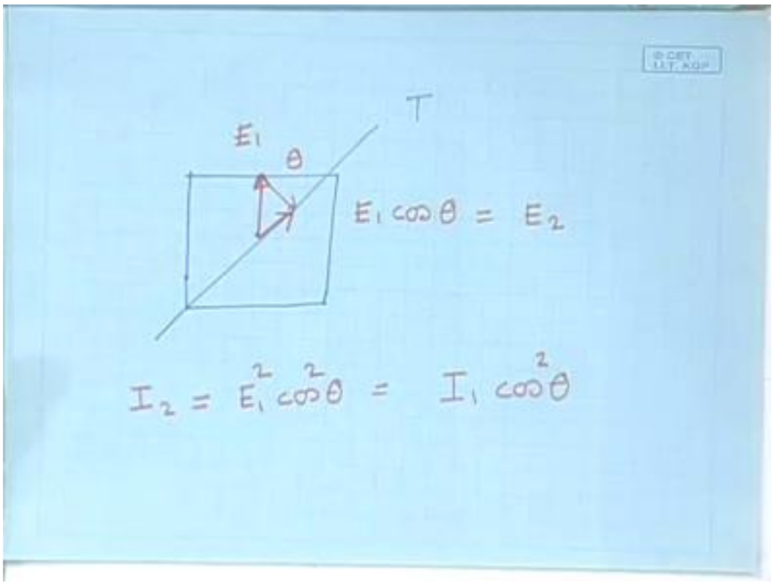

If I have a linearly polarized light at an angle theta to the transmission axis the intensity of the light that comes out is reduced by a factor cos square theta.

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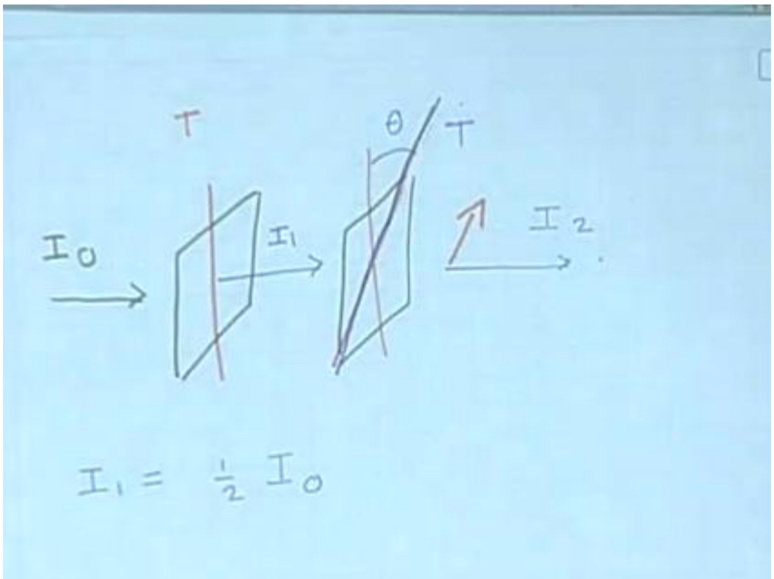

And it is the radiation that comes out is now, polarized in the direction parallel to the transmission axis of the second polarizer.

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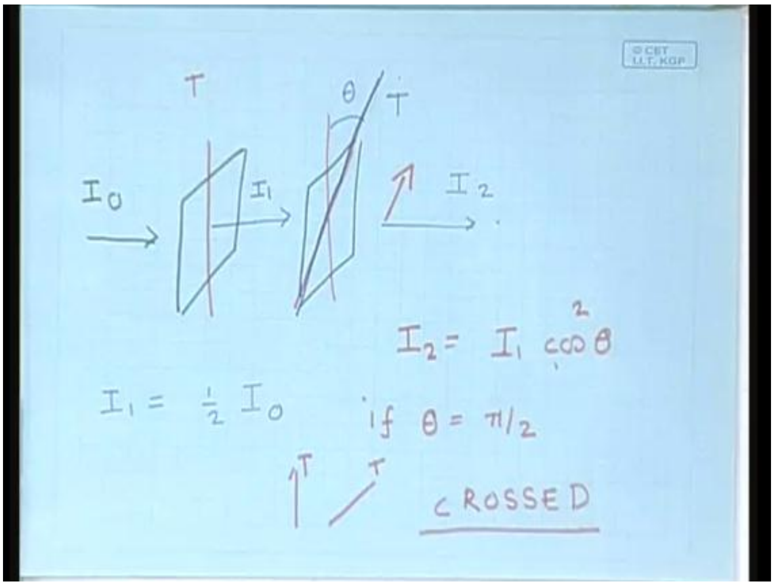

