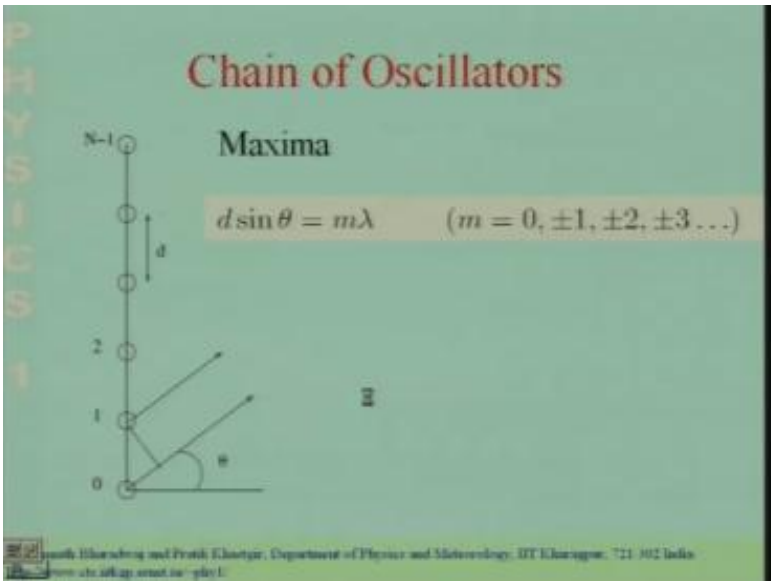
Physics I: Oscillations and Waves
Prof. S.Bharadwaj
Department of Physics and Meteorology
Indian Institute of Technology, Kharagpur

Lecture No-23
Diffraction – IV

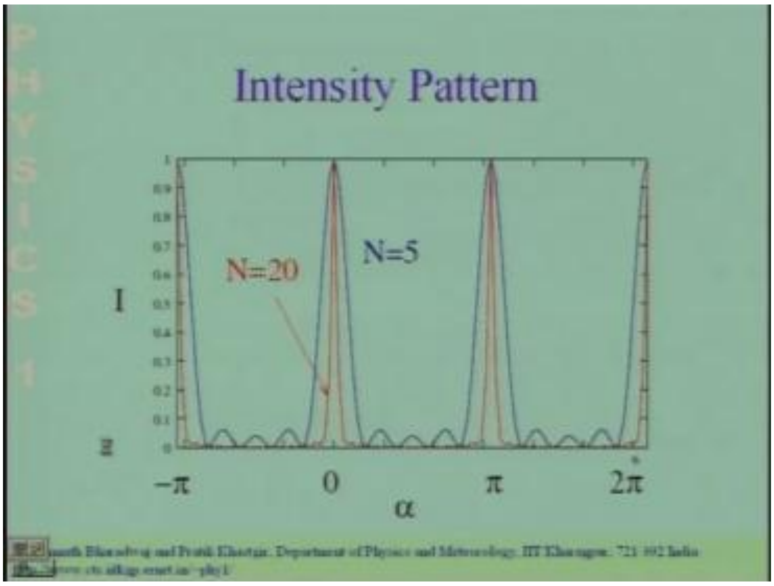
Good morning. In yesterday's lecture, we were discussing a chain of oscillators.

(Refer Slide Time: 01:07)



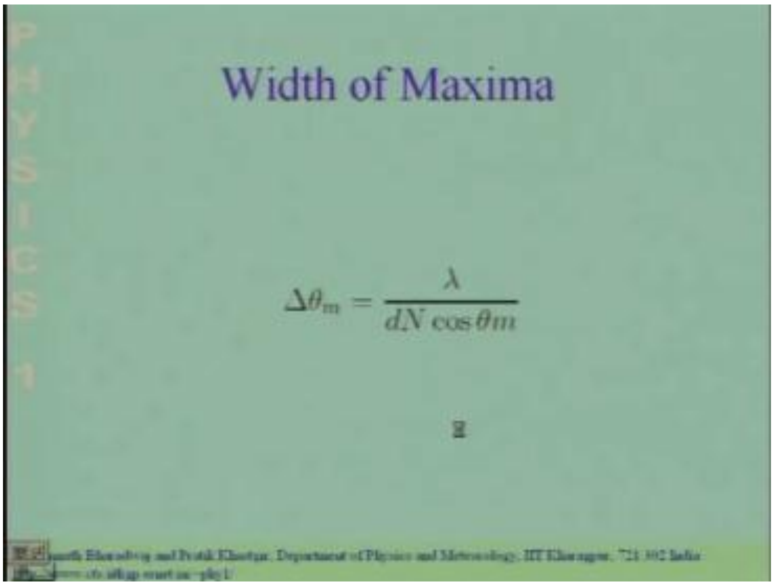
We had a chain of radiators each of which was fed an oscillating signal each radiator we refer to as an oscillator. This radiates in all directions and the total radiation received by a distant observer at an angle theta is the sum of all these radiations sum of all the waves emitted by some of the waves emitted by all of these oscillators. So, we had calculated the directional dependence the theta dependence of the radiation that comes out. And we had found that there will be maximas primary maximas whenever this condition d sin theta is equal to m lambda and could be any integer 0 plus minus 1 2 3 etcetera. Whenever this condition was satisfied there would be a maxima in the radiation that is received in other directions you would receive much less radiation the intensity pattern of the resultant radiation.

(Refer Slide Time: 02:19)



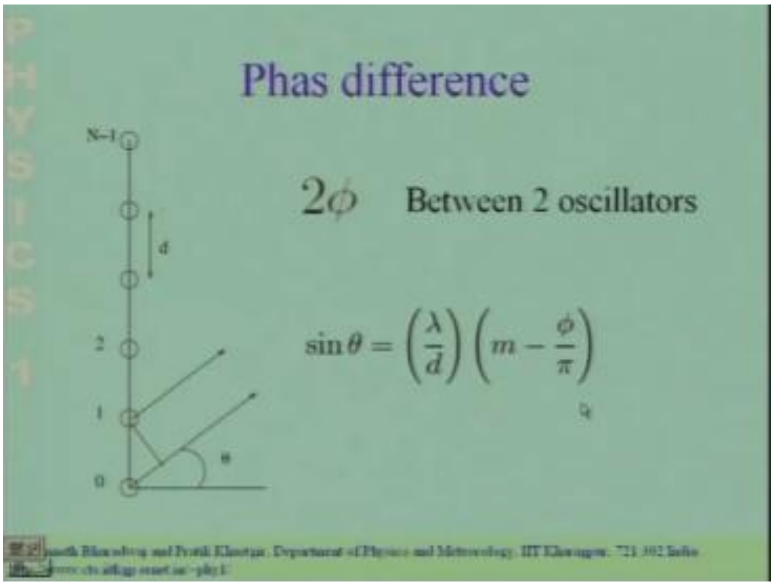
Intensity pattern looked something like this. You had these primary maximas whenever these are satisfied whenever d sin theta is equal to m lambda and this is the primary maxima corresponding to m equal to 0. This is m equal to 1 2 minus 1 etcetera and the width of these primary maximas goes down as you increase the number of oscillators. So, if you have 5 oscillators this is the width if you have 20 the width has g1 down. In between these primary maximas you have N minus 1 minimas where N is the number of oscillators. And you have N minus 2 secondary maximas which you can see here these are much smaller intensity. So, most of the radiation from those oscillators goes out in the primary maximas.

(Refer Slide Time: 03:17)



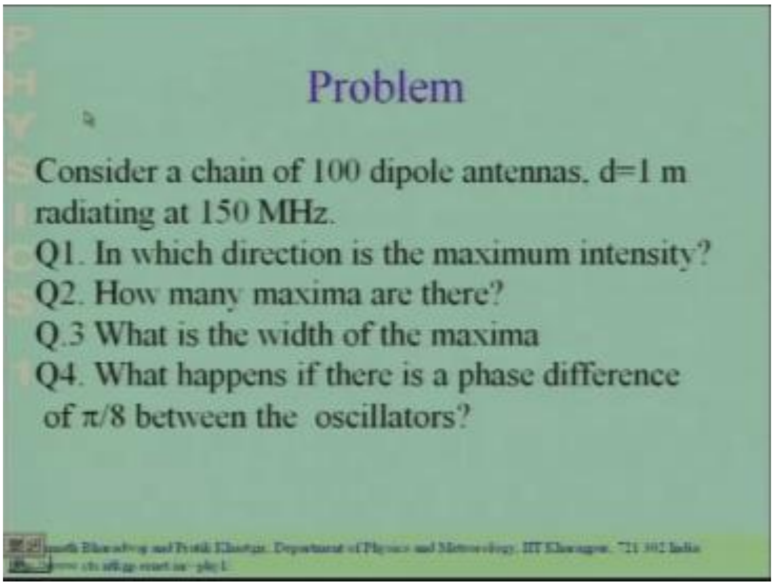
And this is the width of the primary maxima it is delta theta is equal to lambda by d, d is the spacing between the oscillators. N is the total number of oscillator and cos theta m is the angle corresponding to the maxima we are looking at.

(Refer Slide Time: 03:35)



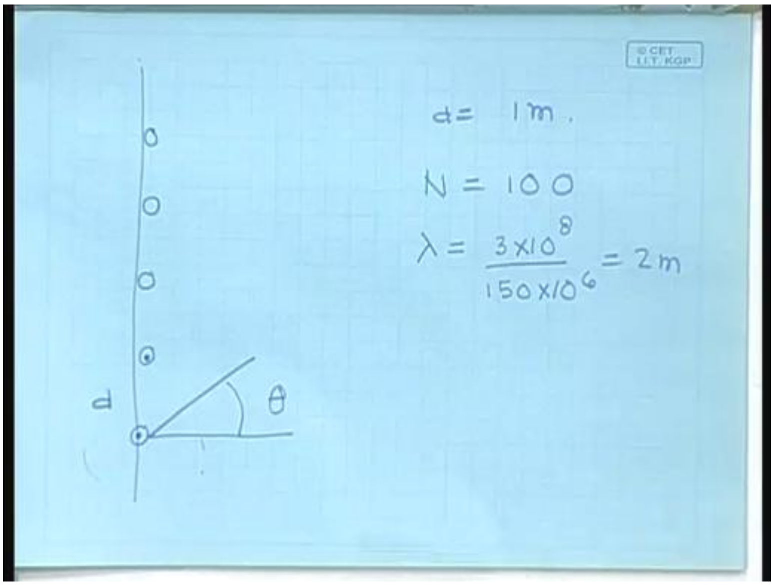
We are also considering the situation where you put a phase difference between 2 conjugative oscillators. And if the phase difference is 2 phi then you find that what happens is that you have to in addition to the phase due to the path difference. You also have to add this and what it does is it shifts the angle corresponding to the maxima and the shift in the angle the shifted angle is the in this situation the maximas satisfy this condition. There is a term in involving phi which appears over here.

(Refer Slide Time: 04:16)



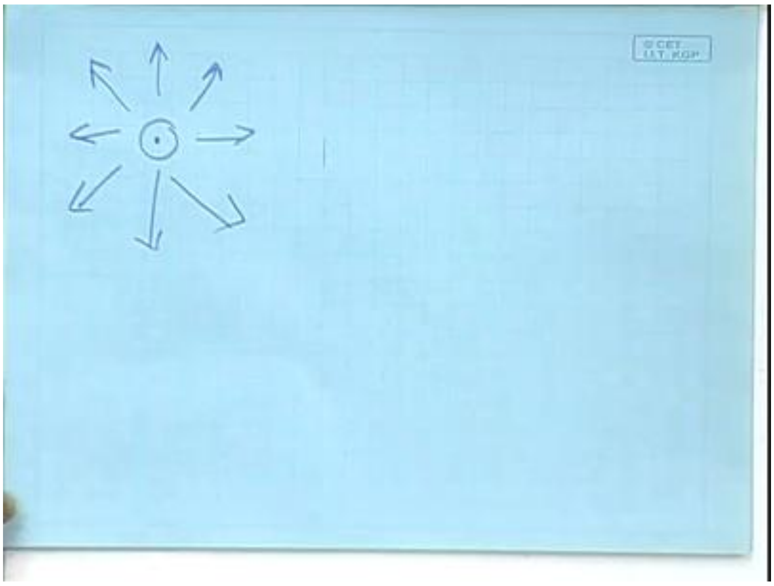
And then in the last class I had given you a problem, and I had asked you to attempt this problem. So, let us now, take up this problem and discusses its solution. So, in this particular problem we have a chain of 100 dipole antennas each at a spacing of 1 meter, emitting radiation at a frequency of 150 Mega Hertz. So, let me draw a picture over here and explain what this thing look s like to you.

(Refer Slide Time: 04:53)



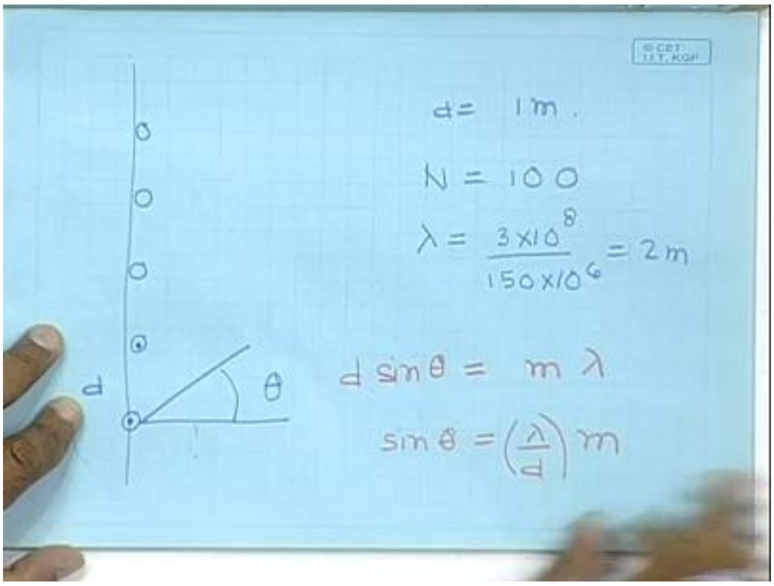
So, you have these 100 dipole antennas. Let me draw a few of them there all at a spacing of d where d equal to 1 meter you are also given the information. So, N is before that N is equal to 100 we have 100 antennas you also gave the information that this radiation is at 150 megahertz. Let us calculate the wavelength that is what we need so, the wavelength is see the speed of light 3 into 10 to the power 8 divided by 150 into 10 to the power 6 which is 2 meters. So, the wavelength is 2 meters and the angle theta is measured with respect to this direction. The dipoles are all the chain of dipoles is aligned like this and the angle theta. Now, before we take up the solution let me just remind you if I had a dipole like this that is vertically upwards like this. Then it is radiation would be such that, it would radiate maximum in the plain over here. And it would radiate equally in all directions on the plain. So, for a single dipole let me draw the picture here.

