Physics I : Oscillations and Waves
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Lecture - 29
Standing Waves

We are discussing standing waves; we have started the discussion in the last class. So, let we first recapitulate what we have done? And then I shall proceed from there.

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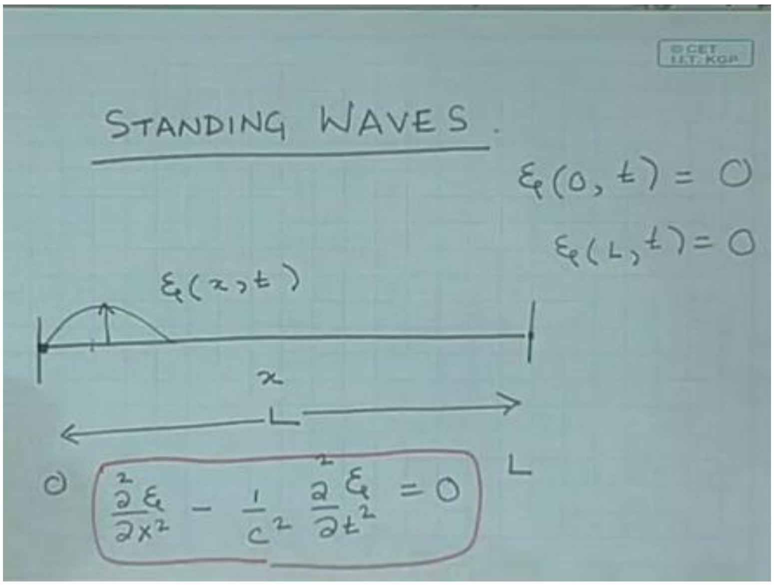

We were discussing a situation, where there is stretched string, the length of this string is L and we use the variable x to denote the distance along the length of string. So, x goes from 0 to L the string is stretched, so, it has a tension t and we consider a situation, where the string is plucked. So, a disturbance is introduced in the string and then the string is lead to vibrate and we would like to study the time evolution of these vibrations that I introduced in the string. So, I had told you in the last class, that the disturbance that transfers displacement of the string, which is the disturbance in this case the transfer displacement xi, which is the function of x and t. The evolution of xi is governed by the wave equation the same wave equation, which we have been studying. So, the evolution of xi is governed by the wave equation.

So, this is the wave equation, which governs the evolution of xi, let me remind you again the xi is the transfers displacement of the string. It is the disturbance in the direction perpendicular to the string and the constant c, which is the phase velocity of this disturbance of the wave in the string. In this case is the square root of the tension divided by the mass per unit length of the string. So, the string has a mass per unit length mu and tension t then, this c over here is a square root of t by mu. So, we are looking for solutions of this wave equation, because the transfers' displacement the disturbance is governed by this wave equation. But the solutions have to now satisfy the boundary conditions that the string cannot vibrate at the 2 ends the 2 ends of the strings are fixed,

So, there will be no displacement at these 2 ends writing these down in terms of xi, this tells us that xi 0 t at all times, the value of xi at x equal to 0 has to be 0, also at all times the value of x xi at x equal to L has to be 0. So, at x equal to L and x equal to 0 the value of xi has to vanish at all times. So, we have to find solutions to this wave equation, which satisfies this boundary condition. And we had proceeded to look for solutions by the method of separation of variables. So, in this method, what you do is you assume that xi the displacement the disturbance is a function of is a product of 2 functions 1 of X alone and 1 of time alone.

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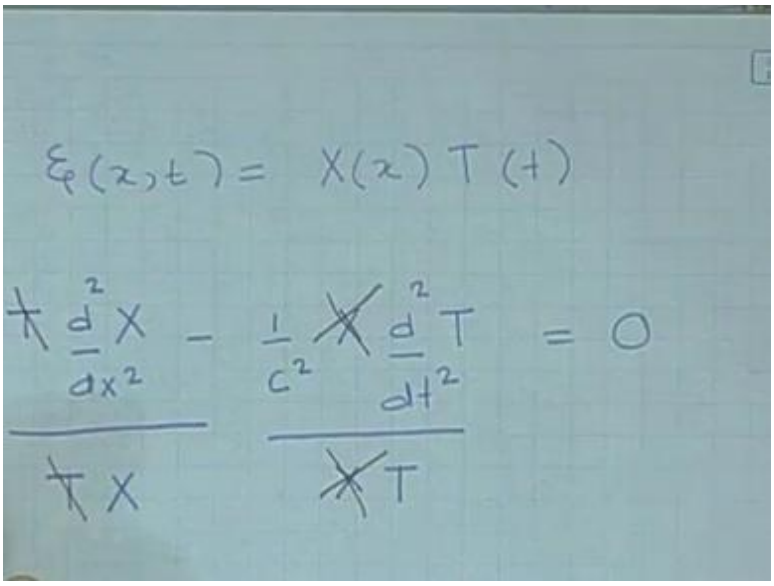

So, xi is a function of X and T we are using the method of separation of variables in this method we assume that xi is a product of 2 functions 1 of X alone and 1 of T alone. So, we take a trial solution, which looks like this and we then.

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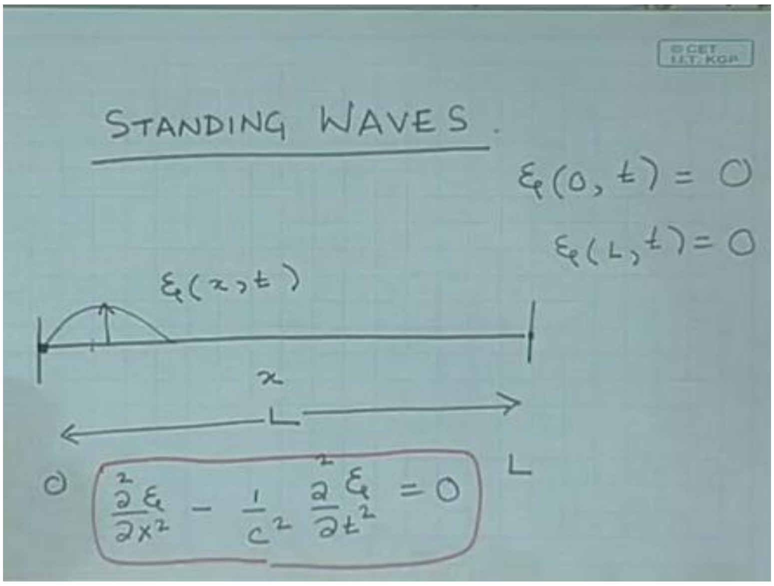

Plug it in into the wave equation; we have done this in last class. So, once you put it in here the X derivative does not act on T.

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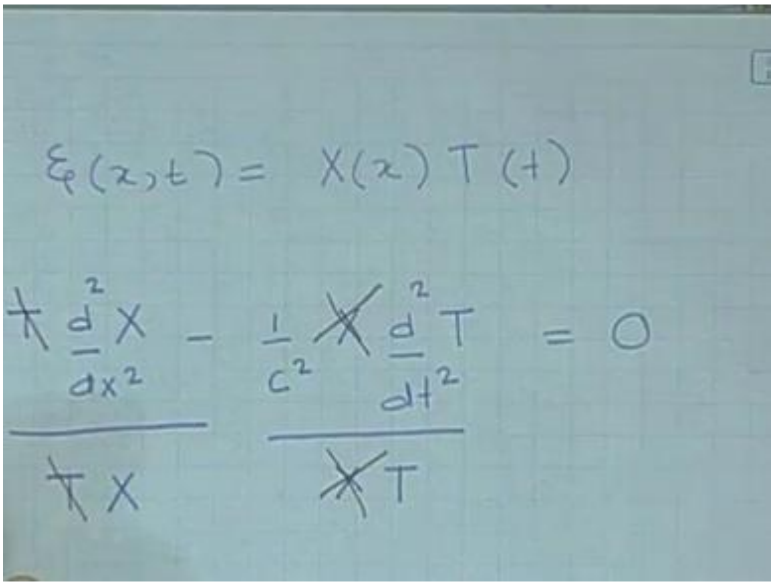

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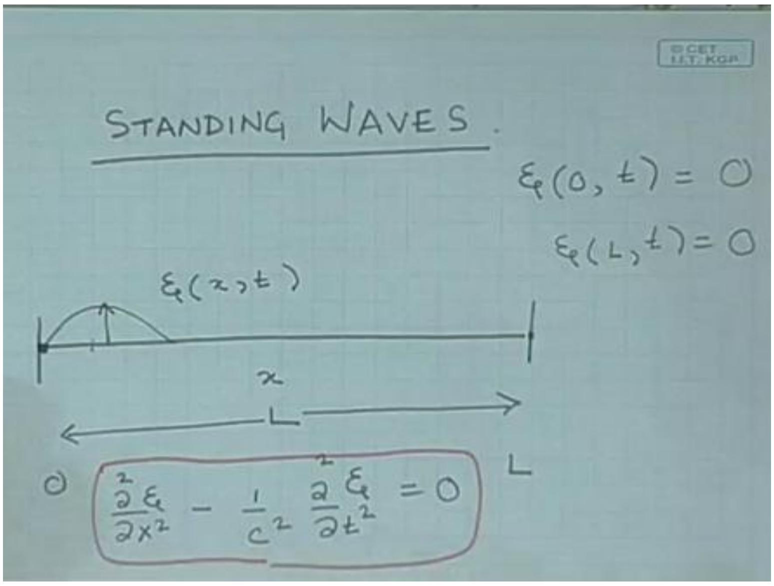

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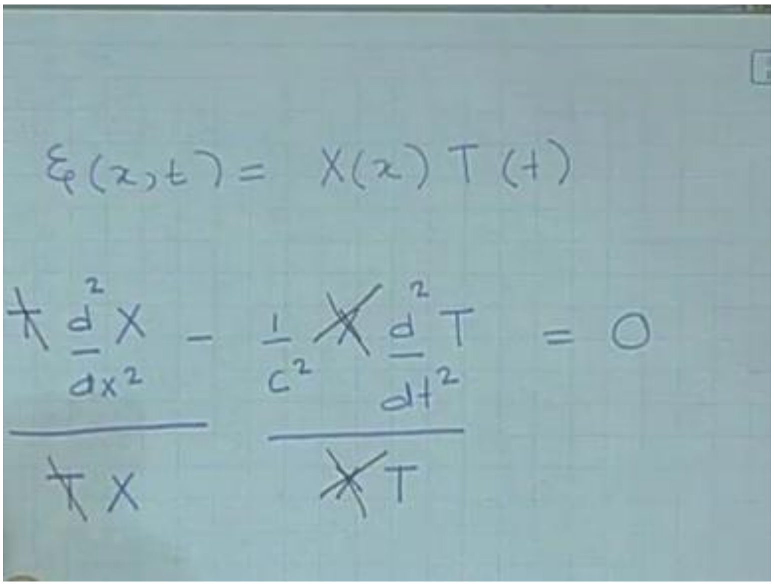

So, the term involving the X derivative, this term over here gives us this T into the ordinary derivative of X the second derivative, the term involving the time.

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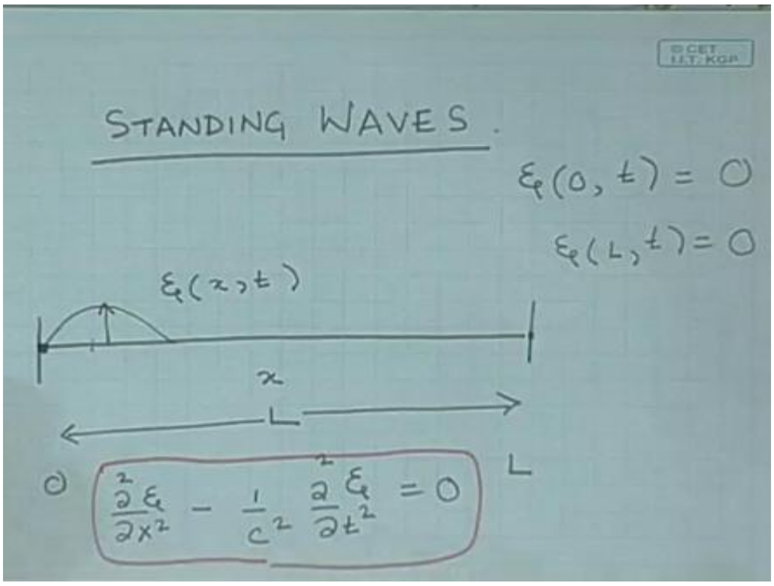

Derivative this term over here now, if I write xi has capital X into capital T this time derivative will act only on capital T.

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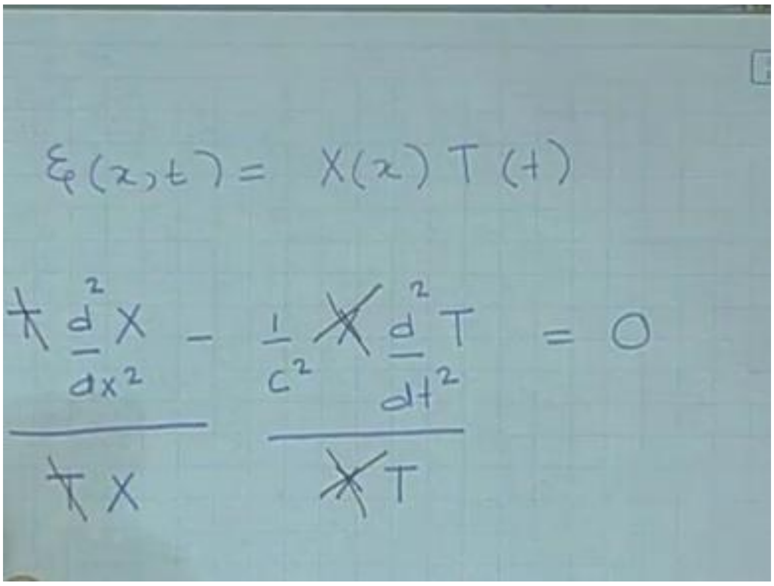

So, this time derivative acts only, on capital T and it is now, an ordinary derivative there is no need to retain partial derivatives anymore and I can take the function X outside. The next thing, which we did in the last class, was we divided this, whole ordinary differential equation. So, this was the equation that we got. Now, we divided this equation by xi which is dividing it by T into X.

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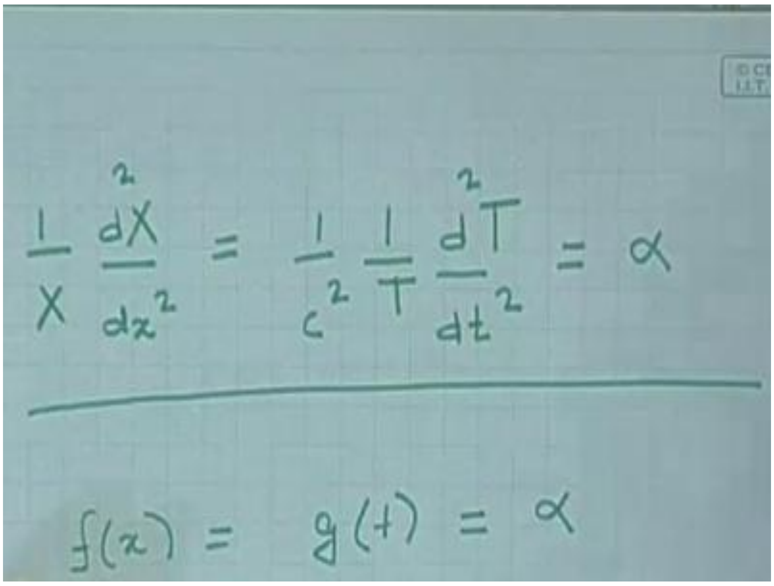

So, the factor of T over here cancelled out and the factor of X over here cancelled out, and you are left with the equation that 1 by X the second derivative of capital X 1 by capital X second derivative of capital X with respect to the variable x small x is equal to 1 by c square c square is the speed. The c is the phase velocity of the wave in that medium into 1 by capital T the second derivative of capital T with respect to time.

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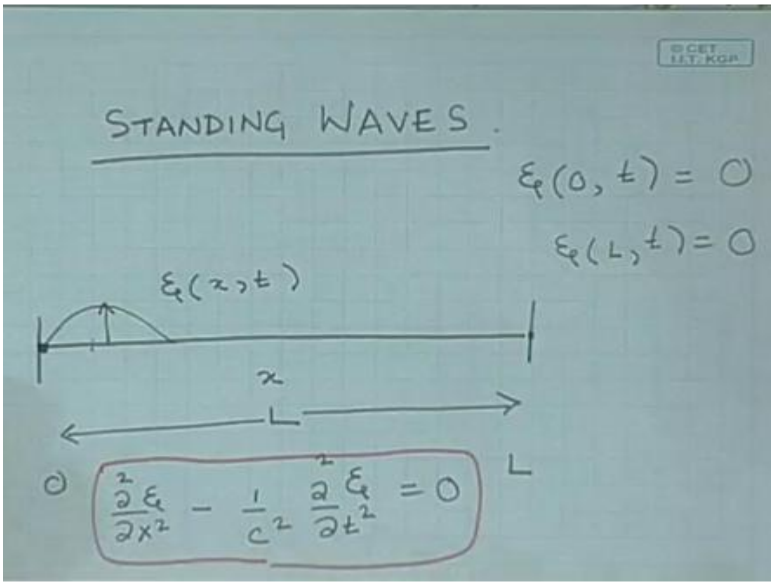

So, this has to be, what we see is that this is equal to this.

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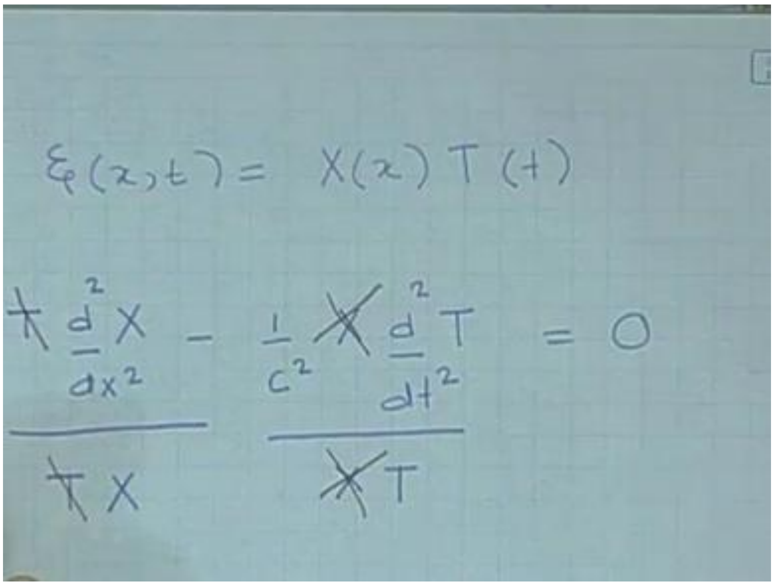

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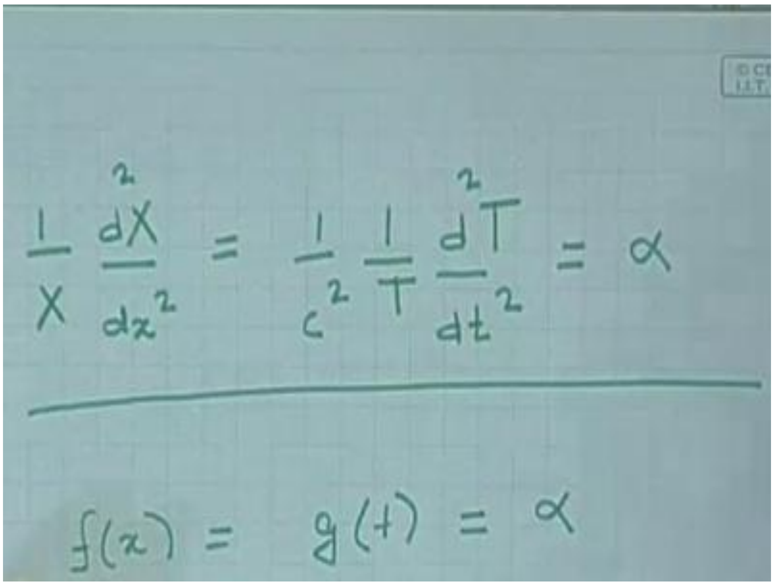

Substitution gives us this equation where, we see that the left hand side of the equation is just the function of x the position, the right hand side this term over here is just a function of time. So, you have a function of x is equal to a function of time, you can vary x randomly, you vary time has you wish, but these are still equal. So, this implies, that both of these should be separately, equal to a constant and I have used alpha to denote this constant. So, what we see? Is that, if you use the separation of variables, you get 2 ordinary differential equations. One where this is a constant and another, where this is a constant the constant is the same for both of these. So, let us now, take up the solution of the spatial part of this equation first. So, we are going to first discuss the solution of the spatial part the x dependence, which is this term over here. So, the equation, that we are trying to solve, so, let me write down the equation, that we are trying to solve the equation, that we are trying to solve is this.

