**Physics I: Oscillations and Waves**
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**Lecture - 26**
**The Wave Equation**

Let us start today is discussion with elastic waves.

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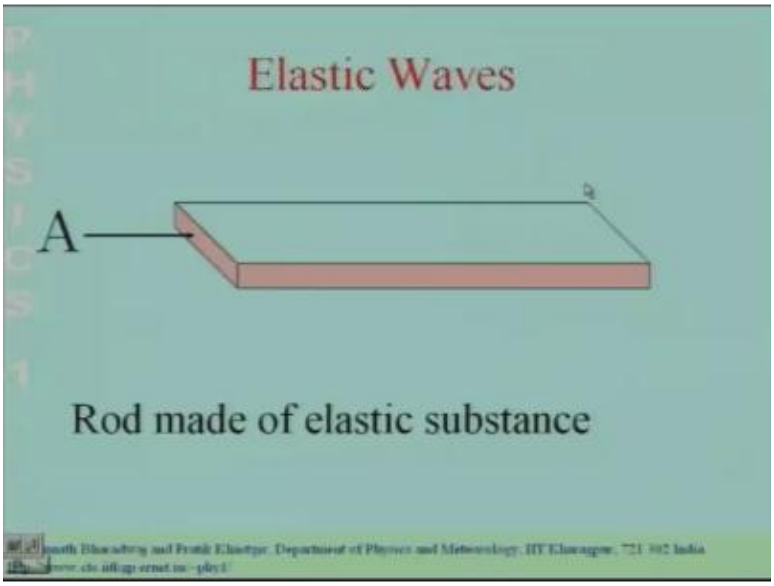


So, this picture over here shows, you a rod made up of elastic substance the length of the rod extents in this direction. And the cross section, the cross the cross section of the rod is has an area A the length of the rod is in this direction. And that the situation, that we are interested in is, where we have disturbed introduced a disturbance in this rod. For example, let me just give you an example, think of this as the rod this as the rod in, which of, which you wish to study the disturbances. So, this is the elastic rod and I put in a disturb it is like this. So, I tap in one end and if I tap it an one end, it will cause a compression over here this compression will then is a is a disturbance, which is which have introduced in this elastic rod. And the question that we interested in is what happens to this disturbance, how does this disturbance evolve?

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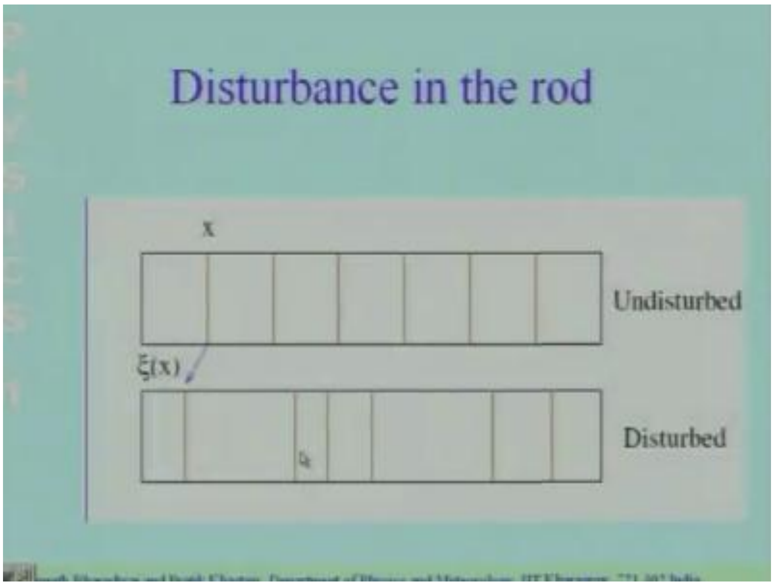


So, picture over here shows you the section, let me.

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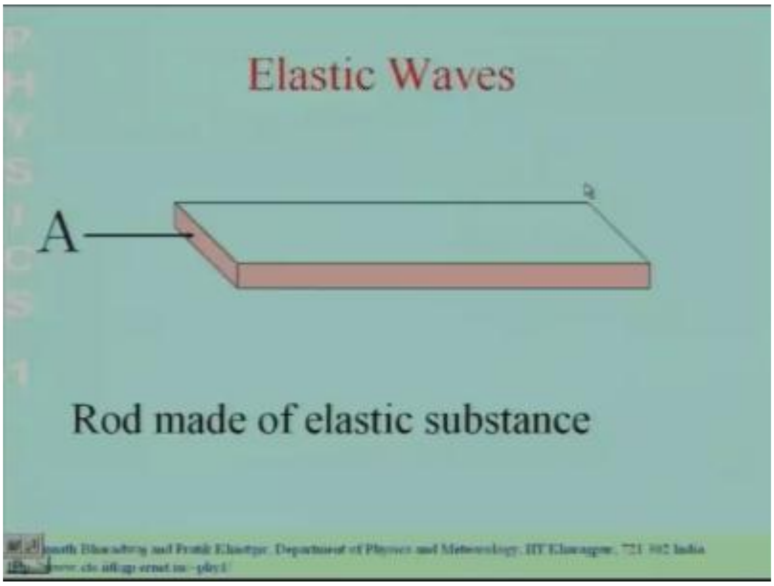


So, what we do is that we have done is that, we have we draw lines at equal spacing in this direction. I should first tell you that, we are interested in disturbances, which are which are there along this direction only. So, to start with we shall discuss a situation, where the disturbance is such that is varies, only in this distraction, it does not vary along the 2 other directions. And we shall refer to this, direction as the x axis the length of the rod x axis along the length of the rod the y and z axis are along these other two directions in, which the disturbance. We are not interested in the variation of the disturbance of the disturbance in these two directions for the time being, we assume that the disturbance does not vary in the other 2 directions. It only varies in this direction and we have drawn lines at equal x intervals over here.

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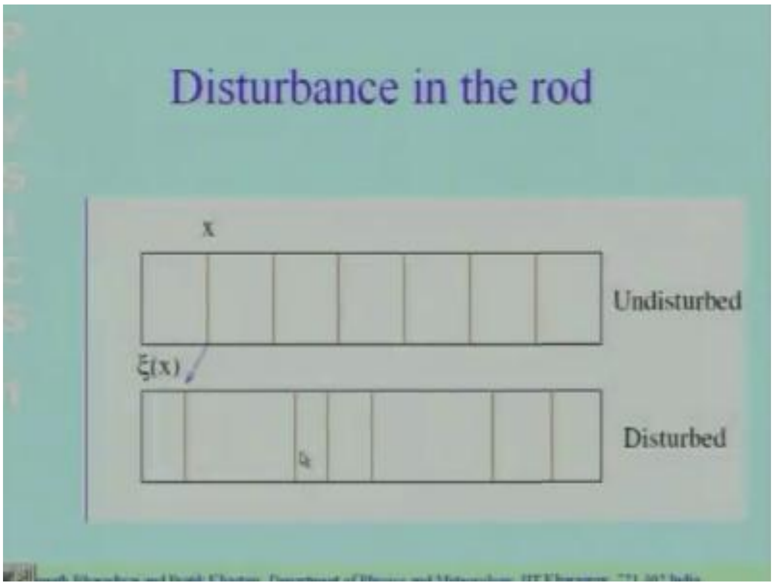


And this picture shows us these lines at equal x interval drawn along the elastic rod in the situation, when the rod is undisturbed. Now, when I introduce a disturbance in the rod the disturbance compresses certain regions and it causes, very fraction in other regions. So, this show, you a particular disturbance, this particular disturbance is such that, it causes the first element over here, the first element. So, it causes this element over here along the length of the rod to get compressed. So, the first element over here has, now got compressed, because of the disturbance and it is only this much the second element has got verified. So, it is it covers a larger length then, it originally was, the third element is compressed the fourth element is compressed. The fifth element again, is the sixth element is the more or less the same may be little compressed and this is compressed.

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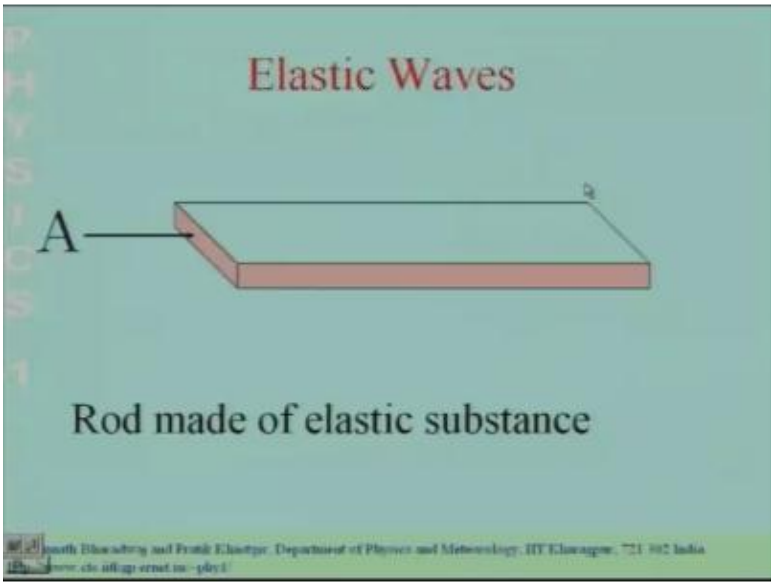


So, what I have shown you here is that we have drawn lines at equal interval along this direction of the rod.

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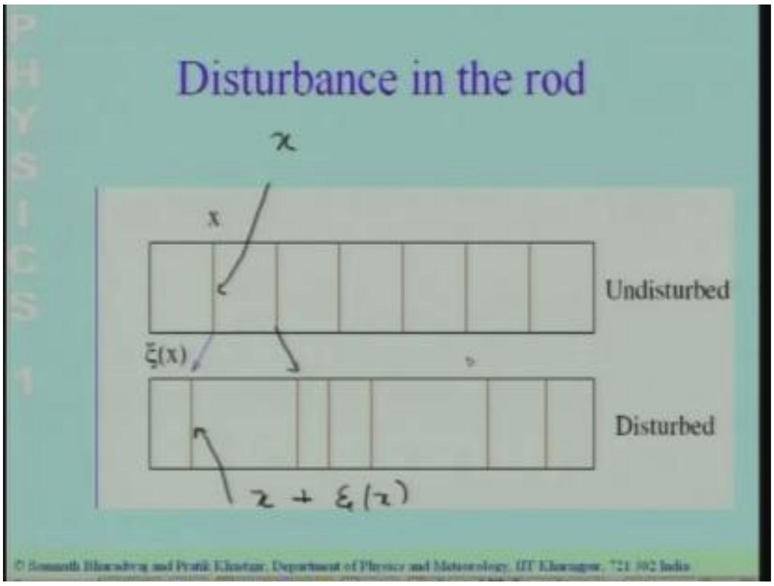


And, because of the disturbance that, we have introduced possibly be a by taping it at one end and whatever means, these lines have got moved around and the different elements have got compressed or verified. So, let us now, focus our attention on the element over here. So, I have drawn the line on the element over here, after the as the consequence of the disturbance. This gets shifted to a new position and the deformation of this element. So, this piece over here this, element over here has got deformed to a new position and the deformation, we denote by xi x. So, the element the element of the rod, which was at x earlier has now, shifted to a position, which is x plus xi of x. So, the element, which was at x earlier this, this particular element, which was at x earlier has now, shifted to a new position, which is x plus xi of x.

And this particular case, it has shifted to the left, so, xi of x is negative this; particular element is an different value of x. And it has not shifted to a new position over here and in this particular element the xi of x the displacement is positive. So, xi of x is the function, which tells me the displacement at any value of the x at any point x and it will vary in general, it will vary with the value of x. Now, we are interested in studying, how this whole disturbance pattern evolves? The disturbance pattern is quantified through this variable xi of x which tells me how each element the displacement of each element and we are interested in studying, how this evolves? So, before getting into this, let me now, briefly recapitulates. So, the properties of an elastic rod, what we mean by elasticity?

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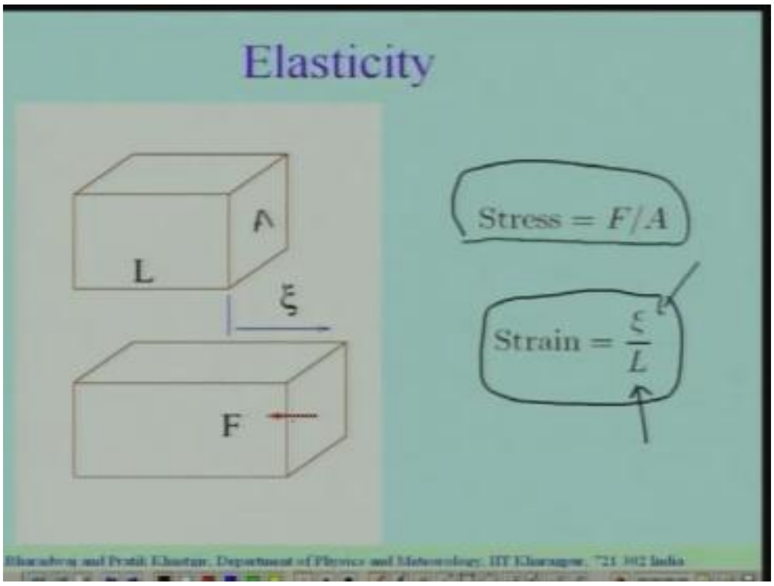


So, let me recapitulate briefly, what exactly do? We mean by elasticity. So, this picture shows you a bit of an elastic medium the cross sectional area of this medium is A. So, this is the cross sectional area of it is, it is A and the medium the elastic substance that we are interested in has a length L length L cross sectional area A. Now, there are 2 quantities, which are defined in elasticity one is called the stress and the other is called strain the stress is the force per unit area. So, if you apply a force on this surface over here or conversely, if you extend this element, if you extend this element by an amount xi. So, if you extend this element by an amount xi. Xi is the deformation of these elements. So, the length, which was originally L has now, become L plus xi this ratio of the deformation to the original length is called the strain.

So, this ration of the deformation the deformation here is xi divided by the original length L. So, the original length of this elastic substance was L, It has got deformed, we have pulled it. So, as to make it length larger and the amount by the, which the length has become it has got elongated as xi. The ration of deformation the deformation xi divided by the length is referred to as the strain. And if you pull this, so, if you elongate this elastic substance by pulling it, if you elongate the, an elastic substance a force will develop, which will oppose the elongations. So, a force will develop in the opposite direction, and there is we define the stress has the force per unit area. So, if you deformed, this elastic substance by elongate it in this direction. A force will develop and the force per unit area is, what we refers to referred to as the stress.

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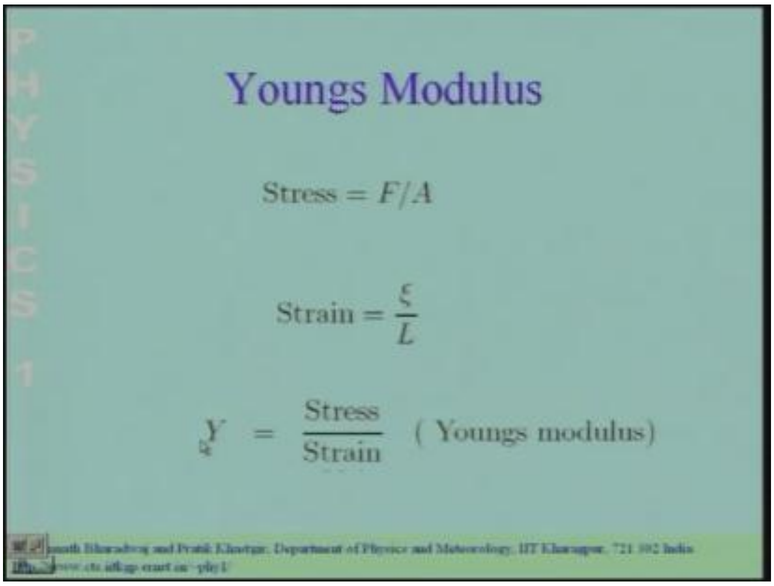


And for an elastic substance the ration of the stress force by unit area and strain is a constant, which dependence only on properties of the material. So, depending on the material, it wills the ration of the stress divided by the strain. The stress divided by the strain will be a constant call the Young’s modulus and this is a property of the material of it is the elastic of that of is the rod is made up. So, every elastic substance, there will be a different value of the Young's modulus and the stress and the strain are related in a linear fashion through the Young's modulus. They are proportional to each other through the Young's modulus.

