Physics I: Oscillations and Waves
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Lecture - 28
Waves (Contd.)

Good morning. In the last 2 lectures, we have been discussing the wave equation and it is solution we started the discussion using an elastic rod as an example. So, we considered a disturbance in the elastic rod and the disturbance was such that there were displacements introduced deformation introduced inside the rod. And we looked at the propagation of these disturbances in the same direction as in which the displacements were introduced. So, we had a situation where we had longitudinal waves there were displacements and the displacements in the rod propagated in the same direction in which we had the displacements. And starting from this we derived an equation which governed the evolution of these displacements. And this equation was the wave equation I then told you that the same equation appears in a large variety of situations where we have the wave phenomena.

So, if you consider a stretch string and pluck it and look at the evolution of the disturbance again it is governed by the same wave equation. That main difference being that we now have a transfer’s wave the displacement the disturbance is perpendicular to direction in which the wave is a moving around. The same wave equation also applies for electromagnetic waves where we have disturbances in the electric and magnetic field. And in the last class, we discussed how to work out solutions for the wave equation. So, we consider two specific situations; the first one was plane waves where we had a planner, symmetry. And the second situation that we discussed we had spherical waves where we had a spherically symmetric situation. Now, today let us first start of by discussing a few problems related to solutions of the wave equation. So, the first problem that we are going to discuss is give the problem is given over here.

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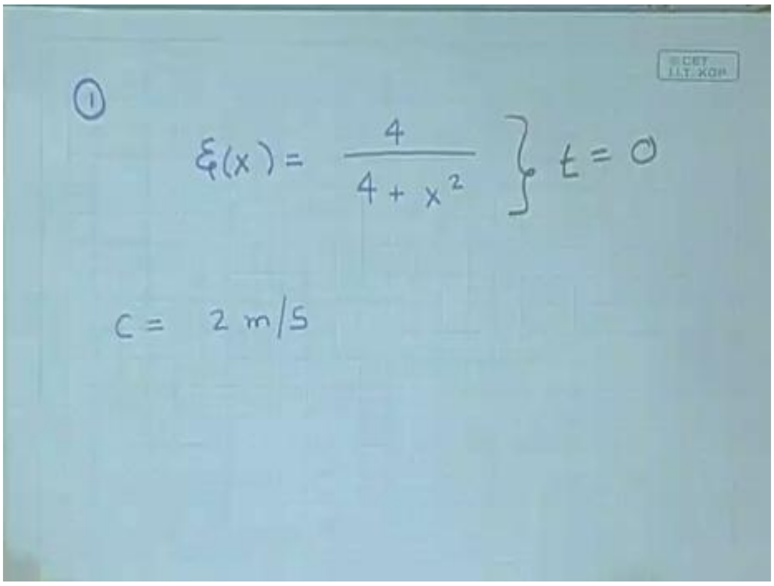

So, you have a displacement xi at the time t equal to 0 at the time t equal to 0 so, this is at the time t equal to 0 the displacement xi the disturbance xi is given by this function 4 divided by 4 plus x square. And the problem is the question is that you have to we have to first draw this function. So, draw the disturbance the form of the disturbance as a function of x at the time t equal to 0. So, let us draw this function it is a very important to be able to draw functions it is also equally important to be able to guess a functional form for a graphs. If you if you see a graph. It is also important to be able to guess the functional form and this intuition develops after you have drawn a large number of functions. So, let us draw this function. So, let us first ask the question what is the value of xi at x equal to 0? So, at x equal to 0 xi has a value 1 it is the ratio 4 by 4 so, let me draw this here.

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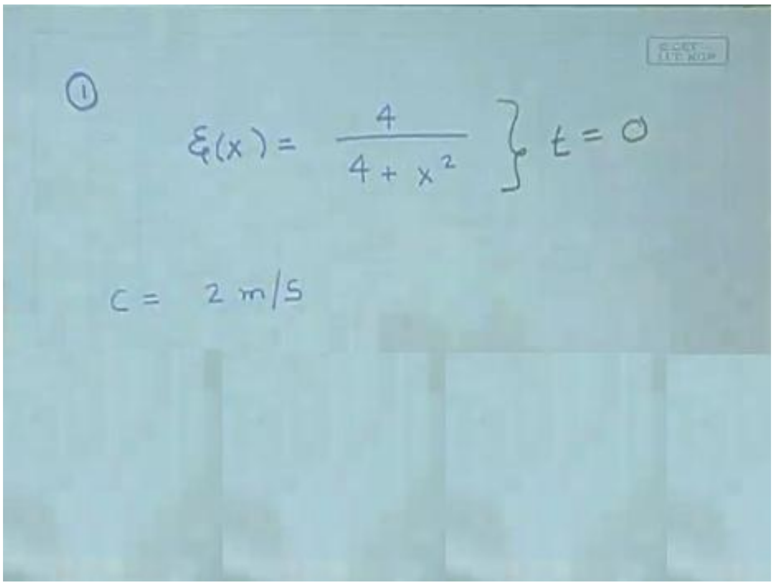

So, this is xi and this is x and at x equal to 0 xi has the value 1 that is the value at x equal to 0 now as x increases just look at this as x increases the value of xi goes down from 1. And as x tends to infinity the value of xi becomes 0 approaches 0 asymptotically. So, this has this function xi has a maxima at x equal to 0 and then the value falls of at a as you go further and further away.

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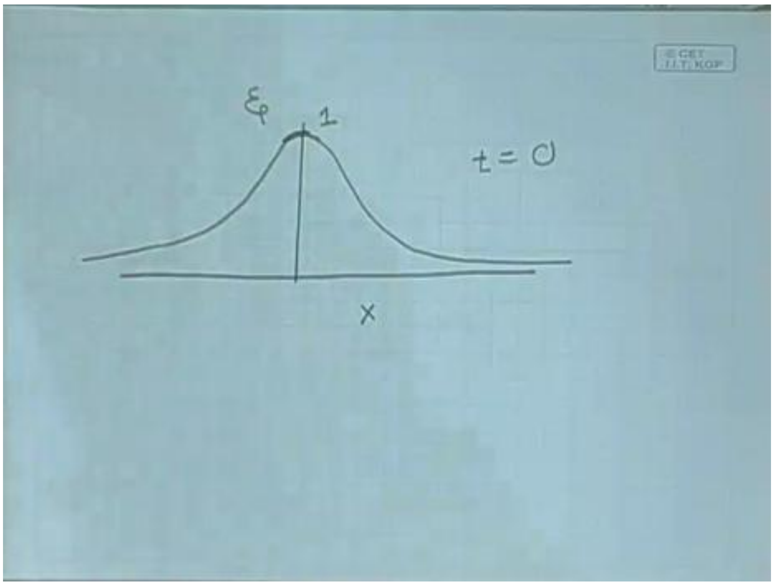

So, the graph looks like this it falls of as 1 by x squared as you go for so, graph looks something like this. So; this is xi at t equal to 0 that is the first part of the problem.

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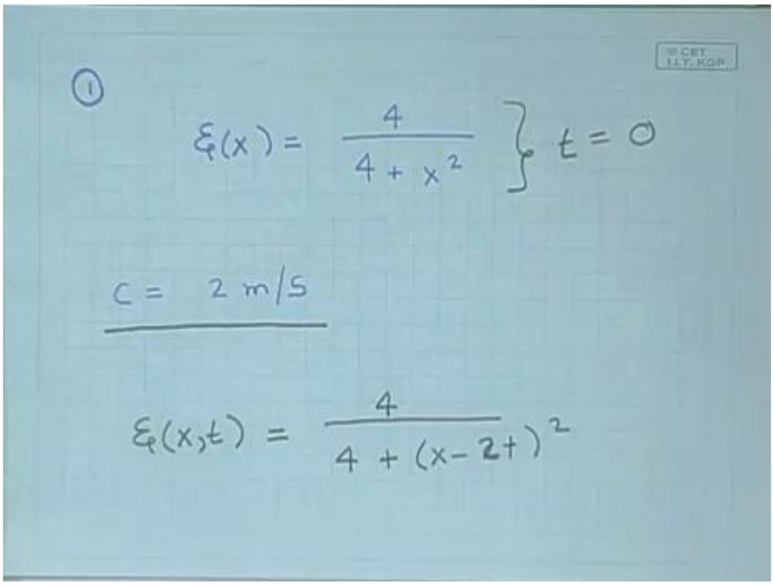

Now, the second part of the problem is that we are given the information that this is a right propagating wave. So, as time evolves this whole it is a right propagating wave propagating towards the right with the speed cs or which I have denoted by c to meters per second. And the problem is that you have to write down the full time dependence of this displacement. So, let me repeat the problem you have been given the displacement xi at the time t equal to 0 the functional form of xi at t equal to 0 is given. And we are also given the information that this disturbance propagates to the right as a right propagating wave with the speed c 2 meters per second. So, the problem question is that now we have to write down what the functional form for the space time for x and t dependence of this displacement.

Now, we know that if you have a right propagating wave for function effects is a right propagating wave then you have to replace x by x minus ct any arbitrary any function of x minus ct represent a right propagating wave. And here with the function has this form 4 by 1 4 plus x square so, it is quite straight forward that the right propagating wave if this function is a right propagates to the right for the speed c. Then the wave is given by this expression this will be 2 t square. So, this is the full time dependence of the displacement if it is a right propagating wave right propagating wave means that you have to replace x by x minus ct and c the speed of sound. Now, has a value the speed of elastic whatever it be the phase velocity now has a value 2 meters per second. So, you once you replace that you have the right propagating wave. Finally the third part of the problem is that we have been asked to draw the displacement this is at t equal to 0. We have been asked to draw the displacement at t equal to 3 seconds.

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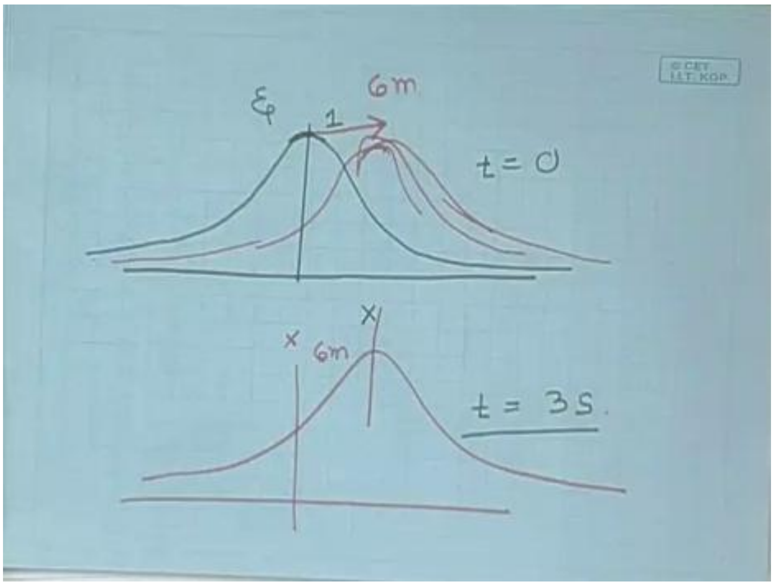

So, what will happen after 3 seconds is that the whole displacement pattern would have shifted to the right by cs or c c into t which is in this case if would have shifted by 6 meter. Because the speed is the phase velocity of the wave is 2 meters per second and you are interested in the wave in the disturbance pattern after 3 seconds. So, the whole thing would have shifted by 6 meters and this is what the pattern is going to look like the whole thing would have shifted here by 6 meters. So, the whole thing is shifted this displacement is 6 meters the whole pattern has shifted I so, I can write for you again over here it has shifted like this. Now, the pattern has shifted by originally centred here it has the centre centre has shifted so, has every point and it has shifted by 6 meters. So, that is the first problem which I wish to discuss.

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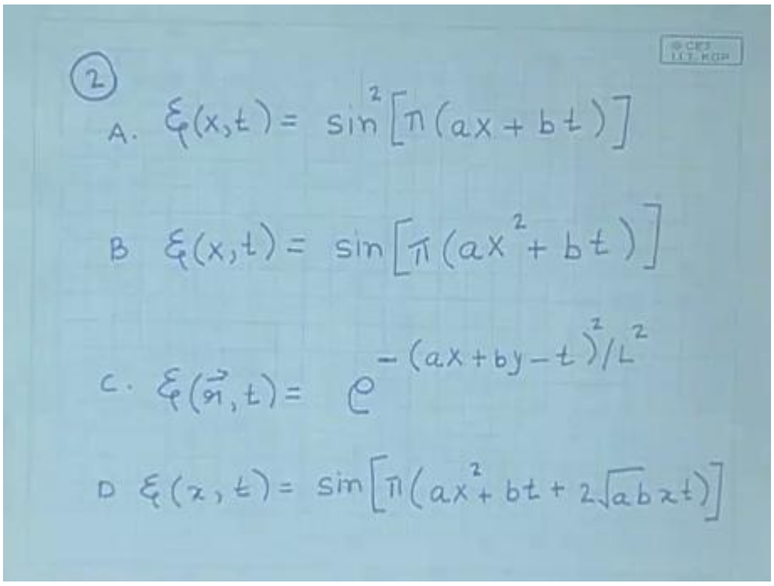

Now, let me take up the next problem. The next problem is as follows we have 4 different functions 4 different functional forms for the displacement xi. And the question is do these represent travelling waves do these functions represent travelling waves if yes then what is the speed of the wave? So, how do you decide if a function represents function position and time if it represents does it represent a travelling wave? In the last class we have seen that you have 2 kinds of travelling waves you have a right travelling wave and a left travelling wave.

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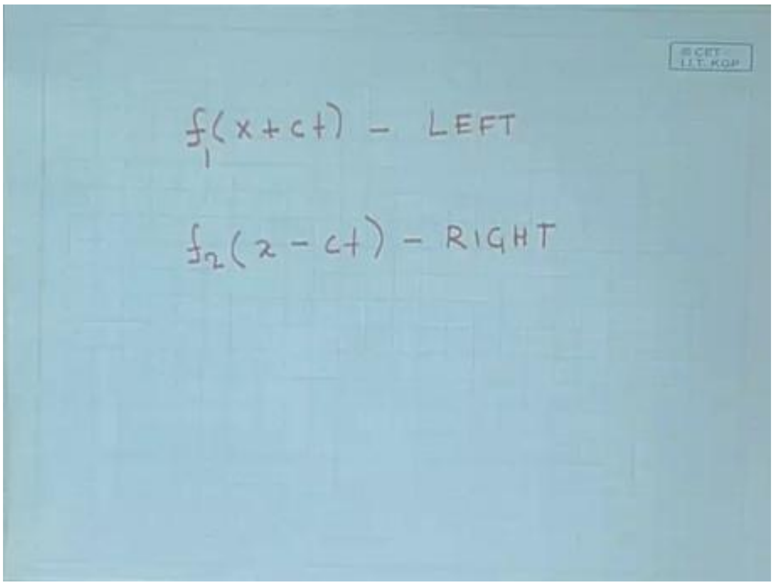

So, if I have some function f of x plus ct f 1 of x plus ct where f 1 could be any arbitrary function provided it is differentiable and you have it is well behaved it is a right it is a solution of the wave equation. And it is a right it is a left travelling wave because we have x plus ct. So, the whole pattern moves to the left this is a left travelling wave and it is a travelling wave solution. And if you have another function any arbitrary function for that matter of x minus ct this represents a right travelling wave. The function should be differentiable and should be well behaved that is the only condition. Then it will be a solution of the wave equation and it is it represent a right travelling wave.

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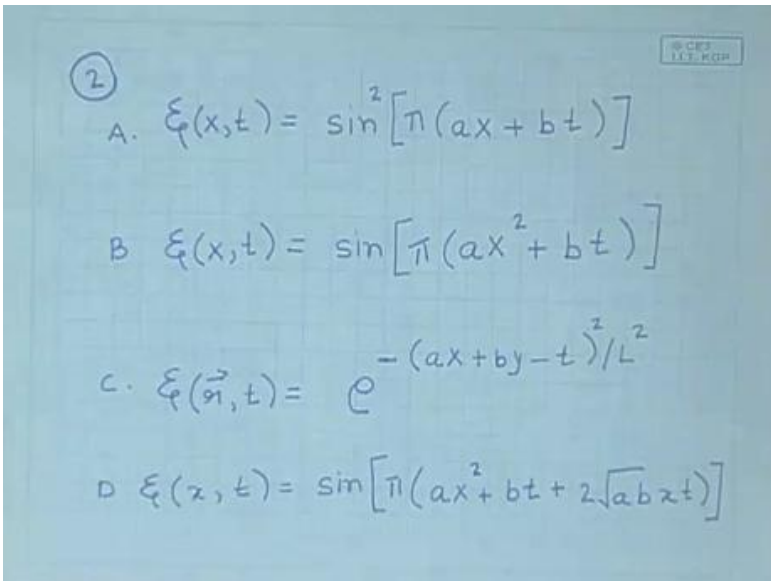

So, what we have to see is does this match with any of these 2?

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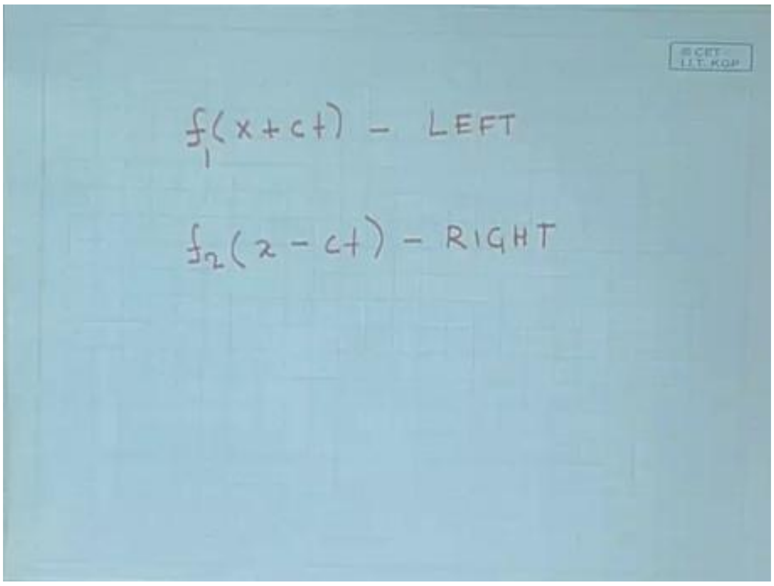

If it matches this it is a left travelling wave, if it matches with this it is a right travelling wave.

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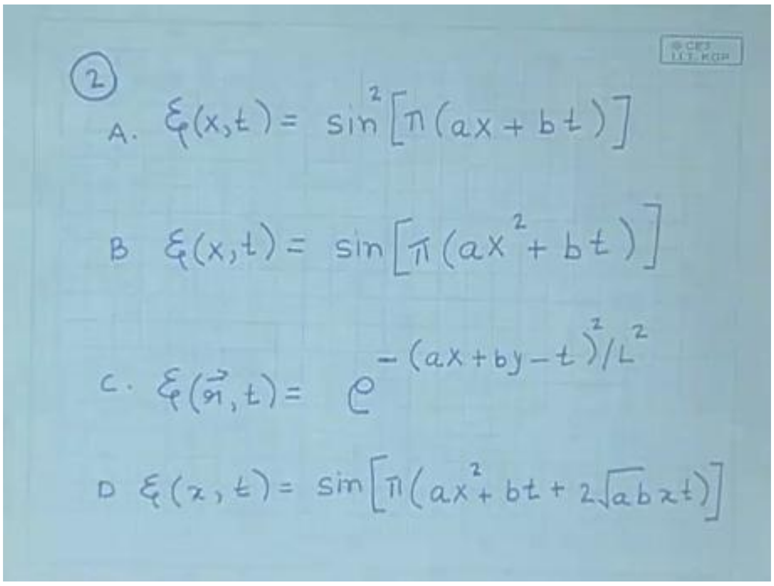

So, let us look at the first problem the first expression that is given A in this expression we have the displacement as a function of x and t. And it is sin square pi ax plus bt does this match with the question is does it match with any of these 2 forms.

