Physics – I:
Oscillations and Waves
Prof. S. Bharadwaj
Department of Physics and Meteorology

Indian Institute of Technology, Kharagpur

Lecture No – 03
Damped Oscillator II

We were discussing, the damped oscillator in the last class and we had taken up a particular situation of an under damped oscillator. Let us, device what we had done in the last class and take up a problem first and then go ahead to discussing the over damped and the critical e damped oscillators.

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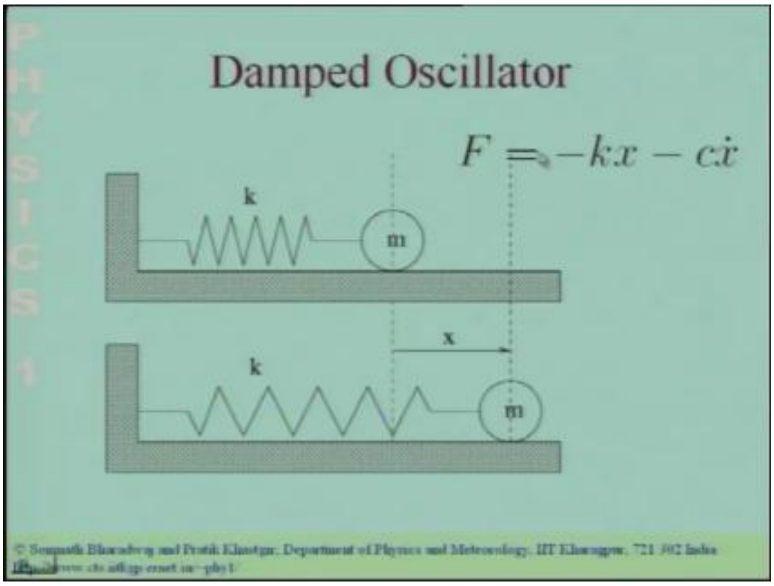

So, the damped oscillator, which we are considering is shown over here we have the good old spring mass system. Now, when you displace the particle and leave it you have 2 forces acting on the mass the first force arises due to the spring and that is the spring constant into the displacement and it has a minus sign. So, it opposes a displacement. But now you also have an additional force, which is proportional to the velocity and this is the damping force which is the main difference which arises when you consider a damped oscillator.

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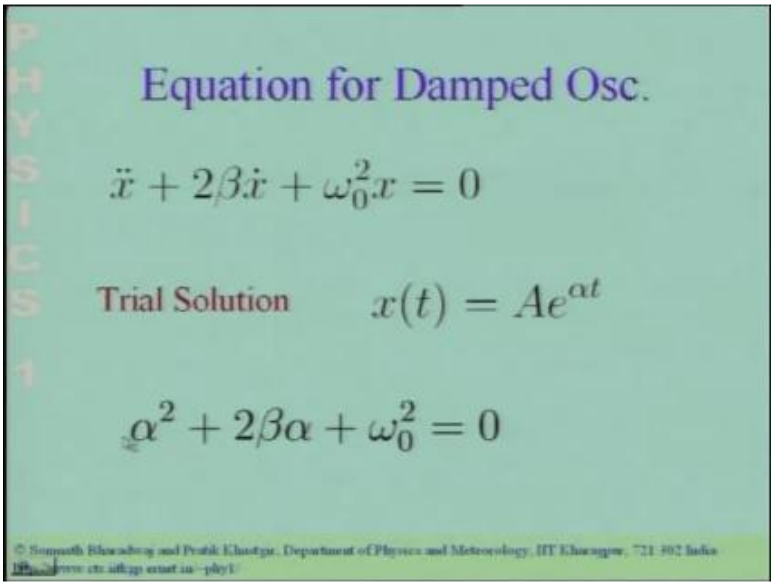

The equation governing the damped oscillator, which we had considered in the last class is again shown over here you have x double dot which is the acceleration the whole equation of motion has been divided by the mass. So, the term the spring constant by the mass has been written as omega naught square and the force due to this is proportional to x. So, this is how you get the over here. And this term is a damping term the damping coefficient the coefficient c which you saw in the previous slide divided by m has been written as 2 beta and we shall refer to beta as the damping coefficient.

So, we want to solve this equation this is what we had looked at in the last class and the way we had proceeded was to take a trial solution x of t is some constant A e to the power alpha t and we had plugged this in into this differential equation. And once you do this the term e to the power alpha t and the constant A all cancel out and you are left with the quadratic equation governing alpha.

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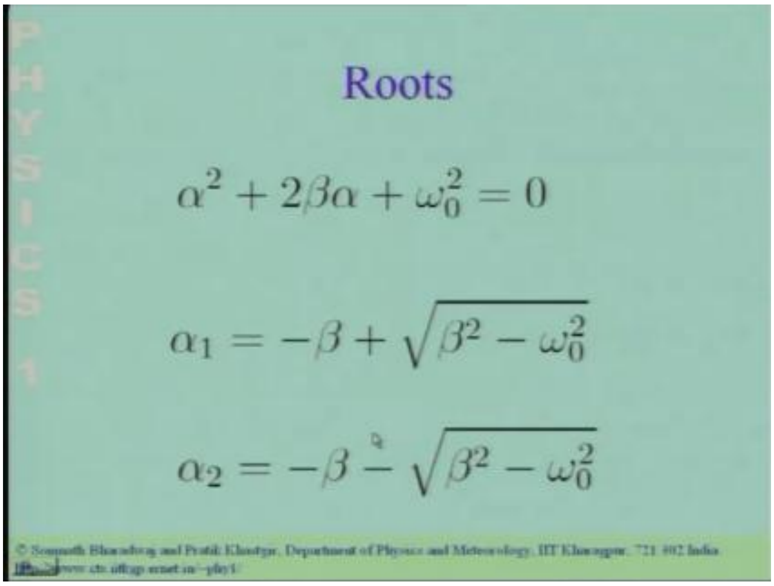

The roots of this quadratic equation. So, this is the quadratic equation it has 2 roots as we all know and the 2 roots alpha 1 and alpha 2 are given over here. We had discussed this in the last class. In the last class we had considered the situation where beta is less than omega naught this situation is what is referred to as the under damped oscillator. So, when beta is less than omega naught they are termed inside the square root over here is negative. And you have an imaginary number over here.

So, the roots are complex and imaginary part gives rise to oscillations and this real part over here is what causes the oscillations to decay with time. So, this was the under damped oscillator.

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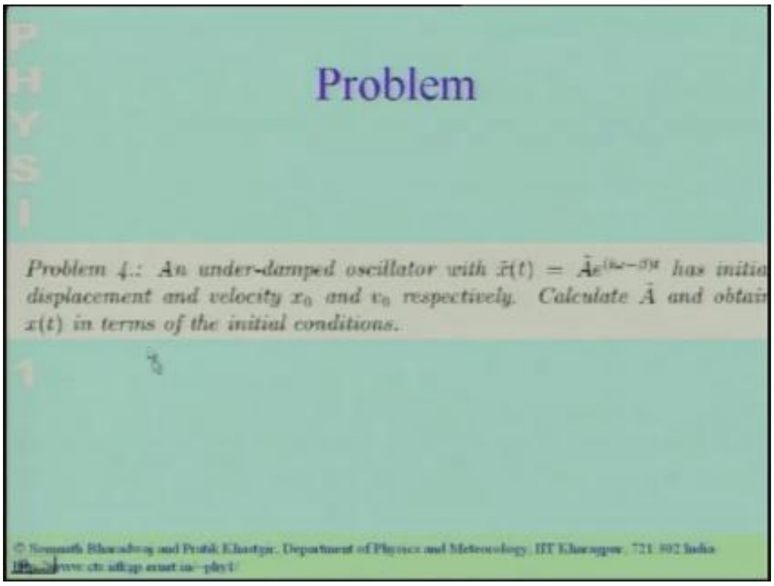

Let us now take up a problem in under damped oscillators. So, we have an under damped oscillator whose motion is given through in this complex notation x tilde t is equal to A tilde e to the power i omega minus beta the whole thing into time. So, this represents an under damped oscillator in the complex notation and we are dealing with the situation where the oscillator has position x naught and velocity v not at the initial time t naught.

Now, the problem which we are given we have to calculate the constant this complex amplitude A tilde in terms of the initial position and the initial velocity. And having calculated this we have to determine the x of t the position as a function of time in terms of the initial conditions. So, this is the problem which we shall take up. So, in the complex notation the position of the particle.

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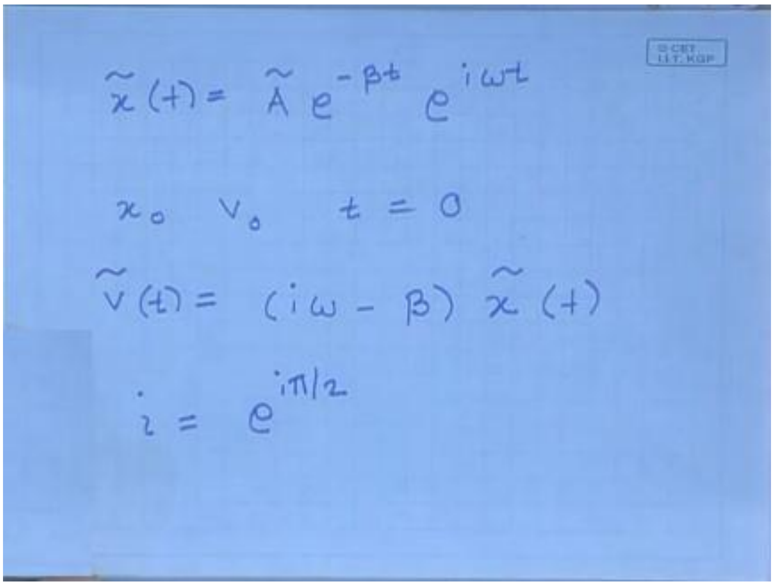

Is represented so, this expression over here which you just saw. So, this represents the motion of a under damped oscillator this A tilde is the complex amplitude and we see that the amplitude decays as e to the power minus beta t as time proceeds. And the whole thing does oscillation at a frequency omega and in this problem we are given x naught and v naught to be the initial positions and velocities at t equal to 0. So, in order to solve this problem, we have to first calculate the velocity from this expression for the position.

So, the way to calculate the velocity is to differentiate this expression for complex the for the complex position variable x tilde if you differentiate this. So, we have velocity tilde which is of a v tilde which is a function of time and if you differentiate this, the whole expression essentially gets multiplied by a factor I omega minus beta. So, this is the expression for the velocity in terms of the complex variable x tilde for the position.

So, the velocity and the position are related through this factor i omega minus beta. Before we go on to the problem which we are discussing let us, spend a few minutes in discussing a very interesting consequence of this expression for the velocity which we have just derived. Now, in the situation where there is no damping when beta is 0 notice that the velocity is i omega into x tilde. Now, i omega i essentially is e to the power i pi by 2.

So, the velocity is essentially omega into e to the power i pi by 2 times x tilde of t. So, we have multiplied x tilde the complex number x tilde with the number e to the power i pi by 2. What it does is it is puts in an extra phase of pi by 2 into this exponent, from this you can conclude that for an undamped oscillator the velocity at the position x they are exactly pi by 2 out of phase so, far an undamped oscillator the position and the velocity these 2 variables they both do oscillation that the same frequency, but, these oscillations are pi by 2 out of phase.

Now, this changes the moment you have damping, the moment you have damping x tilde is no longer multiplied by just an imaginary number to give the velocity it is multiplied by a complex number which has both a real part and an imaginary part. So, the complex number with which x tilde is multiplied to give the velocity is i omega minus beta. Let us write this complex number in terms of an amplitude and the phase. The amplitude of this complex number is the square root of omega square minus beta square.

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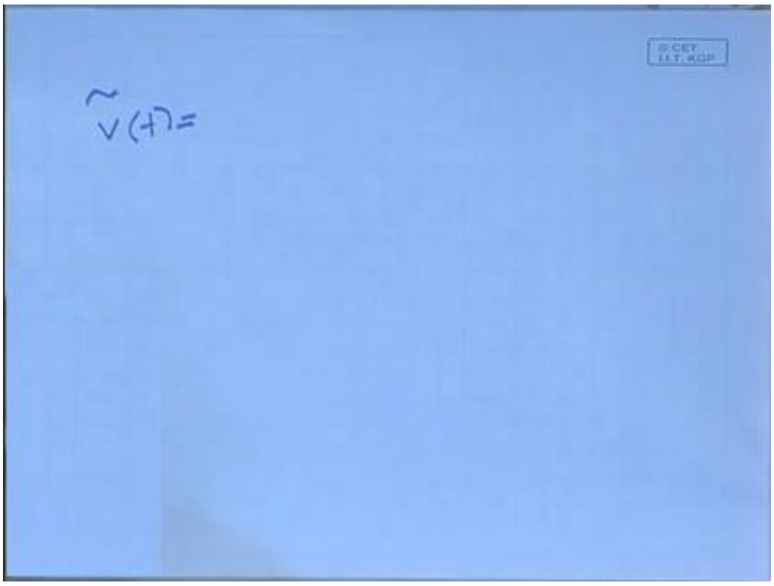

So, the velocity V tilde t is an amplitude the amplitude is the square root of let me, do this little bit of simplification here.