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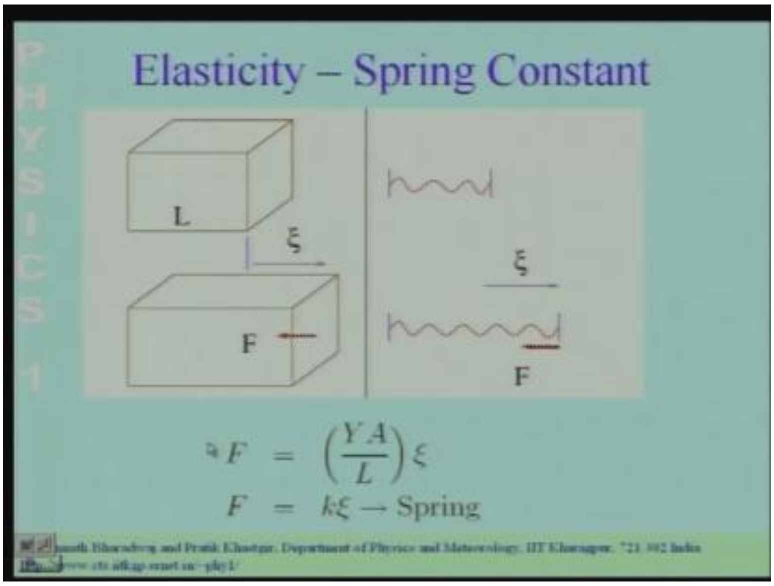


So, what we see is the de form for an elastic body, if I deform it by amount xi there will be force produced. And the ration of stress to the strain is a constant value, which and the value of this constant depends on the medium that, we have chosen the medium of, which this rod is made up. So, this allows, so, this basically tells us, that the force that will be develop. If I elongate this, rod if I deform this rod the force, that is going to develop is proportional to the displacement to the deformation of the rod, which is what we have here.

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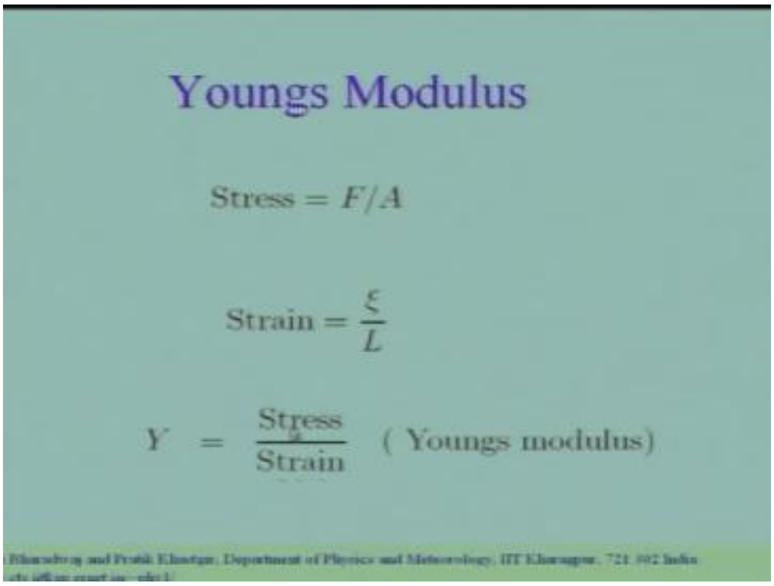


And the constant, which relates the force to the deformation xi is, we arrive at the constant through, this through the fact that the stress divided by the strain is the Young's modulus.

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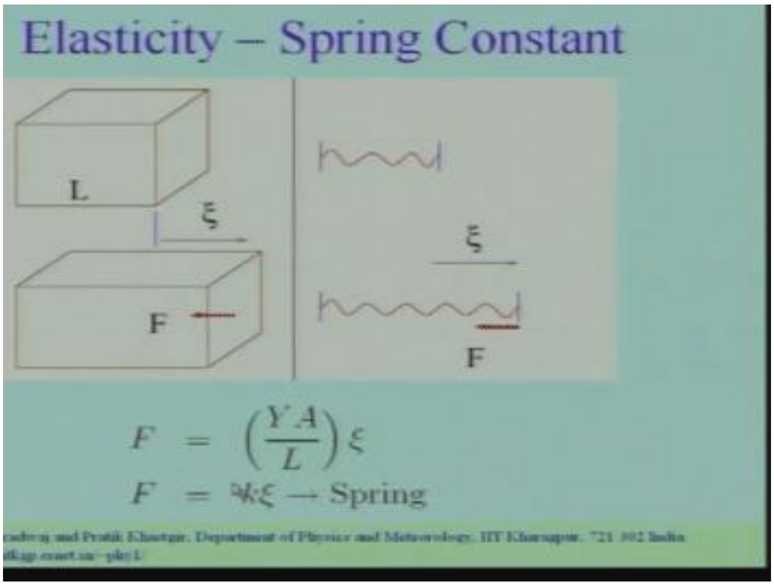


So, it tell us the constant relating the force, which develops, if I deform this object by a by a length xi, this constant is the Young's modulus into the cross sectional area divided by the undeformed length of this object. So, the point, which I am trying to make over here, is that, we can an elastic material. This is an elastic rod of cross sectional area A and of material whose Young’s modulus is Y and it has a deform UN deform length L, we can think of this as a spring. We know that for a spring, if I pull this end of the spring over here by an amount xi. Then there is going to be a force develop, which is going to develop, which is going to oppose this elongation. So, the force is going to act in the opposite direction, if I have pulled elongated it the force is going to try an push, it back to its original position and for a spring the relation between this force. And the deformation is through the spring constant k and if my, if the deformation is applied to a solid elastic object like this. Then we can think of this as a spring whose spring constant k.

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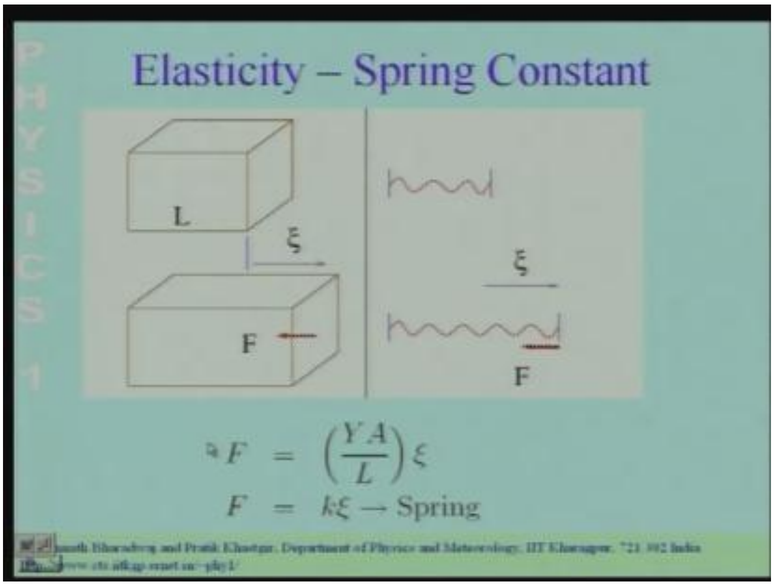


So, the point is that, we can think of this solid as in terms of a spring of spring constant k, which is equal to the Young’s modulus Y into the cross sectional area divide divided by the undeformed length let me repeat again. So, if I have an elastic object elastic body of cross sectional area A Young’s modulus Y and undeformed length L, I can think of this as a spring of spring constant YA by L. So, if I introduce an elongation xi, so, I pull this. So, I deform this by a amount xi, then there is going to be force develop, which is going to oppose. This deformation and the force is the spring constant, which is YA by L into the deformation. So, this act like a spring as far is deformation in this direction is concerned. So, let us now, go back to the, with this brief review of elasticity.

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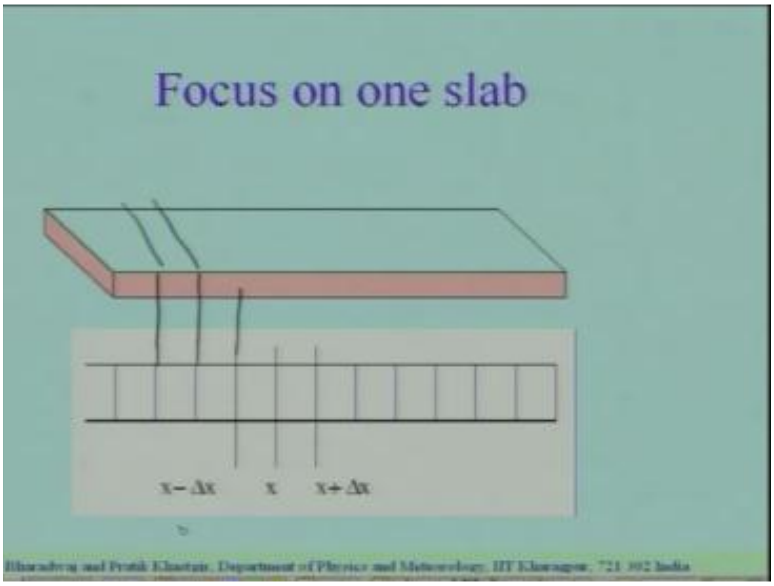


Let us go back to the problem, which we were discussing the problem, which we were discussing, we had an elastic rod in this elastic rod, we had disturbances and in order to study these disturbances. Then the disturbance varies only in this direction the disturbances only, along this direction and it only varies in this direction also. So, in order to study the, this disturbance, we had divided the rod into slabs. So, we had divided into slabs like this, so, this is one slab, I shall be showing only the cross sectional view over here. So, we have labeled, this direction with the x axis and we have divided it into slabs of equal interval delta x. And we shall focus our attention on a particular slab.

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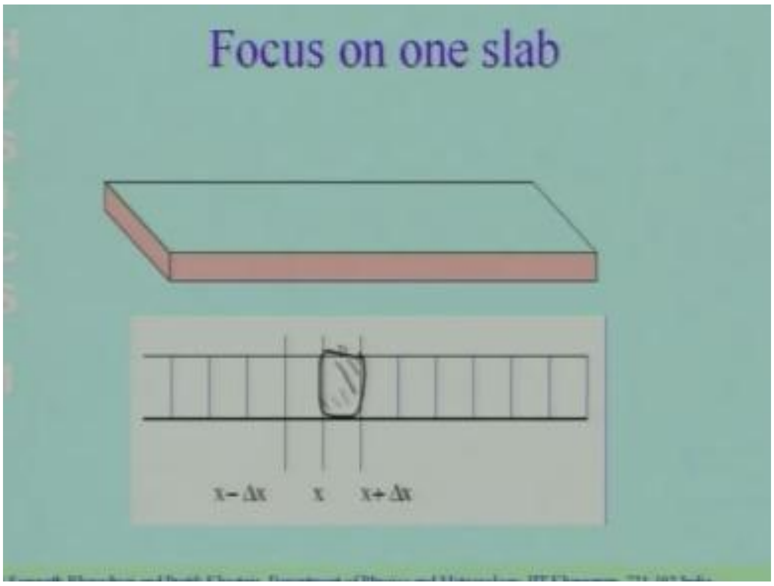

So, we shall choose a particular slab and focus our attention on it. So, we shall look at this particular slab over here, this particular slab and focus our attention on this.

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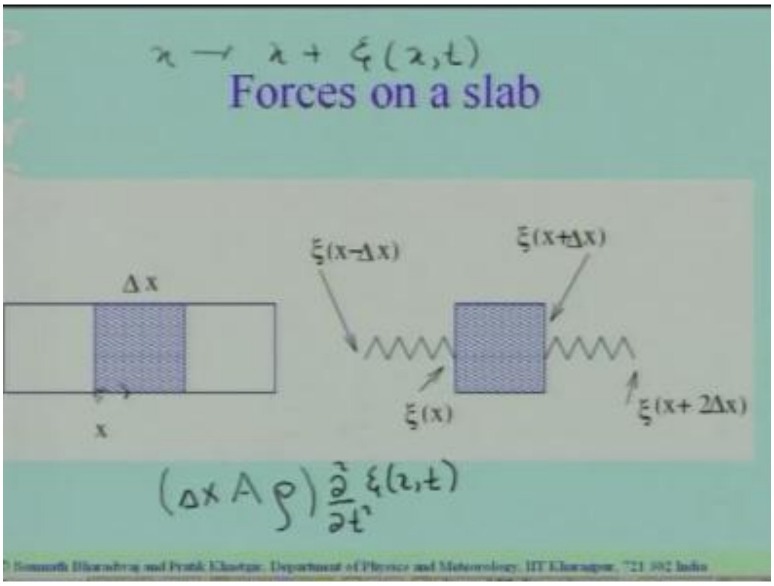


So, this shows you a magnified view of that particular slab, which extends from x to x plus delta x. So, let us ask the question? What is the mass? What is the mass of this slab? What is the mass of this element, which extends from x to x plus delta x? So, if let us first calculate the length the length of this is delta x and it has a cross sectional area A, which I have not shown and it has it the whole medium as a undisturbed density rho. Then the mass of the, this element is delta x into A the cross sectional area, which I have not shown in this figure into the density of this material, which is rho and the mass into the acceleration. So, what is the next question is, what is the acceleration of this element? Now, each point over here, because of the disturbance will get deformed.

So, the point x will move to a point x plus xi xt, that is the, displace the under deformation under the disturbance. The mass, which was at the point x here will move to a different point x plus xi xt. So, it will move to a different point, where the different is xi xt. Now, the question is what is the acceleration? So, you see if you ask the question what is the velocity? It is going to be the rate of change of xi with respect to time and the acceleration is going to be the second derivative of xi with respect to time. So, the acceleration is del xi del t square is of xi xt, so, this is the mass. So, if you focus if you focus our attention on this, particular element of that elastic medium the mass of this element is delta x A rho. And acceleration is del square xi del t square the second partial derivative with respect to time of the displacement xi. So, the mass into the acceleration should be equal to the total force total force some of the total forces acting on this element. So, let me write it down and preserve it for later use.

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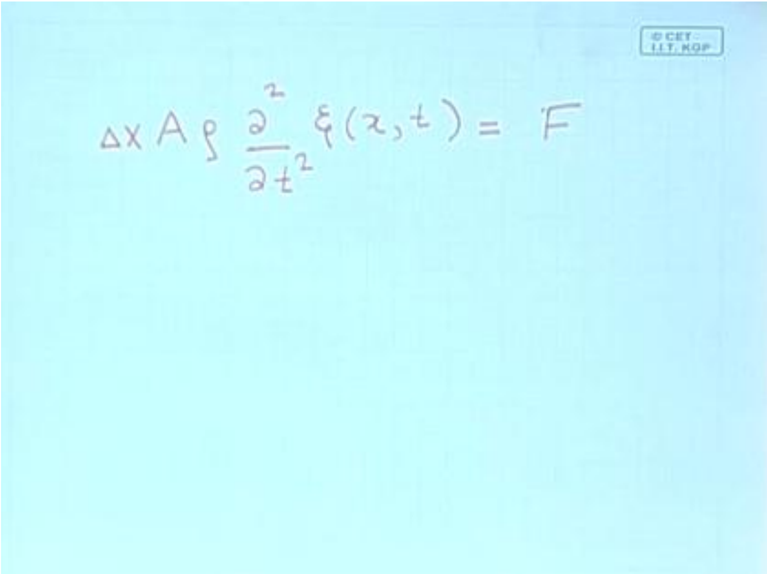


So, we have calculated the mass into the acceleration acting on that particular element, and you see that, it is the mass is delta x into A into rho. This is the mass into acceleration for the element, which we are focusing on for the mass element in the slab the mass element in the rod which we are focusing on. And mass into acceleration should be equal to the total force acting on that element. So, we have to now the, calculate the total force, which is acting on that element. Let us now, discuss how to calculate the total force acting on the element.