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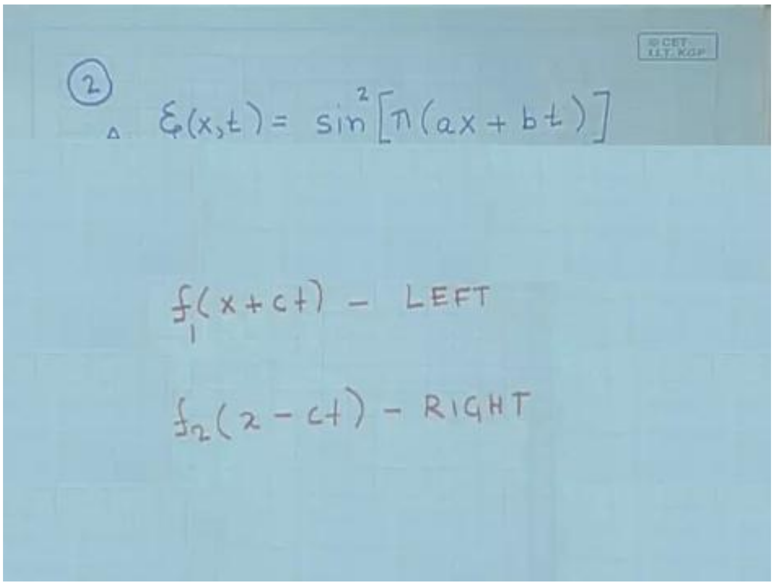

Any of these 2 standard forms for the left or the right travelling wave now, it should be clear that if a and b are both positive. Then this is a left travelling wave travels along the negative x axis if both a, and b are positive if any one of them is negative and the other is positive. Then it is a right travelling wave again if both are negative it is a left travelling wave. So, this does represent a travelling wave it is of the form where it is a function of x plus sub constant into time.

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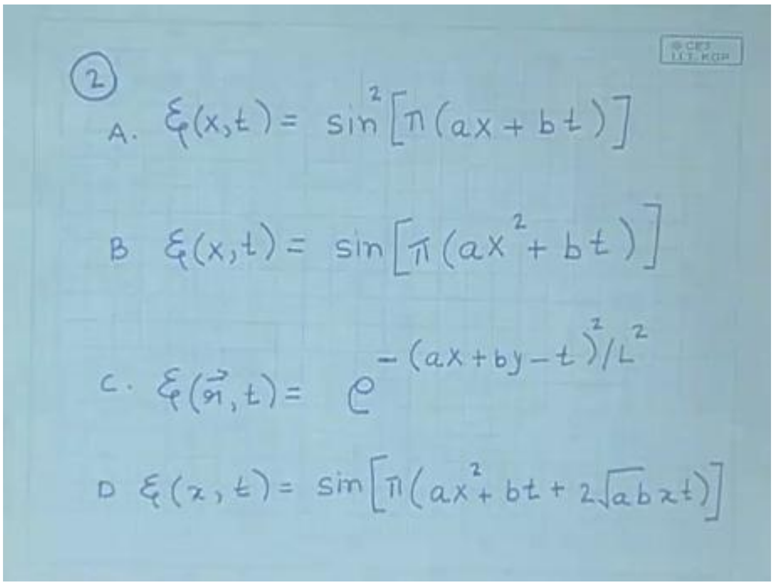

So, it you interpret this as x plus a constant into time a function of x plus a constant into time that constant is the speed of the wave so, for problem A.

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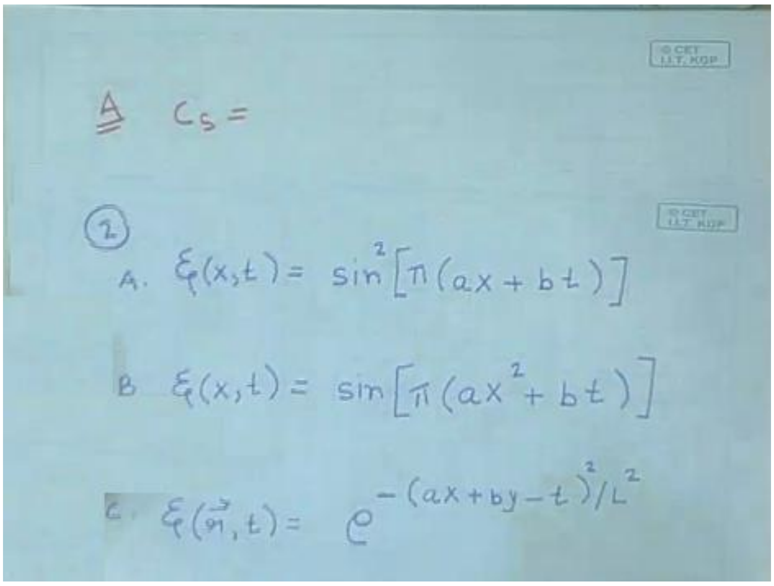

The speed see if you interpret this as x plus a constant into time so, this if you interpret this thing as a function of x plus a constant of time let me do it explicitly for you here the answers will then be clear.

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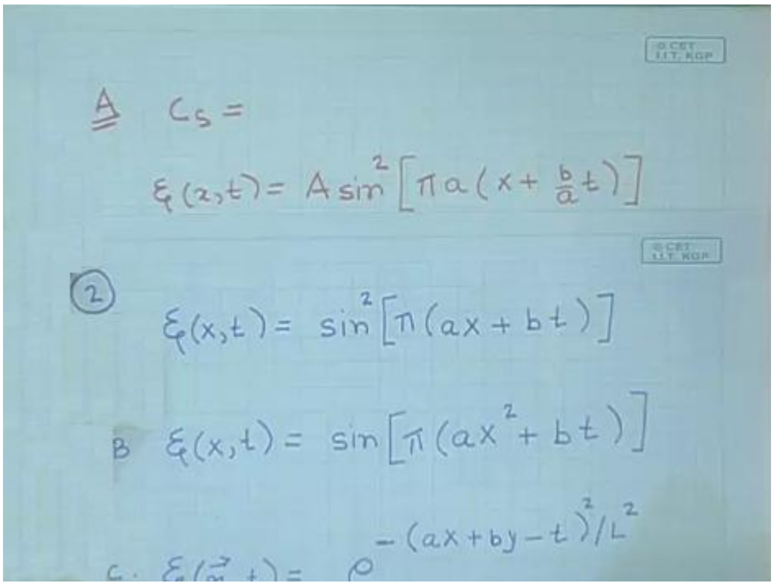

Xi x t is equal to A sin square pi I take a common outside a x plus b by a t so, this is a same function as the 1 which we have been given and you see that this is of the form.

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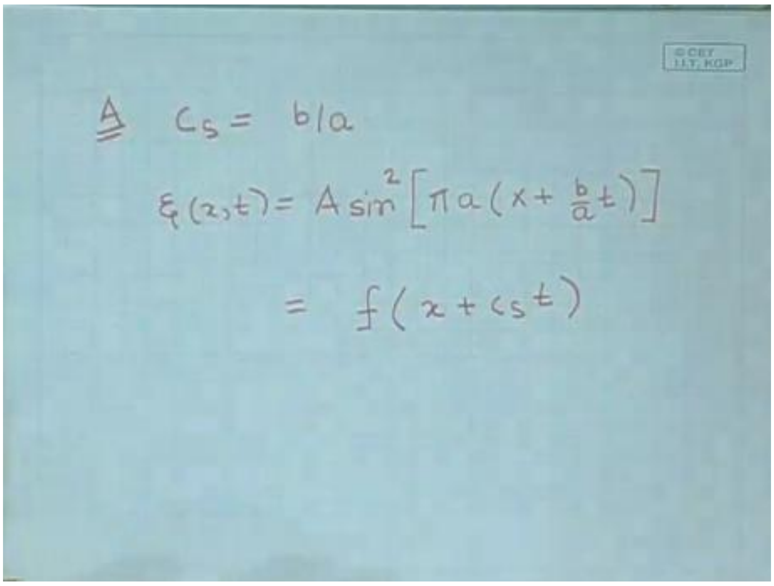

Where it is a function of x plus the speed of the wave into t where you can easily identify what cs is cs is the ratio of the constants b by a. So, this does represent a travelling wave and the wave speed of the wave is the ratio of b by a.

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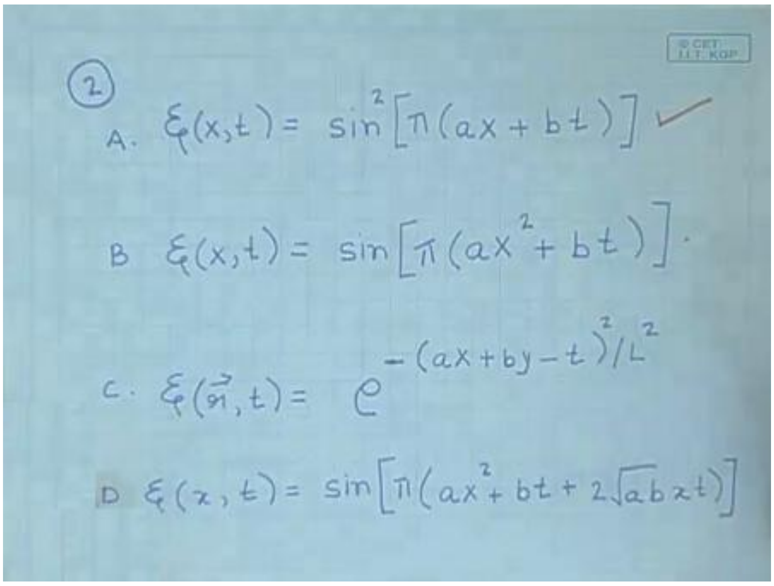

Now, the next question is does this b so, this is a travelling wave does b represent a travelling wave?

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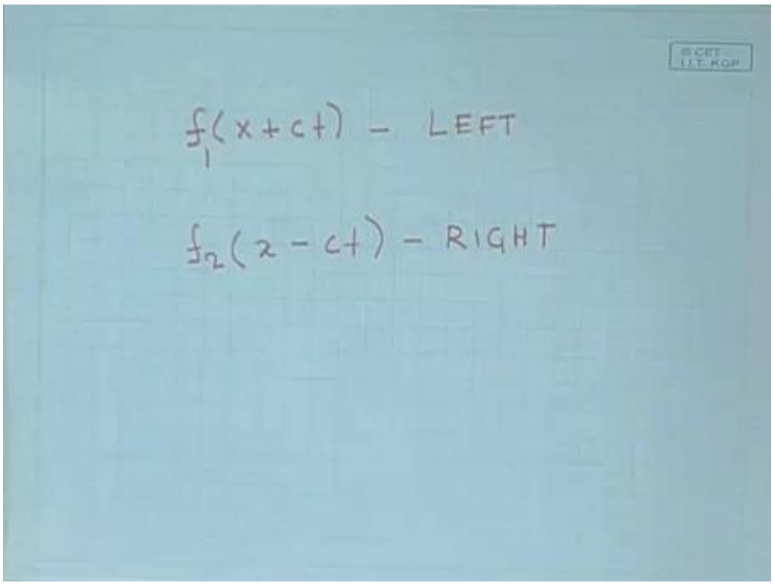

So, if b has to represent a travelling wave it should be either of this form or of this form.

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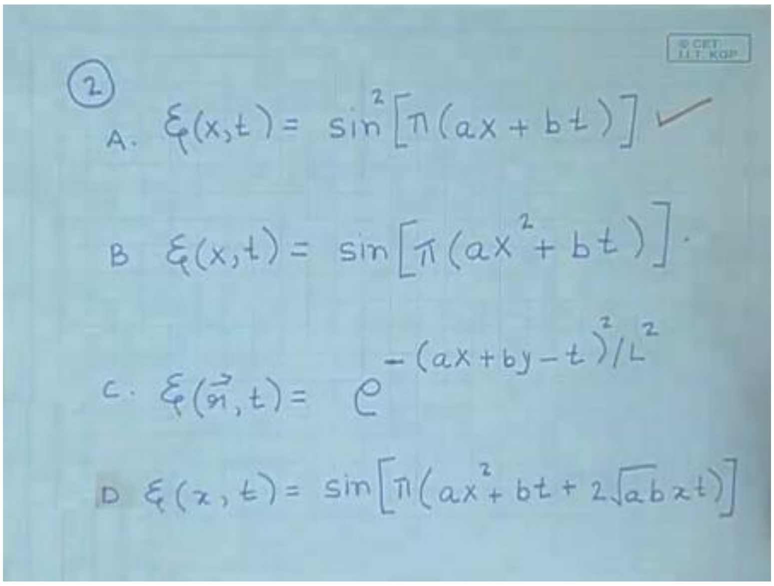

Now, b the function b the problem B the functional form of xi the displacement which is now, a function of x and t is given to the sin pi ax square plus bt.

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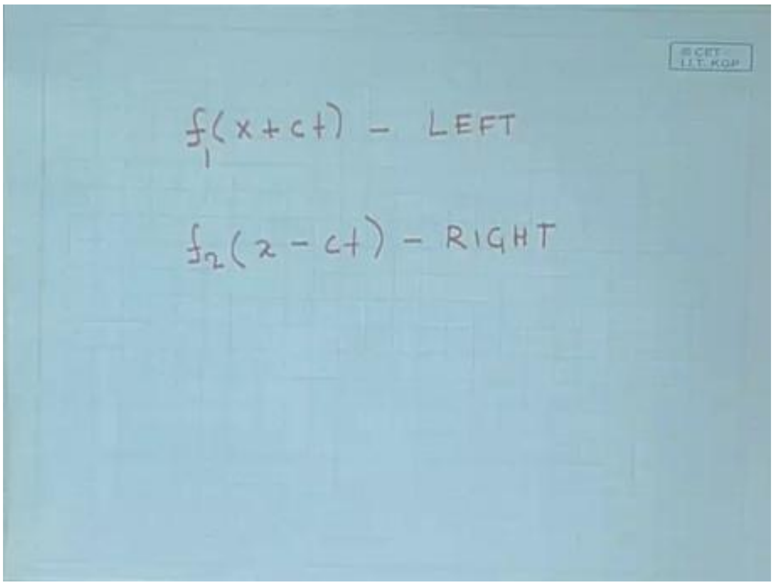

Now, notice that ax square plus bt is neither this it is not x you cannot write it as x plus a constant into time neither can you write it as x minus a constant into time.

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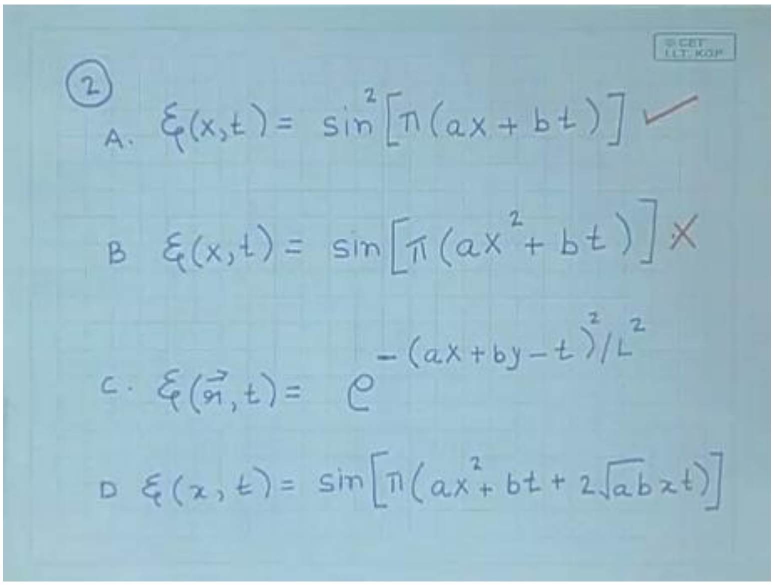

So, it is quite clear that this does not represent a travelling wave let us take up part C of the problem. So, in part C you are given a displacement now the displacement is a function of the position in 3 dimensional space. So, in the earlier 2 problems we had a disturbance which was there only along the x axis in part C in the part C problem the displacement xi is a function of all 3 coordinates. So, we have represented it through a 3 dimensional vector r so, vector r has all 3 x y and z in it. And the functional form of xi is given over here xi is the exponential of minus ax plus by minus t the whole square the square of this whole expression divided by L square. So, L is some number of the dimension of dimension of time because this whole thing has to be dimension less. So, the question is this a travelling wave and if it is a travelling wave what is the speed now?

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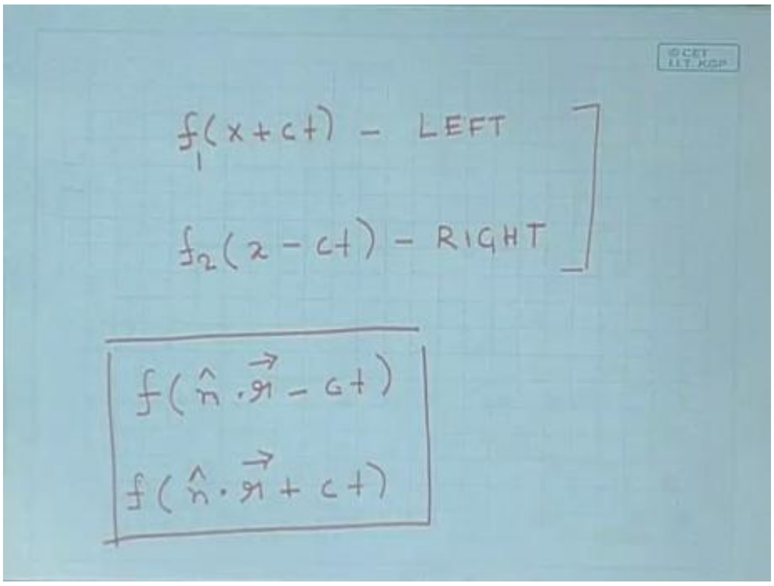

What we have written down here is was in for a wave in 1 dimension for a wave in along the x specifically along the x axis. Now, if I have a plane wave in some arbitrary direction then I can write it as f I can write the left propagating wave as n dot r where n is a unit vector in the direction in which the wave is going. So, this represents a wave propagating along the direction n and this represents a wave propagating along the direction minus n as time increases. The wave goes forward along this direction this wave and this represents a wave which goes along minus n direction as time increases. So, this is how you would represent a plane wave in any arbitrary direction here n is the direction of the plane wave which in this case is along the x axis. So, n would be just the unit vector along the x axis.

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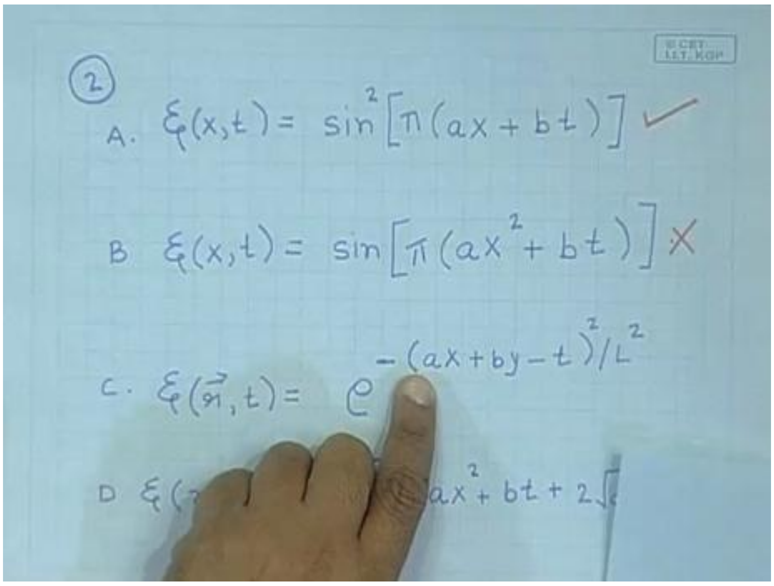

Now, can you write this in this way.