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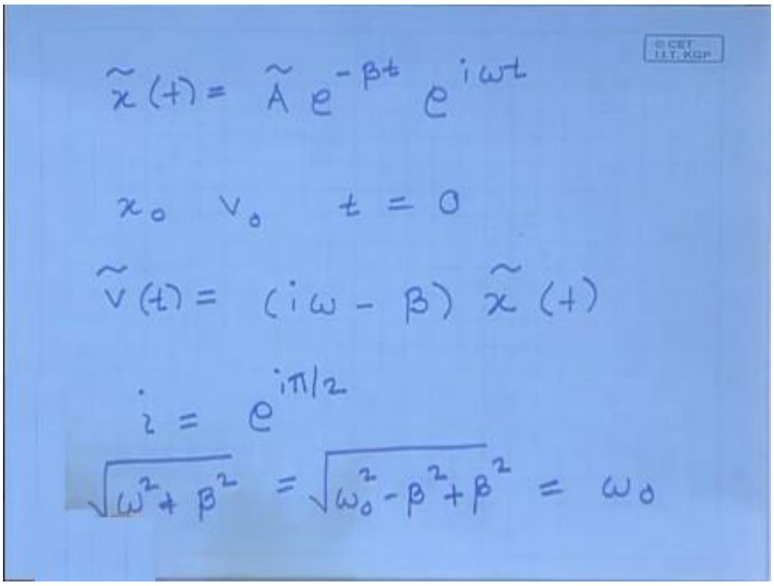

The amplitude is the square root of omega square minus beta square and recollect that the frequency of an under damped oscillator is related to the undamped frequency omega naught as omega naught. This will be sorry this will be omega naught omega square plus beta square not minus and omega square itself is omega naught square minus beta square. So, what we have is a square root of omega naught square minus beta square plus beta square.

This whole thing gives us omega naught. So, the amplitude of the number which multiplies x tilde to give the velocity is omega naught just as if there was no damping. And the phase of this number is thus tan inverse of minus omega by beta.

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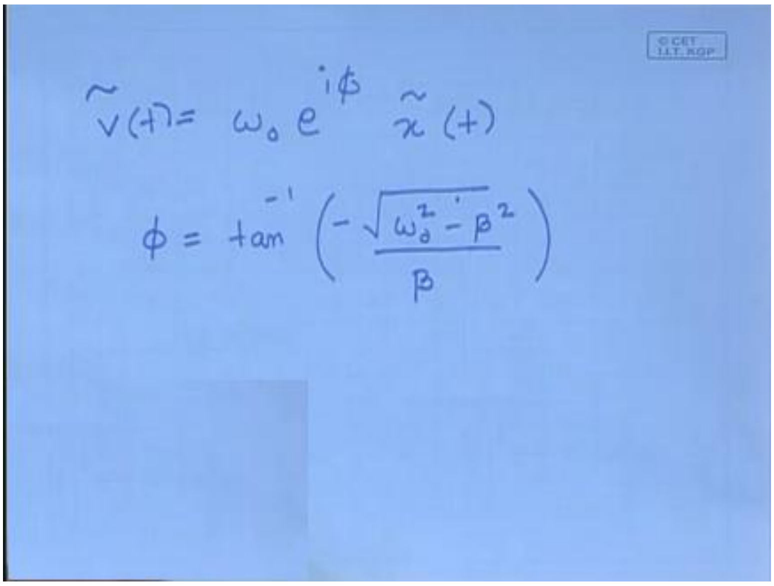

So, we can write v tilde as the amplitude omega naught into e to the power i times a phase phi into x tilde t where phi is tan inverse omega minus omega by beta and omega recollect is omega naught square minus beta square this whole thing divided by beta. So, we see that the velocity and the position variable in complex notation and now related through a phase e to the power i phi which need not be pi by 2. It becomes.

So, let us study, how the phase between the velocity and the position variable changes if you vary a damping coefficient. When the damping coefficient beta is exactly equal to zero you have tan inverse of minus infinity and the tan inverse of minus infinity is minus pi by 2. So, what the situation where there is no damping you see that the velocity and the position are related are differ by a phase of minus pi by 2. Now, what happens when you increase beta.

Now, beta can assume a maximum value of omega naught actually beta equal to omega naught is the case of critical damping you will not have oscillation then this expression will no longer be valid. But let us take the limit as beta approaches omega naught as beta approaches omega naught you find that this expression over here tends to 0. So, the phase difference between the velocity and the position tends to 0. So, the velocity and the position variable tend to oscillate with the same phase, the phase difference becomes smaller and smaller as you increasing the damping.

So, this you see is a very interesting feature which occurs, when you introduce damping into an oscillator as you increase the damping coefficient the oscillations and the position and the velocity is slowly move towards the same phase. From being exactly pi by 2 out of phase when there is no damping as you increase the damping the position and velocity slowly move towards oscillation with the same phase. This is a very interesting feature which occurs, when you introduce damping into an oscillator.

Now let us, go back to the problem which we are dealing with in our problem we have to calculate the coefficient A which occurs over here.

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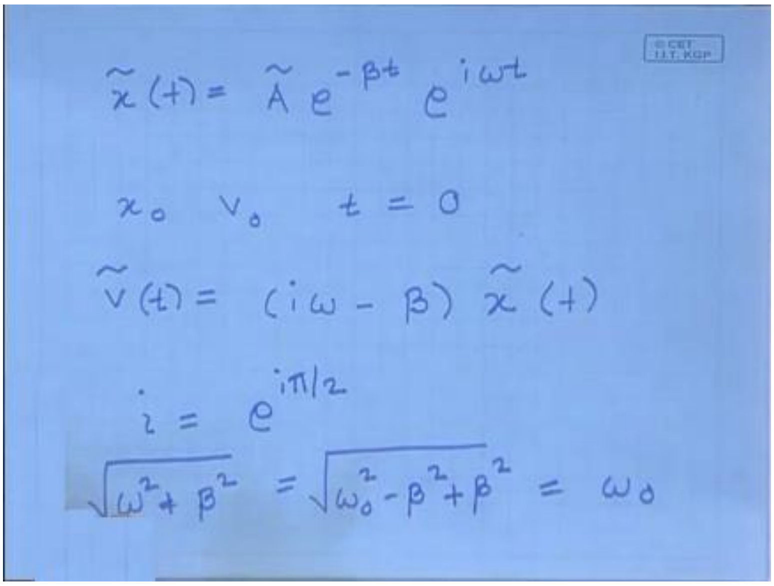

In the expression, for x tilde in terms of the initial position and velocity. So, in order to do this we have to set t equal to 0. So, at t equal to 0 x tilde is exactly equal to A tilde and if we express A tilde.

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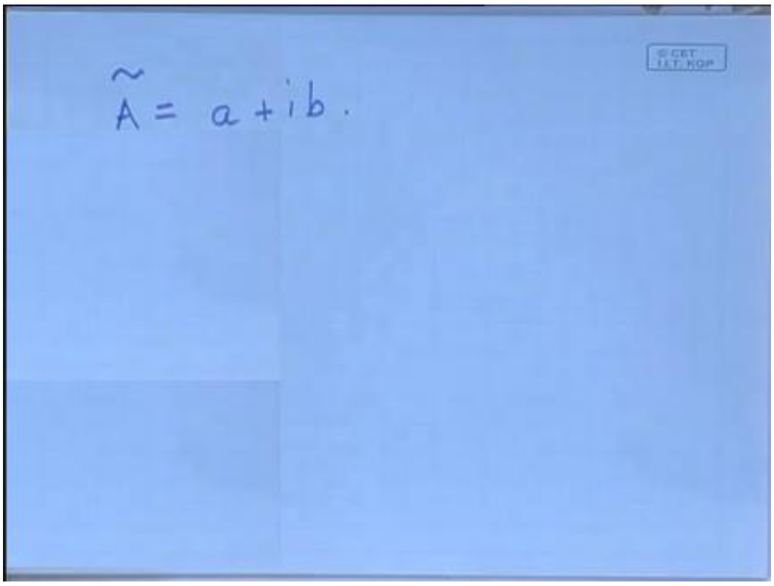

As a plus ib then.

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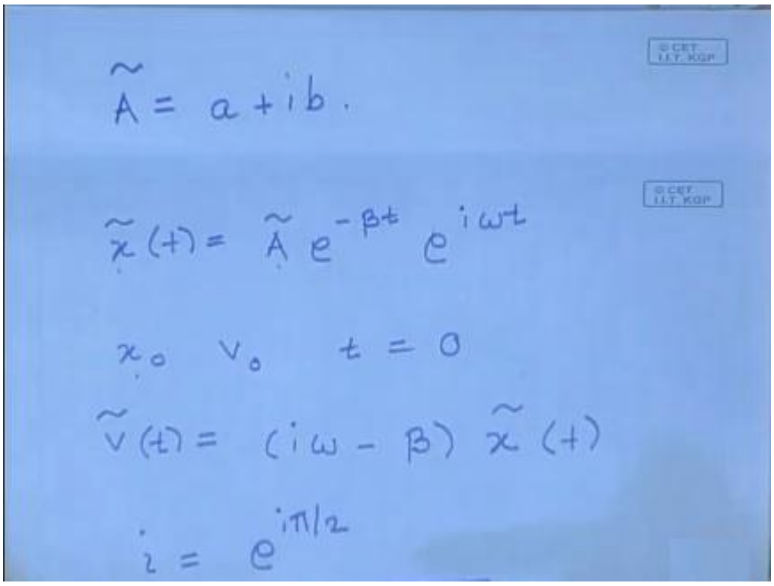

X tilde is equal to A tilde at t equal to 0 x tilde is equal to A tilde and if you express A tilde is a plus ib then when you ask the question. What is the position of the particle at t equal to 0 you should take only the real part of x tilde t equal to 0 which is the real part of A tilde. The real part of A tilde is a. So, you are led to the conclusion that a is the initial position of the particle x naught.

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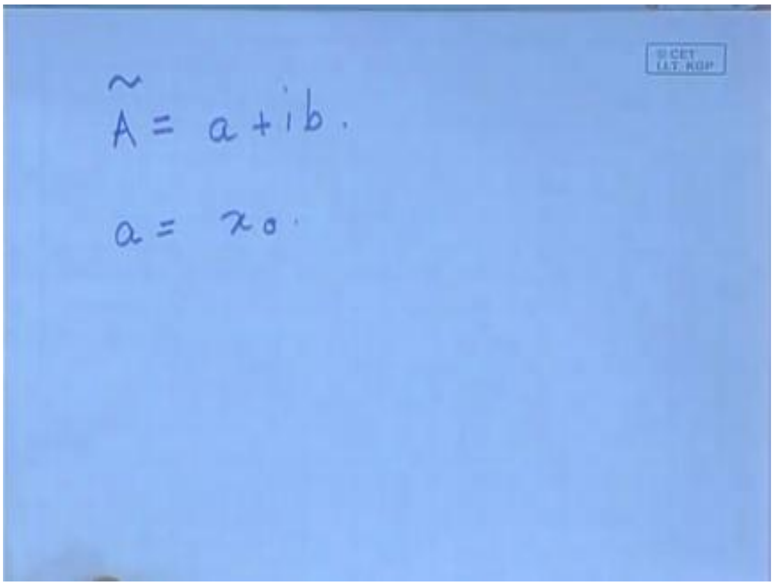

So, we have determined a and a is equal to the initial position of the particle x naught. You determine this by looking at the real part of x tilde at t equal to 0 which is a and setting it equal to the initial displacement. Now, we have to determine b the other unknown part of A tilde. In order to determine b we have to look at the expression for the velocity at t equal to 0.

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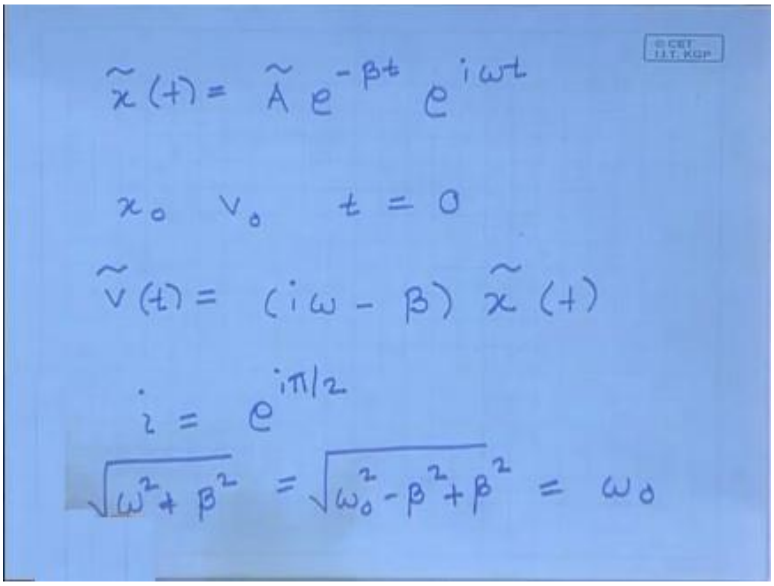

So, at t equal to 0 x tilde is essentially A tilde. So, the velocity is i omega minus beta into A tilde. And when you want to when you ask a physical question what is the actual velocity of the particle you have to only considered, the real part of v tilde that is the rule when you are dealing with complex, when you are representing real quantities using complex variables. The moment you ask a physical question you have to only take the real part of the complex variable.

So, when if you ask the question, what is the velocity of the particle at t equal to 0 you should take only the real part of this expression at t equal to 0.