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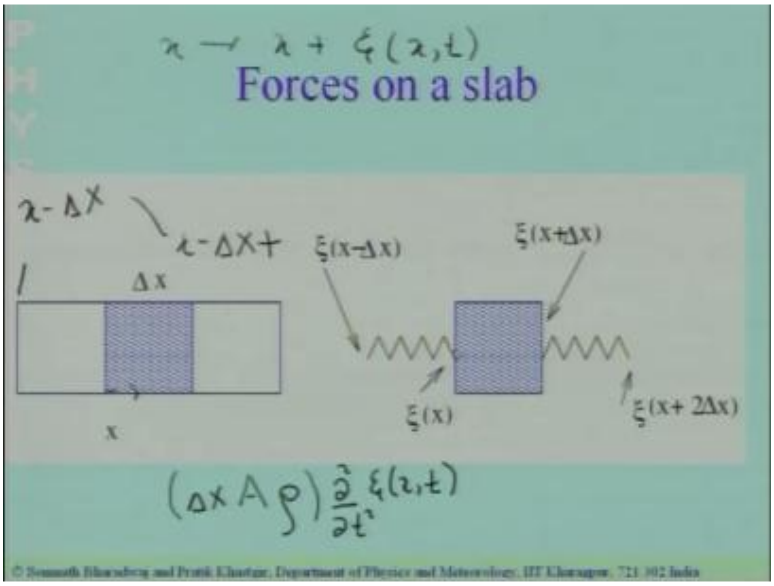


So, we are interested, we are focusing on this particular mass element and there is going to be a force on this particular mass element. One force is going to arise from the deformation of the, of the element of the elastic rod, which is to its left another force is going to arise from the deformation of the element which is on its right. So, let us first consider the element, which is on it left, let us consider this element over here. We can think of this element as a spring, I just told you a shot while ago that we can think of the deformation of each of these elements as that of a spring. So, under the disturbance this point x minus delta x gets shifted to a point x let me write it for here. So, the point x minus delta x gets shifted to a point x minus delta x minus rather plus xi at the point x minus delta x there is t also, which I have not shown explicitly.

So, the deformation of this point this point is originally in the undisturbed situation, this point at x this point corresponds to x minus delta x, because of the disturbance. It goes over to x minus delta x plus xi x minus delta x, that is the deformation that is the deformation of this particular point. Now, what the deformation of the other end of the spring? You see the, we are going to we are thinking of this as a spring, which I have shown over here. This end of the spring originally at x now, goes to x plus xi x over here. So, if I now, want to find what is the deformation of what is the deformation of the spring. The deformation of the spring is the difference between this and this. So, the force, which is going to develop because of the deformation of this, is going to be, so, the force, which is going to develop from the spring on the left.

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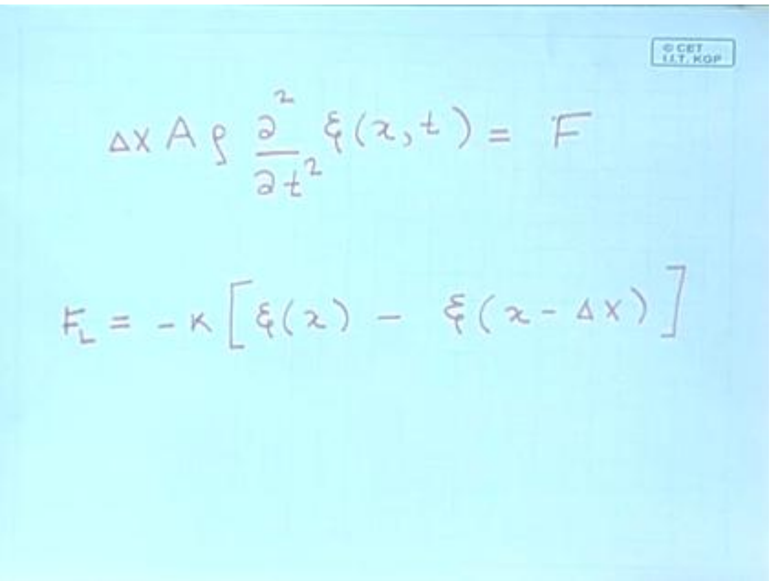


The force, which is going to develop from the spring on the left is the, is going is there is going to be a force, which is going to develop. The force from the spring on the left is going to be proportional to the change in the length of the spring; the change in the length of the spring is the original length is delta x the change in the length is difference between this and this. So, the force is going to minus K into xi x minus xi x minus delta x. Right. So, this is the deformation of the spring the left.

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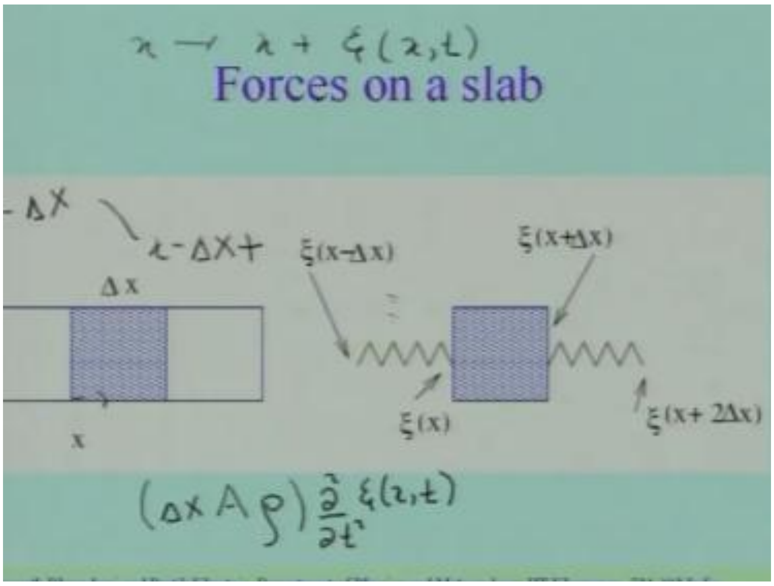

The left tip of the spring gets, displaced to xi x minus delta x the right tip get this, displaced to xi x by an amount xi x. So, the deformation of the spring is the difference between these 2 and there is going to be force, which is going to be opposite, it to this deformation, which is the force due the spring on the left which is what I have shown over here.

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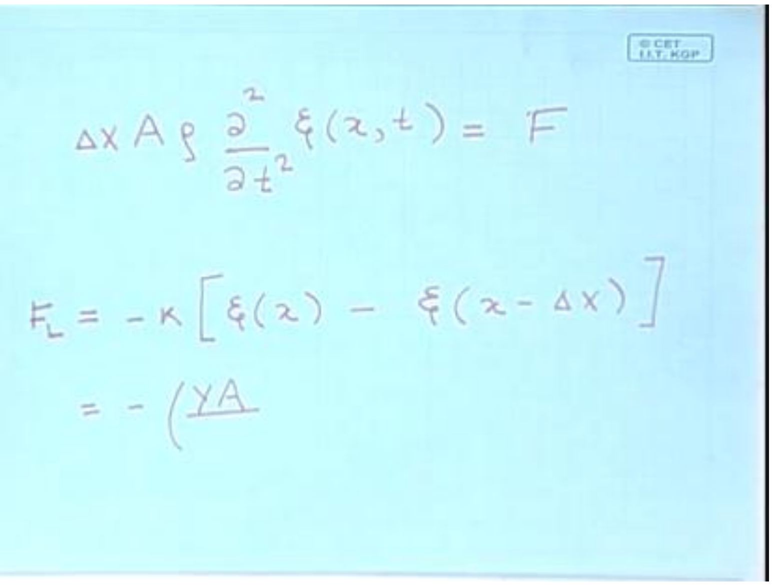


Now, the spring constant k for this element, we have already determined that the spring constant k is Y into the cross sectional area divided by the length the undisturbed length of this element.

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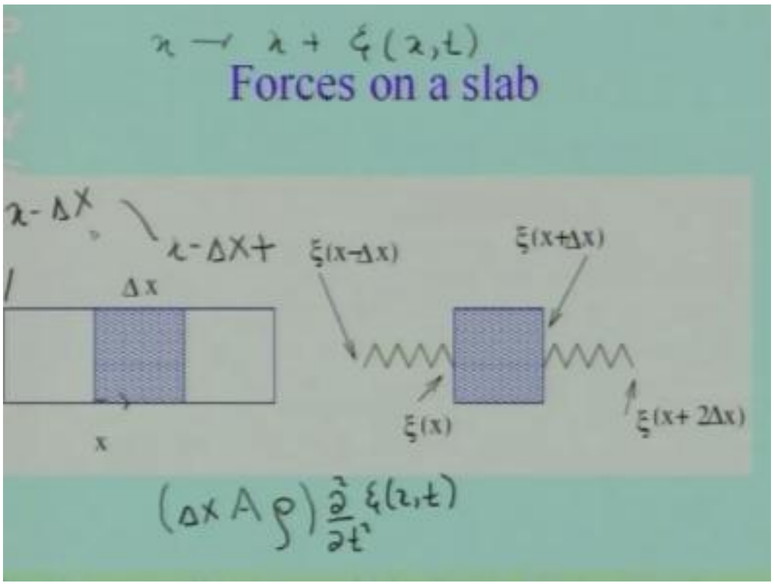


So, this is the element that we have replaced by this spring its undisturbed length is delta X.

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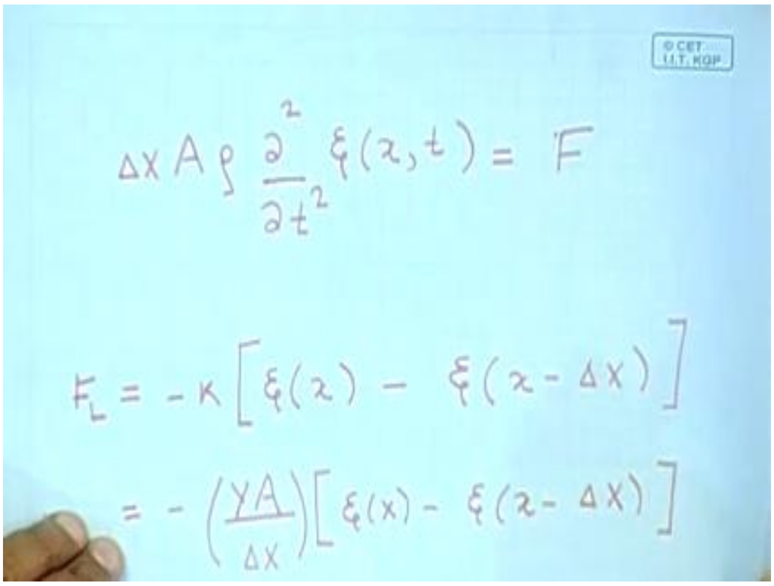


So, this going to be YA by delta x into this factor xi x minus xi x minus delta x. So, we can calculate the force on the left the force on the left is this, whole thing over here. And if I look at the ratio of xi x minus xi x minus delta x divided by delta x, that is the derivative of xi x at the point of the function xi at the point x.

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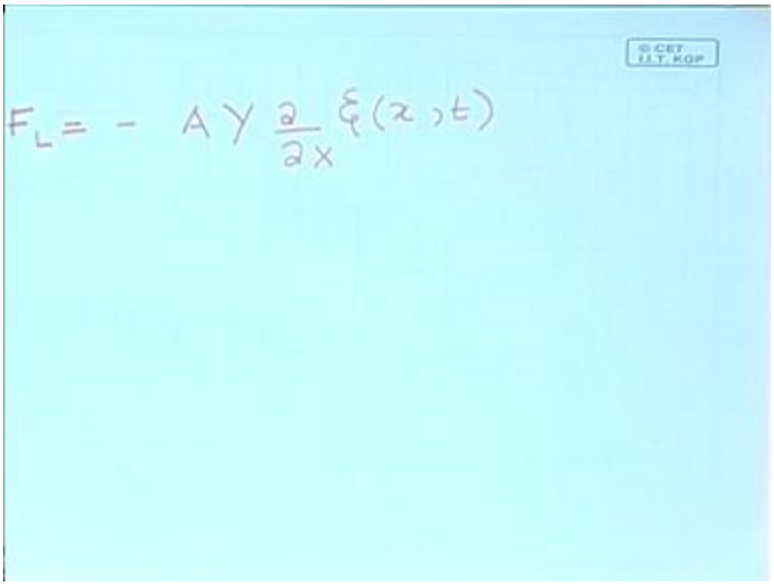


So, what we see is that the force from the left the force from the left is equal to minus A the cross sectional area into the Young's modulus into del by del x of xi x t.

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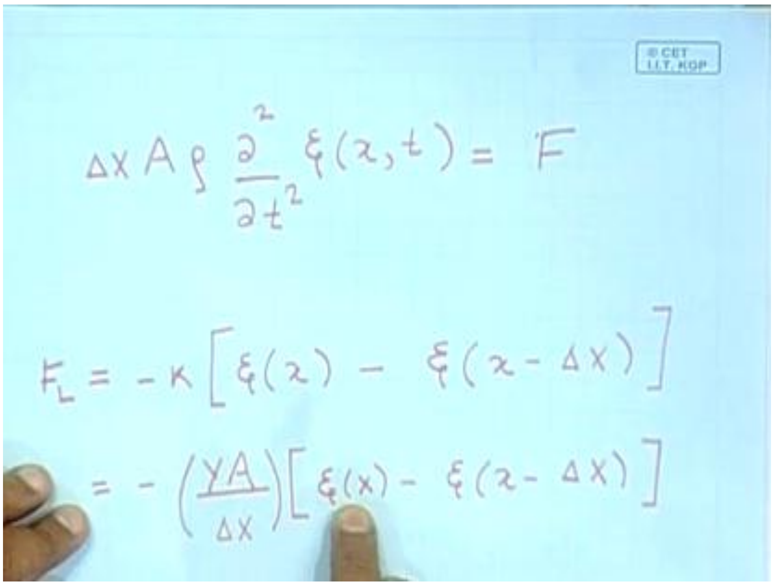


So, the ration of this difference divided by delta x is the derivative of xi at the point x, which we what we have put replace over here. So, we have calculated the force due to the spring on the left and that is proportional to the derivative of xi at the point at this point over here.

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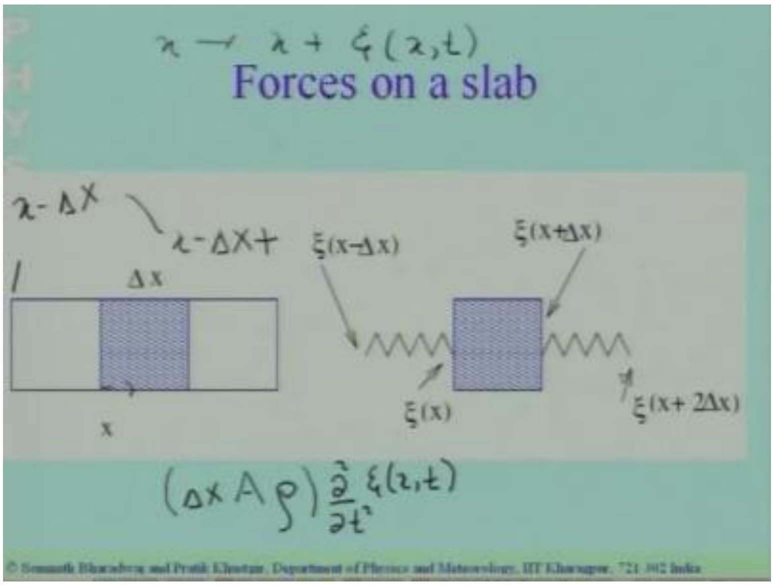


Now, let us calculate the force due to the spring on the right the spring on the right is. So, we have represented this elastic, this bit of the elastic material on the right as a spring over here and there is going to be a force. So, if I displace, this point and if I displace this point the difference between these 2 displacement. The displacement of this point is xi x plus delta x the displacement of the other tip of the spring is xi x plus 2 delta x. The difference between these 2 displacements is going to give a force, which going to act on this. How much is the force, which is going to act from the right. Let me now, focus on that the force, which is going to act from the right.