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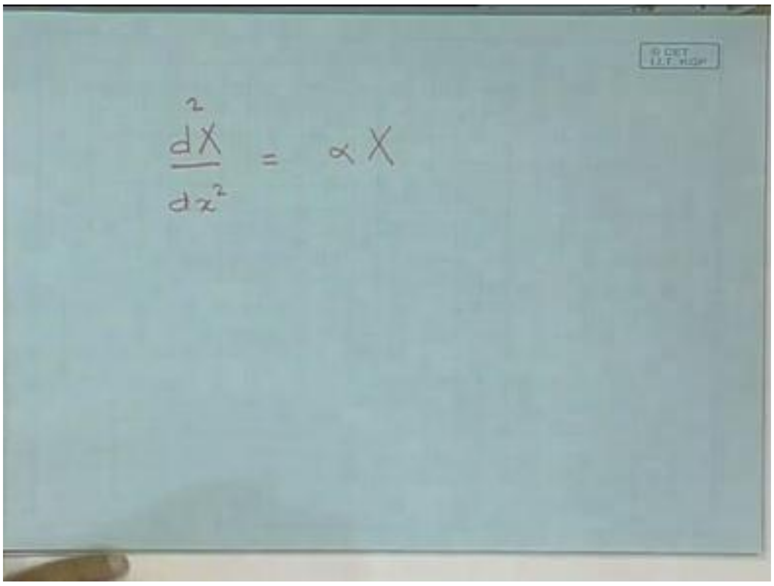

Right.

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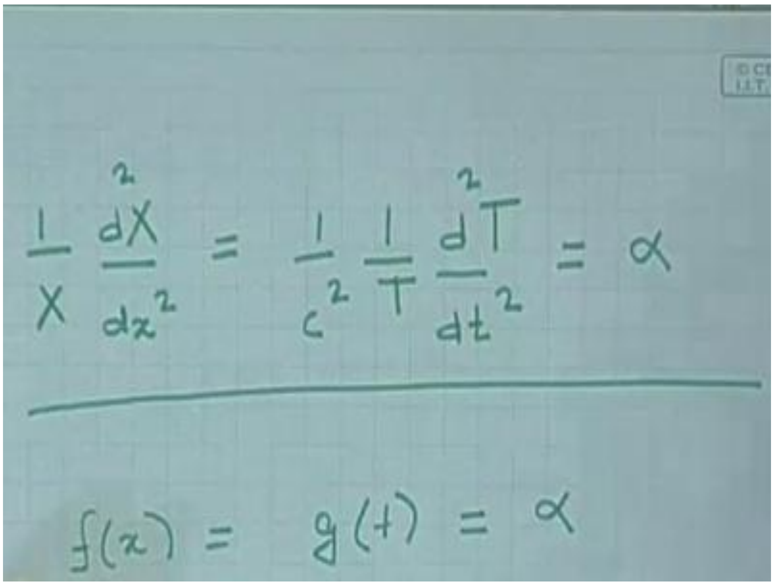

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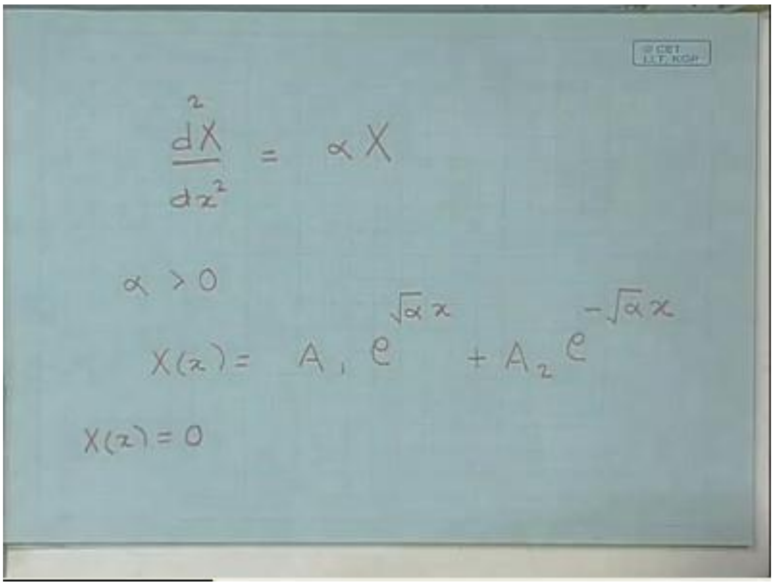

So, this is the equation, that we are trying to solve, we are looking at only the spatial part which is the, this should be equal to this which is what I have written over here. So, this is the ordinary differential equation that, we are trying to solve. And if this constant alpha is greater than 0 then we can write down the solution. The solution is some constant, which I will call A 1 e to the power square root of alpha x plus A 2 e to the power minus square root of alpha x. So, we have worked out the solution to the spatial dependence to the spatial dependent part of the wave of the disturbance xi. And we find that, if alpha is positive it looks like, this it is 1 exponential, it is an exponential of square root of alpha into x. And then I have minus exponential of square root of exponential of minus square root of alpha into x there are 2 terms. And we have to impose the boundary condition that, this function should vanish at x equal to 0. So, we have to impose the boundary condition that, the function has to vanish at x equal to 0. Let me remind you again, where this boundary condition comes from.

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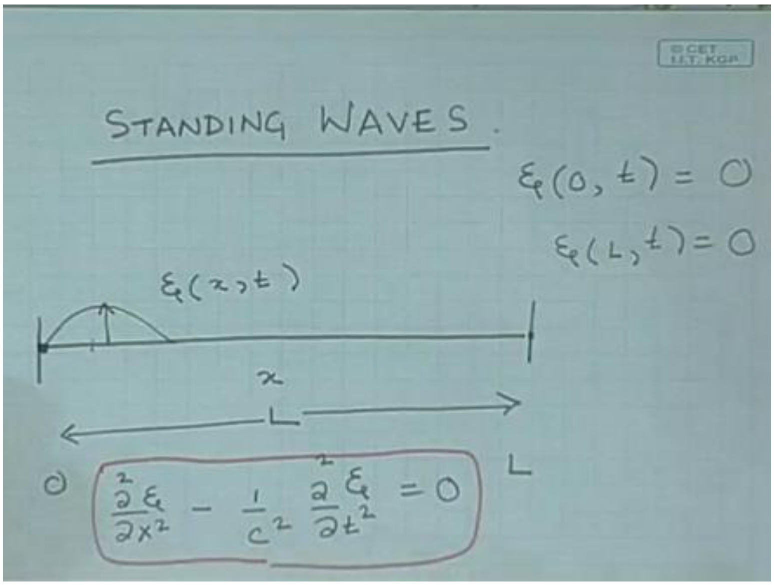

We are dealing with the situation; where we have a string the string is free to vibrate in any fashion expect at the boundary points at the 2 end points, which are fixed. So, xi has to vanish at x equal to 0 and it also has to vanish at x equal to L. So, the fact that xi has to vanish at x equal to 0 and x equal to L should be born in mind.

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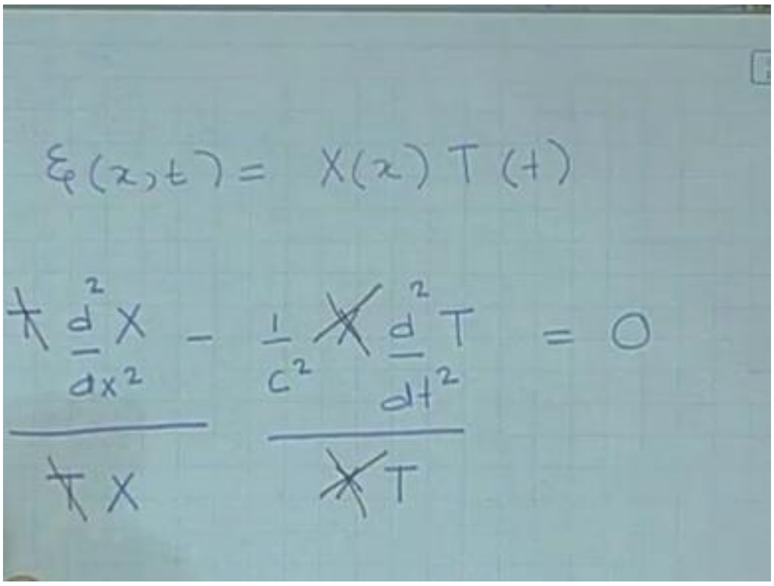

And we have replaced, we have express xi as a product of 2 functions 1 of x and 1 of time. So, if xi has to vanish at x equal to 0 and x equal to L it implies, at all values of time, it implies that this function should be 0 at x equal to 0 and x equal to L. So, we are imposing the boundary condition.

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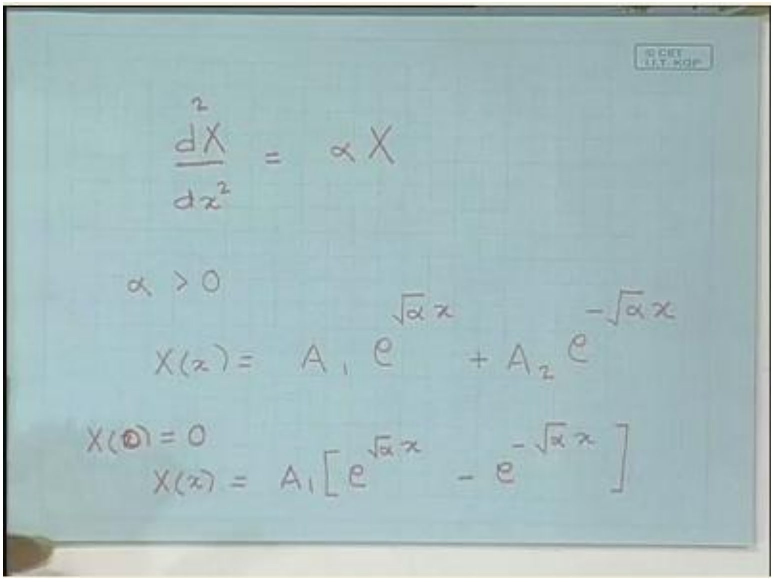

That x equal to this X has to be 0 at x equal to L, so, this has to be 0, which basically, tells us that at x equal to 0. If this function has to be 0 it tells us that A 1 and A 2 are minus of each of are exactly, the same and they have opposite signs. So, they have the same magnitude and opposite signs or it tell us, that X is equal to A 1 e to the power square root of alpha x minus e to the power minus square root of alpha x. So, this you see, has satisfies the boundary condition that at x equal to 0, this is 1 this is also 1. So, at x equal to 0 these 2 terms cancel out. So, this function capital X is 0 at 1 end of the string now, we require to also impose the boundary condition.

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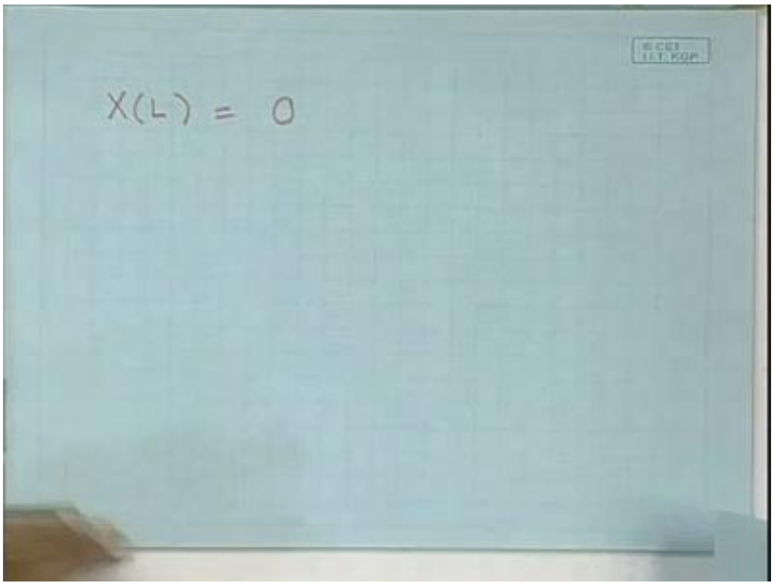

That this, vibration has to vanish at the other end of the string, which is X equal to L the position X equal to L.

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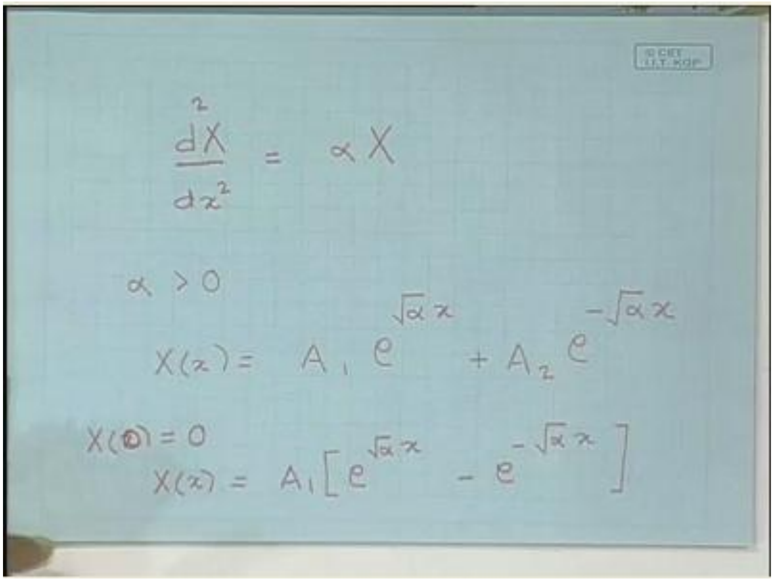

And it is quite clear, that you cannot you do not have a solution; it is not possible to find a solution, which satisfies both boundary conditions.

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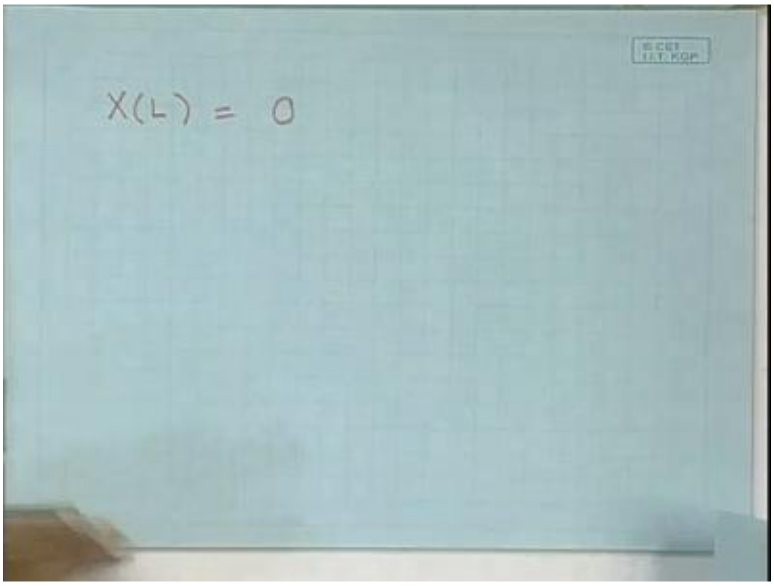

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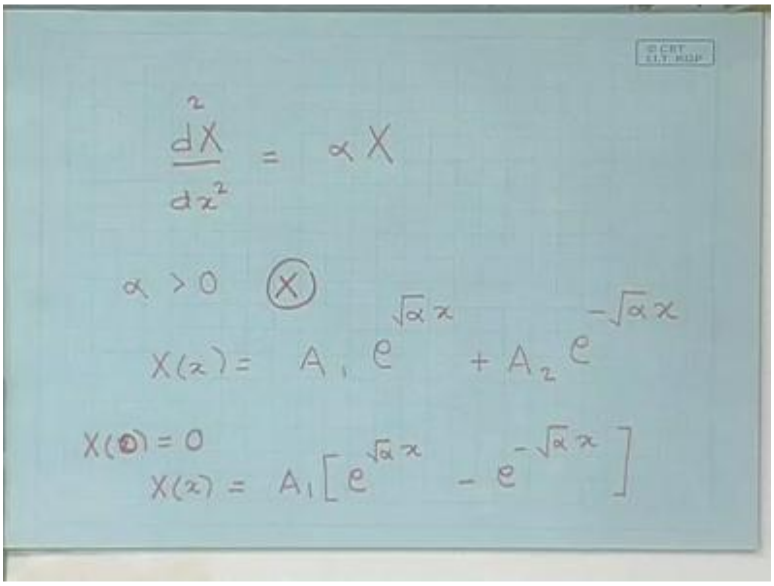

Because the only way, you can satisfy this with this particular solution is by setting this, constant equal to 0 which means, that there is no disturbance, which is not which is a trivial solution where there is no disturbance. So, the case the situation, where alpha is greater than 0 is ruled out this is ruled out. So, this situation is ruled out and we have to look at the situation where alpha is less than 0. So, if alpha is less than 0 that is the situation, we are looking at solution of this equation. Where alpha is less than 0, so, if alpha is less than 0, it is convenient to write alpha. So, we are looking at looking for solutions to this.

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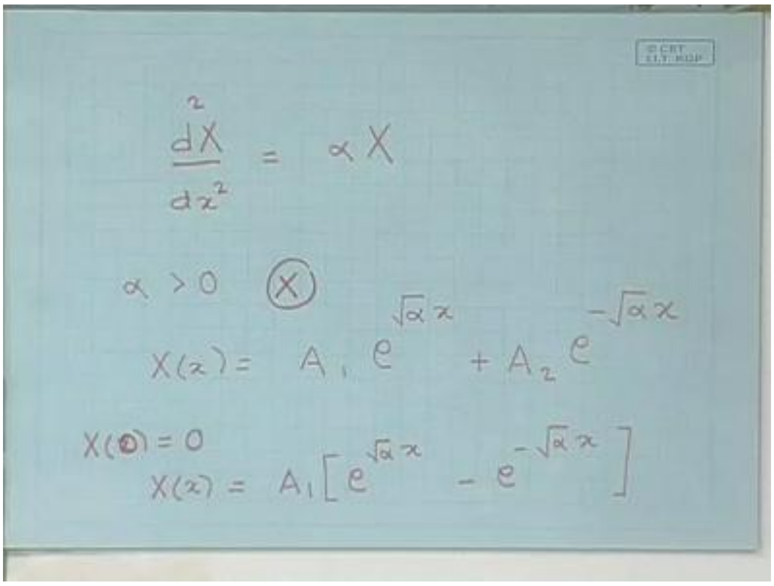

Ordinary differential equation, where alpha is less than 0. So, it is convenient to write alpha as alpha is equal to minus k square.

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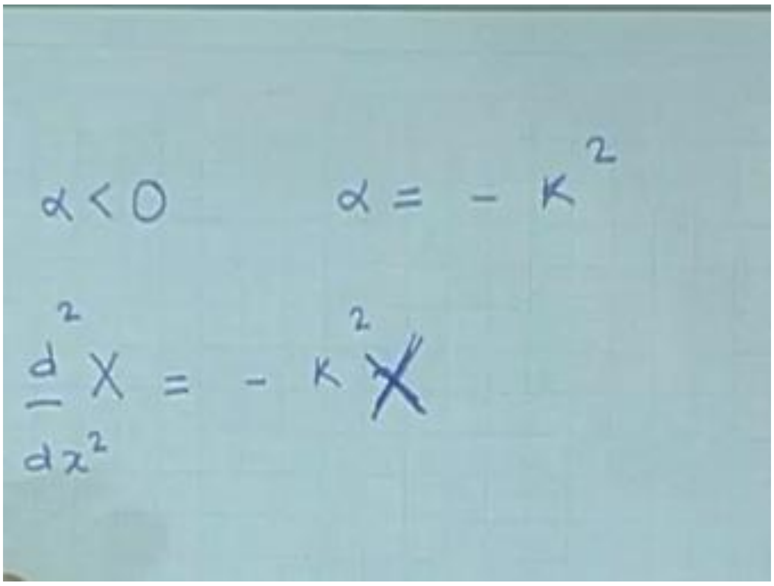

Alpha is less than 0 and alpha is equal to minus k square, we just rename this variable. Since it is less than 0, we can write it like this, where now, k is positive with this substitution the equation governing capital X.

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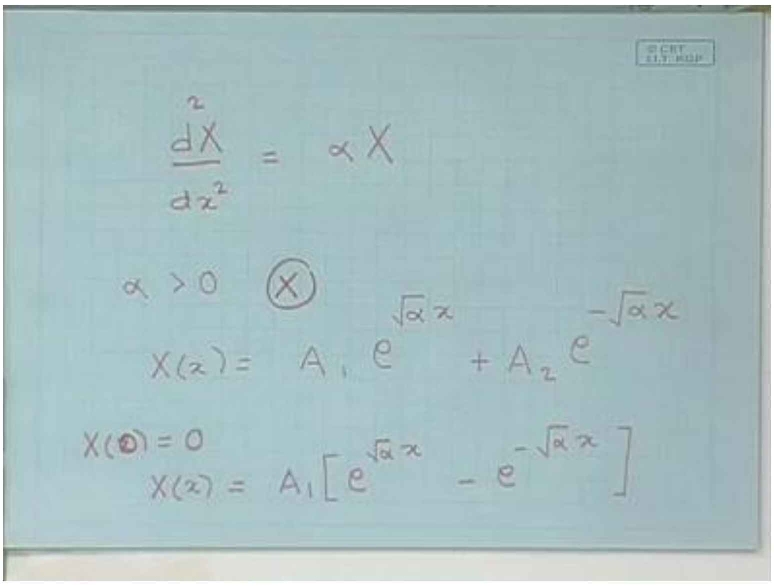

Now, becomes so, this is the differential equation governing the evolution of capital X which is here the X dependence of capital X.