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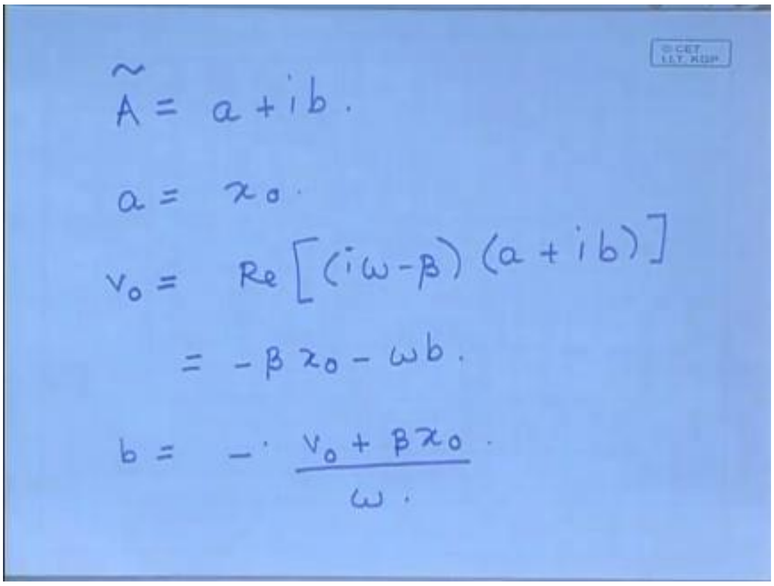

So, to calculate the velocity at t equal to 0 you have to take the real part of this will be the real part of i omega minus beta into A tilde which is a plus ib. So, let us see what the real part of this is: the real part will have there will be 2 contributions to the real part. The first contribution is minus beta into a. So, this is minus beta into a, a we already know is x naught and the other part is i omega into plus ib. So, that gives us minus omega b. And we have to now obtain an expression for b in terms of v naught and x naught.

So, what we get is b is equal to minus v naught plus beta x naught by omega. So, now we have A tilde in terms of the initial conditions, we can put this back into the expression for x tilde.

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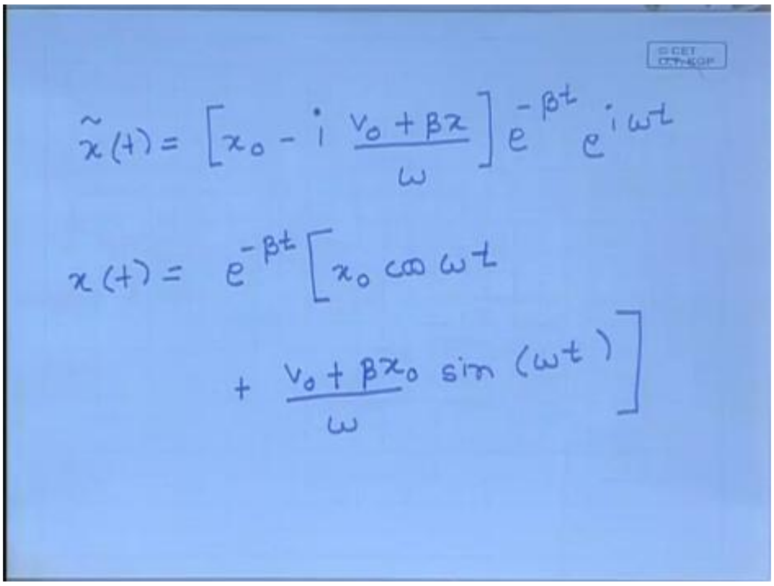

So, what we have is X tilde t is equal to A tilde A tilde is x 0 minus i into v 0 plus beta x by omega that is: A tilde the whole of A tilde this into e to the power minus beta t e to the power i omega t right. So, that is the expression for x tilde the complex representation of the position as a function of time in terms of the initial conditions. Now, if you want an expression for x the real position as a function of time you have to take only the real part of this.

So, let us write down from this what the position as a function of time is going to be. So, there will be an overall e to the power of minus beta t and we will have x naught cos omega t and then we will have 1 more term, which is minus i v naught plus beta x by omega into this will give you i sin omega t i sin omega t and minus i will give you plus. So, we have v naught plus beta x by omega sin omega t.

So, this is an expression for the position of the particle as a function of time expressed in terms of the initial position of the particle at t equal to 0 and the initial velocity of the particle at t equal to 0 this should be x naught. Notice that there is a big difference from the situation where there is no damping and the big difference is that if a particle starts from rest at t equal to 0. So, the particle is at rest it has got only a displacement.

So, I have taken a particle displace it from the origin and left it. And I want to study it is motion I want to see it is position as a function of time, if there is no damping we know that the solution is cos omega t cos omega naught t. It will have an amplitude that the amplitude will be the amplitude of the displacement for an undamped oscillator, but, the moment you put in damping the cos sin itself does not fully describe this.

The cos sin does not fully describe this you also have a sin term and the sin term has a coefficient beta by omega into the initial displacements. So, you still have the cos term there, but, you also have a sin term both of these together give you the position of the particle x. And this fact that we have a sin term here is closely related with the point which we had discussed little bit earlier the fact that, if you introduce damping the position and the velocity are no longer pi by 2 out of phase having discussed, the under damped oscillator.

Let us now, move on to study the situation where omega naught and beta are not beta is not less than omega naught and these situations correspond to the over damped oscillator and the critical damped oscillator.

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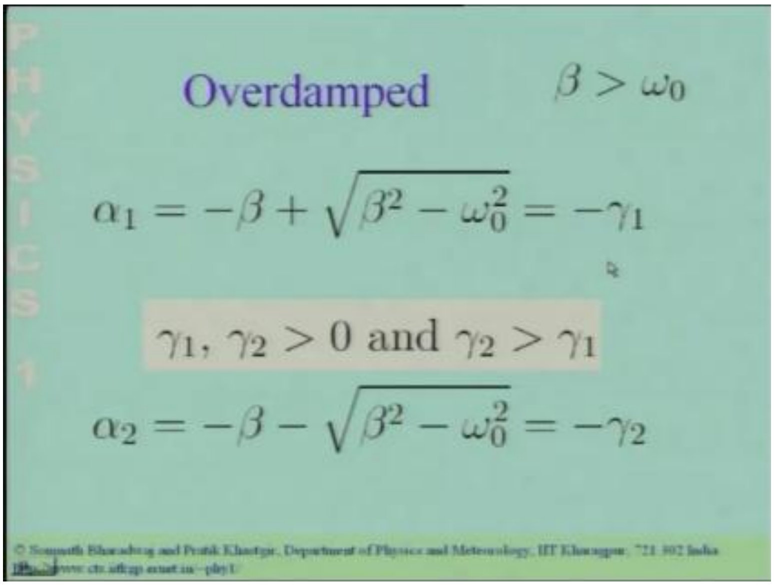

So, let us first take up the over damping refers to a situation where beta is much where beta is greater than omega naught. So, if beta is omega is greater than omega naught the 2 roots alpha 1 and alpha 2 are both real see if beta is greater than omega naught the term inside the square root is in positive. So, the square root is real and you have these 2 roots minus beta plus square root of beta square minus omega naught square and minus beta minus square root of beta square minus omega naught square.

Now, for an over damped oscillator this number over here inside the square root after I have taken the root is going to be less than beta. So, these 2 roots alpha 1 and alpha 2 are always going to be negative both of them are always going to be negative. So, it is convenient instead of dealing with these 2 negative numbers it is convenient to introduce 2 numbers gamma 1 and gamma 2 both of which are positive.

So, we have 2 numbers now gamma 1 and gamma 2, which are related to alpha 1 and alpha 2 through this minus sign. So, these are both positive and gamma 2 is more than gamma 1. So, this value of gamma 2 is more than gamma 1.

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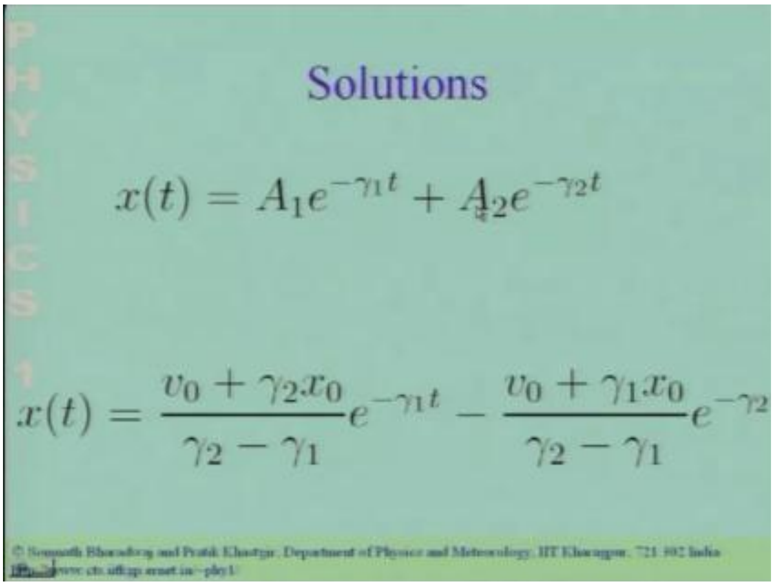

So, if you put write down the solutions in terms of these you have 2 solutions A 1 e to the power minus gamma 1 t plus A 2 e to the power minus gamma 2 t. There in mind that gamma 2 is larger than gamma 1. So, if you ask the question which of these 2 solutions is going to decay faster both of these solutions are; obviously, decaying with time. And if you ask the question which 1 is going to decay faster remember that gamma 2 is more than gamma 1.

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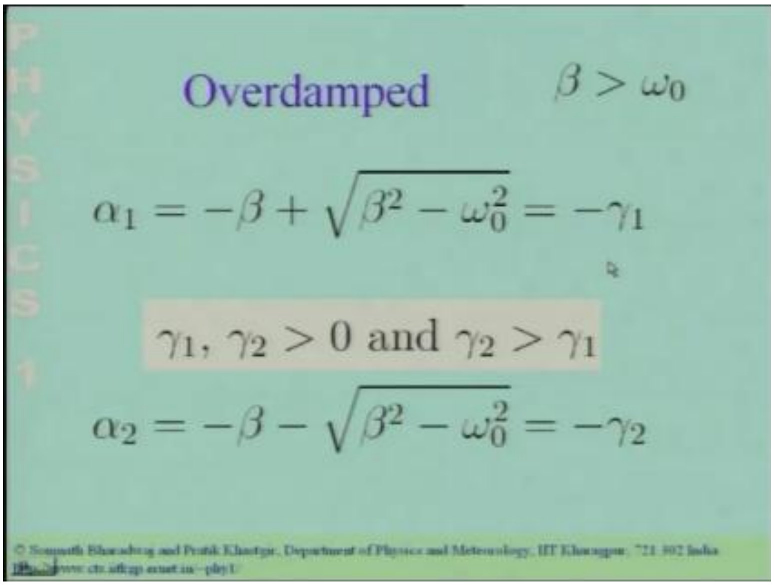

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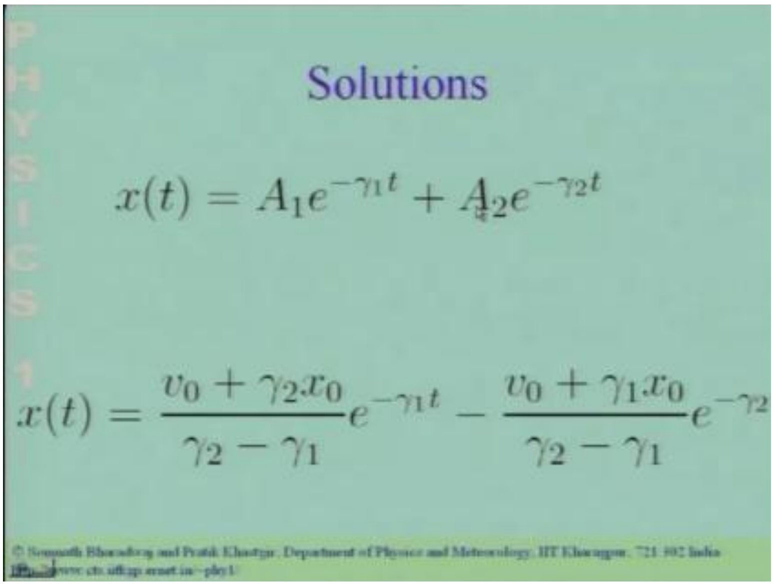

And if gamma 2 is more than gamma 1 you can guess that the second solution is going to decay faster than the first 1. This is the point which we shall come to later on as we along in today's lecture. Now, this is the general solution for an over damped oscillator you have 2 unknown coefficience A 1 and A 2, just like, for the under damped oscillator we had worked out the 2 initial conditions which were there in the complex amplitude A tilde in terms of the initial positions and velocities. You can do exactly the same thing for the over damped oscillator, you can take the expression for x say t equal to 0 and set is equal to the initial position x naught differentiate this you will get an expression for v.

In that expressions say t equal to 0 and equate it to v naught you can then invert these 2 expressions and obtain A 1 and A 0. I will not be discussing that explicitly here, but, if you do the algebra its straight forward if you do the algebra your resulting expression for x of t in terms of the initial position and the initial velocity is given over here. And this can be done by just doing the algebra which I have described earlier.