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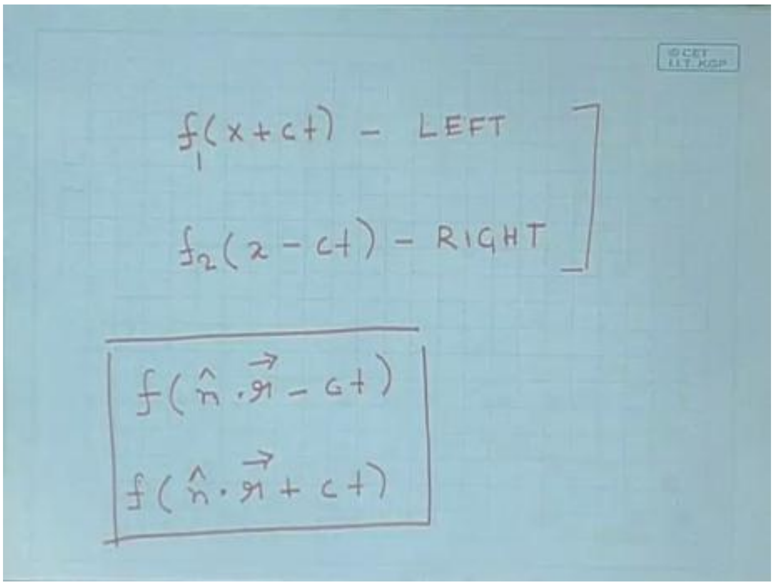

If you can then it is a travelling wave it is a travelling plane wave actually then so, the question is can you write this in that form.

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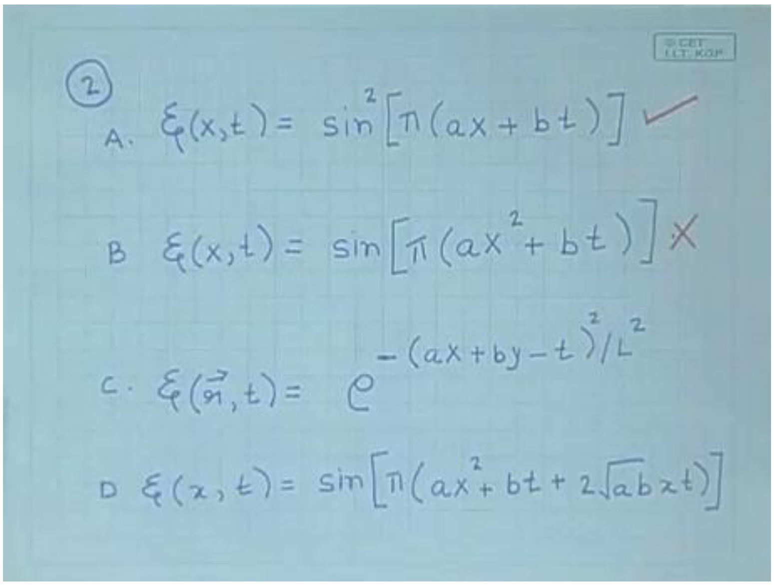

Just look at it for a few minutes and it will be clear whether this can be done or not. So, look at the argument over here the argument over here is ax plus by and then you have a minus t. So, let us just focus on this ax plus by.

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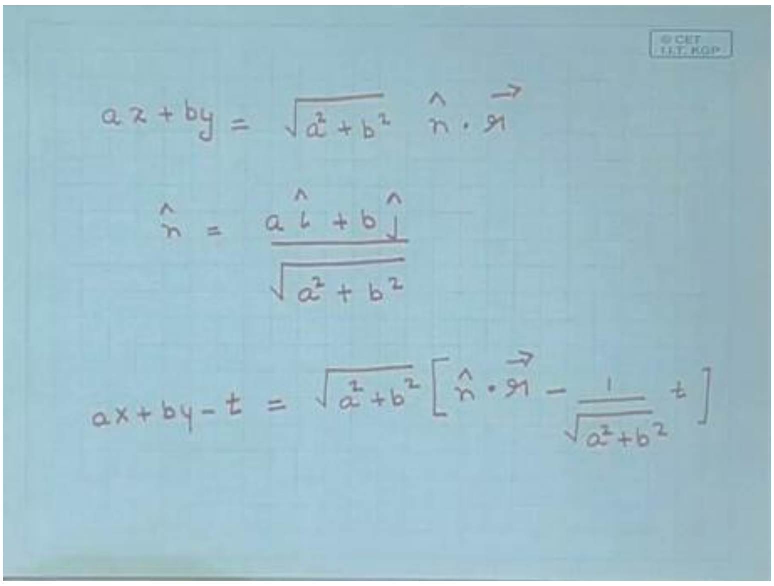

So, we can write this as the square root of a square plus b square into the unit vector n dot r where n the unit vector n is a into the unit vector I along the x axis plus b into the unit vector j along the y axis divided by a square plus b square. So, you can it is quite straight forward to check that this is a unit vector, because if I take the length of this n dot n I will get a square plus b square divided by a square plus b square which is 1. So, this is indeed a unit vector and n dot r into this number is indeed equal to this so, we have been able to write this as a number into a unit vector dot r. So, we can write minus ax plus by minus t as equal to square root of a square b square n dot r minus 1 by square root of a square plus b square into t right. So, all that we have done is we have written ax plus by minus t as a overall factor of square root of a square plus b square into this unit vector n dot r minus this number into t. So, this is exactly the same as this.

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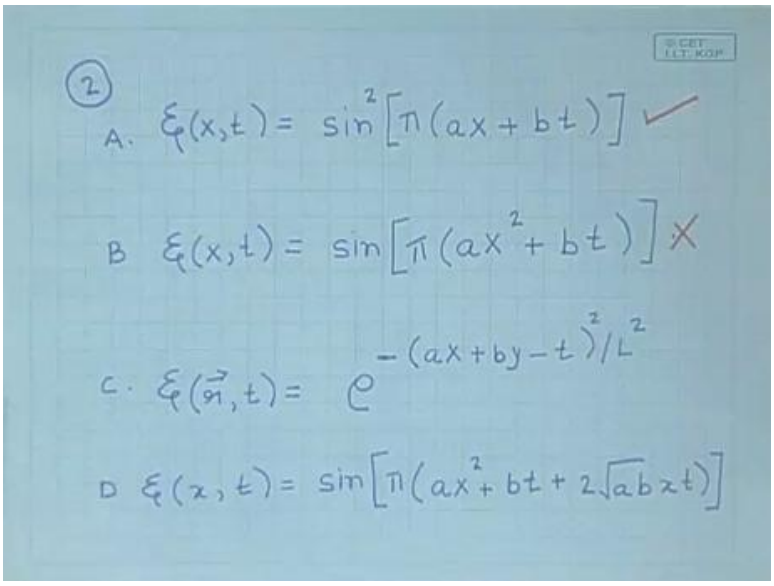

Now, it should be quite clear that the argument of this function the, which is there in the exponent has now, been written in this fashion.

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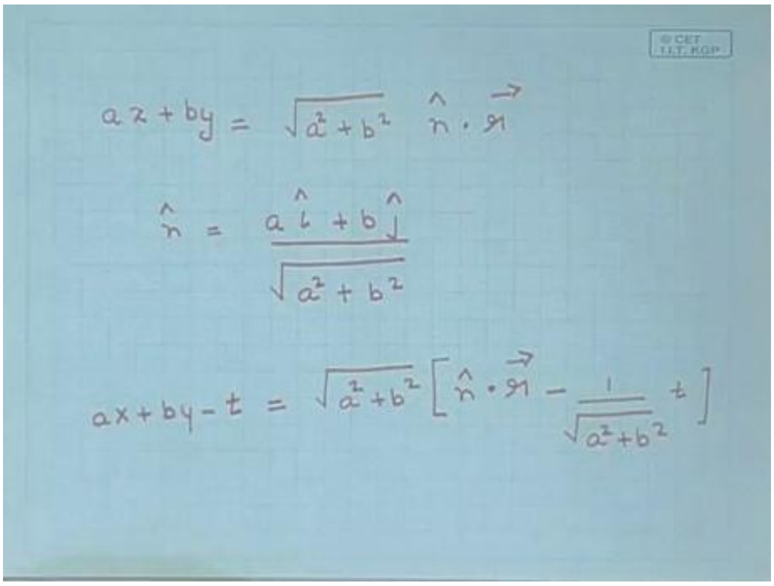

Right.

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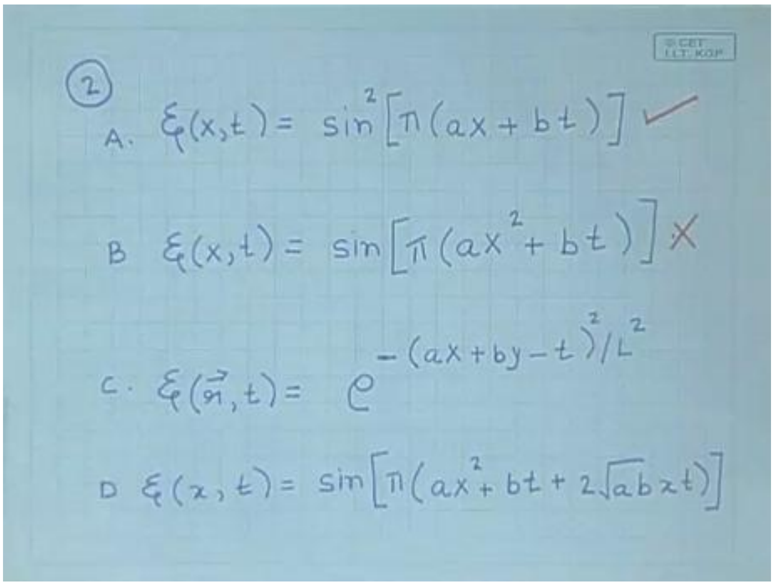

So, the argument of this function has now, been written in this fashion.

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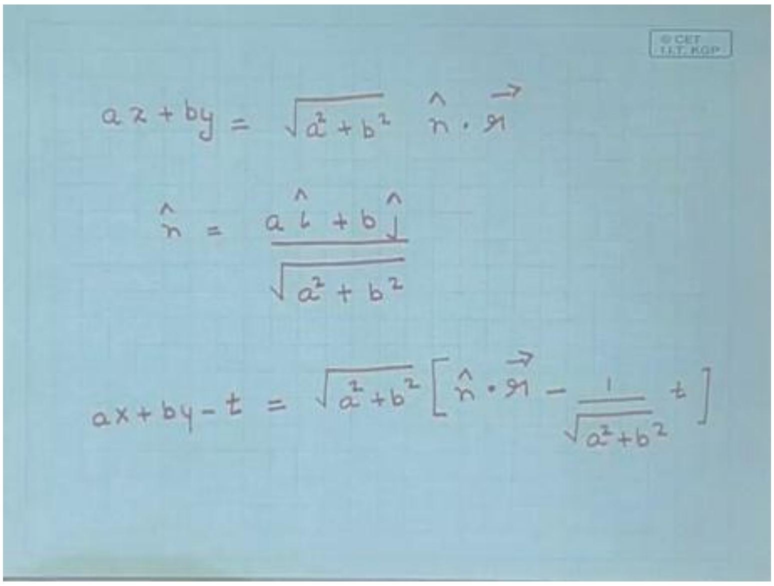

So, it should be clear.

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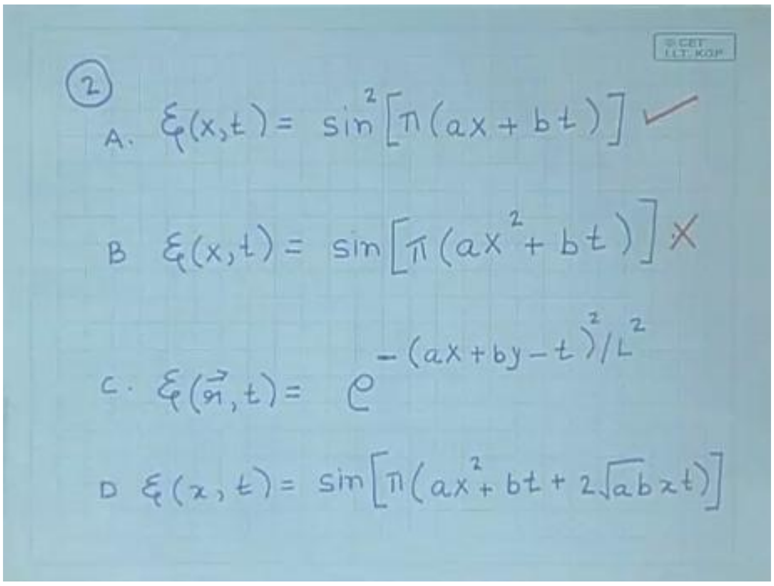

That this is indeed of this form unit vector dot r plus ct.

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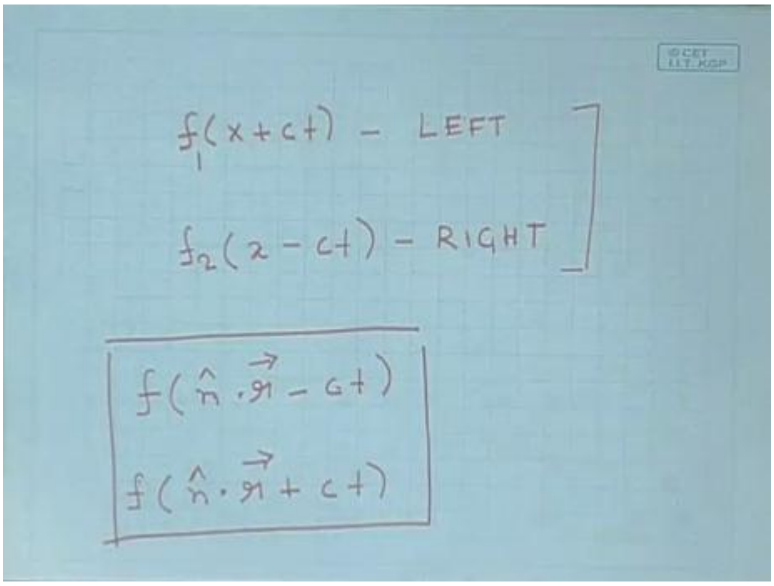

So, what you can say is that this.

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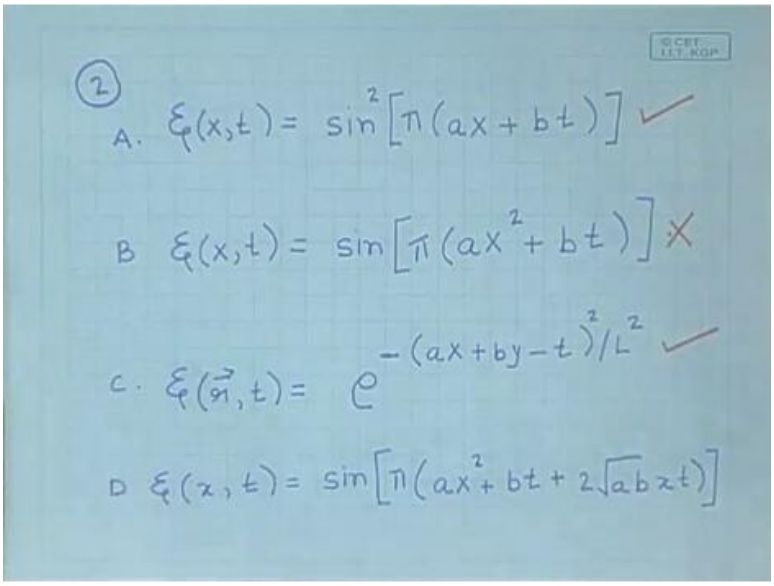

Does represent a travelling wave.

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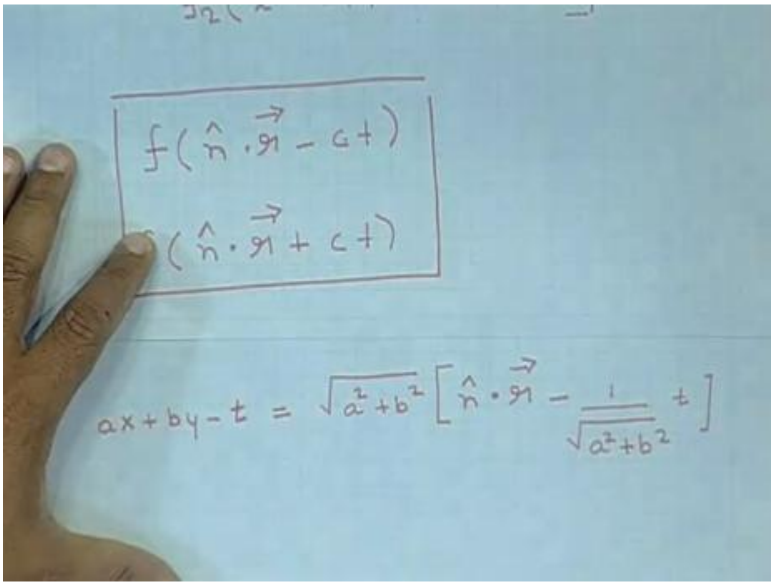

And if you compare this with the way we have written the argument the argument is over here.

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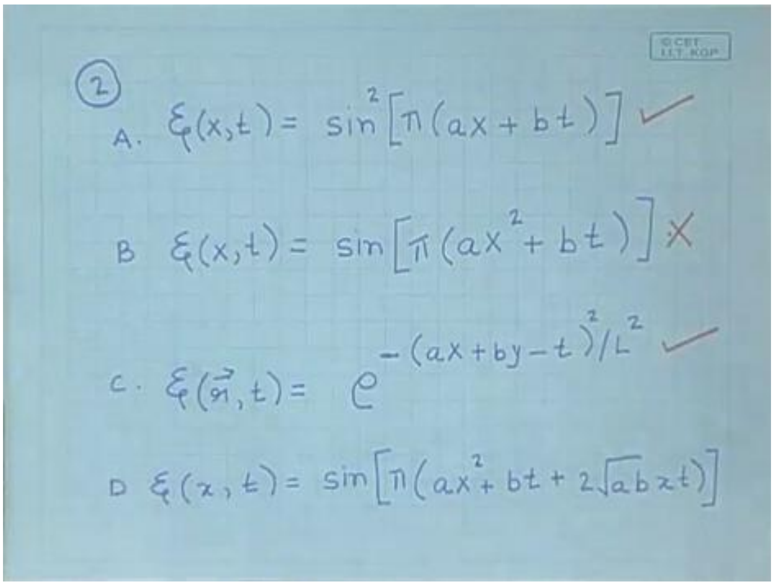

So, this is a function of this quantity.