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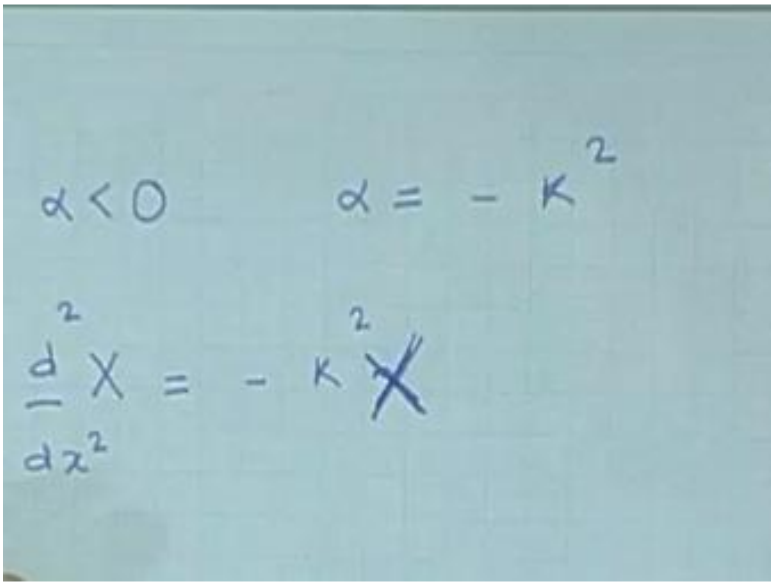

The same equation just that, I have replaced alpha by minus k square now, this differential equation that, you see over here. This particular differential equation is quite familiar to all of us. We have discussed this differential equation right in the first lecture of this course. And it is quite clear that this is the equation governing the simple harmonic oscillator. The position of the simple harmonic oscillator here has been replaced.

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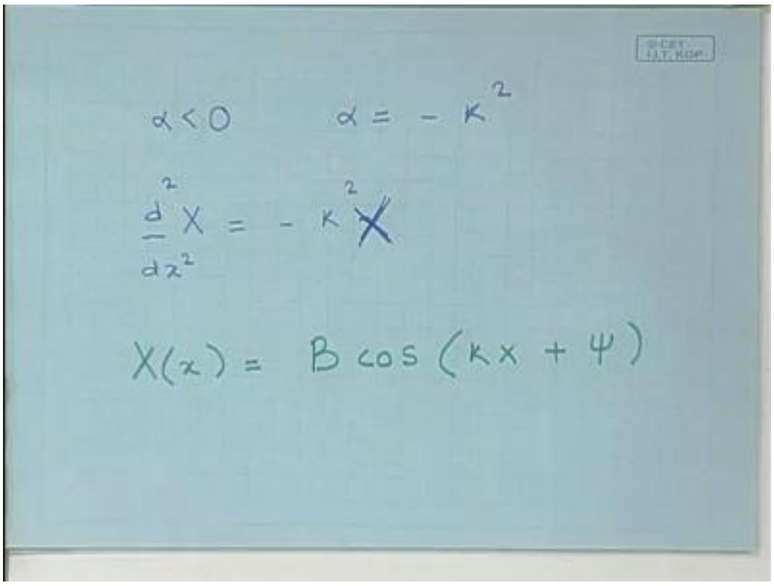

By this variable capital X and time has been replaced by this variable small x. So, this is nothing, but the equation governing a simple differential equation for a simple harmonic oscillator. And the solution could be written in terms of either complex imaginary exponentials or you could write it in this fashion. So, this is the solution to this differential equation, which governs capital X. Now, let us impose the boundary conditions.

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So, the first boundary condition, that we will impose is that the function X should vanish at one end of the string, which corresponds to X equal to 0. So, this is the first boundary condition that we are going to impose.

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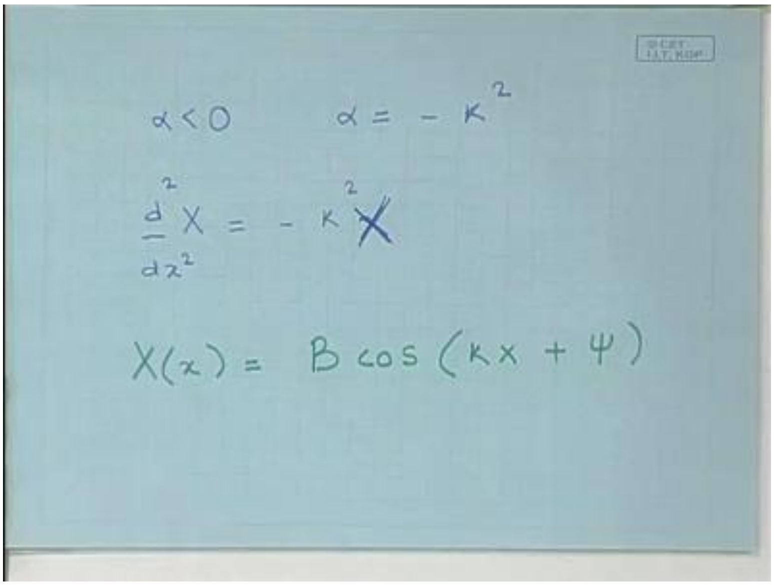

So, with this boundary condition let us see what does, it tell us.

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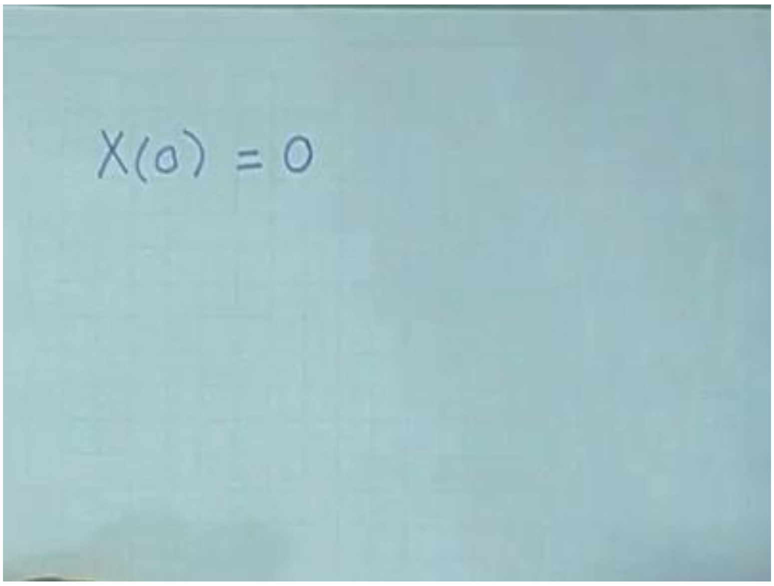

We want this to be equal to 0 and let us see, what this tells us the basically us that the phase psi over here.

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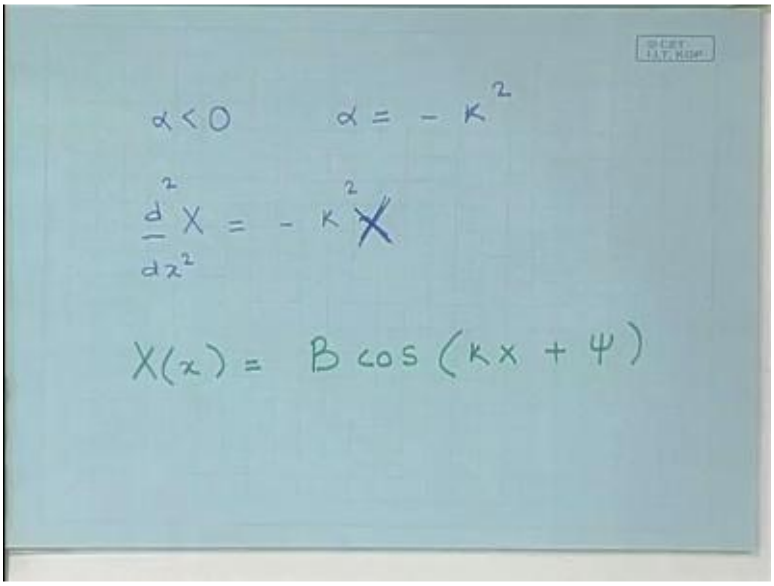

Should be either plus minus pi by 2 right, which makes this into sin.

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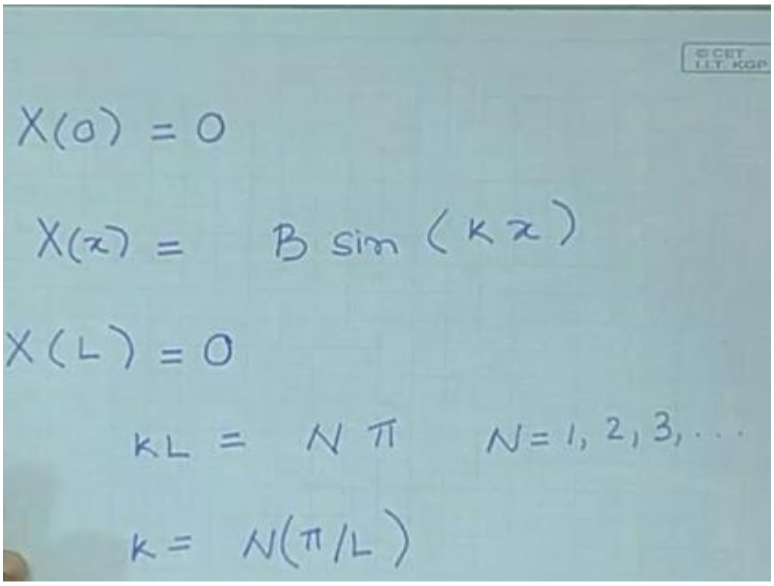

So, what we can say is that? With this boundary condition implies, that the phase psi has to be plus minus pi by 2, which allows I us to write this as sin k x. Right and this is guaranteed to vanish at 1 end of the string x equal to 0 it satisfies this boundary condition. So, this boundary condition basically, sets the phase now, we have to impose the next boundary condition, which is that the string that the vibration should also vanish at the other end of the string, which corresponds to x equal to L. So, we would like this function to vanish whenever x is L now, sin sin we know the sin function, become 0 whenever the argument becomes, a multiple of pi.

So, the boundary condition tells us that k into L x equal to L should be a multiple of pi N times pi, where N could be any integer 1 2 3 etcetera. So, we have obtained the solution of the, for the spatial part of the disturbance xi. And the spatial part of the disturbance xi is some constant into sin kx, where k into L has to be and multiple of pi. Or I could write this in the following way, where k should be a multiple of pi by L k is of the form N times pi by L, where N could be an any integer 1 2 3 4 etcetera. And for any such k, you will have a solution to the differential equation, where we are trying to solve.

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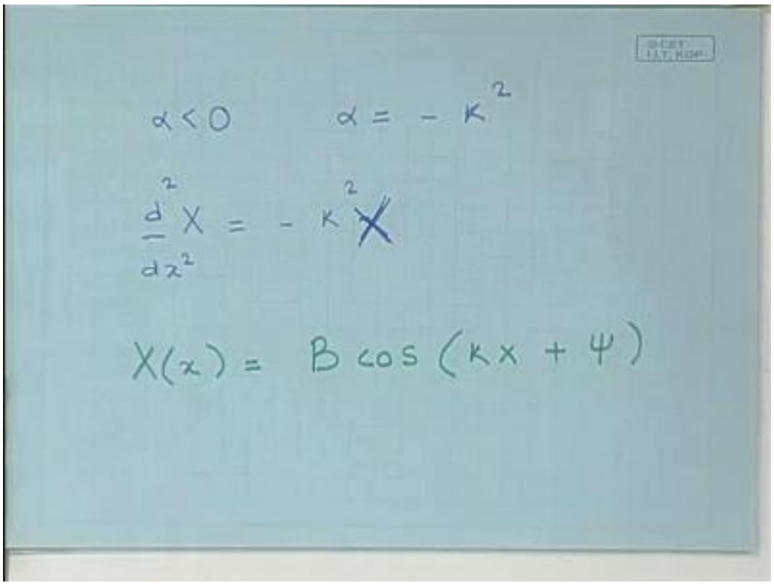

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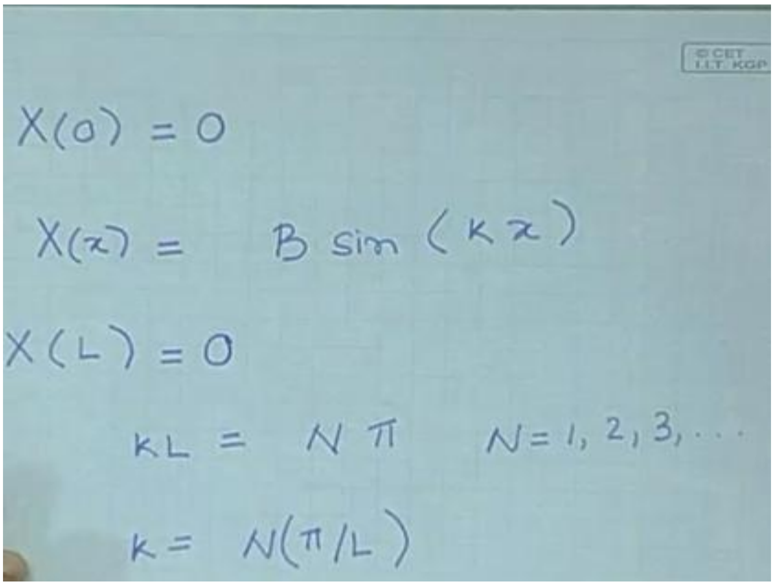

To this differential equation, which satisfies the boundary condition that, it vanishes at x equal to 0 and x equal to L. Let us now, work out the time part of this of xi. So, the time part of xi let us go back to the original expression.

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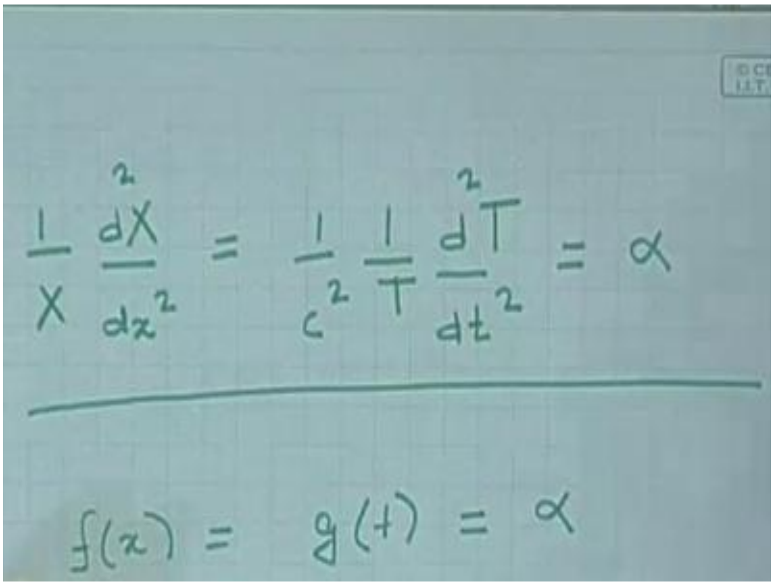

So, we had from the wave equation, through the substitution of through the method separation of variables, we obtained this expression. The left hand the term over here is a function of x the term over here is a function of time, they are both equal. So, they have to be equal to separately, equal to a constant and we have solved. This particular equation, where this is equal to a constant and the constant, we saw has to be negative and it has to be. So, we wrote it as minus k square and you found that k has to be a multiple of pi by L. Now, let us see, what that implies for the time part the time part of this equation.

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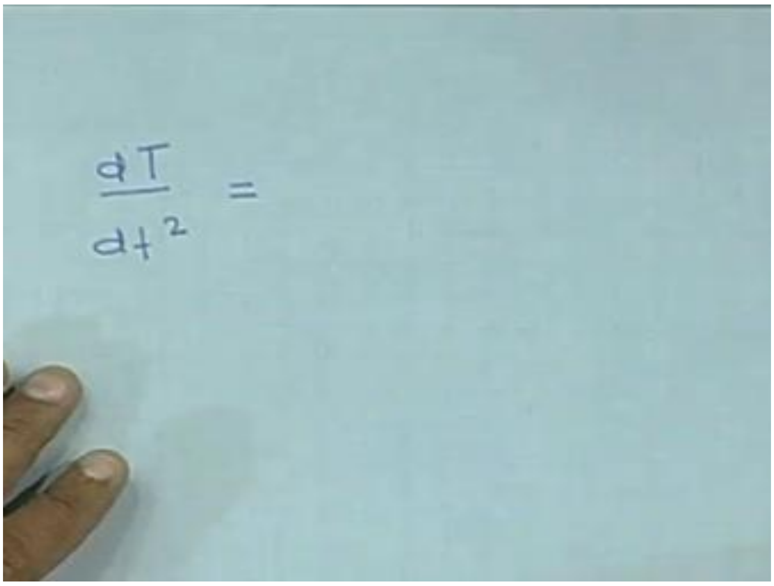

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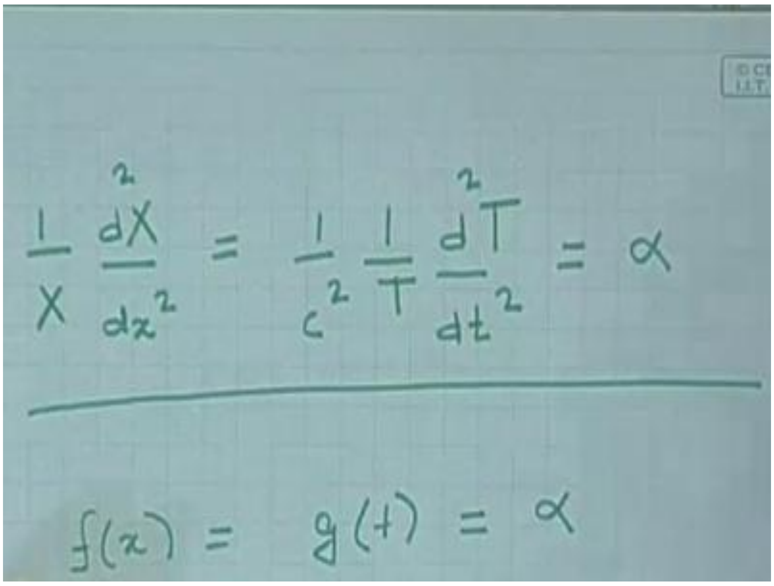

Now, becomes dT by dt square is equal to now, the constant alpha we wrote as minus k square.

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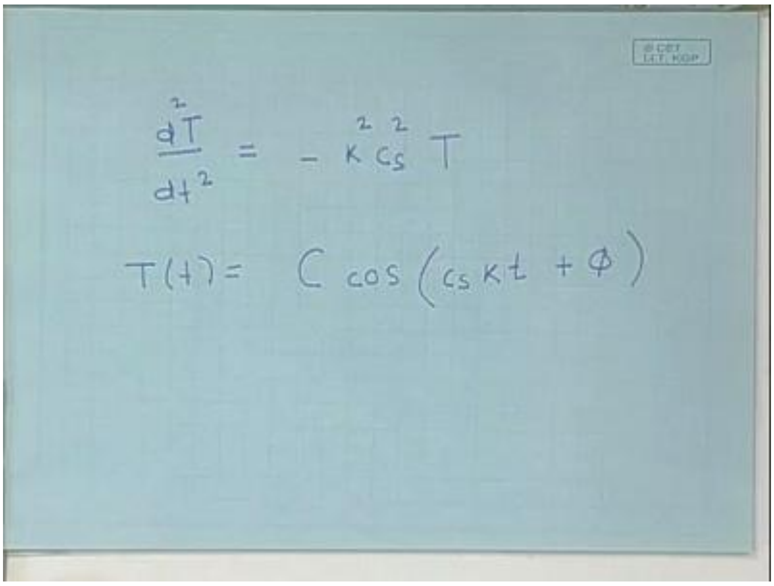

So, we can write this as minus k square cs square into T again, we find that the time dependent part of xi the time dependent part of xi also satisfies exactly, the same equation as the simple harmonic oscillator. This 2 is a simple harmonic oscillator equation and the solution for this is again well known. So, we can straight away write down the solution. It is of this form a constant into cos cs into k into t plus, some phase phi the phase phi and the constant c will be determined by the initial conditions of the vibration. So, let me now, put together the x part of the solution the spatial part of the solution which is here.