You could ask the question what happens if the 2 if these 2 are mutually perpendicular. If these 2 are mutually perpendicular so, we have see that I 2 is I 1 cos square theta. If these 2 polarizer’s were the polarizer and the analyzer were mutually perpendicular then the value of cos square theta is 0. So, if theta is pi by 2 the 2 the 2 transmission axis this is T and the other transmission axis is T they are perpendicular the intensity of the light that comes out is 0. The 2 polarizer's the polarizer and the analyzers are set to be crossed in this situation. So, for crossed polarizer and analyzer there is no radiation that comes out.

So, what we see is that a linearly a linear polarizer produces an output which is linearly polarized in the direction of its transmission axis. The intensity of the radiation that comes out falls and if it is unpolarized like that you have sent in it falls to half the original intensity if it is linearly polarized like that you have sent in then it falls. And it falls to by a factor of cos square theta where theta is the angle between the direction of the 2 transmission axis. Now, next let us ask the question how do you construct a linear polarizer how do you what produces polarize light? Now, polarized radiation linearly polarized radiation is in general polarized radiation is produced these polarizer's are basically based on 3 mechanisms there are three fundamental mechanisms. So, these mechanisms let me write down these different mechanisms here.

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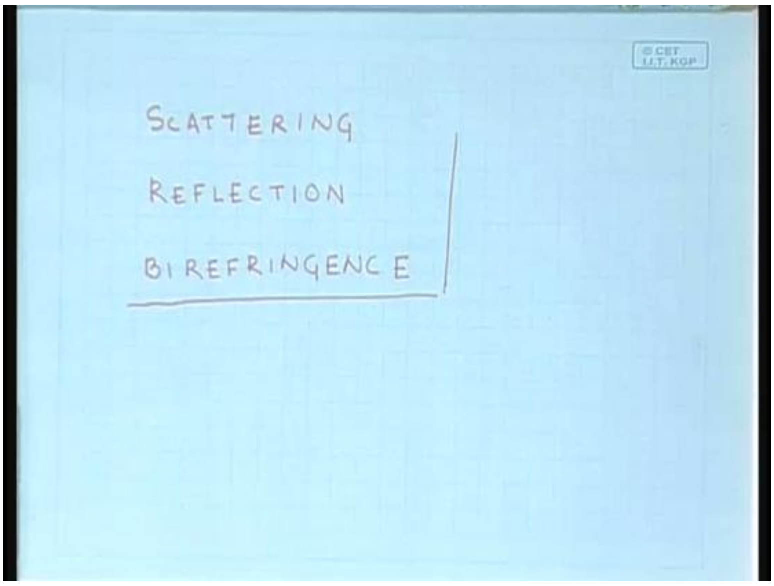

Three mechanisms which generate polarized light, three mechanisms for which you can design a polarizer here. Or by which polarized radiation is produced they are scattering reflection and birefringence. So, these three mechanisms produce polarized radiation these are the 3 main mechanisms which produce polarized radiation. Let us let me give you an example well we have already discussed an example of how scattering produces polarized radiation from natural or unpolarized light? Let we go through this again this scattering which i am referring to occurs in nature. So, we know that the sky is blue because of the scattered light. Light which is scattered and how does scattering take place we have all also discussed this.