(Refer Slide Time: 06:36)



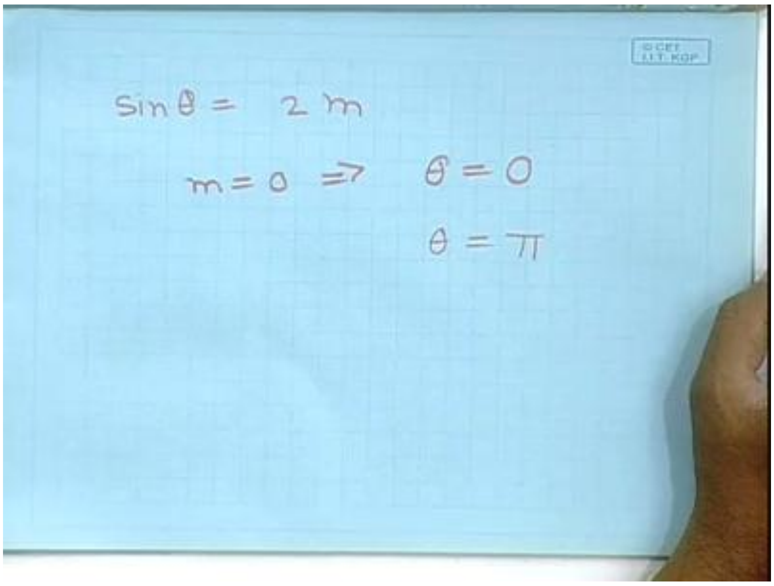
A single dipole like this pointing vertically like this emits maximum radiation in the plain of the paper and it emits equally in all directions. This is for single dipole on the plain it emits equally in all directions. It does not emitting emit anything in this direction on this direction. And it and intensity changes as sin square theta we all know this when we have studied the dipole radiation pattern. Now, in this particular problem,

(Refer Slide Time: 07:15)



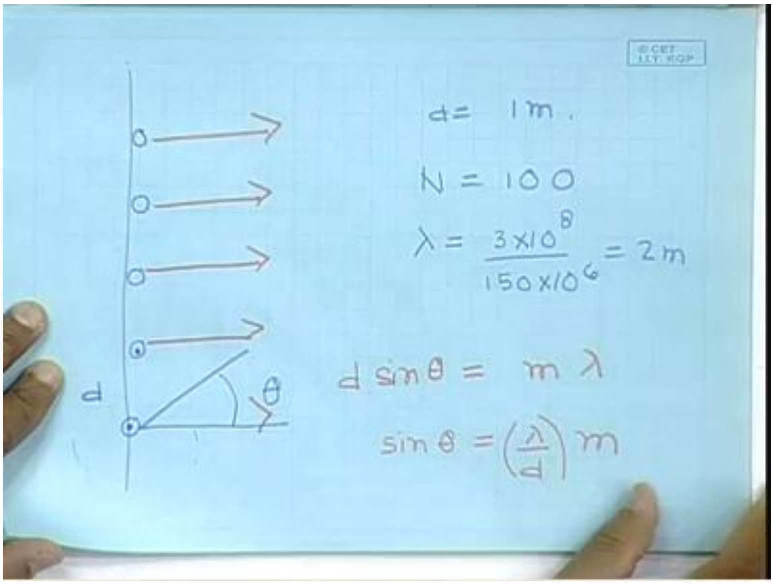
We have 100 such dipoles aligned in a chain. So, individually each dipole emits equally in all directions, but now, we have 100 of them all being fed the same signal. So, now, from the analysis that we have d1 we know that the radiation pattern is not going to be the same in all directions there is going to be maximas and minimas. The maximas are going to occur whenever d sin theta is equal to m lambda. So, sin theta is equal to lambda by d into m which in this case lambda is 2 and d is 1.

(Refer Slide Time: 08:12)



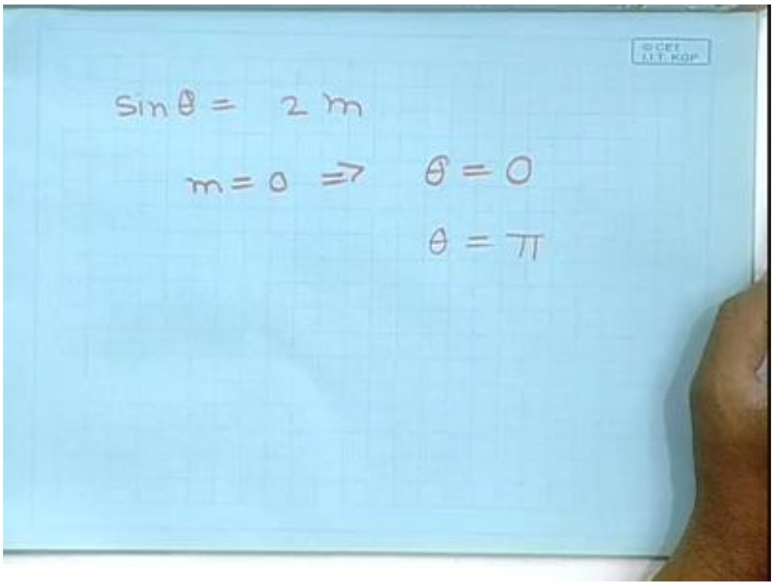
So, in this case sin theta is equal to 2 m and m can have any value 0 plus minus 1 to plus minus 2 plus minus 3 etcetera. So, the question is what are the possible solutions for theta? In this particular case you see lambda by d is this the ratio lambda by d in this particular case lambda by d is more than 1. If lambda by d is more than 1 there is only 1 possible solution which is m equal to 0 which implies theta equal to 0. So, there is only 1 solution in this case all the intensity.

(Refer Slide Time: 08:54)



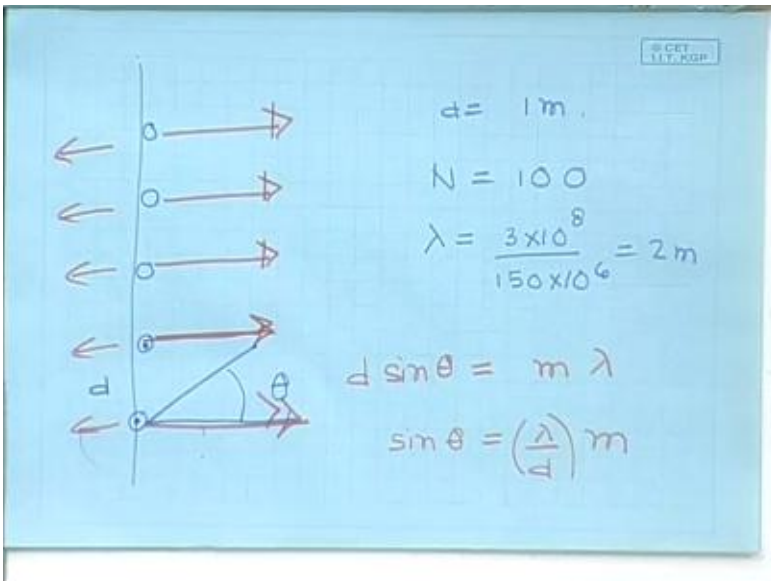
All the radiation comes out in this direction.

(Refer Slide Time: 09:05)



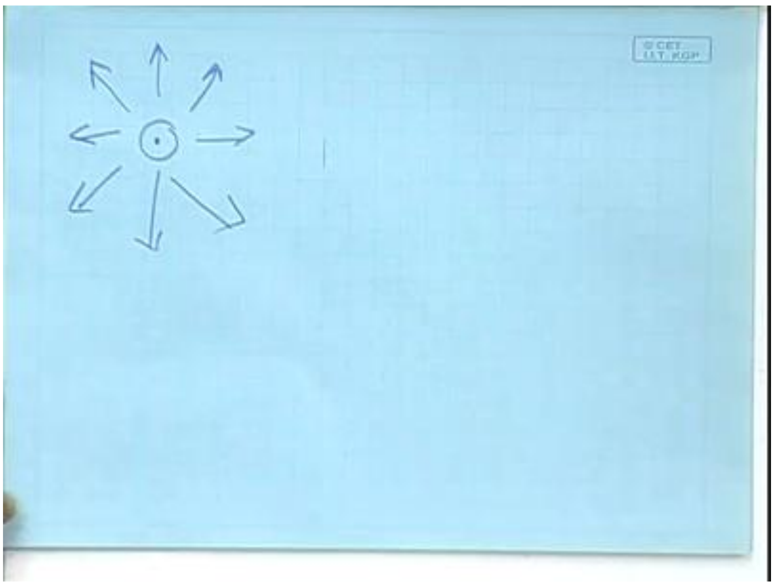
This is the direction of the maxima equivalently sin theta equal to 0 also has a solution theta equal to pi which is the backward direction.

(Refer Slide Time: 09:14)



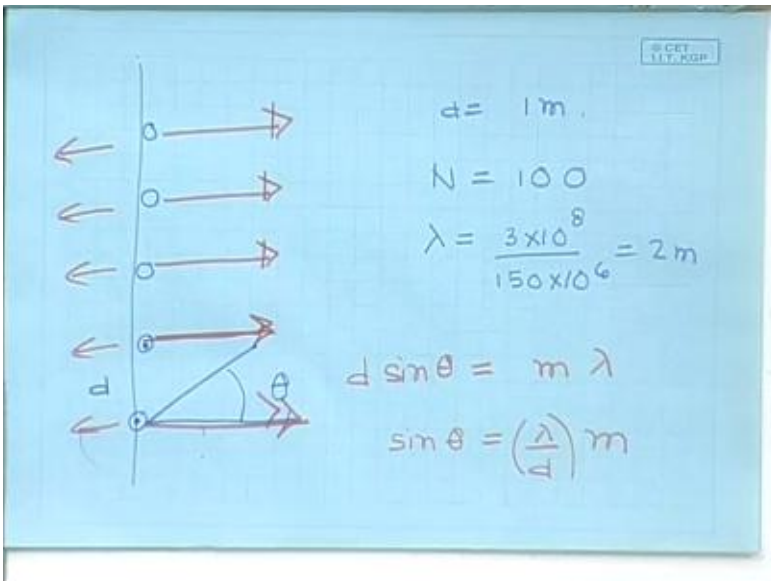
So, what we see is that when we have only 1 dipole.

(Refer Slide Time: 09:27)



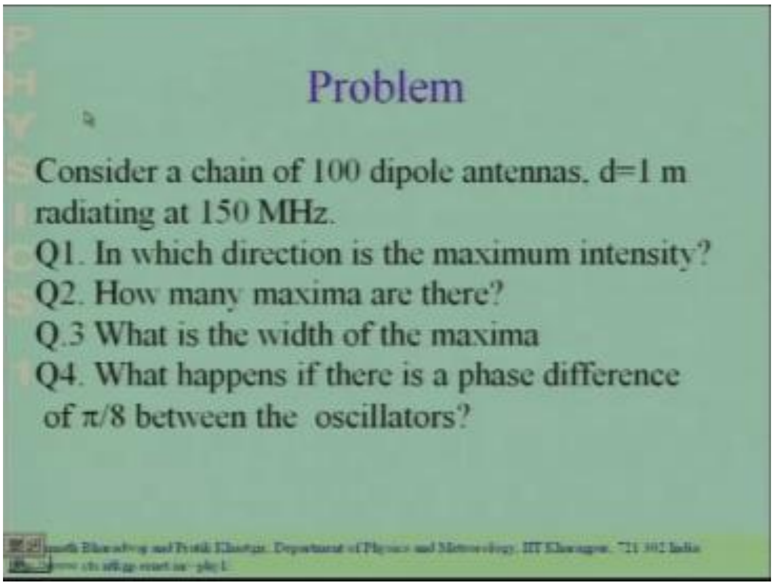
It emits radiation equally in all directions in the plain.

(Refer Slide Time: 09:32)



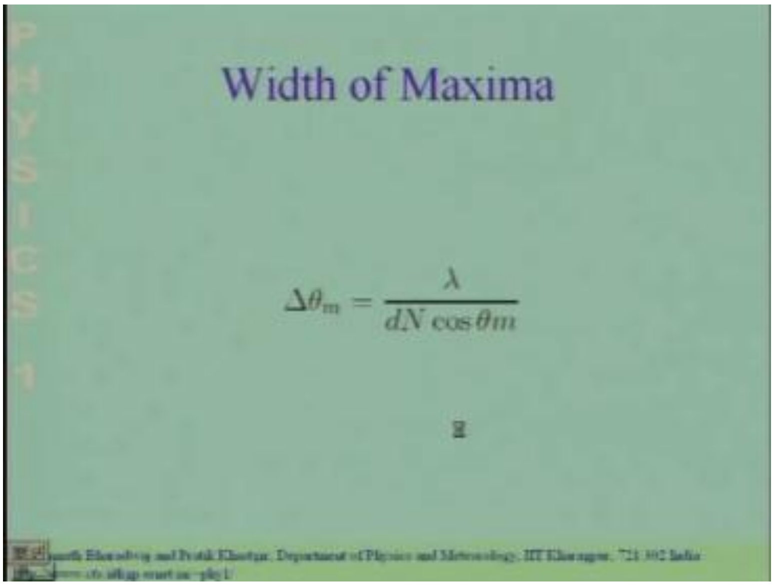
But now, when I have an array of 100 of such dipoles it the radiation pattern has maximas. In this case there are only there is only 1 maxima in the forward direction and 1 in the backward direction. So, the bulk of the radiation goes out towards the directions theta equal to 0 in the forward direction in the backward direction theta equal to pi. And so, you have been able introduce a definite directional dependence in the radiation from this whole configuration. Now, let us take up the next question the next part of the question we have already addressed 2 of the first 2 parts of the question.

(Refer Slide Time: 10:11)



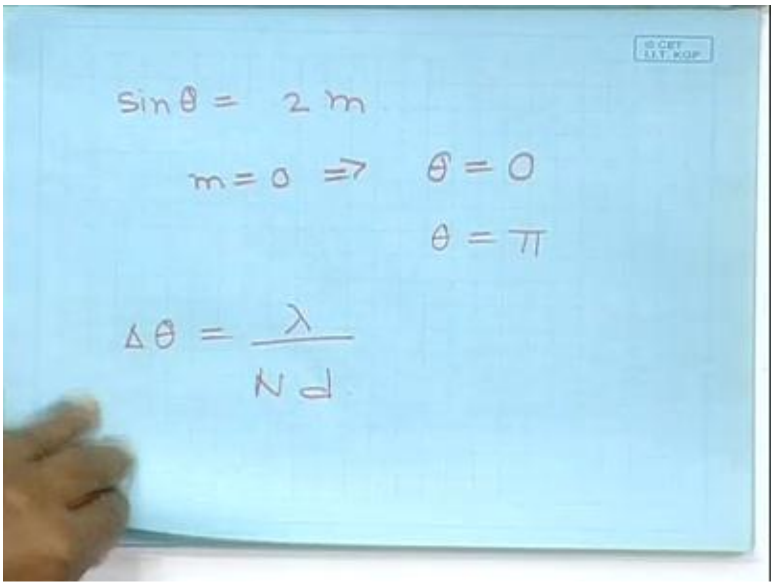
That is in which direction is the intensity maximum intensity how many maxima are there? There are only 1 and then we have the width of the maxima to consider the width of the maxima.