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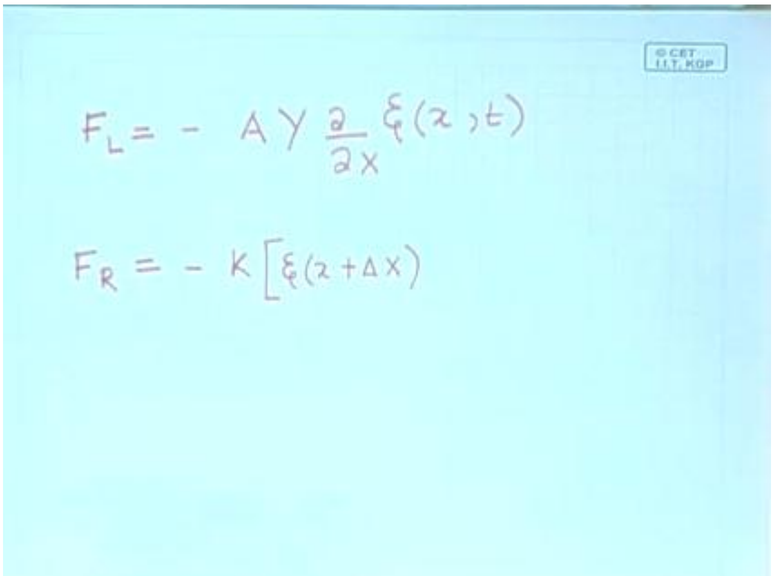

So, if I push this element of the spring on to the right the forces that develop is going to be to the left, which tells us that the force from the right is going to be minus k into xi x plus delta x.

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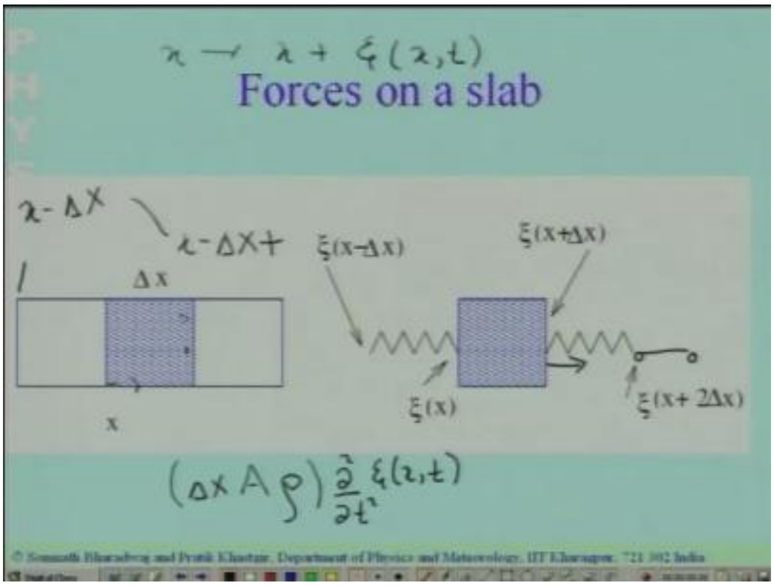


This is the force that arises when I push this tip to the right or to the left there is also going to be a force, when I push this force. If I push, if I pull this end of the spring over here, If I pull this point over here elongated like this, it is going to give a force in this direction on this on this end.

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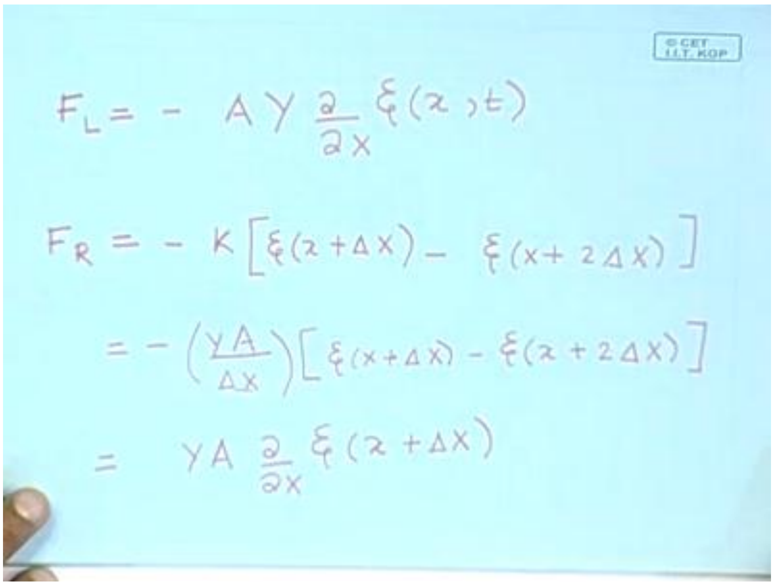

So, we can say that the force arising due to the displacement of this end of the spring is going to be this minus xi x plus 2 delta x. And again, we can write the spring constant in terms of the Young's modulus the cross sectional area divided by delta x. And we have the difference between these 2, which gives us again, you take the minus this divide by delta x. And in this case you get a extra negative sign, because so, what you have s this equal YA del by del x. We have written the force both of the left hand side and on the right hand side in terms of derivatives the force on the left hand side involves the derivative at this point.

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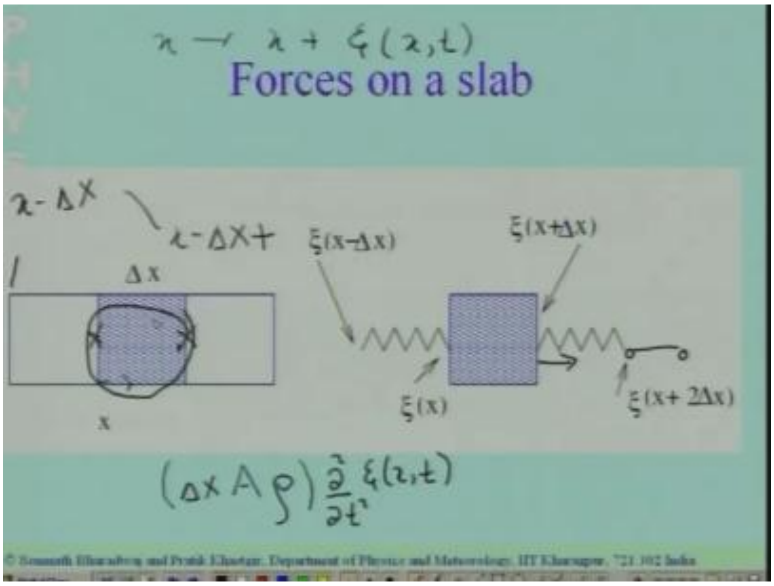


The force on the from the right hand side involve the derivative at this point. To calculate the total force on this mass element, which we are focusing on, we have to add up both of these.

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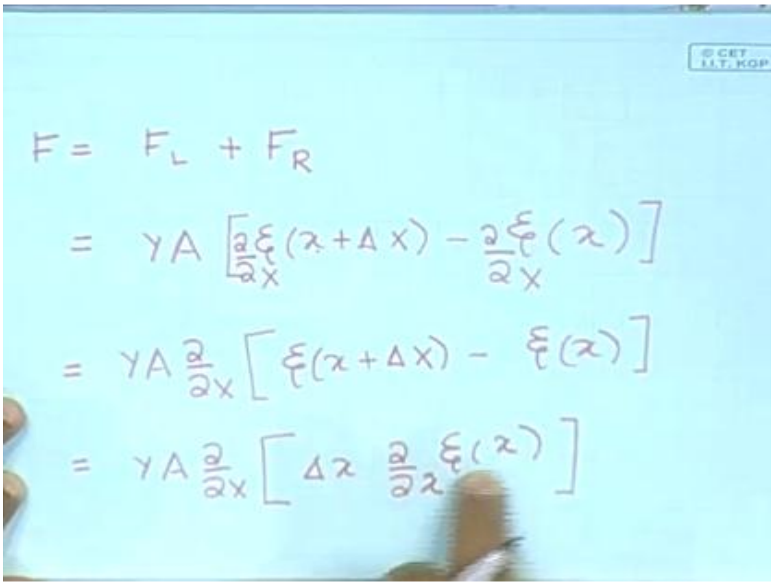


So, the total force F is equal to the force left plus the force from the right and this is equal to YA the cross sectional area A so, let me show us the expressions again.

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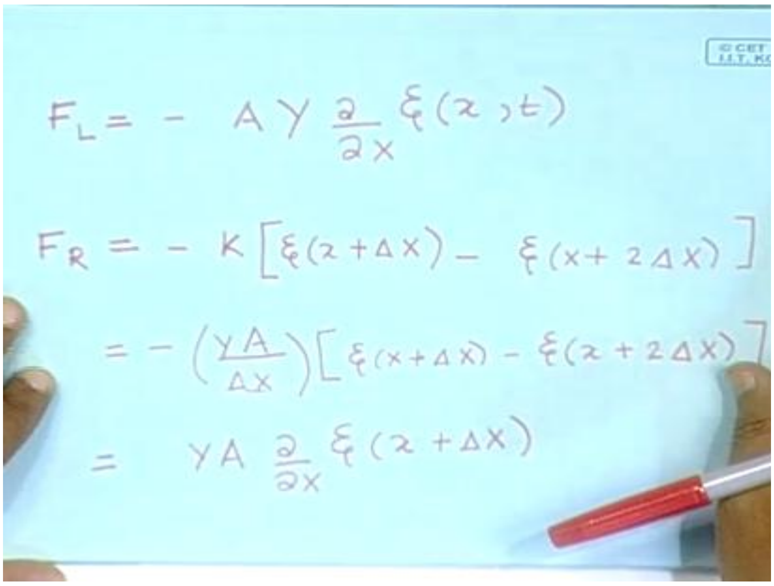


So, you have add up this and this.

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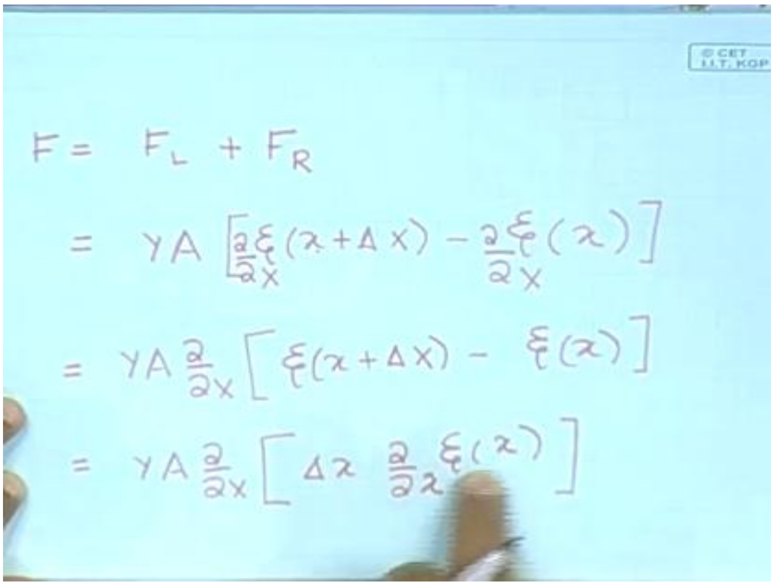


And this is equal to YA xi x plus delta x minus xi x this is the force from the right this is the force from the left. And there is the derivative of both of these del del x del del x, this is the force from the right this is the force form the left. We can write this as YA del del x xi x plus delta x minus xi x and this difference, we can write as YA del del x this difference. We can write as delta x into the derivative of xi with respect to x keeping only the first order term. So, what we see is the, that the force is the second special derivative of xi with the factor of delta x going out.

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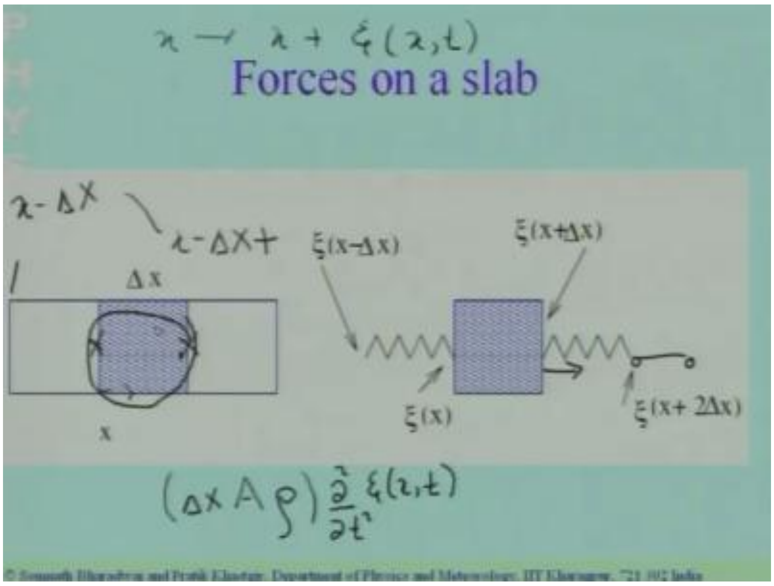

So, the total force due to the 2 elements on the 2 sides.

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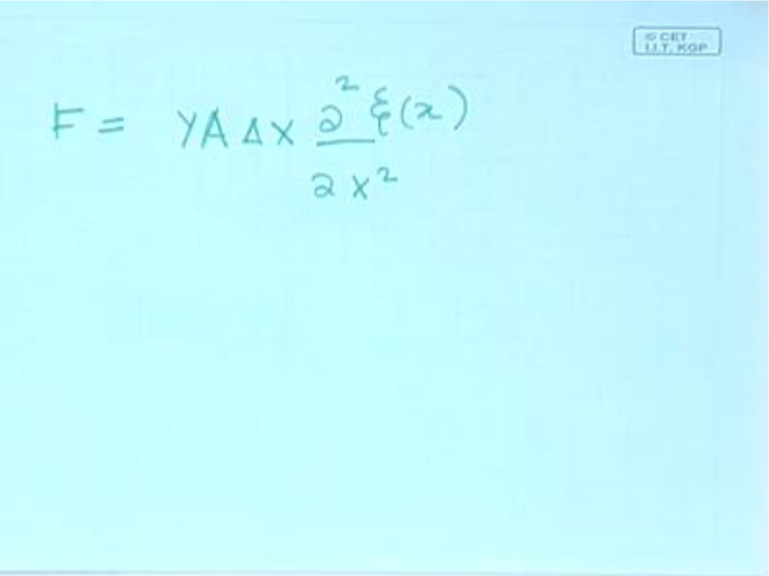


Total force is YA delta x into the second derivative of xi. Now, we had write in the beginning calculated the mass into the acceleration.

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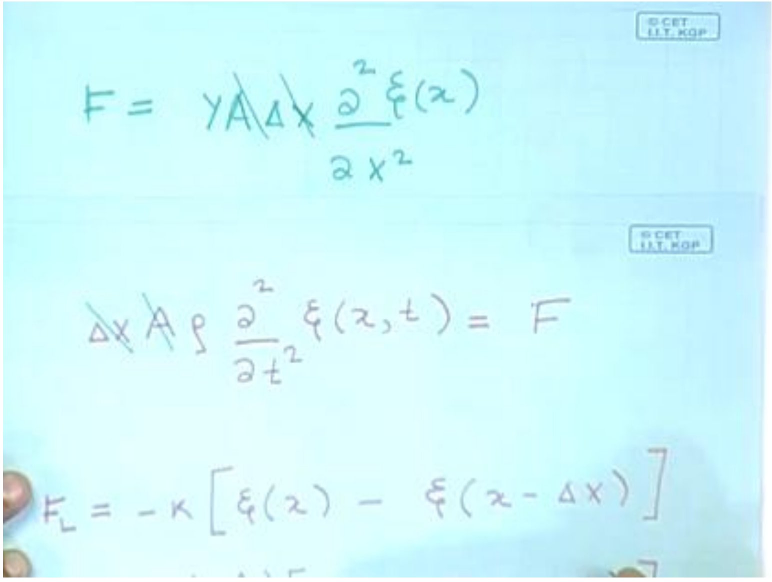


And that had come out to be this, so, now we can equated to the total force, which we have just calculated. And we see that factor of delta x cancels out A cancels out and we have an equation.