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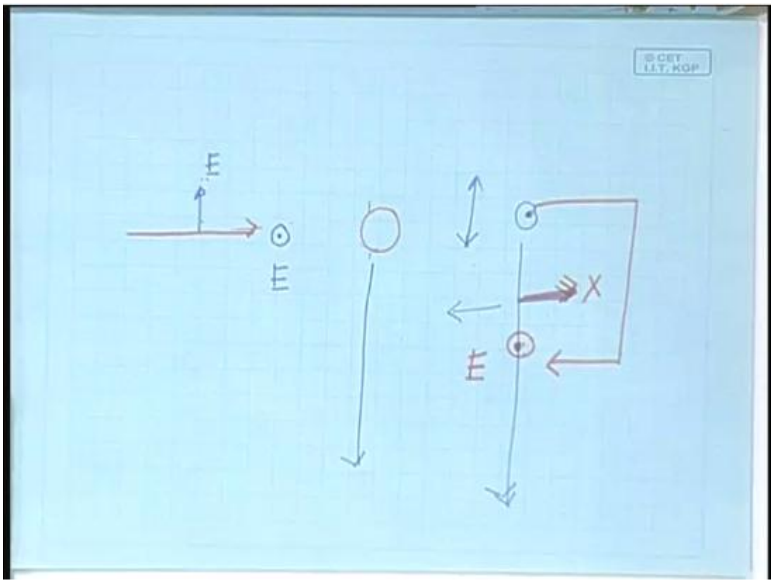

So, if I have radiation like this which is unpolarized so, the electric field can oscillate anywhere in this plane if we have unpolarized radiation incident say on some atom or molecule. The electric field in the incident radiation E will induce dipole movements over here. So, you could think of this it could be a dielectric whatever it could free electrons it could be an atom. Or a molecule it will introduce a dipole movement over here which will be parallel to the direction of the electric field and if the electric filed oscillates the dipole over here is also going to oscillate and it is going to emit radiation.

So, the radiation that is incident on this could be polarized could the electric field could be in two direction 1 like this or perpendicular to the plain of the paper which I am going to denote like this. Now, consider the situation where I observe this scattered light from this direction this polarization is going to set this polarization. So, the incident light electric field is a superposition of 2 polarizations this and this. And it could be varying randomly it could be some random super position of these two and it could be going around in random, because it is natural radiation.

Now, we are going to treat these 2 separately. So, this polarization is going to set the dipole into oscillation in this direction. A dipole oscillating in this direction is not going to emit any radiation in this direction. So, if you look at this whole thing from here you are not going to receive any radiation due to this polarization. This is going to set the dipole oscillating like this and if you look at it from this direction what you will see is that the electric field oscillates back and forth like this. Now, if you ask the question what are the 2 possible states of polarization for radiation travelling in this direction? The 2 possible states of the radiation travelling in this direction are as follows.

You could have the electric field oscillating like this or electric field could be oscillating perpendicular like this sorry no if the dipole oscillates up and down in this page it is going to produce an electric field which is also oscillating up and down So, this dipole I what I told you just now, is wrong this dipole is going to produce an electric field which is oscillating up and down in parallel to the dipole So, this is going to be produce an electric field like this, but this comp1nt of the electric field this possible polarization is going to be absent. Because there is no way that that these kind of dipole oscillations in this direction are going to be produce by this, because there is no electric field comp1nt parallel to the direction in which the incident wave is coming So, let me summarize what I have told you here what I have told you here is that.

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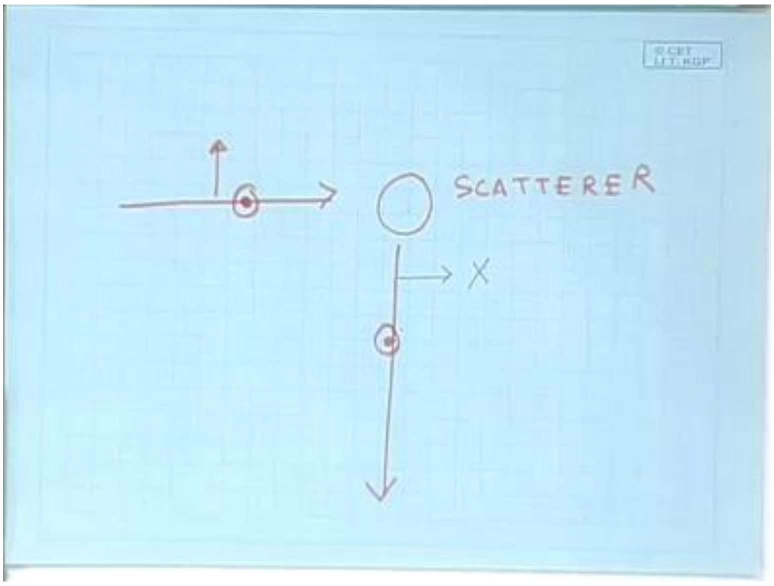