(Refer Slide Time: 10:24)



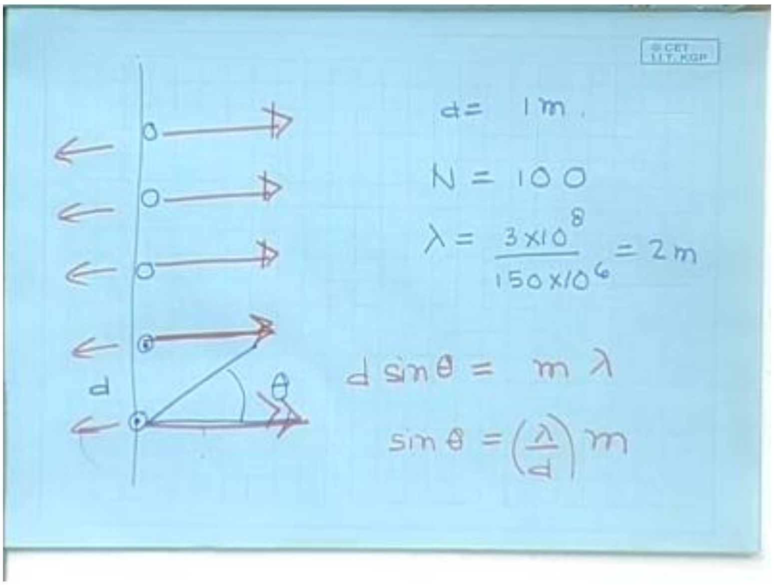
And we had found that the width delta theta corresponding to the mth order maxima is lambda by d N cos theta. So, in this particular problem we have the zeroth order maxima. So, m is equal to theta m is equal to 0.

(Refer Slide Time: 10:43)



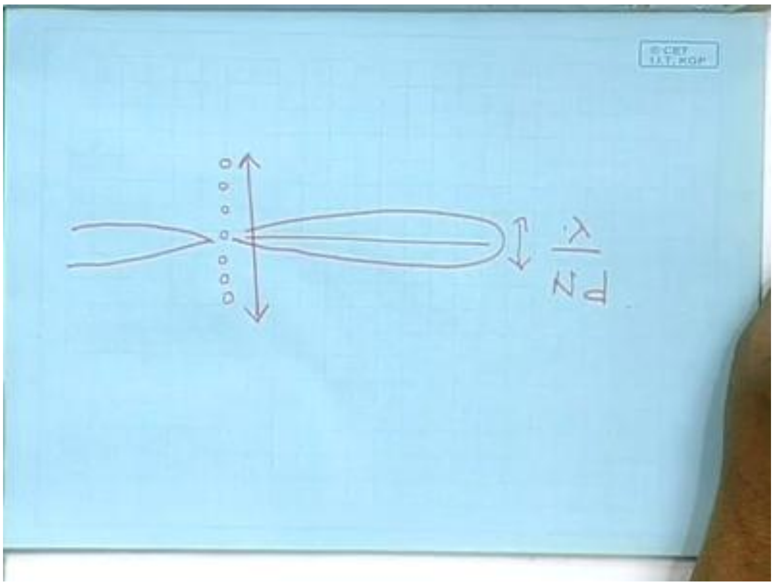
So, we have delta theta is equal to lambda by N d.

(Refer Slide Time: 10:54)



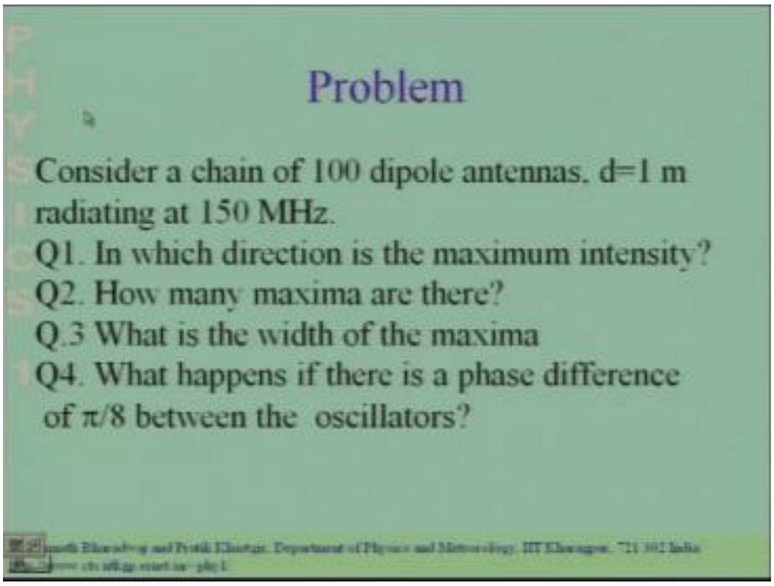
So, what we find is that the radiation pattern from this array of dipoles.

(Refer Slide Time: 11:05)



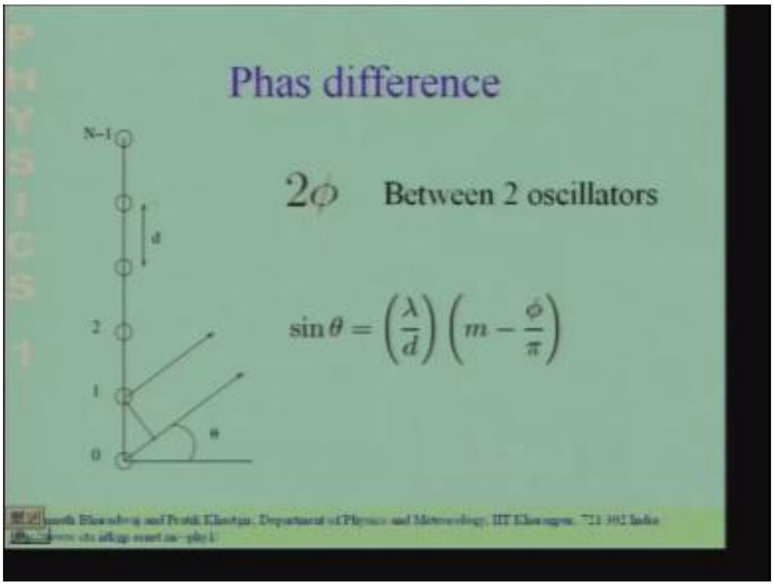
Let me draw the picture for you over here again. So, we have this array of dipoles like this. The radiation pattern from this array of dipoles is like this. So, it emits most of the radiation in either the forward direction or the backward direction and it in this particular case it is towards the angle theta is equal to 0. And it has a width in angle the width is lambda by N d where d is the spacing and N is the number of dipoles in the array in this case this N d is a actually the total length spanned by this chain of oscillators. So, now, let us go back to the problem that we were discussing. We have another part to this to be having another part to this problem that we are discussing. So, let us go back to the problem that we are discussing there the other part which remains to be.

(Refer Slide Time: 12:06)



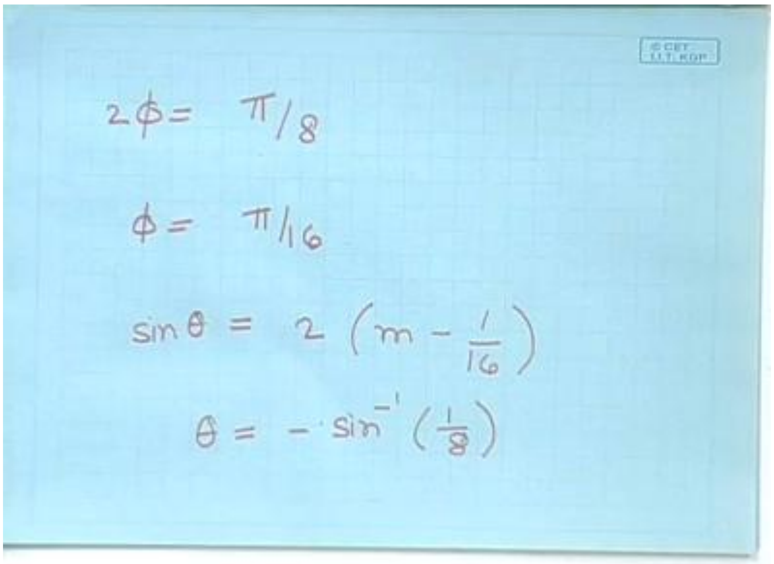
Addressed is that what happens when there is phase difference of pi by 8 between the oscillators.

(Refer Slide Time: 12:13)



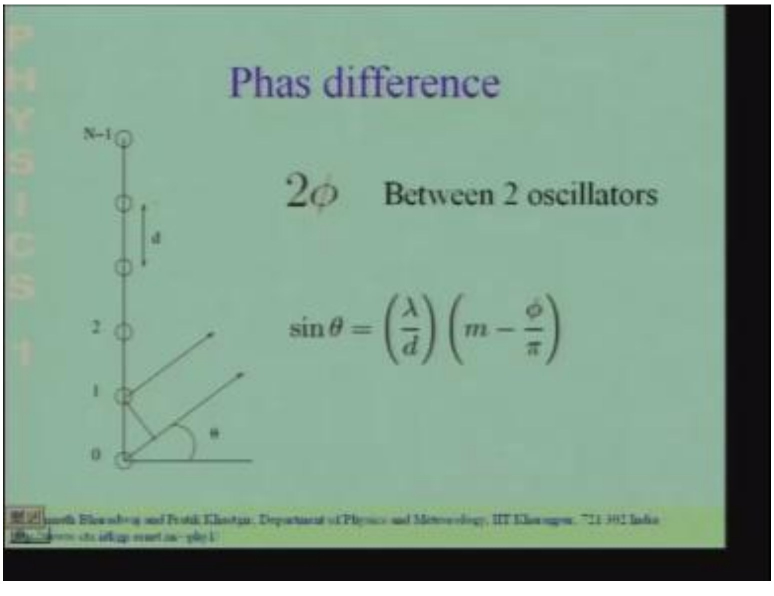
So, remember in the last lecture we had also discussed what happens when you have a phase difference of 2 phi between any 2 successive oscillators and in best this.

(Refer Slide Time: 12:29)



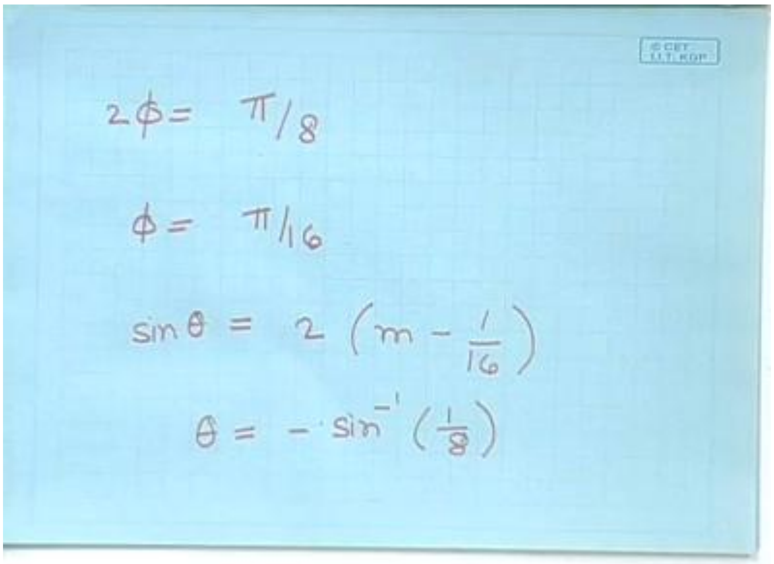
So, this particular case 2 phi is equal to pi by 8 or phi is equal to pi by 16. So, the condition for the maximas is sin theta is equal to lambda by d which is 2.

(Refer Slide Time: 12:51)



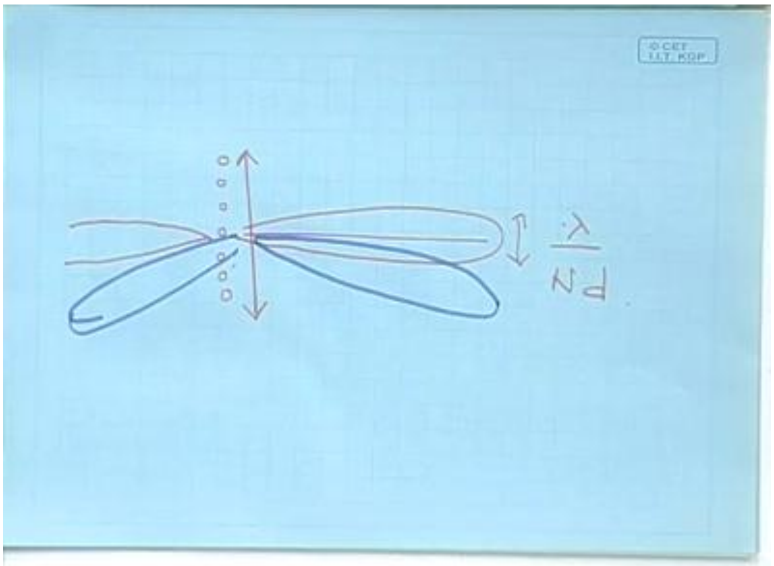
See the condition for the maxima now, gets change because of this phase difference. And the condition now, is sin theta is equal to lambda by d into m minus phi by pi phi. We have already calculated it is pi by 16. So, the maximas now get shifted and the maximas will occur.

(Refer Slide Time: 13:01)



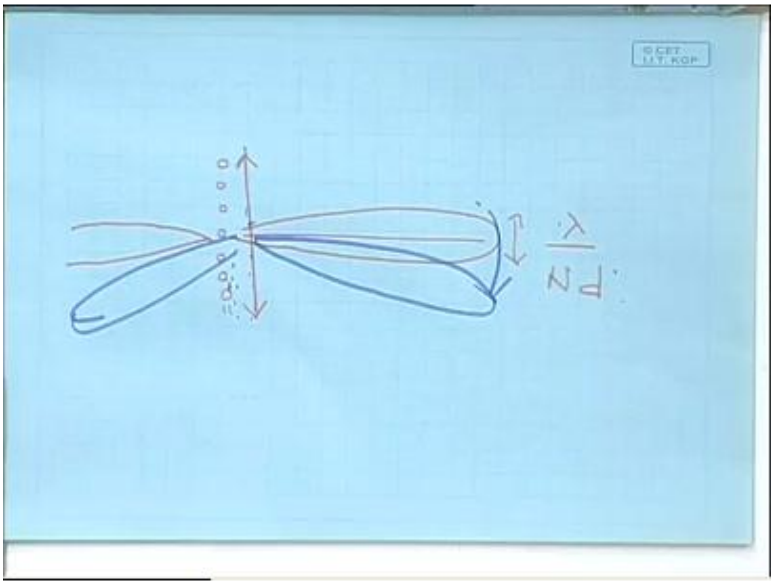
Whenever the condition this is equal to m minus 1 by 16 is satisfied. So, we can see that the maxima will now, occur at an angle which is sin inverse minus sin inverse 1 by 8. So, if you introduce the phase between the any between each of these antennas each of these dipoles.