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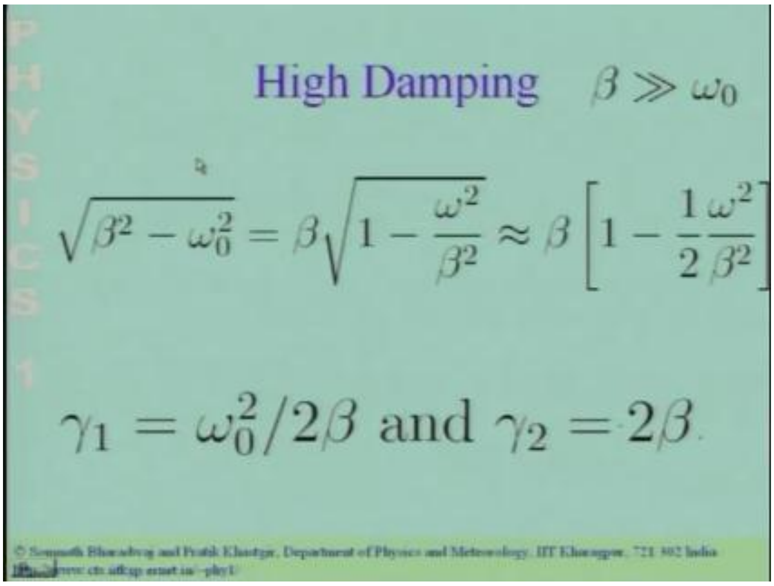

Let us also briefly discuss the situation what happens when the damping is increased. So, that it is much higher than the angular frequency. If the damping b beta damping coefficient beta is much higher than omega naught; the situation can be understood by just considering the following steps over here, we can beta common outside. So, you have minus omega naught this should read omega naught 1 minus omega naught square by beta square.

And since, beta is much larger than omega naught you can do a tailor series and retain only the first term. So, you will get beta into 1 minus half omega naught square by beta square this 2 should beat omega naught. So, in this limit when beta is much greater than omega not when the damping is very much very much larger than the angular frequency of the undamped oscillator; then this term a square root term the term over here the square root becomes beta into 1 minus half beta square by omega omega naught square by 2 beta square.

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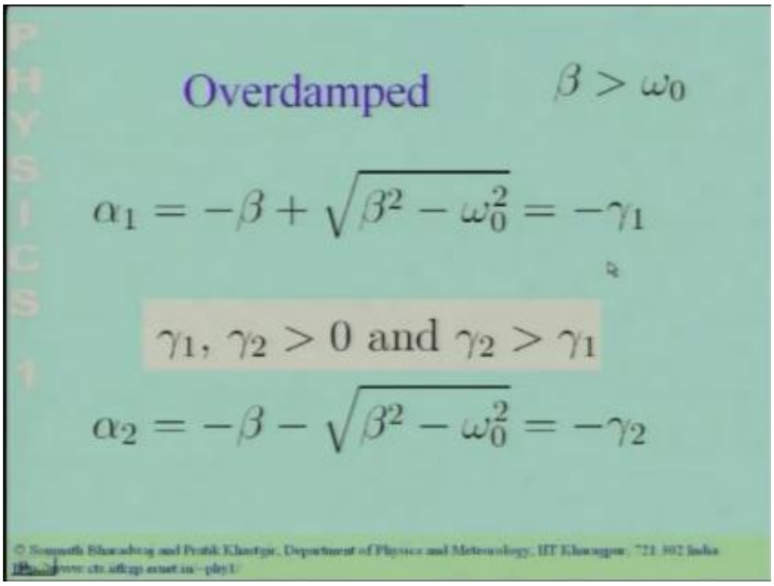

If you, put this into the expression for the 2 roots. So, into this and this.

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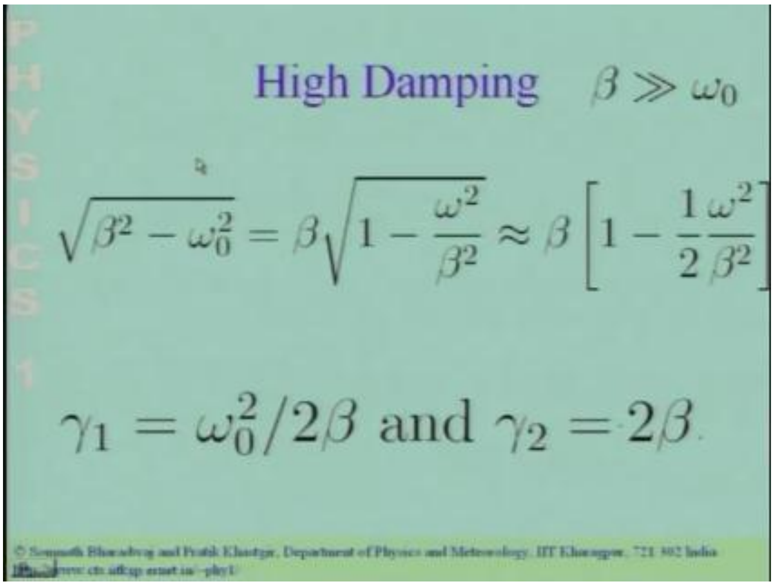

Then you find that gamma naught when you calculate gamma 1 the beta term will cancel out with minus beta and you are left with omega naught square by 2 beta. And gamma 2 is the leading term and gamma 2 is 2 beta this term can be dropped over there. So, the 2 roots now are omega naught square by 2 beta and 2 beta. Now, just let us just discuss what happens when beta becomes very large much larger than omega naught.

So, in that limit gamma 2 tends to become a very large number while the root gamma 1 slowly tends to 0.

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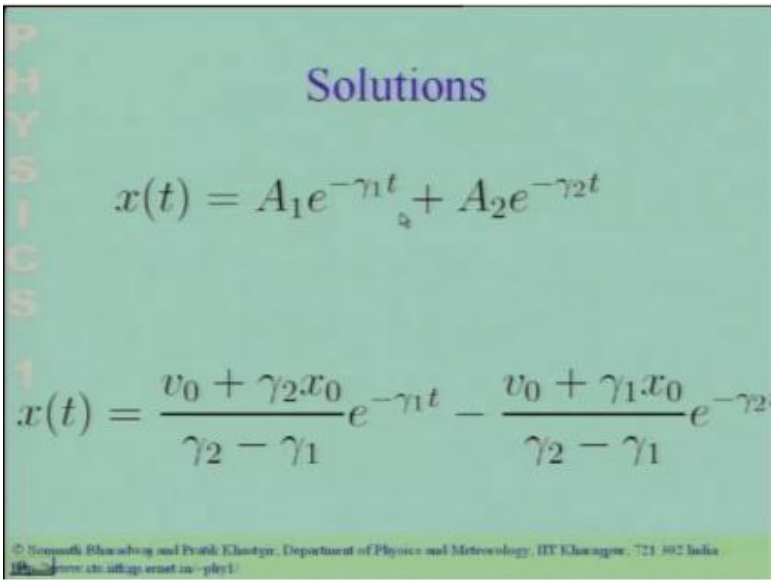

If you ask the question how do the 2 solutions e to the power minus gamma 1 2 t and e to the power minus gamma 2 t behave when the damping is very large notice that 1 of the roots. So, this particular solution when the damping becomes very large gamma 2 becomes very large for very large gamma 2 this exponential function decays very fast. Whereas, this exponential function over here decays extremely slowly because as

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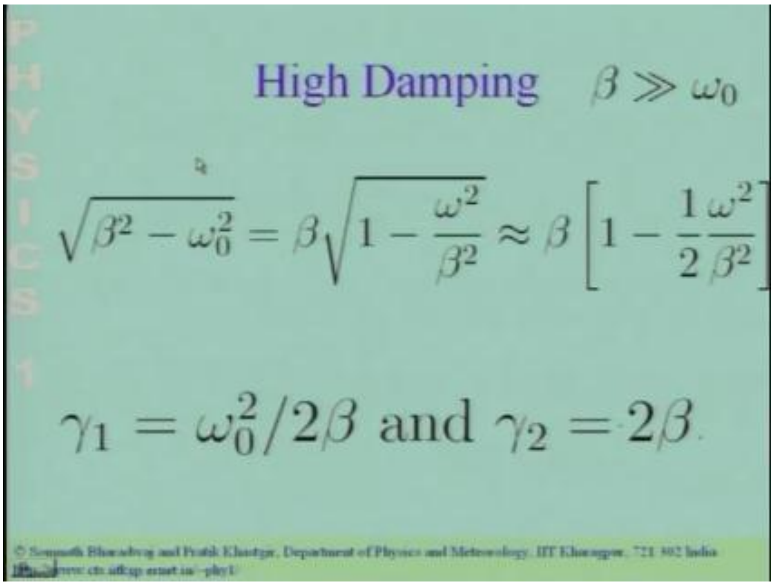

The damping is increased as beta is increased if beta is made much large in an omega naught the coefficient gamma 1 tends to 0.

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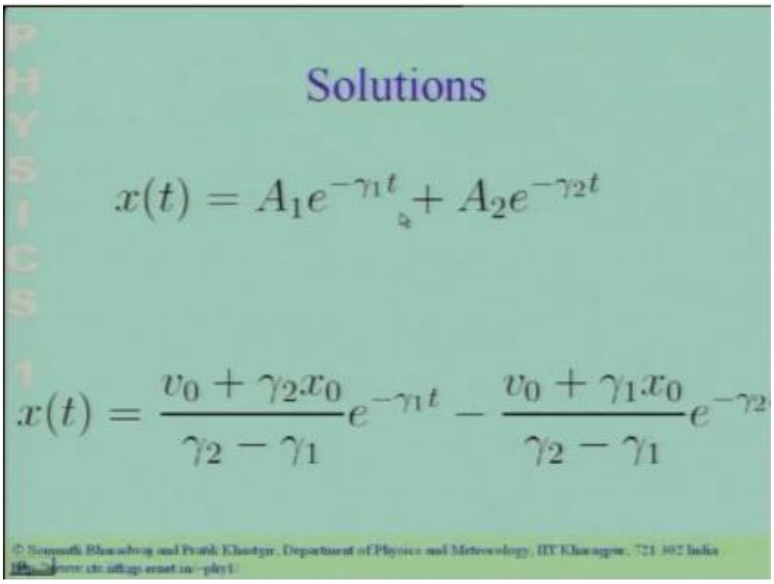

And if the coefficient gamma 1 here it tends to 0 or if it becomes very small the rate at which this exponential term decays becomes extremely slow. So, if you increase a damping very much 1 of the exponential terms over here decays very fast while the other 1 decays very slowly. It is worthwhile to bare this in mind and this is a point which we shall come to again as we go along in today's lecture.

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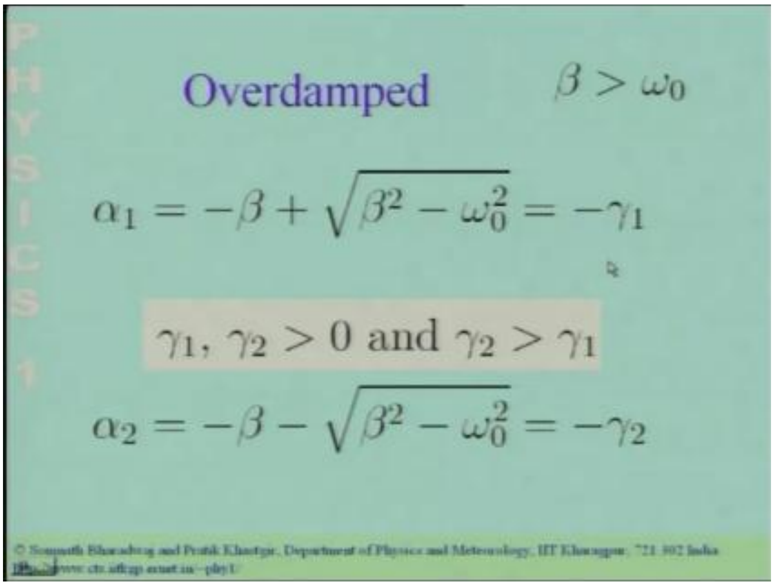

So, we have now studied the over damped oscillator where the damping is more than the angular frequency.

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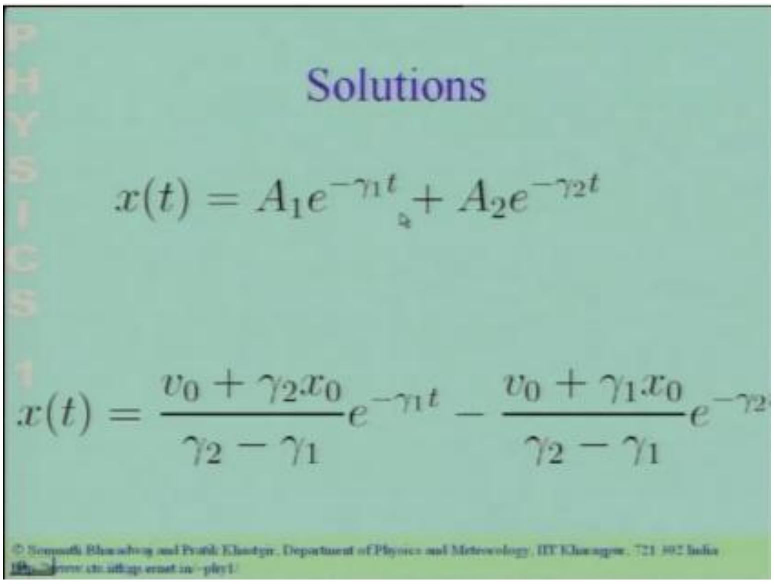

And you have exponentially damped solution in this situation.