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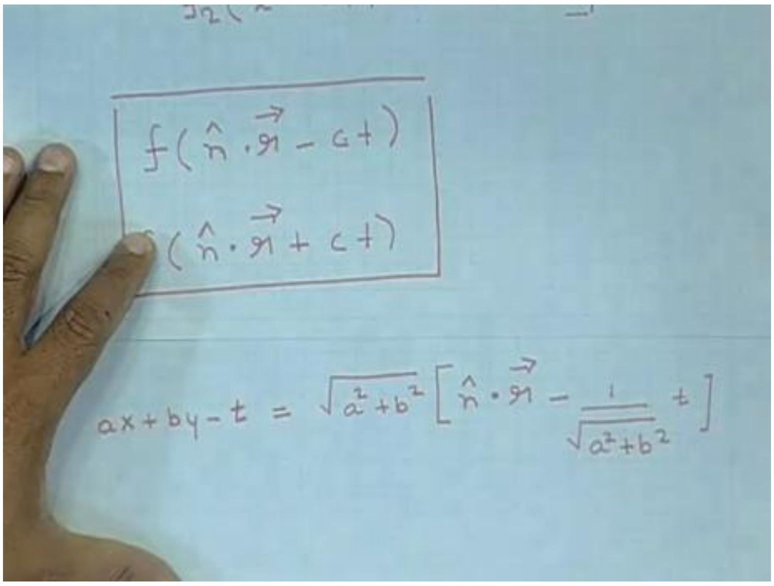

And that quantity which it is a function of can be written like this. So, you see that it is a function of n dot r plus.

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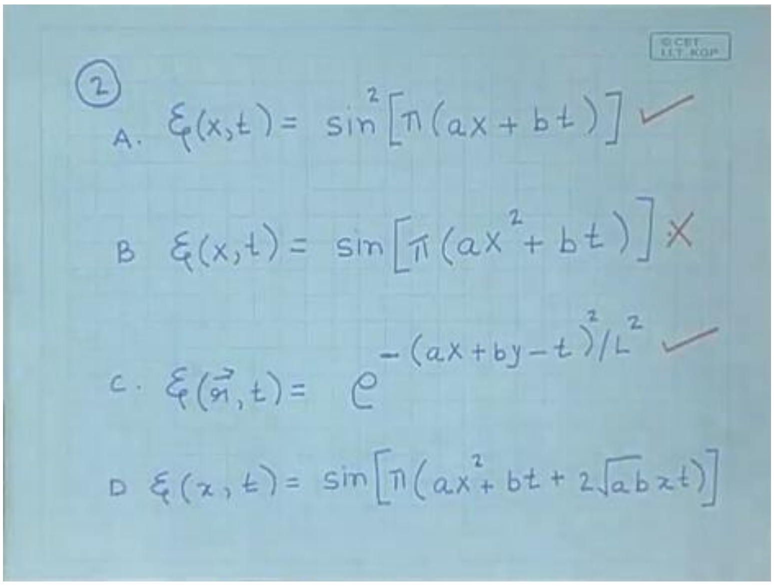

This should be this is minus so, this is.

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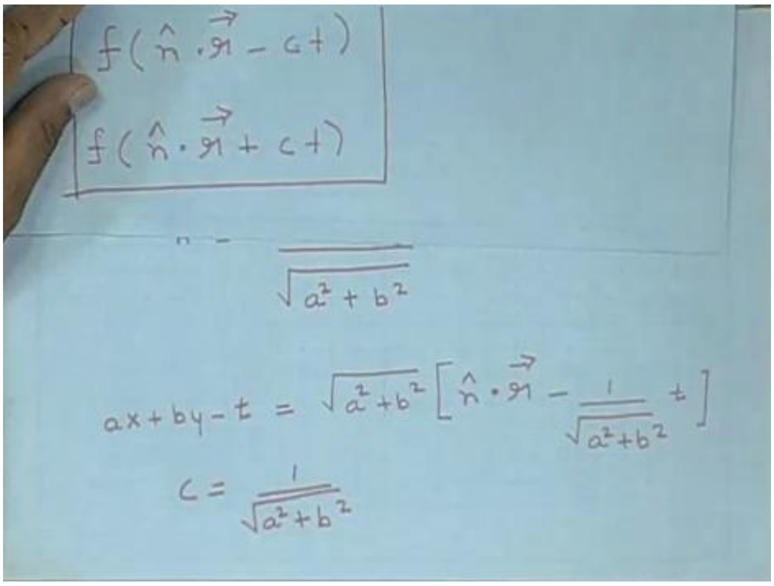

So, we have so, this represents this kind of a wave this represents this kind of a wave and just by just comparing this expression with this expression you can identify that. The speed of the wave is 1 by a square plus b square. So, let me recapitulate what we have learnt.

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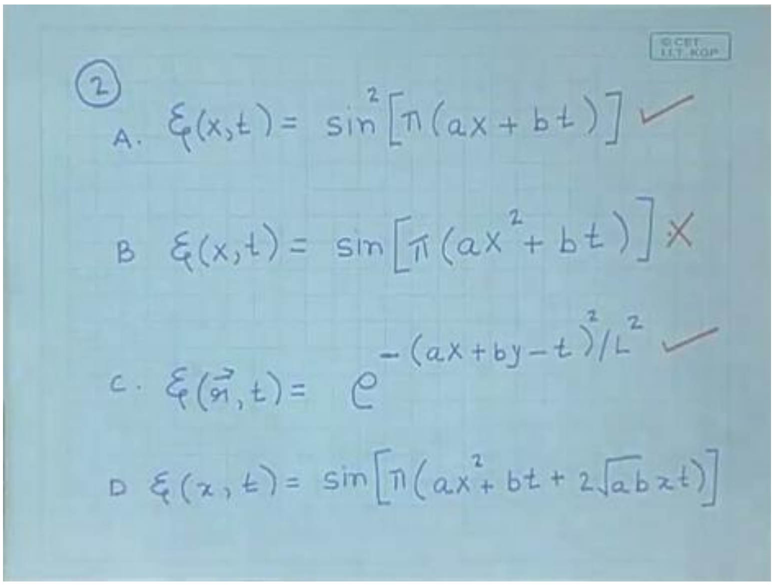

We leant that this does represent a travelling wave it represents a wave travelling along this unit vector.

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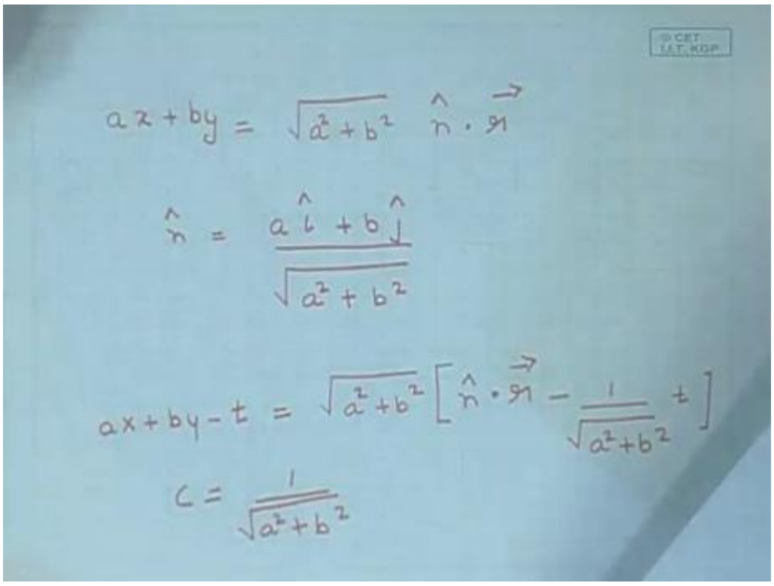

And the speed of the wave is square root of a square plus b square where a, and b are the 2 constants which appear over here.

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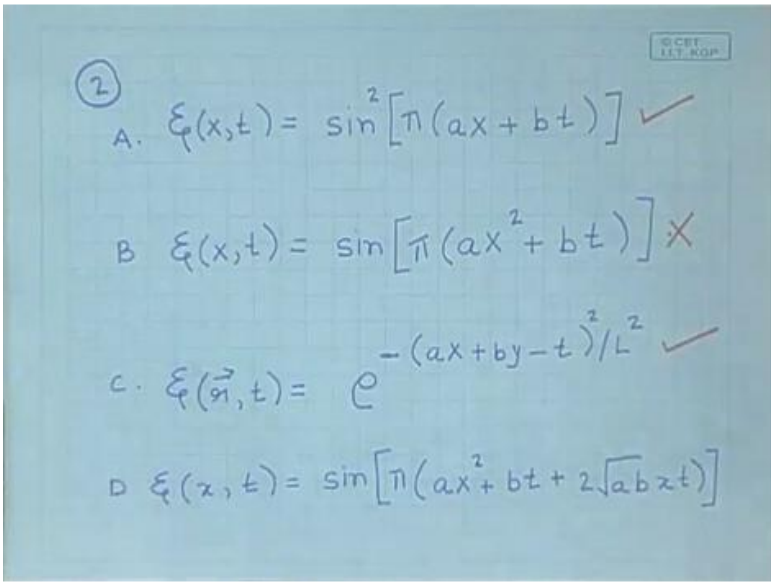

Let me now take up the fourth the fourth problem. The fourth problem is again 1 dimensional the displacement xi is a function of x alone and only 1 spatial variable x and time. And the functional form of xi is given over here if xi is to represent xi this functional form for xi xi xt is to represent a travelling wave.

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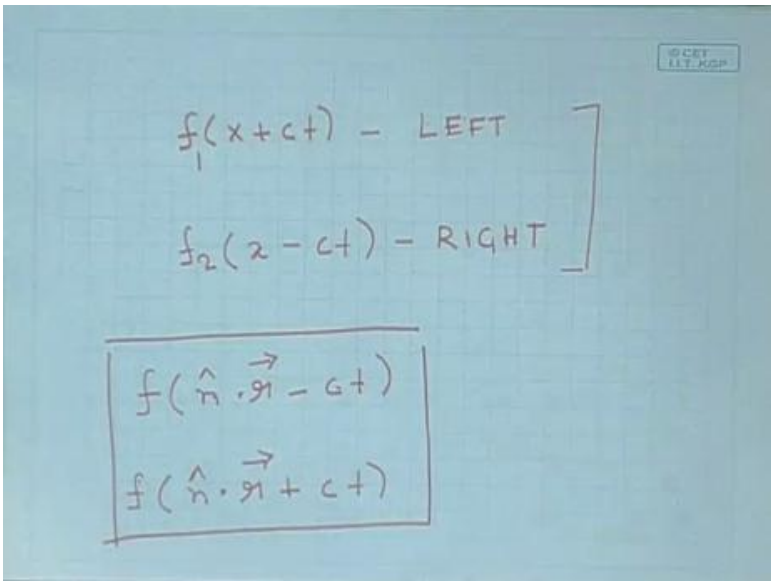

It should be either of function of x plus ct or x minus ct.

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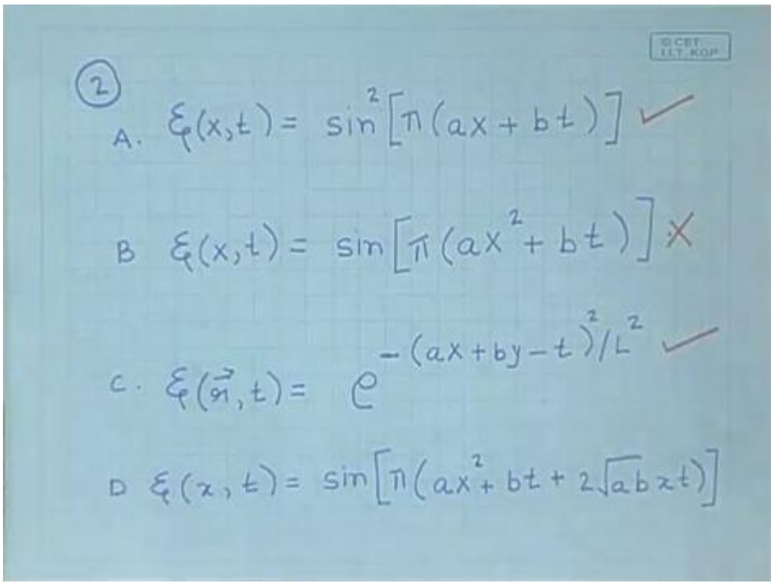

The function that we have over here is sin pi then we have ax square plus b. This should be t square sorry bt square plus 2 square root of ab x t question is it of a form where it is a function of x plus ct or x minus ct. Now, you can simplify this to address this question we simplify this expression a little bit. So, this is the expression that we start with and we can simplify it a little bit.

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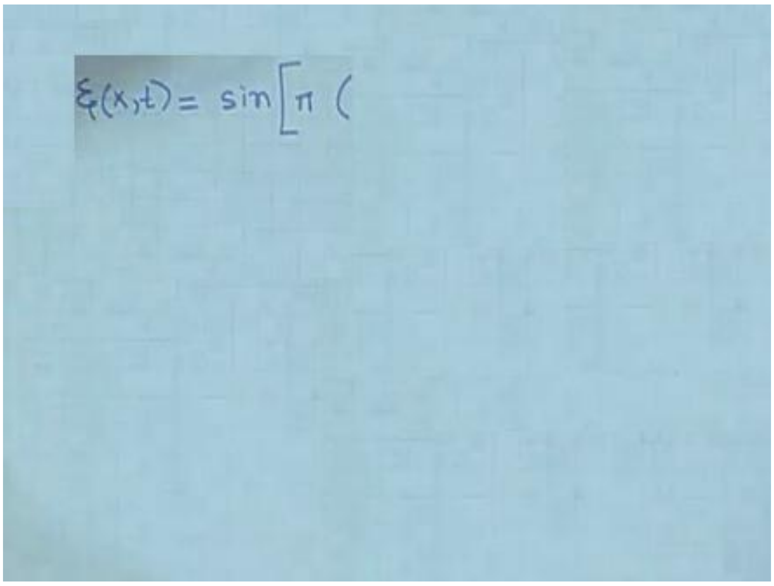

So, xi x t can be written as sin pi.

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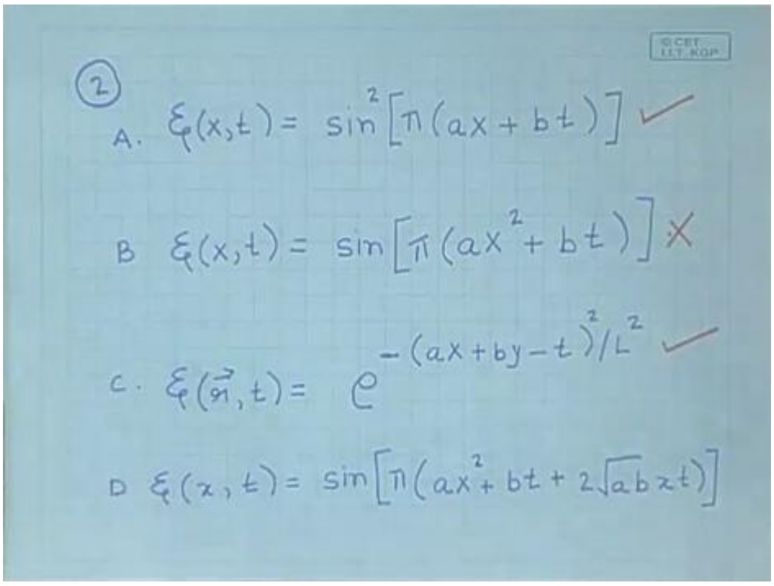

And notice that this whole thing over here is the.

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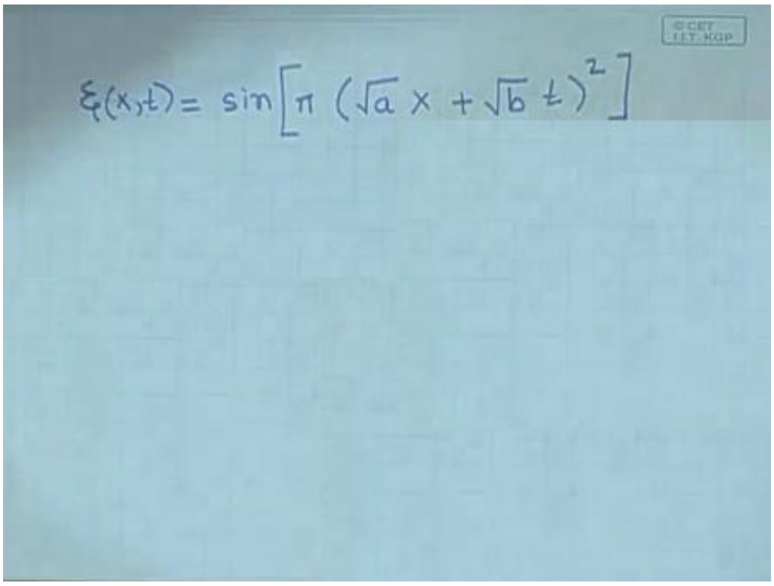

Whole square of ax plus square root of b t.

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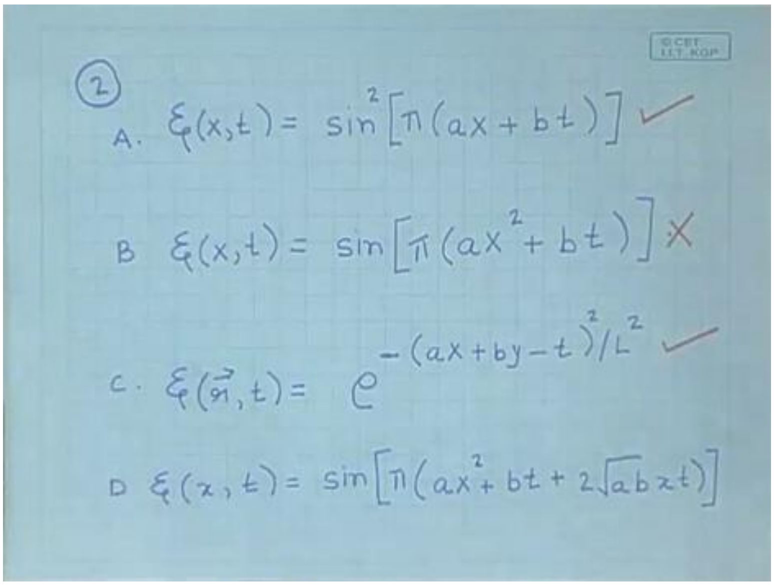

This expression is exactly the same.

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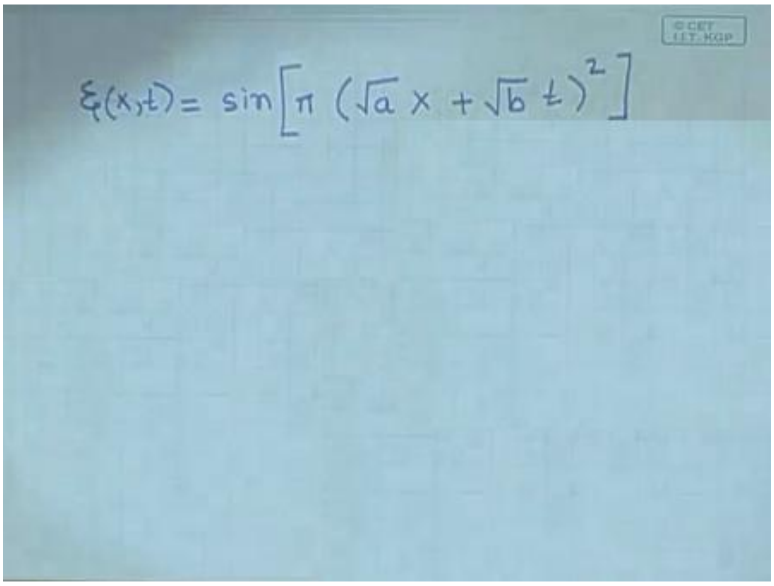

As what I have written over here, because when I squared this I will get the first term.