(Refer Slide Time: 13:38)



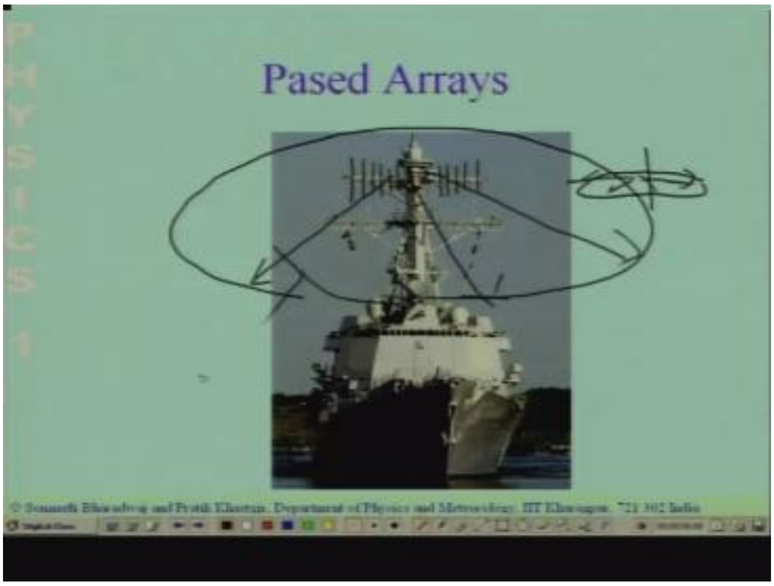
What you do is, you can shift the direction in which the maxima occur in this case it will shifted downwards like this. So, by introducing different phases you can shift the direction in which the maximas occur. So, let us let me just recapitulate for you the implications of what we have just discussed. We have an array of N dipoles each at a spacing d in the situation where I had a single dipole. It would emit radiation equally in all directions in the plain perpendicular to the dipole in the situation where we have 100 of them. There is considerable amount of directivity in the radiation that comes out directionality in the radiation comes out. And in this particular case when the all the dipoles are oscillating at the same phase the bulk of the radiation comes out in the direction perpendicular to this chain.

(Refer Slide Time: 14:51)



And it is spread over a small angle delta theta which is lambda divided by N d where N d is the total N is the number of oscillators in the chain and d is a distance between them. So, if the total chain of oscillators span this distance it is essentially lambda divided by this distance the largest separation between the oscillators. So, all the radiation gets focused gets concentrated into this small angle. And if I introduce a phase difference between 2 between the dipoles then I can make the direction of the maximum move around. So, you see this is what is called a phase array. And this has got tremendous amount of applications.

(Refer Slide Time: 15:40)

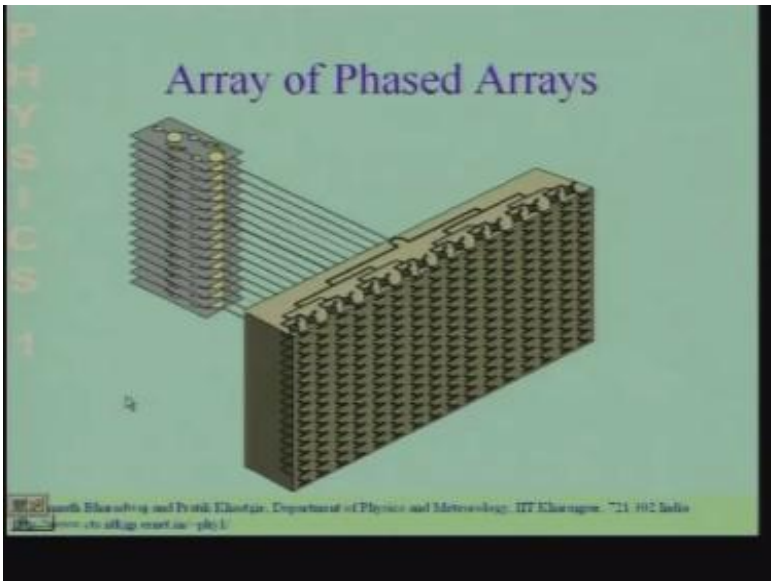


So, let me show you a picture first of a ship. So, the picture here shows a ship. And notice that on the ship you have these rods these vertical rods I do not know if you can see them. But there are these vertical rods over here. So, in case you cannot in case you cannot see them let me reinforce them for you over here on this picture. So, you have these vertical rods like this and these rods are essentially dipoles. So, each rod over there is a dipole. And if you had only a single dipole it would emit radiation equally in all directions in the plain perpendicular to the dipole. So, it would radiate emission equally in all directions like this, but now, we have an array of dipoles.

So, if the same signal is send to all of these dipoles the radiation will be maximum in the direction perpendicular to this chains. So, it will come out forward so, the maximum radiation will come out like this. And if I give a phase difference between all the dipoles then we can change the direction in which the maximum radiation comes out. So, this can be used has a radar and you can send out the signal. And you can receive the signal that gets reflected back if from any source from any distant object and this can be used as a radar. And you can make the radar sweep around the full whole plain by a providing a appropriate phase difference between the dipoles and then slowly changing the phase difference.

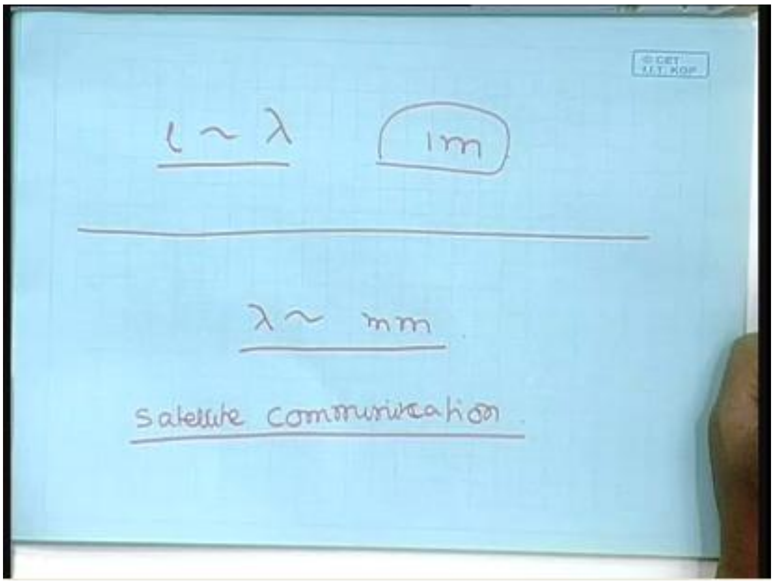
So, if we if you give a phase difference between the dipoles then you can change the direction in which you have the maxima. And if you change the phase difference between the dipoles you can make the direction of the maxima slowly sweep. The whole sky like this and this can act like radar. So, this is a phased array has got applications as a radar as you can see on the ship here. So, this is a battle ship. And on this battle ship we the phased array acts like a like a radar system. So, you can there are various other applications. So, let me show you another application of the phased array.

(Refer Slide Time: 18:30)



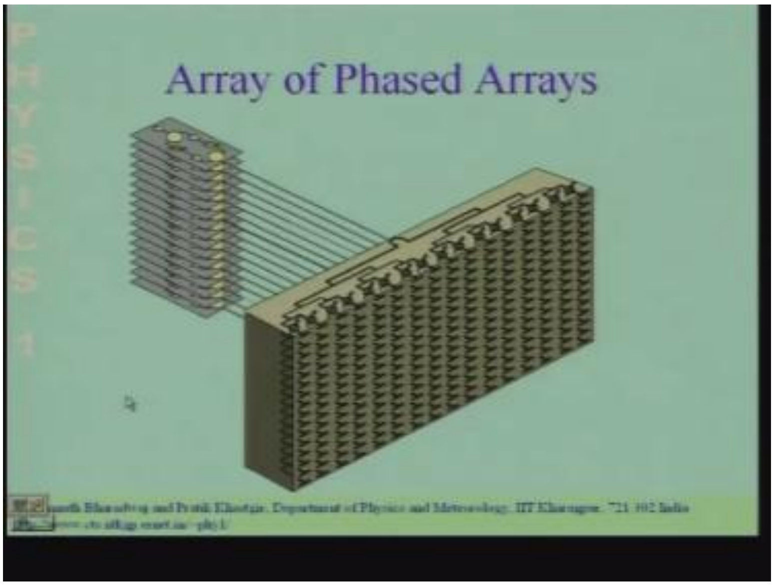
So, the next application shown over here is an array of phased arrays. Now, I had we had already seen in earlier lectures that if you want a dipole to be affective in radiate sending out radiation. The length of the dipole should be comparable to the wavelength at which you are radiating.

(Refer Slide Time: 18:48)



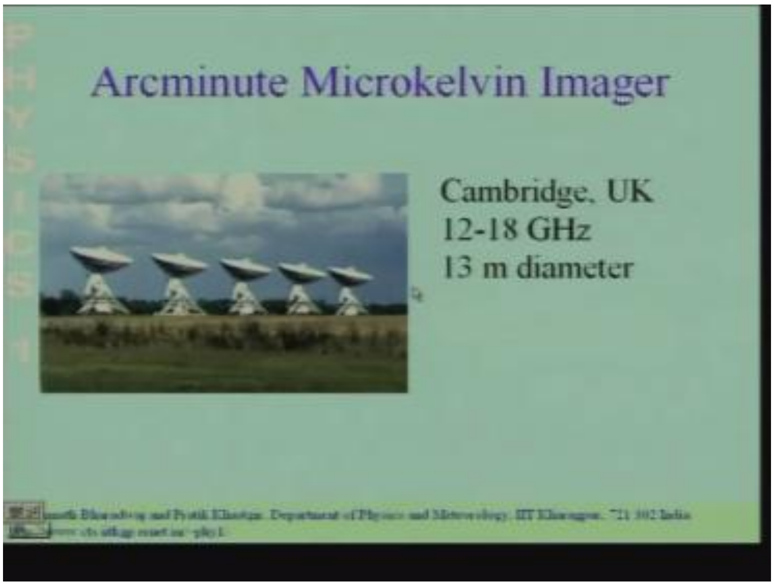
So, if you are working at something like 1 meter wavelength then the dipole should be also of length 1 meter which is what the situation that I had shown you just a little bit little while ago the radar on the ship. That was possibly working in somewhere like a wavelength may be few centimeters and the length of the dipoles were also of the order of meter may be of few centimeter or a meter. Now, if you go to much smaller wavelength let say millimeter wavelengths. So, lambda is of the order of millimeter not possibly even less. This wave this has got several satellite communication and various of the communication applications. So, orders of the wavelengths of the order of from centimeter to millimeter then the radiating elements will also be smaller. And this picture shows you an array of phased arrays.

(Refer Slide Time: 20:00)



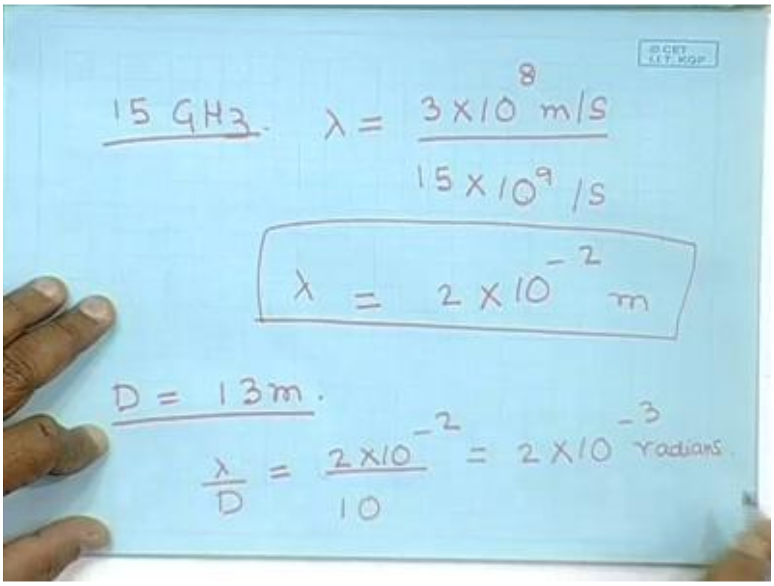
So, this shows you an array of phased arrays. And this is on a much smaller length scale has you can see. Now, each of these so, this is one phased array using a phased array you can make you can ensured that the radiation gets directed in a particular direction. And you can change the direction by introducing different phases between these radiating elements and using an array of such periodic phased array. So, you have one phased array over here you have another phased array over here. So, you have an array of phased arrays using an array of phased arrays you can actually move around the direction of the radiation in both this direction and this direction. So, both horizontally and vertically this gives you 2 degrees of freedom in which you can turn the direction in which the majority of the radiation of the radiation comes out. Let me now show you another application of the same idea of a phased array. So, the picture which I am going to show you now is an application in radio astronomy we had discussed the cosmic microwave background radiation much earlier. And there is a considerable amount of an effort going on to map the cosmic microwave background radiation at high angular resolution.

(Refer Slide Time: 21:40)



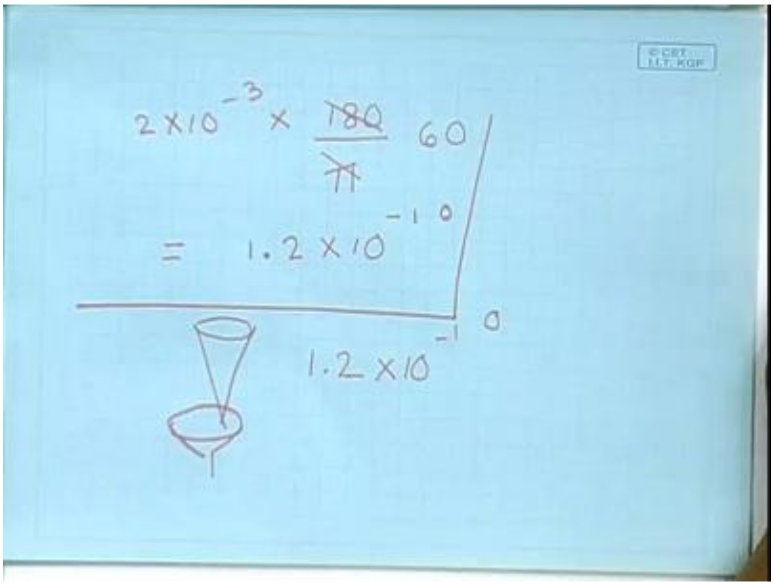
So, this picture shows you this is a picture of the Arcminute Microkelvin imager. This is at Cambridge UK it works in the frequency range 1 to 18 Gigahertz. And it has got antennas which you can see over here there is a linear chain of antennas. A part of the array of the Arcminute Microkelvin imager is a linear chain of an antenna which you can see over here each of these antennas is 13 meter in diameter. So, the question is what is achieved by having a linear chain of antennas over here? So, let me just spend a little time discussing this point each antenna let us say that we are working at 15 Gigahertz.

(Refer Slide Time: 22:42)



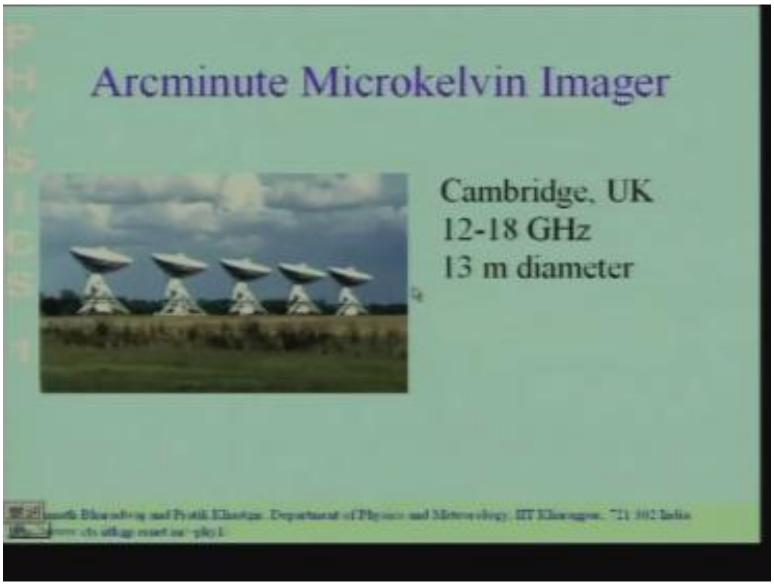
So, at 15 gigahertz let us first estimate the wavelength. So, the wave corresponding to 15 gigahertz will be 3 into 10 to the power 8. This is meters per second divided by 15 into 10 to the power gigahertz will be 10 to the power 9 per second. So, this gives us lambda so, 10 to the power 9 over here 10 to the power 8 over here which gives us a factor of minus 10 to the power minus 1 already. And then if i divide 3 by 15 I get 2 into 10 to power minus 2 meters this is the wavelength. It is 2 centimeters. And in this particular situation we have antennas of diameter D which is 13 meters. And we have studied few lectures ago that, because of diffraction the diffraction is going to set the angular resolution of these antennas. So, the angular resolution of these antennas is of the order of lambda by D. Lambda here is 2 into 10 to the power minus 2 D is 13. Let me just take into be for simplicity to get an order of magnitude. So, lambda by D the angular resolution is 2 into 10 to the power minus 3 radians and to convert this into degrees.