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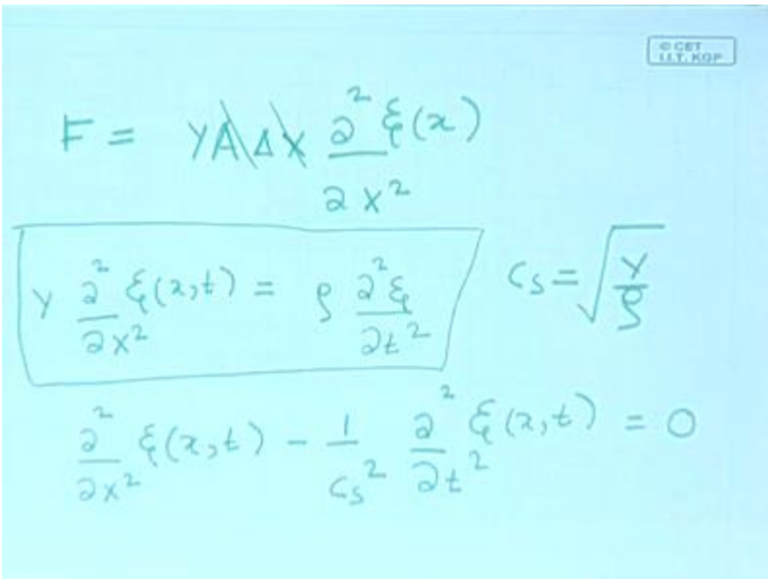


The Young's modulus del square del x square xi is equal to the density into the time derivative of xi second time derivative of xi. This equation can be written as follows, so, let me just rearrange the terms in this equation, this equation can be written as del xi del x square minus one by Cs square del xi del t square. This is equal to 0, where this constant Cs in this case, you can identify what it is? So, we have we have to bring the Y over here and then you can write Cs you can identify Cs with a ratio of rho and Y. So, Cs is Y divided by rho the square root of this. So, we have written this equation in this form, where we have introduced this constant Cs whose value whose significance. We shall discuss shortly Cs is the ratio of the Young's modulus to the density of the material.

Now, until now, we have been discussing our situation, where we have a disturbance which is their only in the along the x axis, we have not considered what happens to the disturbance along the y or z axis. We can easily, generalize the whole discussion now, to a situation, where the disturbance propagates along 3 directions along x y and z. So, if you have a disturbance, which is their along all 3 direction along x y and z. You have to replace the special derivatives with respect to x with special now, you replace it some combination of special derivative with respect to all 3 directions. So, let me now, tell you what you have to do in case you we have a disturbance, which propagates in all 3 directions.

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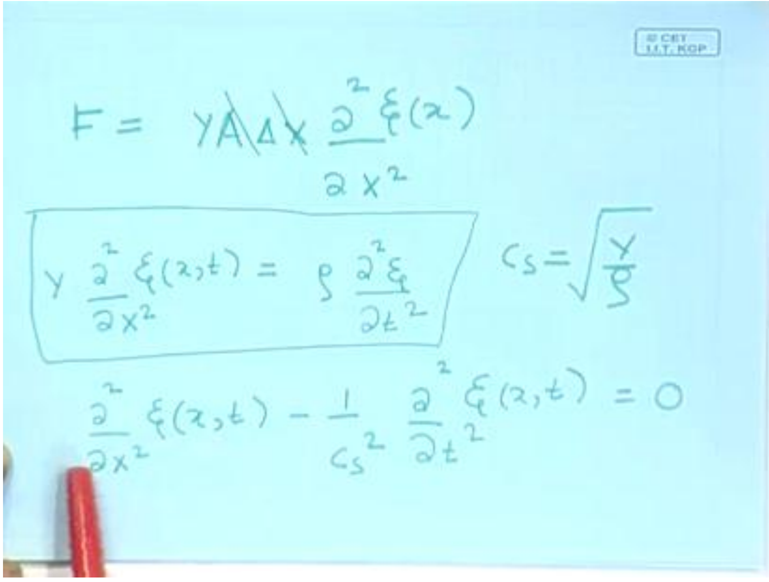


You see there is a special derivative with respect to x xi is the function of x alone. And we have a special derivative with respect to x.

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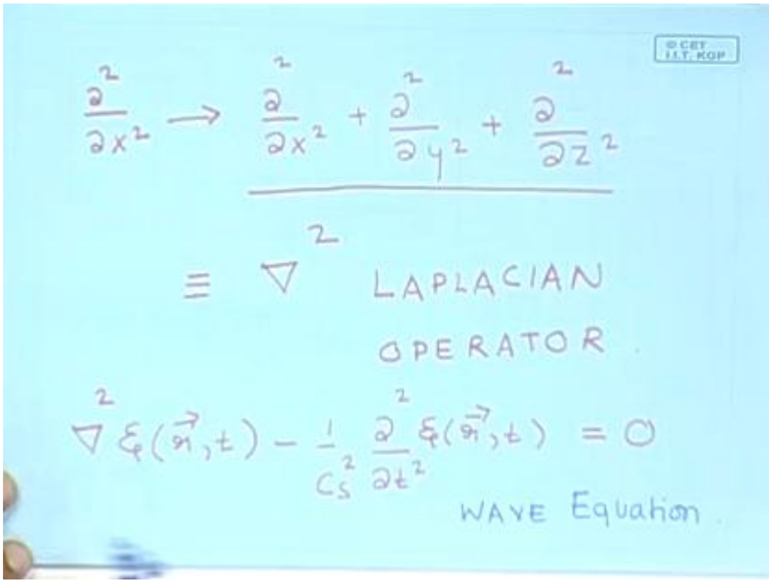


You have to replace this del del x square the second partial derivative with respect to x, we have to now, replace this with del del x square plus del del y square plus del del z partial derivative with respect to x square. Now, get replaced by the partial derivative with respect to x square plus the derivative with respect to y’s the square of that plus the partial derivative with respective z the square of this. This combination of partial derivatives of a x y and z is a very important is a very important operator, which is referred to as the Laplacian. And it is denote by this symbol over here this call a Laplacian operator.

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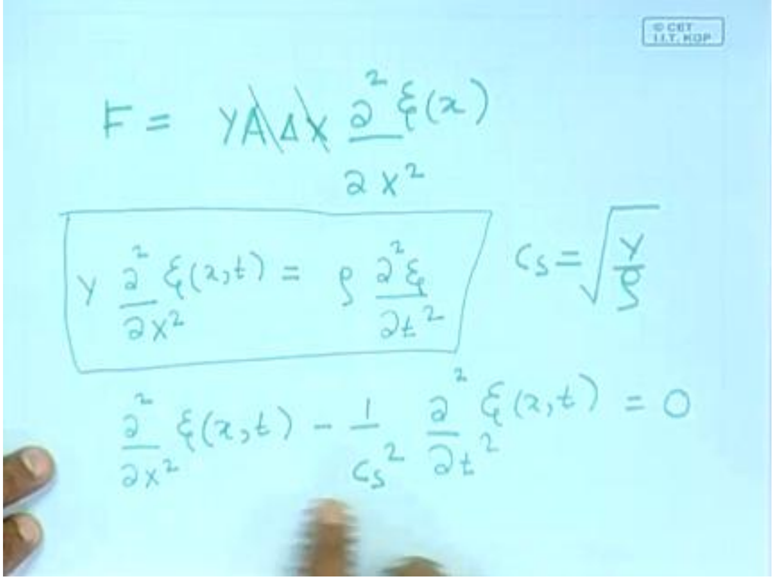


And the equation, that we had earlier equation that we derived for the disturbance in that solid in that elastic solid in the situation, where the disturbance can propagate along all 3 direction.

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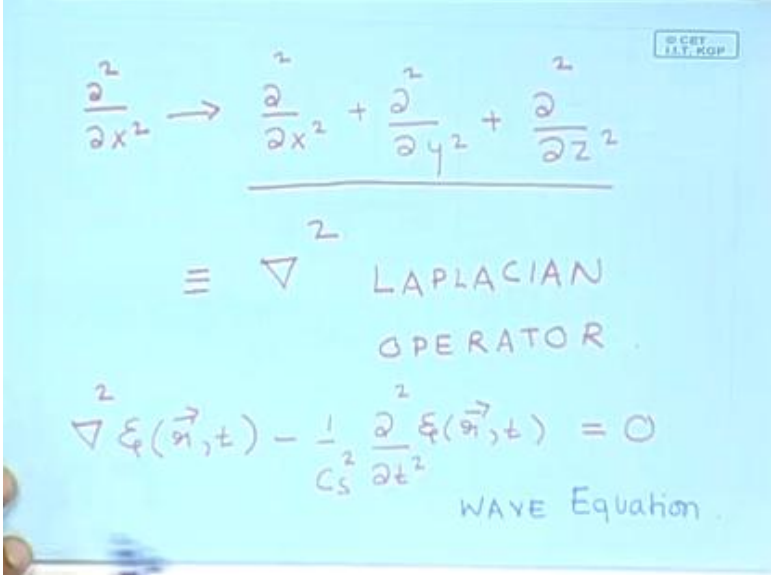

This can be written as the Laplacian del square xi xi is now, a function of r and t vector r and t. It can vary in all possible directions this rest the equation remains unchanged.

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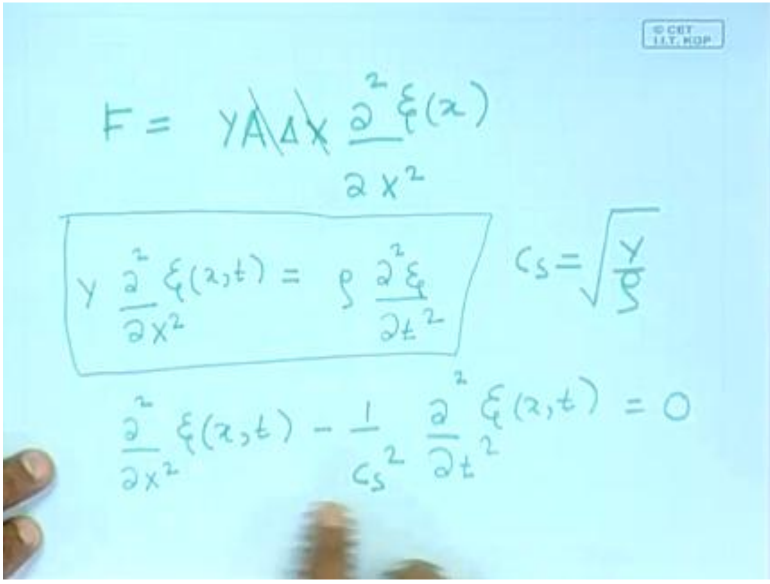


So, let me write down the rest of the equation show, you the rest of the equation. We have minus one by Cs square into the partial derivative second partial derivative so, that remains unchanged.

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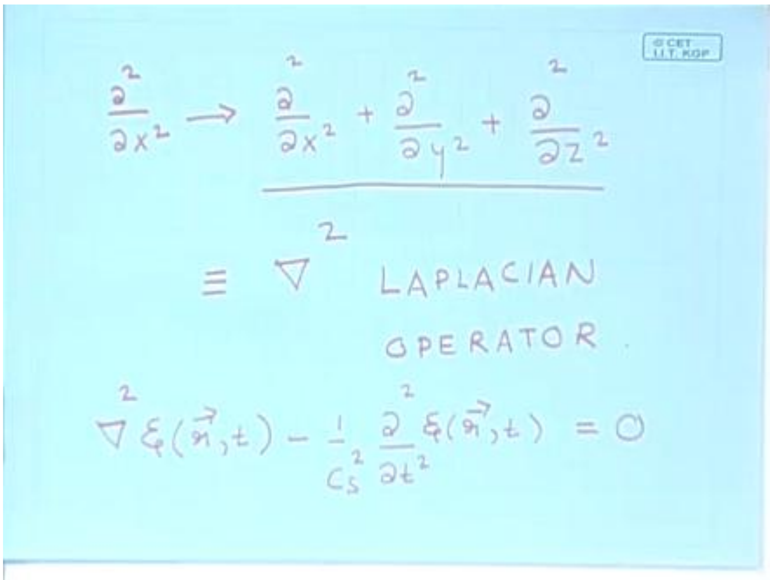


So, the most general equation that, you could have for the disturbance, where the disturbances now, free to propagate in all 3 direction x y and z. All which we denote by this vector r this equation is called the wave equation 3 dimensional wave equation, this is called the wave equation this equation is the wave equation. So, if we have a wave, which can propagate in all 3 directions 3 dimensional situation.

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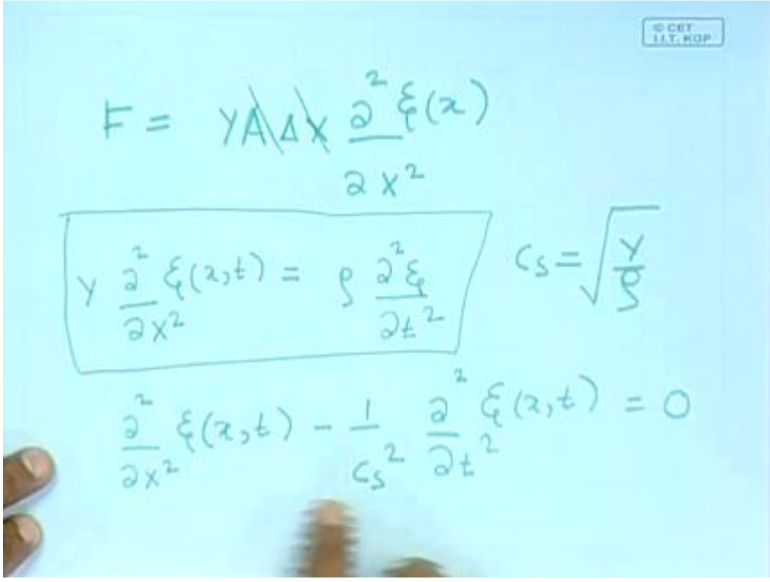


This is the wave equation in the situation where it is only one dimensional this is the wave equation. So, what we see is the evolution of disturbances in that elastic solid. So, we have a solid, which is made up of some kind of an elastic material you put a disturbance in that elastic material. The disturbance the evolution of the disturbance in the elastic medium is governed by the wave equation. So, this is the first thing that we see now, let us look a little closer at the wave equation. The wave equation as we have derived it has one constant Cs which in this for this particular situation, where we have elastic waves. The constant Cs is the square root of ration of the Young's modulus divided by the density of the medium. Now, let us look at this equation a little closely. So, the first thing that we can do is we can ask is the plane wave which we have discussed quiet extensively earlier in this course to the plane wave.

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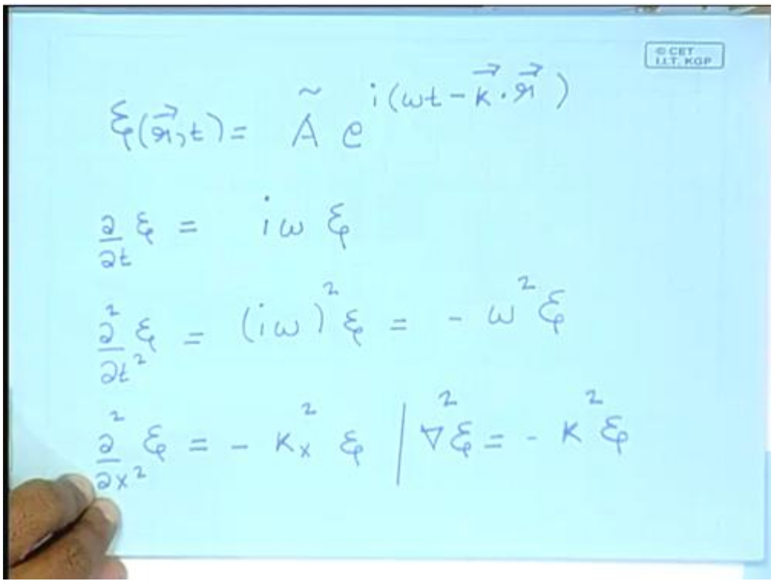


Xi solution xi r t is a constant A, which is not very important amplitude constant amplitude, we are working in the complex notation let me remained you. So, we are going to look at the particular solution, which we have already discussed, which is the plane wave solution e to the power I omega t minus k dot r. So, we have already studied this represents a plane wave.