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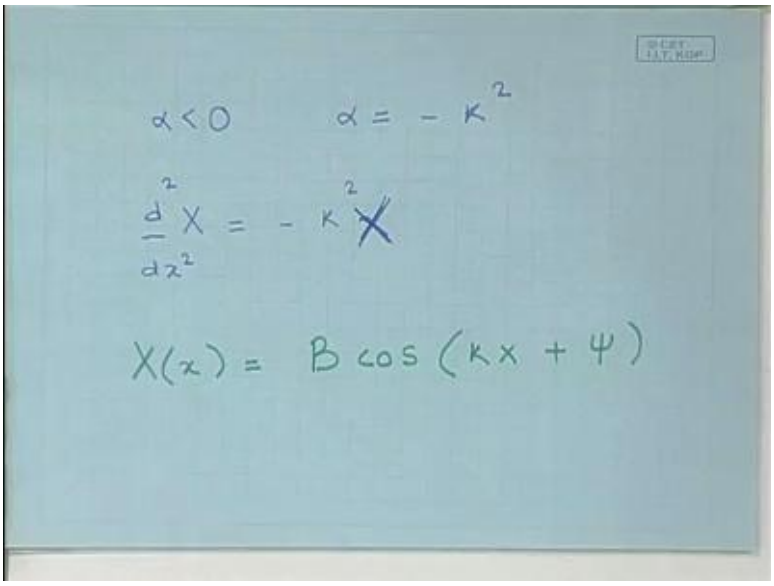

We have rather, we have simplified it and the spatial part of the solution is given over here after the simplification.

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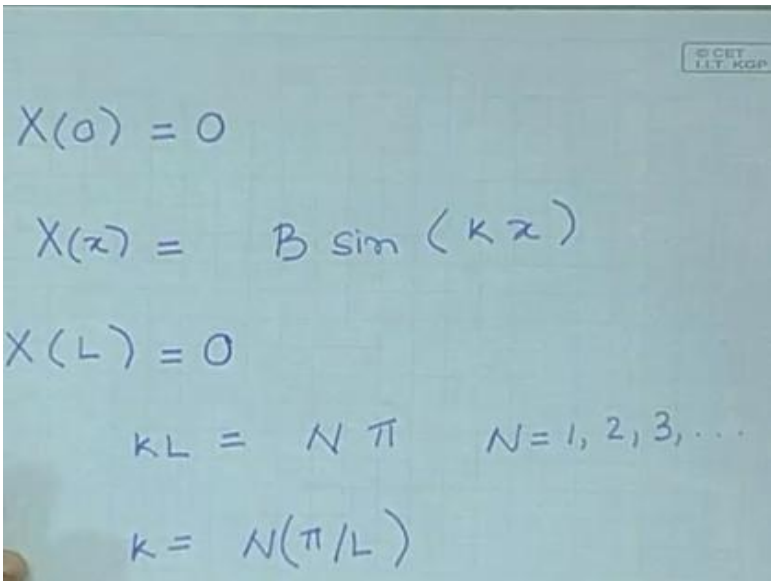

So, let us put back the spatial part of the solution.

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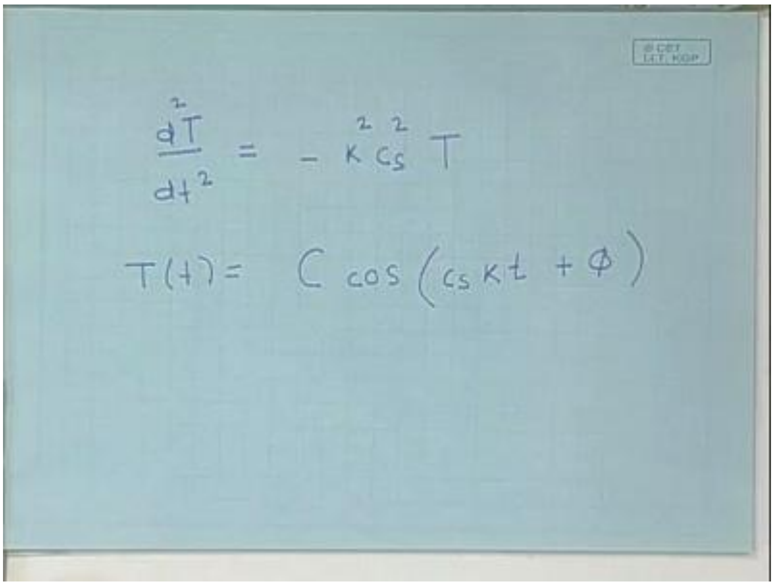

And the temporal part of the solution and obtain an expression for xi.

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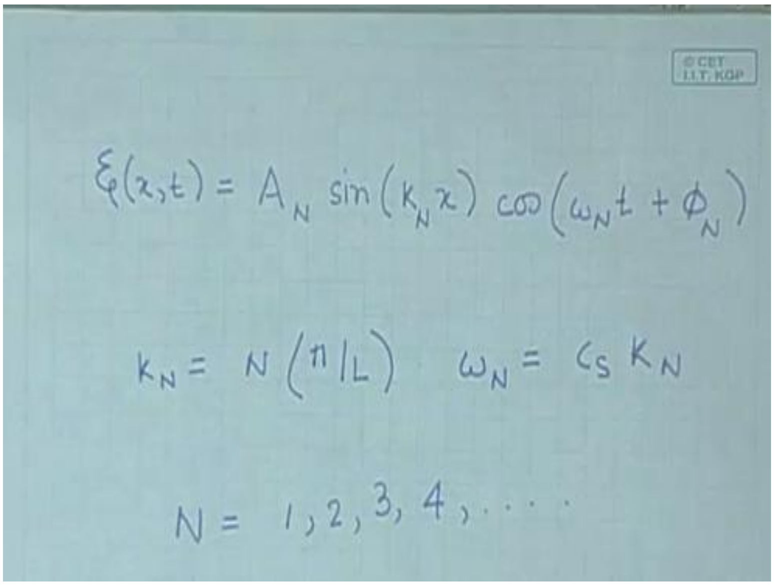

So, what we see is that xi the disturbance in the string is of this form AN sin KN into x cos omega N t plus phi N. Where KN is equal to N pi by L omega N is equal to cs into KN N could have any value any integer value 1 2 3 4 etcetera. So, what we found was? What we what we found was? That the disturbance xi, which satisfies this boundary condition that it has to vanish at the 2 ends has many solutions. So, there are many such many possible solutions, there is a different solution corresponding to each integer N. So, N equal to 1 gives you a particular solution N equal 2 gives, you a different solution N equal to 3 gives, you another solution N equal to 4 gives, you another solution. So, let. So, corresponding to every integer, there is a different solution to the wave equation, which satisfies the boundary condition that the vibration disappears at the 2 end points.

And the solution corresponding to any value of the integer, we can write in terms of an amplitude AN maintaining a subscript here to show that, we are looking at the solution corresponding to a particular value of N if I change N I will get a different solution. So, AN sin KN, so, corresponding to any integer, there will be a particular value of this k and the allowed values of k corresponding to different integers are multiples of pi by L. So, N equal to 1 will be 1 times pi by L N equal to 2 KN will be 2 times pi by L N equal 3 will be 3 times pi by L etcetera. This spatial function will be multiplied by a temporal time dependent function cos omega N t plus phi N. So, phi N is the phase, which is determined by the initial condition omega N is cs this or c here.

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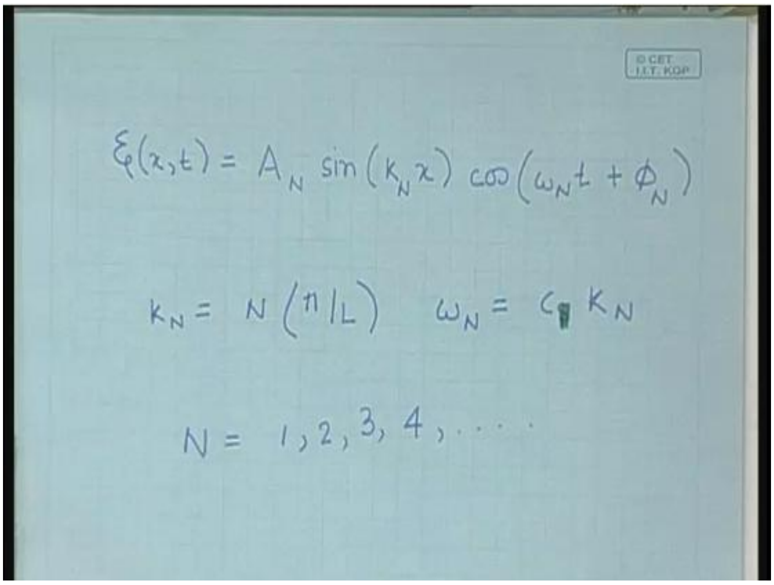

I have been using c. So, let me get rid of the s it is c into KN. C is the phase velocity of waves in that medium the square root of t the tension divided by mass per unit length that c. So, the time part will be omega N, where omega N is c into this k the wave number k plus some arbitrary phase phi N, which is decided by the initial condition. So, I could have many possible solutions, each 1 corresponding 1 corresponding 2 each integer. So, there will be a particular solution corresponding to N equal to 1 a solution corresponding to N equal to 2 another solution corresponding to N equal to 3 etcetera. Let us, look at these solutions 1 by 1. So, the first solution that we look at is N equal to 1.

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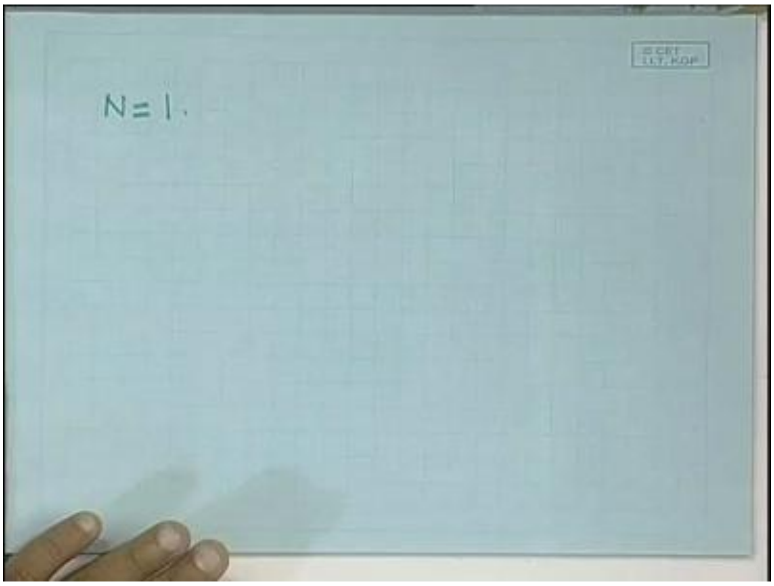

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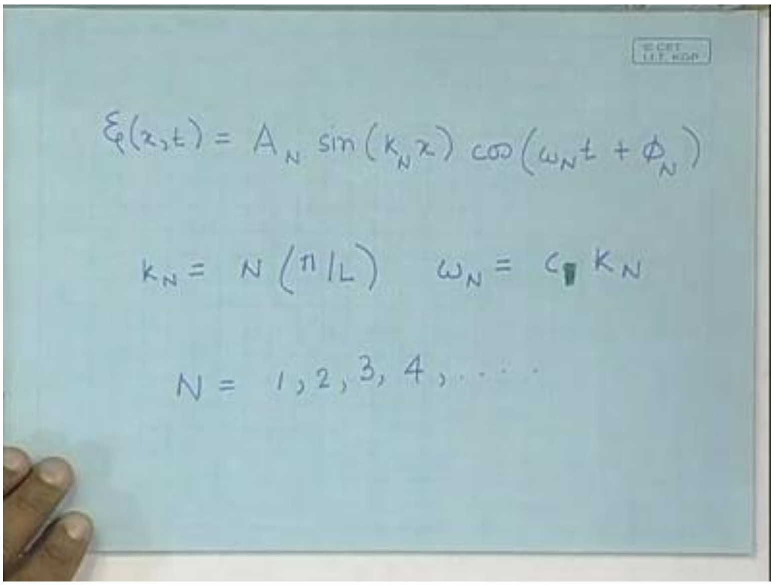

So, what you have to do is, you have to put in Fix N equal to 1 put in all the values AN will be A 1 will be some arbitrary amplitude decided by initial conditions phi. 1 will be sum initial phase arbitrary phase decided by the initial conditions. And then, you have to put in the values of KN and omega N. So, let us do the exercise and we can discuss it then.

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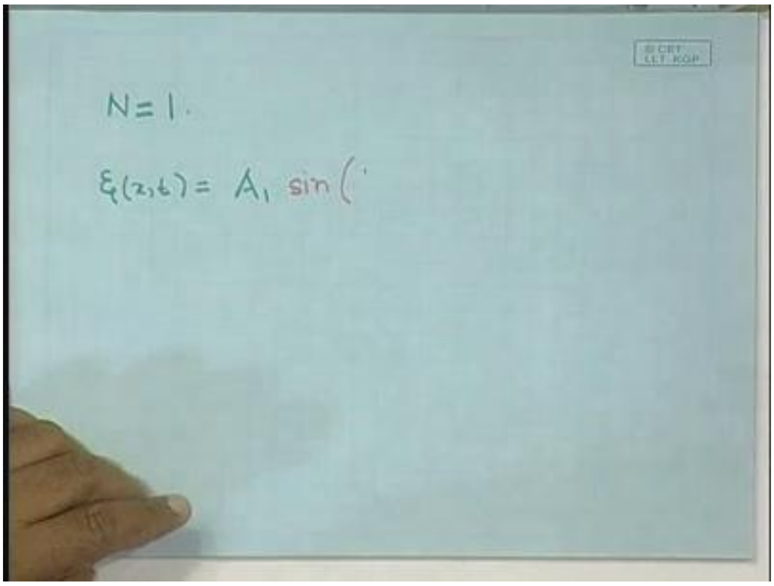

So, what we have is xi x t is equal to A 1 the amplitude into sin.

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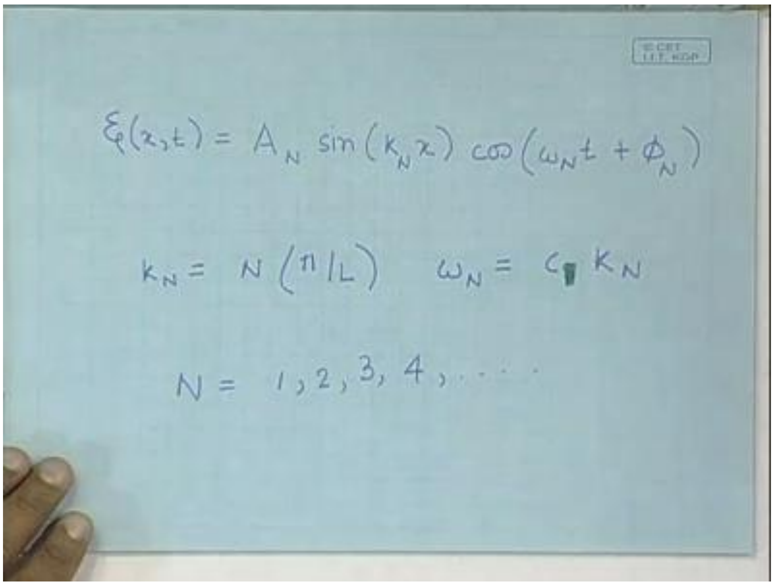

So, k 1 is pi by L. So, this becomes pi x by L into cos omega.

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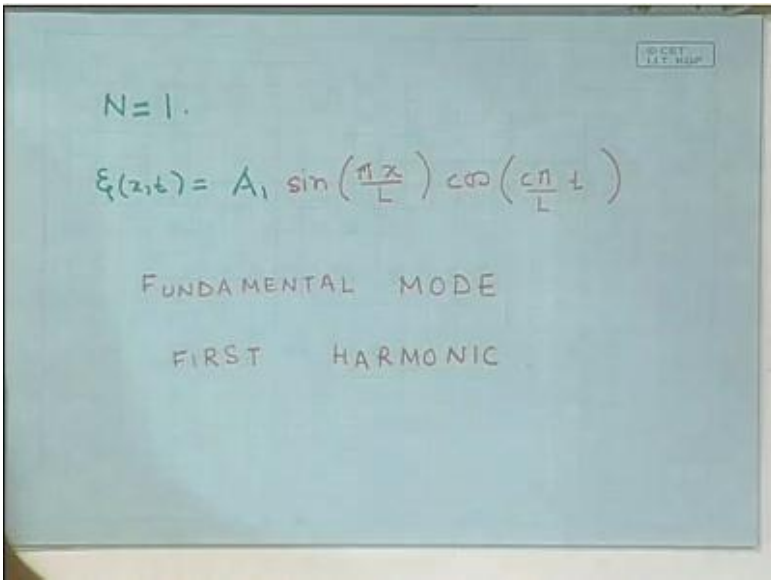

So, omega is going to be c pi c into k 1. So, c pi by L into t plus phi 1 phi 1 let me set it equal to 0. So, this is the solution corresponding to N equal to 1 this solution is called the fundamental mode of vibration. It is also called the first harmonic of the vibration, what does this vibration look like. So, let me plot it for you at t equal to 0 at t equal to 0 this cosine term has a value 1. Let me choose the amplitude A, so, that it is also has a value 1, so, at t equal to 0 this is sin pi x by L. So, it vanishes at x equal to 0 it also vanishes at x equal to pi and this is shown over here.

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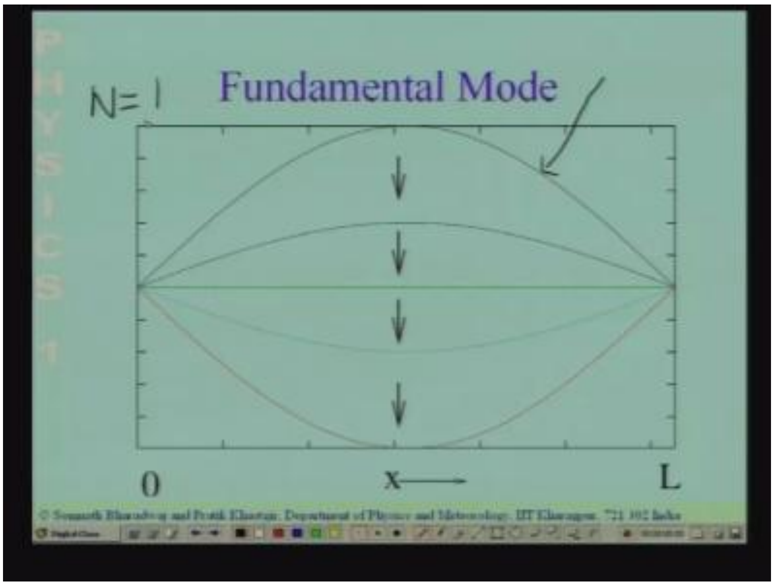

So, this curve over here, this curve over here shows, you the disturbance in the string corresponding to N equal to 1 at the initial time t equal to 0. Let me repeat again, we are considering. So, we have seen that corresponding to every integer, you have a different solution of the wave equation.

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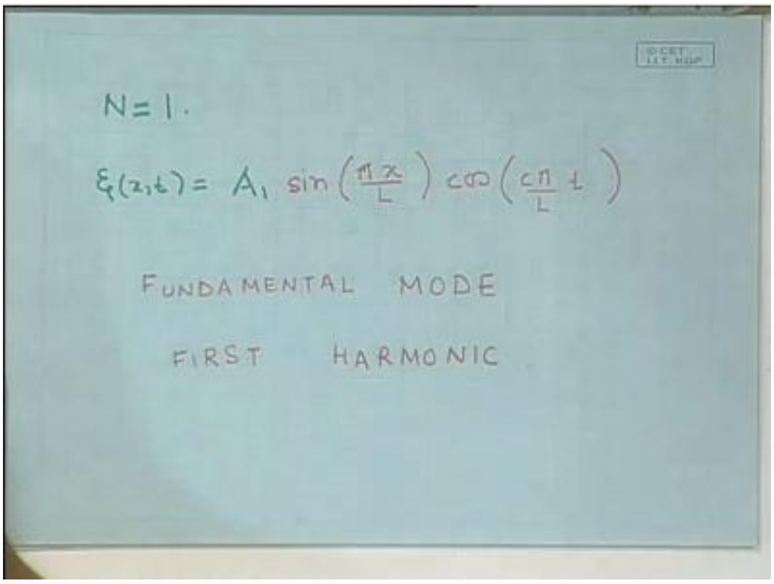

So, we are considering the solution, which corresponds to N equal to 1 this solution is given over here. We have set the phase to 0 the phase is decided by the initial condition, we have set the amplitude to 1 and we are looking at the situation. Where, we want to see the disturbance so, the function of x at the time t equal to 0. At time equal to 0 this cosine term has a value 1 and the disturbance is just given by sin pi x by L, which is what is shown over here the curve on top.