(Refer Slide Time: 24:41)



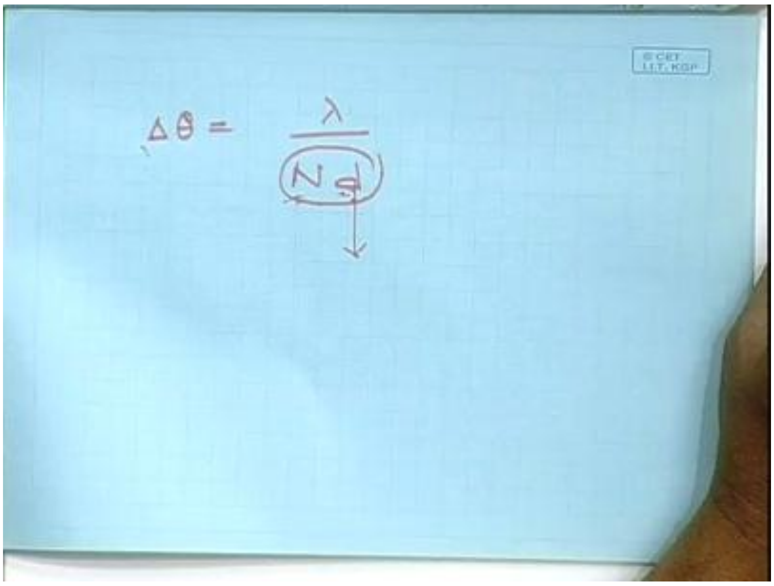
I have to multiply by 2 into 10 to the power minus 3 into 180 divided by pi which I can take of the order of 60. So, this gives me 1.2 into 10 to the power minus 1 minus 1 degrees. So, that is the kind of a resolution angular resolution of this point 1 2 degrees is the kind of angular resolution of a lambda of the single antenna. So, each of these antenna if I had 1 antenna like this, it would a receive radiation it would emit radiation or receive radiation from an angle of the order of 1.2 into 10 to the power minus 1 degrees.

(Refer Slide Time: 26:12)



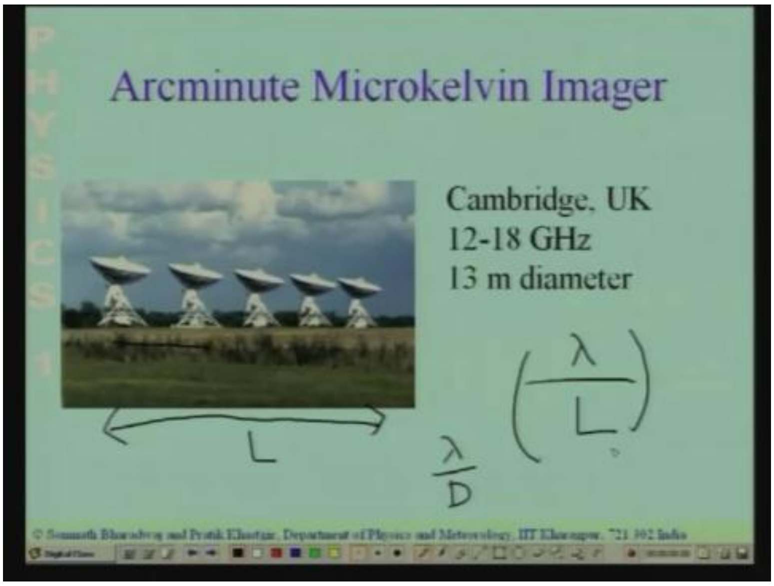
Now, what is the advantage if I combine a set of antennas like this in the form of a phased array? We have seen that if I combine these radiating elements has a phased array and I do not give any phase difference between any of the between 2 of the conjugative elements. Then it is going to emit the maximum radiation in the forward direction assuming that there are no other maximas. If there other maximas there will be other direction also in which you will have primary maximas other primary maximas. But, let us focus on the primary maxima which in forward direction m equal to 0. So, it is going to emit a bulk of its radiation in the forward direction or it is going equally receive bulk of the radiation from the forward direction and from a width.

(Refer Slide Time: 27:03)



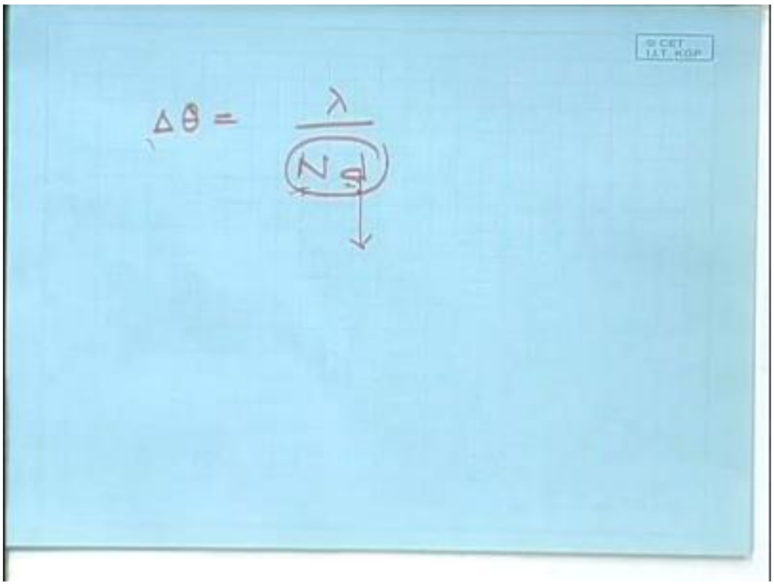
Delta theta which we have seen is going to be lambda by N d where N d is the spacing is the total d is the spacing between any 2 of these oscillators that N is the total number of oscillators.

(Refer Slide Time: 27:27)



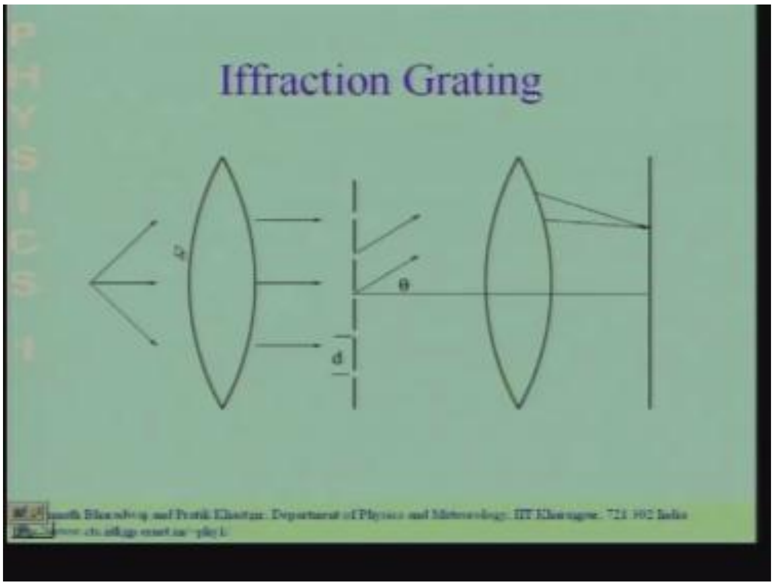
So, with reference to this particular picture let me tell you what is going to happen is that one of these antennas. So, each of these antennas we have seen is going to emit if I had only one antenna it would emit radiation or receive it would have a resolution which is 12 degrees. But now, when I have if when I use this has an array it is resolution is going to be decided by this total length over here. And the resolution of this is now, going to be of the order of lambda into L lambda by L instead of lambda by D and this number is going to be smaller. So, you have you will be able to achieve higher angular resolution. A smaller angular resolution when you use this chain of antennas has an array instead of using them as individual antennas that is the crucial point. And the angular resolution is going to be decided as you can see the angular resolution the is going to be lambda by N d.

(Refer Slide Time: 28:28)



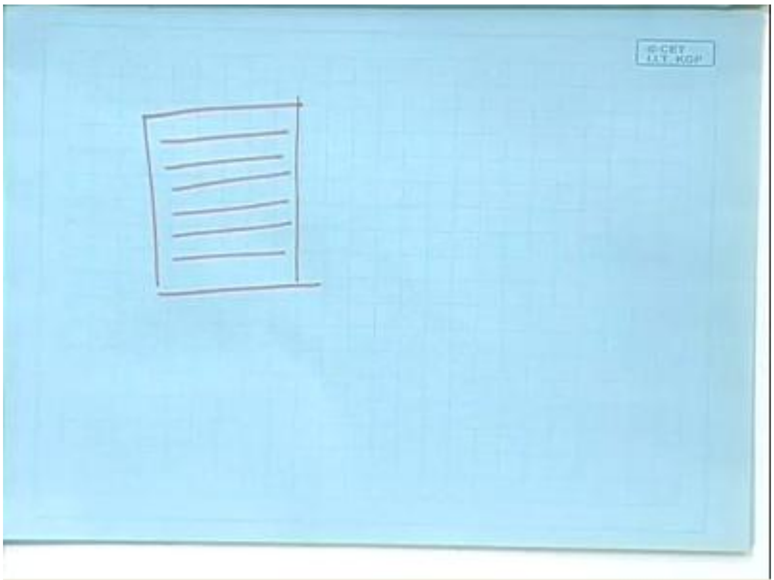
That is the width of the primary maxima so, that is going to be the angular resolution. It is going to be decided by the separation between the 2 for these elements in this chain of oscillators. Let me now discuss another application of this chain of oscillators which we had discussed in the last class. And this particular application is what is called a diffraction grating.

(Refer Slide Time: 29:19)



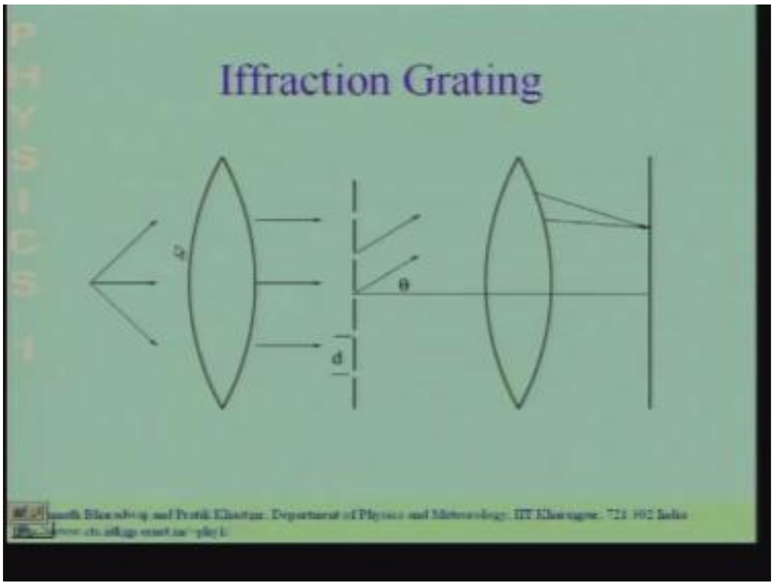
A diffraction grating is essentially an opaque screen so; we have a opaque screen over here. In this opaque screen you have a periodic arrangement of slits which you can see over here. And the spacing between any 2 slits is d any 2 successive slits is d. If I draw a picture for you here it will be clear.

(Refer Slide Time: 29:50)



So, if you look at it phase on from the front it looks like this you have this opaque screen on which you have a periodic arrangement of slits. So, each of these is a slit and you have a plain wave.

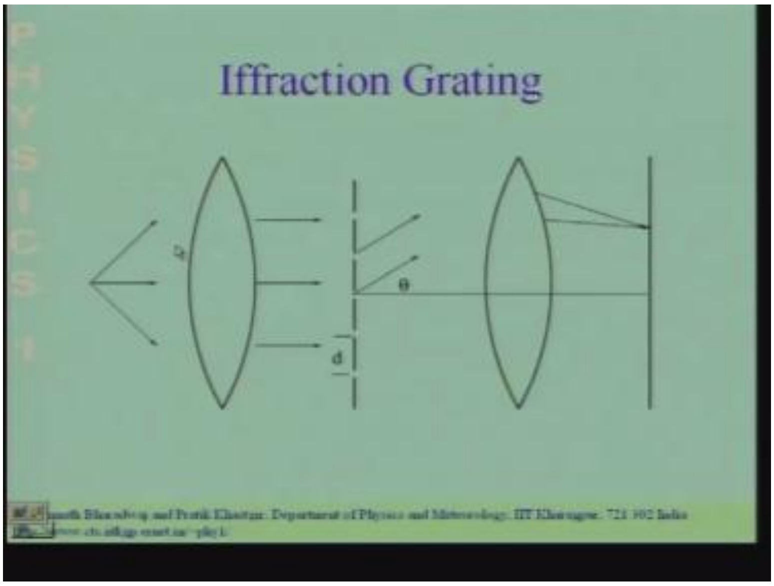
(Refer Slide Time: 30:07)



Incident on this diffraction grating; the so, when the plain wave is incident on this, each slit acts like a source for a secondary wave. So, each of them now emits a secondary wave and this is effectively a chain of oscillators, because you have the same radiation coming out from each of the slit. Before going on to discusses what the intensity pattern will look like? Let me discuss what how we can construct a diffraction grating? You might have seen a diffraction grating in your physics laboratory the typically in if you are working in the optical range you will find that you will what the diffraction grating will be something of this size. And if you look at it you will notice that there are these dot lines and through which the light cannot pass and then there are transparent lines through which the light can pass it will you look like this. So, one way you can realize these this kind of arrangement is, If you draw up black and white lines you paint black and white lines on a wall.