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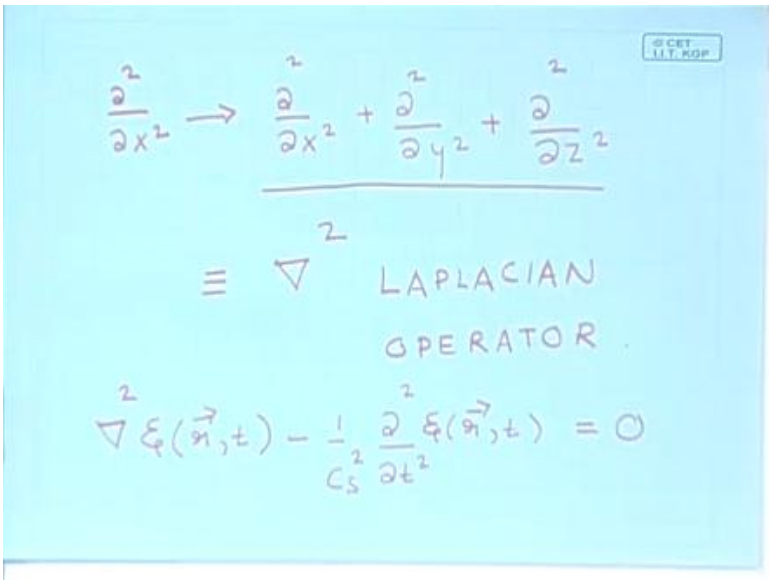


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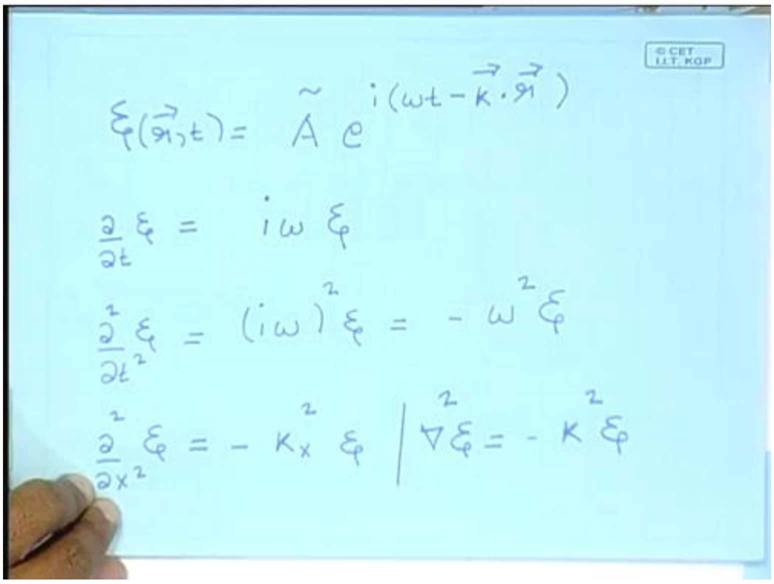


Let us check if this plane wave is a solution to this wave equation that we have just derive the plane wave, which we have studied, has a few unknown constants few constant. So, this A is the amplitude of the plane wave, it has in the complex notation, it has both the amplitude and part of the phase. Omega is the angular frequency of the plane wave, we tells me how fast if I look sit at particular position. It will tell us, how fast it changes with time, how fast a disturbance changes with time k is the wave vector. It has both the information about the direction in the, which the wave is propagating it also tell us that at a fix time after what distance the wave will repeat. What’s the wave length it also tell us the direction in, which the wave is propagating. Now, let us check if this is actually as it whether, this is a solution to the wave equation which we have just derived this wave equation governs the evolution of disturbance in at elastic solid.

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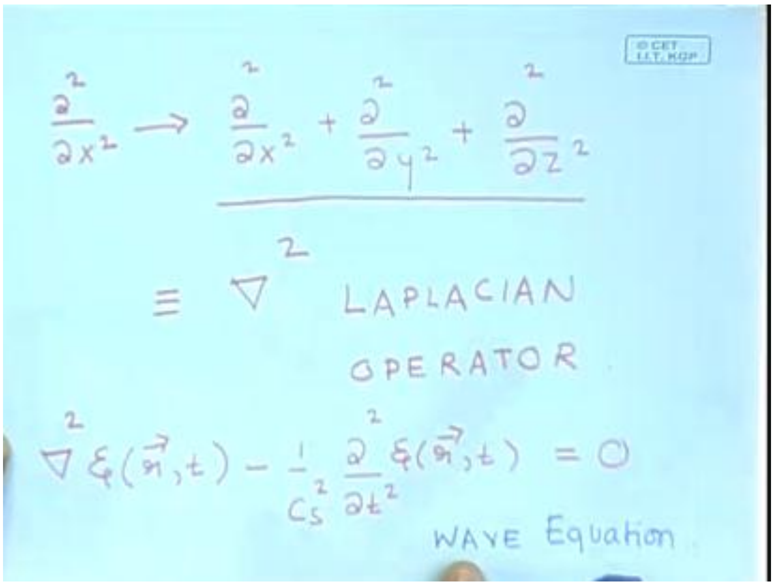


So, let us first calculate the time derivative of this function.

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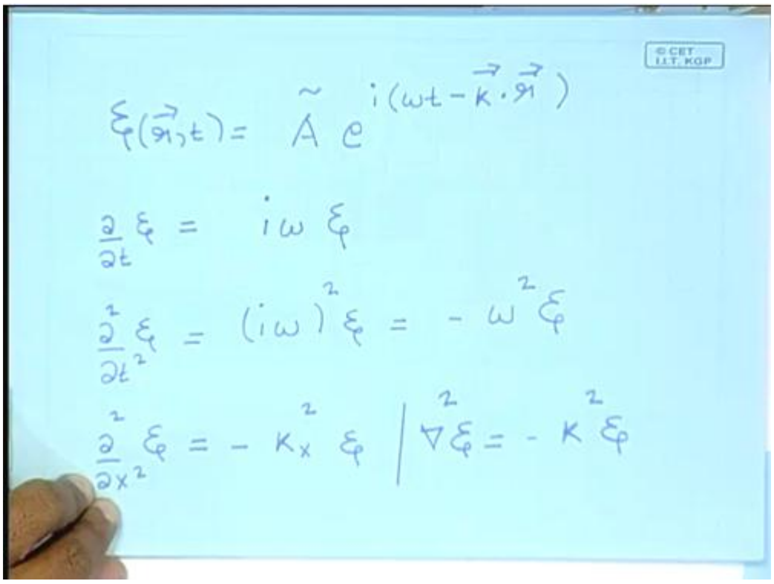

If I differentiate xi with respect to time once you basically, differentiate the e to the power I omega t with respective time. If you differentiate an exponential, you recover the exponential with the factor. So, if I differentiate this function with respective time, what I recover is I omega into xi that is a differentiating, this with time with respective time once brings out the factor of I omega and we have the xi again write that is obvious. So, if I differentiate, it twice with respect to xi we get on more factor of I omega. So, we will have I omega square xi, which is minus omega square xi. So, what we see is that if I differentiate it with respective time, if I differentiate this particular plane wave solution plane wave, which we are which we are trying to see whether, it is solution or not. If we differentiate this form of xi twice with respective time, it is equivalent to multiplying it with the factor of minus omega square.

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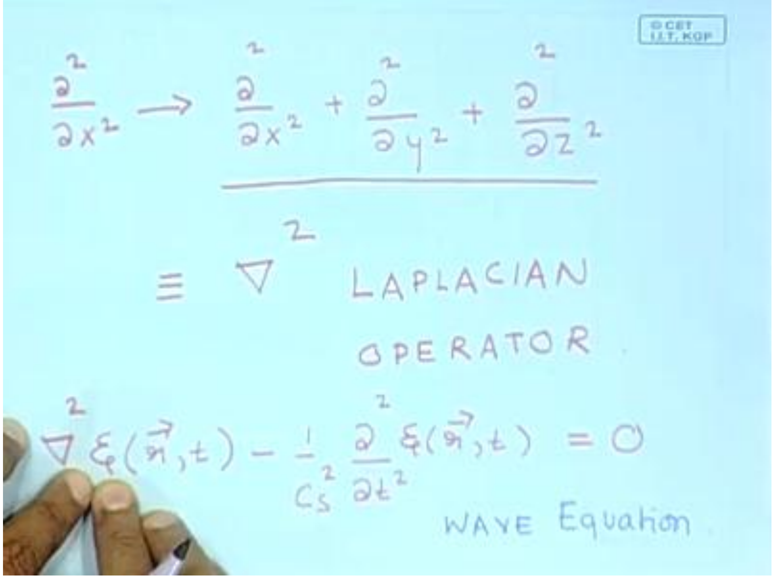


Similarly, you have to also consider the action of the Laplacian operator on this functions xi. Now, the Laplacian operator has 2 derivatives with respect to x 2 derivatives with respect to y and to the 2 derivatives to respect to z.

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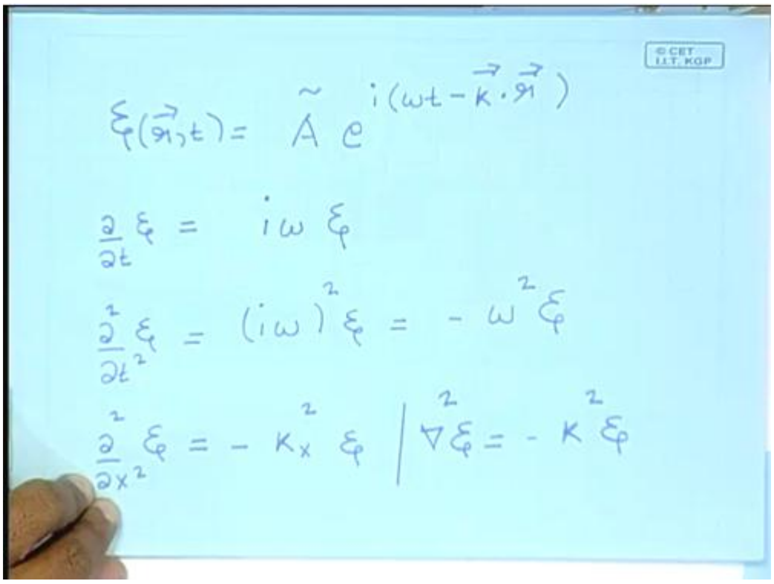

If we differentiate, this with respect to x what happens? That the question. Now, remember, that the wave vector k will have 3 component kx ky and kz if I differentiate this with respect to x twice. So, let me write it down straight away without doing the algebra del del x square xi is going to be see, if you differentiate once you will get a factor of I into kx where x kx is the x component of this vector k. So, if I differentiate it twice I am going to minus kx square xi just like if I differentiate with respective time. Similarly, if I differentiate with respect to y twice, I will get ky square, if I different differentiate with respect to z I will get kz square. So, what we can say? Straight away is that the Laplacian of xi is going to be minus k square xi, where k is the modulus of this wave vector k is the wave vector dot with itself. So, it is quite clear from this, that the Laplacian acting on the xi which the Laplacian is the sum of derivative with respective x square the x square of derivative twice with respect x twice with respect to y twice with respect to z. If I add up all of these, I am going to get minus k square, where k is the modulus of this it is the length of this vector the wave vector into xi.

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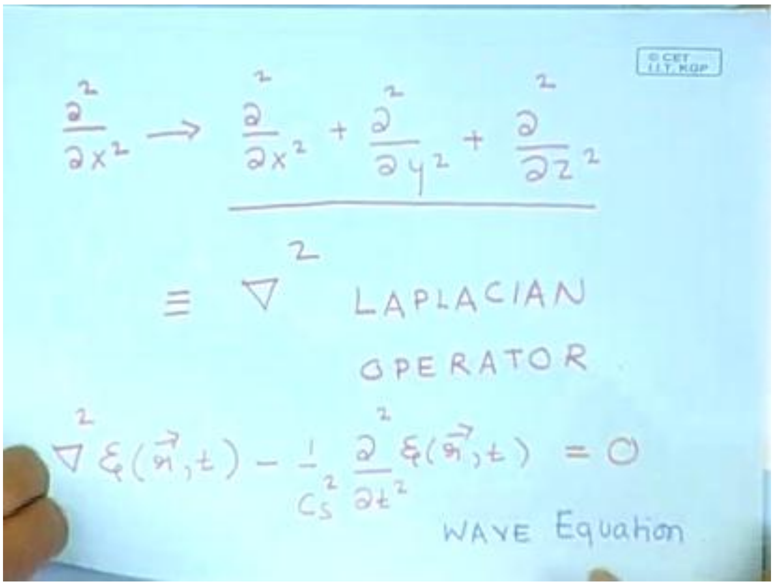


So, if you plug this in into the wave equation this term gets replace by minus k square xi this term gets, replace by minus omega square.

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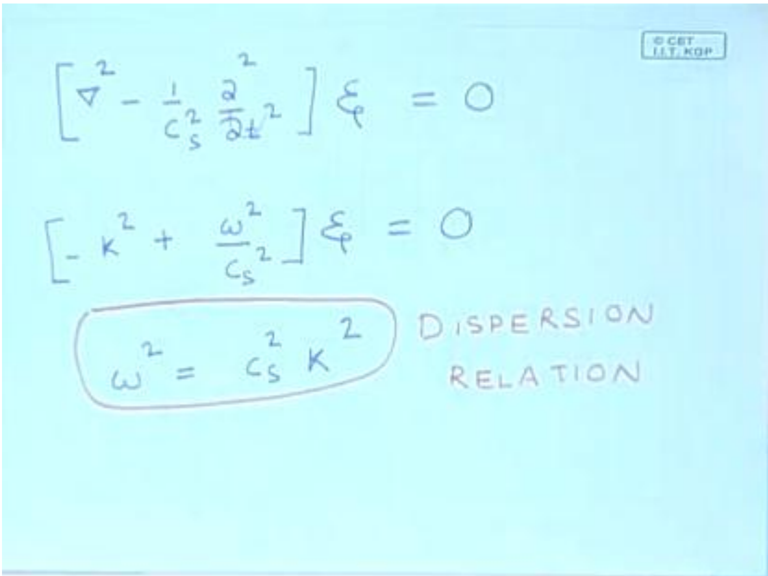


So, what we have is let me combine all of this, so, the wave equation was del square minus one by Cs square del by del t square this acting on xi is 0. This is the wave equation, this acting on xi gives minus k square this second time derivative of xi gives minus omega square. So, I have plus omega square by Cs square xi equal to 0. Now, if xi is 0 there are 2 possible ways this can satisfied one if xi is 0.

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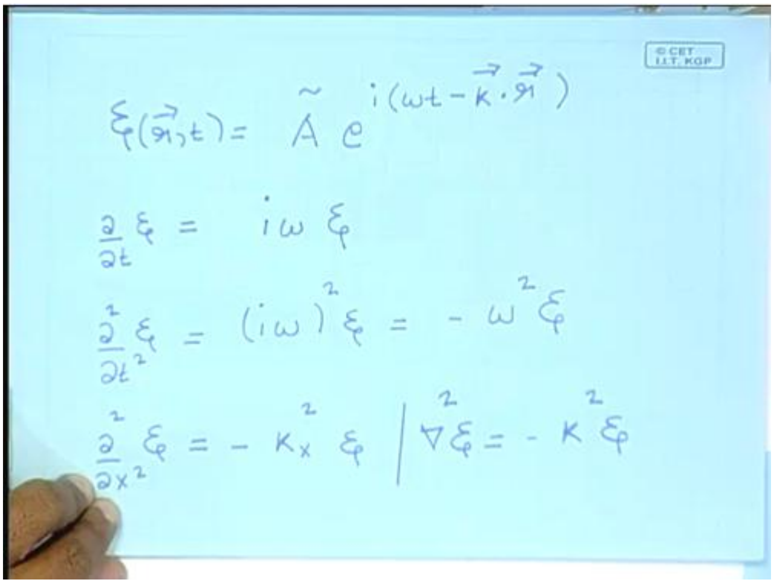


So, you have said the amplitude the wave to be 0, which is the tribal solution of no interest.