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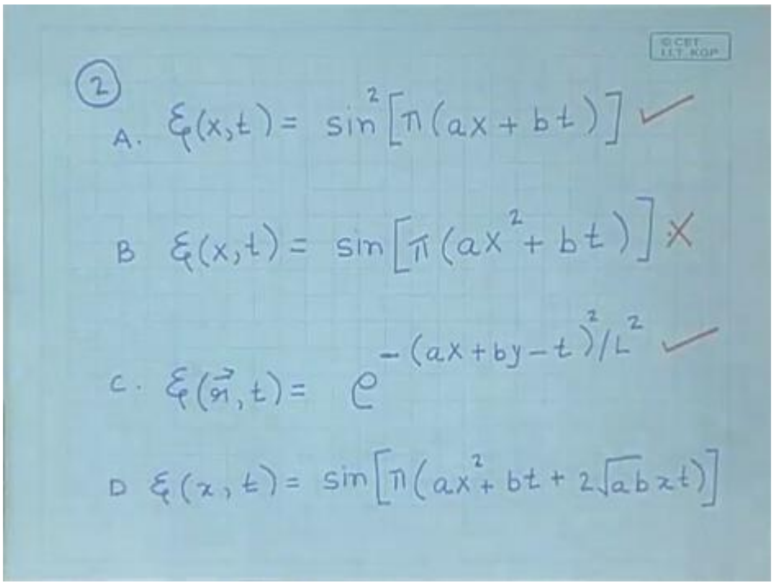

The square of this;

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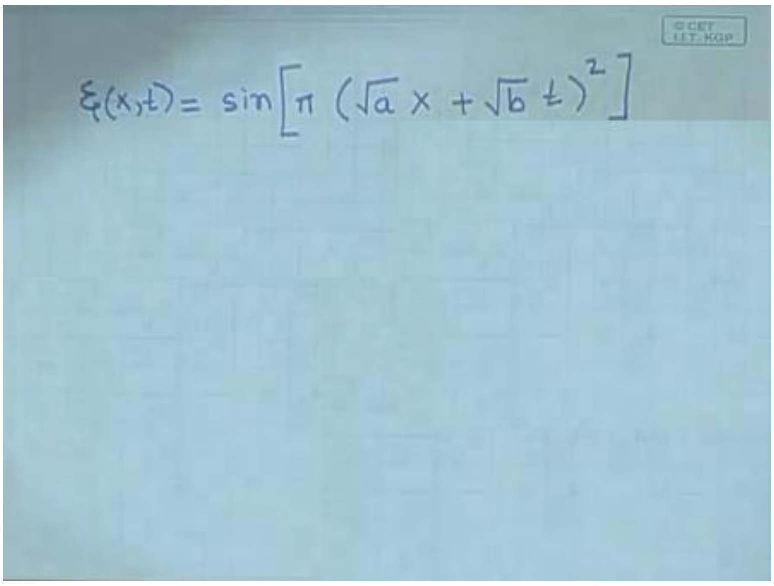

Will give me the second term.

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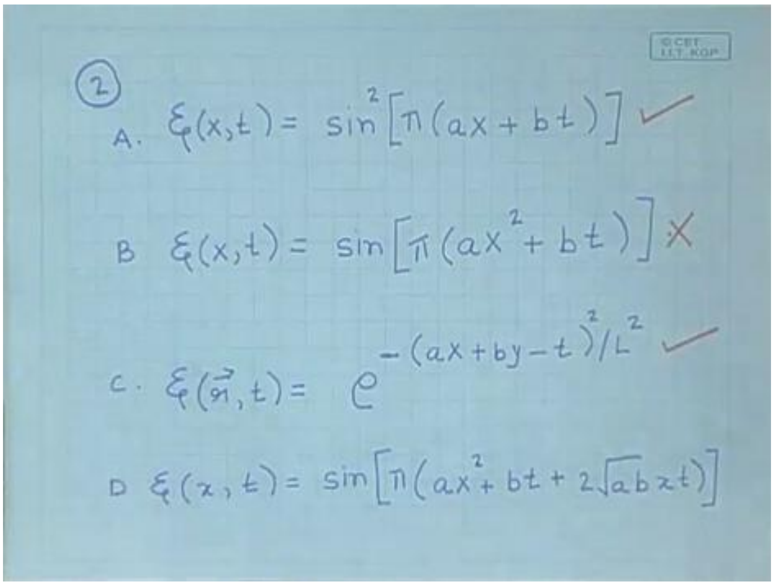

And the.

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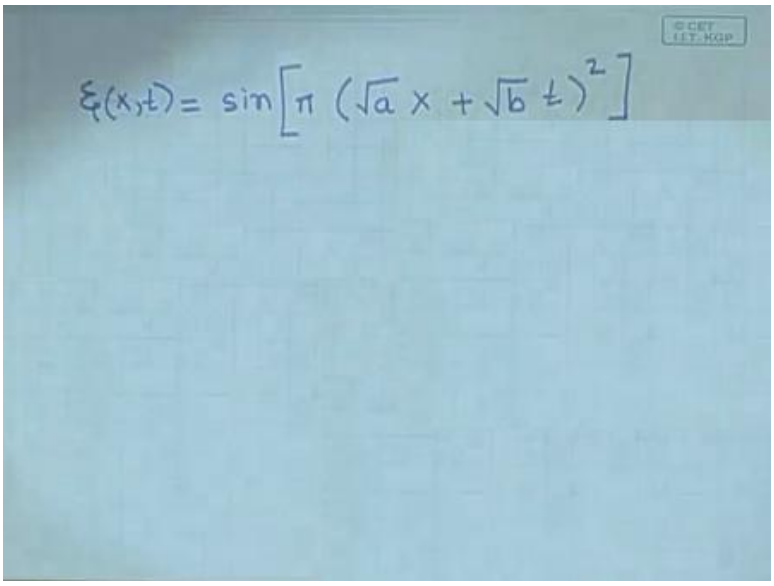

2 times this into this is going to give me the third term.

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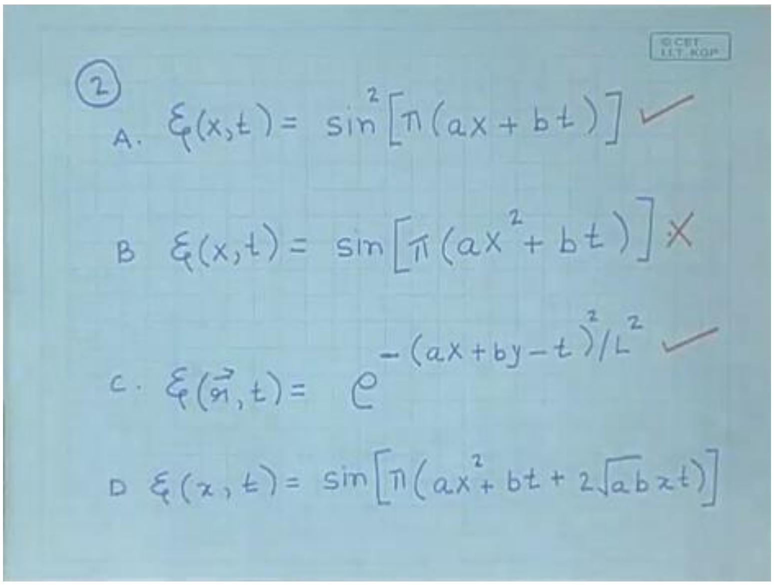

So.

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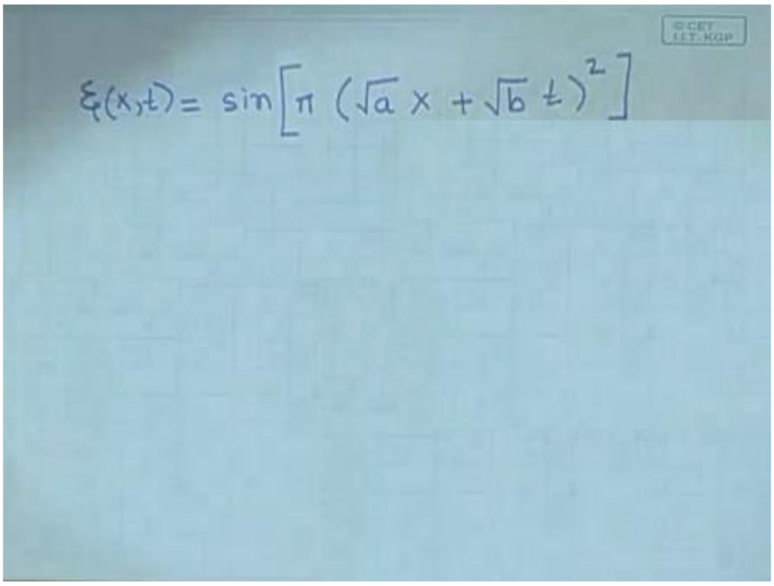

The same displacement is expressed the displacement is now, expressed like this. And it is very clear that this is of the form where it is a function of x plus ct where c has the value square root of b by a right. So, this wave this is indeed a travelling wave and it has the speed the square root of b by a.

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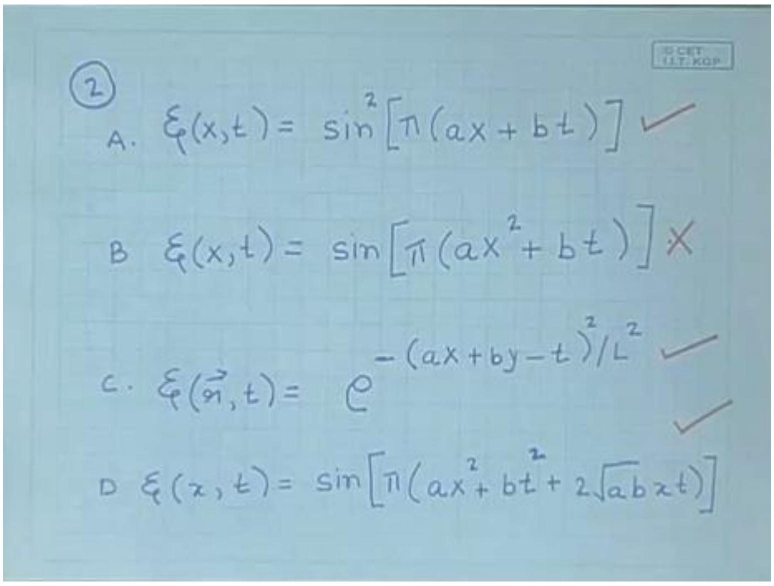

So, this is a travelling wave and it is it is speed has also been calculated. Let me now, take up a different problem the final problem which we shall be discussing today and this problem is as follows.

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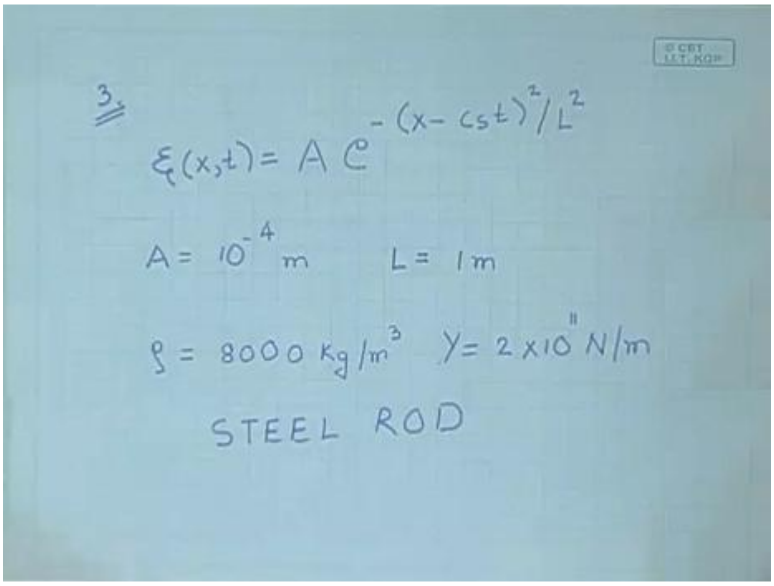

So, in this final problem that we are going to discuss we have a disturbance in a steel rod the the x the steel rod the length of the steel rod is aligned with the x axis. So, the displacement the disturbance varies along the x axis and it evolves with time. And the functional form of the displacement of the disturbance is given over here xi; xi represents the disturbance which is a displacement in the same direction x along the x direction Xi is the function of x t is A e to the power minus x minus cs t whole square divided by L square. So, A e to the power minus x minus cs into t whole square divided by L square and A the magnitude of the displacement is given to be 10 to the power minus 4 meters L over here which has dimensional length is given to be 1 meter. So, the first problem the first the first question that is asked is as follows where is the displacement maximum at the time t equal to 10 to the power minus 3.

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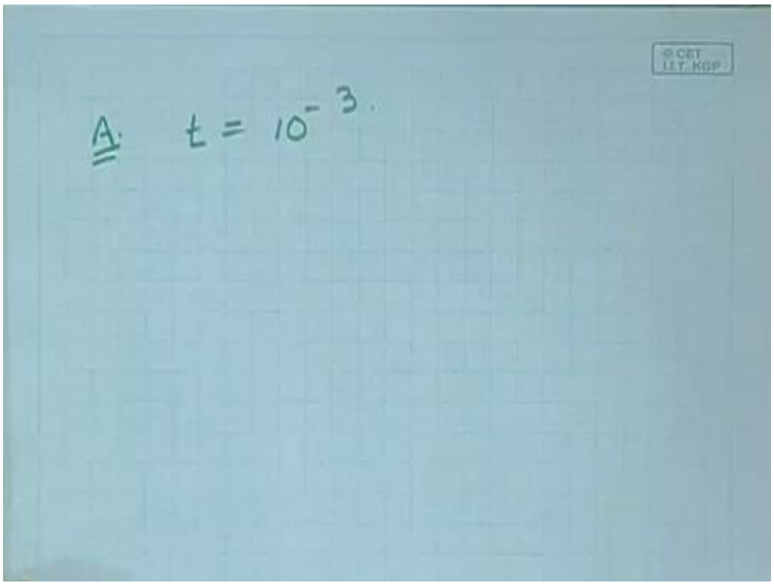

So, the first question that is asked is at the time t equal to 10 to the power minus 3 where is this displacement.

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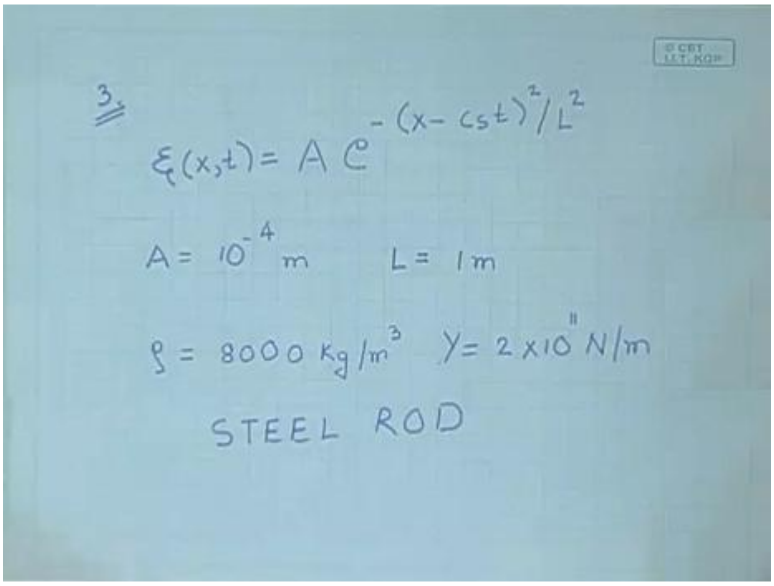

Maximum at what value of x is the displacement maximum at the time.

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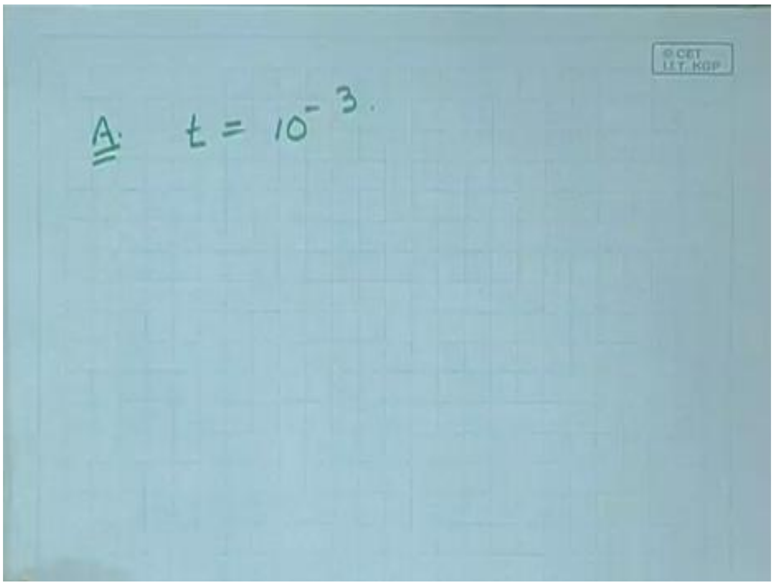

T equal to 10 to the power minus 3 seconds;

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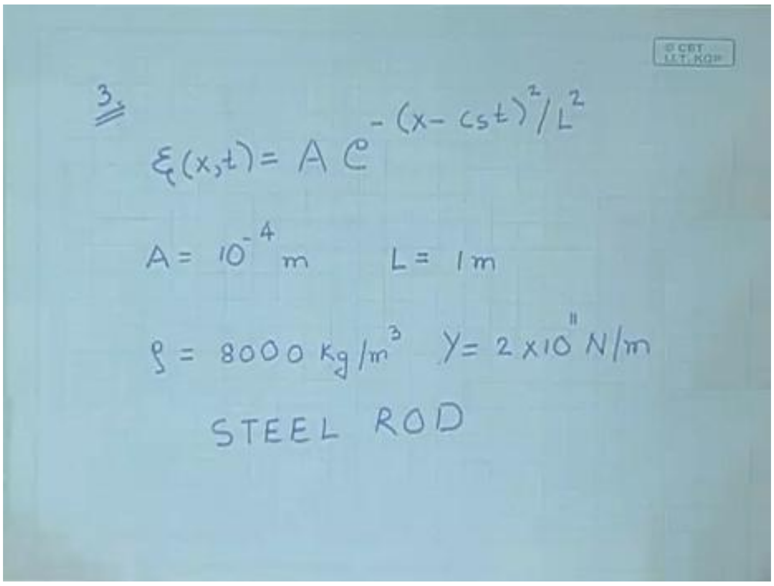

Now, let us just look at this function when does this function have the maximum value it should be quite clear that. This function has the maximum value when this argument x minus cs t becomes 0 when this argument x minus cs t becomes 0. This xi has the largest value largest value is A if x minus ct cs t is more than 0 the value of xi falls. So, the value of xi is maximum when x equal to minus cs into t so, the maximum displacement is going to occur at the point X is equal to cs into t.