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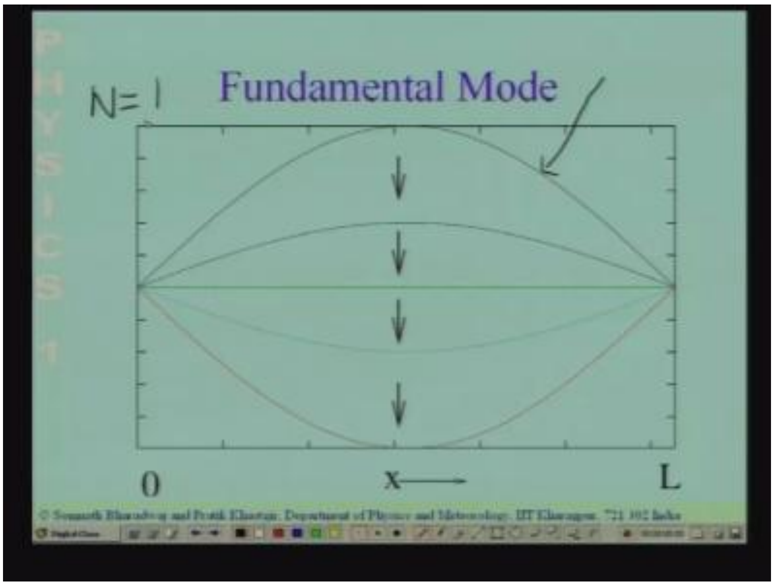

So, this is what the string wills the disturbance in the string will look like. So, the string is actually, look like this the undisturbed string is what is shown over here, at t equal to 0 the string will look like this now.

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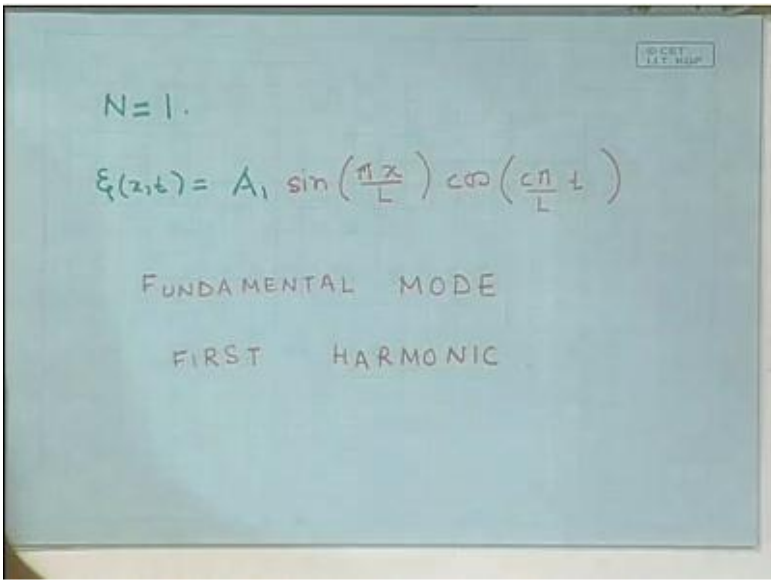

As time increases so, as time increases, this argument over here is going to increase and we know that from as t increases from 0 the value of cosine of this thing, over here is going to fall. So, the value of this cosine term is going to fall this term is going to remain as, it is and this is 1. So, the value of cosine over here falls and at a later time slightly, later time this is the string.

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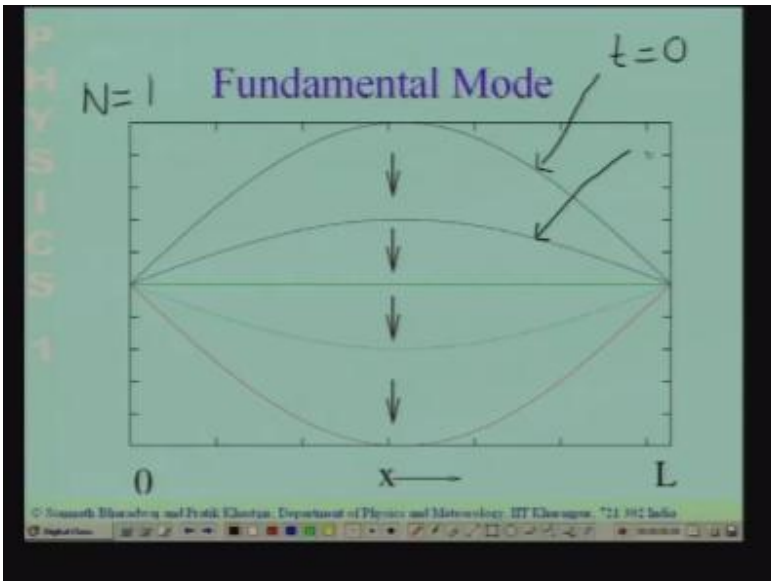

This shows you the position of the string at a slightly later time. So, this is t equal to 0 and this is at some slightly later time.

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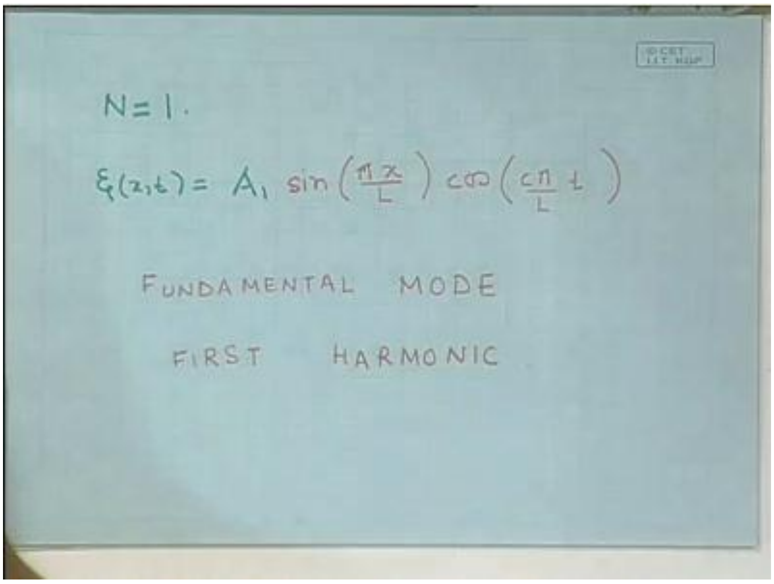

And then as t increases further when this whole argument becomes pi by 2 right, so, you can calculate the time required for this, whole argument to become pi by 2 not very difficult. So, when this whole argument becomes, pi by 2 the value of cosine pi by 2 we know is 0. So, the displacements of the string become 0, so, this is when the argument of the.

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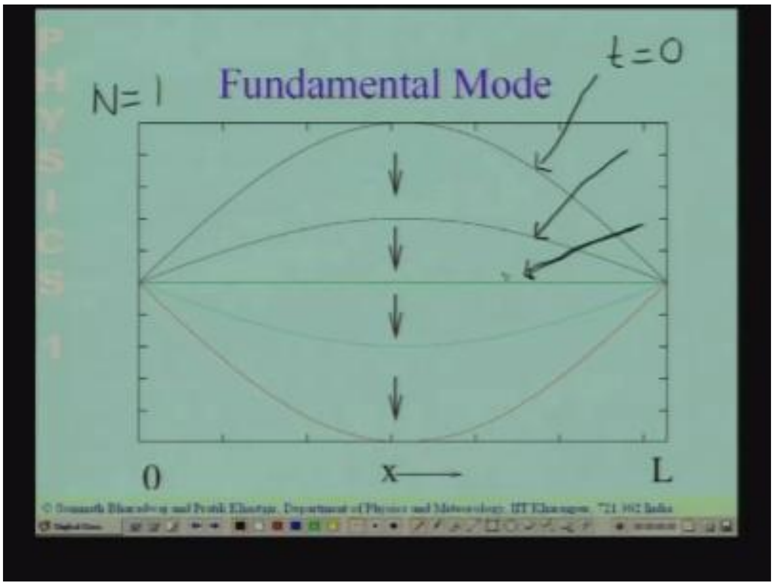

Cosine term, this over here is when the argument of the cosine term becomes, pi by 2 and the cosine term gives, you a 0 the string is in the undisturbed position the displacement of the string become 0 everywhere. And then again, when time increases even further this crosses pi by 2.

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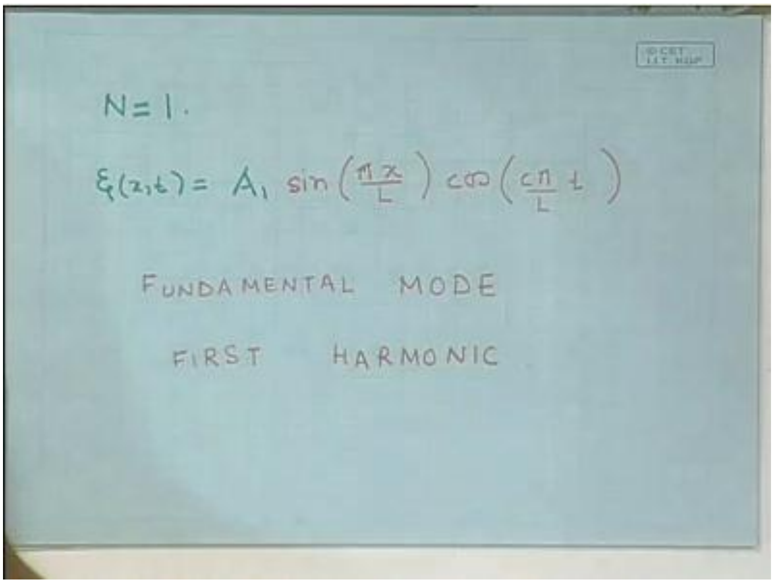

So, the cosine term has a negative value.

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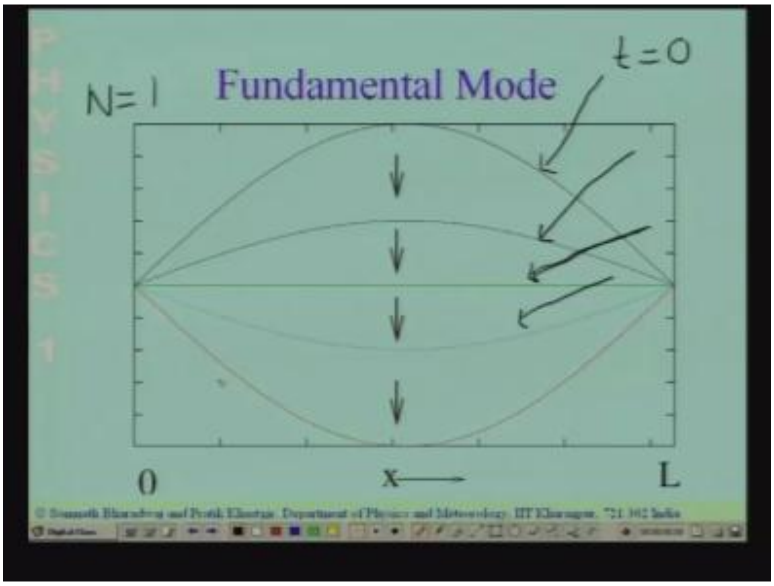

So, this will be the position the disturbance in the string at a later time and finally, when it becomes pi, when this argument over here.

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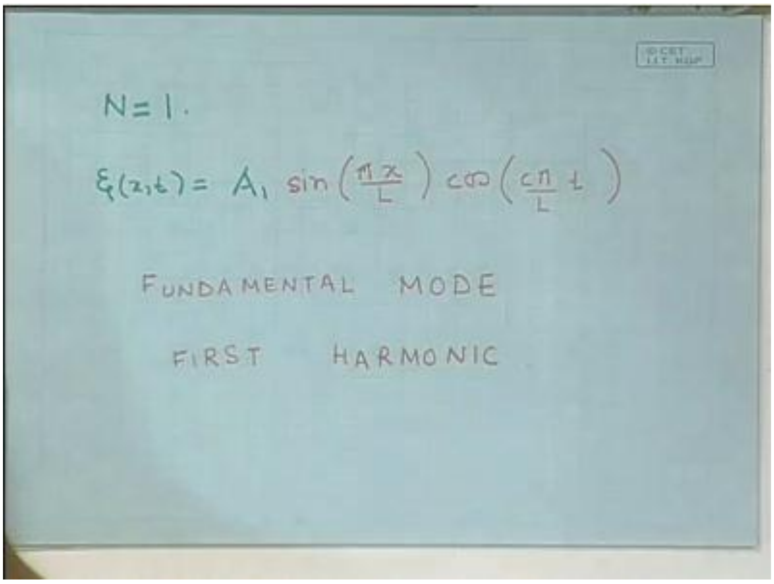

Becomes pi this value will be minus 1.

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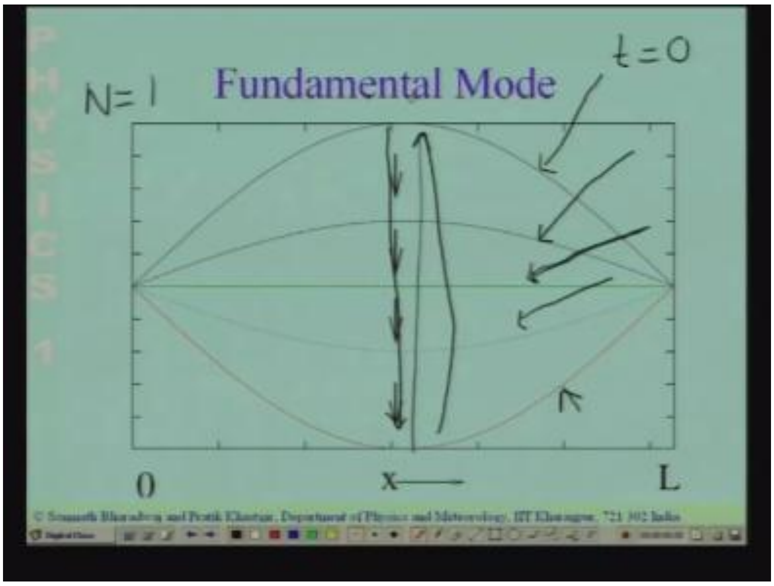

And the String will be here, when it crosses pi again the value of cosine is going to tend to 0, so, the whole thing will going to repeat. So, the string is the disturb the string initially, was disturb to this position is going to go down as, I have shown by the arrows over here is going to come here and then again go back and forth. So, it is going to keep on going up and down, so, it will go down and then it will go up again. And then, it will come down again, it will keep on going back and forth like this.

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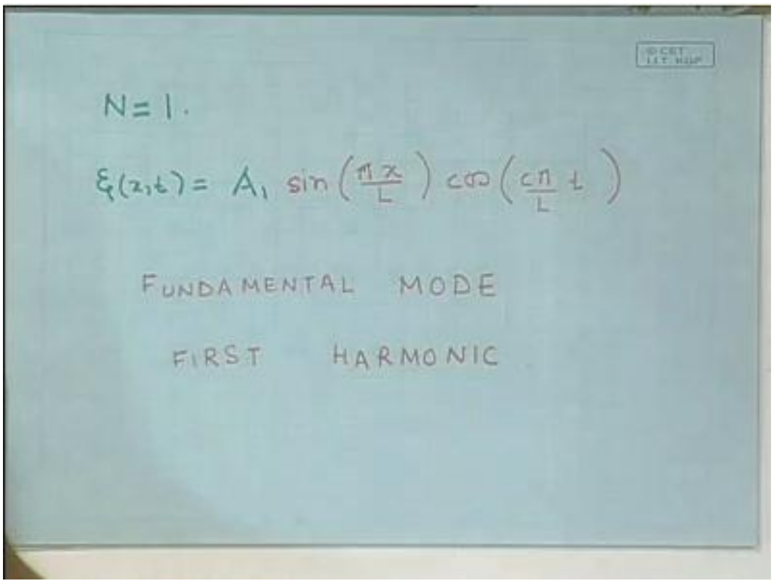

So, I hope this is clear to you, so, this is the solution corresponding to N equal to 1; this is called the fundamental mode or the first harmonic. Let us also ask the question? What is the frequency? What is the wavelength of this? Of this disturbance in the string, so, we can look at this k k has a value.

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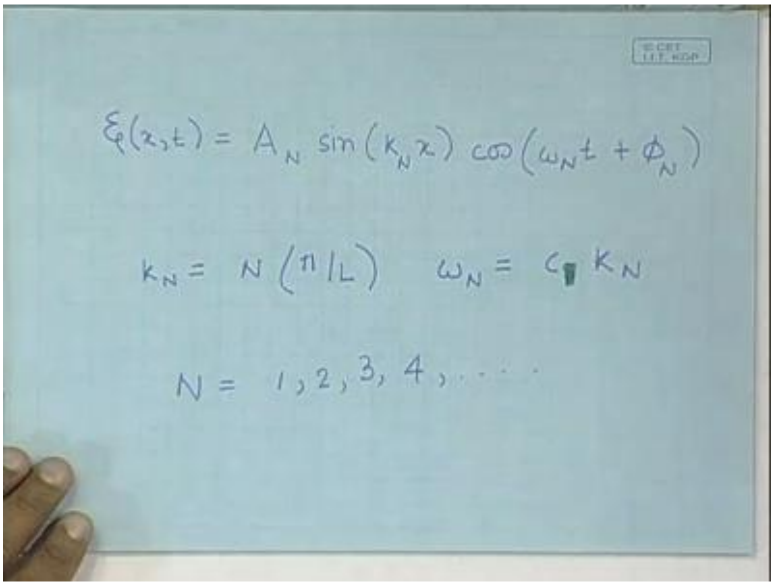

N equal to 1 k has a value pi by L and 2 pi by k gives us the wavelength. So, what you see is that for N equal to 1.

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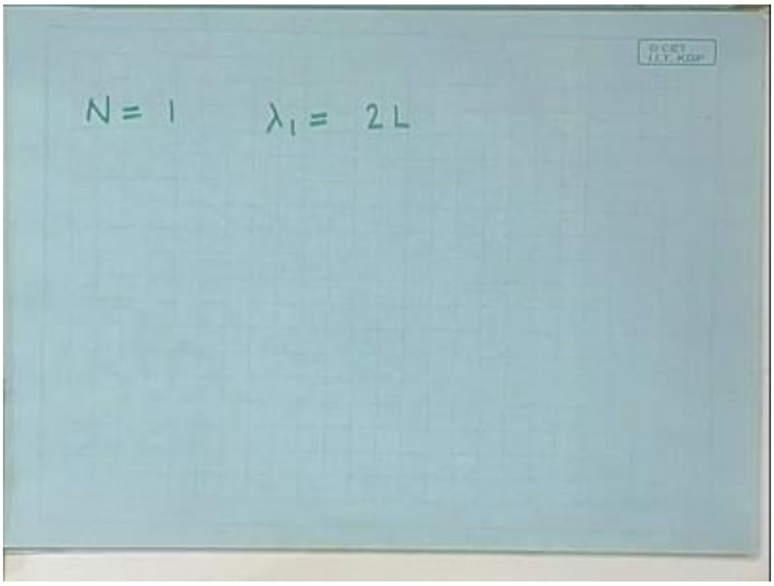

N equal to 1 you have a wave length lambda 1 which is 2 L.

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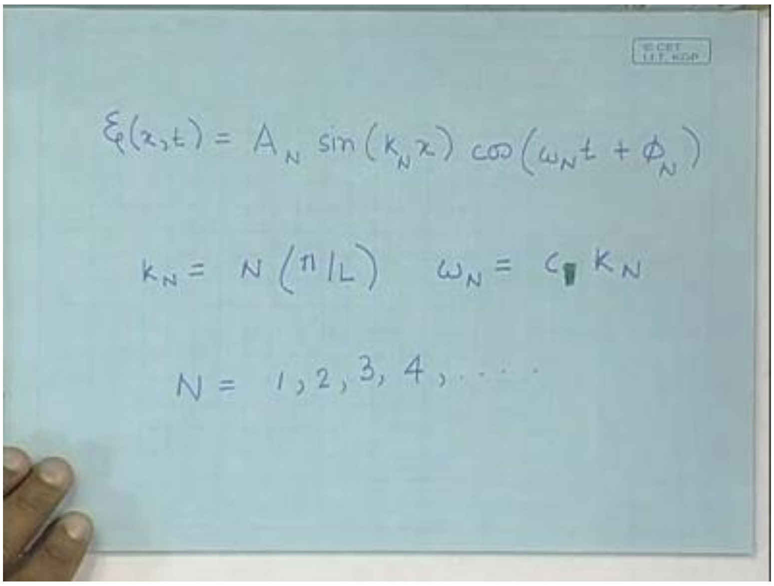

What is the frequency of this vibration? So, omega N omega 1 is c times k 1 omega divided by 2 pi gives us the frequency k 1 we know is pi by L. So, I have to divide it by 2 pi so.