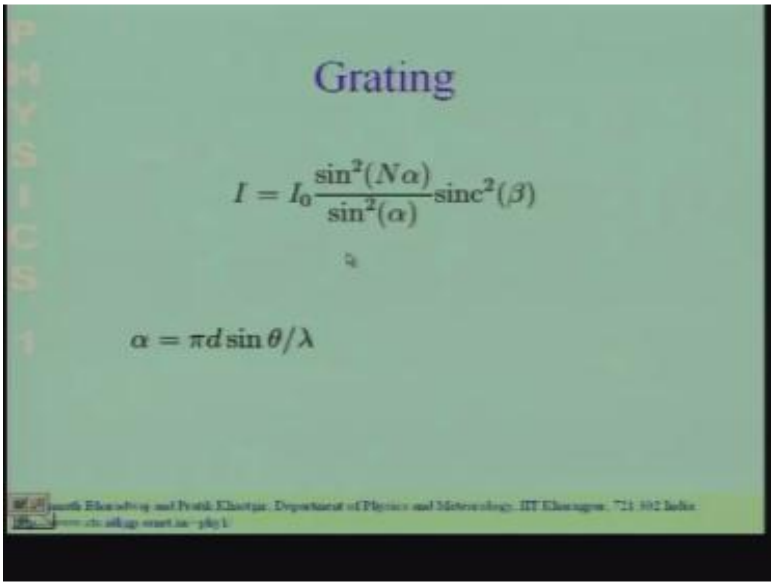
So, take a wall a big wall and paint black and wall white lines on this wall. It is on a large scale big wall paint black and white lines. Now, take a picture of this using a camera go to a distance and take a picture of this using a camera. And take the negative you develop the film you will get a negative in that negative you will have a transparent region. The negative will be the inverse of the picture that you have taken. So, corresponding to the black line you will get a transparent view you will get a transparent region and corresponding to the bright line you get the dark region. And you will have these black and white rulings on your negative which you can use as a diffraction grating. This going to be essentially be a very scaled on version of the black and white lines which you had drawn on the wall. There are many other possible ways in which you can construct a diffraction grating the diffraction resource.

(Refer Slide Time: 32:22)



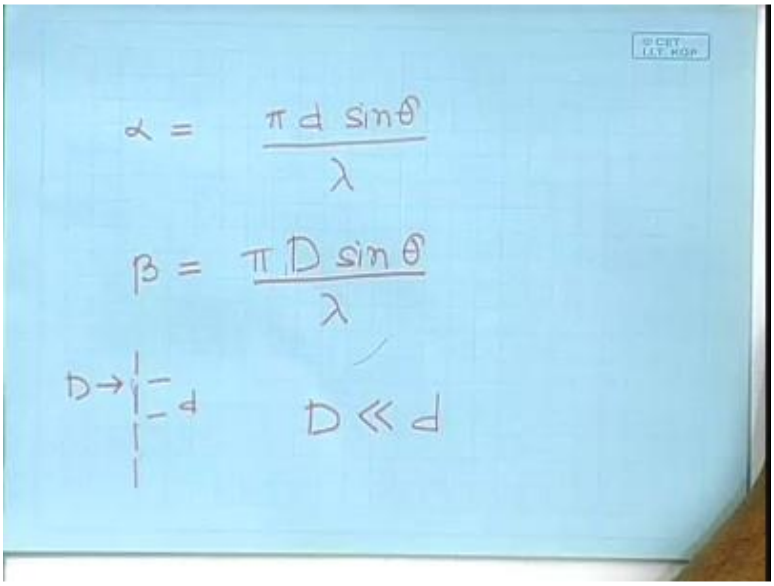
The question now, is what is the intensity pattern that the diffraction grating is going to produce on a screen over here? The intensity patterns, that the diffraction gating, grating is going to produce on the on the screen over here. Let me write down the expression for you. The intensity pattern that the diffraction grating is going to produce is given over here.

(Refer Slide Time: 32:58)



It is I equal to I naught sin square N alpha divided by sin square N alpha into sorry sin square N alpha divided by sin square alpha into sin square beta. So, you see the intensity pattern produced by the diffraction grating is a product of 3 terms. The first term is the overall intensity, the second term is the intensity pattern produced by chain of oscillators here alpha is phi d sin theta by lambda; d is the spacing between each of the oscillator in the chain of oscillators.

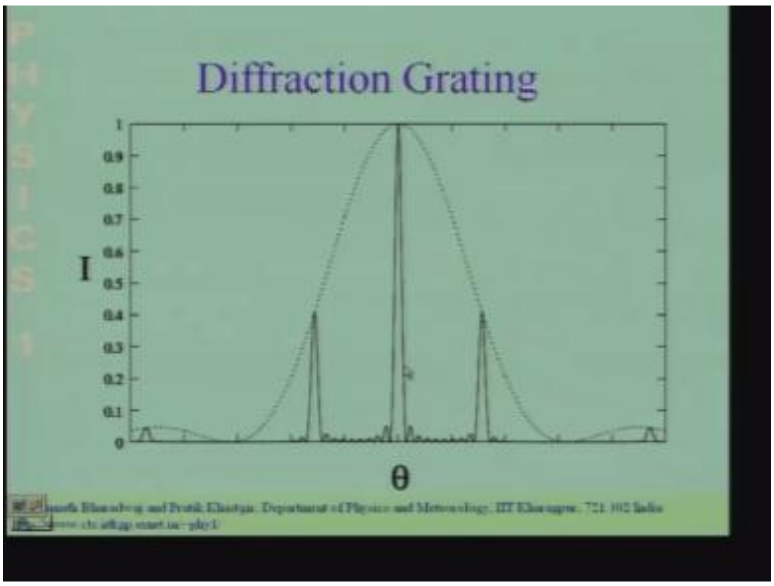
(Refer Slide Time: 33:33)



And we have another term which is sin c square beta which again you must have encountered in this course little few lectures earlier beta is phi D sin theta by lambda. So, this d is the separation between the slits so, grating that we have is a sequence of slits which are equally placed the separation between any 2 slits is d and the width of each slit itself is capital D. And the intensity pattern which this diffraction grating produces is a product of the intensity produce by a chain of N oscillators each at a separates small d multiplied by the intensity pattern produced by a signal slit of width capital D. So, the next question is what does the intensity pattern look like? So, we will consider a situation where capital D the slit width so, the width of each slit is much smaller than the spacing between the slits.

So, we will consider this situation which is the situation that occurs usually in a diffraction grating. So, we will consider a situation where each slit width is smaller than the spacing between the 2 consecutive slits. Now, in this situation let us ask the question if I vary theta which of this turn is going to change faster. And it is quite clear that since this slit width is much smaller than this spacing between the slits. This term alpha is going to change much faster and this term beta is going to change much slower. So, the intensity pattern due to the chain of oscillators is going to change much faster the intensity pattern due to the diffraction pattern of single slit is going to change much slower. And this is what determines the overall final intensity pattern produced by the diffraction grating.

(Refer Slide Time: 36:35)

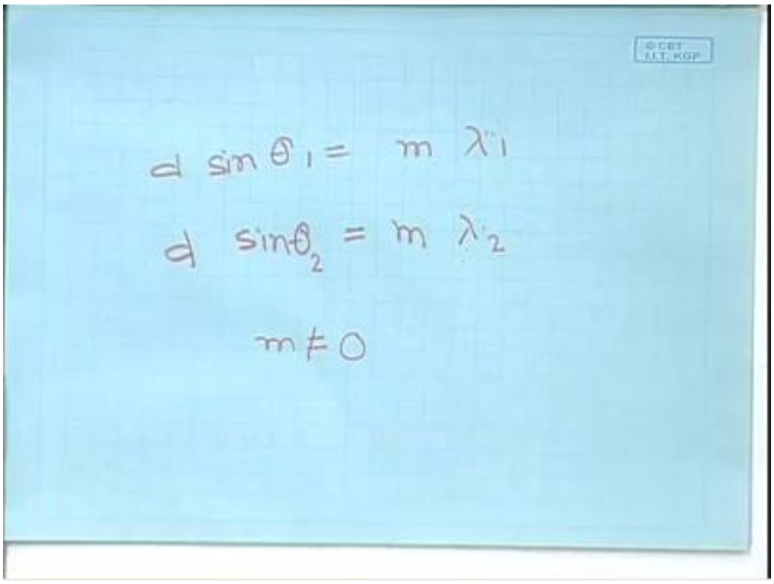


This is what is shown over here the intensity pattern produce by the chain of oscillators this changes much faster as we have just seen. This decides the intensity pattern by the chain of oscillators. This d the spacing is much bigger than the width of each slit. So, this is going change much faster this is what you see here the contribution the pattern due to the chain of oscillators is much more rapidly varying. You have these primary maxima it then you have minima then you have a secondary maxima you have quite a few secondary maxima you have another primary maxima of a higher order again. You have the secondary minimas maximas again you have another primary maximum of a higher order and so forth. This is multiplied by the diffraction pattern of a signal slit which show by the dash line over here this varies more slowly.

The net result of this whole thing is as follows. If the slit with were ignored then all the primary maximas would have the same intensity. This was the situation that we had considered in the last class. All the primary maximas would have the same intensity which would be I naught N square where N is a number of oscillators, but because of the effect the finite width of the slit the higher order maximas are going to have a smaller intensity. So, the m equal to 0 order is going to have the maximum intensity as you go to higher order of m the intensity of the maxima is going to fall which you conceive over here. The first order m equal to one intensity is considerably smaller than m equal to 0 m equal to 2 cannot be seen at all.

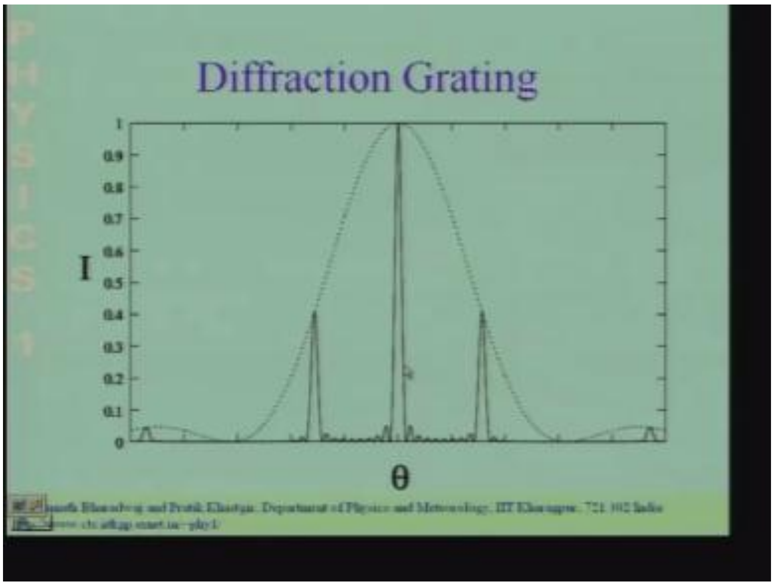
Because it falls very close to the minima of the diffraction pattern of the single slit m equal to 3 can be made out over here and the others will be quite small. So, what we see is that we have both the effects in diffraction grating we have the chain of oscillators. The chain of oscillators produces intensity pattern with many maximas many possible maximas all of the same intensity. But if you take into a account the fact that each of the slit has a finite width. Then this gets multiplied with a diffraction pattern of a single slit which is a sinc square function. And this cause is the higher order maximas that is m equal to 1 2 3 these 2 have more and more I mean to intensity of the maximas to become progressively fainter and fainter which is what you see over here. So, this is the intensity pattern predicted for a diffraction grating. Now, the diffraction grating is a very useful device in spectroscopy it is utility lies in the following.

(Refer Slide Time: 39:59)



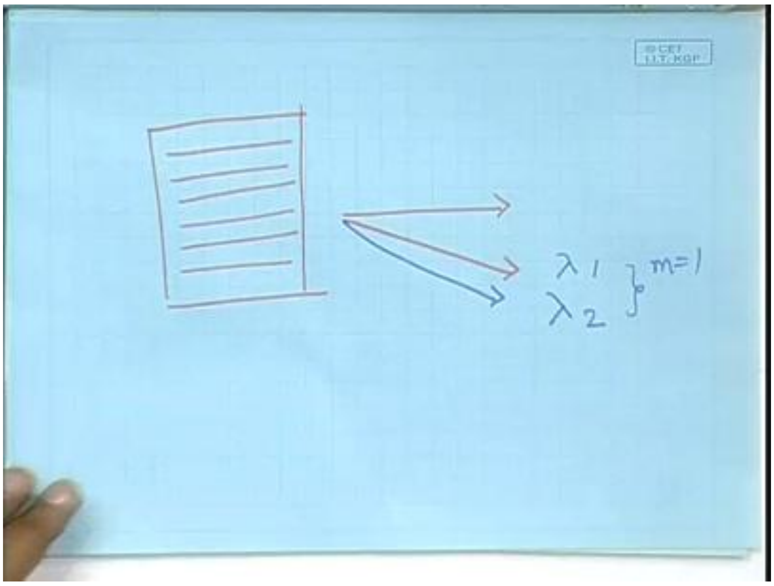
The maximas as we know occur wherever this condition d sin theta is equal to m lambda whenever this condition is satisfied we get a maxima. Now, if I have 2 different wavelengths. So, if I have lambda 1 the maxima will occur at theta 1 if I have lambda 2 the same order maxima will occur at a different angle theta 2. So, if m is not equal to 0 if m equal to 0 then all the maximas independent of wavelength for all wave lengths a maxima occur at occurs at theta equal to 0. But if m is not equal to 0 the angle theta at which you will get the mth order maximum depends on the wavelength. And if you have 2 different wavelengths the maxima will occur at 2 different angles theta and you can use this to determine the spectral composition of light.

(Refer Slide Time: 40:58)



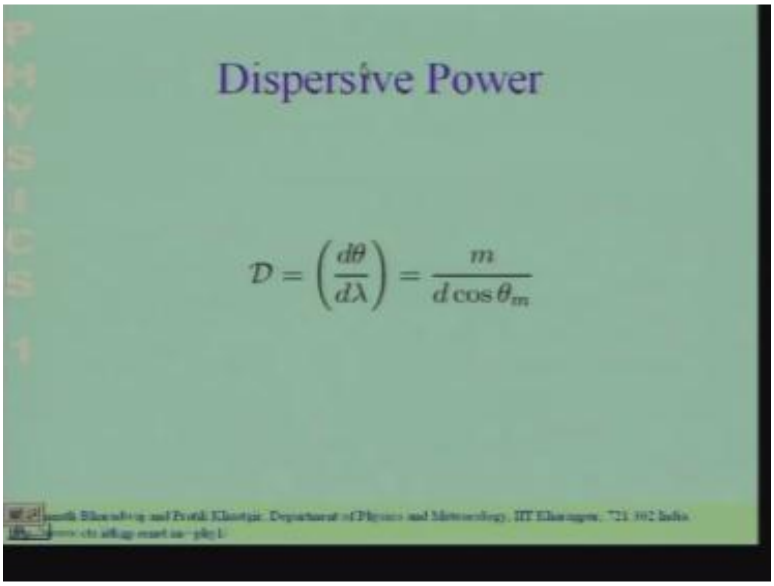
So, if I have light which has got several wavelengths I can send this light into a diffraction grating.