If there is a scatterer over here on which the unpolarized natural radiation is incident like this. This can have two different electric field here can be in 2 different directions like this and like this. And both are going to be equally present, because it is natural unpolarized light. Now if I look at the radiation that goes out in this direction from the scatterer so, this scatterer the radiation which goes out in this direction. This polarization is going to be present, because this is going to set the dipole oscillating up and down which is going to produce an electric field also oscillating up and down. But the other possible polarization over here which is this one this is going to be absent. So, if you look at the radiation scattered out at ninety degrees it is going to be linearly polarized. And the polarization in the plain of the scattering is going to be absent only the polarization perpendicular to the plain of the scattering is going to be present. So, this is one way in which scattering produces linearly polarized light.

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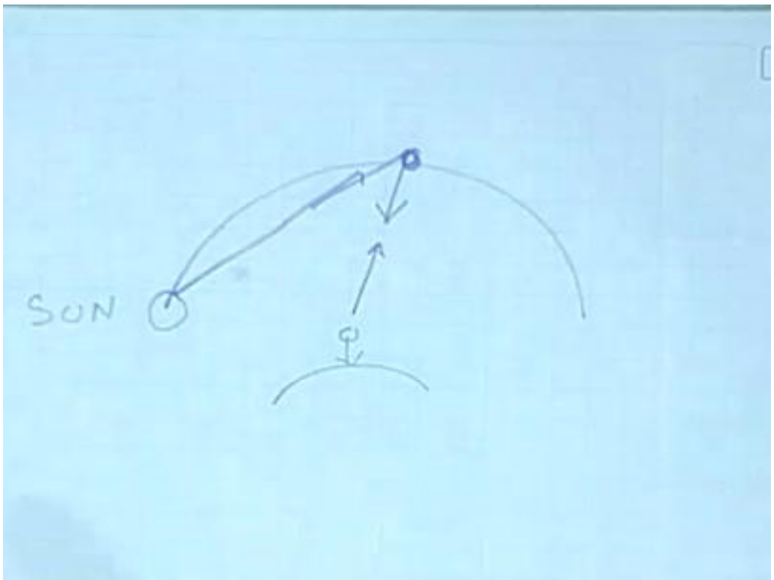

and this occurs in nature because when you look at the sky think of this as a sky and this as the earth surface we are standing here. The sun let us say is here this is the sun when you look in this direction what you will see is you will see sun light which has got scattered and is coming in this direction that is why you see the sky in the first place. So, it is a scattered light from the sun that you see and if this scattering; angle is 90 degrees. You will get linearly polarized light if the scattering angle is different from 90 degrees you will get partially linearly polarized light. So, scattering produces linear polarization that is that is something which we have learnt to today we have discussed this earlier also. So, today again I have told you how scattering produces linearly polarized light. So, in the next lecture we are going to discuss the other two mechanism.

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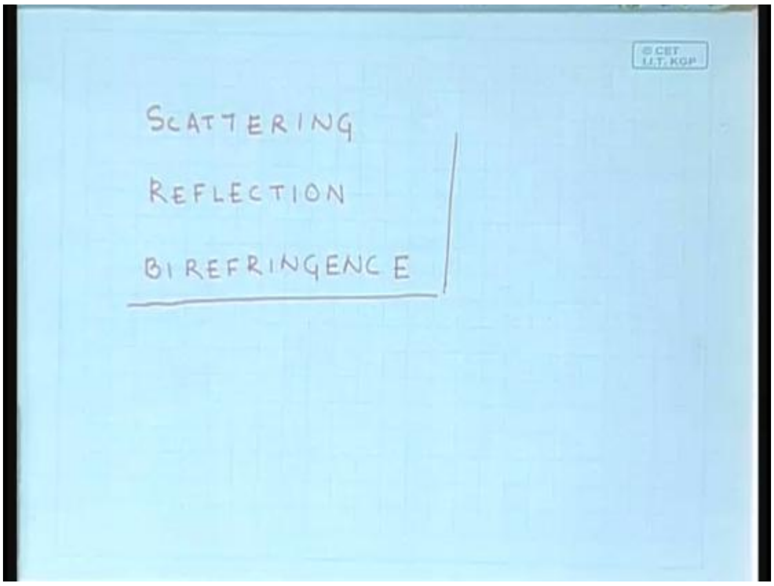

We are going to discuss reflection and by birefringence.