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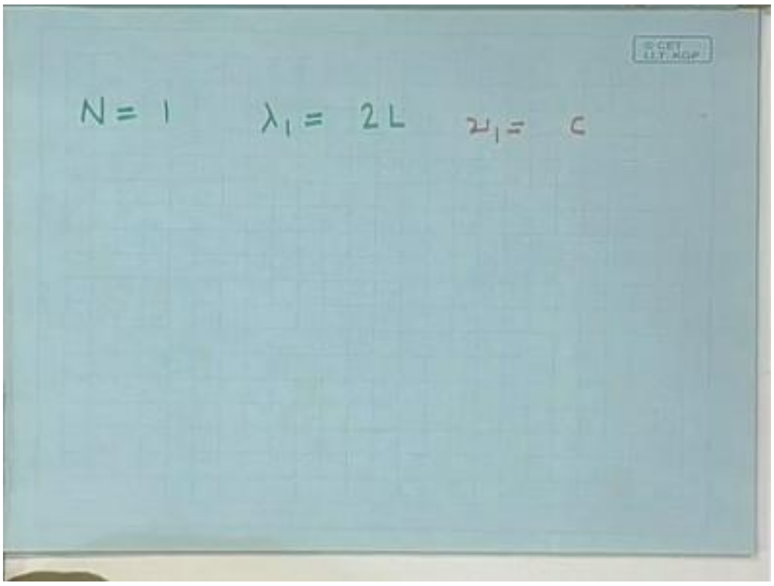

The frequency nu 1 is c.

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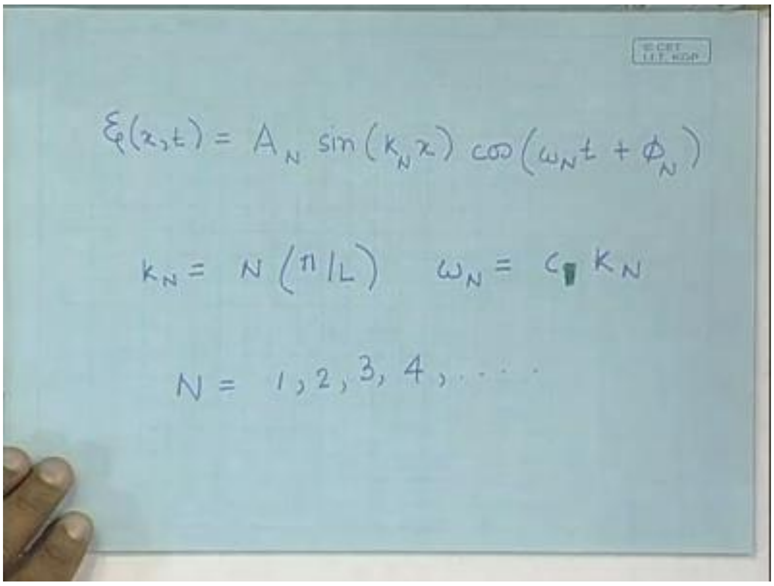

So, omega divided by 2 pi this is pi by L. So, if I divide, this put pi by L here and divided by 2 pi. So, the factor of pi will cancel out, and I will have c by 2 L.

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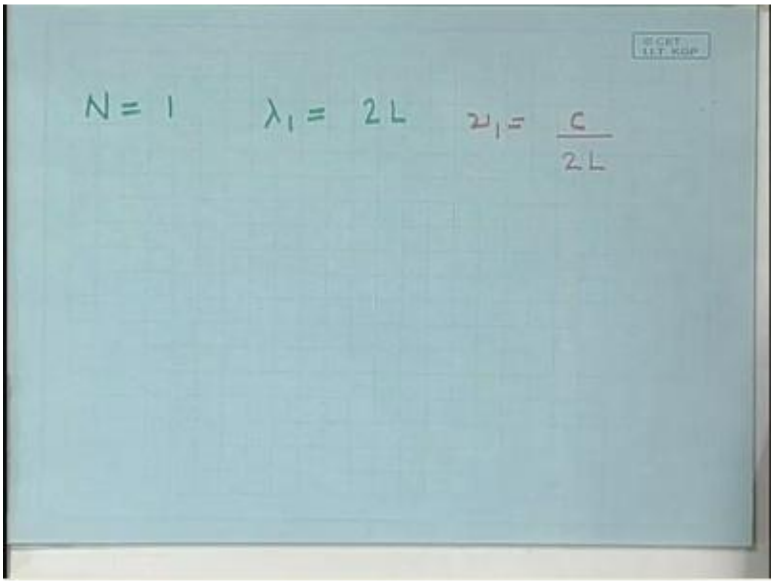

So, the fundamental mode or the first harmonic has a wavelength, which is twice the length of the string. And it has the frequency, which is c divided by 2 L, where c is the phase velocity in that string. Let us now, consider the solution, when N is equal to 2.

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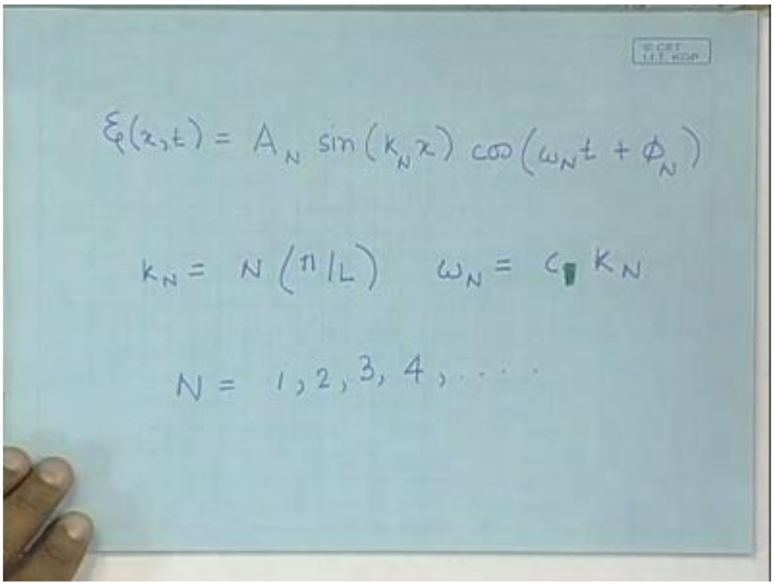

So, this is the expression for the disturbance in a general situation, let us now, consider the solution when N is equal to 2. So, when N is equal to 2 k 2 will become, 2 pi by L omega will become c times that. So, let me write down the solution again, I will choose the amplitude to be 1 and I will set the phase equal to 0. So, let me write down the solution for N equal to 2.

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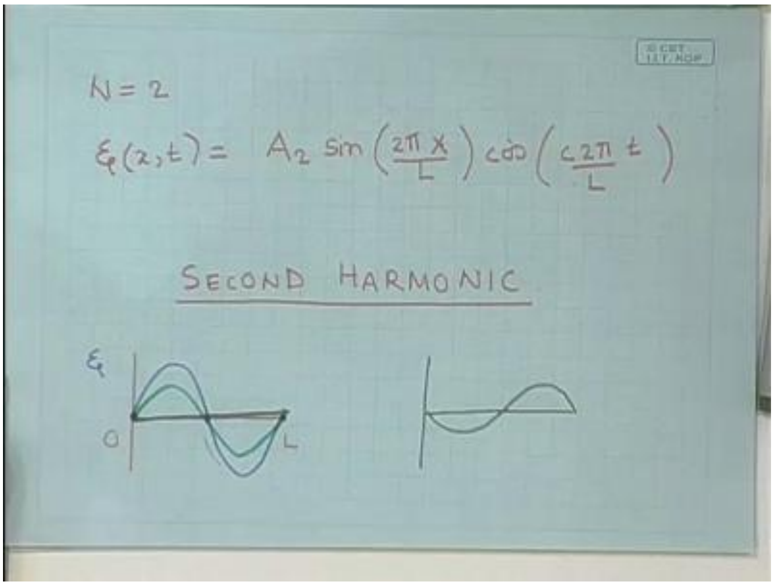

I will have some arbitrary amplitude ,which I will set equal to 1 now, I have sin 2 pi X by L into cos c into 2 pi t by L I have set the phase to be 0. This is called the second harmonic of the vibration or the first overtone. So, let us draw what this looks like and then study it is evolution, so, let me draw it for you here at t equal to 0. So, let us first set t equal to 0 this has a value 1. So, at t equal to 0 this is sin 2 pi X by L, so, we know that, it will have value 0 here. It will have value 0 here it'll also have a value 0, when X is L by 2 at X equal to L by 2 the argument over here becomes pi. So, it will again go to 0, so, we are drawing sin 2 pi X by L and it will at it will look like this. So, this is the disturbance of the string, so, this is what the string will look like at t equal to 0 at t equal to 0, this cosine term is has a value 1.

So, this is what the string will look like at the disturbance in the string, which you can say is the string itself, it will look like at t equal to 0. Now, as t increases, this cosine term is going to fall and the string is now, going. So, the amplitude of this is going to go down and it will look something like this at a slightly later time. And then at a certain value of t, which again you can calculate the cosine term over here is going to be 0. So, the string is now, going to appear like this. And then as time increases further, it is going to become negative; this cosine term is going to be negative. So, the string is going to appear like this. And then with further increase of time is going to get more negative and you will have this pattern repeating itself. So, let me show you what this looks like.

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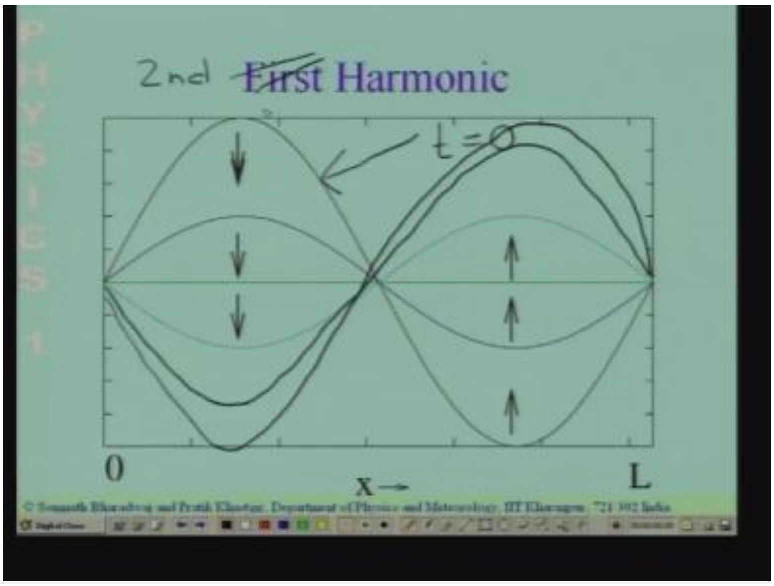

This should be not first, but the second harmonic. So, this shows, you the string at t equal to 0 and then as time increases, this is going to go down. This is also going to go in this direction and then at a certain time the displacement is exactly 0 and then again, this is going to this part becomes negative, this part becomes goes up. And then it is going to become like this, and then it will become like this and then the whole thing is going to go down again and the pattern will repeat. So, this is the second harmonic and then, you will have higher N equal to 3 4 5 etcetera now, I should tell you that the points, where the see these are what are known as standing waves. That is the first point, which I should mention, so, you see as the time.

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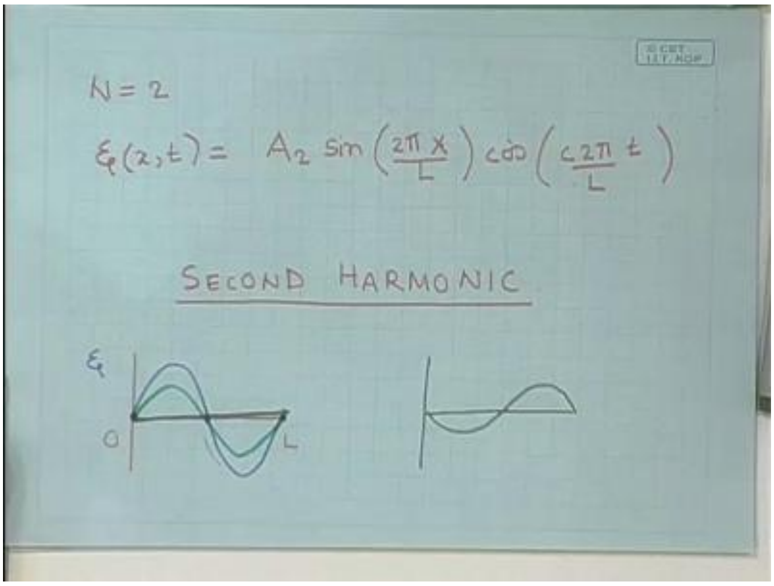

Evolves you have a spatial pattern does not move forward or backward right.

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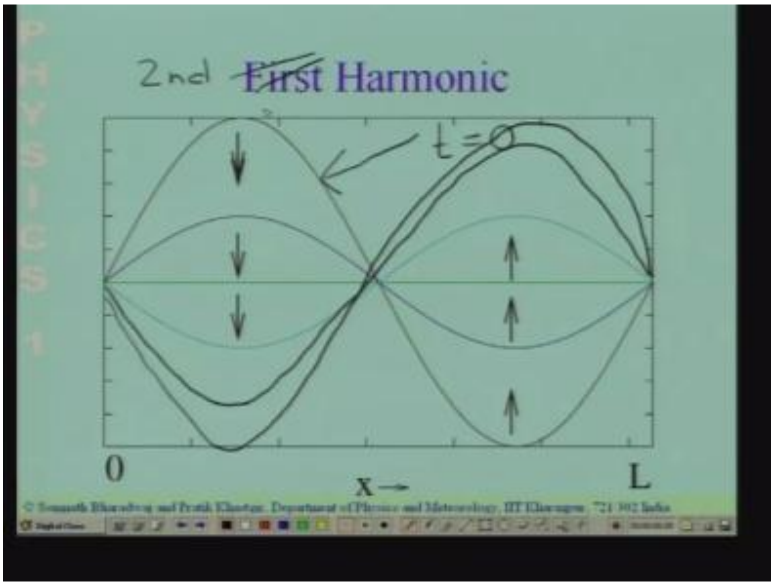

So, what you see is that as time evolves the same spatial pattern the amplitude of it goes up and down, but the pattern, itself does not involve forward or backward this is why these are called standing waves. The same pattern remains in the string, it is amplitudes goes up and down becomes negative and then again, becomes positive and negative and 0. So, the amplitudes oscillate, but the pattern itself does not move either forward or backward in the string. So, this is why it is called as standing wave the standing pattern or a stationary wave the same pattern, just goes the amplitude of same pattern, just goes up and down with time. That is what happens over here the wave does not move forward to the left or to the right, which is why it is called a standing wave in this standing wave pattern, there are places where the this the disturbance has a maximum value. And there are places where the disturbance has a value 0 the place, where the disturbance is 0 it remains, 0 throughout right. So, there are certain places points where the disturbance remains 0 throughout these points are called nodes. So, in this situation we have a node over here we have 2 nodes.

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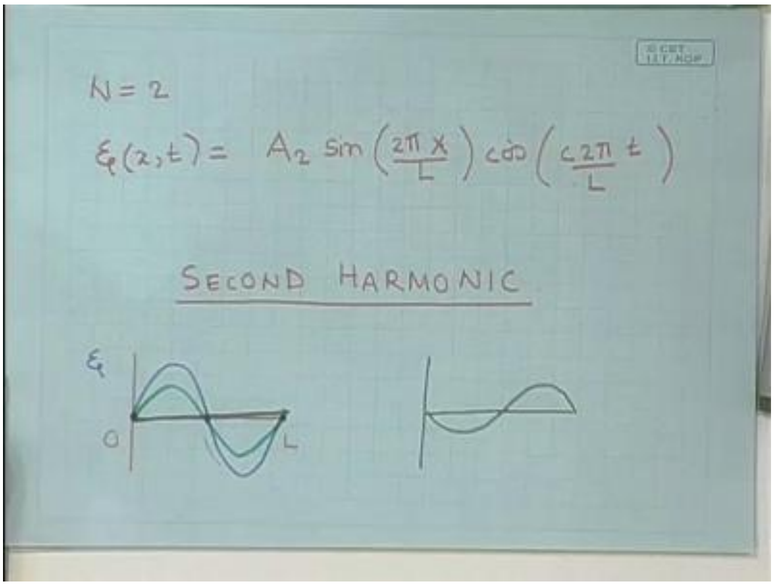

At the 2 end points and we also have a node over here. So, N equal to 1 we only have nodes at the end points which are fixed N equal to 2 we have 1 node at the center in the center N equal to 3 you can guess it will look like this.

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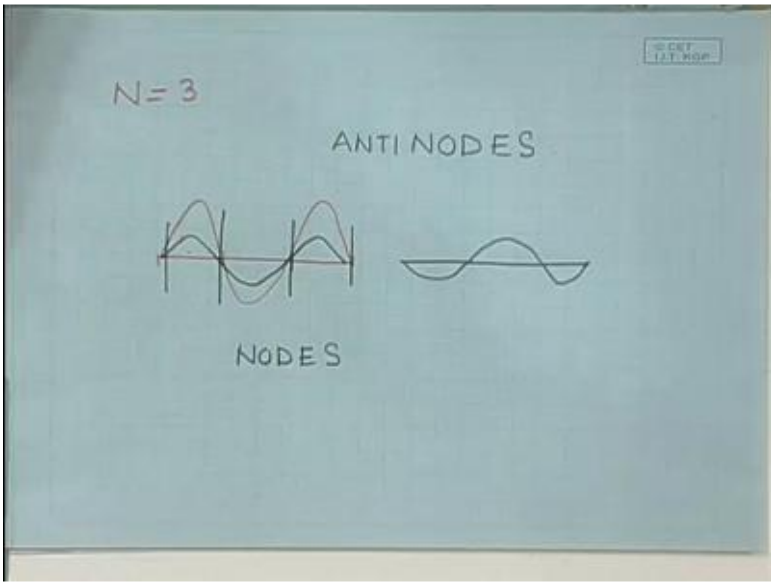

There will be 2 nodes in between and the whole disturbance is going to look like this and the amplitude of this disturbance is going to evolve with time. So, at the later time instant the amplitude is going to go down it will look like this and then at a certain time the amplitude will become 0 and then it will become negative. So, the whole pattern will get inverted it will look like this. So, these are the nodes the value of the disturbance here continues to be 0 throughout the vibration. So, these are called nodes. And the point where the disturbance is maximum are called antinodes let us also ask the question what how the wavelength of the vibration changes as I vary N. So, we have already seen that for N equal to 1.

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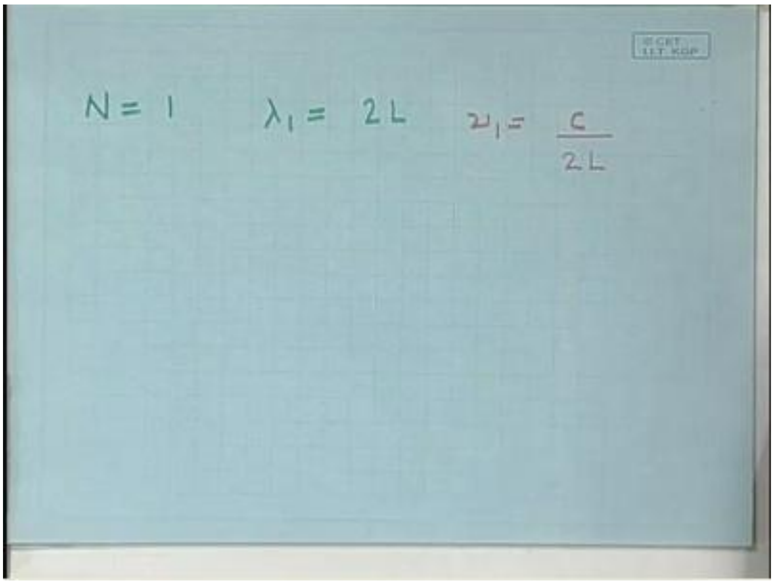

The wavelength is 2 L and the frequency is c by 2 L.

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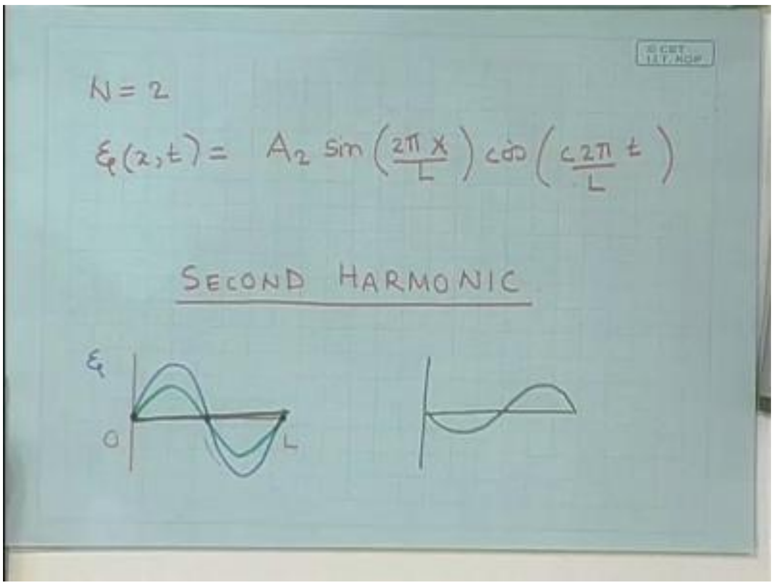

Let us now look at the situation where the N equal to 2 or we could just use this over here.