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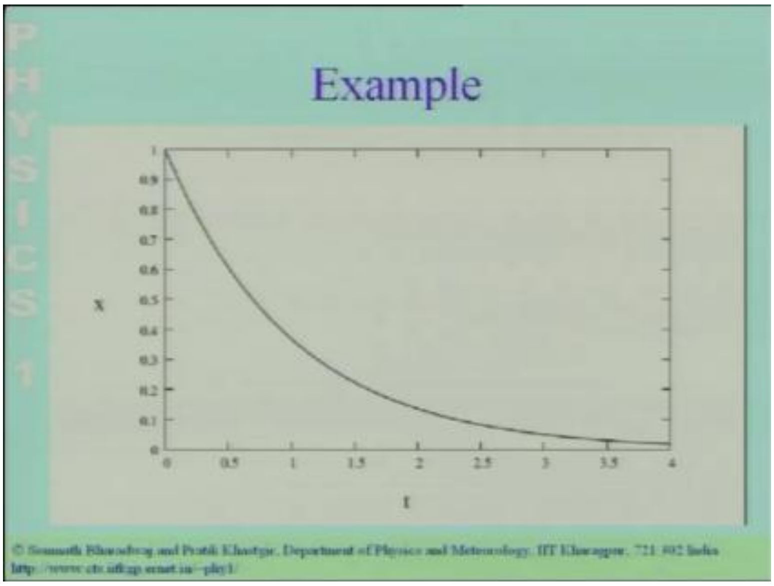

And the solution typically looks like this. So, if you displace the particle by a certain amount and leave the particle there it will slowly exponentially tend, it will exponentially decay to the equilibrium position. So, from the displaced position it will exponentially decay to the equilibrium position that decay has 2 exponential terms with different exponents, 1 of the exponents is larger than the other and the exponent which is larger that particular term decays faster the exponents, which is slower; which is smaller that particular exponential term decays slower.

So, you have a combination of a fast decaying term and a slow decaying term and the particle slowly reaches equilibrium as these 2 terms decay away. So, that is the typical behaviour of an oscillator, which is over damped you no longer have oscillations. The reason why you no longer have oscillation is easy to understand in this problem of a simple harmonic oscillator there are essentially 2 time scales, 1 time scale is decided by the angular frequency of the oscillator when there is no damping omega naught.

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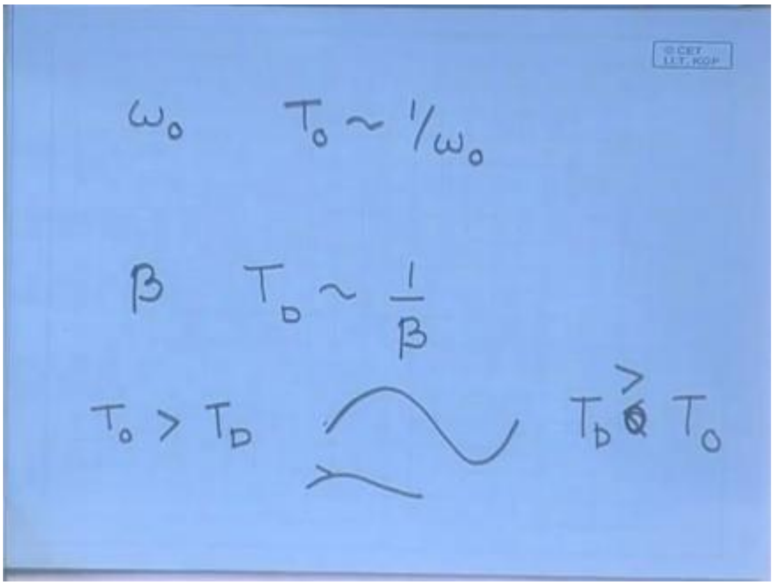

So, the time scale corresponding to this, a time period of oscillation is of the order of 1 by omega naught and it is exactly 2 pi by omega naught. So, this is 1 time scale in the problem that is a time scale of the oscillation. When you introduce damping you essentially introduce another time scale into the problem and the time scale is there in the coefficient beta and the damping time scale is of the order of 1 by beta.

So, there are 2 tendencies in the situation there is 1 corresponding to oscillations and that has a time scale of 1 by omega naught and there a another feature in this whole system that is the tendency to decay and that is of the order of 1 by the damping coefficient beta. Now, if the time scale for oscillation is more than the time scale for the decay. So, if the time scale of oscillation is more than the time scale for decay, which essentially means that this is 1 oscillation and the decay is much faster than has a decay occurs at a time which is smaller than this.

So, before the system can do the oscillation, it has essentially decayed. So, it would have decayed you will not see the oscillations. So, under this situation you will not see the oscillations and this situation is what corresponds to over damped oscillations. So, if the time period for the oscillation is larger than the time scale over which the decay occurs the decay will occur first and you will not see the oscillations.

Whereas, if the time period for the decay is smaller than the time period for the is larger than the time period of the oscillations. So, the decay occurs slower the oscillations occurs faster then you have under damped oscillations. And you will see the oscillations which are slowly damped away due to the decaying the decay the damping over here. So, the damping time scale, if it is larger then you will see the oscillations. The situation where these 2 time scales are equal is what is called critical damping.

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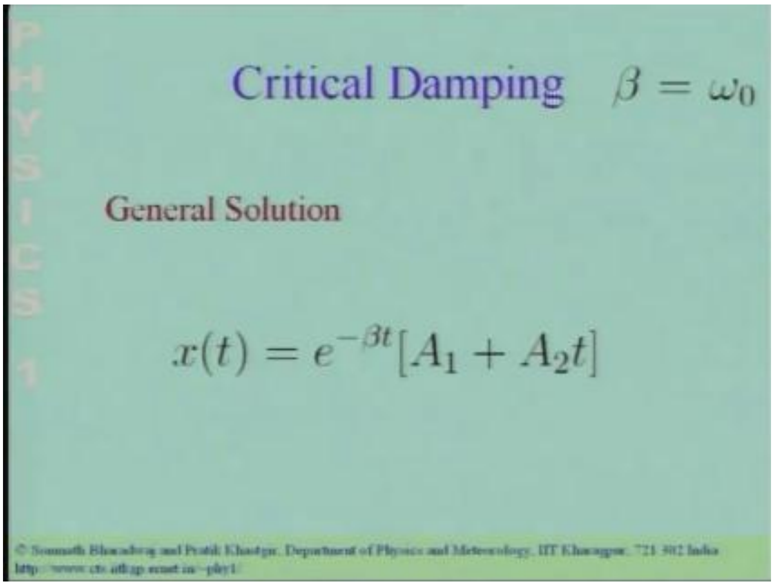

So, critical damping is when the damping coefficient beta is equal to the angular frequency omega naught.

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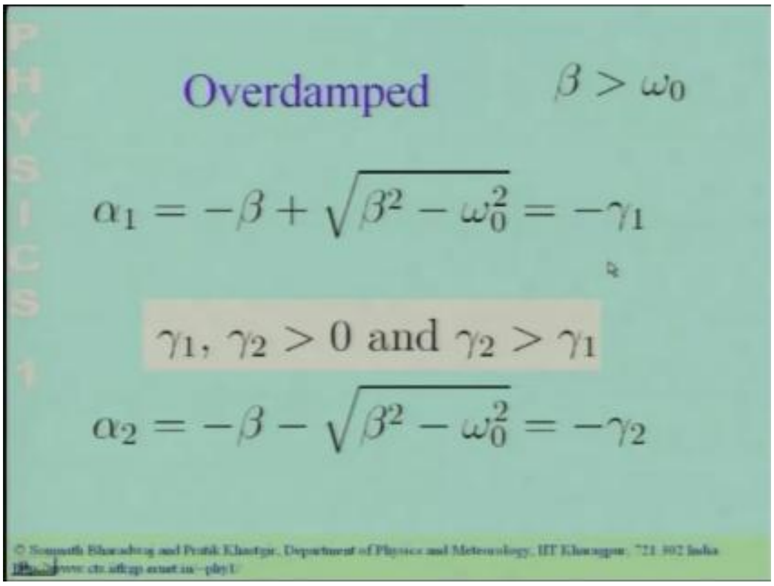

Now, if beta is equal to omega naught just look at this.

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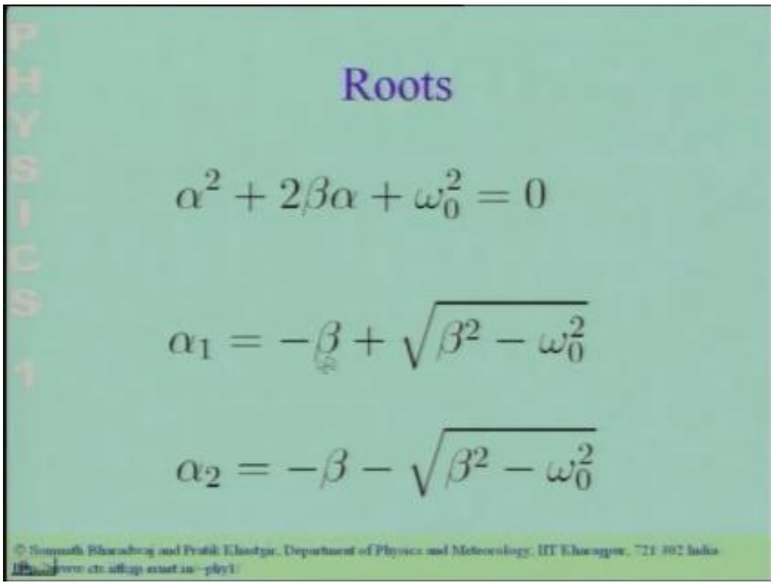

Beta is equal to omega naught, if beta is equal to omega naught then the 2 roots alpha 1 and alpha 2 are both equal to minus beta. So, that there is only 1 root to this quadratic equation.

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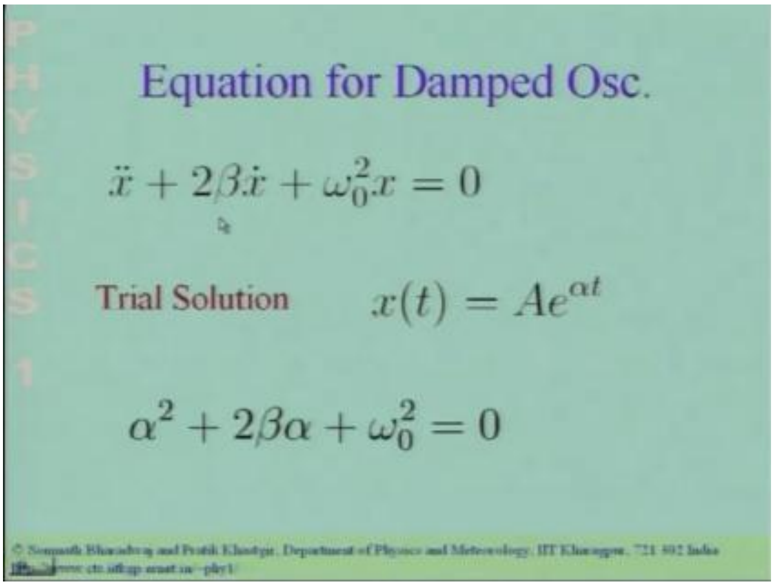

This equation is still a second order differential equation the equation governing the simple harmonic oscillator is still a second order differential equation. The difference now is that the solution is not just an exponential you have to consider a different kind of solution which is what is shown over here.

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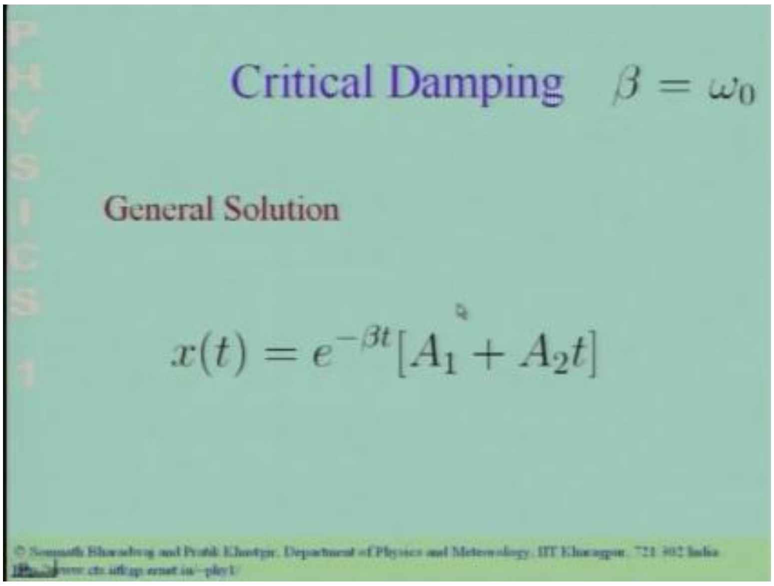

So, when you have a critically damped oscillator you only have 1 root that is beta which is equal to omega naught, but, you now have to consider, a solution e to the power minus beta into a constant plus A 2 of t. So, this constant is A 1 plus another constant into time.

So, you have 2 solutions still 1 of them is the exponential which you had earlier another solution which now comes up is time into the exponential. So, it is t into e to the power minus beta t.

So, you have these 2 solutions which could have 2 different coefficients and the most general solution is a super position of these 2 solutions.