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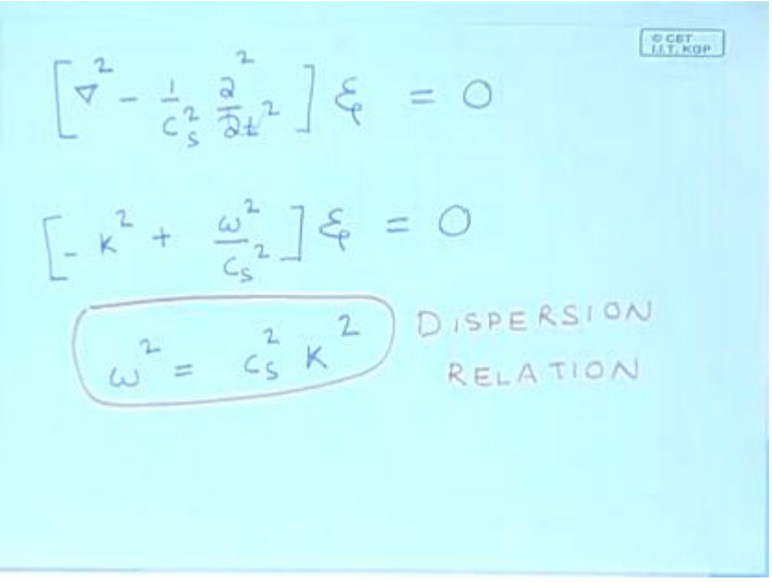


So, the only way that, we you could possibly satisfied this equation in a nontribal way is that. You have to set this term in the square bracket to 0 or what you say is that omega square should be equal to Cs square into k square. So, what we find is that the plane wave.

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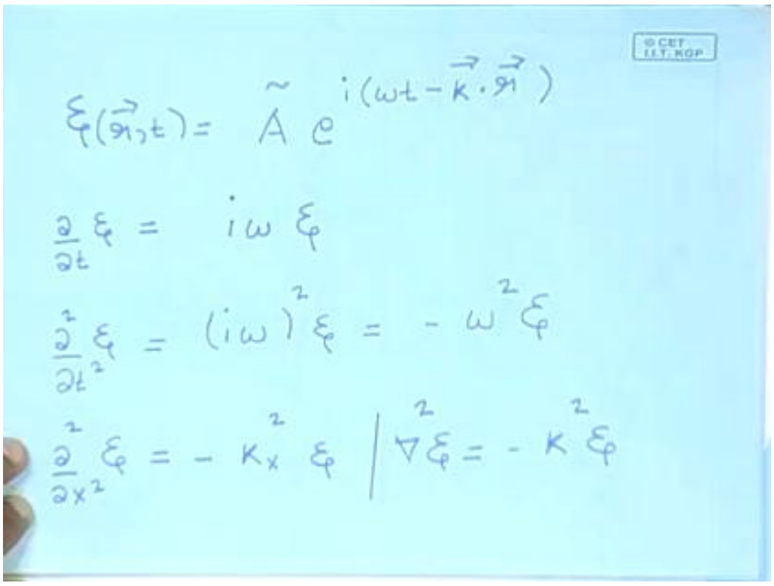


This particular plane wave is a solution is indeed a solution to the wave equation.

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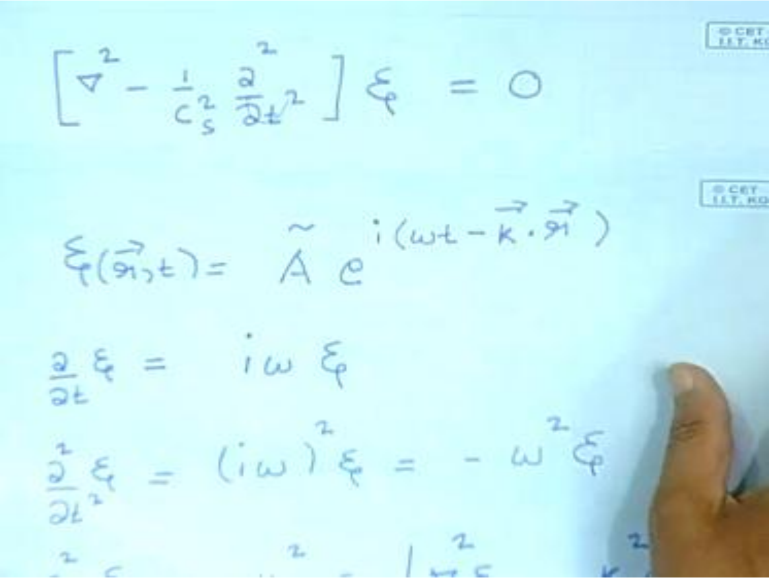


To this wave equation a plane wave like this indeed a solution to the wave equation provided the wave vector.

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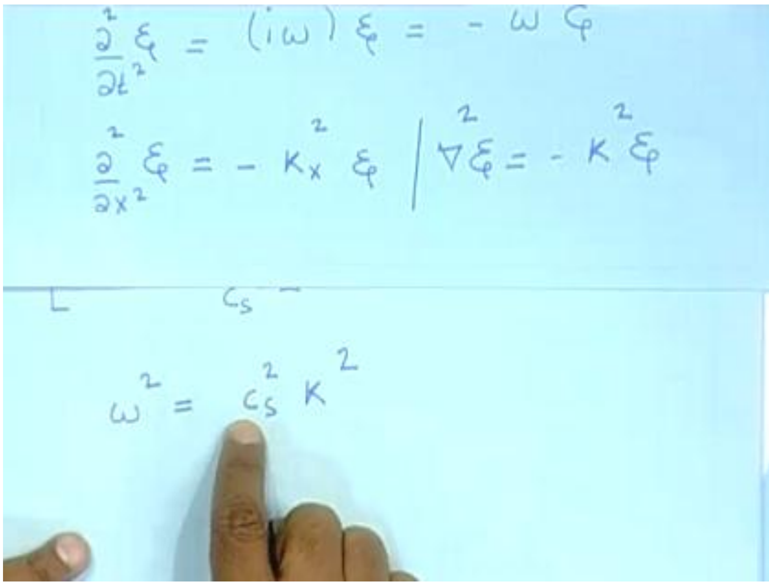


And the wave and the angular frequency are related. So, the wave number and the angular frequency are related through, this constants Cs which appears in the wave equations.

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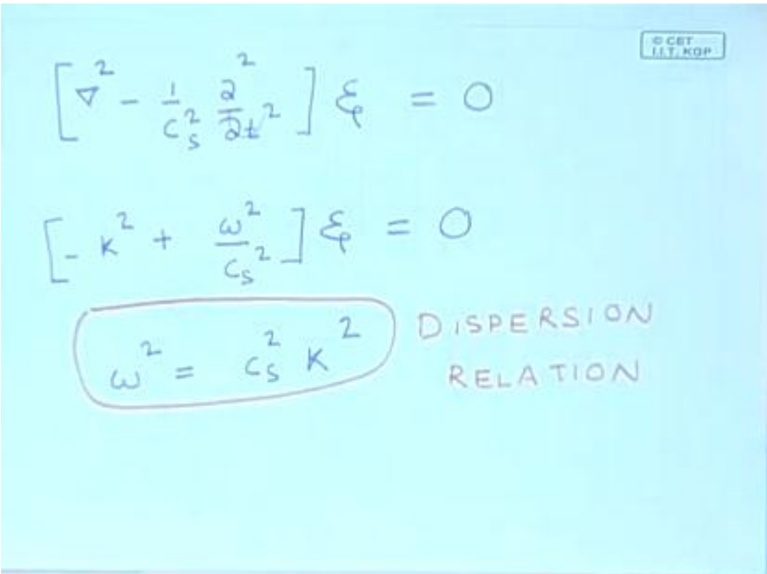


So, there is a constant Cs, which appears in the wave equation. So, provided the wave number and the angular frequency are related like this then the plane wave is a solution.

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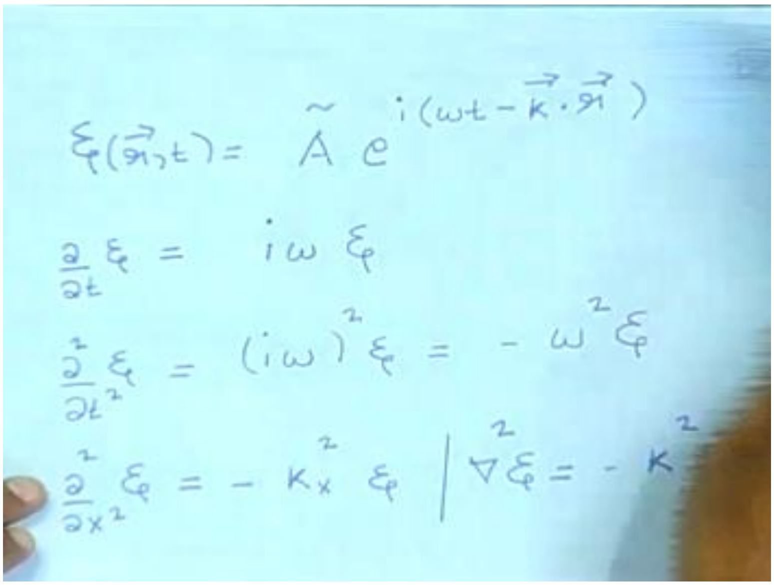


The plane wave over here is a solution to this equation.

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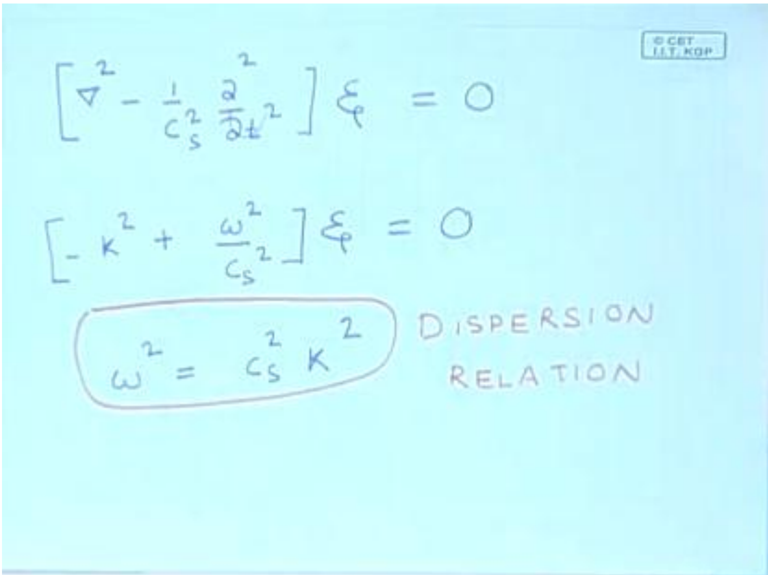


So, the plane wave is the solution provided, this relation is satisfied this relation over here. This satisfied this relation such relation between the wave number and the angular frequency is called a dispersion relation. And the dispersion relation the dispersion relation plays a very important role in the study of waves. So, let me again remind you, what is the dispersion relation? We had a wave equation and for the wave equation? We tried a plane wave as a solution. So, we took a plane wave as a trial solution and we ask the question does it satisfy our wave equation. So, we have this wave equation.

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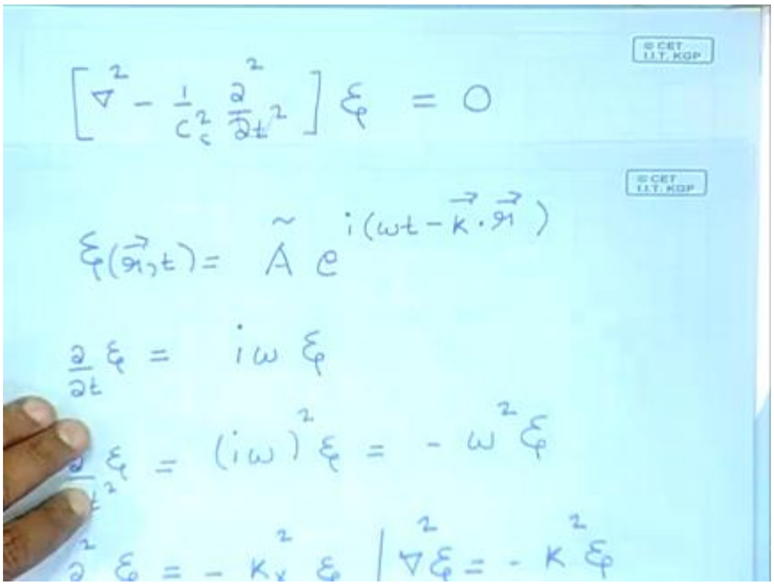

We took a plane wave like this as a trial solution for this wave equation.

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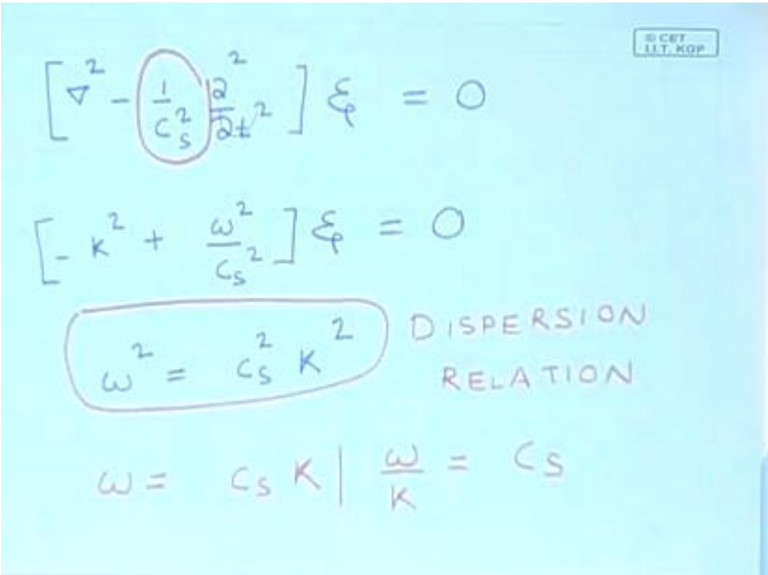


And ask the question does this satisfy the wave equation what we found is that the plane wave satisfies the wave equation provided the dispersion relation is satisfied. So, the plane wave is a solution to the wave equation, provided the angular frequency and the wave number are related like this. So, the dispersion relation has to be satisfied the dispersion relation is a relation between the wave number and the angular frequency. So, the plane wave is a solution to the wave equation provided the dispersion relation. It satisfies the dispersion relation provided the angular frequency and the wave number are related in this fashion. That is what the importance of the dispersion relation; it tells us how the wave number and the angular frequency are related. Now, let us come back to the dispersion relation, this dispersion relation tells us that omega the angular frequency should be equal to Cs into k. We have kept only the positive root of this equation to tells us the angular frequency should be equal to Cs some constant into k.

This straight away tells us that this wave has a phase velocity phase velocity of a wave is omega by k which is equal to Cs right. So, the wave as a phase velocity Cs, so, what do, we learnt from this? What we learnt from this is that if I have a wave equation, like this if I have a wave equation like this. Then the constant, which appears over here this constant they will be a in the wave equation. We will have a constant, which appears in front of the derivative with respect to time, this constant over here one by Cs, we have written this constant in the form one by Cs square. If you write the constant over here in this form then this Cs, which occurs over here is going to tell us the speed the phase velocity of the wave in this particular situation. So, if you have an elastic wave, which is propagating in a particular medium, we have calculated what Cs is.