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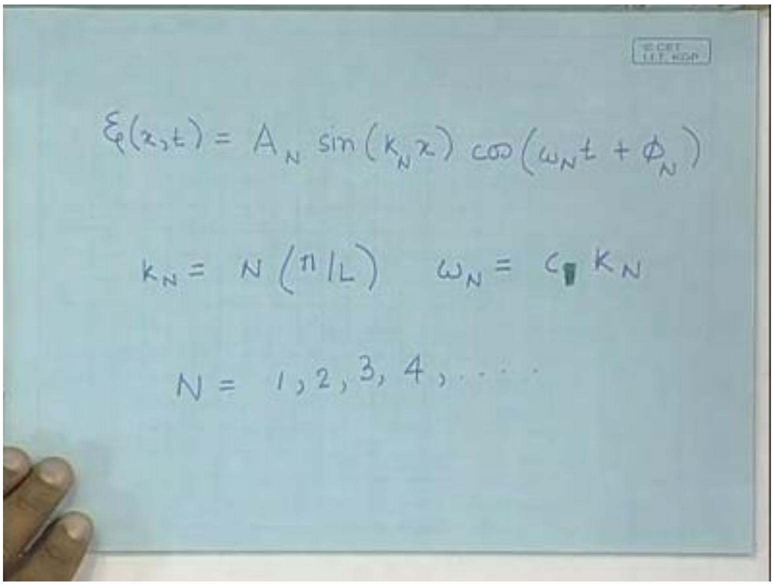

So, k 2 we have k 2. So, this going to be a factor of 2 over here 2 pi by k gives the wavelength. So, when N equal to 2 the wavelength is L right at N equal to 2 the wave number is 2 pi by L 2 pi divided by the wave number gives the wavelength. So, 2 pi divided by 2 pi divided by L gives me L.

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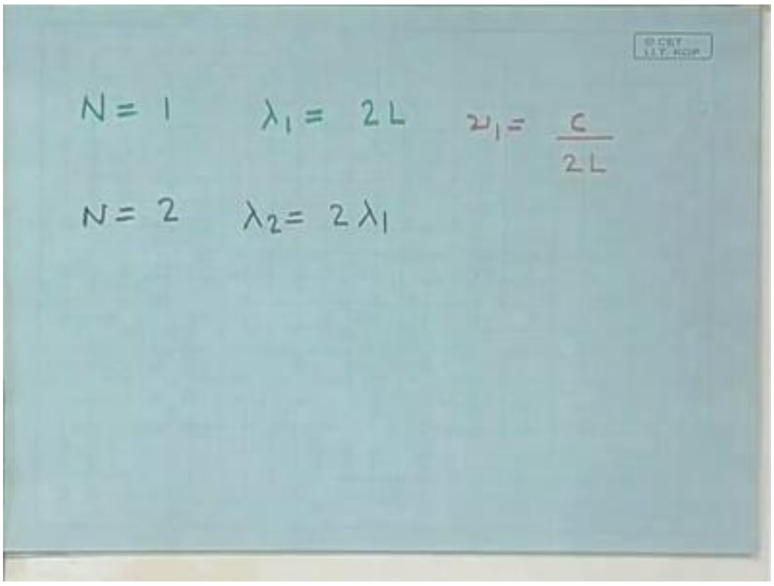

So, for N equal to 2 the wavelength is 2 times lambda 1 lambda 1 is 2 L. So, the wavelength for N equal to 2 is 2 times lambda 1.

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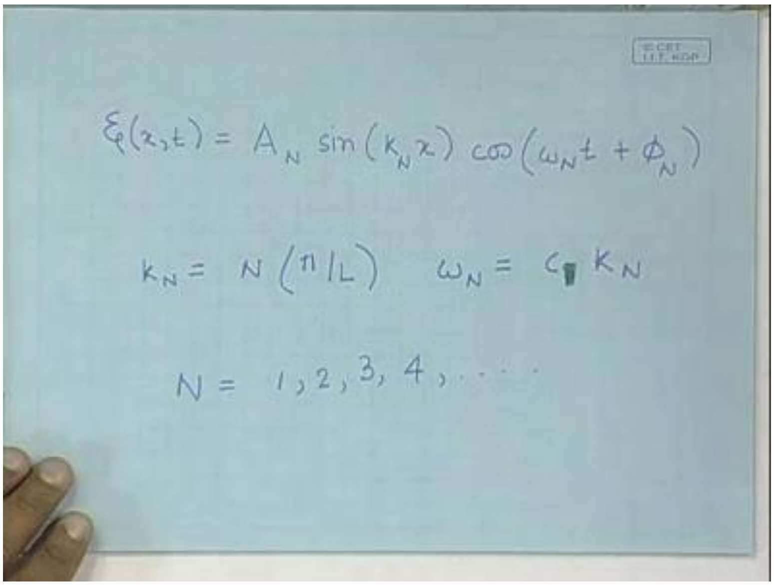

Similarly the frequency is 2 pi 2 pi into the sorry the angular frequency divided by 2 pi. So, N equal to 2 the frequency is going to up twice.

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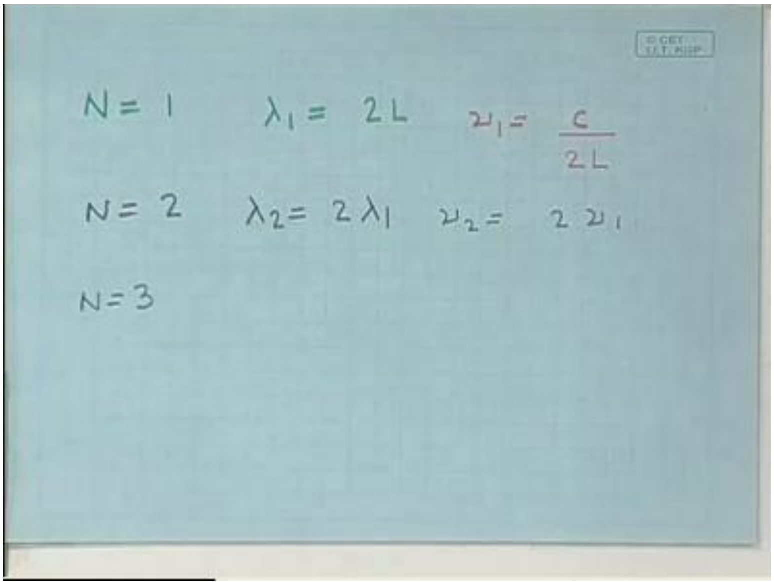

So, nu 2 is equal to twice nu 1 N equal to 3 again let me show you the expression.

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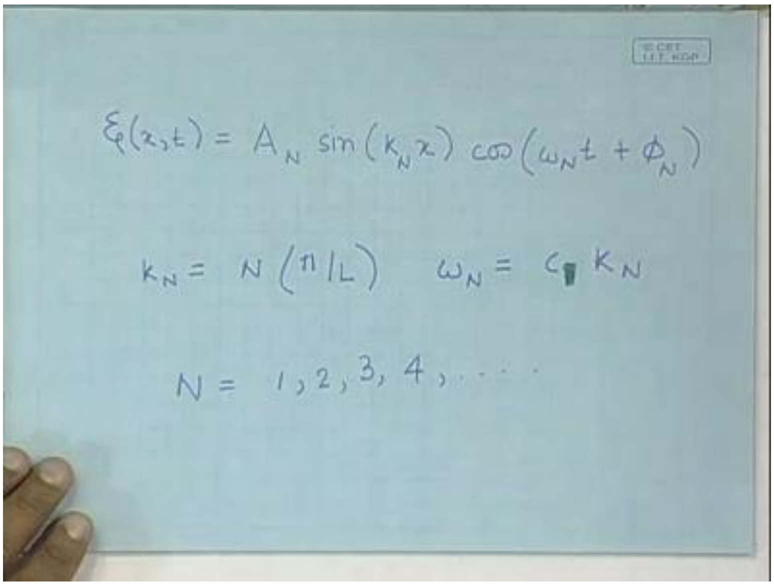

N equal to 3 N equal to 3. So, this is going to be 3 3 pi by L the wave number is 3 pi by L 2 pi divided by wave number gives the wavelength. So, the wavelength is 2 by 2 divided by 3 2 by 3 into L 2 thirds of L.

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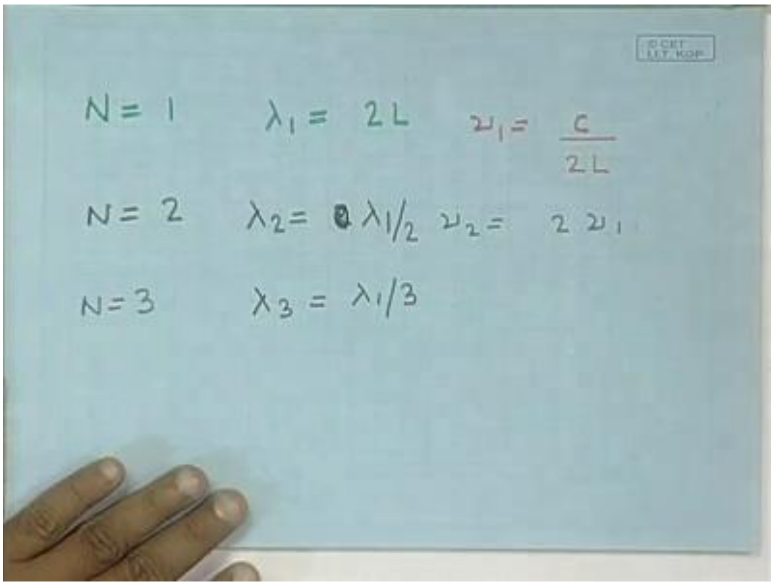

Or what you can say is that this is sorry this should be lambda 1 divided by 2 and this will be right I had made a mistake over here.

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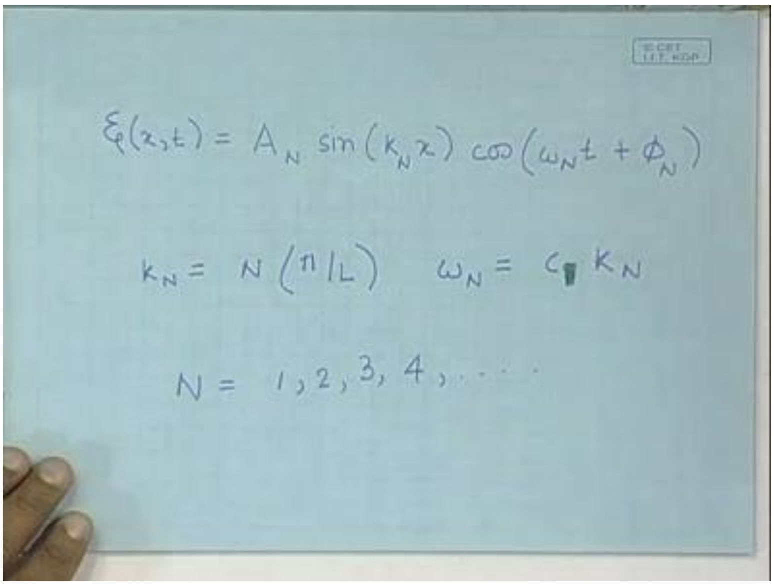

It should have been the this the length the wavelength was for N equal to 2 the wavelength was L which is lambda 1 divided by 2 not multiplied by 2.

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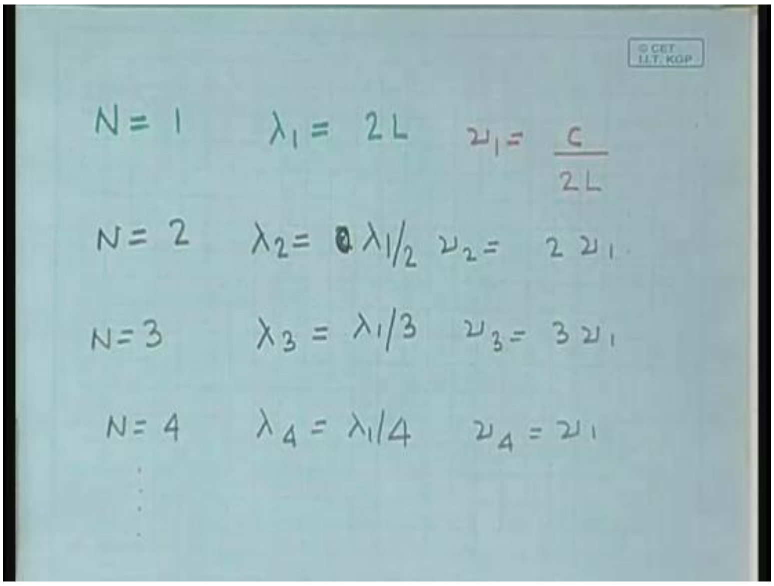

Similarly, for N equal to 3 the wavelength is lambda 1 divided by 3 N equal to 4 the wavelength is lambda 4 is equal to lambda 1 divided by 4 and the frequency is 3 times nu 1 here it is 4 times nu 1 etcetera. So, for higher and higher values of N you will have higher and higher harmonics the higher harmonics all have higher frequencies. So, the first harmonic which is the fundamental mode has a frequency c divided by 2 L twice the length of the string. The first, the second harmonic has the frequency which is twice this the third harmonic has a frequency which is 3 times this.

The wavelength is half one third the fourth harmonic has a wavelength which is one fourth of the fundamental wavelength and the frequency which is 4 times the fundamental frequency. So, these are called the higher harmonics of the wave of the string. Now, a this nearly brings us to the end of our discussion there are 2 points which I should make before we really close the first point which I should make is that any any standing wave. So, let me just is as actually a superposition of 2 travelling waves. So, let me show this to you. So, let us consider.

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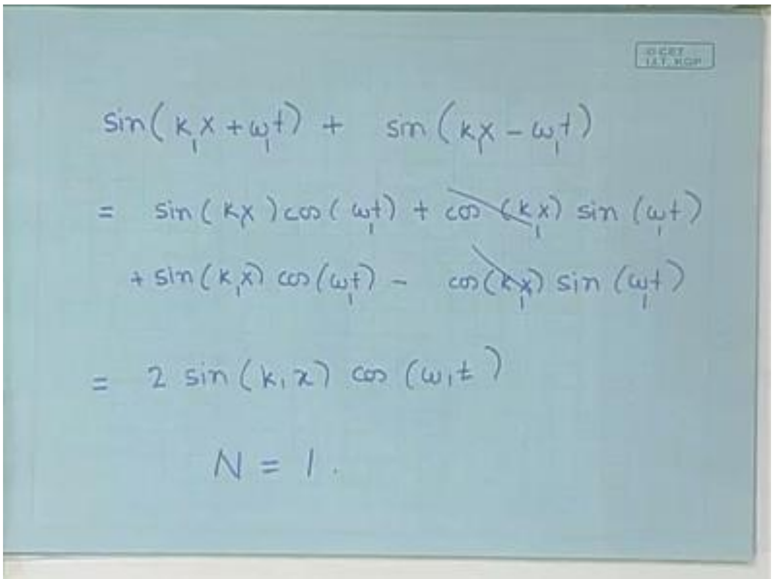

The 2 2 travelling waves sin kx plus omega t plus sin kx minus omega t. So, this is a superposition of 2 travelling waves this is a right travelling wave this is a left travelling wave. Now, this first wave we can write as sin kx cos omega t plus cos kx sin omega t the second one we can write as plus sin kx cos omega t minus cos kx sin omega t. So, what we see is that when I superpose these 2 these 2 terms the last 2 terms will cancel out. And what we have is 2 sin this should be k 1 omega 1 k 1 omega 1 k 1 omega 1 k 1 omega 1 k 1 omega 1 k 1 omega 1. So, finally, this will be give you sin k 1 x cos omega 1 t which you see is the standing wave corresponding to N equal to 1 the first harmonic the fundamental mode of vibration.

So, what I have shown you here is that the super position of a left travelling wave and a right travelling wave give you a gives you a standing wave. So, any standing wave is actually a superposition of a right travelling wave and a left travelling wave. This is the first point which I would like to make the second point which I would like to make before we end our discussion is as follows. So, let me let me just go back to the point where we started our discussion. So, the point where we started our discussion was over here.

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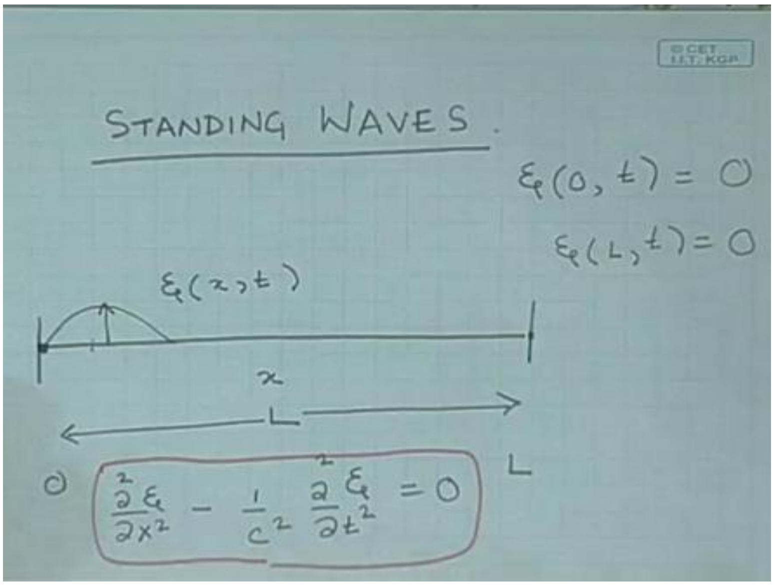

We are interested in following the evolution of a disturbance in a string in a situation where the two end points of the string are fixed they cannot vibrate. And we found that there are solutions to the wave equation which are standing waves which satisfy the, these boundary conditions. Now, here I show you some arbitrary disturbance let me ask you the question does this look like 1 of the standing waves right does this look like 1 of the standing waves. We have seen pictures of the standing wave the first harmonic is 0 here and 0 only at this point it goes all the way like this. The second harmonic is something which goes up and then comes down.

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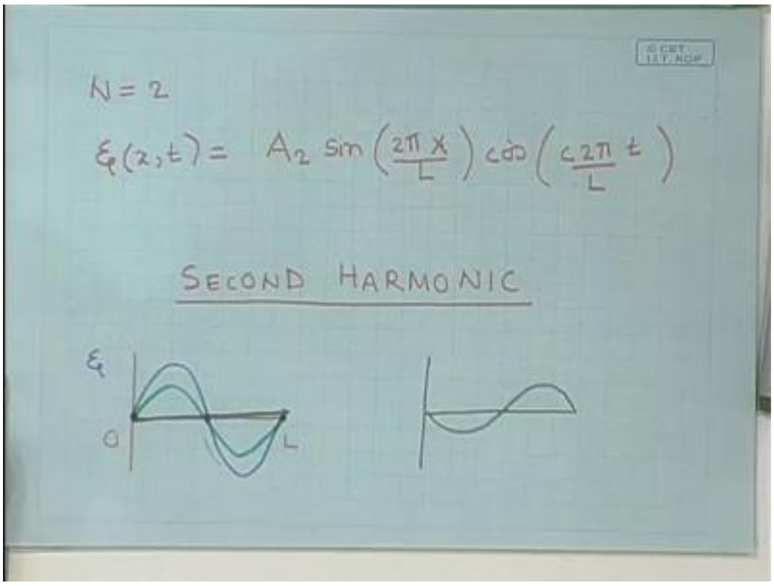

Shown over here the third harmonic I have also told you is going to have 2 0s and then the fourth harmonic is going to have 3 0s and so, forth.

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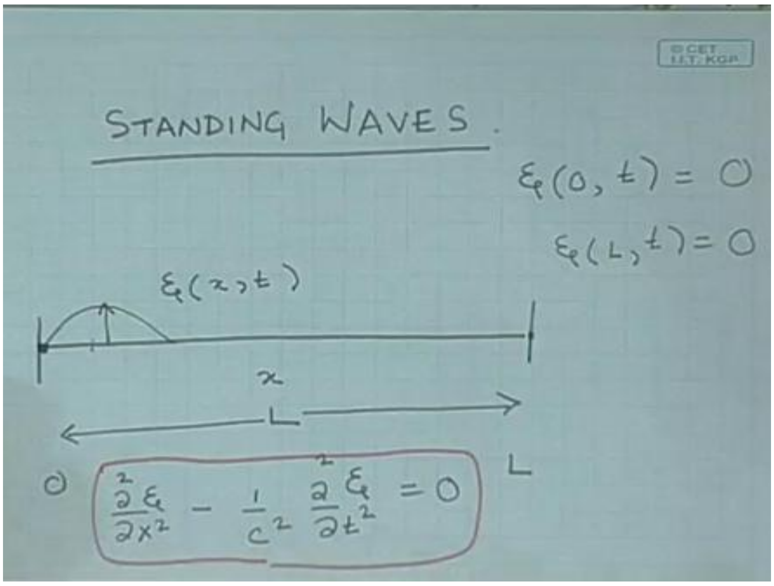

So, the question is this one of those standing waves and the answer is no; obviously, this does not look like sin right the standing waves we saw over sin k NX thus x dependence was sin k NX and it is does not look like that. So, the question is how do we handle this well the the solution to this question is as follows any arbitrary disturbance in the in the string.