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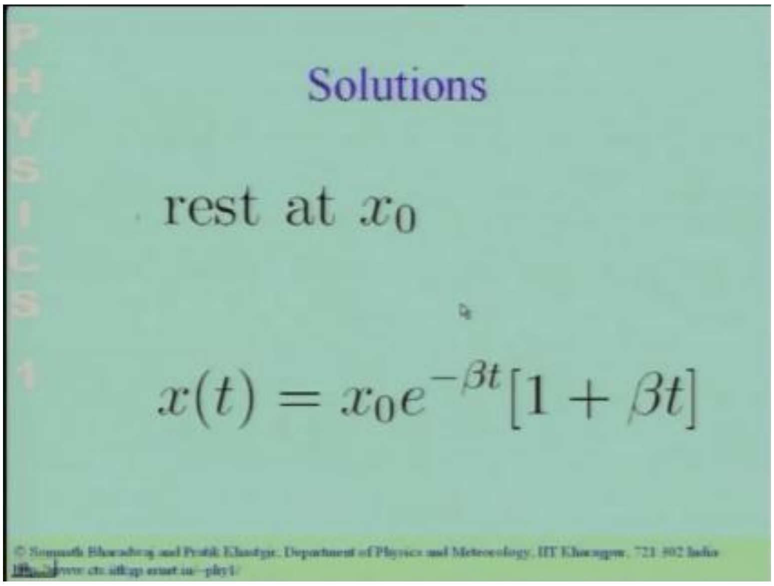

So, these coefficients have to be determined from the initial conditions and I first show you a situation, where the particle is initially at rest and it is displaced from the origin. So, it is at rest and it is displaced to a position x naught. If you calculate the initial conditions using the procedure which I have outlined in some detail for the under damped oscillator. So, you have to take the expression for x of t and the expression for the velocity.

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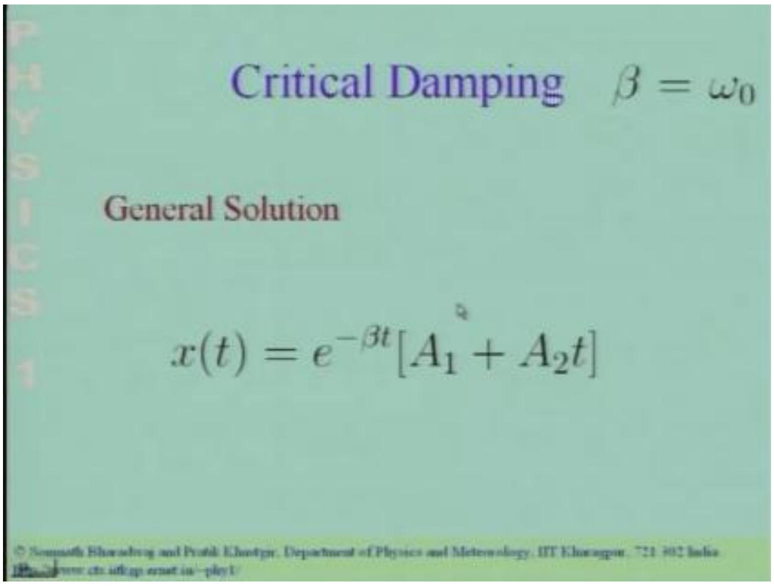

So, you have to take this expression for x of t differentiate it get an expression for v of t and then put in the fact.

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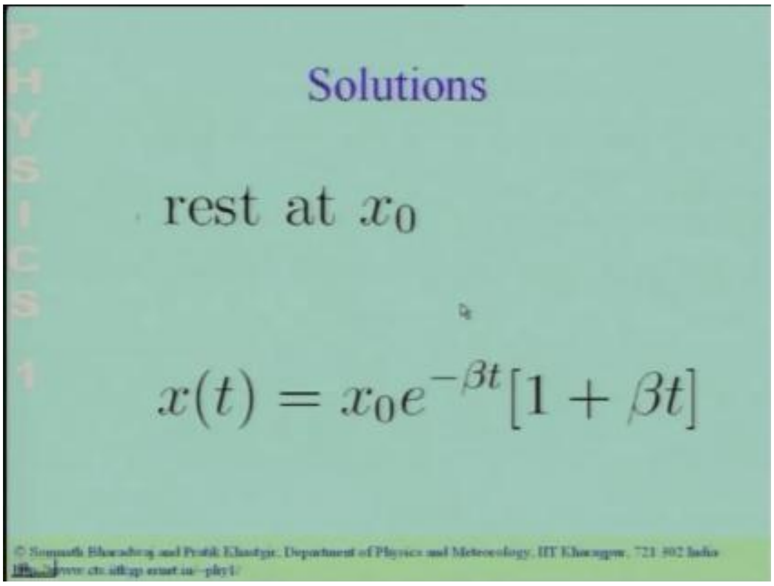

That the velocity is initially 0 and the particle is at x equal to x naught initially in this will give you A 1 and A 2 which you had in the previous slide. And putting in those values you get the solution it is x naught e to the power minus beta t 1 plus beta t.

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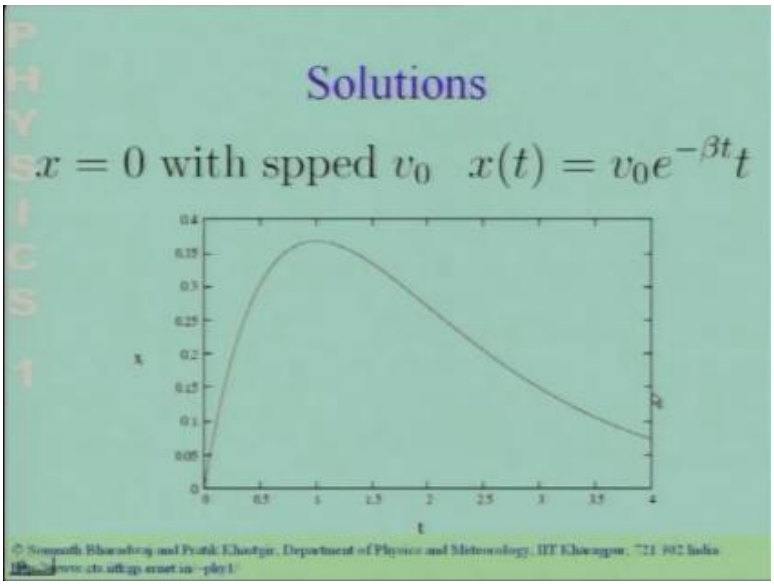

You could also have another situation, where the particle is at the origin to start with, but, it has been given of finite speed v naught. In this case the solution is v naught e to the power minus beta t into t. And this solution is the 1 which is shown over here the particle starts from the origin it goes up to a maximum displacement and then it falls off. And in the general situation where you have some initial position and velocity both you have to work out the general situation depending on from case to case which I have not done over here.

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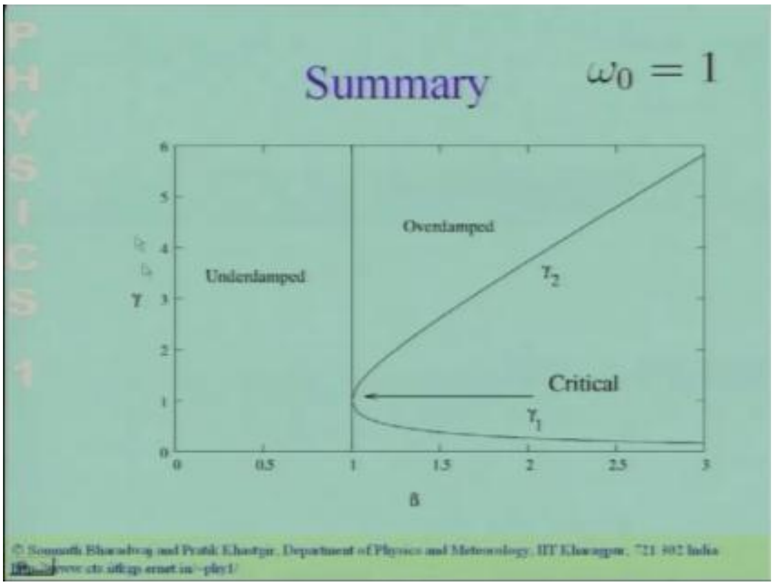

So, in summary we have 3 kinds of oscillations, we have the under damped, the over damped and critical oscillation.

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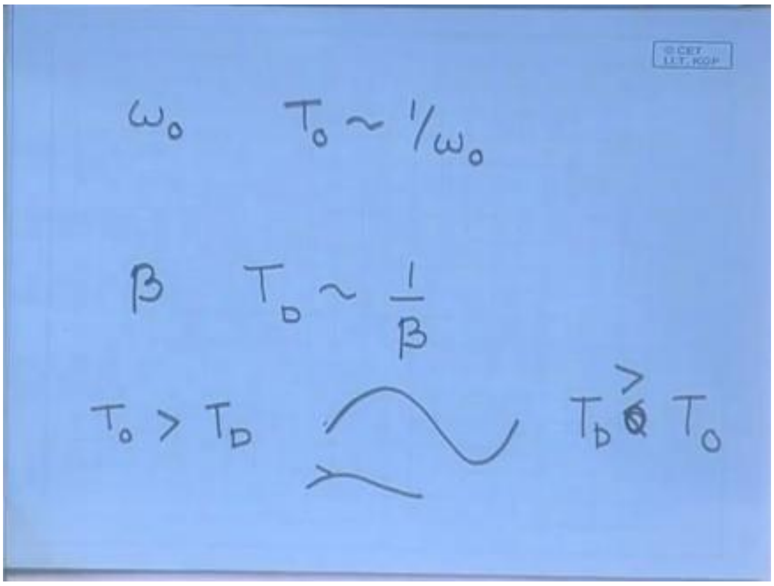

So, let me summarise, what we have learnt from this last 2 lectures.

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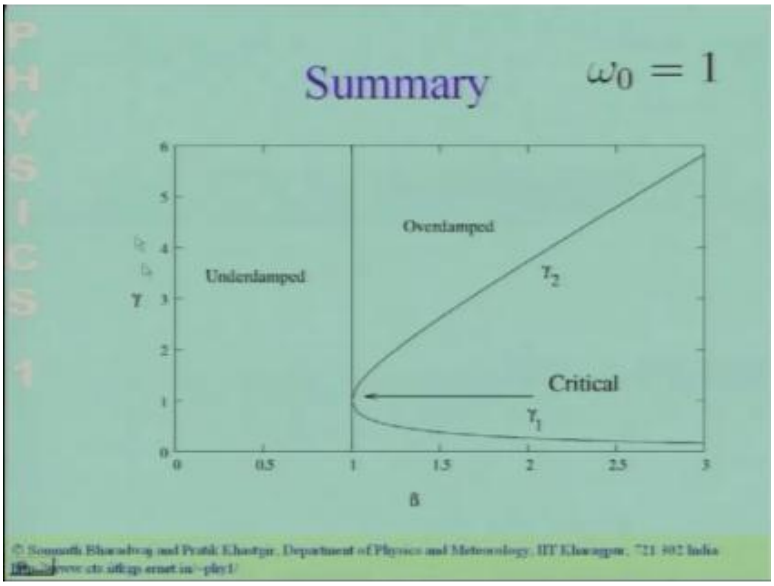

This figure over here summarises our findings. It shows it assumes omega naught the frequency of the undamped, if you have no damping the oscillator would oscillate with an angular frequency omega naught which has been chosen to be 1. So, omega naught is 1 you have the damping coefficient beta which you can vary if beta is less than omega naught that is it is less than the value 1 you have under damped oscillations, which is what is shown over here.

So, beta is plotted on the x axis of the graph, if beta is less than 1 you have under damped oscillations. The oscillator, oscillates the amplitude of the oscillation decays exponentially. If beta the damping coefficient is more than omega naught, which in this case is 1. So, if beta is more than omega naught you have over damped oscillations. In over damped oscillations there are no for an over damped oscillator, there are no oscillations that there are 2 solutions both of which decay exponentially and the exponent is e to the power minus gamma t.

So, there are 2 solutions gamma 1 and gamma 2. So, in this figure I show you the 2 solutions gamma 1 and gamma 2. 1 solution the larger solution as you increase the damping the larger solution increases and it will go to infinity, if you make the damping coefficient infinity. So, it will increase with 1 of the 1 of the exponents increases with beta where as the other exponent decreases slowly with beta it decreases as 1 by beta.

So, if you increase beta the other exponent the coefficient of the other exponent gamma 1 with the smaller exponent goes to 0. The situation where beta is equal to omega 1 you have critical damping the 2 roots become identical and they both have the value 1. So, this is the situation, where you have critical damping. This figure gives you an idea of the behaviour of the oscillator in the critically damped and the over damped situation. It is important to develop some kind of an intuition for what happens in these if you vary the damping coefficient.

Let us just consider, a hypothetical situation where we would like to make a door stopper door; door closer, door shutter rather. So, a door shutter is a device which you can attach to a door and its purpose is that if somebody opens the door after the person has gone through and let go and the person lets go of the door the door shutter will slowly close the door. Now, if you put just a spring fix it to the door frame and fix the other end to the door you then have an under damped oscillator.

Suppose the damping is quite small you would then have an under damped oscillator. So, if a person opens a door and leaves it the door would now be pulled back to the equilibrium position where it is shut, but, then it would swing back to the other side and it would swing back to and flow and you would have an under damped oscillator And a damped the oscillations would get damped slowly, but, it would keep on oscillating for quite some time and slowly it will go to the equilibrium position where it is at rest.