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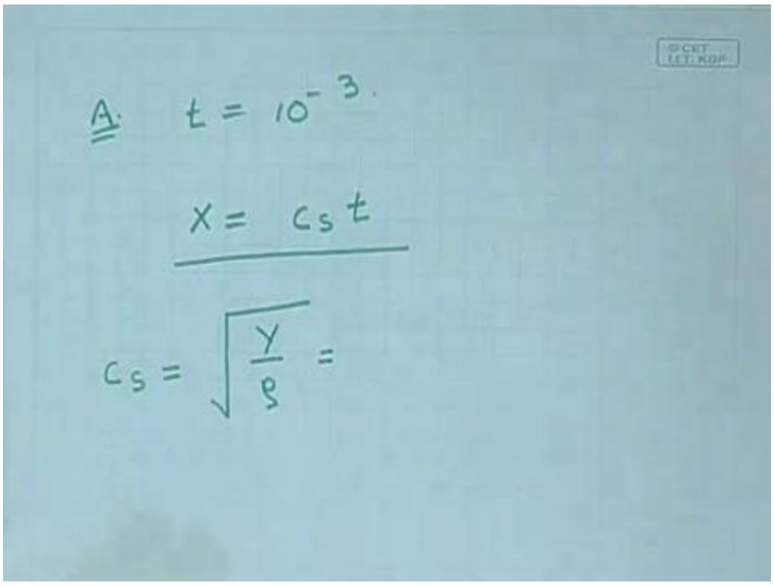

So, this is where the maximum displacement is going to occur so, for this to determine this position at t equal to 10 to the power minus 3 seconds. That 1 millisecond after at the time equal to 1 millisecond if to determine the point where the maximum displacement is going to occur we need the value of this speed of sound. And we know that the speed of this wave the phase velocity of this wave this elastic wave in the solid is the square root of young's modulus divided by the density.

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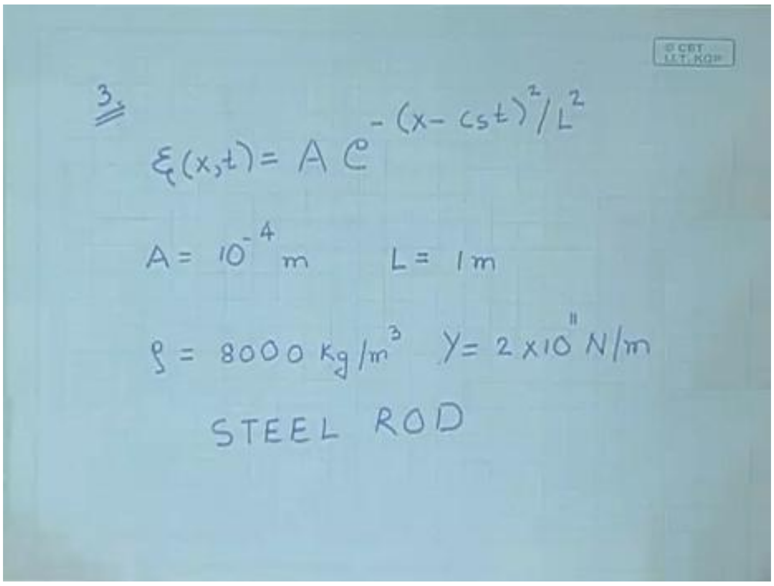

And for this particular solid the steel rod the young's modulus has a value 2 into 10 to the power 11 Newton square meter.

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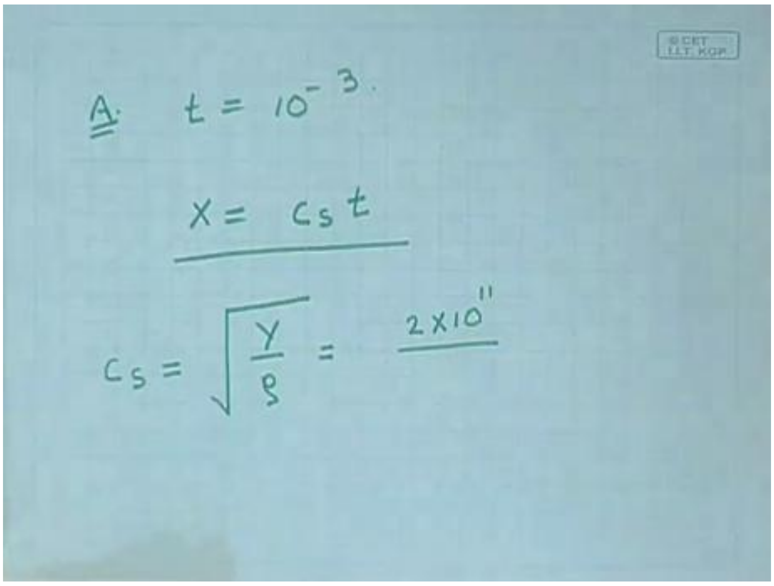

So, this is 2 into 10 to the power 11 and divided by the density of steel is 8000 kg's per meter cube.

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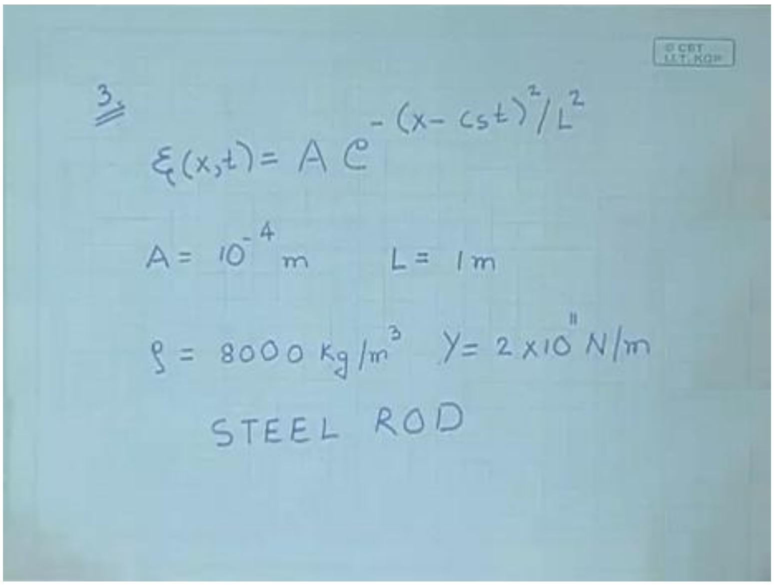

So, this has to be divided by 8 into 10 to the power 3 and then i have to take the square root of this. So, 2 and 8 what I have left is 1 by 4 the square root of 1 by 4 is half and 10 to the power eleven divided by 10 to the power 3. So, what we have is 10 to the power 8 and the square root of 10 to the power of 8 is 10 to the power 4 which gives us 5000 and we are working in the meters si units. So, this is 5000 meters per second this was kg per meter cube this was young Newton’s per meter. So, finally, the speed of the elastic wave inside the steel rod is 5000 meters per second. Just to remind you the speed of sound in air is few 100 meters per second around 300 meters per second. So, the disturbance in the steel which is equivalent to sound propagating in steel propagation are much faster speed it propagates the speed 5000 meters per second right. So, sound propagates this disturbance elastic disturbance propagates at a speed which is much faster inside the steel much faster than sound propagates in air.

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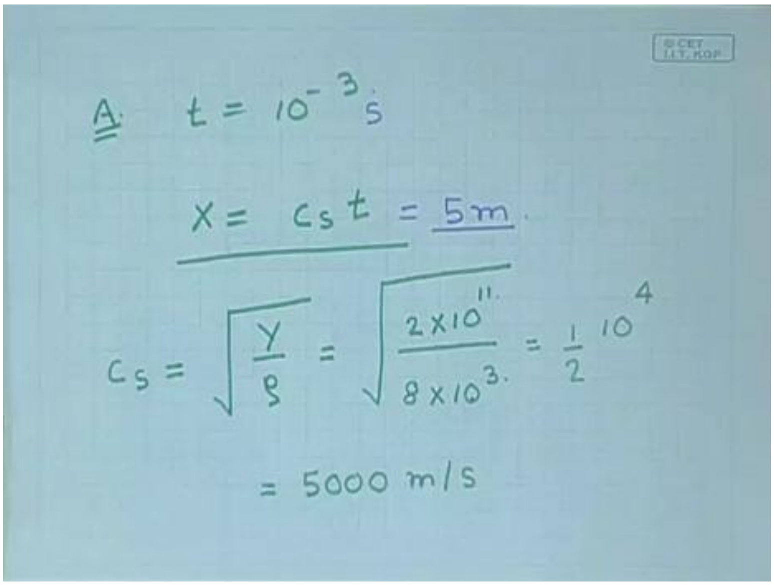

So, we have determine the speed is 5000 meters per second we can now determine the value of x where you have the maximum displacement this is going to be 5000 into 10 to the power minus 3. So, the value of the position where you will have the maximum displacement at 10 1 millisecond and the time 10 to the power minus 3 seconds is going to be 5 meters. So, at a distance 5 meters away from the origin x equal to 0 you are going to have the maximum displacement at the time t equal to 1 millisecond.

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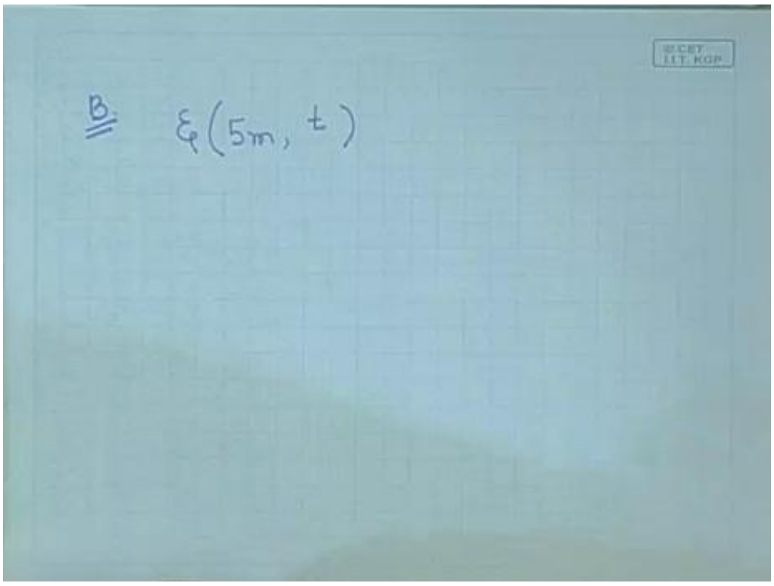

The second part of this problem part B is as follows we have to plot the displacement xi at the fixed point 5 meters as a function of time. So, at the fixed point fixed position x equal to 5 meters we would like to determine the behaviour of the displacement as a function of time.

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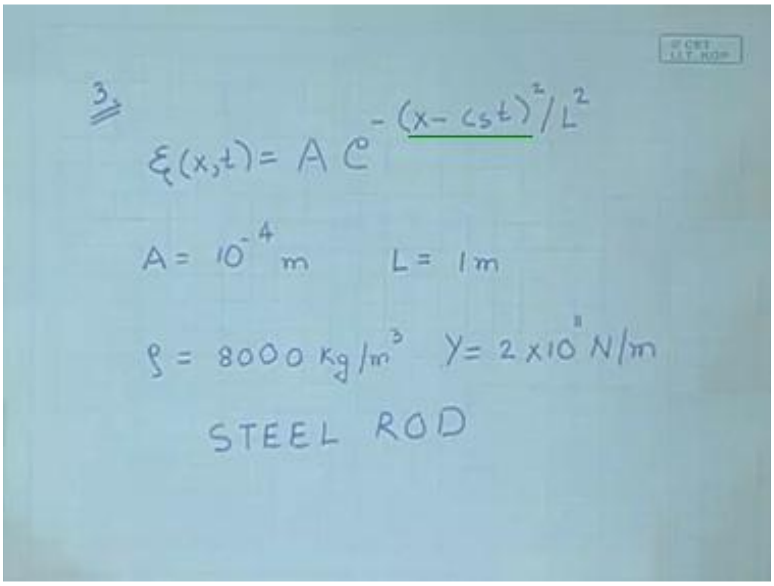

So, the displacement we are told that the displacement has this functional form it has an overall coefficient A amplitude A. And then it is e to the power minus x minus cs t whole square divided by L square and we have to plot this as a function of time at the position x equal to 5 meters. So, let us ask the question at what point at what instant of time of time is the displacement going to be maximum at x equal to 5 meters. So, that is going to occur whenever x is equal to cs t we have just determined the value of t I mean we have just determined this.

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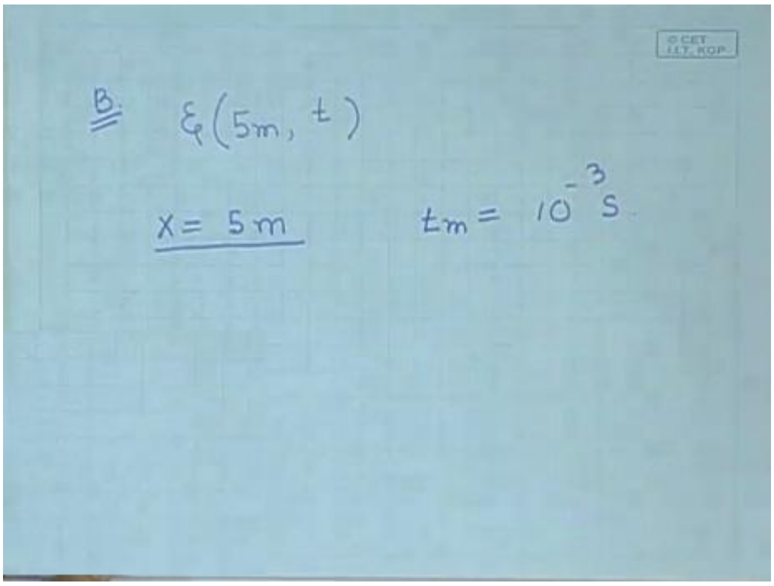

So, at the point x equal to 5 the displacement is going to be maximum I will call this t maximum at 10 to the power minus 3 seconds.

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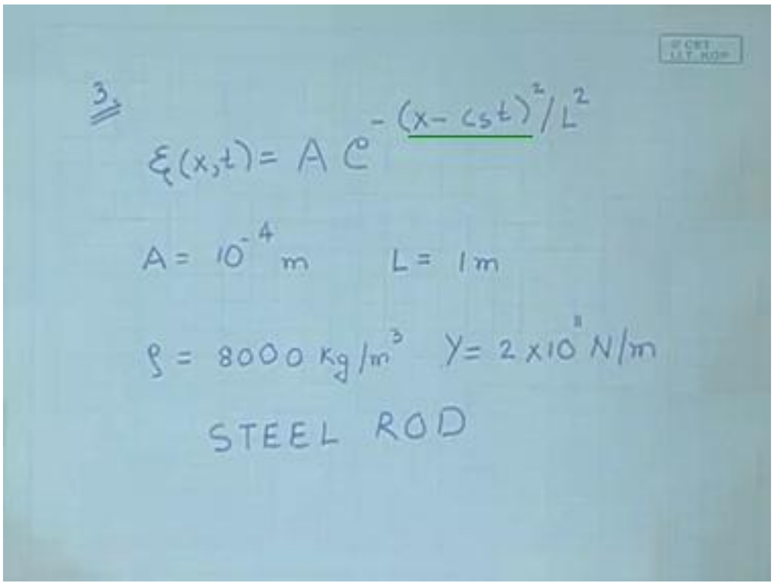

The displacement is going to be maximum when this argument is 0 x we are told to look at the point x equal to 5. And we have already found out that the displacement is going to be maximum this argument is going to become 0 then t equal to 1 milliseconds so, you already know this.

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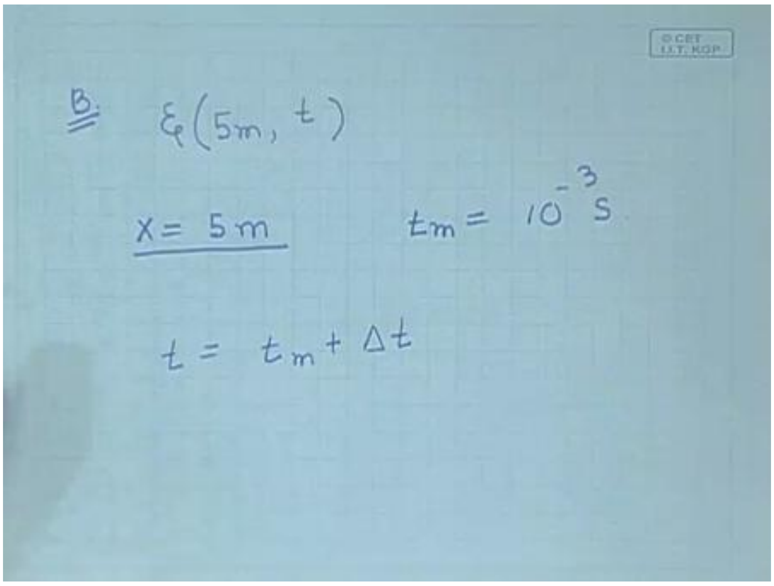

Now, we want go plot this xi as a function of time so, let me write any arbitrary time instant as the difference from the time instant when the displacement is maximum. So, t this t maximum refers to 1 millisecond this is the instant when the xi is maximum. And if you look at any arbitrary time the difference from this time instant when the displacement is maximum I am calling delta t.

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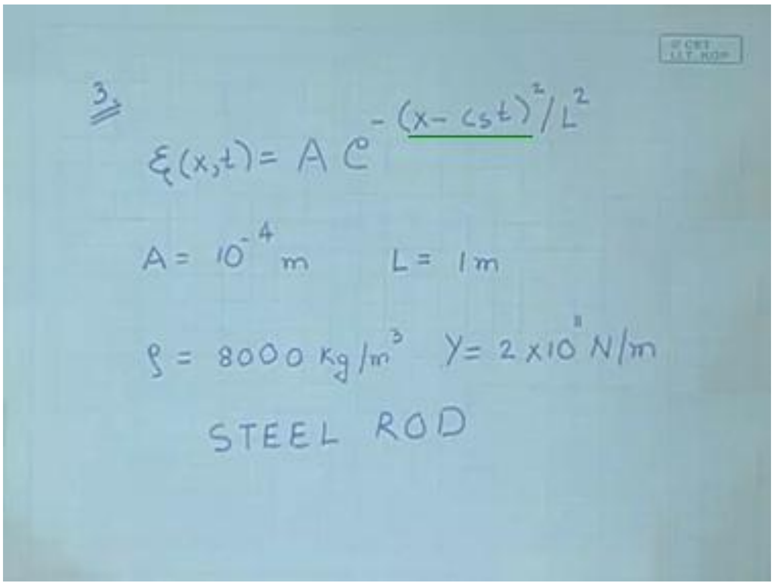

So, I can now represent the displacement at 5 meters as the function of delta t the difference in time from where it is maximum.

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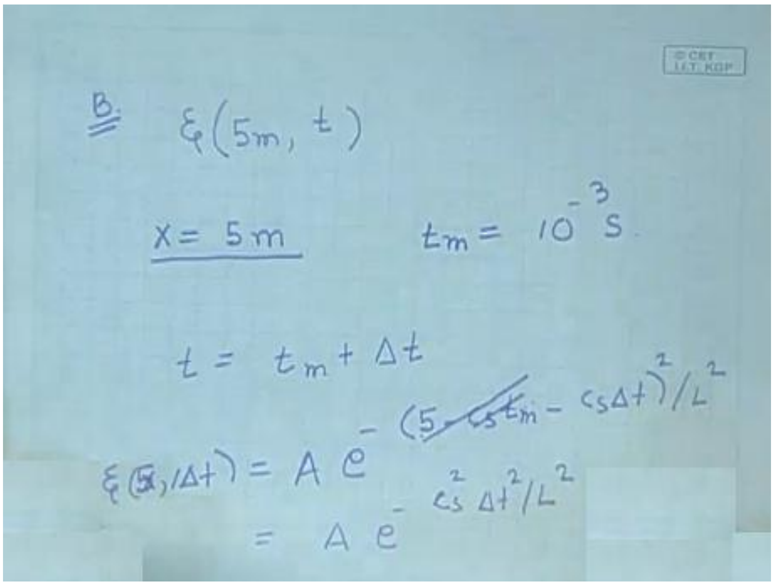

So, if I substitute t by t maximum plus delta t when xi at the point 5 and at the function of delta t now is equal to A e to the power minus 5 minus cs tm minus cs delta t square divided by L square. And the maximum time at the time when tm when the displacement is maximum is to exactly cancel out. So, what we see is that A is e to the power minus cs square delta t square divided by L square. So, as you move away from the time when the displacement is maximum the displacement falls off like a Gaussian E to the minus proportional with some constant of proportionality to delta t squared. How does this function look like?