(Refer Slide Time: 41:13)



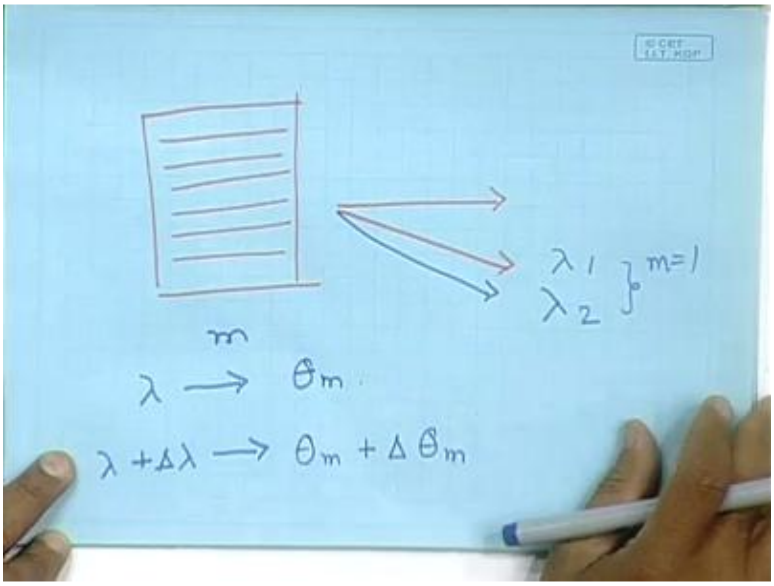
So, if I send light of different wavelengths into a diffraction grating like this the zeroth order maxima for all wavelengths will be at theta equal to 0. But the first order maxima let us say will occur at different wavelength different angle theta for different wavelengths. So, for a wavelength lambda 1 let us say occurs here for a wavelength lambda 2 it may occur at a different angle. It will occur at a different angle it will occur at a different angle for wavelength lambda 3 to rather to different angle this are all m equal to 1 and m equal to 2 will occur elsewhere and so forth. So, this allows you to determine how many different wavelengths there are in the light that you are sending in and this plays a very important role in spectroscopy. This what spectroscopy is all about to determine the frequency components that are have present in the light that you are you wish to analyze.

(Refer Slide Time: 42:13)



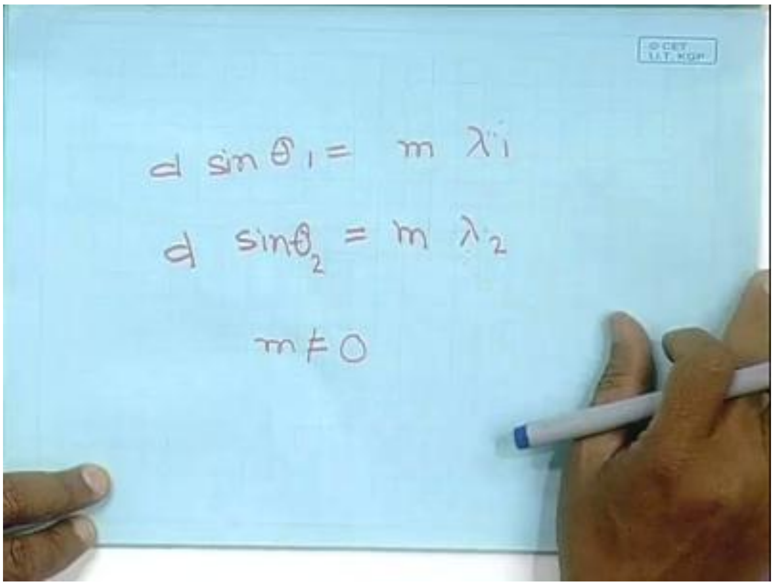
So, this property of a grating is quantify through what is called the dispersive power of grating. So, the question is as follows.

(Refer Slide Time: 42:19)



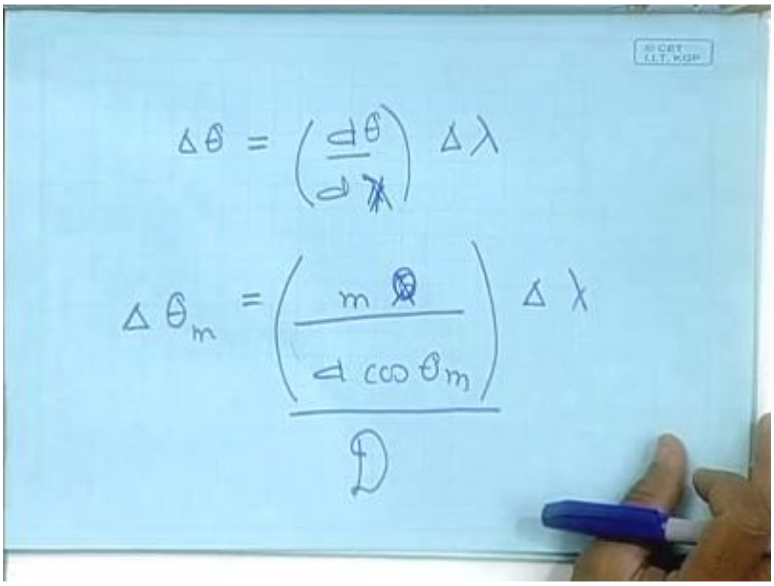
We have light of wavelength lambda it produces a maxima mth order maxima at an angle theta. Now, at an angle theta m so, this is lambda it produces mth order maxima at an angle theta m. Now, instead of lambda if i have a wavelength lambda plus delta lambda at it is going to produce the maxima at different angle. Let us call that theta m plus delta theta m. So, the question is how much is this delta theta m?

(Refer Slide Time: 42:58)



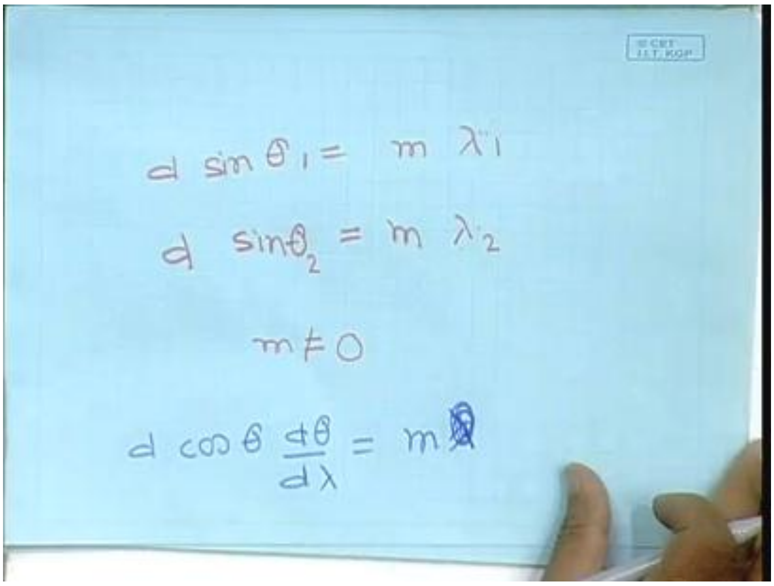
So, the maxima we see is occurs whenever this condition is satisfied d sin theta is equal to m lambda.

(Refer Slide Time: 43:06)



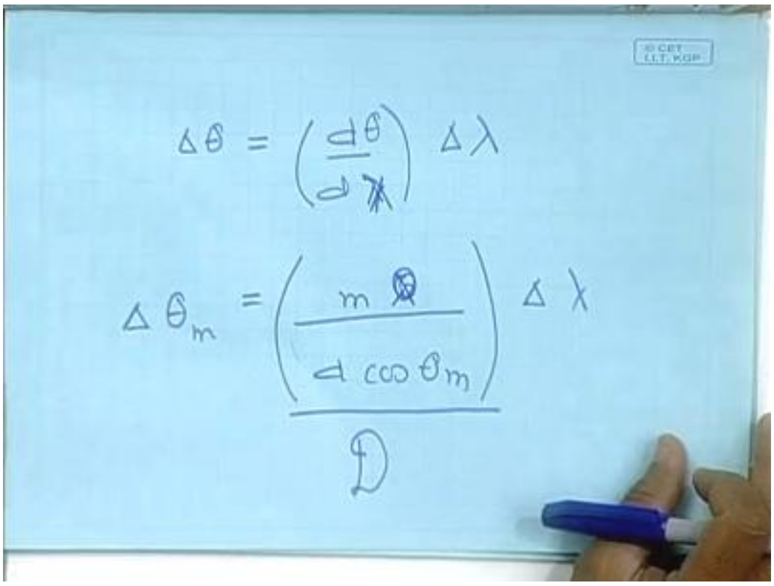
So, delta theta can be calculated the shift in the maxima can be calculated by differentiating this and multiplying it with delta lambda.

(Refer Slide Time: 43:26)



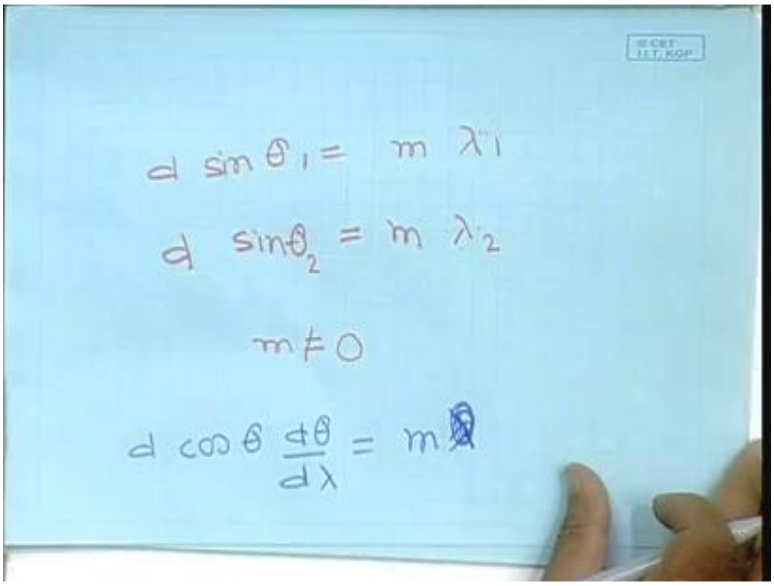
And if you differentiate this expression what you get is d cos theta d theta d lambda is equal to m lambda. So, d theta d lambda is equal to m lambda by d cos theta.

(Refer Slide Time: 43:46)



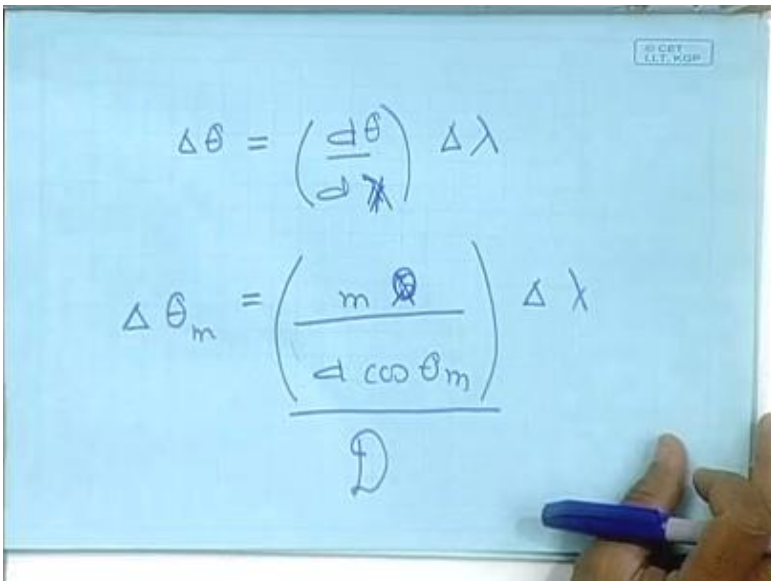
This is equal to m lambda by d cos theta m. This is called oh sorry this lambda will not be there.

(Refer Slide Time: 44:08)



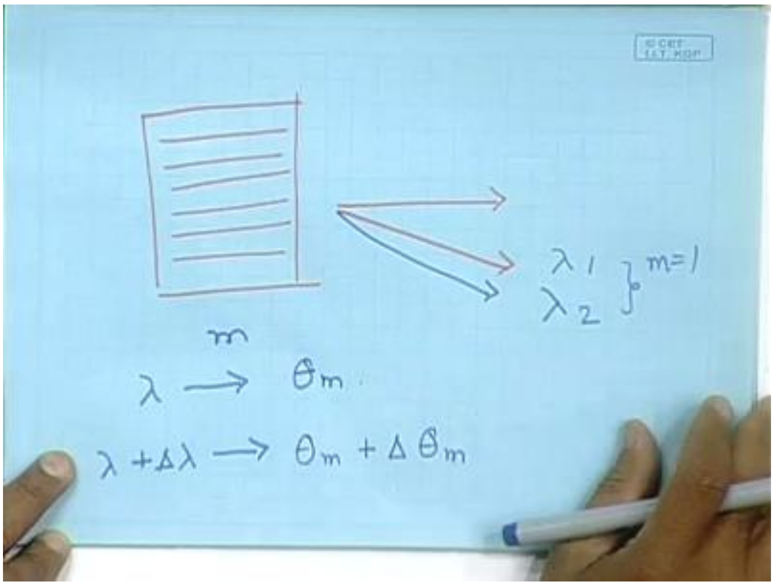
When I differentiate with respect to lambda this lambda is going to vanish.

(Refer Slide Time: 44:19)



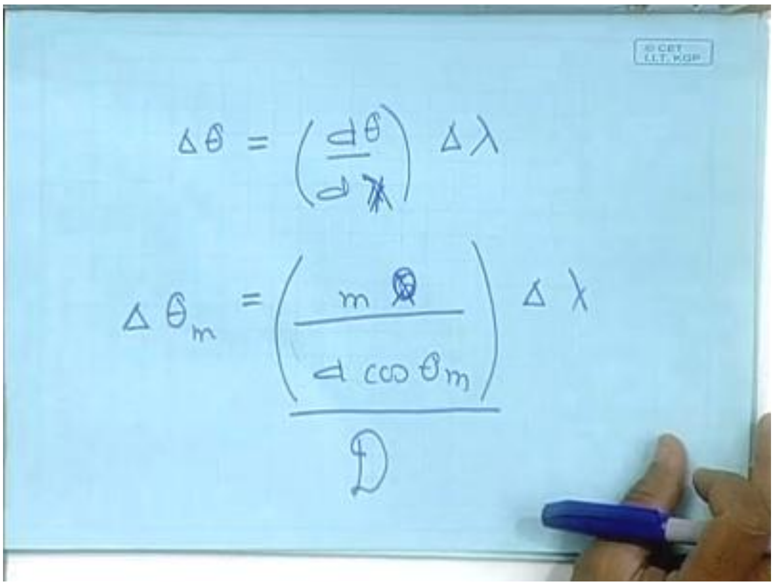
And this going to give me 1 basically and this is the condition. And this term in the brackets is what is called the dispersive power of a grating.

(Refer Slide Time: 44:34)



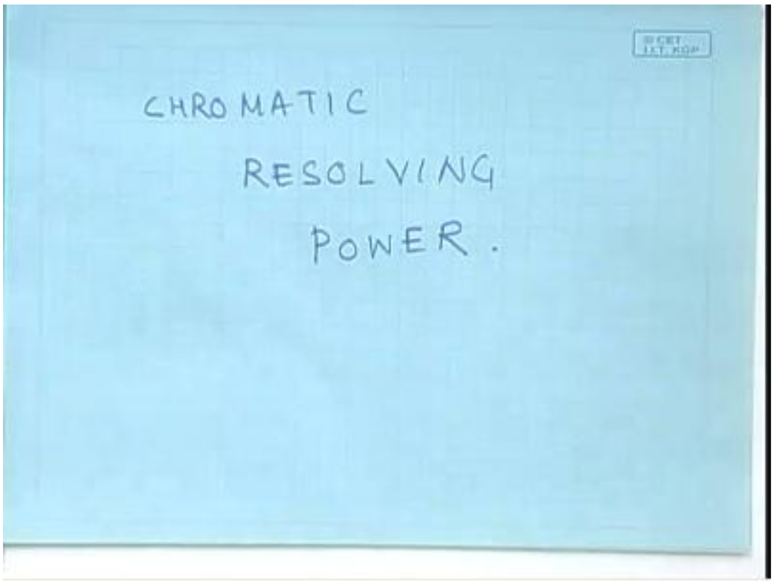
What it tells us is that if I put in light of wave and lambda the maxima will occur at an angle theta m. If I change the wavelength by a small amount to lambda plus delta lambda, how much is the maxima going to shift? The maxima is going to shift by an amount which is proposal to delta lambda.