Now, in most situations we do not want a door shutter to function like this, we would like a door shutter to pull the door slowly to a position, where it is closed. So, we would like to have more damping now, the question is how much of damping should you put in so, that the door shutter functions in a reasonable time. Now, suppose you make the damping very large what happens let us just, look at the situation where you have a very large damping.

In the situation where you have a very large where the damping beta coefficient is much larger than the omega 1 omega naught. The 2 roots gamma 1 and gamma 2, 1 of the roots becomes very large and the other root becomes very small.

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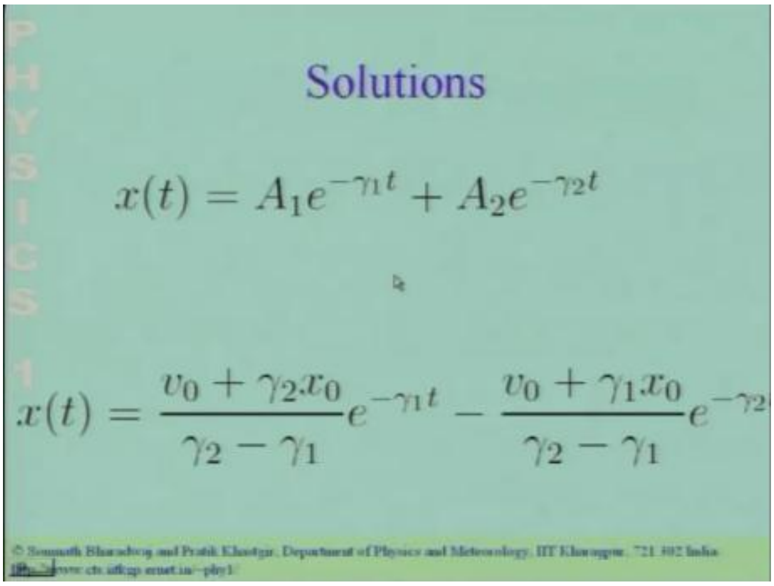

So, if you open the door and leave it the doors goes back to the equilibrium position where it is shut and the motion is governed is a super position of these 2 roots. So, you can set v naught equal to 0 and it will give you the solution you see that you have both the roots you have gamma e to the power minus gamma 1 t and e to the power minus 2 t. Now, gamma 2 is quite large. So, the gamma 2 root this particular solution is going to decay very fast, but, gamma 1 if for very high damping gamma 1 is very small.

So, this particular solution is going to decay extremely slowly. So, if you have very high damping the point to note is that; if you have very high damping the door is going to take an extremely large time to come back to the position where it is shut. This is because of the solution gamma 1 which is very small; if gamma 1 is very small this particular solution e to the power minus gamma 1 t is going to decay very slowly and it is going to take a long, long time to come to the position where the door is shut.

Now, if you ask the question at what time is the door going to be exactly shut it will take infinite time for the solution to reach x equal to 0. The exponential tk never really reaches x equal to 0 never reaches a value 0, it takes infinite time to reach a value 0. So, it is not really fruitful to ask the question when the when the door is exactly shut when x exactly equals to 0. A more relevant question would be when the for the time, when the door would be 90 percent shut or 99 percent shut as your case would be.

So, if you ask such a question then in this situation the door would be 90 percent shut after a long time because of the root gamma 1 which is extremely small.

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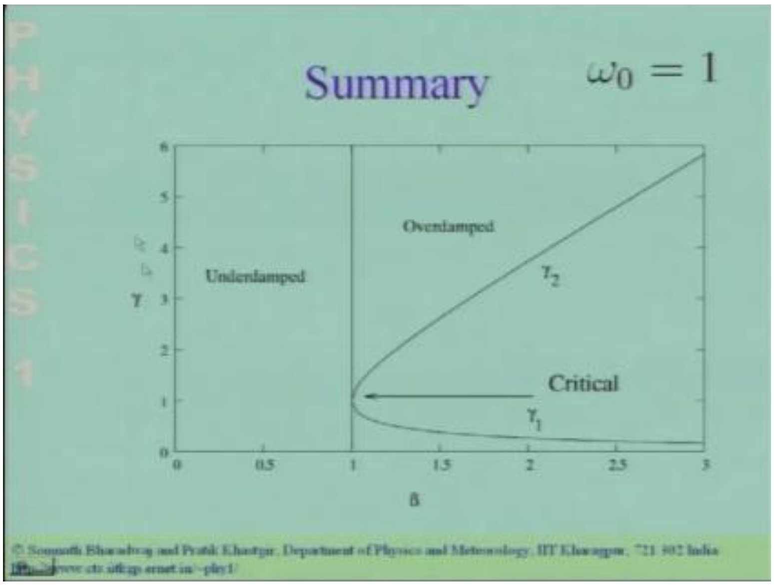

So, if you would like a, like your door like to design your door shutter. So, that it shuts in a reasonably small time then you should choose the value of beta to be close to the critical damping situation. The critical damping situation, the situation when you have critical damping is the 1 which corresponds which where the situation will come back to the equilibrium position to come back near the equilibrium position the fastest.

Whereas, if you have a very high damping the system will take a long time to come back to the equilibrium position, it may even get struck. So, it will essentially be struck far away from equilibrium and it will the e to the power minus gamma 1 t term will decay very slowly as it tends to equilibrium. So, you would like to have if you want the door shutter to shut in the fastest possible time you will choose it to be near the critically damped value. But again that may pose new dangers a person coming behind the person who has just gone through the door may have the door close on his face and he may end up with the he or she may end up with the broken nose.

So, you have to judiciously choose the damping parameter. So, that the door closes in a safe and a reasonable time.

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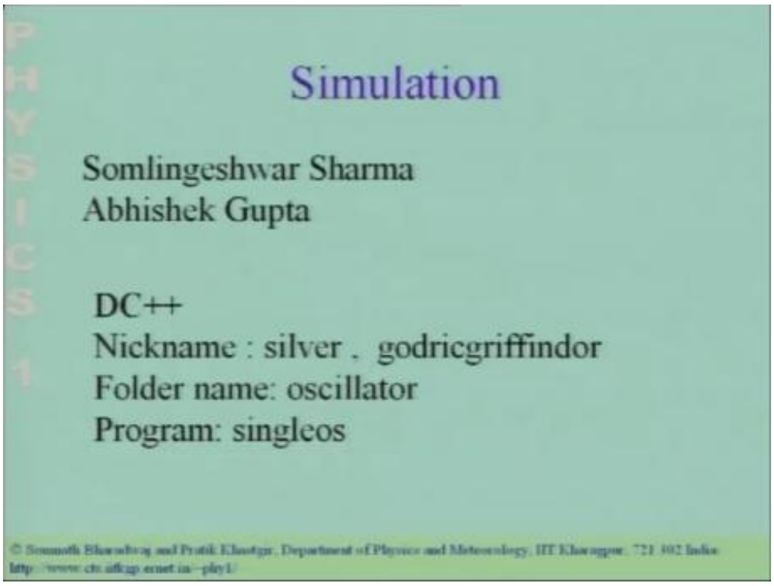

Let me, now show you simulation the simulation has been developed by 2 of our physics students Somlingeshwar Sharma and Abhishek Gupta they are now third year of the integrated MSC in physics. And if you are interested you can download the code using DC plus plus from the sight which is given over here. So, let me now move over to the simulation.

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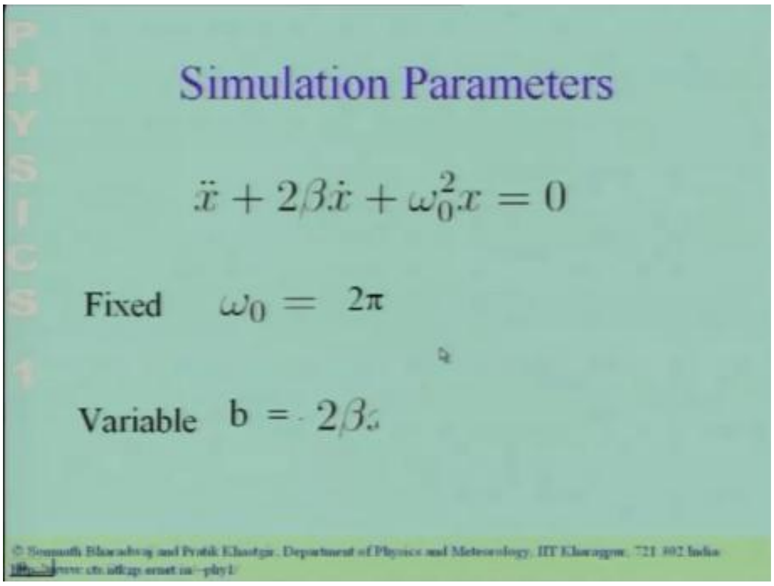

The simulation simulates the differential equation governing the damped oscillator the differential equation is shown here again. In this simulation the value of omega naught is fixed it has a value 2 pi this is fixed through the throughout the simulation you can vary the damping coefficient beta. The parameters which is available for you to vary in the simulation is the parameter b which is twice beta and to get the numbers straight. The situation corresponding to critical damping beta should be equal to 2 pi.

So, b should be equal to 4 pi 4 times pi has a value around 12.5. So, in the simulation omega naught is fixed I have another parameter b which I can vary. The value of b around 12.5 corresponds to critical damping, a value of b less than 12.5 corresponds to under damped and more than 12.5 corresponds to over damped. So, let us now move over to the simulation, which I had just mentioned.

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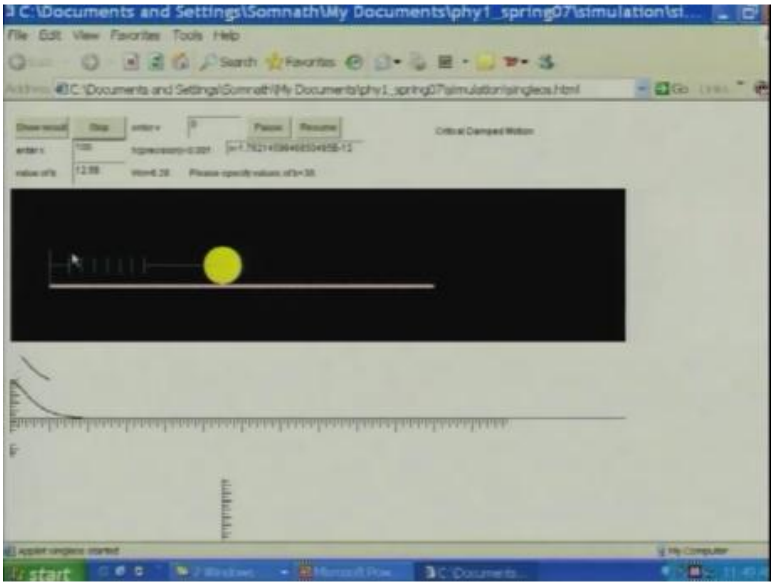

So, this is the spring mass system whose motion, we are going to simulate and in the simulation you can give in the initial position over here. So, it has been set the value is now 100. The initial velocity is 0 and let us, set a value for the value for b which let us, set the value of b to 0. So, there is no damping right now and let us see, how the oscillator behaves. So, this shows you the oscillator, the simple harmonic oscillator where there is no damping and it does the sinesoidal oscillation which you can see here as you expect it.

So, this figure over here, the figure below shows you x as a function of time. So, along the x axis over here you have the time and along the y axis you have the displacement of the particle.

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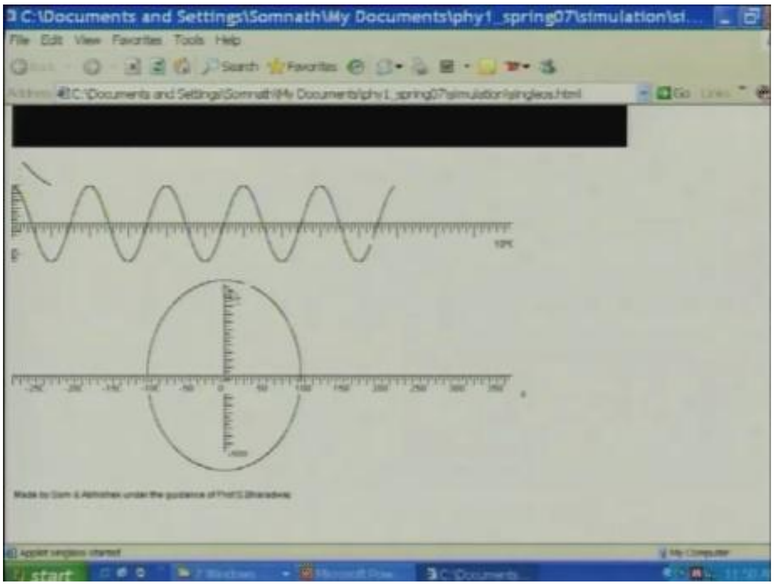

The figure which is there below shows you something called the phase plot. So, it shows you the trajectory of the particle in phase space. Phase space is very interesting and very useful the study of trajectory and phase space is very interesting and useful if you are studying dynamics of different kinds of systems

We shall not go into this in detail over here let me just tell you what this figure shows you this figure shows you the position of the particle along the x axis and along the y axis is the momentum or velocity. The velocity in this case the velocity of the particle. So, this shows you a trajectory in a space whose x axis is the position whose y axis is the velocity. In this space the particle trajectory here for a simple harmonic oscillator is an ellipse which you can see over here.