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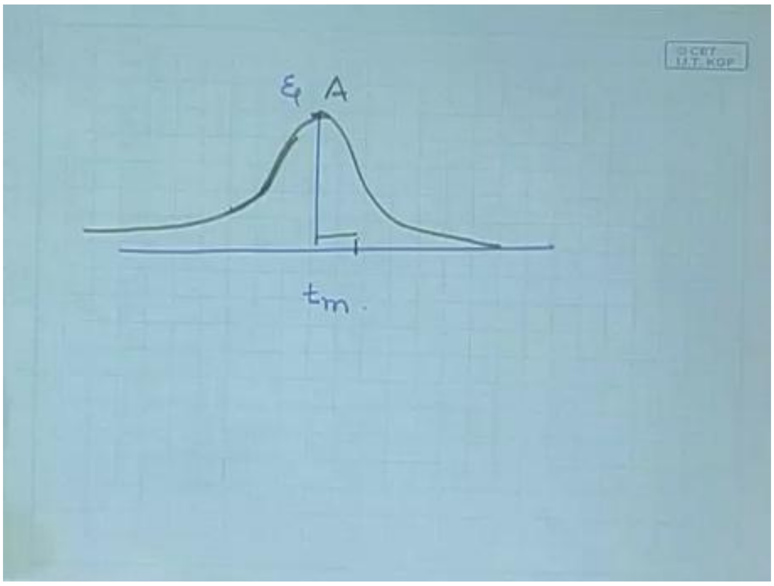

So, let me draw it for you this is the displacement this is tm as we move as we look at a time in other different time instant. So, let if you look at a different time instant say this time instant this difference is delta t and when delta t is 0 this has the maximum value A. And as delta t increases this falls as e to the power minus delta t square with some constant on top over there so, it falls like this. So, that is the displacement does a function of time at the point x equal to 5 to make sure. We have really understood everything how would the displacement as a look as a function of time at the point x equal to 6. The point is that you would get exactly the same functional dependence except for the fact that everything would have shifted by cs. So, at the time at the point equal to 6 the maxima, is going to occur at a slightly later time 1 by cs. And the whole curve would have shifted by the same amount 1 by cs same amount of time.

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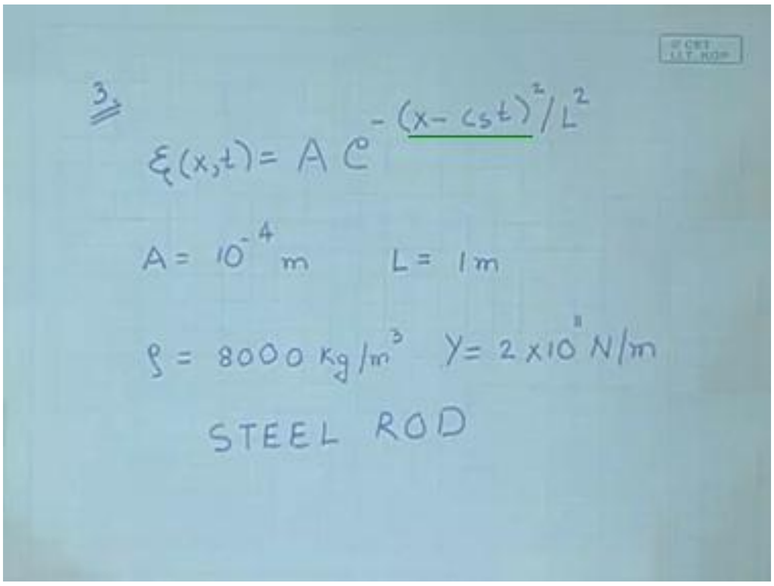

The next part of this problem is to determine the velocity of this elastic medium as a function of time at the point x equal to 5. Let so, we have to determine xi the, are been filtered in the velocity of the elastic medium the velocity of the elastic material as a function of time at the same point x equal to 5. So, let us calculate the velocity how will, you calculate the velocity this gives the displacement. So, if you differentiate this once with respect to time let me remind you again what it gives us.

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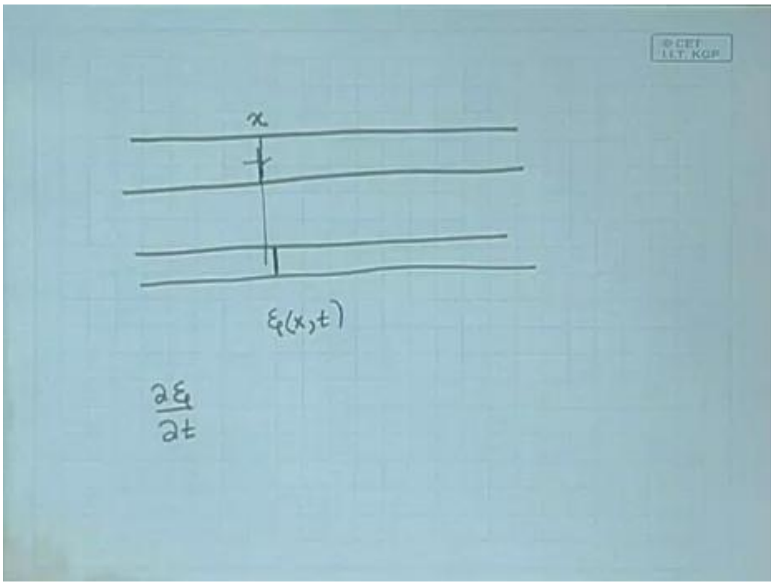

This is the elastic rod and because of the disturbance this element of the elastic rod will move around. So, because of the disturbance it may move here or it may move here and as it moves around. So, this is the same rod and because of the disturbance this particular element here has now, moved over here. And this is the disturbance xi x t this is the point x. So, the time derivative of this del xi del t tells us the speed with which this point in the material at this point is moving this particular bit of the elastic material is moving.

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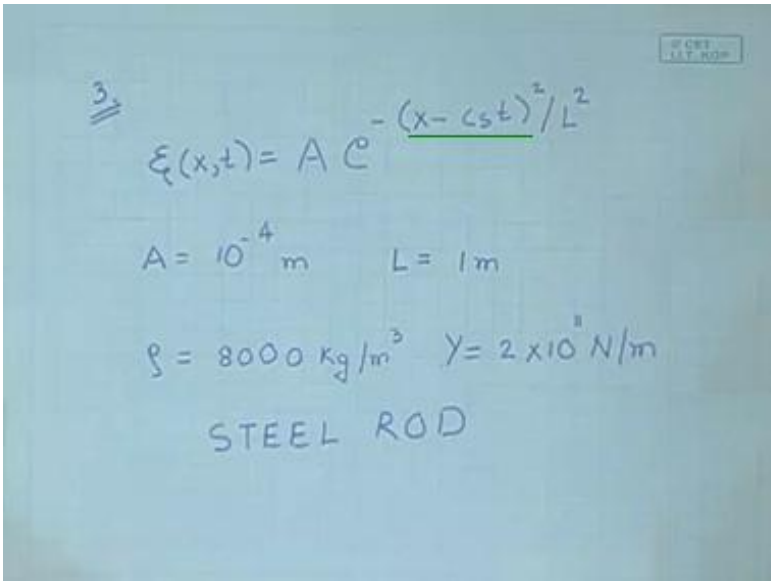

So, we have to calculate the time temporal derivative of this function will gives the displacement. So, when you differentiate this you will pick up the say you have to differentiate this exponential. So, you will get this whole thing the, you will get the same exponential back and then you will multiply it by the derivative of this function. The derivative of this function is going to first have a minus sign factor of 2. Then you have to differentiate this which gives us another minus sign into cs.

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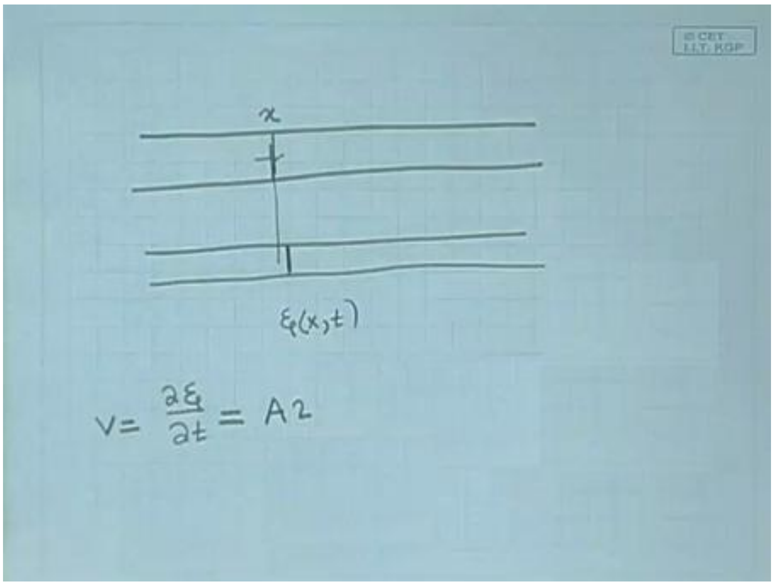

So, the velocity this is the velocity at any point x is going to be A 2 2, because of this.

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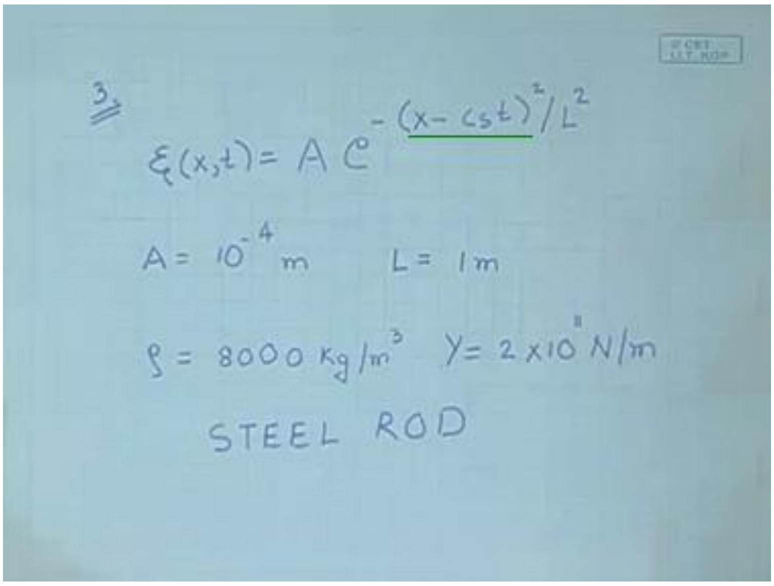

Then.

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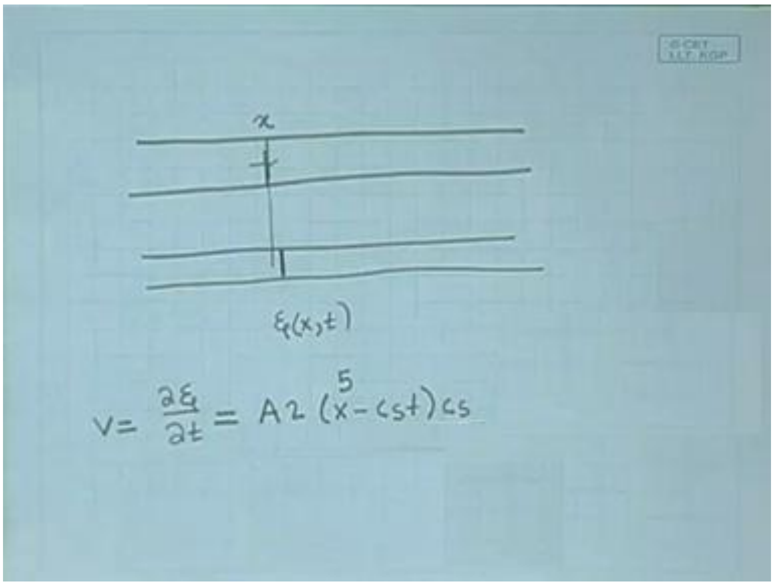

X minus cs t into this minus sign over here.

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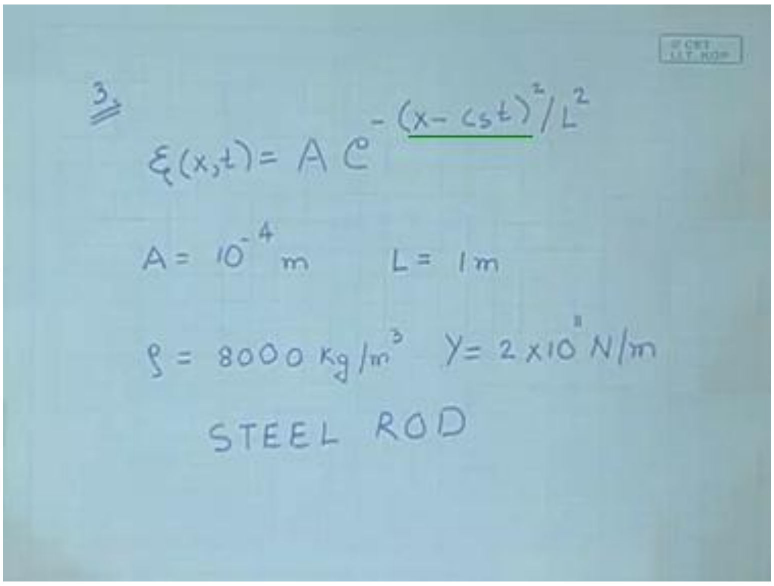

And when I differentiate this I will get another minus sign into cs.

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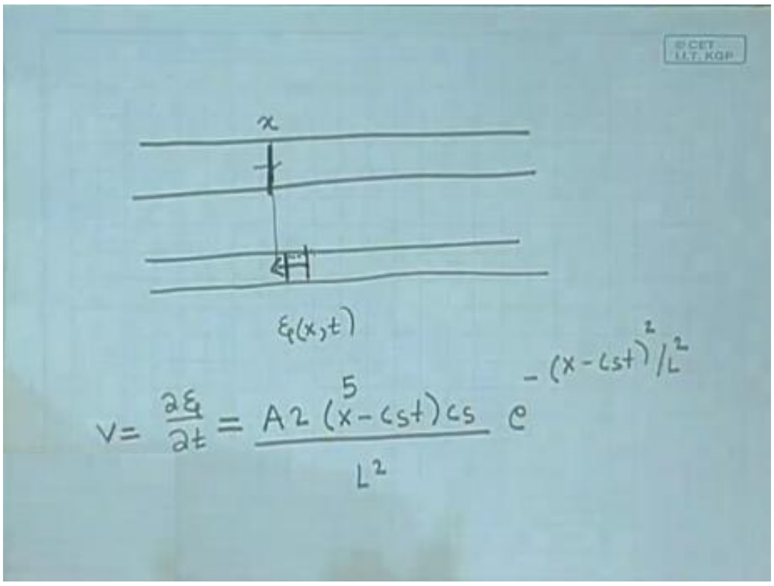

This divided by L squared into xi into e to the power minus x minus cs t. So, we have to now, plot v as a function of time at the point x is equal to 5. Now, so, we have to focus at the point where x is equal to 5. Now, the point to note is this that when t is equal to tm that is the time instant when the displacement is maximum the velocity is exactly this argument becomes 0. So, the velocity is 0 when t is less than tm the time instant when their displacement is maximum T is less than tm. So, this argument is positive the velocity is positive when t is more than tm the velocity this is more than this. So, the velocity is negative right. So, we see that the velocity is positive at all times before the displacement reaches its maxima and then it becomes negative.

So, this is very easy to interpret this is the we are looking at this particular element of the elastic medium it moves after sometimes it moves here. And then the motion will even later time it may have moved further away and then at some instant it will meets the maximum displacement and then it will move back. So, the velocity is positive until the maximum displacement is reached at the maximum displacement the velocity is 0 that is how the maxima is defined and then the velocity becomes negative. So, this is what do you also see here if you plot this curve I will request you to sit down and put in the values and actually plot it right now, I will just plot it schematically.

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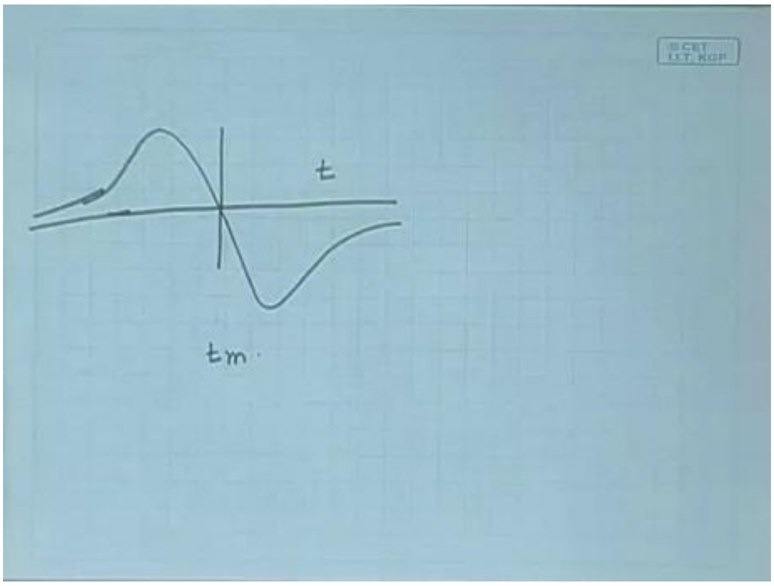

And it looks like this this goes to 0 at large time and this instant of time where the velocity is 0 is the, is tm the instant when the displacement is maximum okay. So, that brings us to the end of the problems which I wanted to discuss. So, for the rest of today's class, we shall we shall be discussing we shall discuss standing waves. So, the next topic that we are going to take up is standing wave it is a continuation of of solutions of the wave equations. So, we are going to discuss the topic the topic that we are going to discuss is standing waves. So, let me now get rid of this.

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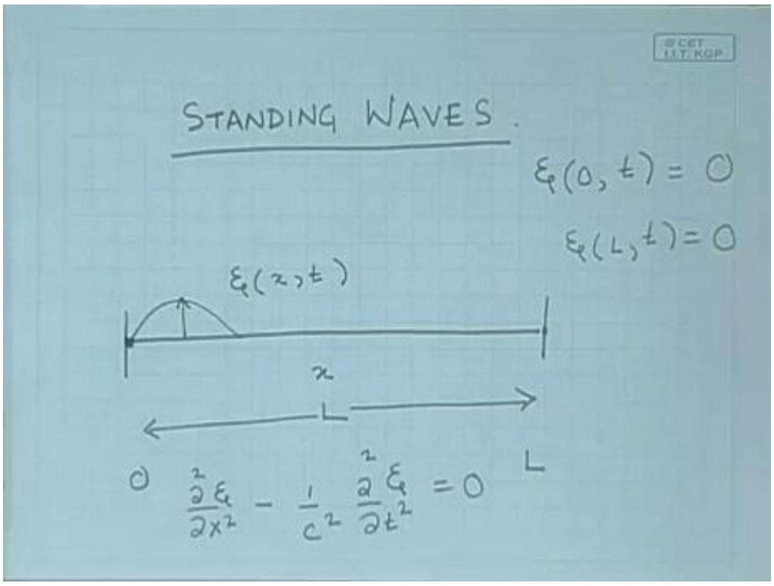

And so, the situation that we are going to consider is as follows we have a, we will discuss it standing waves in the context of a stretch string. So, we have a stretch string which I show over here and the string is attached at 2 end points. So, this is one end point and this is the other end point the string is attached at these 2 end points. It is held tight by the 2 end points the 2 end points cannot vibrate. And we shall be considering the transverse disturbances of the string. So; the string can be disturbed like this in the transverse direction. This displacement in the transverse direction could vary with x this is x and it could also vary with time. So, we are interested in the vibrations of the string the string has a length L and this point corresponds to x equal to 0 this point corresponds to x equal to L. So, we are interested in studying the solutions of the wave equation so, these displacements are governed by the same wave equation which we had which we have been studying all along. The equation is the same the sound speed the speed sorry that the sound speed.