(Refer Slide Time: 44:53)



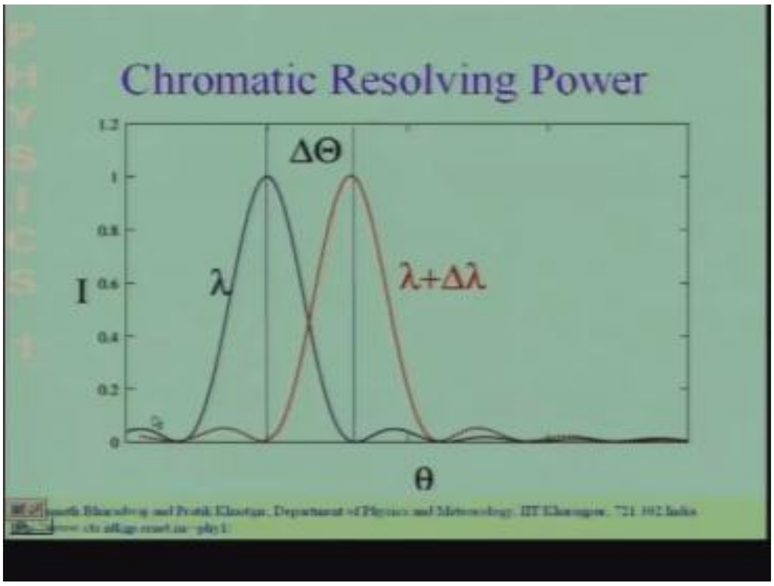
And this constant of proportionality is d theta d lambda this is what is call the dispersive power of a grating. It tells us that if I change how much the angle is going to shift angle at which the maxima occurs, how much this is going to shift if I change the wavelength by a slight amount? Let us take another quantity which is of interest when we are when we use the diffraction grating as in spectroscopy. So, there is another quantity which is of interest when we use the diffraction grating as in spectroscopy and this quantity is called the chromatic.

(Refer Slide Time: 45:37)



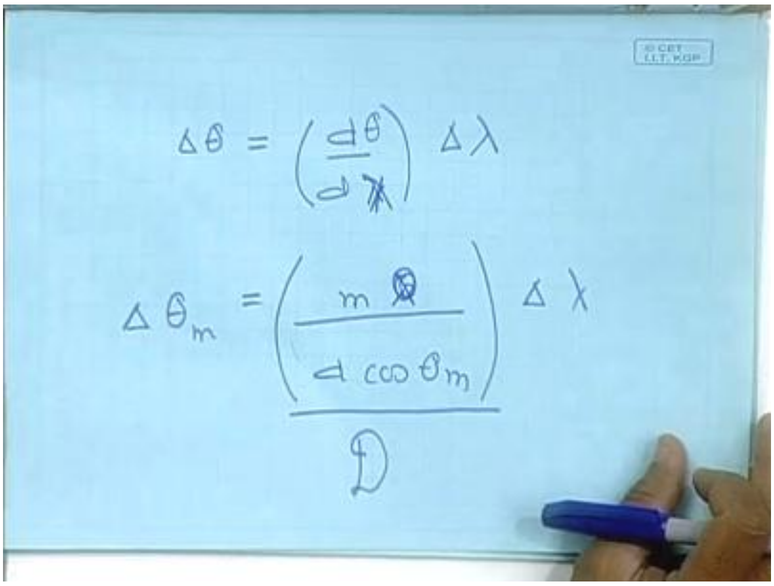
Chromatic resolving power is as follows.

(Refer Slide Time: 45:56)



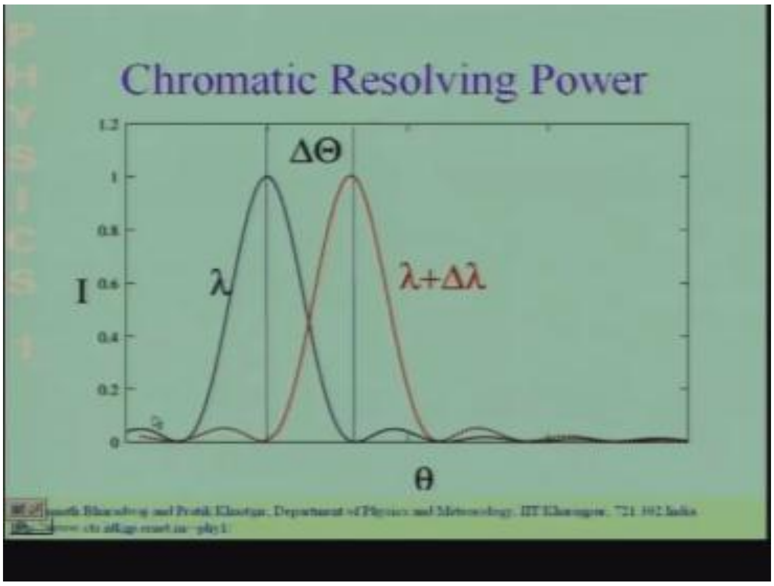
We have light of 2 wavelengths 1 lambda and another lambda plus delta lambda. The mth order maximum of the wavelength lambda occurs at a particular angle theta. So, this shows you the mth order maxima corresponding to a lambda wavelength lambda it occurs at angle theta. And for the wavelength lambda plus delta lambda the maxima is shifted by an angle delta theta, where we have a just calculated how much it will be shifted.

(Refer Slide Time: 46:37)



And we saw that it will be shifted by an amount which is m divided by d cos theta m into delta lambda.

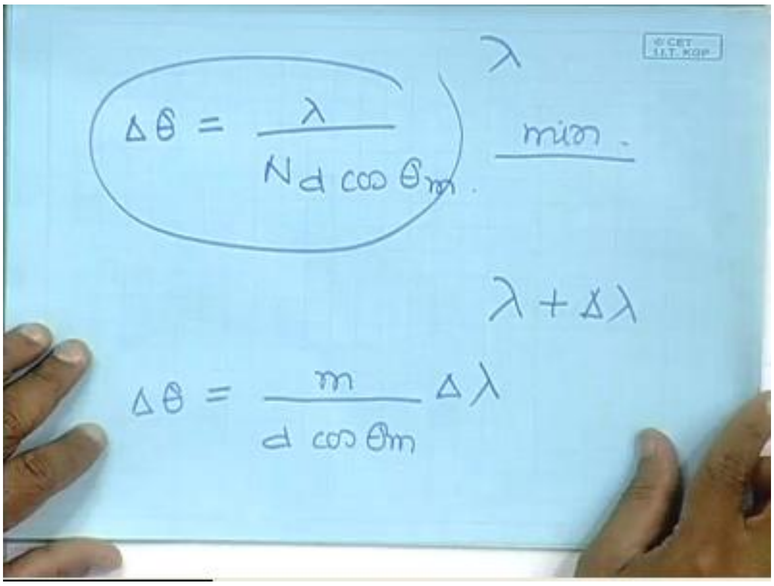
(Refer Slide Time: 46:46)



So, for the wavelength lambda plus delta lambda the maximum is going to shifted by this amount. Now, the question is under what condition we can say that there are 2 different wavelengths and not 1. So, this is what is called refer to as a 2 lines 2 different spectra lines 2 different wavelengths being resolved. So, there are 2 different spectral features over here. In order to resolve in order to say that we can resolve these 2 spectral features, the shift in the angle should be such that the maxima of this wavelength lambda plus delta lambda coincides at least coincides the shift is sufficient. So, that it at least coincides with the minima of the, of the intensity of the wavelength lambda.

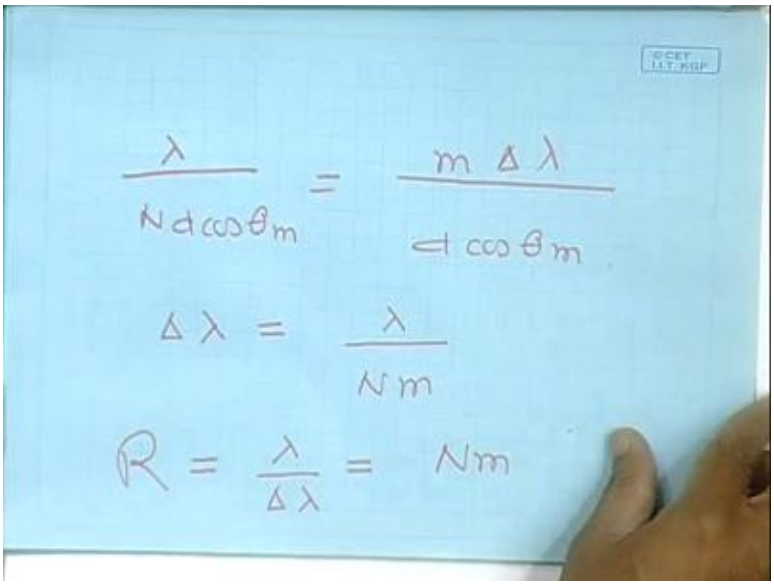
So, this shift should be such that this at least coincides with the minima of this. If it is more if shift is more than the minima then you can distinguish between this curve and this curve. But in the shift is less than the minima you cannot distinguish between this curve and this curve that is the Raleigh resolving criteria. So, you can resolve these 2 lines these 2 wavelengths provided the shift in the angle is more than the minima of this particular curve. And we have calculated where the minima of this curve should occur. the minima of the of the intensity profile of the diffraction pattern correspond to corresponding to a wavelength lambda.

(Refer Slide Time: 48:31)



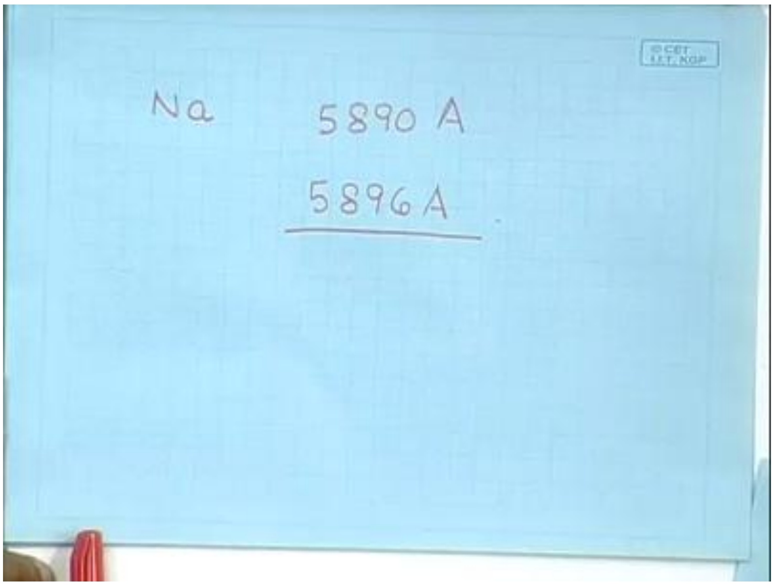
That will occur at an angle delta theta, the minimum will occur at lambda divided by N d cos theta m. So, this is where the minimum will occur this, the minima of the wavelength lambda. And the maxima of the wavelength lambda plus delta lambda is going to be at delta theta is equal to m divided by d cos theta m into delta lambda. So, the Raleigh criteria for resolving these 2 different to be able to distinguish to be able to resolve these 2 is that this should be equal to this only then can resolve these 2 different spectral features.

(Refer Slide Time: 49:36)



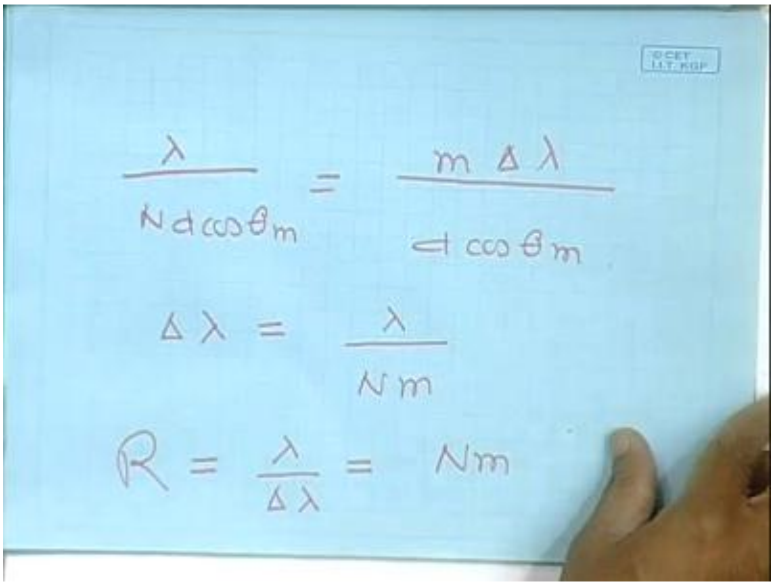
So, if I equate this 2 it tells me let see what it tells us lambda by N d cos theta m is equal to m delta lambda by d cos theta m. So, what it tells us is that delta lambda is equal to lambda divided by N m R. So, this is how the chromatic resolving power is defined if I have 2 different spectral lines if I have a radiation which has 2 different wavelengths at a. So, the wavelengths are at around the wavelength lambda and they are separated by delta lambda.

(Refer Slide Time: 50:50)



For example, remember sodium; sodium has 2 different wavelengths 1 at 5890 Armstrong another at 5896 Armstrong. So, the question is under what condition shall we able to distinguish that there are 2 different wavelengths and not 1 we have worked on the condition.

(Refer Slide Time: 51:16)



If this condition is satisfied we shall be able to say that there are 2 different wavelengths and not 1. And for a grating there is this quantity called the chromatic resolving power which is defined as the ratio of lambda by delta lambda; which for diffraction grating is the number of splits into the order of the maxima that you are looking at. So, the number of slits that you have in your grating into the order of the maxima that you looking you are looking at this decides the chromatic resolving power. The larger the chromatic resolving power the better is your diffraction grating and the smaller chromatic resolving power the less is your diffraction grating it. The chromatic resolving power is essentially the inverse of the ratio of the separation that you can distinguish divided by the value of the wavelength around which these 2 wavelengths are distributed. So, this is how you can quantify the resolving the chromatic resolving power the ability of grating to distinguish between 2 different wavelengths.

So, in yesterday's lecture and today's lecture we have been essentially studying the radiation from a chain of oscillators or a periodic arrangement of oscillators. So, whenever we have a periodic arrangement of radiation sources we get the kind of diffraction pattern that we have been discussing over here. So, all of this that we have been discussing is all valid whenever we have periodic arrangement of radiation sources. In the next lecture, we shall take up another very interesting application. In today's lecture, we took up several applications of this chain of oscillators or a periodic arrangement of a radiating source. We had the phased array which can be used as radar which has can be several application in communication radio astronomy. And then we considered the diffraction grating which is also a periodic arrangement of radiations sources. And in tomorrow's lecture, we shall take up another periodic arrangement of radiation sources. And this particular periodic arrangement occurs in nature so that we shall take up in the next class, tomorrow which is next class.