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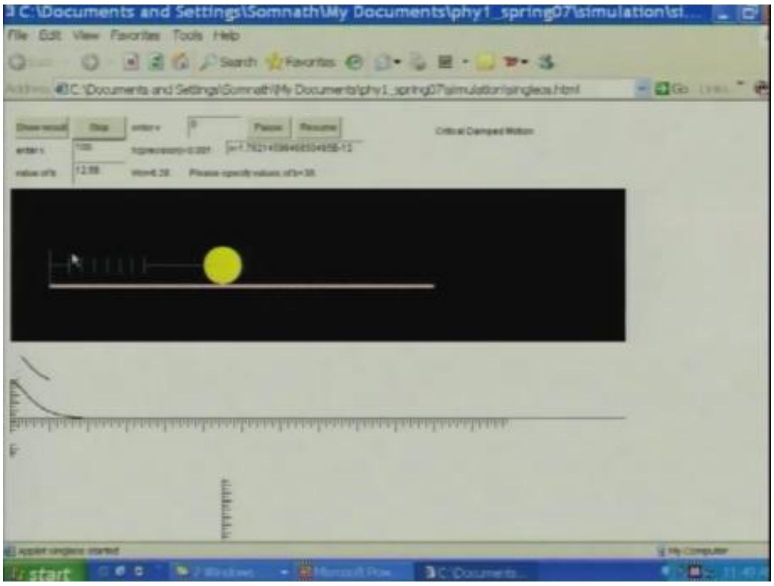

So, the particle continues to oscillate the amplitude of the particles oscillation remains the same it goes back and forth around the equilibrium position here there is no damping. Now, let us introduce some amount of damping and see how it behaves. So, let me introduce small damping. So, b the value of b which I have chosen is 1 let me, start the simulation again.

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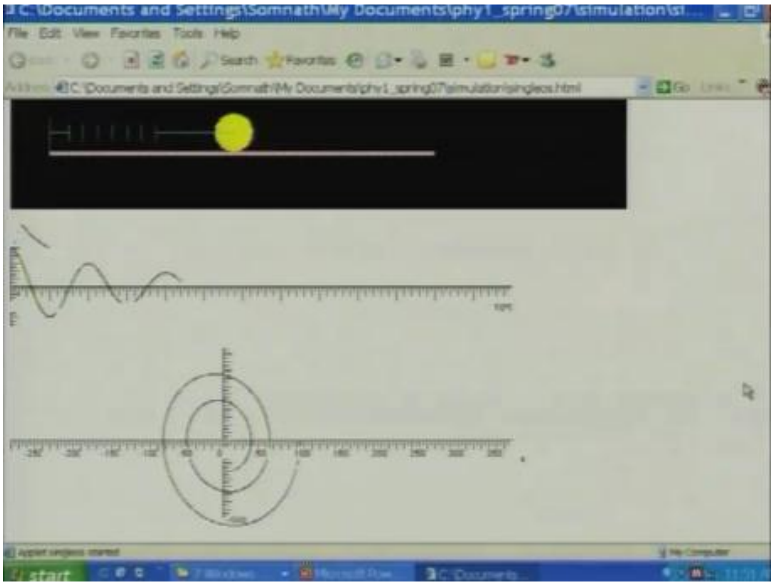

So, this shows you an under damped oscillation notice that the amplitude of the oscillation, as the system evolves with time the amplitude of the oscillation decays exponentially as we expect it for an under damped oscillator. And in phase space the particle slowly spirals into towards the centre, it is no longer an ellipse. The particle slowly spirals in towards the centre of phase space towards the equilibrium position where x is equal to 0 and v is equal to 0. It never really gets that it take infinite time to reach that, but, it slowly spirals in.

So, notice that their amplitude decays exponentially as the oscillation proceeds. Let's see what happens; if I increase the amplitude further. So, I will make it 4 increase the damping further. So, I have made the coefficient b which is twice beta equal to 4. So, twice beta is 4 beta is equal to 2. So, notice that the damping occurs much faster now, the frequency of the oscillation is also changed and it has more or less died away within 2 oscillations whereas, it had you could the oscillations for longer time in the earlier situation.

The c can no longer see the particle moving its oscillating, but, the oscillation is very small. Now, let us next take an over damped situation an over damped situation would correspond to b the value of b which is more than 4 pi. So, a value which is more than 12.5. So, let us choose a value for b which is around let us say, 20. So, let us choose a value for b which is around 20.

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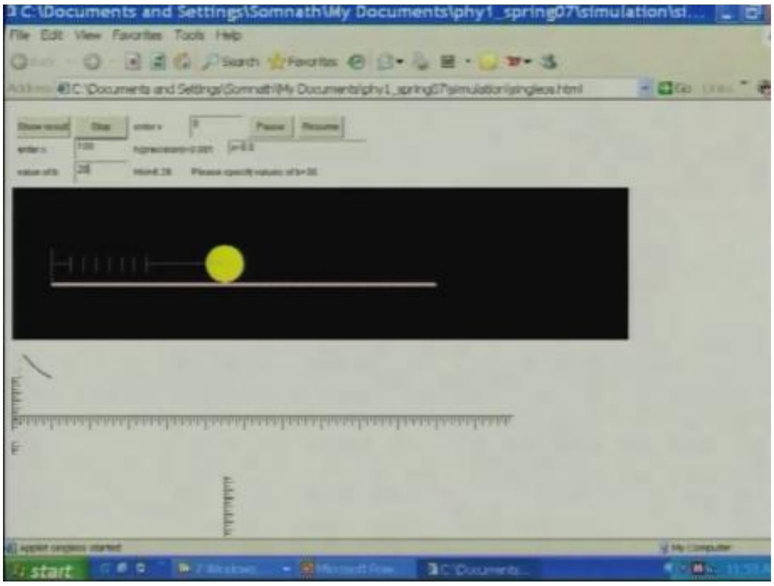

So, the value of b has been set to 20 and let us run the simulation again, see notice that the oscillations have been totally killed and the particle slowly tends to the equilibrium position as time evolves.

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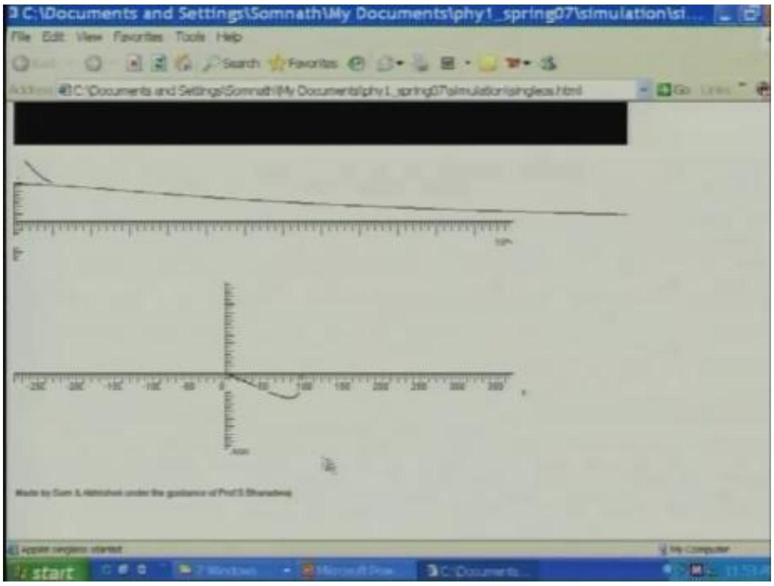

So, this is the situation in phase space the particle starts from here and it slowly tends to the equilibrium position as time evolves.

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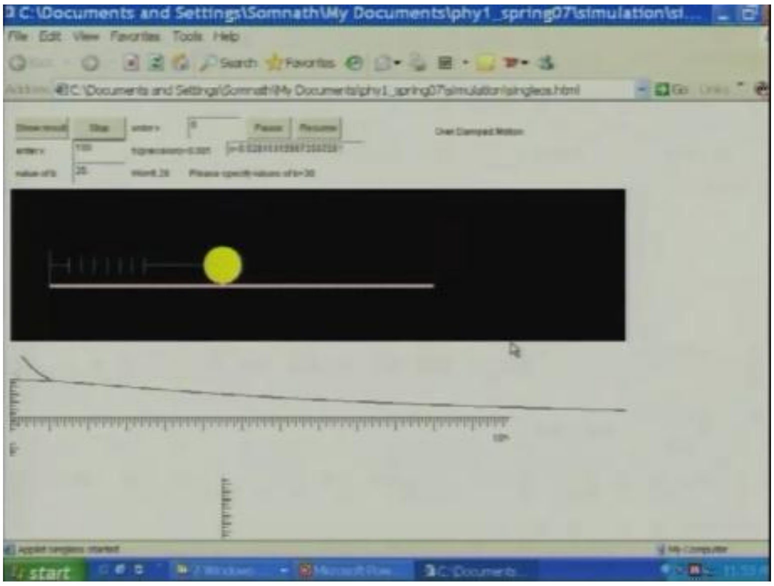

Now, let us see what happens if you increasing a damping. So, let us make the damping 100 the coefficient be 100. So, what we expect is that the: decay will occur much slower as I increase a damping coefficient beta this is what you see here you hardly notice any damping. So, the particle approaches the equilibrium position very slowly it is more or less stuck. So, it very slowly approaches the equilibrium position.

If I increase a damping even further oh there is an upper limit to be anywhere, if I increase a damping even further if you would notice that it would approach the equilibrium position even slower. Now, let us consider the situation where I have very close to critical damping. So, when I have very close to critical damping which is a situation where we are which we are considering now. The coefficient b the parameter b has a value 12.5 which is very close to the value 4 pi let us see what happens now.

So, notice that the particle, the displace particle reaches the equilibrium position reaches very close to the equilibrium position much faster than the situation where we had a large damping coefficient. Let me, show you the situation with large damping coefficient again. So, this is a situation with a large damping coefficient, it approaches the equilibrium position extremely slowly this is because of the e to the power minus gamma 1 t, gamma 1 is extremely small.

So, the particle approaches the equilibrium that gamma very slowly the term decays very slowly whereas, if you have very close to critical damping. So, if you have a situation which is very close to critical damping, which we are considering again it decays to the equilibrium position very fast and there are no oscillation they decays to the equilibrium position very fast. And it slowly and keeps on approaching it with as a time increases.

Let me finally, show you 1 situation where we also have starting velocity and then I will stop. So, we have both initial position. Let us do away with the initial displacement only a initial velocity and no initial displacement. So, this is a situation let me make the amplitude a little larger this was very difficult to see. So, at the amplitude of the initial velocity is 1000. So, notice the particle in this situation starts from the origin with a initial velocity, it moves to a extremum and then slowly decays to the equilibrium position which is what you see here.

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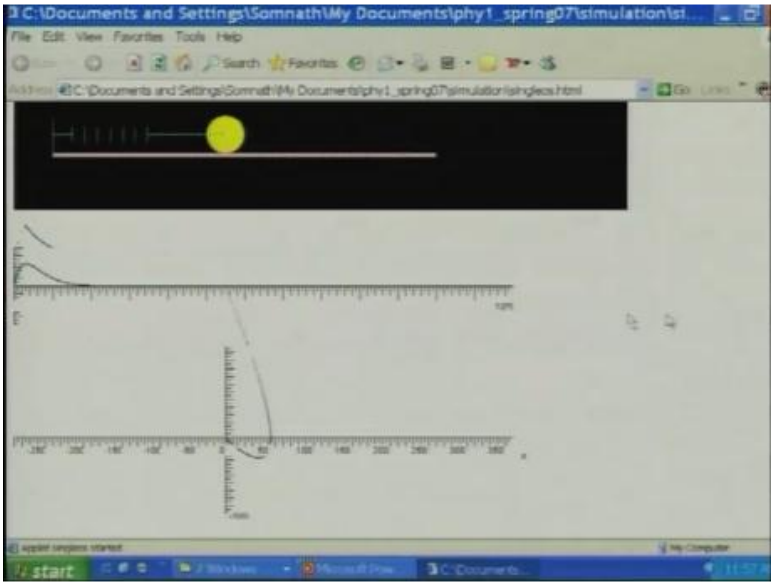

The particle starts, from the origin it has a velocity. So, it will go it will move away from the origin reach an extremum and then slowly decay to the equilibrium position with time. So, in the in these simulations I have shown you all of the situations which we had worked out analytically and if you are interested you can download the simulation and try out various other values of the parameters and the initial conditions. And see for yourself how things behave.

So, in summary damped oscillators you can have 3 different situations, if the damping coefficient beta is less than omega naught you have an under damped oscillator, the oscillator oscillates the amplitude decays exponentially. If beta is more than omega naught then oscillations are killed. So, if you displace the particle and leave it the displacement decays exponentially and the particle approaches the equilibrium there are no oscillations.

The more the damping, the more time the particle takes to approach the equilibrium. Critical damping you have critical damping, when beta is equal to omega naught in this case there are no oscillations again, if you displace a particle and leave it will slowly approach the equilibrium position there will be no oscillations whatsoever and this is a situation, where the particle reaches the equilibrium position fastest.

So, these are the main features of what happens to an oscillator when you introduce damping. So, today we shall stop here in the next class, we shall consider what happens to an oscillator if you put in an external time dependent force.