Now, the speed of this of these transverse vibrations in this stretch string are different it depends on the tension and mass per unit length of this string. But the equation governing it is the same only that the speed of sound whether the speed the phase velocity is different. So, we want to look for solutions to this equation so, the transverse displacement disturbance is in the string satisfy. This equation Xi represents the transverse disturbance c here is the speed for this wave which depends on the tension of the string. And the mass per unit length of the string the value of c depends on that and these displacement are governed by this equation. We now, have 1 additional thing that we have to take into account it is the boundary condition. Until now, we have not discussed the boundary conditions at all. So, in our discussion of waves still now nowhere have we discussed what the boundary conditions it could be the wave could be any arbitrary function.

We have not discussed any boundaries here in this, particular problems we have to impose the boundary conditions. So, the boundary conditions here are the fact that xi is 0 at the 2 end points so, at all time instance xi is 0 at this point where the string is attached. This point does not get disturbed similarly the other end of the string the other extremity of the string xi as a function of L at the sorry at the other end point L is a function time is also 0. So, at the 2 end points the string is fixed the string is fixed here and here these 2 points are not disturbed there are several string can be disturbed. So, we want to find solutions of the wave equation which have this boundary these, this boundary condition. And we could take whatever that we have discussed still now, the plane wave solution that we have discussed until. Now, and apply this boundary conditions, but we shall proceed to solve this equation in a different way. So, that it gives you exposure to another method of solving the wave equation. So, we are going to use the method of separation of variables. So, we are going to look for solutions xi x t we are going to look for solutions to the wave equation xi.

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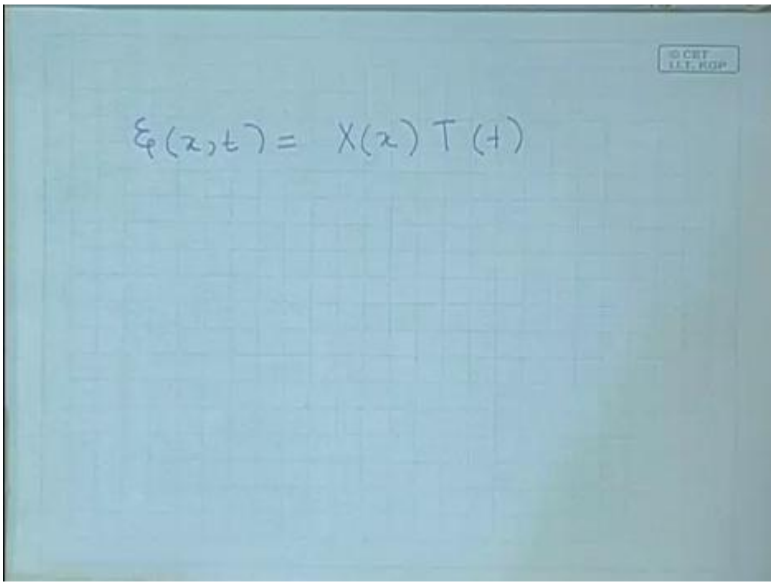

So, you are going to look for solutions which are a product of 2 functions 1 of x I am calling this capital X and another of time alone. So, we are expressing xi the disturbance as a function as a product of 2 of 2 functions. The first 1 capital X is a function of the position alone the second function capital T is a function of time alone. So, we are expressing the disturbance in the string as a product of 2 functions one of position alone and one of time alone. So, we are taking a trial solution that looks like this.

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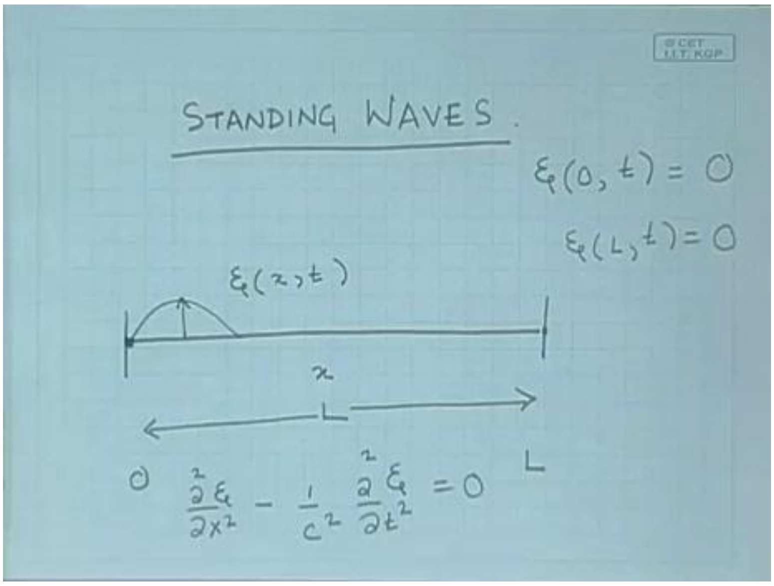

So, if you take such a trial solution and put it in the wave equation what do you get? So, let us look at this term by term; the first term over here is the spatial derivative of xi.

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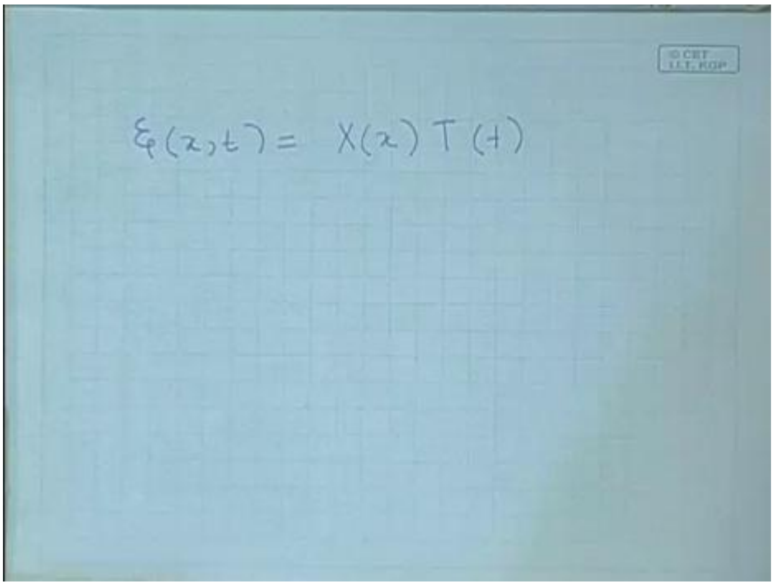

So, when you differentiate xi with respect to X xi has this form you know have only derivatives of this function derivative with respect to X does not effect this function at all. And for this function the derivative with respect to X is an ordinary derivative there is no need to talk about a partial derivative anymore, because it is the derivative of a function of only 1 variable.

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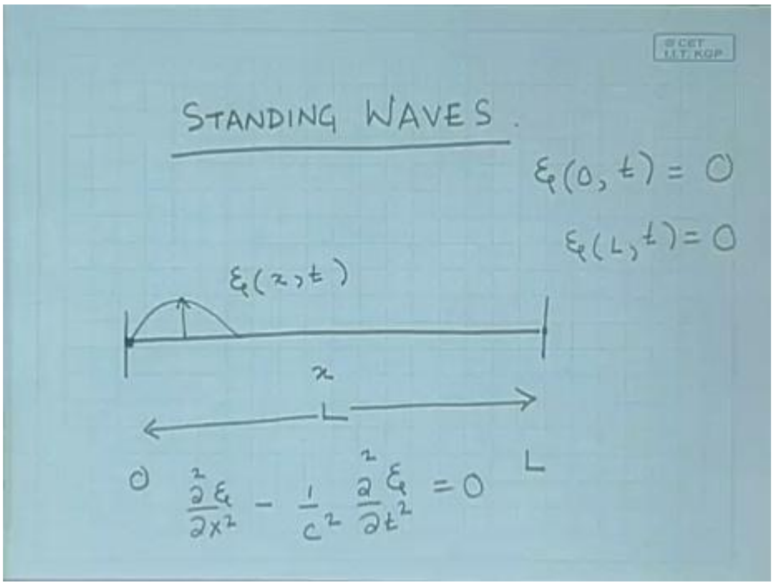

So, the term the first term over here becomes T into the derivative.

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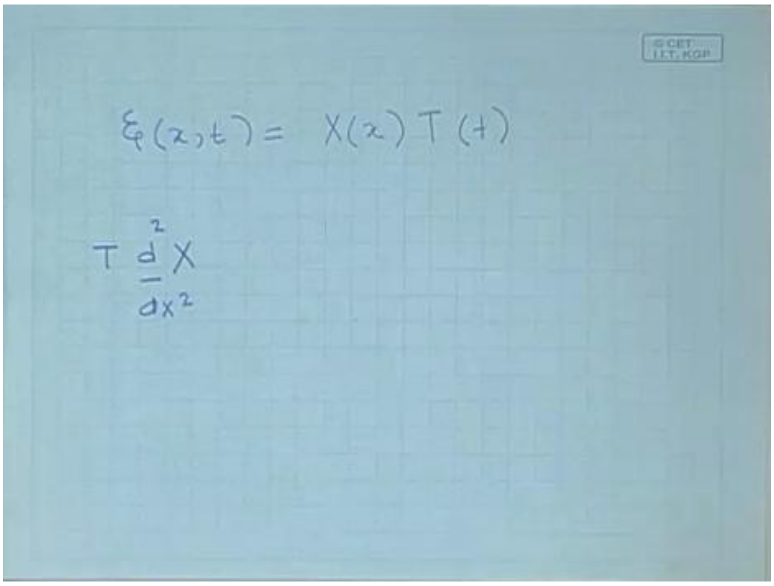

Ordinary derivative with respect to x of this function X which is a function of x alone.

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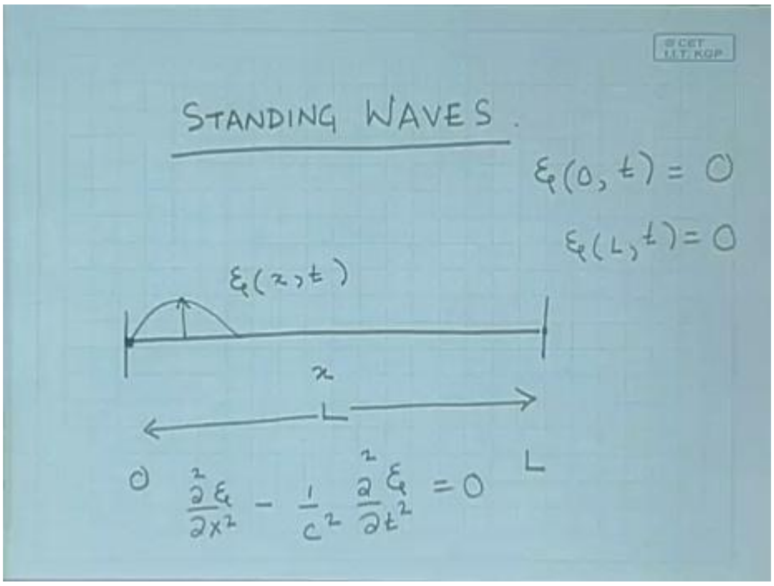

And similarly the second term in the wave equation which has a partial derivative with respect to time when I take a Partial derivative with respect to time.

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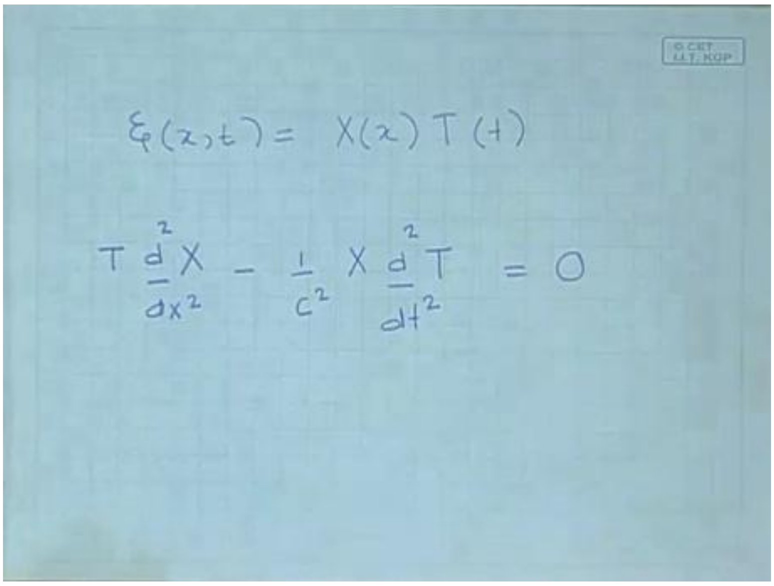

I have to keep X fixed. Now, when I take a partial derivative with respect to time here it does not affect this function, because it has no time do it is only on this function that the derivative acts. And you can replace it with the total ordinary derivative, because this there is no variable other than t in this function. So, the differential equation now becomes this.

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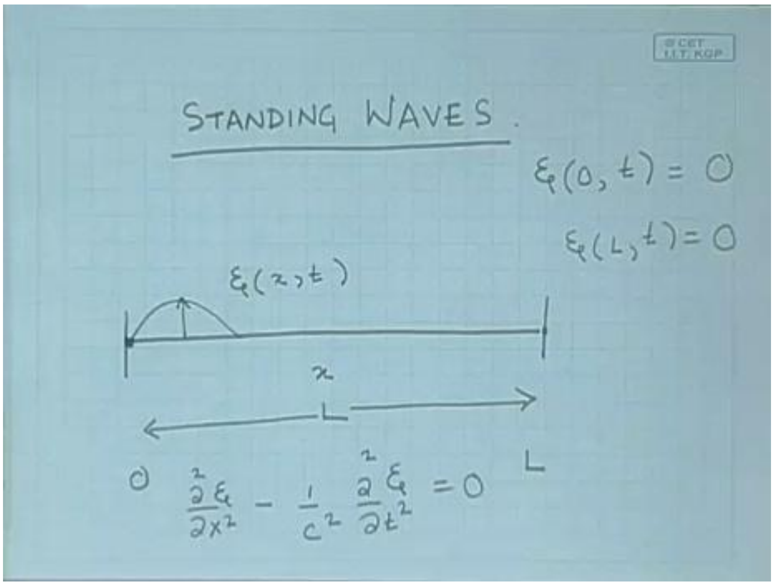

So, the partial differential equation that we had started out with if you write the.

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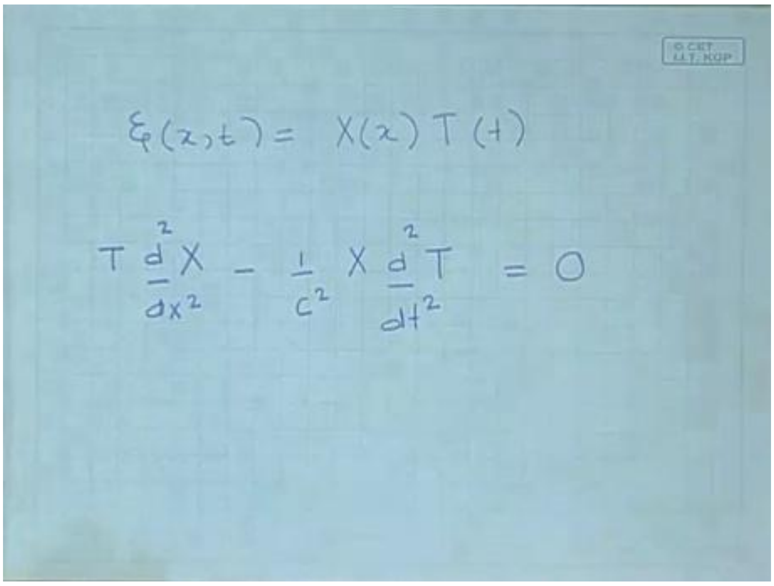

Take a trial solution which you write as a product of a function of X and a function of T the equation now becomes this. And we can divide this equation throughout by X T. So, I am going to divide it by xi which is X into T i am going to also divide this by xi which is X T. So, I have divided this whole equation by xi which is T into X. And if I divide this by xi then we see that the term T cancels out from this. So, this the time dependence cancels out from this and a term X over here cancels out from this. So, the equation that we are left with let me write it down over here. So, this is what we have is only these 2 terms over here on the left hand side.

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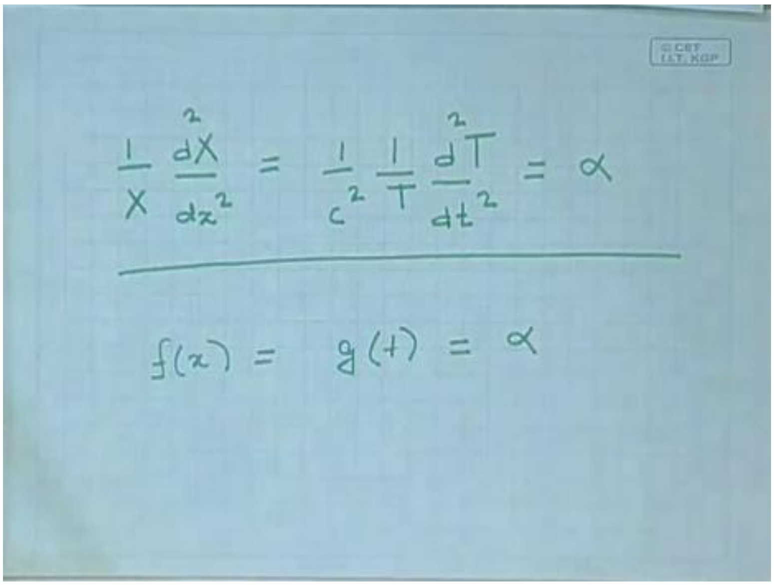

So, the equation that we are left with is 1 by X d square X dx square is equal to 1 by c square 1 by T d square T dt square. So, the ordinary differential the partial differential equation that we had now gets simplified considerably. And we have an equation in terms of ordinary derivatives only on the left hand side. We have the partial differential equation essentially now gets converted into this under the assumption that the solution is a product of 2 different functions 1 of X alone and 1 T alone the partial differential equation gets simplified to this. The term on the left is the function of X alone and the term on the right is a function of T alone now, here we have a function of X.

And here we have a function of time I can vary x in the arbitrarily I can vary time arbitrarily, but still this equality holds. What this tells us is that this has to be equal to a each function has to equally be individually be equal to a constant. So, we will call this constant alpha. So, what this tells us that is that this function of X this function of T if they have to be equal they have to be separately equal, equal to a constant individually equal to a constant and that constant we are going to call alpha. So, let me stop here for today’s lecture and we will resume our discussion here tomorrow. So, we are going to work out the solutions of this we will get 2 different separate differential equations their ordinary differential equations. So, let us work out the solutions to these ordinary differential equation in the next class and then impose the boundary conditions.