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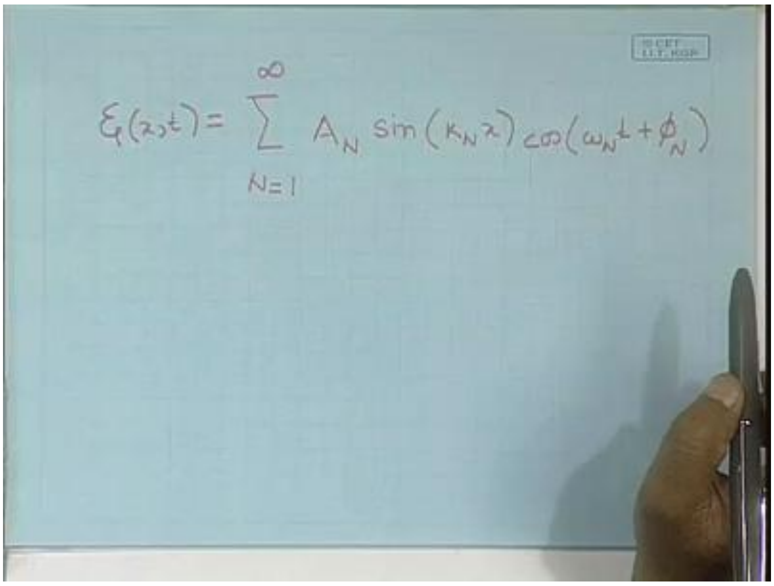

So, any arbitrary disturbance in the string can be decomposed as a superposition of standing waves with different values of the integer N.

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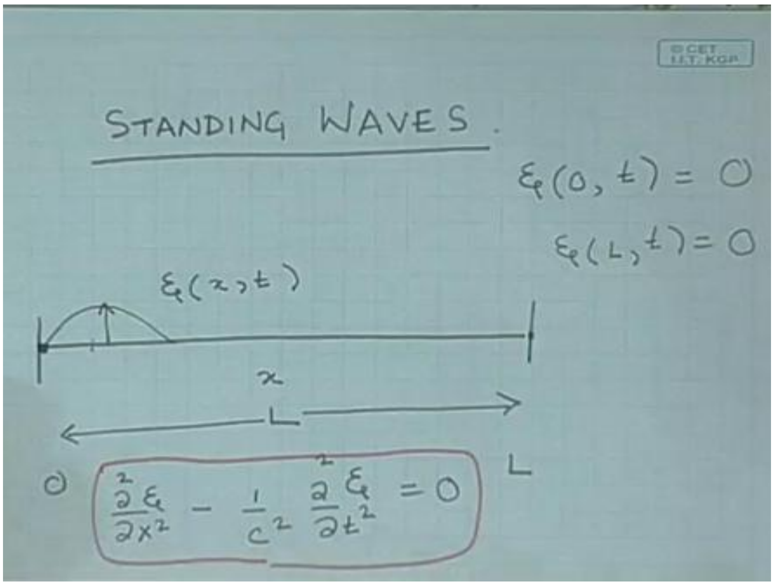

So, any arbitrary disturbance for example this particular disturbance can be decomposed into a superposition.

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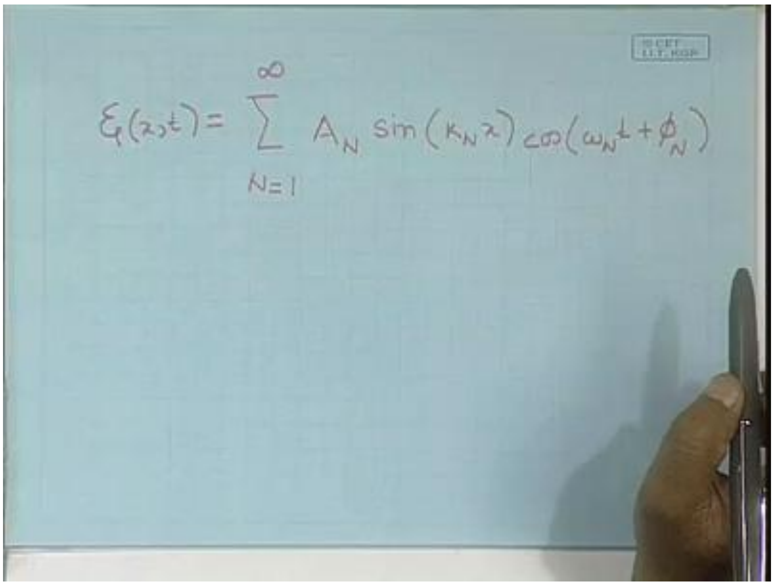

Such standing waves and the time evolution of this can be obtained by following the time evolution of this expression which is a sum of different standing waves. And this in general will the sum of such standing waves in general will not be a standing wave it will be something that goes back and forth along the string.

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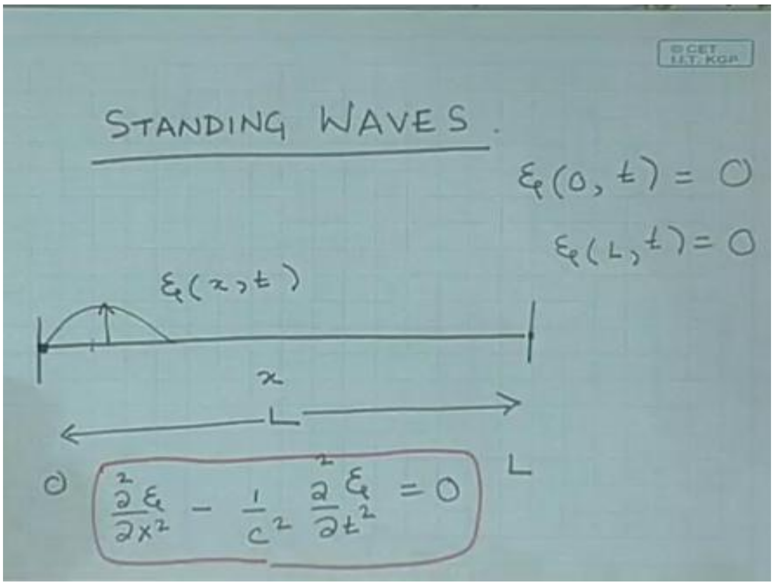

So, let me show you first let me show you one situation where...

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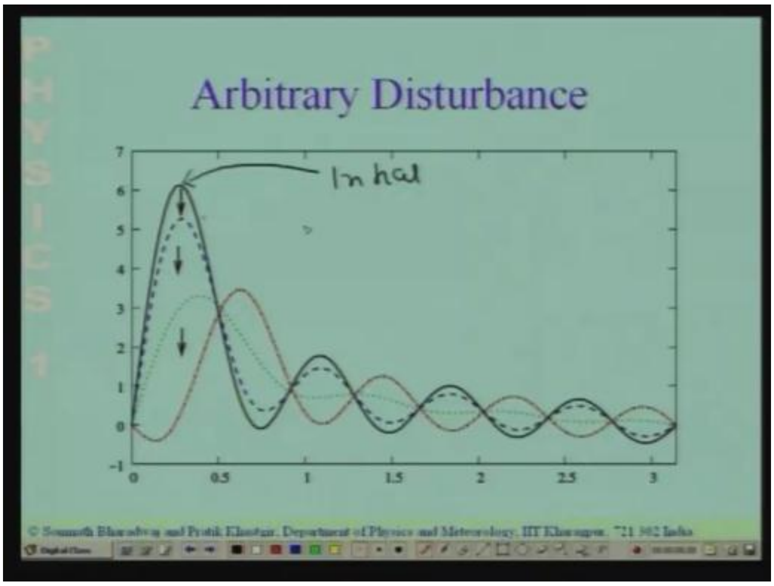

So, we have a disturbance in a string which initially looks like this. So, we have a disturbance in a string which initially looks like this. So, this is the initial disturbance in the string the black curve over there. And this is a superposition of many standing waves with different values of N different amplitudes different phases.

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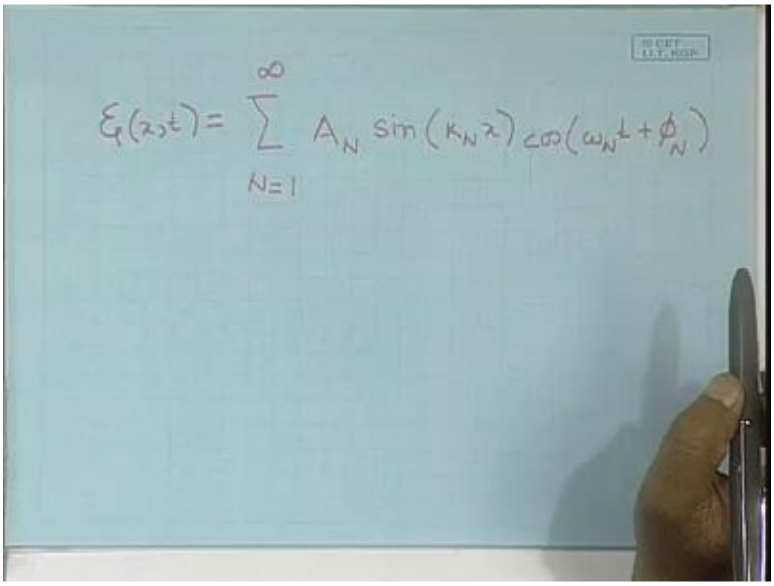

Here I have shown you 1 example, but it is a general mathematical fact that any arbitrary disturbance in the string any arbitrary disturbance in such a String with the boundary conditions at the disturbance is 0 at the 2 ends can be decomposed into a sum of such standing waves.

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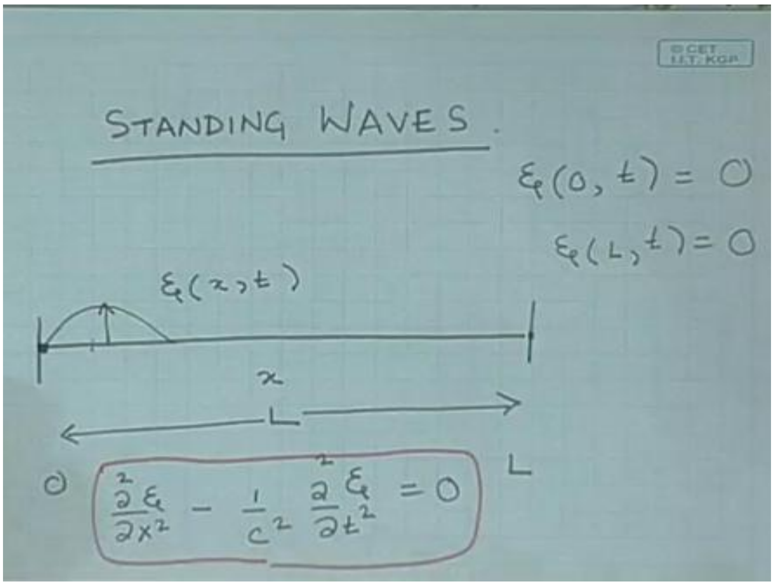

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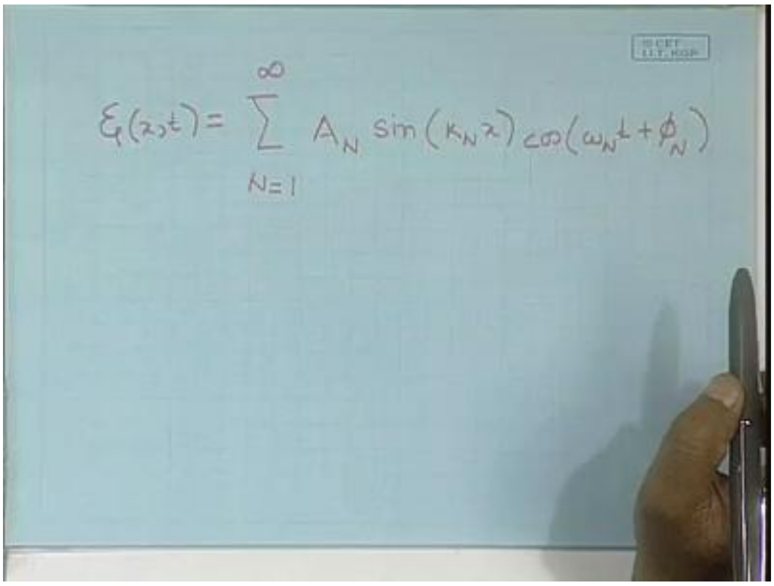

So, this is the initial disturbance.

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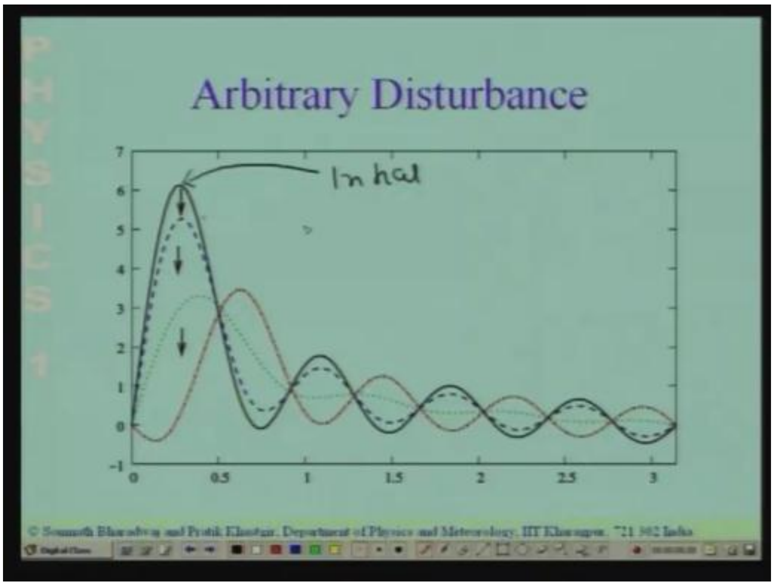

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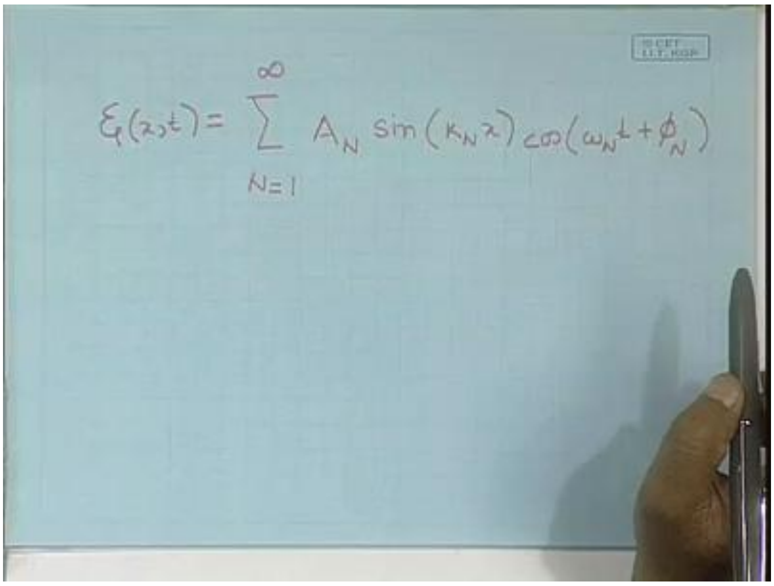

In the string now as time evolves each standing wave the Cosine term of each standing wave is going to change with a different frequency omega N is different for each standing wave. So, it is going to change differentially. So, this xi x t is going to look different after some time.

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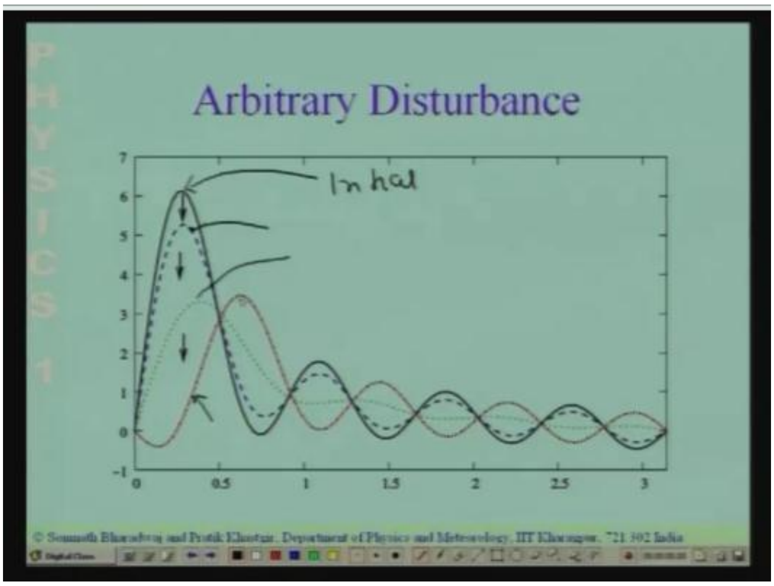

So, this shows you this curve over here this curve over here shows you the form of xi x t at some slightly later time you see the form as changed. And then this shows you the form of xi x t some even later time and then this curve over here shows you the form of xi x t at some even later time. And this whole pattern is going to change it will evolve with time. So, you can see that the peak has shifted over here. And it is going to evolve with time and then after some time it is going to repeat over again and the time required to repeat again is decided by the smallest frequency smallest value of Omega N which is there over here right this decide by the full.

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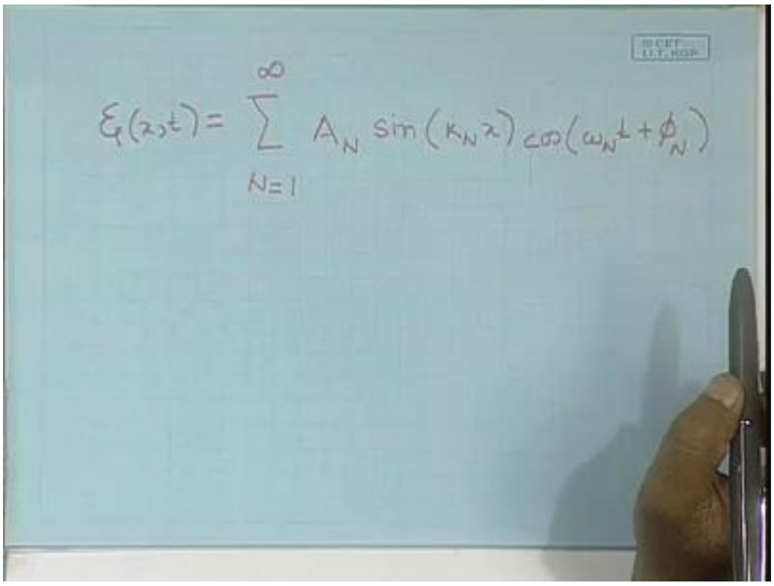

So, it is decided by all of these values of omega N which are here they all have integer values. So, you can determine when the whole pattern is going to repeat. So, the point here is that we have learned about standing waves in this lecture a standing wave pattern does not move around it is only it is amplitude changes with time it has nodes. At nodes the vibration the disturbance remain 0 throughout any arbitrary disturbance in the string with the boundary conditions. Then the disturbance should vanish at the end any arbitrary such disturbance can be decomposed into a sum of standing waves. And we can follow it is time evolution by following a time evolution of the individual standing waves and adding them up together.

These standing waves are very important say for example, if you want to look at the vibration of a string in your guitar. The guitar we know produces sound and the smallest frequency sound you can produce with the string is basically the fundamental mode. You will also have the higher harmonics being produced if you plug the guitar string. It depends on how you plug the string depends on which mode you excite these standing waves are also important. For example if you are studying electromagnetic waves in a cavity if you have a metallic cavity and you have an electromagnetic wave inside the metallic cavity.

We know that at the boundaries on the metal surface the electric field has to vanish right the electric field vanishes on the metal surfaces. So, if you have a electromagnetic inside a metallic cavity it has to vanish at the boundaries and again there you have to solve the wave equation with the appropriate boundary conditions where it vanishes at the ends. And this gives you standing waves standing electromagnetic waves inside the cavity. So, this basic idea of standing waves has various application and it appears in a large variety of situations the sum of which we will be encountering later on in this course. And there are many more which we shall not be discussing any let me stop here for today and we shall continue in the next class.