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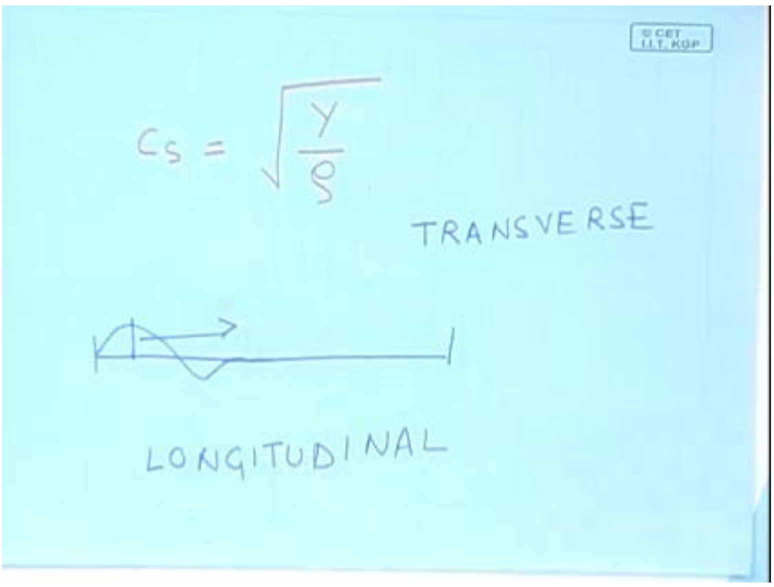

So, for an elastic wave the phase velocity Cs in the medium is equal to square root of the Young's modulus divided by rho the density of the medium. And if I have the different kind of wave in a different context then, they then the wave equation is usually going to be just the same as this.

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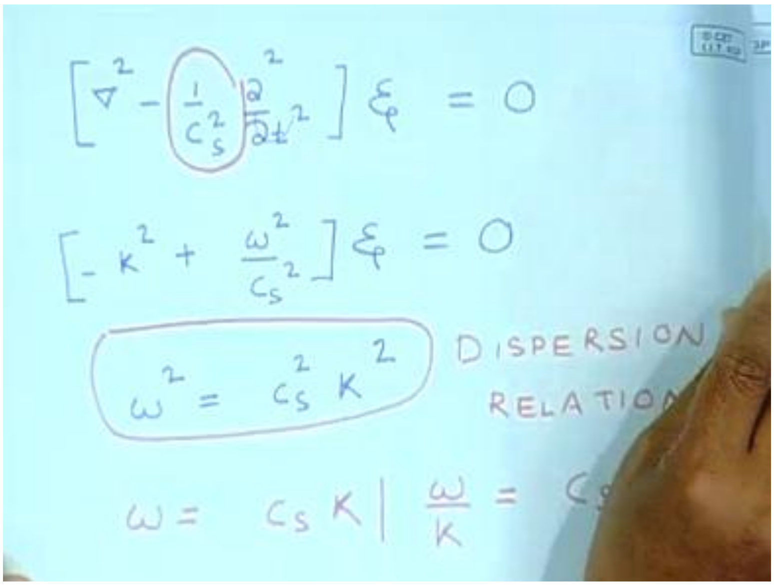


But the you will have some constant, which will appear over here and the constant will be will have a different dependence on the properties of the medium etcetera, but you interpret this constant as being the phase velocity of the wave. So, let me recapitulate again, for you what we have done in today is lecture, what we did was as follow. Let me recapitulate, we had an elastic rod in this elastic rod we introduce a deformation.

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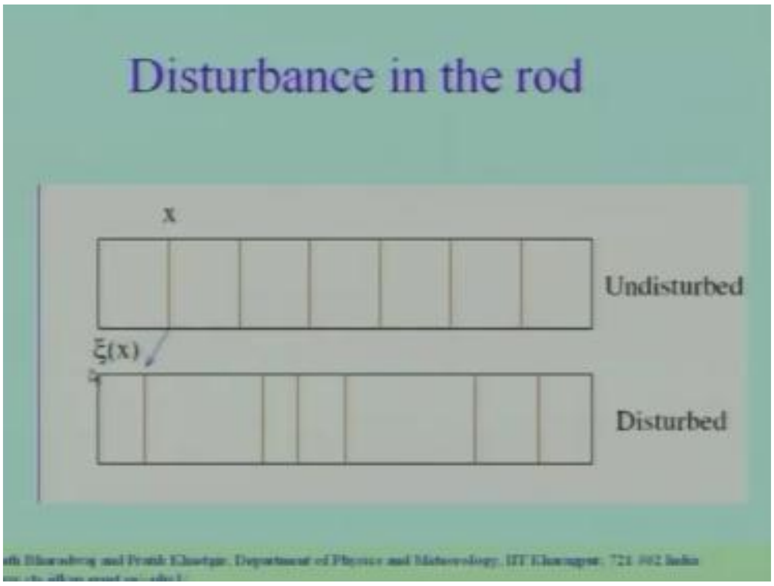


So, here I show the deformation again, we have this elastic rod in this elastic rod, we have a deformation the deformation moves around the elastic medium. So, if I have drawn lines in the elastic medium, when the medium was undisturbed the movement I have a disturbance, these lines have been moved around. So, this element over here has moved here this, element has moved here. So, this introduces compressions this, has got compressed this has got verified extra the displacement of each element. We denote by xi x and the displacement varies, it is different at different points along the rod. And the question that, we address is what governs the evolution of the xi x. And we saw that for an elastic medium the displacement xi x is governed by an equation, which is partial differential equation a second order partial differential equation.

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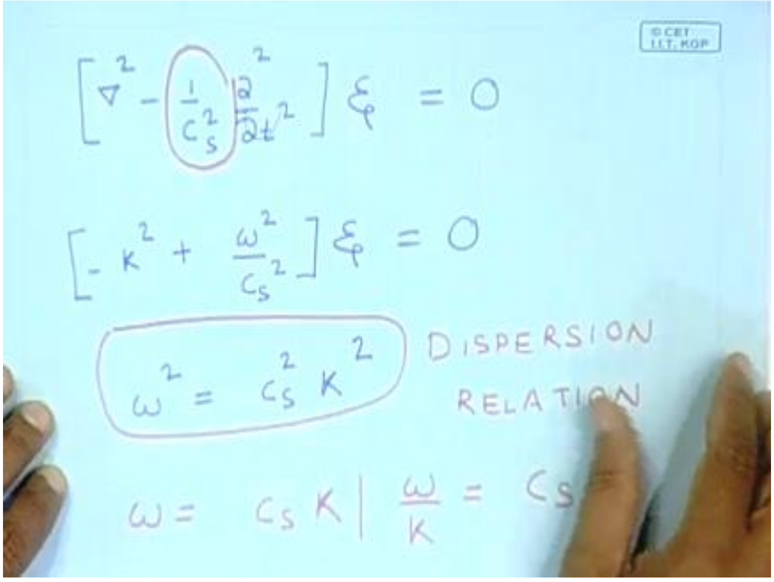


It is the wave equation and we could so, this is the wave equation, this xi the displacement follows the wave equation.

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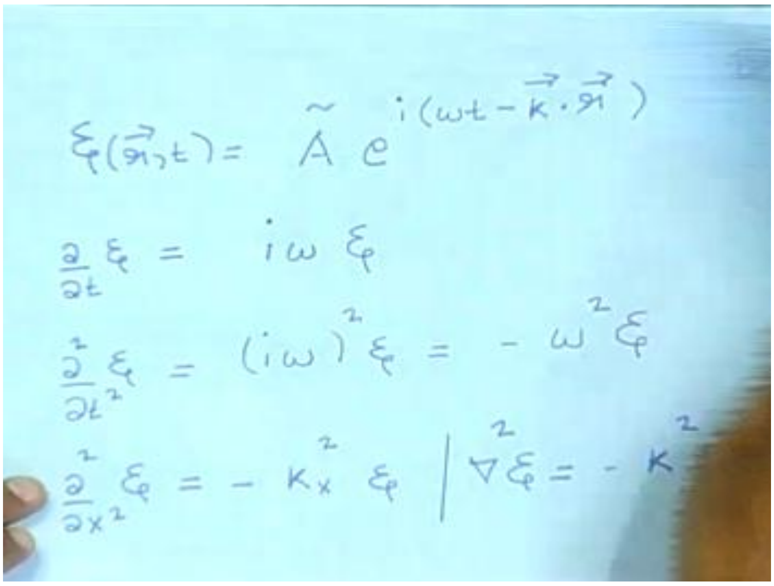


We checked that the plane wave the plane wave, which we had considered earlier, which we had discussed extensively earlier is indeed.

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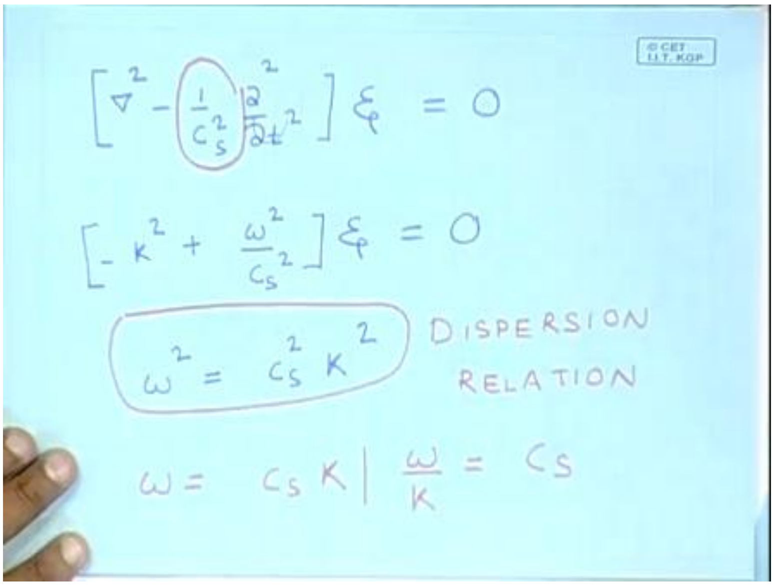


The solution of this wave equation provided the angular frequency.

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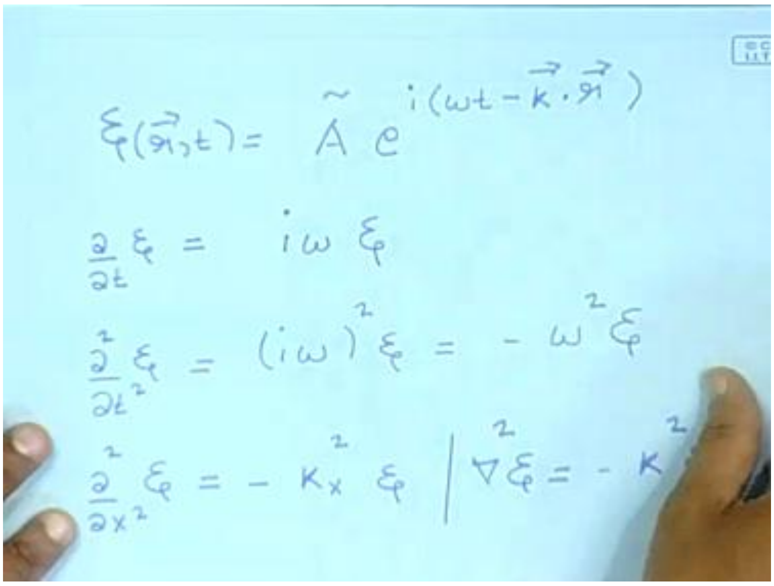

And the wave number are related in this fashion, where Cs is the constant, which appears in the wave equation,

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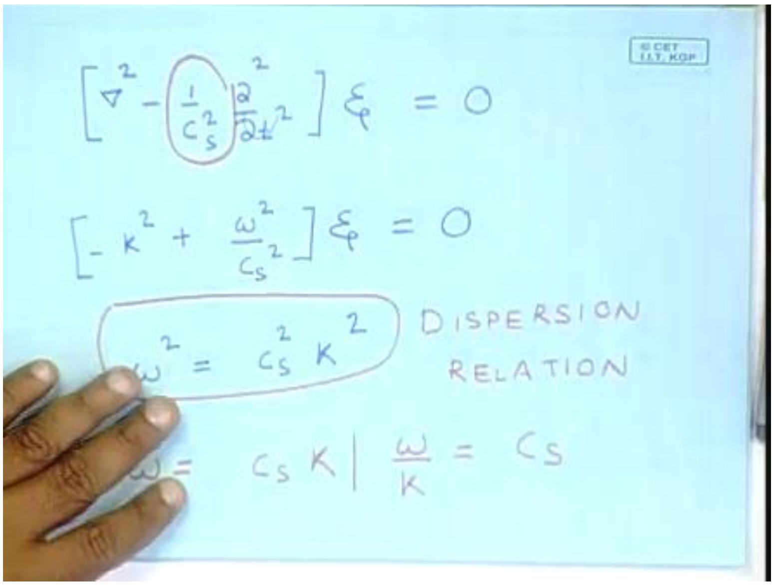


It depend on the Young's modulus and the density of medium for an elastic solid. Such relation such a relation is called a dispersion relation, it is very important, when you wish to study the wave equations and waves. So, in today lecture, we have considered particular situation of elastic of an elastic medium and we have studied in the waves in that. And what we essentially did was we derive the equation governing the evolution of waves in this elastic rod we. So, we derive the wave equation governing evolution of disturbances in this elastic rod. Now, the wave equation that we have derived here is very general it applies in a large number of situations, where we have waves.

For example, electromagnetic waves, which we studied the eclectic magnetic field satisfies exactly the same wave equation all that changes is the value of the phase velocity. So, for an elastic medium, it is the phase velocity is the speed of sound in that medium for eclectic magnetic waves; it will be the speed of light in the vacuum. So, for a large verity of waves this same wave equation holds the only difference is that the quantity governed by that the wave equation that appears in the wave equation is different for an elastic medium. The quantity that appears is the displacement the displacement here is horizon longitudinal, it is in the same direction in which the wave is propagating. If you had an elastic string and you plucked, it you would have transfers displacement.

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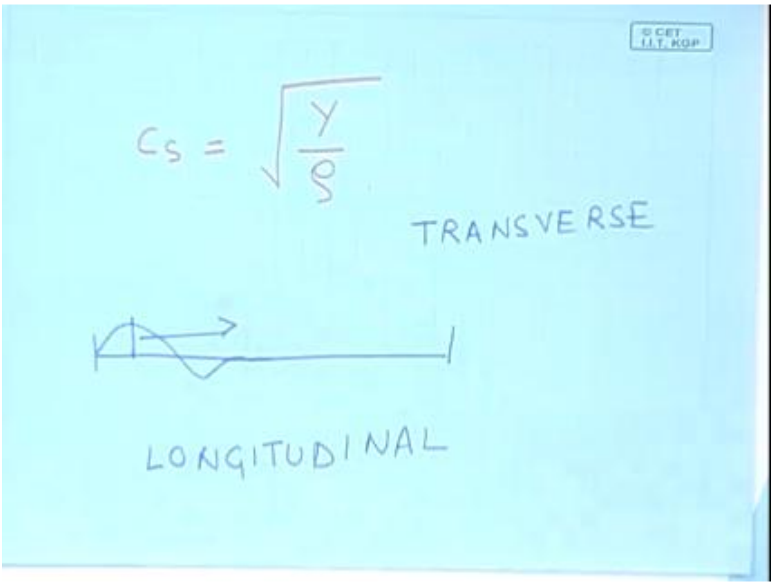


So, for example, if I had an elastic string like this and I introduce the disturbance in this string in the string the disturbance would propagate. In this case the disturbance is in the direction perpendicular to the direction in which the wave would propagate. It is a transfers wave the wave that we had considered just now the elastic wave is a longitudinal wave. We can have 2 kind of wave is transverse, where the displacement the disturbance is perpendicular to the direction in, which the wave propagates. Example the electromagnetic wave the electric field and magnetic field are both mutually perpendicular. And they are perpendicular to the direction in, which the wave goes transfer vibration in a string, which had jus mentioned.

And you could have longitudinal waves, which we have just discussed in today is lecture, where the disturbance when the displacement is parallel to the direction? In which the waves propagates. So, the wave equation is very general our derivation here is for a specific situation, but the equation that, we have derived is very general. It can be applied in the large verity of situations the 2 major changes that, you have to remember is that the variable changes. The variable could be the electric field could be the magnetic field, could be the transfers displacement for a string are it could be the longitude to the displacement for the elastic wave that, we have considered. So, it varies from situation to situation further phase velocity the constant Cs that appears over here, will be different in different context. So, let me stop today is lecture over here. In tomorrow’s lecture, we are going to discuss, how to derive solutions for the wave equation.